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INTRODUCTION

The economy and light weight of flexural members of light gage cold-formed steel has resulted in their increased use throughout industry. If the cross-section is closed, as in standard square and round pipe, torsional rigidity is high and secondary torsional-flexural coupling is of little concern. However, if the cross-section is open, as in the channel, zee and angle sections, flexural-torsional coupling may contribute to buckling and excessive deflections (18)[4]. This type of member is often used where dynamic machine loads are encountered, but because of the complexity of dynamic analysis little guidance is available to the designer as to the importance of the cross-section parameters such as torsional rigidity, warping rigidity or ratio of bending stiffnesses. Parametric flexural-torsional coupling and inertial effects may lead to buckling in such beams loaded dynamically parallel to a nonsymmetric axis, even if the external load passes through the shear center so as to cause no torsion.

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[4]Numerals in parentheses refer to corresponding items in Appendix I.--References.

This paper presents an investigation of the dynamic response of such beams with an open thin-walled shape. Torsional-flexural coupling is considered. The beam is assumed to be slender, elastic, uniform in cross-section, simply supported and local buckling is assumed to be negligible. Loading is a sunusoida force at midspan. Various geometric, and material parameters are studied to determine their effect on beam deflections.

The method of analysis is presented in the following sections. The governing equations of motion are uncoupled using Budnov-Galerkin's method and numerical integration was accomplished on a CDC 6400 computer to determine displacements.

The general theory regarding the strength and stability of thin-walled members was derived by Goodier (5,6), Timoshenko (13), Vlasov (17), and others. Vibrations of thin-walled bars and beams has been discussed by Vlasov (17), Gere (3), Gere and Lin (4), and Tso (15, 16). The parametric excitation phenomena in the dynamic stability of such members has been studied by Bolotin (1).

GOVERNING EQUATIONS OF MOTION

Consider the portion of a uniform thin-walled member with an asymmetrical, open cross-section shown in Fig. 1. The triad of axes, x_1, x_2, and x_3, is taken as a Lagrangian reference frame whose origin is considered to be fixed at point 0, which is the shear center of the section, at one end of the beam. Axes x_1 and x_2 are parallel to the principal axes of the cross-section, and x_3 is the longitudinal axis of the beam. Axes $\bar{x}_1$, $\bar{x}_2$, and $\bar{x}_3$ form an Eulerian reference frame, at a distance x_3 from the end of the beam, which translates and rotates with the cross-section. The moving frame is oriented such that $\bar{x}_1$ and $\bar{x}_2$ are always parallel to the principal axes of the section. The centroid C of the sections is located by the coordinates (C_1,C_2) relative to the x_1-x_2 axes or the $\bar{x}_1$-$\bar{x}_2$ axes.

A general section defined by the coordinate x_3 can move to a new position as illustrated in Fig. 2 in which positive displacements are shown. Translations of the shear center are depicted by $u_1(x_3)$ and $u_2(x_3)$. Angular displacement of the section is denoted by $\phi(x_3)$. Positive internal stress resultants $\bar{p}_i$, $(i = 1,2,\ldots6)$, relative to the moving reference system at an arbitrary, displaced section are shown in Fig. 2. These forces and moments are located in a manner such that $\{\bar{p}_i\}$ is a generalized force system.

To describe the general flexural-torsional response of a thin-walled beam, three equations are employed. Bending effects in two orthogonal directions are accounted for by the moment-curvature equations

$$EI_1 u_2'' = \bar{p}_4 \qquad (1)$$
$$EI_2 u_1'' = -\bar{p}_5$$

in which E is the modulus of elasticity, I_1 and I_2 are the principal moments of inertia of the cross section, $\bar{p}_4$ and $\bar{p}_5$ are the internal bending moments, and primes denote differentiation with respect to x_3. The nonuniform torque-twist equation (14) is

$$EC_W\phi''' - GJ_e\phi' + (\bar{p}_3\frac{Jo}{A} + \bar{p}_4 Z_1 - \bar{p}_5 Z_2)\phi' = \bar{p}_6 \tag{3}$$

