The Design of the Pallet Program

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Abstract

This paper describes the procedures underlying the development of the Pallet program which has been produced to design regular pallet racks according to the FEM code. The program determines the buckling load of the equivalent free sway structure and, using stability functions, calculates the axial and shear forces and the bending moments within the structure including the non-linear $P-\Delta$ effects. Twelve different combinations of load are analysed and design checks given in the FEM code applied.

The paper discusses the different modes of operation of the program. Finally the accuracy of the program is discussed together with future developments.

Introduction

Pallet rack structures are used in factories and warehouses for the storage of palletised goods. Such structures often have a large number of beams and columns. Figure 1 shows a typical pallet rack.

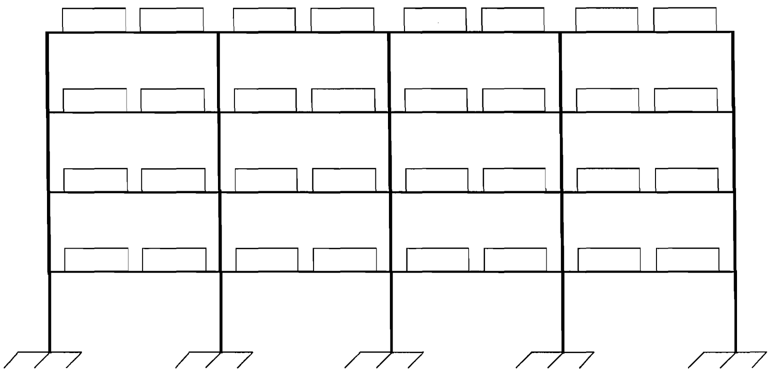

Figure 1: Typical pallet rack

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Traditionally pallet racks have been analysed by constructing a finite element model of the rack. However, as beam-upright connections and upright-base-plate connections are usually semi-rigid the resulting analysis models are large with many joint and beam elements. For example the small frame shown in Figure 1 would require 37 joint elements and, if only four beam elements meshed each column or beam section, 144 beam elements. As the semi-rigid joint stiffnesses are often a different order of magnitude to the beam element stiffnesses the programs often have difficulty in converging to the correct results. The design of pallet racks contains several load cases and when using the FEM code (Federation Europeene de la Manutention (2000)) many different design checks. When a finite element model has been constructed and tested special programs often have to be written to enable the many design checks to be efficiently performed on all elements. The FEM code requires sway calculations to be performed using a second order $P-\Delta$ analysis.

In order to overcome these difficulties the authors have developed the Pallet program (Pallet 2000) The basic philosophy underlying the design of the program has been to have efficient analysis algorithms so that different load cases can be carried out efficiently and quickly in order that the designer will be able to determine the most economic design satisfying the required geometry and loading. In addition the program has been designed so that performance tables can be produced and so that the pallet rack designer can investigate 'what – if scenarios' such as 'What will happen to the performance of my rack if I increase the rotational stiffness of my base-plate?'

Down-aisle Analysis models

Pallet racks are regular multi-bay, multi-level sway structures. In a pallet rack each beam has normally the same design load and hence a single column model, based on a single bay, can be used in the analysis model (Feng et al (1993) and Godley et al (2000)). The structural analysis model is shown in Figure 2.

Each section of column upright is analysed by writing down the slope-deflection equations involving the shear forces, Q_i, axial forces, P_i, and moments M_i at the top and the bottom of each upright in terms of the horizontal deflections, v_i, and nodal rotations θ_i. At each beam level the rotational stiffness can be shown to be (Feng et al (1993))

$$k_i = \frac{1}{\dfrac{l}{12EI_{bi}} + \dfrac{1}{4k_{li}} + \dfrac{1}{4k_{ri}}} \quad [1]$$

where l is the bay width, EI_{bi} the flexural rigidity of each beam. k_{ri} and k_{li} are the rotational stiffnesses of beam-to-upright connection from the two beams, one on the left and one on the right of the connection. The connections between beams and uprights in pallet racks are often hooks inserted into slots. If holes are perforated unsymmetrically into the uprights then the rotational stiffness can be different for clockwise and anti-clockwise rotations. In (Feng et al (1993)) and (Godley et al (2000)) the single column model has been shown to give results which are within 2% of the corresponding results from a finite element model of the whole rack structure. The program contains three down-aisle analysis models:

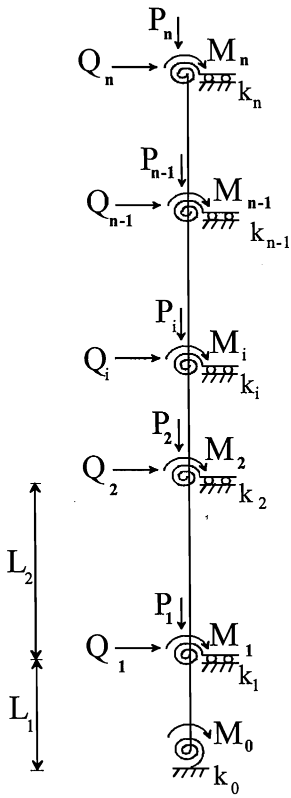

Figure 2: Analysis model

(i) Buckling analysis

The design of pallet rack structures must consider the elastic stability of the rack. In the analysis model shown in Figure 2 the rack will buckle in a sway mode with zero horizontal shear at each beam-to-upright connection. In this case the slope deflection equations only involve the axial forces in each section of upright together with joint rotational stiffness and the rotations at each beam level. In each section of upright the equations relating moments M_{ab}, M_{ba} at the top and bottom of section *ab* to the corresponding rotations θ_a, θ_b are:

$$M_{ab} = i\theta_a \frac{\nu}{\tan\nu} - i\theta_b \frac{\nu}{\sin\nu} \qquad [2]$$

and

$$M_{ba} = -i\theta_a \frac{\nu}{\sin\nu} + i\theta_b \frac{\nu}{\tan\nu} \qquad [3]$$

where $i = EI / L$ and $\nu = \sqrt{P / EI}L$. P is the axial force in the section of upright with flexural rigidity EI and length L. Note that the second moment of area, I, of the column model is given by $I = I_{up}n_{up} / (n_{up} - 1)$ where I_{up} is the second moment of area of the rack uprights and n_{up} is the number of uprights in the rack. These equations are the standard stability functions found for example in Horne and Merchant (1965).

At each beam-to-upright connection compatibility at joint b requires that

$$M_{ba} + M_{bc} = -k_b\theta_b \qquad [4]$$

When all the equations are assembled into a global stiffness matrix the resulting system of equations is tri-diagonal. Buckling loads are obtained by finding the least value of axial load in each member which makes the determinant of the stiffness matrix equal to zero. In the Pallet program the procedure described in Feng et al (1993) was adopted. The procedure may be summarised:

- Find an upper bound to the buckling load of the frame by calculating the least buckling load of all the column sections, each considered to be a strut with fixed ends but free to sway. i.e. $k_b = \infty$.
- Divide the interval from zero to the upper bound into 100 and starting from zero find the first interval in which the determinant of the stiffness matrix changes sign.
- Use the bisection algorithm to evaluate the determinant to a relative precision of 0.0001.

Although the above procedure may appear to be inefficient an explicit formula for the evaluation of the determinant can be derived for any load increment ensuring that this method yields the buckling load of the frame very quickly.

(ii) Non-linear, down-aisle, sway analysis including $P - \Delta$ effects

The FEM code (Federation Europeene de la Manutention (2000)) requires that all down-aisle analyses of racking structures include the $P - \Delta$ effects relating lateral displacements and axial forces. In this program these effects are incorporated by using stability functions. For each section of upright between two beam levels the equations are

$$M_{ab} = c_1\theta_a - c_3 v_a + c_2\theta_b + c_3 v_b \qquad [5]$$

$$Q_{ab} = -c_3\theta_a + c_4 v_a - c_3\theta_b - c_4 v_b \qquad [6]$$

$$M_{ba} = c_2\theta_a - c_3 v_a + c_1\theta_b + c_3 v_b \qquad [7]$$

$$Q_{ba} = c_3\theta_a - c_4 v_a + c_3\theta_b + c_4 v_b \qquad [8]$$

which relate the end moments M_{ab}, M_{ba} and shears Q_{ab}, Q_{ba} to the end rotations θ_a, θ_b and lateral displacements v_a and v_b. The constants c_1 to c_4 are given by

$$c_1 = P\frac{\nu\cos\nu - \sin\nu}{\mu(2\cos\nu - 2 + \nu\sin\nu)},\ c_2 = P\frac{\sin\nu - \nu}{\mu(2\cos\nu - 2 + \nu\sin\nu)},\ c_3 = P\frac{1-\cos\nu}{2\cos\nu - 2 + \nu\sin\nu}$$

$$c_4 = -P\frac{\mu\sin\nu}{2\cos\nu - 2 + \nu\sin\nu} \text{ and } \mu = \sqrt{P/EI},\ \nu = \mu L$$

