Stabilisation of Beams Against Lateral Buckling

K. C. Rockey

D. A. Nethercot

STABILISATION OF BEAMS AGAINST LATERAL BUCKLING

by

K.C. Rockey* M.Sc.(Eng.), Ph.D., C.Eng., F.I.C.E., M.I.Mech.E.

and

D.A. Nethercot** B.Sc., Ph.D.

SYNOPSIS

The design of slender beams is frequently controlled by the tendency of the beams to buckle laterally. This paper examines the effect which various types of semi rigid bracing have in stabilizing beams against lateral buckling. Theoretical relationships,dealing with both discrete and continuous restraints, are presented which enable a designer to determine the buckling load of beams stabilized by restraints possessing both lateral and torsional stiffness. It is shown that the method of attachment is of particular importance when dealing with bracing designed to provide a torsional restraint.

Good correlation is obtained between these theoretical results and the results of an experimental study which has been conducted to examine the effectiveness of corrugated sheeting as a stabilizing medium.

1. INTRODUCTION

When designing slender beams, special attention has to be taken to ensure that the member does not buckle. This is particularly true in the case of cold formed sections due to the thinness of the material. The buckling of structures fabricated from thin walled members may be categorised into three main types : overall buckling, member instability and local buckling of plate elements. This paper is concerned with a particular type of problem in the second category - lateral (torsional - flexural) buckling of beams.

A beam when loaded in its plane of greatest stiffness, may collapse sideways at loads considerably below those corresponding to the attainment of the fully plastic moment M_p. Because of its importance in design this form of buckling has been the subject of much research, with the result that there are now available to the designer a wide variety of formulae (1, 2) dealing with the buckling of slender beams when subjected to different forms of loading and end support conditions. Recently the writers (3, 4) have shown that this research data can be presented in a unified form which enables a designer to readily allow for the influence of different loading and support conditions.

It was shown that the load required to initiate elastic buckling may be obtained from equation (1) :

$$M_{cr} = \alpha (EI_y GJ)^{\frac{1}{2}} \gamma \tag{1}$$

where M_{cr} is the maximum moment in the beam

EI_y is the minor axis flexural rigidity

GJ is the torsional rigidity

$\gamma = \frac{\pi}{L}(1 + \frac{\pi^2}{R^2})^{\frac{1}{2}}$ is a slenderness parameter

$R^2 = \frac{L^2 GJ}{E\Gamma}$

$E\Gamma$ is the warping rigidity

and α is a lateral buckling coefficient which varies with the type of loading and support conditions and which has the value of unity for the case of a simply supported beam loaded by equal end moments.

Simple expressions relating α to R^2 for a large number of different types of loading and conditions of lateral support are available (3, 4). Because it affects the values of two quantities in equation (1), α and γ, the significance of the ratio R^2, the magnitude of which depends not only upon the beam's span but also upon the ratio of its torsional and warping rigidities, is considerable. For cold formed sections twisting is usually resisted principally by warping in which case R^2 adopts a low value, whilst for hot rolled sections the contribution of St. Venant torsion is much greater and R^2 will usually adopt much larger values.

Equation (1) shows that the lateral stability of any beam is influenced by several factors. These are : its material properties, its geometry including the span, the type of loading to which it is subjected and the conditions of lateral support provided. It is the last of these factors which offers the greatest possibility for increasing a beam's stability against lateral buckling, since additional restraints may be provided either in the form of partial or complete end fixity or in the form of bracing located within the span. By the use of sufficient restraints the possibility of failure by lateral buckling can be precluded and the design based on strength considerations alone.

Recently this subject has been extensively studied at Cardiff and this paper presents a summary of this research and, in particular, presents test results which show that corrugated sheeting is particularly effective in stabilizing cold formed members.

2. HISTORICAL REVIEW

The earliest study of lateral buckling to include any reference to the possible increases in stability resulting from the use of restraints located within the beam's span was that by Winter (5) in 1943, who considered the buckling of a member rigidly supported against lateral deflexion along the length of the tension flange. Later work by Schmidt (6) has shown that Winter's expression for the critical load for such a beam may be considerably in error on the unsafe side

* Professor and Head of Department of Civil and Structural Engineering, University College, Cardiff, Wales.

** I.C.I. Fellow, Department of Civil and Structural Engineering, University College, Cardiff, Wales.

due to the representation of the buckled shape by a single half sine wave.

Discrete restraints were first studied by Flint (7) in 1951, who presented simple relationships between the increase in stability and the restraint stiffness for a central restraint possessing either lateral or torsional stiffness. This work is of limited application in the field of cold formed structures, however, since the effects of warping were not included. Experiments conducted by Sergev and Moore (8) on model rectangular section beams provided with an elastic lateral support have also indicated that comparatively flexible bracing is capable of producing significant increases in stability. This work also demonstrated how the influence of a restraint varied with the level of its attachment to the beam. Taylor and Ojalvo (9) extended Flint's work on torsional restraint to include the effect of warping, their results showing that the restraint stiffness necessary to induce second mode buckling increased with decreasing values of R^2. Hartmann (10) reached a similar conclusion for the case of a centrally loaded beam provided with a central elastic lateral restraint. The same author has also given some consideration to the problem of providing adequate lateral bracing at the interior supports of continuous beams (10) and to the knees of portal frames (11) and has verified his findings experimentally (12). More recently, Butler (13) has experimentally examined the influence of lateral support to tapered cantilever beams.

When a sufficient number of torsional restraints are provided, the solutions of Taylor and Ojalvo (9) for a continuous restraint may be used to estimate the beam's increased stability. Dooley (14, 15) has also shown that the assumption of a continuous restraint is permissible in certain cases involving the use of discrete restraints.