in which C_W is the warping coefficient, G is the shearing modulus of elasticity, J_e is St. Venant's torsional constant, A is the cross-sectional area, J_o is the polar moment of inertia of A relative to the shear center axis, $\bar{p}_3$ is the axial load, $\bar{p}_6$ is the internal torque about the $\bar{x}_3$ axis, and Z_1 and Z_2 are constants which depend on geometric properties of the section given by

$$Z_1 = \frac{1}{I_1}\int_A R^2(x_2 - C_2)\, dA = \frac{1}{I_1}\int_A R^2 x_2\, dA - \frac{J_o C_2}{I_1} \tag{4}$$

$$Z_2 = \frac{1}{I_2}\int_A R^2(x_1 - C_1)\, dA = \frac{1}{I_2}\int_A R^2 x_1\, dA - \frac{J_o C_1}{I_2} \tag{5}$$

in which

$$R^2 = x_1^2 + x_2^2 \tag{6}$$

For a dynamic loading condition, the internal force system at any section can be considered to be composed of two contributions so that

$$\bar{p}_i = \bar{f}_i + \bar{g}_i \qquad (i = 1,2,\ldots 6) \tag{7}$$

That is, $\bar{p}_i$ is composed of forces and moments due to ($\bar{f}_i$) the externally applied loads, and ($\bar{g}_i$) the inertial loads. Employing d'Alembert's principle of dynamics and differentiating Eqs. 1, 2, and 3 approximately results in the following linear system of equations:

$$EI_1 u_2'''' = \bar{f}_4 - m(\ddot{u}_2 + c_1\ddot{\phi}) \tag{8a}$$

$$EI_2 u_1'''' = -\bar{f}_5 - m(\ddot{u}_1 - C_2\ddot{\phi}) \tag{8b}$$

$$EC_\omega \phi'''' - GJ_e\phi'' + [(\bar{f}_3\frac{Jo}{A} + \bar{f}_4 Z_1 - \bar{f}_5 Z_2)\phi']'$$

$$= \bar{f}_6 + m(C_2\ddot{u}_1 - C_1\ddot{u}_2 - \frac{Jo}{A}\ddot{\phi}) \tag{8c}$$

The mass per unit length of the beam, which is considered to be uniform, is denoted by m, and dots above a function indicate differentiation with respect to time.

Concentrated Load on a Simple Beam. -- The particular mathematical model for which response data are presented is shown by Fig. 3. A constant-direction, concentrated force, P, at midspan, designated by point "a", of a simply supported beam is investigated. The load is considered to act in the direction of the x_1 axis. For a general loading condition, the principle of superposition of forces can, of course, be utilized.

The kinematic and dynamic boundary conditions for the simple supports are defined by

$$u_1(0) = u_2(0) = \phi(0) = u_1(\ell) = u_2(\ell) = \phi(\ell) = 0 \tag{9a}$$

$$u_1''(0) = u_2''(0) = \phi''(0) = u_1''(\ell) = u_2''(\ell) = \phi''(\ell) = 0 \tag{9b}$$

where ℓ is the length of the beam. The location of P relative to the shear center is specified by the $\bar{x}_1$-$\bar{x}_2$ coordinates (x_1^o, x_2^o).

Allowing the beam to take on a general deformed configuration and employing matrix methods of structural mechanics (7), the internal force system due to the external load can be conveniently expressed. Singularity functions expressed by the Clebsch-Macauley method (9,10) are employed.