[9]

Equation [4] above gives the compatibility equation at each beam level for moment. In addition the compatibility equation for shear is

$$Q_{b,below} + Q_{b,above} = Q_b \qquad [10]$$

where Q_b is the applied external shear at level b.

Full details of the sway analysis are found in Godley et al (2000). The assembled global stiffness matrix is solved by the Gauss elimination procedure to give the bending moments, axial forces and shearing forces at each end of a column segment.

(iii) Pattern loading

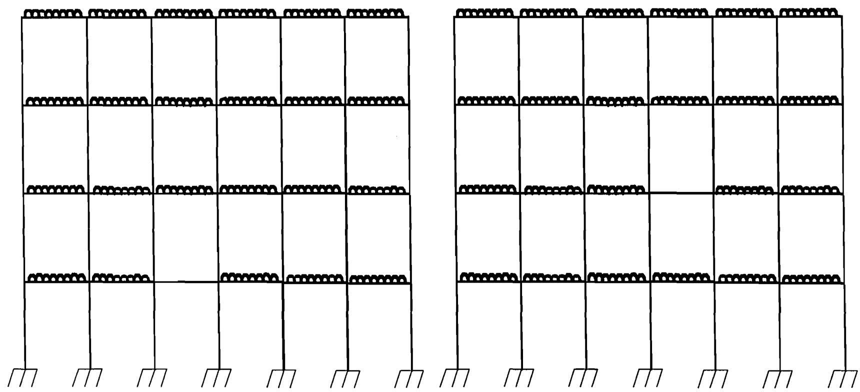

Figure 3: Pattern Loadings

The non-linear analysis described above assumes that the frame is loaded symmetrically. The program must also consider the implications of pattern loading. Figure 3 shows two examples of pattern loading, where elements of the frame are not loaded in the bottom two levels. To incorporate these effects linear analyses are made of the effects of pattern loading and the resulting moments added to the non-linear sway moments. For these analyses the lateral displacements, v, in Figure 2 are set to zero. In this case the slope deflection equations reduce to

$$M_{ab} = 4\frac{EI_a}{L_a}\theta_a + 2\frac{EI_a}{L_a}\theta_b \qquad [11]$$

and

$$M_{ba} = 2\frac{EI_a}{L_a}\theta_a + 4\frac{EI_a}{L_a}\theta_b \qquad [12]$$

The compatibility equations at each beam level b are

$$M_{ba} + M_{bc} = -k_b'\theta_b \qquad [13]$$

In the linear analyses the second moment of area of section i is the unmodified second moment of area of the upright. Each column is analysed independently.

The resulting tri-diagonal system is solved using an explicit formula derived from the Gauss elimination procedure.

Cross-aisle analyses

To simplify analyses in the cross-aisle direction currently the program only considers the bracing pattern shown in Figure 4, the most commonly occurring case. In addition, as cross-aisle failure due to non-linear $P-\Delta$ effects can only occur in slender tall racks, the analysis is limited to cases where the elastic critical ratio (maximum design axial force/elastic buckling load) is less than 0.3 so that amplification factors can be used to estimate non-linear effects. The cross-aisle frame is considered to be pin-jointed and an explicit approximate formula for the maximum deflection used. The areas of the diagonal bracing elements are reduced to take account of the discrepancy between the theoretical and experimental shear stiffness of the frame.

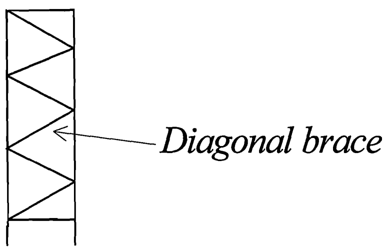

Figure 4: Bracing pattern

Bending moments in the cross-aisle direction are calculated assuming that horizontal point loads due to imperfections and placement are applied solely at the mid-point of the bracing gate. No account is taken of the secondary bending moments arising as a result of continuity of the upright section in the cross-aisle frame.