For columns, Winter (16) has shown how the bracing stiffness necessary to provide complete support i.e. to form a node at the restrained cross section, for an imperfect member is a function of both the imperfection and the bracing stiffness required to provide complete support to a similar perfect member. His tests on cold formed columns braced only by cardboard strips showed that an increase in bracing stiffness is accompanied by a decrease in required bracing strength. An extension of this work to consider some aspects of beam buckling has been made by Zuk (17), who again demonstrated the benefits obtainable by the use of relatively weak bracing.

A considerable amount of work has been performed at Cornell University on the buckling of either columns or beams braced together by shear rigid diaphragms. Earlier tests (18 - 20) had shown that diaphragms consisting of corrugated or troughed sheeting derived approximately 80 per cent of their resistance to in-plane deformation from shearing action. Using this concept of shear rigidity, a process has been advanced (21 - 24) for determining the increased load carrying capacity of columns or beams braced in this manner, the shear rigidity for a given diaphragm being determined experimentally.

3. THE INFLUENCE OF DISCRETE RESTRAINTS

Previous investigations (7, 9) of the influence of torsional restraints on the stability of beams have assumed that on buckling, the beam deforms with a uniform twist. This implies that the method of attachment of the torsional restraint is such that no cross sectional distortion can occur. Using the Finite Element method (25, 26), the writers have obtained results for the buckling of beams provided with torsional restraints attached only at discrete points on the beam's cross section and a set of these is shown in Fig. 1.

These results, which relate to a simply supported beam possessing a value of R^2 of 32 loaded by equal end moments, show that incomplete attachment of a torsional restraint decreases its effectiveness. For the particular case considered in Fig. 1, these reductions are of the order of 35 per cent for values of bracing stiffness $\beta (= \frac{K_T L}{GJ})$ in the region of 80. Additional results, which are available (27) for other values of R^2, show that although the magnitude of the effect decreases with an increase in R^2 it remains significant even for rectangular sections when R^2 is equal to infinity.

Tests performed by the writers (28) on cold formed channel sections provided with either one or two discrete torsional restraints have shown that the provision of a vertical stiffener at the restrained cross section so effectively diffuses the restraint's effect that complete attachment may be assumed.

In practice the type of discrete bracing member which is employed will frequently be such that it supplies both a lateral and a torsional restraint, e.g. a purlin member would supply lateral support due to its axial stiffness and torsional restraint due to its bending stiffness.

Fig. 2 shows how the value of the buckling parameter α varies with the lateral stiffness parameter λ for different values of the torsional stiffness parameter β. It will be noted that a restraint possessing both torsional stiffness and lateral stiffness is much more effective than a restraint possessing only lateral stiffness. For the example shown, increasing the value of the torsional restraint β from 2.72 to 6.8 produces a decrease of several hundred per cent in the lateral restraint stiffness necessary to provide a complete support. The results given in Fig. 2 were obtained assuming the influence of both restraints to be concentrated at the beam's shear centre. These α, λ relationships, therefore, represent a lower bound to the benefits which can be obtained by utilizing the torsional stiffness of a bracing member, see Fig. 1. In addition it should be noted that Fig. 2 relates to a value of R^2 of 12, which, as stated earlier, implies a beam for which warping effects are important. Although an increase in the value of R^2 tends to lessen the magnitude of this effect (27), in the case of cold formed sections, whose R^2 values are always low, it is an important factor. Theoretical relationships between the three parameters α, λ and β have also been obtained (27) for the buckling of beams

