New Design Method for Cold-formed Purlins

Frans Soetens

A. W. Toma

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by: ir. F. Soetens and ir. A.W. Tomà
Steel Structure Department of the Institute TNO for Building Materials and Structures, Delft, The Netherlands.

1. INTRODUCTION

In Europe, a growing tendency exists for the application of cold-formed sections (e.g. Z-, Σ- or C-sections) as purlins in combination with trapezoidal sheeting. For such systems, the following items are characteristic:

- the load can be due to gravity (dead and snow load) or uplift (wind-suction);
- the non-symmetrical cross section of these purlins implies a sensitivity to torsional behaviour;
- the sheeting connected to the purlin gives a restraint against this torsion.

Figure 1 shows the deflections of C- and Z-purlins under uplift loading. It may be clear that a design procedure should be based on the composite action between sheeting and purlin.
References [1] (which is based on references [3] and [4]) and [2] give such design procedures. Table 1 gives a comparison of both procedures with results from tests on cold-formed Z-sections of SAB Profiel BV (Dutch manufacturer).
This paper describes a more optimal procedure, using a calculation model based on a research programme executed at TNO-IBBC.
A litarature study has been carried out in the beginning of the programme (ref. [5]). A number of references have been listed, of which [2], [3] and [4] seem to be the most important. Based on this study, a design hypothesis has been derived. To check the hypothesis, a testing programme has been executed (ref. [6]). The comparison of test results and calculations has been given in references [7] and [8]. Finally, in reference [9], a survey of the whole programme has been given, including recommendations for a design procedure. This paper will focus on the hypothesis and the comparison with test results. The hypothesis has been based largely on the work by Peköz and Soroushian (lit. [2]).
The main additions are:
- for uplift loading, the load transfer between sheeting and purlin via the fasteners will be taken into account;
- second order effects caused by compressive stresses in the free flange have been taken into account in the buckling curves and not introduced through initial imperfection.

2. DESIGN HYPOTHESIS FOR DIAPHRAGM BRACED BEAMS

2.1. Single span beams

Figure 2 shows schematically the hypothesis for the calculations on a diaphragm braced beam under uplift loading. For gravity loading, a comparable scheme can be made. According to the scheme, the stresses in the section are a combination of:

- stresses from in-plane bending of the entire section due to the load q. These generate an axial force N (x) in the free flange of the section (see fig. 2b and 2c). This axial force varies along the length of the member due to the in-plane bending moment M (x); with uplift, N (x) is a compressive force and with gravity, N (x) is a tensile force.
- stresses from lateral bending of a part of the section due to the lateral load $k_h q$. The value of k_h is shown in figure 3.

For determining the in-plane bending stresses the effective widths of compressed parts of the section are applied to account for local buckling effects. The stresses caused by the lateral load of the free flange will be determined without reducing the width of the free flange.

With diaphragm braced beams, the rotation of the beam is restrained by:
- the section properties of the diaphragm,
- the section properties of the beam,
- the connection between diaphragm and beam.

Usually this rotational restraint is converted into a lateral restraint as indicated in fig. 2a (taken from [2]), being a linear extensional spring with stiffness K located at the level of the free flange. This means that the part of the section due to lateral bending (and with uplift loading also a compressive load, as explained later) can be calculated as a beam on an elastic foundation (see figure 2b and 2c). Reference [9] gives the procedure to determine K.
With the energy method, the combination of stresses will be applied. In the energy equation, the following are taken into account:
- energy due to lateral load
- energy due to axial force
- flexural strain energy of the free flange
- elastic foundation strain energy (caused by the rotational restraint of the sheeting)

This leads to equations for actual stresses depending on the edge conditions as given in appendix II. This means that imperfections (e.g. initial deflections, residual stresses) have been neglected.

As criteria for the ultimate limit state, the actual stresses will be smaller than the yield stress or the ultimate stress for flexural/torsional buckling of the free flange when it is under compression. The ultimate stress for flexural/torsional buckling will be determined in a model based on a beam-column behaviour of a part of the section.
The load-bearing capacity of the beam-column is checked as follows:

$$\omega\,\sigma_c + \frac{M_{(y)}}{W_{fy}} \le f_{ty}$$

Where:

ω = buckling coefficient
σ_c = compressive stress due to in-plane bending of entire (effective) section
$M_{(y)}$ = lateral bending moment acting in the free flange plus $\frac{1}{6}$th of the height of the web (see appendix II)
W_{fy} = section modulus based on moment of inertia (I_{fy}) of a part of the section
f_{ty} = yield stress

The buckling coefficient ω depends on the slenderness $\bar{\lambda}$, for which following have to be taken into account:
- a variable axial force along the length of the bar,
- an elastic foundation, and
- appropriate end conditions.

