Generalized Geometric Programming in Cold Formed Steel Design

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COLD FORMED STEEL DESIGN

by

S. Ramamurthy* and R. H. Gallagher**

SUMMARY

Relationships governing the minimum weight design of two cold formed steel cross-sections, the hat and the channel, are constructed. The generalized geometric programming (GGP) algorithm, which is especially suited to this class of problem, is described and applied to it. Numerical results are presented for problems involving the action of positive or negative moments and transverse shear.

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1. INTRODUCTION

Cold-formed steel structures represent an attractive potential for the application of optimum structural design procedures. The key dimensions of structural forms of this manner of construction are open for selection by the design analyst who can use this latitude to achieve the objective of structural integrity at low cost or weight. Optimum structural design procedures represent organized, programmable approaches to the achievement of such goals.

The overall problem of optimal structural design has attracted considerable attention in the past fifteen years. An account of much of this work can be found in Ref. 9. A large share of this activity has involved the amalgamation of methods of computerized structural analysis and mathematical programming, particularly feasible direction and gradient methods. Other efforts have been devoted to the adaptation of classical minimization procedures to various structural design problems. Still other activities have sought "optimality criteria" procedures, which exploit some fundamental property of an optimal solution, e.g., fully-stressed design.

Seaburg and Salmon (15) have treated the problem of minimum weight cold-formed steel design, employing a mathematical programming approach. Although this type of approach is workable for the subject class of problem, it does not take advantage of any special characteristic of this class of problem. The high computational cost of mathematical programming makes it highly desirable to seek efficiencies via methods which do exploit such special characteristics.

One method which is appealing in this regard is geometric programming. Devised originally by Peterson, Duttin and Zener (6) the method applies to conditions under which the quantity to be minimized and the constraints on the problem can all be written as the sum of polynomials with positive coefficients. If this and certain other conditions can be met then a transformation can be applied which yields a problem that is much simpler to solve. Templeman (16, 17), Morris (11, 12), and others have adapted these "standard" geometric programming concepts to numerous optimum structural design problems.

Unfortunately, many significant engineering optimization problems -- including cold formed steel design -- do not conform to all of the restrictions of standard geometric programming. Such non-standard problems have been cast into the format of standard geometric programming by Templeman (16), and Morris (11), who employed the approaches at Avriel and Williams (4). More recently, a new class of less restrictive geometric programming algorithms, known as "generalized geometric programming", symbolized here as GGP, have been developed by Avriel and his associates (Refs. 2, 3), Dawkins et al (Ref. 5), and Ecker and Zoracki (Ref. 7).

The advantages of GGP, of the form developed by Avriel, Dembo and Passy (3) are brought to bear upon the cold formed steel design problem in this paper. Two specific problems are chosen to describe the procedure, these bring the design of a hat section and a channel section, respectively, under the action of applied moments and a shear force. The details of these problems are given in the next section. Then, the pertinent concepts of GGP, including the strategy for treatment of equality constraints, are outlined. Finally, the results of numerical optimization studies of the subject problems are presented.

2. DESCRIPTION OF PROBLEM

2.1. Section Geometry and Loading

The geometry of the hat section and the channel section are shown in in the top flange. $\bar{y}_p$ and $\bar{y}_b$ are the distances from the centroidal (1-1) axis and the top center lines of the top and bottom flanges, respectively. I_e is the moment of inertia of the effective cross-section.

For the hat section problem, the effective width of the top flange, b, which is used in the calculation of I_e is obtained from the effective width equation (Eq. 2.3.1.1 Ref. 1),

$$\frac{b}{x_4} = \frac{253}{\sqrt{f}}\left(1 - \frac{55.3}{(x_2/x_4)\sqrt{f}}\right) \tag{3}$$

Equation 3 is valid only when

$$x_2/x_4 \geq 171/\sqrt{f} \tag{4}$$

otherwise, $b = x_2$.

b. <u>Stress Constraints Due to Negative Bending Moment M_n</u>:

The condition that the direct stresses in the flanges due to the applied positive bending moment are less than the allowables can be written as

$$\frac{M_N \bar{y}_N}{I} \leq 0.6F_y \qquad \text{(top flange)} \tag{5}$$

$$\frac{M_N \bar{y}_{Nb}}{I} \leq F_c = \text{Allowable Compressive Stress} \tag{6}$$

where, $\bar{y}_N$ is the distance between the centroid and the top center line,

$\bar{y}_{Nb}$ is the distance between the centroid and the bottom center line,

and I is the moment of inertia of the entire section.