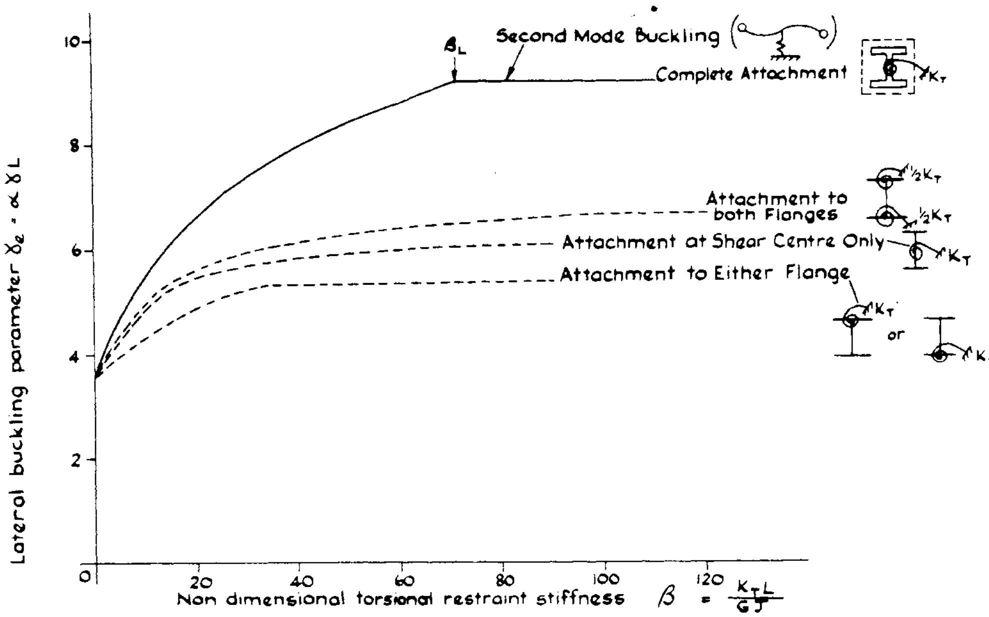

FIG. 1 INCOMPLETE ATTACHMENT OF A TORSIONAL RESTRAINT FOR A BEAM FOR WHICH $R^2 = 32$

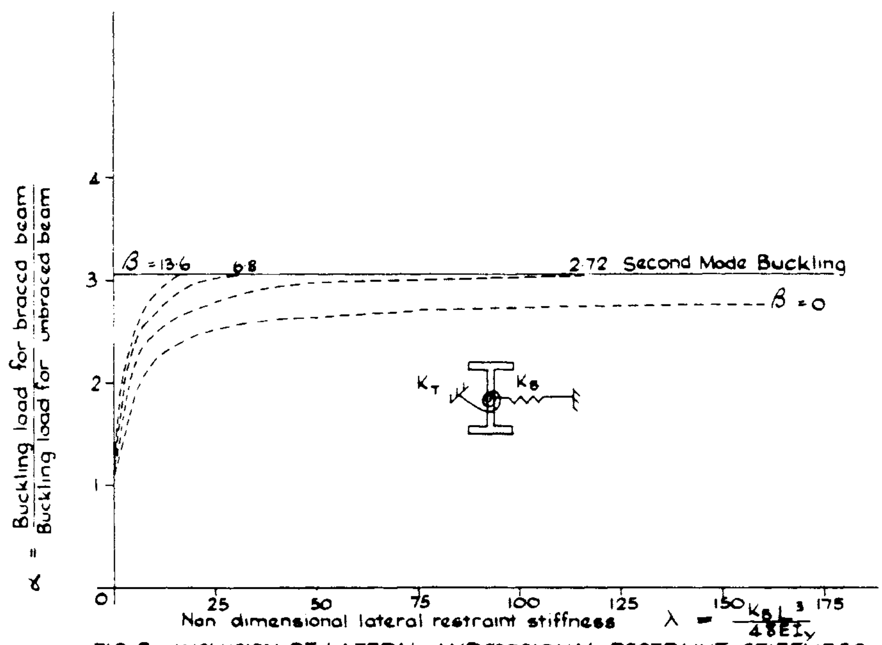

FIG 2 INCLUSION OF LATERAL AND TORSIONAL RESTRAINT STIFFNESS FOR A BEAM FOR WHICH $R^2 = 12$

provided with two equally spaced combined restraints. From these results it has been possible to derive relationships between the values of λ_L and β_L which will just ensure complete support. However, it must be borne in mind that these results relate to perfect beams and the work of Winter (16) on columns has shown that for a given increase in stability an initially bowed member requires a stiffer restraint than a similar perfectly straight member. Fig. 3 shows the results of three tests (28) on cold formed channel sections provided with a central, combined restraint having the same values of λ and β in each test. The initial bow of the top flange of each beam was measured prior to testing and the values expressed as a fraction of the span length L are given in Fig. 3. It will be seen that the stability of each of the beams was reduced by the presence of an imperfection, the amount of the reduction in buckling load increasing with the magnitude of the initial imperfections.

4. THE INFLUENCE OF CONTINUOUS RESTRAINTS

In the case of a continuous torsional restraint, theoretical results have been obtained which show that incomplete attachment again results in significant decreases in the restraint's effectiveness. Fig. 4 shows this effect for a beam of narrow rectangular cross section in which case $R^2 = \infty$. It is considered that the attachment effect may be of greater importance for continuous restraints than for discrete restraints since for the latter the horizontal cut-off in the γ_e, β relationship, which occurs when a complete support is provided, means that reductions due to incomplete attachment can never exceed about 35 per cent in the case of shear centre attachment. However,

