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V. V. Acharya

R. M. Schuster

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ANALYSIS OF HAT SECTIONS WITH MULTIPLE INTERMEDIATE LONGITUDINAL STIFFENERS

V.V. Acharya[1] and R.M. Schuster[2]

ABSTRACT

This paper is the second paper by the authors on the subject of the behavior of cold formed steel hat sections in bending with multiple intermediate longitudinal stiffeners. The first paper, also contained in these Proceedings, deals with the test results only (Acharya and Schuster, 1998). Presented in this paper is the analysis associated with the test program that was conducted at the University of Waterloo.

The main objective was to develop a consistently accurate and practical method of predicting the ultimate bending strength of sections that fail in overall plate buckling. Recent testing carried out by previous researchers indicates that the bending resistance of multiple stiffened cold formed steel members which fail in overall plate buckling is too conservatively predicted by the current Canadian design standard (S136-94). These researchers have also shown that the American design specification (AISI 96) is also unconservative for the same sections. This investigation primarily includes a theoretical study that is substantiated with the experimental data summarized in the first paper. The current North American design methods were evaluated with respect to their ability to predict the strength of the test specimens.

Based on the work by Lind (Lind, 1973), which is shown to adequately predict the strength of sections that experience overall plate buckling, an alternate design method for strength determination is presented.

INTRODUCTION

Analysis of current North American Approaches

The Canadian Design Standard (S136-94) and the American Design Specification (AISI 96) have different procedures for calculating effective section properties and corresponding section strengths. There are three basic sub-procedures within each document to calculate the ultimate strength of sections that incorporate multiple intermediate stiffeners. The governing sub-procedure depends on the moment of inertia of the stiffener (I_s) and the largest section slenderness ratio (W = w/t). A more detailed explanation of the North American Standards can be found in the Thesis by Acharya (Acharya, 1997).

The moment of inertia of the stiffener (I_s) about it's own neutral axis is compared to that of what is considered to be an adequate stiffener (I_a). The adequacy of a stiffener was originally

[1] Partner, Durisol Building Systems Inc., Former Graduate Student, University of Waterloo
[2] Professor of Structural Engineering, Department of Civil Engineering, University of Waterloo

investigated by Winter and later by other researchers including Desmond (Desmond et al, 1981). Winter used energy methods (Winter, 1968) with respect to longitudinal edge stiffeners, and determined the required minimum stiffness of an adequate stiffener. If the edge stiffener is of sufficient flexural rigidity such that the ultimate strength of the flange equals that of an identical flange which is stiffened by webs on both longitudinal edges, the stiffener is termed adequate. Winter's analysis gave the necessary dimensions to make the critical buckling stress of an edge-stiffened flange equal to that of the identical flange, but stiffened by webs along both edges. With respect to an intermediate stiffener, the value of I_a was doubled to reflect the two adjacent plates being stiffened by one stiffener.

The sub-element slenderness ratio, w/t, is compared to the limiting slenderness ratio (W_{lim}). W_{lim} represents the limit when the effective width equals the actual width and corresponds to the configuration of geometry and stress which induces elastic local buckling prior to the onset of yielding. For plates with a w/t ratio less than W_{lim} , the plate/section is considered fully effective, and gross section properties can be used. The extension of the effective width concept to multiple stiffened sections has led to the use of the W_{lim} value to determine the extent to which the intermediate stiffeners are capable of influencing the section strength. The assumption used in S136 and AISI is that if the w/t ratio of any flat sub-element in the stiffened plate assembly is larger than W_{lim} , then only the stiffeners immediately adjacent to the webs are considered effective. This assumption is based on the premise that once the slenderness of any sub-element becomes sufficiently large such that $W > W_{lim}$, the effectiveness of the stiffeners in the middle of the plate is diminished, due to the effects of shear lag (AISI Commentary, 1968).

The major difference between the two design documents is the manner in which the equivalent section thickness is calculated when considering the overall buckling mode of failure. The two different procedures are briefly outlined in Tables 1 and 3.