Thus, the external, time-dependent load can be represented by

$$q = P < x_3 - \frac{\ell}{2} >^{-1} \tag{10}$$

where the shorthand notation

$$< x_3 - \frac{\ell}{2} >^n \equiv < >^n \quad (n = \text{integer}) \tag{11}$$

will be adopted in this paper. This is equivalent to the Dirac Delta function for $n = -1$. Substituting the necessary expressions for the internal forces into Eqs. 8 leads to the three, coupled, fourth-order, linear, variable coefficient, nonhomogeneous, partial differential equations as follows:

$$EI_2 u_1'''' = -\frac{P}{2}[(C_1 u_1''' - u_2''' x_2^o)\chi_1 - 2(1 + 2C_1 u_1'' - 2u_2'' x_2^o) < >^{-1}] - m(\ddot{u}_1 - C_2 \ddot{\phi}) \tag{12a}$$

$$EI_1 u_2'''' = \frac{P}{2}[(2\phi' - u_1''' x_2^o - u_1''' C_2)\chi_1 + \phi''\chi_3 - 2(\phi - 2u_1'' x_2^o - 2u_1'' C_2) < >^{-1}] - m(\ddot{u}_2 + C_1 \ddot{\phi}) \tag{12b}$$

$$EC_w \phi'''' - GJ_e \phi'' = \frac{P}{2}[(u_2'' + Z_2 \phi'')\chi_3 + Z_2 \phi' \chi_1 - 2(\chi_2 + x_1^o \phi_a) < >^{-1}] + m(C_2 \ddot{u}_1 - C_1 \ddot{u}_2 - \frac{J_o}{A}\ddot{\phi}) \tag{12c}$$

in which

$$\chi_1 = 1 - 2 < >^o \tag{13a}$$

$$\chi_2 = u_{2a} - u_2 + x_2^o \tag{13b}$$

$$\chi_3 = x_3 - 2 < >^1 \tag{13c}$$

and u_{2a} is the displacement of the shear center at midspan in the χ_2 direction. The external force P is harmonic with a frequency of ω.

Elimination of the external load terms in Eqs. 12 leads to Gere's form of linear free-vibration equations of motion for coupled bending and nonuniform torsion (4).

SOLUTION OF EQUATIONS OF MOTION

Separation of the time and spatial coordinates involved in Eqs. 12 is accomplished by using the Bubnov-Galerkin averaging technique (8). Approximate solutions of these equations can be expressed as

$$u_1(x_3,t) = \ell \sum_{n=1}^{N} \eta_{1n}(t) \sin\frac{n\pi x_3}{\ell} \tag{14a}$$

$$u_2(x_3,t) = \ell \sum_{j=1}^{N} \eta_{2j}(t) \sin\frac{j\pi x_3}{\ell} \tag{14b}$$

$$\phi(x_3,t) = \sum_{k=1}^{N} \eta_{3k}(t) \sin\frac{k\pi x_3}{\ell} \tag{14c}$$

in which the dimensionless functions of time, η_{ij}, $(i = 1,2,3;\ j = 1,2,\ldots N)$, are the generalized coordinates which are undetermined. The chosen spatial functions are linearly independent and satisfy the boundary conditions given by Eqs. 9. Each of the N spatial functions or mode shapes in each of the series of Eqs. 14 represents a degree of freedom. Multimode solutions of the governing equations have been obtained by the senior author (2). It was determined that for the problem under investigation, the first term in the series of Eqs. 14, a single-mode solution, was adequate in predicting the response of the system. Thus, fundamental-mode solutions are presented and discussed in this writing, and the generalized coordinates become η_i ,(i = 1,2,3).

Since the assumed approximate solutions are unlikely to satisfy the complex equations identically, substitution of the first term in each of Eqs. 14 into Eqs. 12 yields three equations which are not equal to zero but to some respective residual $R_e(j)$, (j = I,II,III). In accordance with the averaging technique, the following equations evolve:

$$\int_0^{\ell} R_e(j) \sin\frac{\pi x_3}{\ell}\, dx_3 = 0 \qquad (j=I,II,III) \tag{15}$$

This procedure yields three, coupled, second-order, ordinary differential equations with time as the independent variable.