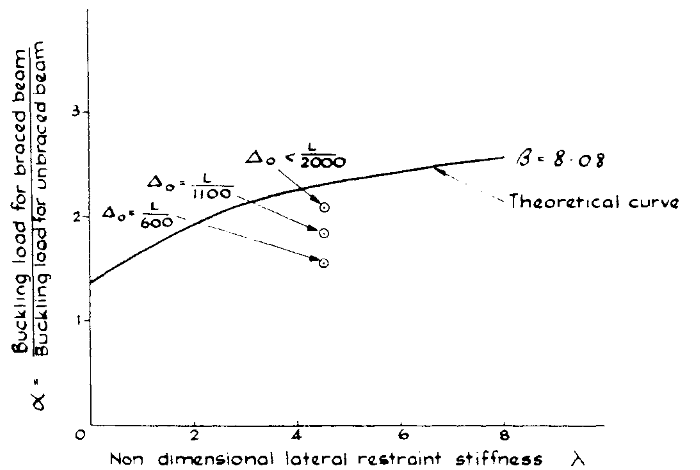

FIG. 3 EFFECT OF INITIAL IMPERFECTIONS ON EFFECTIVENESS OF A COMBINED RESTRAINT

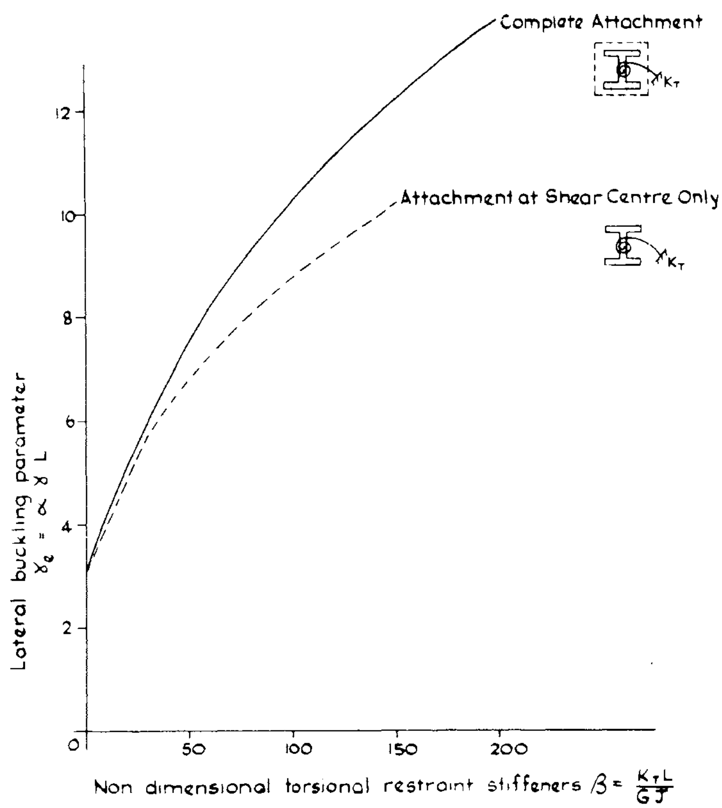

FIG. 4 INCOMPLETE ATTACHMENT OF A CONTINUOUS TORSIONAL RESTRAINT FOR A BEAM FOR WHICH $R^2 = \infty$

as will be noted from Fig. 4, for a continuous support, at high values of β the discrepancy between the two γ_e, β curves is far larger. A limit is placed upon the range of applicability of the present work, however, by the condition that buckling is elastic and for beams provided with very stiff continuous restraints the ultimate load carrying capacity will be governed by inelastic failure.

When a continuous combined restraint is provided, the relationship between α, λ and β is of the form shown in Fig. 5, which relates to a value of R^2 of 100. Since the relationship between α and λ is such that an increase in λ always produces an increase in α, the possession of any torsional stiffness by the lateral restraining medium serves to increase the value of α corresponding to a given value of λ. Thus a family of curves each corresponding to a different value of β is obtained. By taking into account the combined action of restraints it is therefore possible to achieve significant economies in the amount of bracing used.

5. EXPERIMENTAL INVESTIGATION

Two series of tests will be described in this section : the first on beams braced by cross members having the shape of a single trough of a corrugated sheet and the second on beams braced either continuously or at intervals by panels of corrugated sheeting. The aim of the first series was to show that the torsional restraint provided by a corrugated section was significant, whilst that of the second test series was to investigate the effects of the extent and method of attaching the sheeting to the beam on its ability to act as a stabilising medium.

Experimental Apparatus

Fig. 6 gives a general view of the test apparatus used to conduct the first series of tests. The test beam, which was a 5.937 in. by 2.75 in. by 0.100 in. cold formed channel section, was simply supported with respect to both bending and twisting over a 15 ft. span and was loaded by means of four equally spaced hydraulic jacks, the load being applied to the test beam via specially designed yokes. These loading yokes acted through a ball race mounted on the side of the channel so that the load was applied through the shear centre, see Fig. 7. Thus the loading corresponded closely to that of a uniformly distributed load acting through the beam's shear centre at the level of its top flange. This arrangement, a fuller description of which is provided in reference (28), was designed to ensure that the loading provided little or no resistance to lateral movement of the beam and as was observed during the tests on unbraced beams, functioned satisfactorily.

Series I Tests

The cross beams consisted of top hat section members supported at each end on roller bearings which gave complete freedom to horizontal lateral movement, and provided a rigid support vertically, see Fig. 8. This meant that these members, if the slight friction acting on the roller bearings is ignored, acted principally as torsional restraints, and by varying their cross section and/or their length their bending stiffness and therefore the amount of torsional restraint they supplied could be adjusted. Seventeen tests were performed, ten on centrally restrained beams and seven on beams supported at the third points. The results obtained have been given in Tables 1 and 2.

In all tests collapse occurred by buckling of the main beam towards the shear centre. Buckling always occurred in a single half wave and was usually a fairly gradual process, the load versus lateral deflexion curve exhibiting a well rounded knee.