Design Criteria

The program was written to analyse and check pallet racks according to the FEM code. In order for this to be done the following load combinations are analysed:

- vertical loads only – an analysis of the frame when subjected to vertical forces with no sway. This is a simple check which is used in the performance checks as it quickly gives maximum values of allowable load.

- three pattern load combinations – the two combinations shown in Figure 3 together with a third which is obtained by finding the worst combination of the two cases. Note that when pattern loading is considered that the vertical loads applied to the column are modified to ensure that the correct average value of load is applied to each section of the upright.

- vertical loads in combination with frame imperfections and pattern loads. Frame imperfections take the form of an initial sway, ϕ, and an equivalent horizontal load, $P\phi$, at each beam level. The initial sway is given by

$$\phi = \sqrt{\left(\frac{1}{2}+\frac{1}{n_c}\right)\left(\frac{1}{5}+\frac{1}{n_s}\right)}\left(2\phi_s+\phi_e\right) \qquad [14]$$

  and must satisfy the conditions

$$\phi \le \left(2\phi_s+\phi_e\right)\text{ , }\phi \ge \left(\phi_s+0.5\phi_e\right)\text{ and }\phi \ge \frac{1}{500} \qquad [15]$$

  ϕ_s is the erection tolerance of the frame (out-of-plumb error) and ϕ_e the beam-end-connector looseness. n_s and n_c are respectively the number of storeys and the number of columns in the frame. The combination of vertical load and frame imperfections is analysed using the non-linear $P-\Delta$ effects. Superposition is used to add the pattern loads.

- Vertical loads in combination with frame imperfections, pattern loads and placement loads. The placement loads are found from the FEM code and are given by:

$$\text{Placement load} = \begin{cases} 2Q_{ph}\gamma_f / \left(n_b(n_c-1)\right) & \text{H} \le 3 \\ Q_{ph}\gamma_f \left(\text{3-H/3})/n_b(n_c-1)\right) & 3 < \text{H} \le 6 \\ Q_{ph}\gamma_f / n_b(n_c-1) & \text{H} > 6 \end{cases} \qquad [16]$$

  where H is the total height of the pallet rack, Q_{ph} the design placement load (700N) and γ_f the design load factor (1.4 for service load condition).

When all the combinations of vertical load, pattern loads, frame imperfections and placement loads are taken into account there are currently twelve load combinations. Work is being carried out to include the effects of accidental load on the system.

To ensure that users of the program are able to understand the results of the design checks used in the program all results are scaled to the maximum allowable value of the check. Hence a design check less than or equal to 1 implies that this check is satisfactory; a design check greater than 1 implies that the pallet rack has failed to satisfy this particular check. Values greater than 1 may occur when an engineer is running the program in the 'design mode' in order to investigate the manner in which a given rack configuration may fail.

The following design checks are made:

- the maximum horizontal deflection due to sway of the top of the column must be less than 0.005 times the height of the column under service loads in both cross-aisle and down-aisle directions. (Clause 2.3.4 of the FEM code). This condition is only critical when full loading is applied to the rack. Hence this condition is not checked for pattern loaded down-aisle combinations.

- for non-placement service load cases the maximum vertical displacement of a beam must be less than beam length/200. (Clause 2.3.4) Users are able to change the standard division factor of 200 for special cases. Note that although the program only analyses uprights the approximate expression in the FEM code for central deflection Δ_b is used. The expression is

$$\Delta = \frac{5P\ell^3}{\lambda_f \cdot 384 EI_b}\left\{1 - \frac{0.8}{1+\dfrac{2EI_b}{k_e \ell}}\right\} \qquad [17]$$

  where k_e is the effective rotation stiffness of the beam-end connector, EI_b the rigidity of the beam of length ℓ with a uniformly distributed load P applied to it. The load factor λ_f is normally equal to 1.4. k_e is the minimum effective stiffness of the connector to the upright, experienced by a beam at the top level and at one end of the rack. It is given by

$$k_e = \frac{k_b}{\left(1+\dfrac{k_b h}{3EI_c}\right)} \qquad [18]$$

  k_b is the beam-end connector rotation stiffness, h the maximum distance between beam levels and EI_c the rigidity of the upright.

