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by

Shien T. Wang[1] and George E. Blandford[2]

INTRODUCTION

Stability analysis and design of plane frames has been a subject of extensive investigation for the past decade due to the rapid development of high speed computers and the matrix methods of analysis. The previous investigations in this general area were summarized in a report by Yucel et al (1) in 1973 for the ASCE - IABSE Joint Committee on Planning and Design of Tall Buildings. Based on this extensive literature survey, it indicates that the elastic and plastic behavior of braced and unbraced plane frames can be predicted rather accurately. But none of the previous analyses considered the effects of local buckling of the component structural members on the behavior of the framework.

Due to the weaking effects of local buckling, overall buckling takes place at a lower load than the frame would carry in the absence of local buckling. Interaction of local and overall buckling therefore, must be considered in the analysis. This is an important subject not only for frames composed of thin-walled members but also for frames composed of hot-rolled and built-up sections due to the high yield strength used. There is an apparent need of research in this area. The need of developing analytical solution schemes for the stability analysis of locally buckled framework was also stressed in a survey paper by Gallagher (2) on framework finite element sta-

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[1]Associate Professor, Department of Civil Engineering, University of Kentucky, Lexington, Kentucky.

[2]Research Assistant, Department of Structural Engineering, Cornell University, Ithaca, New York; formerly Research Assistant, Department of Civil Engineering, University of Kentucky, Lexington, Kentucky.

bility analysis.

Recently, the first author has developed solution schemes for the analysis of locally buckled continuous structures, (3, 4, 5, 6). These solution schemes are based on the stiffness as well as flexibility matrix methods of analysis in conjunction with the concept of effective width and have been used successfully in predicting the post-local-buckling behavior of continuous beams and plane frames. More recently, a procedure based on the finite element method of analysis has been developed to predict the post-local-buckling behavior as well as to determine the buckling load for the locally buckled framework.

The purpose of this paper is to present this solution scheme to investigate the overall stability problems of locally buckled framework. Results on example problems are presented.

METHOD OF ANALYSIS

Postbuckling Strength of Locally Buckled Frames.- In this paper, the following expression for effective width is used to account for the post-local-buckling strength of the buckled compression plate elements (7):

$$\frac{b}{t} = 0.95 \sqrt{\frac{KE}{\sigma_{max}}} \left(1 - 0.95\, \xi\, \frac{t}{w} \sqrt{\frac{KE}{\sigma_{max}}}\right) \tag{1}$$

for

$$w/t \geq 0.64 \sqrt{EK/\sigma_{max}} \tag{2}$$

in which b = effective width of compression plate element; t = thickness; E = modulus of elasticity; w = flat width of the compression plate element; σ_{max} = maximum edge stress; K = coefficient determined by boundary conditions and aspect ratio of compression plate element; and ξ = modification factor based on experimental evidence and engineering judgement. For values of w/t smaller than $0.64\sqrt{EK/\sigma_{max}}$, b = w. Eq. (1) has been shown through experimental verification to be applicable to both stiffened and unstiffened plate elements if K is appropriately adjusted. The value of K can be evaluated by considering the relative dimensions of the section. For sections under uniform compression, K varies from 4.0 to 6.97 for stiffened plate elements and from 0.425 to 1.28 for unstiffened plate elements. For practical design purpose, however, ξ may be taken as 0.22 and K may be taken as 0.5 and 4.0 for unstiffened and stiffened elements, respectively.

Consider the rigid frame shown in Fig. 1(a). The compression plate elements of the members in the frame will buckle locally and the neutral axis will shift away from the buckled compression plate element as shown in Fig. 1(c) if the external moment at the section is larger than the local buckling moment, m_{cr}. The initial local buckling stress in the compression plate element can be computed from Eq. 2 which is a function of w/t. The corresponding local buckling load is w_{cr}. For the regions along the member length with external moments larger than m_{cr}, the reduced effective flexural rigidity $(EI)_{eff}$ varies along the member length depending upon the magnitude of the moment at the section. No reduction is necessary at locations with moments less than m_{cr}. In the post-local-buckling range, the frame is, therefore, composed of nonprismatic members (Fig. 1(d)). The stiffness of the buckled member continues to be reduced as the applied load is increased.