In appendix III the resulting stability check equations are formulated.

It may be noted that in the model to check the beam-column capacity the influence of the energy due to the axial force has been taken into account twice (in ω and in $M_{(y)}$). For reasons of uniformity with the column philosophy, an "ω" formulation is choosen (e.g. same initial deflections and residual stresses) because in $M_{(y)}$ the imperfections have not been taken into account.
Furthermore, the value of $\omega\sigma_c$ in the equation is always higher than the value of the second term (lateral bending contribution).

2.2. Multiple span beams

a. For gravity loaded systems the ultimate limit state has been defined by appearance of a failure mechanism. Ultimate moment capacity in the mid-span should be determined theoretically (in principle according to 2.1, although in practice this means yield stress multiplied by the section modulus of the effective cross section) or by testing (single span tests with a span comparable with the length of the positive moment area). For the behaviour over the support, detailed support tests are necessary. These tests should provide information about the rotation over the support after reaching the maximum moment. The governing mechanism of the system will be reached at ultimate moment capacity in the mid-span and compatibility of moment and rotation over the support. As serviceability limit state has been defined reaching maximum moment capacity over a support with a load factor of 1.1. and deflection at mid-span.

b. For uplift loaded systems the ultimate limit state has been defined by the smallest of following loads:
   - The load at which the maximum moment at the support is reached. Only local buckling should be taken into account according to reference [1]. The interaction of the support reaction with the moment may be neglected because it is introduced as a "tension" force.
   - The load at which the maximum moment in the span is reached according to 2.2.a.

   For the force distribution an elastic behaviour may be assumed.
   As serviceability limit state a deflection requirement at mid-span governs.

c. For overlap or sleeve systems the design procedure is as follows:
   - Detailed support tests should be executed. Only the increasing part of the load-deflection curve is of interest.
   - From the tests, the following can be derived:
     i. the stiffness of the overlapping or sleeved part
     ii. failure combination of bending moment + support reaction (in overlapped or sleeved part) or bending moment + shear force (besides overlapped or sleeved part).

- With item "i" the force distribution in the system can be determined (also where local buckling of the cross section in the span is taken into account).
- The force distribution shall be checked for:
  * failure combinations of bending moment + support reaction or bending moment + shear force (near support)
  * maximum moment capacity in span according to 2.2.a., neglecting the influence of overlap or sleeve
  * the allowable deflections

TEST PROGRAMME

In reference [6], the testing programme on C-, Z-, and Σ-sections has been described. The report also comprises the results of the tests and a comparison of these results.
The specimens have been built up as two parallel purlins with sheeting over it. The specimens are placed in a box and loaded by suction due to a vacuum.
The choice of test specimens has been determined in such a way that almost every test will be executed in two-fold. Between the different specimens only one parameter has been varied. The combinations of parameters which have been used are:
- single span and double span
- spans of about 4 m and 6 m
- shape of the section of the purlins Z, C and Σ
- section height of the purlins h = 140 mm and h = 240 mm
- section thickness of the purlins t = 1.5 mm at the height h = 140 mm and t = 2.0 mm at the height h = 240 mm
- two types of torsional restraint delivered to the purlins by sheeting (type A and B)
- type of loading; gravity and uplift (The test specimens were to be acted upon only by vertical uniformly distributed loading)

Table II gives a survey of the total of 28 test specimens. The failure loads in the test are summarized in table III.