- the central moment of the beam M_c under ultimate limit state loads must be less than $W_{eff} f_y / 1.1$ where $W_{eff} f_y$ is the moment of resistance of the beam about its major axis. (FEM Clause 4.4.3.2). M_c is determined by the formula

$$M_c = \frac{PL}{8}\left\{1 - \frac{\frac{2}{3}}{1 + \frac{2EI_b}{k_e \ell}}\right\} \quad [19]$$

- the total moment at the beam-end connector in the ultimate limit state due to both beam and pattern loads must be less than the connector resistance moment. The resistance moment must be determined by tests and is input to the program. (FEM Clause 5.5). Plastic redistribution of excess moment at the connector to the central moment is allowed by the code and incorporated into the program. The redistribution has a maximum of 15% of the design resistance of the beam-end connector. When the total moment is calculated the worst combination of beam fixed end moments and moments introduced into the beam by pattern loads are considered for this check.

- the shear force at the beam-end connector in the ultimate limit state must be less than the characteristic beam-end connector shear strength which is found from test (FEM Clause 5.7). The shear force at the connector has two components. Firstly the end shear load on the beam and secondly the shear due to sway of the rack. The presence of pattern loading, because it only causes single curvature bending in the beams, does not contribute to shear.

- the down-aisle interaction between bending moment and axial forces at the ultimate limit state must satisfy the interaction formula

$$\frac{1.1N_{Sd}}{\chi A_{eff} f_y} + \frac{1.1M_{Sd}}{W_{eff} f_y} \leq 1 \quad [20]$$

where $A_{eff} f_y$ is the squash load of the column determined from stub column tests. N_{Sd} is the axial load in a section of upright. χ is the stress reduction factor and is determined from tests or from calculation. In both cases flexural-torsional effects are accounted for. It is expressed in polynomial form, typically

$$\chi = a_0 + a_1\bar{\lambda} + a_2\bar{\lambda}^2 + a_3\bar{\lambda}^3 + \ldots + a_n\bar{\lambda}^n \quad [21]$$

where $\bar{\lambda}$ is the non-dimensional slenderness ratio corresponding to the effective length of the storey. The effective length is found for each storey from the maximum of the storey. The moment M_{Sd} is the maximum moment occurring at either the top or bottom of a section of upright between adjacent beam levels or between the base-plate and first beam level.

- the moment at the base of an upright must be less than the maximum design moment in the ultimate limit state. The resistance of a base varies with the design axial moment in the bottom length of a column and is determined experimentally so that

$$M_{base} = b_0 + b_1N_{Sd} + b_2N_{Sd}^2 + b_3N_{Sd}^3 + \ldots + b_nN_{Sd}^n \quad [22]$$

For a pinned base the axial moment is zero and so this test is not applied. It is interesting to note that if base-plates have low resistances that this test often governs the maximum load carrying capacity of the rack. We then have the apparently anomalous result that racks with semi-rigid base-plates carry less load than racks with pinned bases. This result has been reported in Beale and Godley (2001).

The total number of checks in any design exceeds 70.

Modes of program operation

The program has been written so that it works in four different modes. They are:

(i) calculate the maximum load that a rack of a particular geometry can carry

The buckling load of the frame is first determined so that an upper bound to the maximum load that the frame can carry is found. A bisection algorithm is then adopted to find the maximum capacity of the frame so that all design checks are satisfied. Figure 5 shows the structure of the performance algorithm. The bisection algorithm is extremely efficient finding the maximum load that a rack can carry taking less than 1 second. Having determined the maximum load the program prints all the forces and moments within the column followed by the results of each design check.

(ii) determine the relative safety of a pallet rack under a given design payload

The second mode of operation of the program is to enable a designer to enter the geometry of a rack and specify the unfactored payload that the rack is to carry. The program then prints out the results of all design checks, even those which exceed the maximum safe value of 1. The designer can then identify those load combinations which are critical and the sections of the rack which may be under-performing. He/she is then able to identify and investigate quickly the implications of small design changes. For example, the influence of the base-plate is often critical in performance. The designer can see if changing the base-plate will significantly improve performance. He/she can also experiment with 'what-if' scenarios to see where the product may need to be improved.