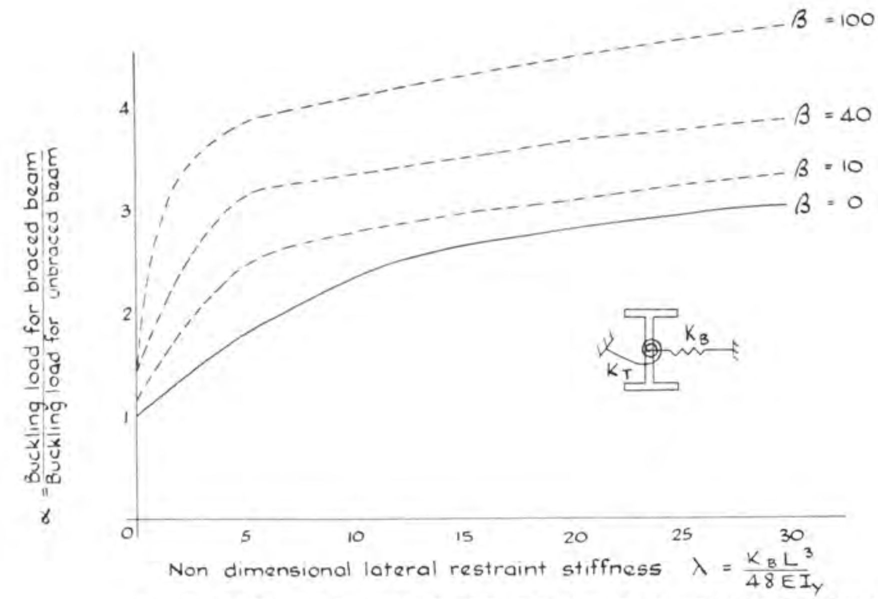

FIG. 5 EFFECT OF A COMBINED CONTINUOUS RESTRAINT ON A BEAM FOR WHICH $R^2 = 100$

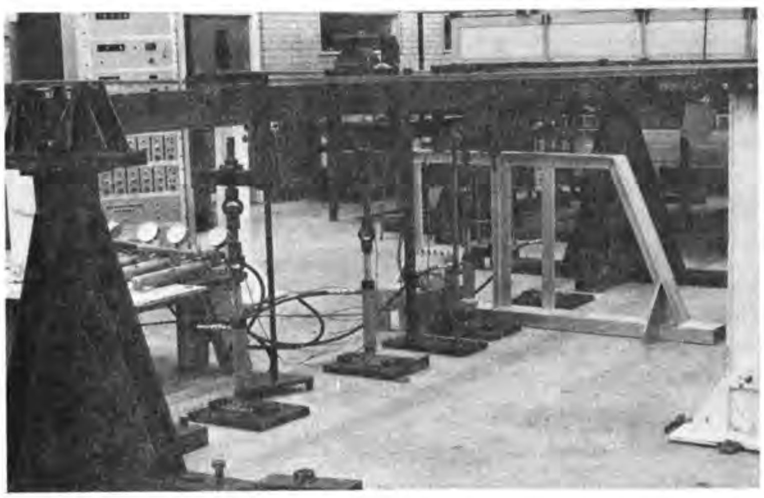

Fig. 6. General View of Apparatus.

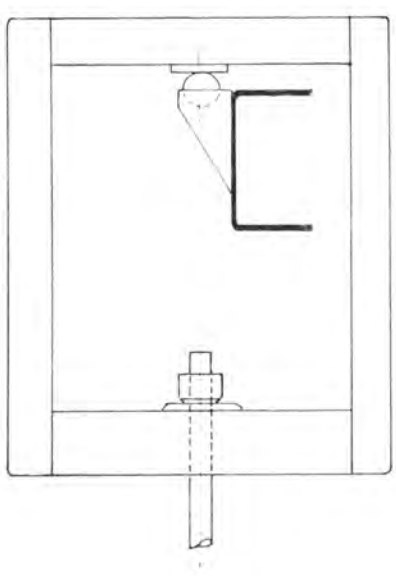

FIG. 7 LOADING YOKE

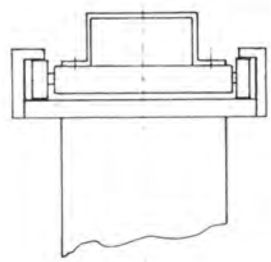

FIG. 8 END SUPPORT FOR CROSS BEAMS

Fig. 9 compares the experimental results obtained from the tests on beams provided with a single restraint with three theoretical finite element solutions. It may be seen that, providing due allowance is made for the width of the attachment of the restraint to the beam's top flange, the theory is in good agreement with experiment. The assumption that the restraint is concentrated at a point on the flange, corresponding to the centre line of the attachment, underestimates its effectiveness, whilst any assumption of complete attachment over the depth of the main beam is clearly unsafe. The importance of correctly allowing for the method of attachment of the restraint is also indicated in Fig. 10, which gives the results of the tests on beams provided with two restraints applied at the one third position. In this case the attachment effect is greater, but by making due allowance for it in the theory, fairly accurate predictions of the increase in stability are again possible. It is considered that in both cases the increase in the values of α above that given by the theory is due to the presence of some frictional restraint in the bearings which would amount to the provision of a slight lateral restraint. Moreover, the friction at the junction of the cross beam and the test beam would tend

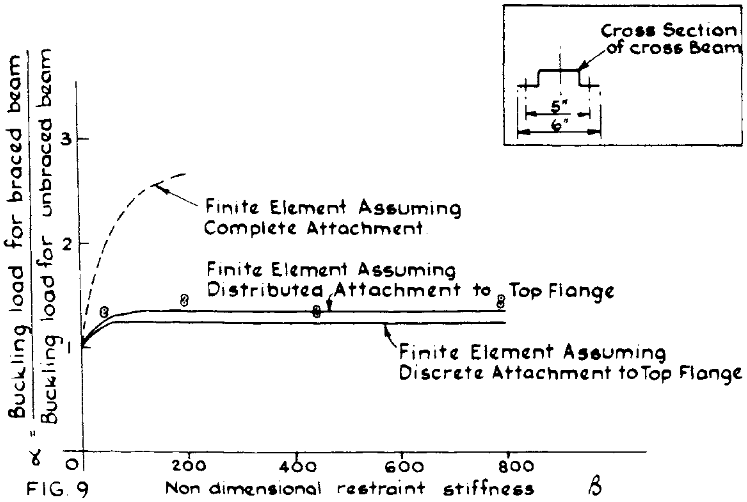

FIG. 9 COMPARISON OF TEST RESULTS FOR BEAMS PROVIDED WITH A SINGLE CROSS-BEAM WITH THEORY.

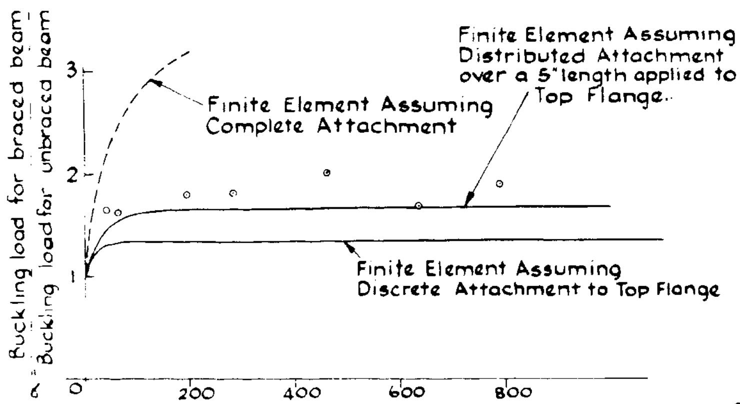

FIG 10 COMPARISON OF TEST RESULTS FOR BEAMS PROVIDED WITH TWO CROSS-BEAMS WITH THEORY.

to increase the width over which the restraint was applied. In the theory, the restraint was assumed to act over the 5" width between the bolt centres, whereas the restraint has an overall width of 6".