4. COMPARISION OF CALCULATION PROCEDURE AND TEST RESULTS

The behaviour of the test specimens has been predicted by the calculation procedure given in paragraph 2. The comparison is given in table III as ratio q_{th}/q_{test}. With respect to these results it can be observed that:
- For single span beams all test results are very well approximated for gravity loading (ratios: 0.96 - 1.03).
- For double span beams the failure loads for gravity loading are higher than the theoretical results (without redistribution of moment).
  If yielding at mid-support, observed during the test, should be taken as failure, the moment capacity of the mid-support is overestimated by 13% in test 25 and 3% in test 27. This overestimation is due to the support reaction. However, in the tests redistribution of forces after yielding at mid-support occurs, which allows yielding/failure in the span, while theoretically failure is defined as yielding at mid-support. Taking into account the moment redistribution the results will be as shown in the figures 4 and 5, which shows a very good approximation.
- For single span beams and uplift loading, all test results are very well approximated (ratios: 0.87 - 1.00).

- For <u>double span</u> beams and <u>uplift loading</u>, theoretical failure occurs with yielding at mid-support, while during the tests failure occured simultaneously at mid-support and in the span. Therefore, in accordance with the above described situation with gravity loading, there could have been a redistribution of forces at mid-support, which explains the higher test results particulary with test 28. To allow for a small amount of conservatism ($q_{theor.}$ is smaller than $q_{failure}$ for the mid-support) and also because the edge conditions at mid-support for stability check in the span will change rigorously when mid-support fails the procedure as shown in 2.2.b. has been proposed.

5. SUMMARY

The research carried out in the project described in this paper concentrates on the behaviour and load carrying capacity of diaphragm braced beams using cold-formed sections.

A hypothesis for a calculation model, partly based on reference [2] has been developed. To check the resulting model, a number of tests has been executed:
- 24 tests on simply supported beams,
- 4 tests on double span beams and
- 4 detailed support tests.

On basis of these tests the calculation model has been improved. Finally this has led to a recommendation for a design procedure for diaphragm braced beams which is formulated in detail in reference [9].
Furthermore, it is recommended that some detailed support tests be used as one of the input parameters for designing purlin-systems. Only then can the ultimate load bearing capacity of the system be predicted. It is also sensible to do tests for the torsional restraint of the purlin delivered by the sheeting. The values for the torsional restraint given in the reference [9] are on the conservative side.

ACKNOWLEDGEMENTS

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- Stichting Koudprofileurs (Dutch foundation of cold rolling manufacturers);
- Staalbouwkundig Genootschap (Dutch foundation for coordination of steel research).

Table 1: Comparison of design procedures with testresults

	simply supported, single span beams			
	Z-section span length [m]	ultimate testload [kN]	load ratios: $\frac{\text{calculated}}{\text{test}}$ ECCS approach	USA approach
uplift loading	l = 4.0	20.3	0.59	0.70
	l = 5.0	16.5	0.57	0.86
	l = 6.0	13.3	0.67	1.57

<u>Table II</u>: Test programme, combinations of parameters

Test no.	system	appr. span [m]	shape	h [mm]	t [mm]	diaphragm	loading
1, 2	S	6	Z	140	1.5	A	G
3, 4	S	6	Z	140	1.5	A	U
5, 6	S	6	Z	140	1.5	B	G
7, 8	S	6	Z	140	1.5	B	U
9,10	S	6	Z	240	2.0	A	G
11,12	S	6	Z	240	2.0	A	U
13,14	S	4	Z	140	1.5	A	G
15,16	S	4	Z	140	1.5	A	U
17,18	S	6	Σ	140	1.5	A	G
19,20	S	6	Σ	140	1.5	A	U
21,22	S	6	C	140	1.5	A	G
23,24	S	6	C	140	1.5	A	U
25	D	4	Z	140	1.5	A	G
26	D	4	Z	140	1.5	A	U
27	D	6	Z	140	1.5	A	G
28	D	6	Z	140	1.5	A	U

For the system S means single span and D means double span.
For the loading G means gravity and U means uplift.