Moreover, in Eq. 6, the allowable bending stress in compression, F_c, is a function dependent on the ratio, x_1/x_4 (Eqn. 3.2.b Ref 1). For the present problem x_1/x_4 must not exceed $144\sqrt{F_y}$, and the corresponding expression for F_c is

$$F_c \leq F_y\ [.767 - \frac{2.64}{1000}\ (\frac{x_1}{x_4})\ \sqrt{F_y})\tag{7}$$

c. Stress Limitations Due to Shear, V

Similar to F_c, the allowable stress for shear, F_v, is a function of the web depth-to thickness ratio, x_3/x_4, and the expressions for F_v are dependent upon the range of values of $x_3/x_4 = k$, given by Eq. 3.4.1 a and b Ref 1. The chosen range of k for the hat section is

$$k \geq 547/\sqrt{F_y}\tag{8}$$

For the ranges considered in Eq. 8, F_v is restricted thus

$$F_v = \frac{83,200}{(x_3/x_4)^2}\tag{9}$$

2.3. Behavior Constraint Conditions - Channel Section

a. Constraints due to Positive Bending Moment M_p

The constraints representing the condition that the direct stresses in the flanges due to the application of M_p are less than the allowables are the same as those for the hat section and are therefore given by Eqs. (1) and (2). No constraints for negative bending moment (M_N) are considered for the channel section.

b. Stress limitations due to shear

In the case of the channel section for the chosen range of $k=x_3/x_4$, we have from Eq. 3.4.1a and 3.4.1b of Ref. 1

$$k \leq 547\sqrt{F_y} \tag{10}$$

Also, for the chosen range of k, the average shear stress, f_v, is limited to be less than F_v where,

$$F_v = 152\sqrt{F_y}/k \text{ with a maximum of } 0.4F_y \tag{11}$$

c. Web-crippling constraint

Assuming bearing length is equal to the web-depth, Eq. 3.5.2 of Ref. 1 requires that

$$R_B \leq x_4^2\,[305 + 1.8(h/x_4) - .009(h/x_4)^2] \times [1.22 - 0.22\,F_y/33]\,F_y/33 \tag{12}$$

where h is the distance between the inside of the flanges.

d. Constraint Relating to the Effectiveness of the Lip-Stiffener

The minimum depth of the lip stiffener of the hat section should not be less than either

$$2.8\,x_4\,\left[\left(\frac{x_2}{x_4}\right)^2 - \frac{4000}{F_y}\right]^{1/2} \tag{13}$$

or

$$\leq 4.8\,x_4$$

e. Combined Bending and Shear Stresses in Webs

The webs subjected to combined bending and shear should satisfy the following equation:

$$(f_{bw}/F_{bw})^2 + (f_v/F_v)^2 \leq 1 \tag{11a}$$

where f_{bw} = actual bending stress in the web and

$F_{bw} = 520{,}000/k^2$ with a maximum of $0.6F_y$

f. Constraints Due to Artificial Variables:

In addition to the above, four artificial variables, x_5, x_6, x_7 and x_8 are introduced to simplify the algebraic manipulation of the design equations that are expressed in a form amenable for GGP format. These equations are

$$x_5 \geq (\frac{x_2}{x_4})^2 - \frac{4000}{F_y} \quad (14)$$

$$x_6 \geq 0.5\ x_3^2 + x_2x_3 + x_1x_3 \quad (15)$$

$$x_7 \geq .6667\ x_1^3 + x_1x_3^2 + x_2x_3^2 + .333\ x_3^3 - x_1^2\ x_3 \quad (16)$$

and $x_8 \geq x_3 - x_4.$ (17)

In the above, x_5 relates the expressions within the square root sign of Eq. 13, x_6 bounds above the first moment of the section about the top center line of the cross section, and x_7 is restricted to be less than the moment of inertia of the entire section about the axis passing through the top center line. Lastly, x_8 bounds above the distance between the inside of the flanges parallel to the webs.

2.4 Bounding Constraints on Design Variable Magnitude

In addition to the above behavior constraints, bound constraints, which can set limits on the smallest and largest acceptable values of x_i's, can be introduced. These bound constraints, which are inequalities, can be part of the GGP procedure, and these bounds may be due to manufacturing limitations. In the present GGP procedure, bound constraints, however, are introduced as an artifice to incorporate the equality constraints which are not a part of the GGP procedures published heretofore. This procedure will be discussed subsequently.