Series II Tests

For these tests panels of corrugated sheeting with the corrugation profile shown in Fig. 11 were employed in various configurations as bracing against lateral instability. The ends of the panels were simply supported in the vertical plane using the roller arrangement illustrated in Figs. 12 and 13. This arrangement allowed the sheeting complete freedom of horizontal lateral movement within the tubular guides. Thus the method of support, which could be carefully controlled in the laboratory, was such that the sheeting provided a torsional restraint and is less restrictive than that normally encountered in practice. Therefore its stabilising influence represents a lower bound on that that might be expected in an actual structure. Details of the results of the series of tests are given in Table 3.

Ten tests were performed in which the sheeting was used as discrete panels. Tests were conducted using either one, three or five panels, each panel being located in the sections of the test beam between loading points, see Fig. 12. The sheets were attached directly to the beam's top flange using self tapping screws and tests were carried out for the cases of fasteners employed in every second or every fourth valley.

The results shown in Table 3 indicate that, whilst the provision of a greater area of sheeting did increase the beam's stability, the use of only a single panel was capable of providing an increase in load carrying capacity of over 250%. It will also be noted that the alteration in the fastening arrangement did not significantly alter the restraint's effectiveness.

In almost every test local buckling of the beam's compression flange was observed beneath the two central loading yokes at loads of approximately two thirds of the collapse load. However, this was not sufficient to cause failure in itself although at collapse this region of the beam was badly deformed.

Because of this, three tests were conducted in which stiffeners were bolted between the beam's flanges at each load point. In one of these tests the central, most heavily stressed region of the beam was left unbraced, only two sheeting panels being employed, see Fig. 12. Although local buckling of the compression flange was observed in this region at approximately the same load i.e. at about twice the unbraced lateral buckling load, collapse did not occur until a load of more than three times the unbraced buckling load was reached. In the case of the test on the beam provided with three sheeting panels, this early local buckling occurred between the fasteners in the central panel and final failure, which occurred at a load at least 20% greater than that for the corresponding test without stiffeners (CS 2), was precipitated by the growth of large buckles in the regions between the stiffeners at the loading points and the ends of the central panel.

Five tests have also been conducted in which the sheeting was continuous over the whole length of the beam. In all of these tests stiffeners were provided at the loading points, the sheeting being cut to accommodate the loading yokes. In three of the tests fastening was employed in every second valley and seam fasteners were employed at the transverse joints between adjacent sheets.

Fig. 14 gives plots of load versus lateral and vertical deflexion for one of these tests. Throughout the test virtually no lateral deflexion was observed, the beam behaving linearly in the vertical plane up to the load at which local buckling took place. Failure was not accompanied by any significant lateral deformation and occurred when a buckle developed in the web adjacent to one of the more well developed flange buckles. Similar behaviour was observed in each of these three tests.

Using the effective width formula given by Winter (29) for plates supported along one edge, in conjunction with the experimentally determined (28) value of the critical local buckling stress for the compression flange, the moment at which yield is reached in the compression flange has been calculated. This moment corresponds to an applied load of

TABLE 1. Results of Tests on Beams Provided with a Central Cross-Beam (see Fig. 9)

Test No.	Depth of Cross Beam ins.	Length of Cross Beam ins.	Value of Torsional Restraint Parameter β	Critical load W_{cr} lbs.	Increased Stability $\alpha = \frac{W_{cr}}{W_o}$ *
CB1	1.0	11.5	197.0	1519	1.49
CB2	0.5	11.5	42.6	1449	1.36
CB3	1.5	11.5	443.0	1421	1.34
CB4	2.0	11.5	788.0	1512	1.42
CB5	2.0	11.5	788.0	1564	1.48
CB6	1.5	11.5	443.0	1383	1.30
CB7	1.0	11.5	197.0	1563	1.47
CB8	0.5	11.5	42.6	1424	1.34
CB9	1.5	11.5	443.0	1472	1.39
CB10	2.0	11.5	788.0	1504	1.42

TABLE 2. Results of Tests on Beams Provided with Two Cross-Beams (see Fig. 10).

Test No	Depth of Cross Beam ins.	Length of Cross Beam ins.	Value of Torsional Restraint Parameter β	Critical load W_{cr} lbs.	Increased Stability $\alpha = \frac{W_{cr}}{W_o}$ *
CB11	0.5	11.5	42.6	1760	1.65
CB12	1.0	11.5	197.0	1913	1.80
CB13	1.5	11.5	443.0	2143	2.01
CB14	2.0	11.5	788.0	2013	1.90
CB15	1.0	7.5	284.0	1930	1.81
CB16	0.5	7.5	61.3	1724	1.62
CB17	1.5	7.5	638.0	1780	1.68

* W_o is the experimental critical load for the unbraced beam (28), W_o = 1060lbs.

TABLE 3. Test for Beams Braced by Sheeting.

Test No.	Sheeting Arrangement	Fastening Arrangement	Seam Fasteners	Stiffeners at Loading Points	Ultimate Load W_{cr} lbs	Increase in Stability $\alpha = \frac{W_{cr}}{W_o}$
CS1	1 panel	Every second valley	-	NO	2863	2.69
CS2	3 panels	Every second valley	-	NO	3130	2.95
CS3	5 panels	Every second valley	-	NO	3505	3.29
CS4	5 panels	Every second valley	-	NO	3422	3.21
CS5	1 panel	Every Fourth valley	-	NO	3014	2.83
CS6	3 panels	Every Fourth valley	-	NO	3134	2.94
CS7	5 panels	Every Fourth valley	-	NO	3247	3.06
CS8	2 panels	Every second valley	-	YES	2913	2.74
CS9	2 panels	Every second valley	-	YES	3307	3.10
CS10	3 panels	Every second valley	-	YES	3861	3.62
CS11	Continuous	Every second valley	YES	YES	4168	3.92
CS12	Continuous	Every second valley	YES	YES	4018	3.79
CS13	Continuous	Every second valley	YES	YES	4257	4.01
CS14	Continuous	Every second ridge	NO	YES	3648	3.44
CS15	Continuous	Every fourth ridge	NO	YES	3371	3.17