(iii) generate performance tables for pallet racks

Salesmen require tables of performance data for racks where the height to the first beam level, storey heights, beam span, number of storeys and number of bays are all varied. Traditionally the data for these tables was produced by calculating performance data for a small number of standard racks using finite element software and interpolating. The Pallet program can generate the data for many cases by looping on the performance version. The results are output in ASCII format in a tabular form suitable for input into costing and spreadsheet programs.

(iv) Generate a list of all combinations of beams and uprights satisfying design requirements

When salesmen are trying to satisfy a client's requirements for a particular rack many programs will only produce what the program considers to be the optimum rack. However, frequently, the theoretical optimum may not be the correct solution as it can not take into account availability of stock, the effects on production of changing sections or even the cost benefit of reducing the specification by a small margin and using smaller sections.

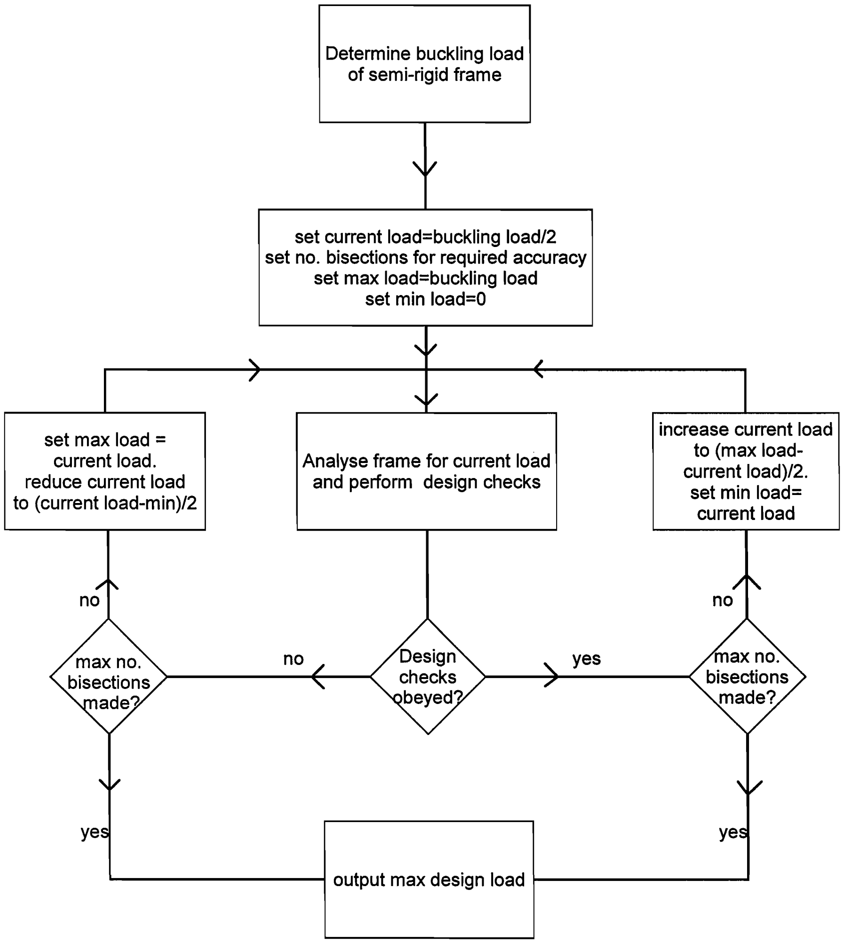

Figure 5: Structure of the performance program

The final mode of the Pallet program is to determine all configurations of beams, uprights and beam-end-connectors which satisfy the design requirements. Currently the program produces some invalid combinations but version 2 which is due out in the Summer of 2002 will only generate correct combinations whose specification is close to satisfying design requirements (within $\pm$10% of the payload).

It is intended that a version of the Pallet program will be developed which will enable salesmen to use the Pallet program for a given user product. This version will use this mode but will be tailored to individual customers so that commercially confidential structural performance data is encoded.

Program input and output

To enable the program to be easily used it has been written in Compaq Fortran using Windows routines. Figures 6 and 7 show examples of a couple of the input Windows.