Table 1 - Summary of Procedures to Calculate Ultimate Strength (S136-94)

Condition	Procedure	Implied Failure Mechanism
$I_s < I_a$	S-I Ignore all intermediate stiffeners. Use basic effective width equations on the the approximated single plate	Local buckling of entire compression flange with no contribution from any of the intermediate stiffeners.
$I_s > I_a$ & $W_{max} > W_{lim}$	S-II Ignore all intermediate stiffeners except the two immediately adjacent to the webs. Use basic effective width equations on the three divided sub-elements separately	Local buckling of plate sub-elements between intermediate stiffeners (Only the two exterior intermediate stiffeners are considered to contribute section strength because of shear lag effects.)
$I_s > I_a$ & $W_{max} < W_{lim}$	S-III Substitute entire compression flange with a plate of equivalent thickness.	Overall buckling of entire compression flange assembly with all intermediate stiffeners contributing to section strength.

* Equivalent thickness based on equivalent elastic buckling stress

Figure 1 shows the general relationship between the three S136-94 procedures and their ability to accurately predict the section strength. The test data was obtained from an extensive testing program conducted at the University of Waterloo (Acharya et al, 1998). It is apparent that the three procedures in S136-94 divide the test data into three distinct regions of accuracy, with a slight transition between Procedure S-I and S-III. The degree of accuracy of the Standard is dependent upon the procedure used, which in turn is dependent upon the two parameters; I_s/I_a and W_{max} / W_{lim}. A breakdown of the current data according to each procedure is contained in Table 2.

Procedure S-I ($I_s < I_a$), which does not consider the presence of intermediate stiffeners, is as expected, the most conservative in strength prediction. Obviously, disregarding any beneficial effect of the stiffeners would result in rather conservative strength predictions. Procedure S-III also produces considerably conservative predictions, as is illustrated in Figure 1. It was assumed at the outset of this investigation that Procedures S-I and S-III produce conservative approximations of section strength. A better prediction for sections that fail in overall buckling (Procedure S-III) is the main objective of this work. It was however not known at the outset that Procedure S-II would produce unconservative results.

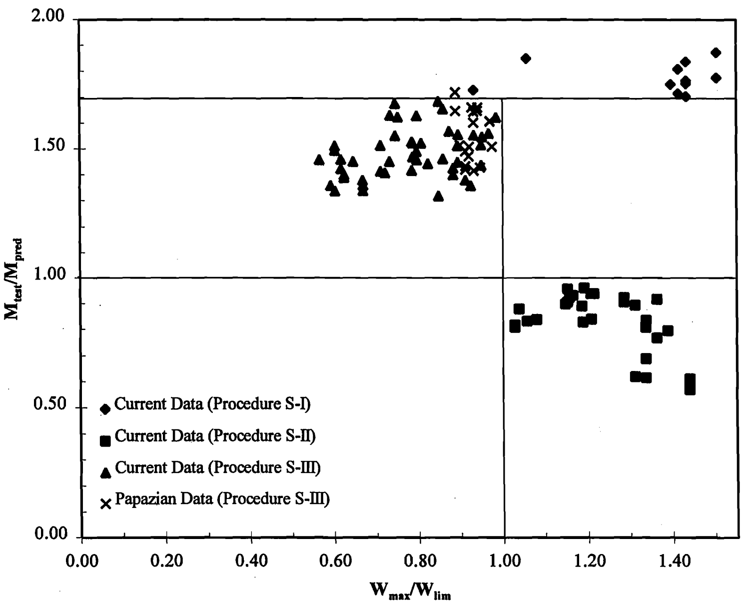

Figure 1 - M_{test} / M_{pred} vs. W_{max} / W_{lim} (S136-94)

Table 2 - M_{test} / M_{pred} for Test Data Subdivided By Procedure (S136-94)

Test Source	Procedure	No. of Tests	M_{test} / M_{pred} (Average)	M_{test} / M_{pred} (Coeff. Var)
Current Research	S-III	50	1.49	0.06
Papazian	S-III	18	1.55	0.06
Combined	S-III	68	1.50	0.07
Current Research	S-II	30	0.83	0.14
Papazian	S-II	n/a	n/a	n/a
Combined	S-II	n/a	n/a	n/a
Current Research	S-I	11	1.78	0.03
Papazian	S-I	n/a	n/a	n/a
Combined	S-I	n/a	n/a	n/a

This procedure, which accounts for local buckling of the plate sub-elements, predicts strength by substituting a single flat plate element in the interior of the compression flange. It was believed that a large w/t ratio of this approximate flat plate would result in an overly conservative effective width when used in the basic effective width expression. The data presented does not support this reasoning and shows that this procedure overestimates the section strength by 17% on average.