For frames composed of sections with relatively small width to thickness ratios, the frame may be failed by yielding or by overall buckling before the occurrence of local buckling depending upon the geometry of the frame. The overall buckling load for the frame composed of fully effective members is defined as W_{cr}. If W_{cr} is larger than w_{cr}, on the otherhand, the frame will buckle locally before failure by yielding or overall buckling. Due to the weakening effects of local buckling, however, the overall buckling load for the locally buckled frame, $(W_{cr})_{\ell}$, is smaller than W_{cr}. Therefore, it is possible that the weakened frame due to local buckling may buckle as a whole before reaching failure by yielding.

Finite Element Formulation.- For elastic buckling, the critical load is obtained from the following determinantal equation:

$$\left| [K_E] + \lambda [K_G] \right| = 0$$

in which $[K_E]$ = elastic stiffness matrix; $[K_G]$ = geometric stiffness matrix; and λ = load factor. Explicit forms of the matrices $[K_E]$ and $[K_G]$ are available. The stiffness matrices for each element can be formed and they can be assembled by the direct stiffness method to give the complete structure stiffness matrix. In this investigation, the "code" procedure outlined by Rubinstein is used (8).

In the post-local-buckling range, the stiffness of the member depends upon the effective width in the buckled compression plate element. This effective width is a function of stress in the plate element. Consequently, the coefficients in $[K_E]$ change with the applied load, and an iterative solution procedure becomes necessary. For nonprismatic members, the stiffness coefficients can be evaluated by numerical integration (4). Since the moment distribution in the frame and the stiffness of the buckled member are interdependent for a given load, it is necessary first to establish the moment distribution in the structure by iteration. In each iteration, the Gauss elimination procedure accounting the banded symmetry of the stiffness matrix is used to calculate the unknown displacements. Similar iterative procedures were used earlier by the first author (4, 5, 6). The geometric stiffness coefficients in $[K_G]$ can then be computed using the established moment and force distributions in the frame. The coefficients in $[K_E]$ are computed based on the effective corss-section corresponding to this moment distribution. Therefore, both $[K_E]$ and $[K_G]$ must be revised continuously according to the current moment and force distributions in the structure during the process of iteration.

Solution Procedure.- The proposed solution scheme involves the following steps:

(1) For the first cycle of iteration $(i=1)$, the structure is considered elastic. The local buckling load w_{cr} for the frame may be computed based on the buckling stress σ_{cr} in the compression flange of a member with the largest width to thickness ratio. The elastic overall buckling load W_{cr} for the fully effective frame is obtained by the quadratic interpolation procedure similar to that used by Dhillon (9). If $W_{cr} > w_{cr}$, local buckling will take place before the occurrance of overall buckling.

(2) Assume a value of w_i $(i=1$ for the first cycle) which is slightly in excess of w_{cr}. The subscript i indicates the cycle of computation. Calculate displacements, member forces.

(3) Compute moments at a number of discrete points along a structural member based on member forces and moments of previous cycle to determine the effective flexural rigidity, $(EI)_{eff}$.

(4) Calculate member stiffness based on effective flexural rigidities obtained in the previous step using a numerical integration procedure. Calculate fixed end forces also using the numerical integration procedure.

(5) Compute displacements and member forces based on the new stiffness. The procedure is completed if the difference between the consecutively calculated member forces are within a predetermined quantity. Otherwise, steps (3) through (5) are repeated.

(6) Determine axial forces in columns. These axial forces are denoted

as P's. The geometric stiffness coefficients are then computed. The largest P is defined as P_{max} and each P is divided by P_{max}.

(7) The element stiffness matrices are then assembled to form the structure stiffness matrices $[K_E]_i$ and $[K_G]_i$ using "code" procedure outlined by Rubinstein. The following determinantal equation is then obtained.

$$\left| [K_E]_i + \lambda_i [K_G]_i \right| = 0 \tag{4}$$

(8) Using the quadratic interpolation procedure, the lowest eigenvalue is determined, hence $(P_{cr})_i$. $(P_{cr})_i$ is defined as the critical load in column when the frame is buckled.