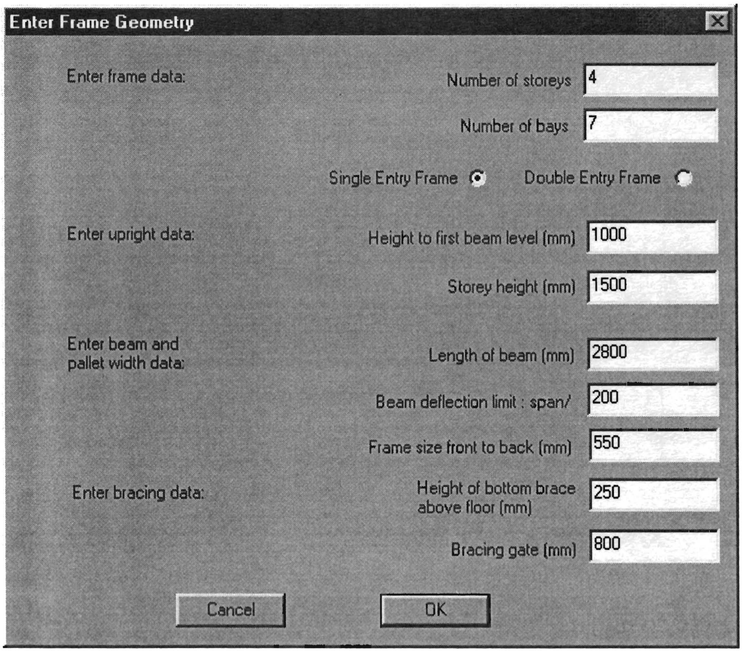

Figure 6: Sample input window for the performance program

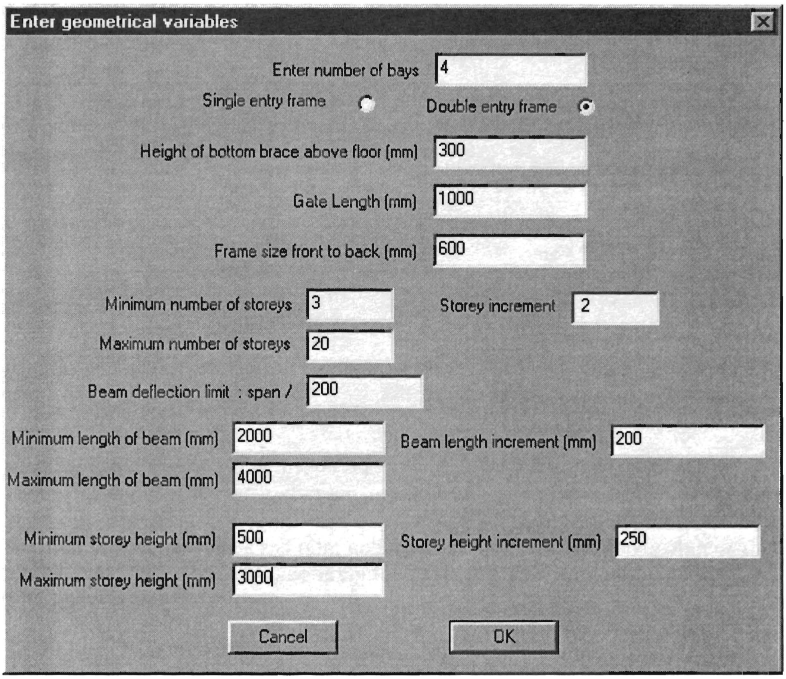

Figure 7: Sample input from performance table generation program

The output from the program is a small window which gives the value of the design criteria and an indication of the mode of failure. A full ASCII table is generated showing all forces, moments, deflections and rotations as well as the results of the design checks.