The current AISI 96 procedures were also evaluated and, as expected, the results were similar to the results of S136-94. The primary difference between the two methods is with respect to the A-III (versus the S-III) procedure and the basis for calculating the effective thickness. Summarized in Table 3 is the overall procedure of the AISI approach and Figure 2 shows a graph of the moment ratio for each individual test. The W_{max}/W_{lim} ratio is used again as the independent variable in this graph, even though the W_{lim} term is not explicitly used in the AISI 96 Specification. The W_{lim} term as used in S136-94 is implicitly included in AISI 96 by the presence of λ (to determine whether or not an element is fully effective or not). Figure 2 would be identical if the W_{max} / W_{lim} ratio was replaced with λ. In this case the vertical line separating Procedures A-II and A-III would cross the horizontal axis at $\lambda = 0.673$ rather than $W_{max} / W_{lim} = 1$.

Comparison of Figures 1 and 2 illustrates the difference between the two North American approaches. The moment ratios of test specimens that fall within the domain of procedure A-III are not all conservative, as was observed with Procedure S-III. This confirms that the use of an equivalent moment of inertia in Procedure A-III is not an adequate method for determining the equivalent thickness. The A-III Procedure results in unconservative strength predictions for most of the sections examined with an average moment ratio of 0.94; approximately 56% lower than with the corresponding S136-94 procedure (S-III). A breakdown of the current data according to each AISI 96 procedure is contained in Table 4.

Table 3 - Summary of Procedures to Calculate Ultimate Strength (AISI 96)

Condition	Procedure	Implied Failure Mechanism
$I_s > I_a$	A-I Ignore all intermediate stiffeners. Use basic effective width equations on the the approximated single plate	Local buckling of entire compression flange with no contribution from any of the intermediate stiffeners.
$I_s > I_a$ & $b < w$	A-II Ignore all intermediate stiffeners except the two immediately adjacent to the webs. Use basic effective width equations on the three divided sub-elements separately.	Local buckling of plate sub-elements between intermediate stiffeners. (Only the two exterior intermediate stiffeners are considered to contribute to section strength).
$I_s > I_a$ & $b > w$	A-III Approximate entire compression flange assembly with a plate of equivalent thickness.	Overall buckling of entire compression flange assembly with all intermediate stiffeners contributing to section strength

* * Equivalent thickness is based on equivalent moments of inertia.

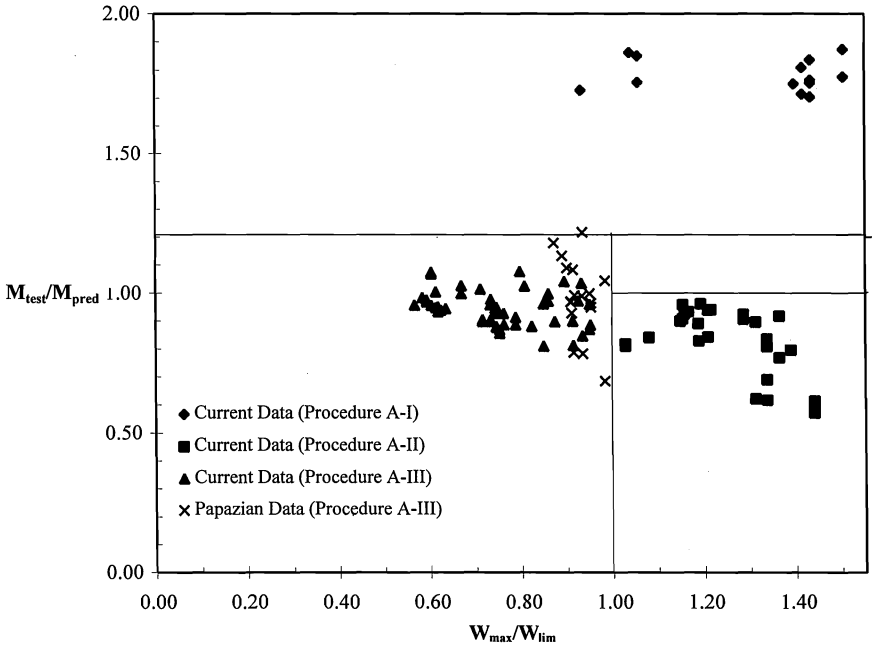

Figure 2 - M_{test} / M_{pred} vs. W_{max} / W_{lim} (AISI 96)