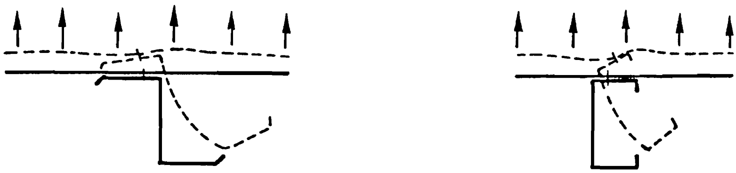
Table III: Checking of the proposed design procedure and the results of the test programme

Test no.	shape of cross-section	Span length [mm]	Failure load q_{test} per m purlin length [N/m']	Theoretical load q_{th} [N/m']	Ratio $\frac{q_{th}}{q_{test}}$	G = gravity U = uplift
1			1253	1197	0.96	G
2	Z-140	5890				
3			903	893	0.99	U
4						
5			1223	1197	0.98	G
6	Z-140	5890	1214	1196	0.98	G
7			902	903	1.00	U
8						
9			4000	3928	0.98	G
10	Z-240	5890				
11			2376	2230	0.94	U
12						
13			2169	2228	1.03	G
14	Z-140	4390	2218	2164	0.98	G
15			1590	1573	0.99	U
16						
17			1678	1673	1.00	G
18	Σ-150	5905				
19			1311	1140	0.87	U
20						
21			1263	1219	0.97	G
22	C-140	5890				
23			896	840	0.94	U
24						
25			2218*)	2501***)	1.13	G
26	Z-140	4195	2366	2309	0.98	U
27			1125**)	1156***)	1.03	G
28	Z-140	5945	1282	1155	0.90	U

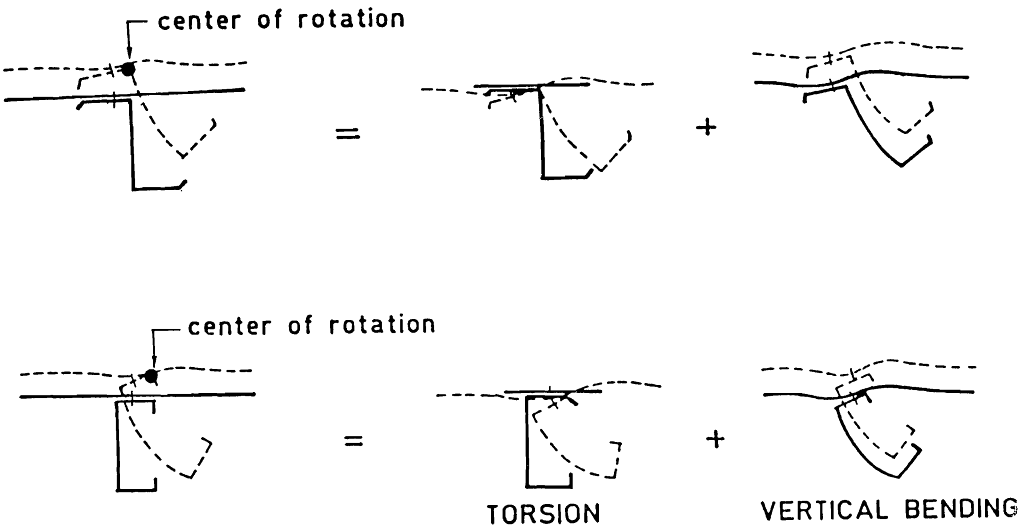
*) yielding midsupport: q_{test} = 2218 N/m', failure midsupport/span: q_{test} = 2661 N/m'