To manipulate the resulting equations more easily, certain dimensionless system parameters are introduced as follow:

$$\begin{aligned} C_1^* &= \frac{C_1}{\ell} , & C_2^* &= \frac{C_2}{\ell} , \\ C_w^* &= \frac{C_w}{I_2 \ell^2} , & E^* &= \frac{GJ_e}{EI_2} , \\ I^* &= \frac{I_1}{I_2} , & J_o^* &= \frac{J_o}{A\ell^2} , \\ x_1^{o*} &= \frac{x_1^o}{\ell} , & x_2^{o*} &= \frac{x_2^o}{\ell} , \\ Z_1^* &= \frac{Z_1}{\ell} , & Z_2^* &= \frac{Z_2}{\ell} , \end{aligned} \tag{16}$$

(16 cont'd.)

$$\omega^*_{B1} = \frac{\omega}{\omega_{B1}} = \frac{\omega}{(\frac{\pi}{\ell})^2\left(\frac{EI_1}{m}\right)^{1/2}}\ , \quad \omega^*_{B2} = \frac{\omega}{\omega_{B2}} = \frac{\omega}{(\frac{\pi}{\ell})^2\left(\frac{EI_2}{m}\right)^{1/2}}\ ,$$

$$K^* = [C^*_\omega + \frac{E^*}{\pi^2}]\ , \qquad \omega^*_{T3} = \frac{\omega}{\omega_{T3}} = \omega^*_{B2}\left(\frac{J^*_o}{K^*}\right)^{1/2}\ ,$$

$$T^* = \omega_{B2}t\ , \qquad P^* = P\left(\frac{\ell^2}{\pi^2 EI_2}\right)\ ,$$

$$P^*_t = P^* \sin(\omega^*_{B2}T^*)\ .$$

The time-dependent, external force, P^*_t, is considered in the following to be a sinusoidal function of time as indicated above. The second-order acceleration equations can be uncoupled by considering the equations as being algebraic in nature and by using Cramer's rule of mathematics. This leads to the following three, coupled, second-order, linear, variable coefficient, nonhomogeneous, ordinary differential equations in matrix form:

$$\begin{bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \\ \ddot{\eta}_3 \end{bmatrix} = \frac{1}{J^*_o - C^{*2}_1 - C^{*2}_2} \begin{bmatrix} (J^*_o - C^{*2}_1) & -C^*_1C^*_2 & C^*_2 \\ -C^*_1C^*_2 & (J^*_o - C^{*2}_2) & -C^*_1 \\ C^*_2 & -C^*_1 & 1 \end{bmatrix} \begin{bmatrix} (T1) \\ (T2) \\ (T3) \end{bmatrix} \qquad (17)$$

in which

$$(T1) = -\eta_1 - 4P^*_t\left[C^*_1\eta_1 - x^{o*}_2\eta_2\frac{1}{2\pi^2}\right] \qquad (18a)$$

$$(T2) = -I^*\eta_2 - 4P^*_t\left[(C^*_2 + x^{o*}_2)\eta_1 + \left(\frac{1}{16} + \frac{1}{2\eta^2}\right)\eta_3\right] \qquad (18b)$$

$$(T3) = -K^*\eta_3 - \frac{1}{4}P^*_t\left[\eta_2 + \left(Z^*_2 + \frac{8}{\eta^2}x^{o*}_1\right)\eta_3 + \frac{8}{\eta^2}x^{o*}_2\right] \qquad (18c)$$

The system of equations is now dynamically uncoupled. However, it remains statically coupled as can be noted by the expressions for (T1), (T2) and (T3) in Eqs. 17.

By defining appropriate new variables, these equations can be reduced to a system of first-order, ordinary differential equations. Hamming's modified predictor-corrector method (11, 12) was adopted to integrate the equations numerically.

RESULTS

The parameter studies presented here were made for cross-sections such as channels and zees which are symmetric about one axis (taken as the x_2 axis here), hence C_1^* and Z_1^* are zero. The external sinusoidal force P^* was located at the span centerline acting parallel to the x_1 axis and fixed to act through the shear center (x_1^{o*} and x_2^{o*} equal zero). P^* was taken as having a maximum magnitude of 0.06 of the Euler buckling load for the beam as an axially loaded member. Initial displacements and velocities were set at zero.