Table 4 - M_{test} / M_{pred} for Test Data Subdivided By Procedure (AISI 96)

Test Source	Procedure	No. of Tests	$\frac{M_{test}}{M_{pred}}$ (Average)	$\frac{M_{test}}{M_{pred}}$ (Coeff. Var)
Current Research	A-III	50	0.94	0.07
Papazian	A-III	18	0.97	0.16
Combined	A-III	68	0.95	0.10
Current Research	A-II	28	0.82	0.15
Papazian	A-II	n/a	n/a	n/a
Combined	A-II	n/a	n/a	n/a
Current Research	A-I	13	1.78	0.03
Papazian	A-I	n/a	n/a	n/a
Combined	A-I	n/a	n/a	n/a

ALTERNATE APPROACHES OF STRENGTH DETERMINATION

A number of alternate approaches were examined by Acharya (Acharya, 1997), including other energy based methods by Timoshenko (Timoshenko et al, 1961), Lind (Lind, 1973) and Schafer (Schafer, 1996). It was determined that best results were obtained using minor modifications to the approach first developed by Lind (Lind, 1973). The energy method that is presented in this section is based on Lind's refinement of Timoshenko's energy method (Lind, 1973). Lind's energy formulation varies from Timoshenko in that his approach is based on orthotropic plate theory, while Timoshenko looked at discrete stiffeners. Lind utilized this formulation to calculate the equivalent thickness (t_s) of the compression flange. This is the approach that is currently used in S136-94 for multiple intermediate stiffeners, and consequently, there is considerable advantage to using this same basis in developing a new method of strength prediction. Minimizing the variation in S136-94, and incorporating existing methodology, would maximize the ease of using a new methodology.

A variation of Lind's formulation was developed to calculate the buckling coefficient ($k_{overall}$) of the compression plate with multiple intermediate stiffeners. A detailed explanation of Lind's work can be found in the referenced material (Lind, 1973).

Lind showed that the buckling load N_{cr}, which is considered valid except for extremely short spans, can be expressed as

$$N_{cr} = \frac{1}{2}\frac{4\pi^2 D}{w_m^2}\left(\frac{w_m^2}{p}\right)\left(1+\sqrt{\frac{12pI_s}{w_m^2 t^3}}\right) \quad \textbf{(1)}$$

where: D = $Et^3 / (12\text{-}12v^2)$
w_m = overall compression flange width (as per S136-94)
p = perimeter of compression flange including stiffeners
I_s = moment of inertia of the entire compression flange assembly about it's neutral axis.

Equating Equation 1 with the critical load for a simply supported plate of width w_m and solving for k we get

$$k = k_{overall} = \frac{2w_m}{p} + \sqrt{\frac{48I_s}{pt^3}}. \quad \textbf{(2)}$$

Using this buckling coefficient, which is an "equivalent" buckling coefficient, in the S136-94 effective width equation is analogous to replacing the compression flange assembly with an unstiffened flange having a much higher buckling coefficient. This higher buckling coefficient is then calculated such that the analogous unstiffened compression flange and the *original* compression flange assembly have the same critical elastic buckling load. The effective width equation would now be rewritten as

$$B = 0.95\sqrt{\frac{kE}{f}}\left[1-\frac{0.208}{W}\sqrt{\frac{kE}{f}}\right] \quad \textbf{(3)}$$

where: B = effective width to thickness ratio of the flat plate (b_{eff} / t)
k = $k_{overall}$ as per Equation 2
E = Young's Modulus [203 000 MPa] (29443 ksi)
f = calculated stress in extreme compressive element ($\leq F_y$)
W = Actual width to thickness ratio of flat plate (w_m/t)

Using this effective width equation and denoting the procedure as L-I, Figure 3 illustrates the accuracy in determining the actual effective width. Several variations of Equation 3 are also plotted in Figure 3. Better results can be obtained when the 0.208 coefficient is replaced with zero (L-II), and best results were possible when using the following equation:

$$B = (1.18)0.95\sqrt{\frac{kE}{f}}\left[1-\frac{0.198}{W}\sqrt{\frac{kE}{f}}\right] \quad \textbf{(4)}$$