(9) If $[(P_{cr})_i - (P_{max})_i]/(P_{cr})_i$ is smaller than a predetermined value, the procedure is completed. Otherwise, increase w_i to $w_{i+1} = w_i + \Delta w$ and repeat steps (2) through (8). The load incrementation process is based on the secant method. Alternatively, the quadratic interpolation process is used. The applied concentrated forces are increased by multiplying by w_{i+1}/w_i.

Computer Program.- A computer program following the preceding solution procedure has been prepared for an IBM 370/165 computer. The local buckling load and the elastic overall buckling load for the fully effective frame are first computed. The condition for the overall stability for the specified load factor is examined. The overall buckling load for the locally buckled frame is searched until the iterative procedure has converged. In the preceding procedure outlined, beam-column and P-Δ effects were not considered. The beam-column and P-Δ effects have been included, however, in another version of the computer program prepared. In this case, additional iterations are required in order to include these effects.

NUMERICAL RESULTS

Local Buckling Stress.- In this investigation, the local buckling stress is derived from Eq. (2). Replacing σ_{max} by σ_{cr} in Eq. (2), the following equation is obtained

$$\sigma_{cr} = 0.41 \frac{KE}{(w/t)^2} \quad (5)$$

This critical stress is smaller than that calculated from classical equations on buckling stress. Based on this critical stress, local buckling moment for a given section, m_{cr}, can be calculated. The corresponding buckling load, w_{cr}, can also be computed.

Comparison with Existing Results.- The computed elastic overall buckling loads for a two-story, one-bay frame are compared with the analytical value (10) in Fig. 2. The frame is laterally braced in the plane of the frame and the columns were divided into one, two, and four elements. No local buckling was considered in this example. From the results presented, it appears that for braced frames each column should at least be divided into two elements. With four elements for each column, the calculated buckling load is less than 1% from the analytical solution.

Figs. 3(a) and 3(b) show a braced and an unbraced two-story and two-bay frame, respectively. The moment number and direction are shown in Fig. 3(c). The dimensions of the rectangular tubular section used are shown in Fig. 3(d). For the given loading conditions shown, the bending moments at the ends of several members are shown in Table 1. These values are obtained when the iterative procedure has converged in the post-local-buckling range. The program prepared in this investigation and an earlier program reported by Wang and Jsa (5) were used to generate the moments presented in Table 1. The computed values from these two programs are in excellent agreement. In the table, the moments for the elastic prismatic frame (without local buckling) are also presented for comparison. The amount of moment redistribution in the post-local-buckling range depends upon moment-curvature relations of member sections, loading levels, loading conditions, and type of frames.

Overall Buckling Loads for Frames with Local Buckling.- A single bay multi-story frame subjected to uniformly distributed loads was investigated. The calculated overall buckling loads for the elastic frame without local buckling, (W_{cr}), and the buckling loads for the locally buckled frame, $(W_{cr})_{\ell}$, are shown in Fig. 4. The length for the member for both beam and column is 100" and the dimensions for the section used are shown in Fig. 3(d). The buckling load for the locally buckled frame is smaller than that for the elastic frame (without local buckling) although the difference is small for the case considered. However, this difference depends upon loading conditions and the geometry of member and frame considered. For instance, the calculated W_{cr} and $(W_{cr})_{\ell}$ and 4.677 lb/in and 4.570 lb/in, respectively, for the 7-story frame shown in Fig. 4 using the cross-section as shown in Fig. 3(d). The corresponding values are changed to 8.903 lb/in and 6.883 lb/in, respectively, if the flange width, w, has been increased from 2.296" (w/t = 70) to 4.920" (w/t = 150"). For certain combinations, the effects of local buckling on the overall buckling strength of the frame can be severe.

CONCLUSIONS

A solution scheme for the stability analysis of locally buckled frames based on the finite element method and the concept of effective width has been presented. It has been found that the solution scheme converges rapidly. Based on the results obtained, it appears that the method is well suited for the type of problem considered.

Due to the weakening effects of local buckling, it has been shown that overall buckling takes place at a lower load than the frame would carry in the absence of local buckling. The amount of post-local-buckling strength of the frame depends upon dimensions and types of member and frame considered.

Further studies on stability analysis of locally buckled framework are underway including that subjected to torsional-flexural buckling (11, 12, 13). Experimental verifications would be helpful to confirm the method presented and the findings in this investigation.