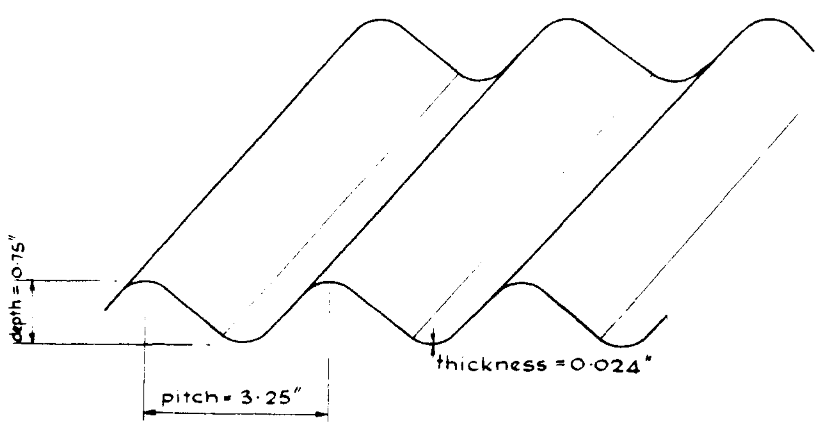

FIG 11 PROFILE OF CORRUGATED SHEETING

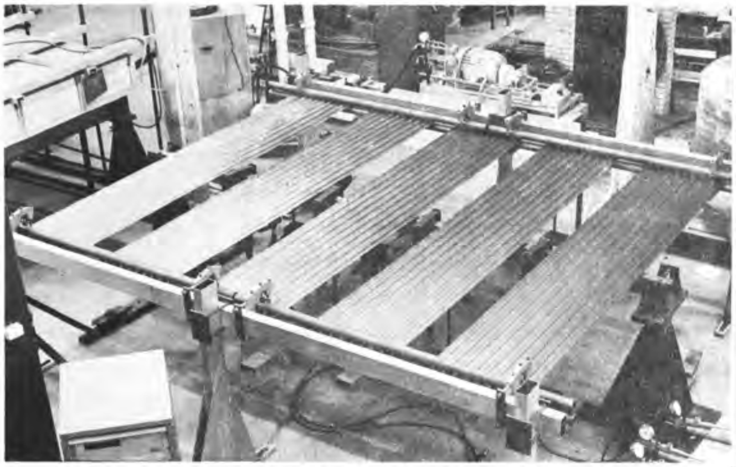

Fig. 12. General View of Apparatus with Sheeting in Place.

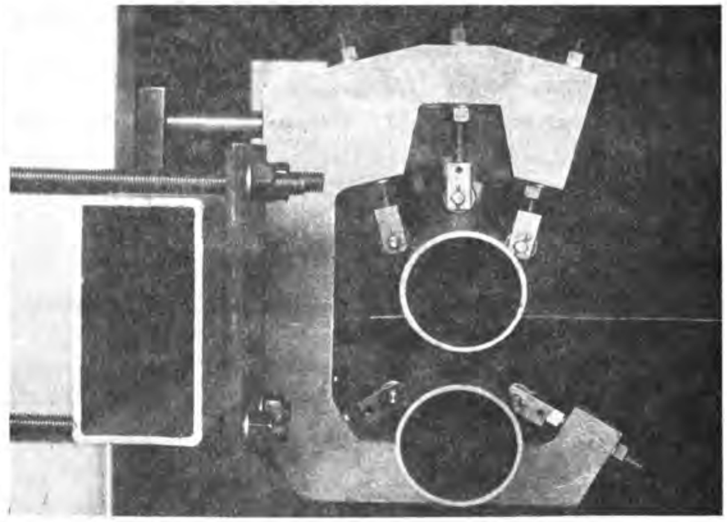

Fig. 13. Details of Edge Supports for Sheeting.

3975 lb. which, bearing in mind the fact that the sheeting acts as a flexible cover plate thus providing some slight additional stiffening corresponds quite closely to the failure load for each of these three tests. Thus with the sheeting employed the full plastic moment of the section is virtually achieved.

Employing the beam to sheet fasteners on the peaks of the corrugated sheet and eliminating the seam fasteners decreased the sheeting's effectiveness as shown by the results of tests CS14 and CS15. This was due to two factors : the sheeting was now not so effective in preventing the growth of local buckles in the compression flange and together with the bending of the exposed shank of the fastener permitted more lateral deformation of the beam. These two effects may be observed in the load-deflexion curves given in Fig. 15, the first on the curve of vertical deflexion which exhibits a faster growth of deflexion after the departure from linearity than does Fig. 14, and the second on the lateral deflexion curve where significant deflexions may be observed as the collapse load is approached.