**) yielding midsupport: q_{test} = 1125 N/m'; failure midsupport/span: q_{test} = 1356 N/m'

***) theoretical failure at midsupport

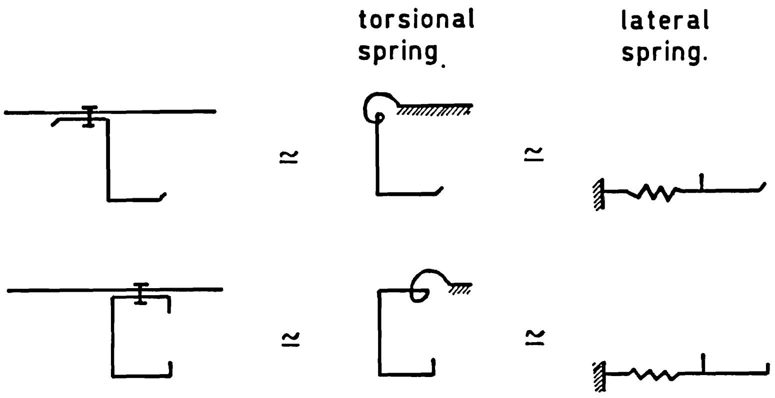


a. Total deflection

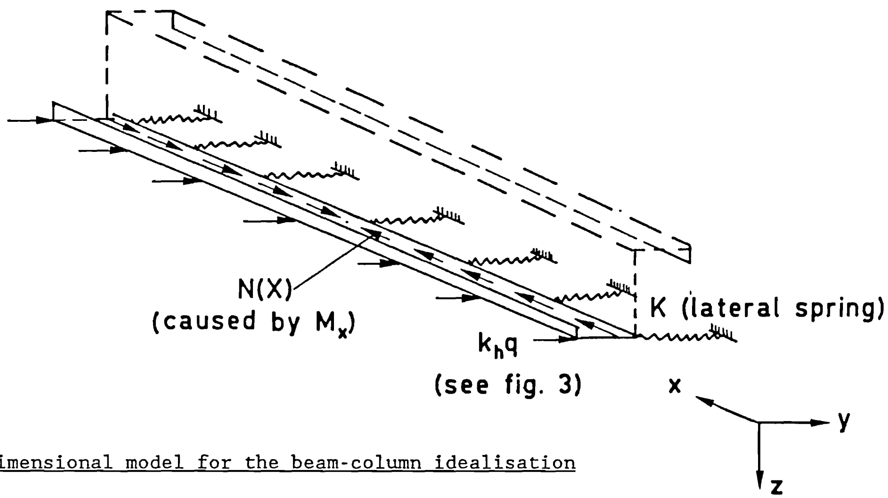


b. Components of total deflection with fixed center of rotation

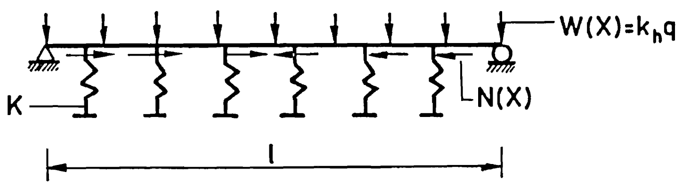
Figure 1: deflections of C- and Z-purlins under uplift loading
(figure taken from lit. [2])



a. Idealisation of rotational restraint



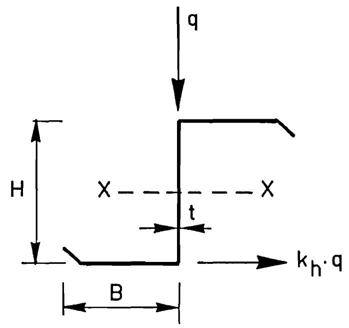
b. Three-dimensional model for the beam-column idealisation



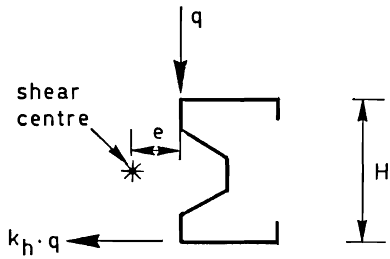
c. Translation to a two-dimensional model

Figure 2: Calculation model for a diaphragm braced beam under uplift loading (calculation model taken from lit. [2])

Gravity load



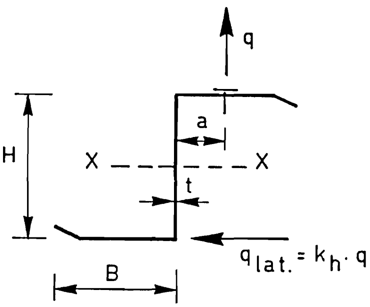
$$k_h = \frac{S_{fl} \cdot B}{2 I_x} = \frac{B^2 Ht}{4 I_x}$$



$$k_h = \frac{e}{H}$$

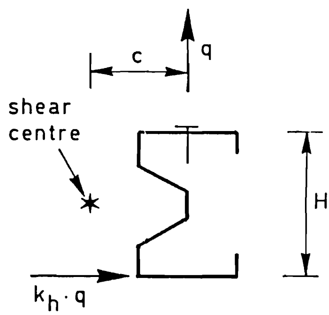
When shear centre at right side of q than lateral load is working in opposite direction

Uplift load



If $\frac{a}{H} < \frac{B^2 Ht}{4 I_x} \longrightarrow \overleftarrow{k_h} = \frac{B^2 Ht}{4 I_x} - \frac{a}{H}$

If $\frac{a}{H} > \frac{B^2 Ht}{4 I_x} \longrightarrow \overrightarrow{k_h} = \frac{a}{H} - \frac{B^2 Ht}{4 I_x}$



$$k_h = \frac{c}{H}$$

When shear centre at right side of fastener, than lateral load is working in opposite direction.