APPENDIX I.- REFERENCES

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APPENDIX II.- NOTATION

The following symbols are used in this paper:

b	=	effective width of compression plate element;
E	=	modulus of elasticity;
EI	=	flexural rigidity;
$(EI)_{eff}$	=	effective flexural rigidity;
H	=	concentrated load;
i	=	subscript, cycle of iteration;
K	=	buckling coefficient;
$[K_E]$	=	elastic stiffness matrix;
$[K_G]$	=	geometric stiffness matrix;
L	=	member length;
m_{cr}	=	local buckling moment for member;
P	=	axial force in columns;
P_{cr}	=	critical load in column;
P_{max}	=	largest axial force in column;
Q	=	applied concentrated load;
Q_{cr}	=	critical buckling load;
t	=	thickness;
W_{cr}	=	overall buckling load for fully effective frame;
$(W_{cr})_\ell$	=	overall buckling load for locally buckled frame;
w	=	flat width of compression plate element exclusive of fillets, or uniformly distributed load;
w_{cr}	=	local buckling load;
w_i	=	uniformly distributed load;
Δw_i	=	load increment;

ν = Poisson's ratio;

λ = load factor;

ζ = modification factor based on experimental evidence and engineering judgement;

σ_{cr} = critical buckling stress; and

σ_{max} = maximum edge stress.

Table 1.- Comparison of Moments - Braced and Unbraced Frames (Fig. 3)

Loading Case (1)	Moment Number (2)	Moment of Elastic Prismati: Frame; in Pound-inches (3)	Moment of Locally Buckled Frame in Pound-inches	
			This Study (4)	Wang and Jsa (5)
1	1	-2206.9	-2322.7	-2322.9
	2	4869.6	4743.5	4743.9
	5	-2956.5	-3022.3	-3022.5
	6	4521.7	4401.5	4402.1
	9	521.7	535.5	535.5
	10	1043.5	1070.9	1071.0
	11	1913.0	1951.4	1951.5
	12	2260.9	2322.7	2322.9
2	1	- 166.6	- 156.1	- 156.2
	2	3235.0	3201.7	3201.9
	5	541.9	551.5	551.3
	6	3971.6	3896.3	3896.3
	9	-1972.7	-2016.2	-2016.2
	10	- 935.3	- 957.7	- 957.7
	11	393.4	406.3	406.4
	12	166.6	156.1	156.2