The system was assumed to have become unstable if the dimensionless displacement variables η_1 or η_2 reached 0.2 or if η_3 reached 0.3. This defines instability as occurring when the translation of any point reached 0.2 of the span or when rotation exceeded approximately 17 degrees.

Parameters Used in the Study. -- The particular class of beams as outlined above was studied, thus C_1^*, P^*, x_1^{o*}, x_2^{o*} and Z_2^* are specified constants and are not varied. Seven other parameters were studied over a range of values as outlined in Table 1. These values were picked to represent a range encountered in channel and zee shapes. The parameter ω_{B2}^* which was chosen to determine the frequency of the midspan dynamic force was set at two values, 1.2 and 2.0.

TABLE 1. -- SELECTED VALUES OF DIMENSIONLESS PARAMETERS

Parameter (1)	Range of Parameter (2)	Mean Value[a] (3)
C_1^*	0.0	0.0
C_2^*	0.0 - 0.025	0.014
C_w^*	0.0 - 0.005	0.003
E^*	0.001 - 0.015	0.008
I^*	0.03 - 0.14	0.085
J_o^*	0.001 - 0.025	0.014
P^*	0.0	0.06
x_1^{o*}	0.0	0.0
x_2^{o*}	0.0	0.0
Z_1^*	0.0 - 0.015	0.008
Z_2^*	0.0	0.0

[a] Parameters with no range in magnitude were specified by the associated constants listed in the mean-value column.

The studies were conducted by setting all parameters at their mean value and then studying beam displacements with a single parameter set at maximum, then at mean, and then at minimum value. This was done independently for each of the parameters, and for the two values of ω^*_{B2}. Since the intent was to provide guidance as to relative effects of each of these parameters, a simplified presentation of results is used. The responses of the beam with the study parameter set at its various values and with other parameters at mean value are compared. Each of the three displacement parameters were individually compared and the range of the effect was noted as follows:

no effect = (--) = 5% or less difference,

slight effect = (-) = 20% or less difference,

significant effect = (+) = 100% difference,

very significant effect = (++) = the system becomes unstable.

The results are summarized in Table 2.

One significant result can be noted in the lower line of this table. Z^*_1 the geometric parameter derived from torsional-flexural coupling has no effect for either loading condition or for any displacement and could be ignored throughout the problem.

J^*_0, representing the polar moment of inertia, seems to have the most significant effect and its variation leads to instability of the rotational displacement at minimum parameter values.

C^*_2, representing the distance from the centroid to the shear center of the cross-section, will be a direct measure of the inertial force moment arm about the shear center and has significant effect since at its minimum value (moment arm 0.0), rotational and translation displacements in the x_2 direction will be

TABLE 2. — EFFECT OF VARIOUS VALUES OF PARAMETERS AT SELECTED FORCING FREQUENCIES ON RESPONSE DATA[a]

VARIATION OF	η_1		η_2		η_3	
	$\omega^*_{B2}=1.2$	$\omega^*_{B2}=2.0$	$\omega^*_{B2}=1.2$	$\omega^*_{B2}=2.0$	$\omega^*_{B2}=1.2$	$\omega^*_{B2}=2.0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
C^*_2	+	+	+	+	+	+
C^*_W	-	-	+	--	+	+
E^*	-	--	-	--	+	-
I^*	+	--	++	+	+	--
J^*_o	+	-	+	+	++	++
Z^*_1	--	--	--	--	--	--

[a]The effects on the response data are denoted as follows: (--) ≡ essentially none, (-) ≡ slight, (+) ≡ significant, and (++) ≡ very significant.

zero and the problem is reduced to plane bending, and as C_2^* is increased, inertial torsional moment increases directly.