Figure 3: Model description bending + torsion converted into in-plane bending + lateral bending of a part of the section.

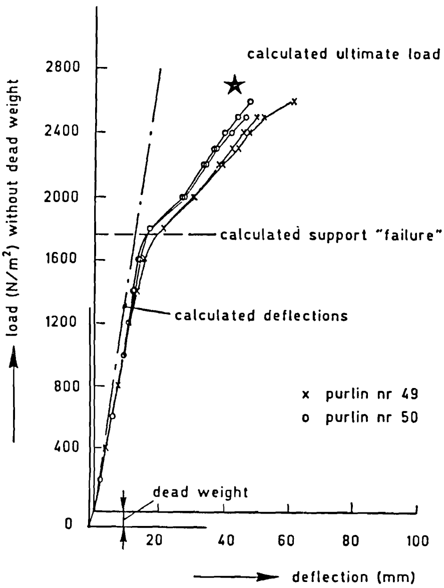


Figure 4: Test and calculation results of a double span beam, gravity load (testno 25 of ref. [6])

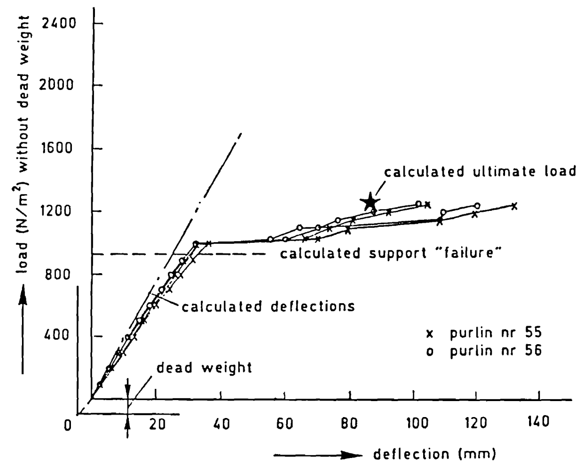


Figure 5: Test and calculation results of a double span beam, gravity load (testno 27 of ref. [6])

Appendix I: References

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Appendix II: Determination of actual stresses (based on lit. [2] and figure 3)

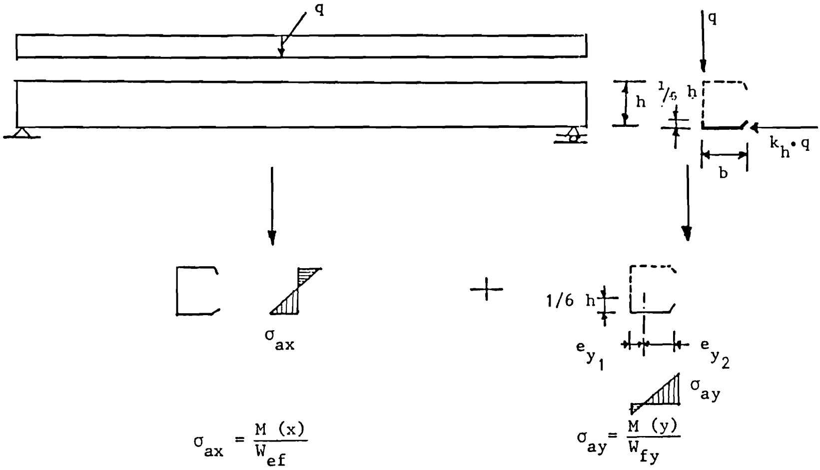
The actual stresses in a cross section in the field follow from:

$$\sigma_a = \frac{M(x)}{W_{ef}} \text{ (for braced flange)}$$

$$\sigma_a = \frac{M(x)}{W_{ef}} + \frac{M(y)}{W_{fy}} \text{ (for free flange)}$$

Herein:

- M (x) : bending moment at a place x in the field due to the component of the design load acting in web direction
- W_{ef} : section modulus for the effective cross section according to ref. [1]
- W_{fy} : section modulus of the free flange plus 1/6 of the height of the web against lateral bending (for gross section of free flange)



$$M(y) = \frac{EI_{fy}\,\pi^2}{\ell^2} A_1 \text{ (lateral bending moment)}$$

with:

- E = Young's modulus
- I_{fy} = moment of inertia (of gross section) of the free flange plus 1/6 of the height of the web against lateral bending
- ℓ = span of the purlin
- A_1 = constant depending on edge conditions of the purlin in lateral direction

At midspan and compression stress in the free flange, for A_1 may be taken:

- Simply supported beam:

$$A_1 = \frac{4k_h\, q\, \ell^4}{EI_{fy}\, \pi^5 + K\, \ell^4\, \pi - 1.8\, \frac{S_f}{I_{ef}}\, q\, \ell^4}$$

- Beams, both ends fixed:

$$A_1 = \frac{4k_h\, q\, \ell^4}{16\, EI_{fy}\, \pi^4 + 3\, K\, \ell^4 - 3.54\, \frac{S_f}{I_{ef}}\, q\, \ell^4}$$

- Beams, one end fixed and one end simply supported:

$$A_1 = \frac{4\, k_h\, q\, \ell^4}{EI_{fy}\, \pi^5 + K\, \ell^4\, \pi - 1.22\, \frac{S_f}{I_{ef}}\, q\, \ell^4}$$

Herein:

k_h : according to figure 3
q : the component of the design load acting in web direction
ℓ : span of the beam
I_{fy} : see before
K : lateral spring stiffness according to ref. [9]; depending on place of centre of rotation of the beam
S_f : static moment of the free flange plus 1/6 of the height of the web about the neutral axis (the effective cross section is governing)
I_{ef} : the moment of inertia of the effective cross section of the whole beam.

Remark

When the free flange is in tension, then the "-" sign in the denominater should be a "+" sign.

Appendix III: Stability check of free flange in compression

The stability of the free flange in compression shall be checked as follows:

$$\omega \frac{M(x)}{W_{ef}} + \frac{M(y)}{W_{fy}} \leq f_{ty}$$

Herein:
M (x), M(y), W_{ef}, W_{fy} and f_{ty}: see appendix II

ω : buckling coëfficiënt

$$\omega = \frac{Q}{F - \sqrt{F^2 - \frac{Q}{\bar{\lambda}^2}}}$$

$$Q = \frac{A_{eff}}{A_g}$$

$$F = 1/2 \{Q + \frac{1 + \eta (\bar{\lambda} - 0,2)}{\bar{\lambda}^2}\}$$

$$\eta = 0.34 (4 - 3Q) \not> 0.76$$

$$\bar{\lambda} = \frac{\ell_{cr}}{i_{fy}} \cdot \frac{1}{\pi}\sqrt{\frac{f_{ty}}{E}}$$

A_g = area of gross cross section

A_{eff} = area of effective cross section belonging to M(x)

i_{fy} = radius of gyration of gross cross section of free flange plus 1/6 of web height against lateral bending

ℓ_{cr} = buckling length depending on edge conditions in lateral deflections

- simple supported beam

$$\ell_{cr} = \frac{\ell}{\pi}\sqrt{\frac{\frac{2}{3} n^2 \pi^2 - 2}{n^4 + R}}$$

$$n_a = \sqrt{0.3 + \sqrt{0.09 + R}}$$

n = next higher integer value of n_a

- beams, both ends fixed

$$\ell_{cr} = \frac{\ell}{\pi}\sqrt{\frac{\frac{8}{9} n^2 \pi^2 + \frac{2}{3}}{\frac{16}{3} n^4 + R}}$$

$$n_a = 0.66 \sqrt[4]{R}$$

n = next higher integer value of n_a

- beams, one end fixed and one end simply supported

$$\ell_{cr} = \frac{\ell}{\pi}\sqrt{\frac{\frac{20}{27} n^2 \pi^2 - \frac{48}{27}}{n^4 + R}}$$

$$n_a = \sqrt{0.24 + \sqrt{0.06 + R}}$$

n = next higher integer value of n_a
ℓ = span of the beam

$$R = \frac{K \ell^4}{\pi^2 E I_{fy}}$$

K = lateral spring stiffness according to ref. [9] depending on place of centre of rotation of the beam

I_{fy} = moment of inertia of gross cross section of free flange plus 1/6 of web height against lateral bending