1 lb-in = 0.113 N-m

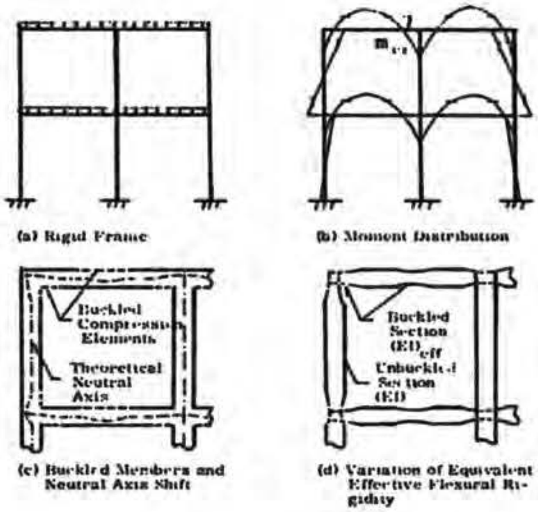


Fig. 1. - Rigid Frame in the Post-Local-Buckling Range

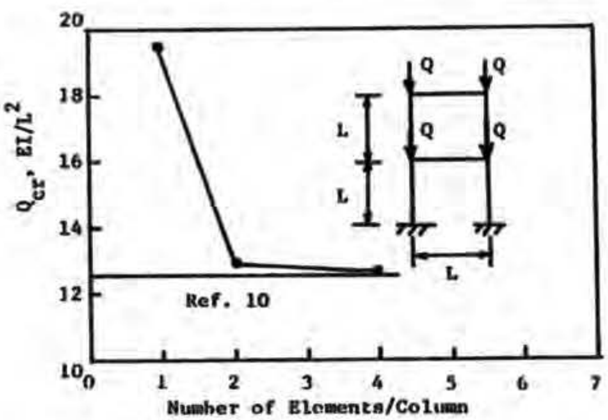
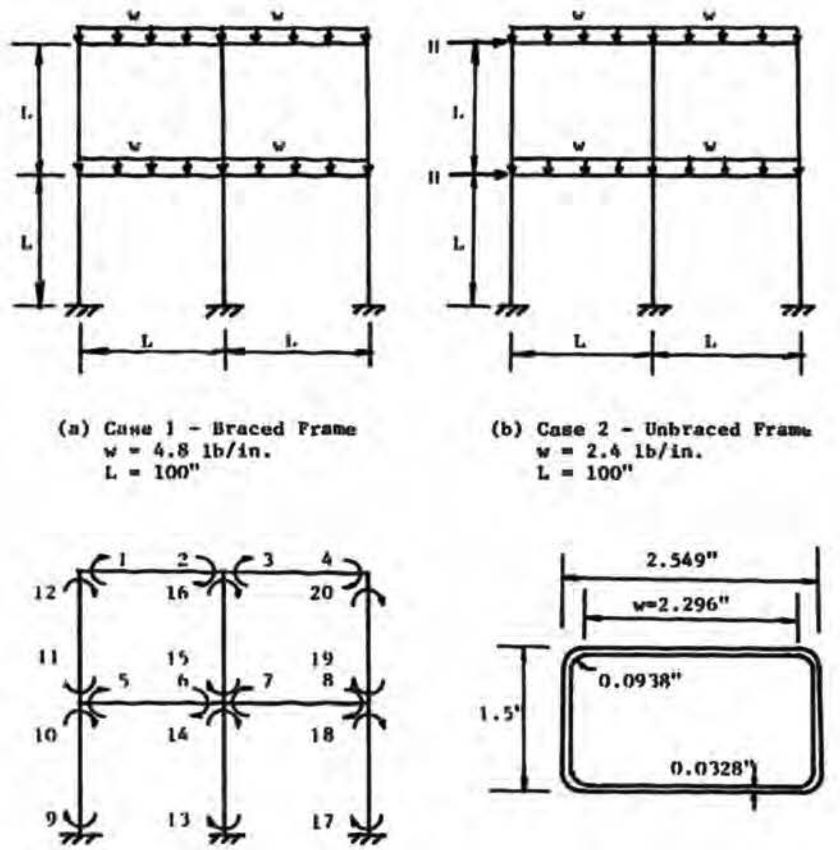


Fig. 2. - Convergence of Buckling Loads for a Braced Frame



(a) Case 1 - Braced Frame
w = 4.8 lb/in.
L = 100"

(b) Case 2 - Unbraced Frame
w = 2.4 lb/in.
L = 100"

(c) Moment Number for Braced and Unbraced Frames

(d) Dimensions for Rectangular Tubular Section

Fig. 5. - Post-Local-Buckling Analysis of Braced and Unbraced Frames (1 lb/in. = 175 N/m, 1 lb = 4.45 N, 1 in. = 25.4 mm)

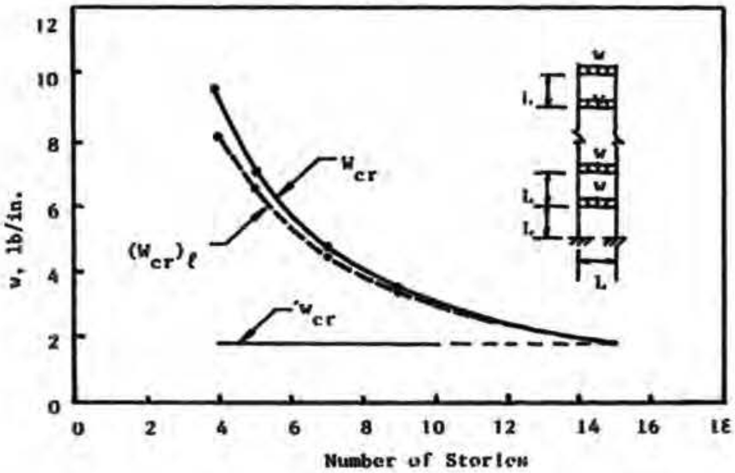


Fig. 4. - Frame Buckling Loads in Post-Local-Buckling Range (1 in. = 25.4 mm, 1 lb/in. = 175 N/m)