Although this test program is not yet completed it is possible to make certain general observations regarding the results obtained so far :

1. The additional stability provided to laterally weak beams by corrugated sheeting used as cladding is considerable. For the tests reported, the use of only one 3 ft. panel at the centre of a 15 ft. span was capable of increasing the beam's load carrying capacity, by more than 250%.
2. When such sheeting is continuous, the use of seam fasteners between adjacent panels increase its effectiveness as bracing.
3. Fastening the sheeting to the beam in the valleys of the corrugations is preferable to fastening on the ridges since, with the latter method, some bending of the fastener is possible. Fastening in the valleys is also superior in inhibiting the

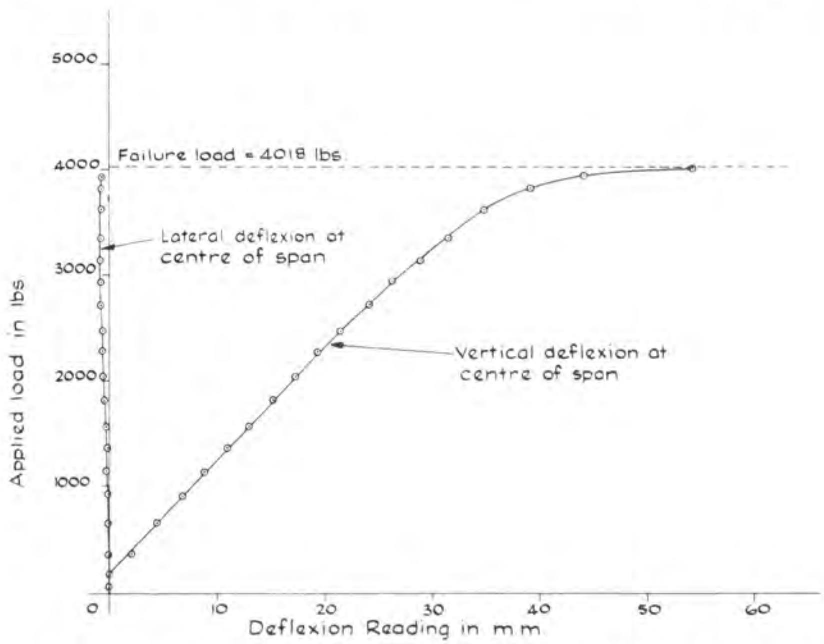

FIG. 14 LOAD DEFLEXION CURVES FROM TEST ON BEAM PROVIDED WITH CONTINUOUS SHEETING RESTRAINT - FASTENERS IN VALLEYS

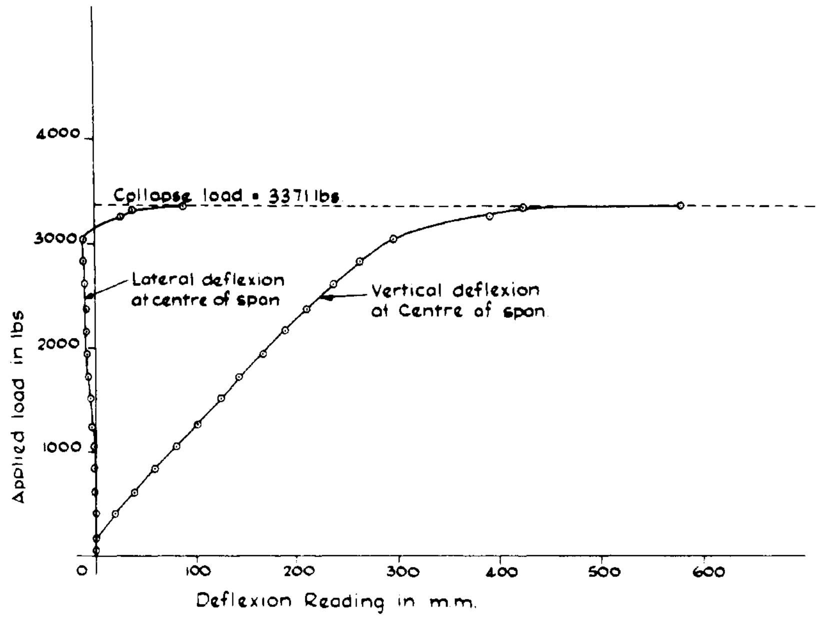

FIG 15 LOAD DEFLEXION CURVES FOR BEAM CONTINUOUSLY RESTRAINED BY SHEETING – FASTENERS ON RIDGES AND NO SEAM FASTENERS.

growth of local buckles in the compression flange.

4. By employing a continuous restraint it was possible to achieve the maximum plastic moment for the section after due allowance for local buckling had been made.

6. CONCLUSIONS

The paper has dealt with the influence of various types of 'elastic' restraints on the lateral buckling of beams, both discrete and continuous bracing systems having been considered. For torsional restraints the method of attachment has been shown to be of importance, whilst the benefits of allowing for both lateral and torsional restraining actions has also been discussed. The importance of the imperfections present in real members in decreasing the effectiveness of lateral bracing has also been indicated.

Tests on cold formed channel sections braced by slender cross beams have been reported and the results correlated with theory. Further tests on beams braced by sheeting have shown that the continuous restraint provided by the sheeting can considerably increase the beam's load carrying capacity. The tests have also shown that when such sheeting is continuous, it is essential to ensure that adequate fastening both to the beam and to adjacent sheeting is provided. If this is done then lateral buckling is prevented and a beam can develop its full bending strength.

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APPENDIX 1

NOTATION

E	Young's modulus
G	Shear modulus
I_y	Minor second moment of area
J	Torsion constant
K_B	Stiffness of lateral restraint
K_T	Stiffness of torsional restraint
L	Overall span
M_{cr}	Maximum moment in beam
$R = \sqrt{\frac{L^2 GJ}{E\Gamma}}$	Torsional parameter
α	Lateral buckling coefficient
$\beta = \frac{K_T L}{GJ}$	Non dimensional torsional restraint stiffness parameter
Γ	Warping constant
$\gamma = \frac{\pi}{L}(1 + \frac{\pi^2}{R^2})^{\frac{1}{2}}$	Slenderness parameter
$\gamma_e = \alpha\gamma L$	Lateral buckling parameter for restrained beam
Δ_o	Initial bow of beam's top flange
$\lambda = \frac{K_B L^3}{48EI_y}$	Non dimensional lateral restraint stiffness parameter

APPENDIX 2

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