I^*, the ratio of bending stiffnesses has significant effect and leads to instability in the direct translation displacement at low parameter values, when the loading frequency approaches the fundamental bending frequency of the beam. This would generally be expected. C_w^*, the torsional warping stiffness parameter, and E^*, the St. Venants torsional stiffness parameter, have significant effect, primarily on the rotational displacement, although a significant coupling effect on translational displacements is noted. No instability results for the range of values studied for these parameters.

This investigation has not studied the effects of joint variation of parameters in the innumerable combinations possible and the response is limited to consideration of the displacements of the centerspan cross-section for shear center transverse translations and cross-section rotation. This ignores local buckling and similar localized displacements. The forcing function frequency parameter was chosen based on the fundamental beam frequency in the stiffest bending direction and, since coupling effects are emphasized, a study with the frequency related to the weak axis bending frequency and the torsional frequency would be considered appropriate for future investigation.

SUMMARY AND CONCLUSIONS

An investigation of the effect of certain geometric-material parameters on the dynamic response of beams with thin-walled, open, monosymmetric cross-sections has been presented. Torsional-flexural coupling was considered and the concentrated, sinusoidal force at midspan was taken as acting through the shear center so as to produce no direct torsion and to emphasize parametrically induced displacements.

It was found that the geometric parameter Z relating to flexural-torsional coupling has little effect and probably could be ignored. The product of inertia was found to have considerable effect in the range of values studied and led to torsional instability at minimum values. The torsional stiffness, torsion-warping stiffness and ratio of bending stiffnesses have significant effects and all are considered important in limiting cases. The distance between the shear center and centroid was found to be significant and should be kept to a minimum. The frequency of the dynamic force has considerable effect as it approaches the fundamental beam frequency with low bending stiffness in the beam.

The study provides guidance to the designer and analyst working with problems of this general type.

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APPENDIX II. — NOTATION

The following symbols are used in this paper:

a = a point at midspand of member where P is applied:

A = cross-sectional area;

C, C' = undisplaced and displaced centroid of cross section, respectively;

C_1, C_2 = coordinates of centroid in the x_1-x_2 or $\bar{x}_1$-$\bar{x}_2$ directions, respectively;

C_1^*, C_2^* = ratios of C_1 and C_2 to ℓ, respectively;

C_w = warping torsional constant;

C_w^* = ratio of C_w to $(I_2\ell^2)$;

E = Young's modulus;

E^* = ratio of (GJ_e) to (EI_2);

$\bar{f}_i(i=1,2,\ldots 6)$ = components of $\bar{p}$, $i=1,2,\ldots 6$, respectively, due to the externally applied loads;

$\bar{g}_i(i=1,2,\ldots 6)$ = components of $\bar{p}_i$, $i=1,2,\ldots 6$, respectively, due to the inertial loads;

G = shearing modulus;

i = integer index;

I_1, I_2 = second principal moments of intertia of A;

I^* = ratio of I_1 to l_2;

j = integer index;

J_e = St. Venant's torsional constant;

J_o = polar moment of inertia of A about x_3;

J_c^* = ratio of J_o to $(A\ell^2)$;

k = integer index;

$K^* = [C_w^* + \frac{E^*}{\pi^2}]$;

APPENDIX II. — NOTATION (Continued)

ℓ = length of member;

m = mass per unit length of the member;

n,N = integer;

0,0' = undisplaced and displaced shear center of cross section; respectively;

$\bar{p}_i (i=1,2,\ldots 6)$ = equivalent, generalized force system composed of three forces and three moments associated with $\bar{x}_1$-$\bar{x}_2$-$\bar{x}_3$;

P = constant-direction, time-dependent, concentrated load;

P^* = ratio of P to the Euler buckling load, $(\pi^2 EI_2/\ell^2)$;

$P^*_t = P^* \sin(\omega^*_{B2} T^*)$;

R_e = residual of equation when approximate solution is substituted into the equation;

t = time;

T^* = dimensionless time, product of t and ω_{B2};