Here the coefficient of 0.208, presently used in S136-94 for all effective width calculations, was replaced with the 0.198, which resulted in the third variation (L-III). However, in order to retain the current effective width expression (Equation 3) in S136-94, Equation 5 was ultimately chosen for the proposed design expression with the plate buckling coefficient given in Eqn 2.

$$B = (1.18)0.95\sqrt{\frac{kE}{f}}\left[1-\frac{(0.208)(1.18)}{W}\sqrt{\frac{kE}{f}}\right] \qquad \textbf{(5)}$$

Another format that could be used is to express the plate buckling coefficient as follows:

$$k = k_{overall} = (1.18)^2[2w_m/p + (48I_s/p/t^3)^{1/2}]\,. \qquad \textbf{(6)}$$

This plate buckling coefficient of Equation 6 can now be used with Equation 3, the basic effective width expression used in S136-94.

A summary of the accuracy of these procedures is contained in Table 5. L-I which was significantly inaccurate had an average moment ratio of 1.13 with a coefficient of variation of 0.10 for the combined current Research and the data by Papazian.

It should also be noted that removal of test specimens with supposedly "inadequate" stiffeners did not produce more accurate results because the strength of the stiffener is accounted for in the energy formulation of the plate assembly. The stiffener "adequacy" is properly reflected in the calculation of the plate buckling coefficient, k.

Therefore, it is possible that procedure L-III can also be used as an alternate method of predicting the ultimate capacity of a section with multiple intermediate stiffeners. Again, this procedure is limited to sections that fail in overall plate buckling, but includes partially stiffened sections.

Since procedure (L-III) provides an accurate method of determining the strength of sections that fail in overall buckling, it is also necessary to be able to first determine the governing mode of failure for a section. This was investigated by the author (Acharya, 1997) with a method of determining the failure mode being suggested as basis for further study. The method was developed using test results from the limited number of specimens (six) that actually underwent local buckling mode of failure. Further testing is required to establish the validity of the proposed method.

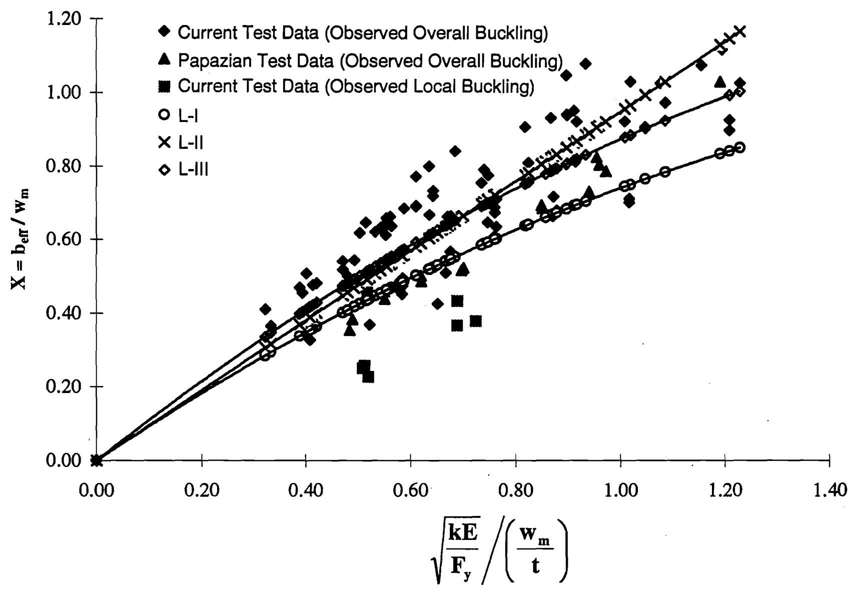

Figure 3 - Compression Flange Width Reduction Factor χ vs. Modified Lind Procedures

Table 5 - Summary of Test Results (Modified Lind)

Test Source	Observed Failure Mode	Modified Lind Procedures: L-II No. of Tests	L-II M_{test}/M_{pred} (Average)	L-II M_{test}/M_{pred} (Coeff. Var)	L-III No. of Tests	L-III M_{test}/M_{pred} (Average)	L-III M_{test}/M_{pred} (Coeff. Var)
Current	Overall	85	1.04	0.11	85	1.03	0.10
Papazian	Overall	18	0.97	0.08	18	0.97	0.09
Combined	Overall	103	1.03	0.11	103	1.02	0.10