2.5 Objective Function

The objective function can be either weight or cost. Due to lack of

b. Stress limitations due to shear

In the case of the channel section for the chosen range of $k=x_3/x_4$, we have from Eq. 3.4.1a and 3.4.1b of Ref. 1

$$k \leq 547 \sqrt{F_y} \quad (10)$$

Also, for the chosen range of k, the average shear stress, f_v, is limited to be less than F_v where,

$$F_v = 152 \sqrt{F_y}/k \text{ with a maximum of } 0.4F_y \quad (11)$$

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Assuming bearing length is equal to the web-depth, Eq. 3.5.2 of Ref. 1 requires that

$$R_B \leq x_4^2 \; [305 + 1.8(h/x_4) - .009(h/x_4)^2] \times [1.22 - 0.22 \; F_y/33] \; F_y/33 \quad (12)$$

where h is the distance between the inside of the flanges.

d. Constraint Relating to the Effectiveness of the Lip-Stiffener

The minimum depth of the lip stiffener of the hat section should not be less than either

$$2.8 \; x_4 \quad [(\frac{x_2}{x_4})^2 - \frac{4000}{F_y})] \; 1/2 \quad (13)$$

or

$$\leq 4.8 \; x_4$$

e. Combined Bending and Shear Stresses in Webs

The webs subjected to combined bending and shear should satisfy the following equation:

$$(f_{bw}/F_{bw})^2 + (f_v/F_v)^2 \leq 1 \quad (11a)$$

where f_{bw} = actual bending stress in the web and

$F_{bw} = 520{,}000/k^2$ with a maximum of $0.6F_y$

f. Constraints Due to Artificial Variables:

In addition to the above, four artificial variables, x_5, x_6, x_7 and x_8 are introduced to simplify the algebraic manipulation of the design accurate cost data, and due to the judgement that cost and weight approximately correlates in this problem, the weight per unit length is chosen as the objective function. This weight is obtained by multiplying the area of the cross section by the unit weight of the steel. Thus, for the hat section

$$W = 6.8\ x_1x_4 + 3.4\ x_2x_4 + 6.8\ x_3x_4 \tag{18}$$

and, for the channel section

$$W = 6.8\ x_2x_4 + 3.4\ x_3x_4 + 6.8\ x_1x_4 \tag{19}$$

3. SOLUTION ALGORITHM

In the standard form of geometric programming the objective function and the constraints must each be of the same form in the n design variables $\underset{\sim}{X} = x_1, \text{---}\ x_n$. It is customary to designate the objective function as $g_o(\underset{\sim}{X})$, while the p constraints are designated as $g_1(\underset{\sim}{X}),\ldots g_p(\underset{\sim}{X})$. The required form is, for a constraint g_i consisting of q_i terms

$$g_i(\underset{\sim}{X}) = \sum_{j=1}^{q_i} c_j^i \prod_{k=1}^{n} (x_k)^{a_{jk}} \leq 1 \quad (i = 1,\ldots p) \tag{20}$$

where the multipliers c_j^i must each be positive. (The ≤ 1 condition does not, of course apply to g_o, the objective function) The exponents a_{jk} need not be positive.

Consider, for example, the constraint represented by Eq. (4). This can be rearranged to read

$$171\, x_4 - \sqrt{f}\, x_2 \leq 1 \qquad (21)$$

Now, neglecting in this example any dependence of f on $\underset{\sim}{x}$, each term on the left is of the form $C_j \prod_k (x_k)^{a_{jk}}$. The sign of C_2, however, is negative, which violates the requirement given above.

When the above requirement is met the constraints (and the objective function) are in the form of a posynomial, a designation coined to describe a sum of polynomial terms each of which has a positive multiplier. When some of the terms have negative multipliers the expression is called a signomial. The objective of Generalized Geometric Programming is to transform a problem given in terms of signomial constraints and objective function into a posynomial suitable for treatment by standard geometric programming. This cannot be accomplished in an exact manner and the solution process is therefore iterative.

To develop this scheme we first identify a typical individual term of a posynomial as a monomial, $u_r(\underset{\sim}{X})$.

$$u_r(\underset{\sim}{X}) = c_r \prod_k (x_k)^{a_{rk}} \qquad (22)$$

so that the corresponding posynomial is of the form

$$U_i = \sum_r u_r = \sum_r c_r \prod_k (x_k)^{a_{rk}} \qquad (23)$$

A signomial constraint $g_i(\underset{\sim}{X})$ can be arranged as the difference between two posynomials, say U_i and $\hat{U}_i$, where

$$\hat{U}_i = \sum_s u_s = \sum_s c_s \prod_k (x_k)^{a_{rk}} \qquad (24)$$

and $r + s = q_i$. Thus,

$$g_i(\underline{x}) = U_i - \hat{U}_i \leq 1 \tag{25}$$

With the above designations at hand, the generalized geometric programming (GGP) problem, applicable to minimum weight cold formed steel design, can be expressed symbolically as

$$\text{Minimize} \quad g_o(\underline{X}) = U_o(\underline{X}) - \hat{U}_o(\underline{X}) \tag{26}$$

Subject to

$$g_i(\underline{X}) = U_i(\underline{X}) - \hat{U}_i(\underline{X}) \leq 1 \quad (i = 1, ---p) \tag{27}$$

and side constraints

$$0 < x_j^{LB} \leq x_j \leq x_j^{UB} \quad j = 1, ---N \tag{28}$$

Here, the subscripts LB and UB stand for specified lower and upper bounds, respectively.