(T1),(T2),(T3) = quantities defined by Eqs. 18;

u_1, u_2 = translational displacements of shear center in the x_1 and x_2 directions, respectively;

= displacement of shear center at midspan in x_2 direction;

x_1, x_2, x_3 = principal, orthogonal, Lagrangian axes; origin at 0; x_3 is the longitudinal axis

$\bar{x}_1, \bar{x}_2, \bar{x}_3$ = principal, orthogonal, Eulerian axes which translate and rotate with the cross section; origin at 0';

x^o_1, x^o_2 = $\bar{x}_1$-$\bar{x}_2$ coordinates of the point at which the external load is applied;

x^{o*}_1, x^{o*}_2 = ratios of x^o_1 and x^o_2 to ℓ, respectively;

Z_1, Z_2 = geometric properties of the cross section expressed by Eqs. 4 and 5;

Z^*_1, Z^*_2 = ratio of Z_1 and Z_2 to ℓ, respectively;

APPENDIX II. — NOTATION (Continued)

$\eta_{1i},\eta_{2i},\eta_{3i}$ = generalized coordinates for displacements in the x_1 and x_2 directions and about the x_3 axis, respectively;

$\eta_i,(i=1,2,3)$ = dimensionless, translational displacements of shear center in the x_1 and x_2 directions, and the rotational displacement ϕ, respectively, of the section at midspan

$\eta_{im},(i=1,2,3)$ = absolute, maximum, dimensionless displacements of shear center at midspan in the x_1 and x_2 directions and about the x_3 axis, respectively, during the response time considered;

ϕ = rotational displacement of cross section about the x_3 axis;

$\chi_i(i=1,2,3)$ = quantities defined by Eqs. 13;

ω = circular frequency of forcing function;

$\omega^*_{B1},\omega^*_{B2}$ = ratios of ω to the fundamental natural bending frequencies $\omega_{B1} = [(\frac{\pi}{\ell})^2.(\frac{EI_1}{m})^{1/2}]$ and $\omega_{B2} = [(\frac{\pi}{\ell})^2.(\frac{EI_2}{m})^{1/2}]$, respectively;

ω^*_{T3} = ratio of ω to the fundamental natural torsional frequency $\omega_{T3} = \frac{\pi}{\ell}\{\frac{A}{mJ_o}[EC_w\frac{\pi}{\ell} + GJ_e]\}^{1/2}$; and

$< >^n$ = Macauley notation for singularity functions defined by Eq. 11.