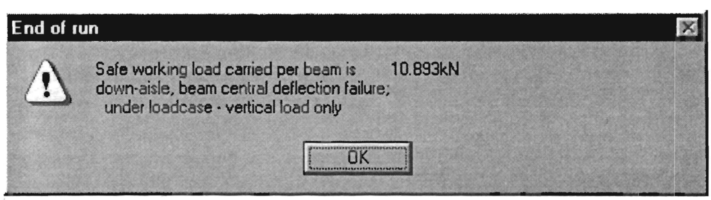

Figure 8: Sample window output

The output not only gives the maximum safe load but also clearly defines the mode of structural failure.

```
Load case vertical + frame imperfections

 Element        Mab           Mba           Disp           Rot          NSd
 Number       (kNmm)        (kNmm)          (mm)          (Rad)         (N)
                                                base   -0.1049E-02
   1         179.8         74.01           1.761      -0.1523E-02  0.6100E+05
   2         114.3         138.3           4.284      -0.1388E-02  0.4575E+05
   3         33.23         117.3           6.218      -0.9225E-03  0.3050E+05
   4        -3.208         67.00           7.402      -0.5420E-03  0.1525E+05
 beam central deflection ratio         =   1.0000
 beam central moment ratio             =   0.6706
 Beam-end connector moment ratio       =   0.6649
 Down-aisle interaction                =   0.8207
 Base moment ratio                     =   0.1040
 Beam end connector shear ratio        =   0.4231
 sway ratio                            =   0.1537

 maximum test factor for down-aisle tests =     1.0000
```

Figure 9: Section of output produced by the performance program

Figure 9 shows a section of the tabular output produced by the program. Each design check is printed out so that the designer can clearly see, not only which design check for the particular load combination was a maximum, but can also see if any other cases are close to maximum. In the example given the beam central deflection ratio has achieved its maximum value and is the limiting condition for the chosen rack configuratioin.

Program validation

To ensure that the program produces results which can be relied upon by the designer the program has been validated in the following ways:

- Comparisons of the results of the analyses have been made against racking frames analysed by non-linear structural analysis programs such as LUSAS (LUSAS 13.2) and SAND (SAND User Manual). The results of the LUSAS comparisons are given in Feng et al (1993), Godley et al (2000), Beale and Godley (2002a and 2002b). Both LUSAS and SAND have shown that in general the use of a single column model gives results that are normally conservative with unity factors about 2% higher than corresponding frame results. This difference is in fact within the differences in repeatability of different structural analysis analyses of the same problem, particularly pallet racks, where numerical instability associated with flexible semi-rigid connections attached to relatively stiff beam and column elements often causes convergence problems.

- The results of the analyses have been independently checked against manual calculations of the design checks associated with the FEM code. These checks have shown complete agreement.

Future Developments

The program will be extended to include accidental damage and improvements in the fourth mode of operating the program, that of generating all combinations of beams and uprights satisfying design constraints. The number of valid combinations will be restricted to those whose performance is close (within $\pm$10%) of the specified beam load. These enhancements will be delivered in 2002.

Currently the program only handles a single, common bracing pattern. Versions of the program will be developed which include alternative bracing patterns.

New analysis routines have been developed to consider the effects on the structure of including splices in the uprights of columns. Details of the analyses can be found in references 9 and 10. Splices will also be included in the second release of the program.

In the long term, the program will be modified so that shelving systems can also be included.

Conclusions

This paper has described the development of a program to analyse and design regular pallet racks according to the FEM code [1] and the methods adopted to obtain performance data. At all stages the program output has been validated against non-linear, finite element, structural analyses of complete pallet racks. The different analysis models used have been summarised and the design criteria explained.

Future enhancements of the program have also been described.

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Appendix – Notation

$A_{eff}f_y$	squash load of column
c_i	Constants defined in equation 9
E	Young's Modulus of Elasticity
i	EI/L

I	second moment of area
k_i	rotational stiffness of beam-end-connector
ℓ	length of beam
L_i	length of column element *i*.
M_{ab}	moment at top of section
M_{ba}	moment at bottom of section
M_i	external moment applied at node *i*.
n_b	number of beams
n_c	number of columns
N_{Sd}	axial load in a section of column
P_i	beam load at level *i*.
Q_{ab}	shear force at bottom of section
Q_{ba}	shear force at top of section
Q_i	external shear force applied at level *i*
Q_{ph}	Placement design load
v_i	horizontal displacement of upright at beam level *i*.
$W_{eff}f_y$	moment of resistance of beam about central axis
γ_f	Load factor
χ	stress reduction parameter
Δ	beam central deflection
$\bar{\lambda}$	non-dimensional slenderness ratio
ϕ	imperfection parameter defined in equation 14
ϕ_e	beam-end-connector looseness
ϕ_s	erection tolerance
μ	$\sqrt{P/EI}$
ν	μL
θ_i	rotation of upright at level *i*.