PREDICTION OF FAILURE MODE

Consider the current S136-94 approach, whereby local buckling is considered to occur when

$$W > W_{lim} \qquad (7)$$

where: $W = w_{max} / t$

$$W_{lim} = 0.644\sqrt{\frac{kE}{f}}$$

w_{max} = largest sub-element width ; k =4

The specimens that were observed to have failed in local buckling had W/W_{lim} in excess of 1.32. However, there were also specimens that had $W/W_{lim} > 1.3$ which did not experience local sub-element buckling. At the very least, it would appear that when considering multiple intermediate stiffeners, the maximum allowable W/W_{lim} ratio for applying overall plate buckling procedures should be increased from 1 to 1.3. Based on the limited data (six specimens), this increase would be considered conservative since some specimens would be subjected to local sub-element buckling equations even though their actual failure mode would be overall plate buckling.

A purely empirical analysis reveals that if the sub-element width in Equation 5 is taken as the spacing between stiffeners and the maximum allowable W/W_{lim} ratio is increased to 2.4, then only the specimens that actually did undergo sub-element buckling would be subjected to the local sub-element buckling equations. Although this procedure fits the data well, it would be more prudent at this time, due to the limited data available, to simply increase the W/W_{lim} ratio (as currently defined in S136-94) to 1.3.

CONCLUSIONS

Based on the evaluation of current design documents (S136-94 and AISI 96), it has been established that the current S136-94 procedure is not adequate in predicting the ultimate bending strength of sections with multiple intermediate stiffeners. Furthermore, it was shown that the current procedure isolates sections into three distinct regions depending on the strength of the stiffener and slenderness of plate sub-elements. When considering specimens with inadequate stiffeners, the current procedure produces overly conservative estimates (approximately 70%) of section strength. For specimens that are considered as failing in local sub-element buckling ($W > W_{lim}$), the S136-94 procedure actually *overestimates* the section strength and predicts strengths that are approximately 17% unconservative. With the specimens that are considered as failing in overall plate-buckling ($W < W_{lim}$), the S136-94 approach yields predictions that are conservative by a factor of about 50% (on average).

The AISI 96 procedure is similar to the S136-94 approach except for the manner in which the equivalent thickness is calculated for sections subjected to overall plate-buckling. Consequently, the same results as with S136-94 are obtained when considering sections with inadequate stiffeners and sections with large plate sub-elements ($W > W_{lim}$). For sections with $W < W_{lim}$, the AISI 96 procedure uses an equivalent plate thickness approach which is based on an equal moment of inertia philosophy. With the S136 approach on the other hand, one calculates an equivalent thickness based on an equal elastic buckling load. It was found that the AISI 96 approach (equal moments of inertia) yields unconservative results for section strength (5% on average).

A different method of strength prediction was developed based on the energy formulation of Lind (Lind, 1973). This previous work provided the basis for the current equivalent thickness approach used in S136-94. The resulting predictions of section strength were found to be sufficiently accurate with an average test to predicted moment ratio of 1.02 and a coefficient of variation of 10 percent.

Through the course of the investigation it was found that the current method of predicting the failure mode using the W/W_{lim} ratio is considerably conservative. The range of sub-element slenderness (W = w/t) over which the standards assume local buckling as the governing failure mode were found to be incorrect. The limiting value of 1 was found to commit sections to sub-element buckling equations when in fact the sections were observed to fail in overall plate buckling. Based on the data available, it was found that increasing the limiting W/W_{lim} ratio to 1.3 would provide a more accurate assessment of the actual failure mode without sacrificing safety (i.e. predicting overall failure when local buckling occurs). This modification would still improperly consign some sections to local buckling equations, but allow for a 30% increase in sub-element slenderness.

Another empirical method of predicting the failure mode was also developed as a part of this research (Acharya, 1997). This method involves using the existing W/W_{lim} ratio with a minor variation. It was determined that by increasing the limiting ratio to 2.4 and redefining W in W / W_{lim} to be equal to the ratio of the stiffener spacing to the thickness, accurate predictions of failure mode could be made.

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