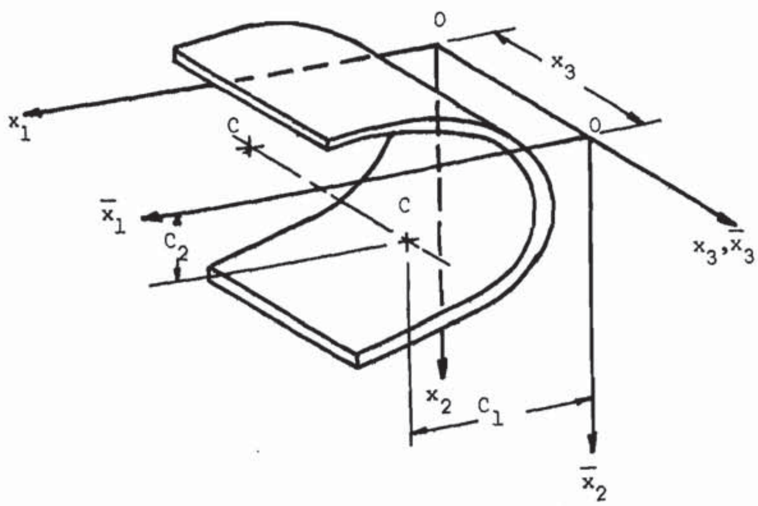

Fig. 1. Segment of a Thin-Walled Member with an Open Cross Section

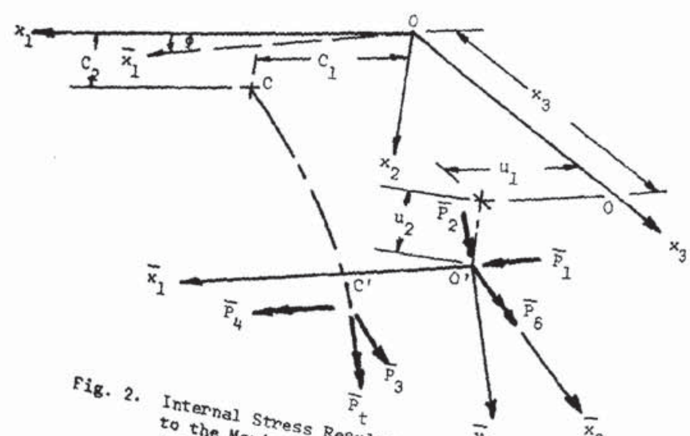

Fig. 2. Internal Stress Resultants Relative to the Moving Reference System at an Arbitrary, Displaced Section

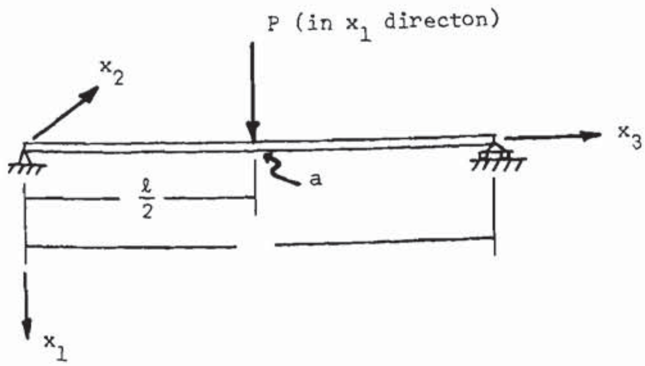

Fig. 3. Constant-Direction, Concentrated Load at Midspan of a Simple Beam

BENDING STRENGTH OF DEEP CORRUGATED STEEL PANELS

by

James L. Jorgenson[1] and Chingmiin Chern[2]

1. INTRODUCTION

The corrugated metal panels under study are fabricated from rolled sheet galvanized steel. The fabrication process consists of: unrolling, cutting, punching, and then going through a rollformer which permits the panels to take on the corrugated shape. If curved panels are desired a final operation, stretch forming, is used. This consists of placing the panel in tension and then stretching it around a mold with the desired radius.

These panels are used in the construction of metal buildings. The buildings are either of an arch shape incorporating the curved panels or are planer walls and roof incorporating the straight panels. The metal panels serve as both a covering of the building and as a structural frame.

1.1 Material

The material used for testing was supplied in accordance with ASTM A525 "zinc-coated steel sheets of commercial quality." It is deficient in that the material does not

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meet a minimum strength requirement. Future steel should be purchased in accordance with ASTM A 446 which is similar to A 525 only it does satisfy minimum strength requirements.

1.2 Code Evaluation

The usual method of strength evaluation for these panel sections is to apply the criteria of the appropriate building code. The code used here was the "Specification for the Design of Cold-Formed Steel Structural Members", 1968 Edition, by the American Iron and Steel Institute. This section comments on the problems in directly applying the code and suggests that a laboratory testing program is necessary to determine the true strength of the panels.

The allowable bending moment on the panel is dependent on the shape of the panel and the yield strength of the steel. The shape of the panel is shown in Fig. 2. It is proportioned such that each flange permits full effective design width for the compression elements. Using 33 ksi yield steel this will permit a flexural stress of 20 ksi. However, when consideration is given to the deep thin web the allowable flexural stress is significantly reduced. The Code limit for h/t is 200, however, the panel under study has values of 175, 232, 280, and 350 for the 18, 20, 22 and 24 gage panels. When these web thickness ratios are used to determine the allowable bending stress, the values are 16.4, 9.3, 6.4, and 4.1