To explain the details of the solution process beyond this point, we observe that the constraint conditions (Eq. 27) can be written in the following alternative form

$$g_i(\underline{X}) = \frac{U_i}{(1 + \hat{U}_i)} \leq 1 \tag{29}$$

Also, we note that the function $(1 + U_i)$ can be condensed into a monomial via approximate procedures. Although such procedures represent a critical step in this approach their development is beyond the scope of this paper; the interested reader is referred to Refs. (3, 14).

The GGP problem is solved via a sequence of GP^m $(m=1,2,\ldots,\bar{m})$ problems. To obtain the posynomial constraints of the GP^m problem, the posynomials U_i's are divided by the condensed monomial of the function $(1 + \hat{U}_i)$ at $\bar{x} = \overline{x}^{m-1}$ where $\overline{x}^{m-1}$ is the optimal point of the preceding GP (i.e., GP^{m-1}) iteration. Here, the starting point, $\underline{x}^o$, must be feasible (meet all constraints).

The computer algorithm (Ref. 14) used in this paper to solve the GP^m problem is done by a process of linearization as used by Avriel et al (Ref. 3). The constraints, approximated as monomials as stated above, are linearized by using a logarithmic transformation. If the solution obtained is feasible the iteration will terminate. The optimal solution of the resulting linear programming program, however, may not be a feasible solution to the parent GP problem. The infeasibility for the parent GP problem will necessitate the introduction of additional linearized constraint, to be appended to the previous LP problem, which is obtained by approximating the most violated posynomial constraint of the parent GP problem. This process of linearization and the addition of linearized constraints is continued until the optimal point of the LP problem is feasible to the GP problem.

A flow chart as shown in Fig. 3 illustrates the above discussed computer algorithm for the GGP.

It should be noted that the GGP algorithm does not take into account equality constraints, as represented by Eq. (28). The means of dealing with this condition in the present application is described in the next section.

4. NUMERICAL EXAMPLES AND RESULTS

4.1 Hat Section

The design conditions chosen for the case of the hat section are adapted from a problem described by Yu (18) where a positive moment Mp = 135 in. K. (1.54×10^4 Nm) is specified. We add to this a specified negative moment M_n = 65. in. K (7.4×10^3 Nm) and a shear force V = 20.3 K (9.0×10^4 N). F_y is 2.78×10^8 N/m^2 40 K/in.2. Yu's design was x_1 = 2.35" (59.7 mm), x_2 = 14.5" (368 mm), x_3 = 9.5" (241 mm) and x_4 = 0.105" (2.65 mm), giving a weight of W = 13.637 lb. (61.0N). The calculated M_p in this case was 138.4 in. K (1.58×10^4 Nm).

The expression of the objective function is given by Eq. (18). Giving this a GGP designation we have:

$$\text{Minimize } W = g_{10} = 6.8x_1x_4 + 3.4x_2x_4 + 6.8x_3x_4 \tag{18}$$

The applicable constraints $(g_1, \ldots g_9)$ were defined in Section 2.3, Eqs. (1) - (9) and, after numerical evaluation based on the above data, they have the form given in Table 2.

It is important to note that in the constraints g_1, g_2 and g_9 the coefficients of some terms are multiplied by b. b is a function obtainable from the following implicit equation: (Ref. 1).

$$b = 253 \frac{x_4^{3/2}}{\left\{\dfrac{3M_p(x_3^2 + 2x_1x_3)}{b\,(2x_3^3 + 6x_1x_3^2) + (x_3^4 + 4x_1x_3^3)}\right\}^{1/2}} \times \left\{1 - \frac{55.3}{\dfrac{x_2}{x_4^{3/2}}\left\{\dfrac{3M_p(x_3^2 + 2x_1x_3)}{b\,(2x_3^3 + 6x_1x_3^2) + (x_3^4 + 4x_1x_3^3)}\right\}^{\frac{1}{2}}}\right\} \tag{30}$$

This equation is applicable only if (by combination of Eqs. 3 and 4)

$$\frac{x_2}{x_4} \geq 171 x_4^{1/2} \left\{ \frac{b \ (6x_1x_3^2 + 2x_3^3) + (x_3^4 + 4x_1x_3^3)}{3M_p(x_3^2 + 2x_1x_3)} \right\}^{1/2} \tag{31}$$

otherwise, $b = x_2$

Although it is possible to solve Eq. (30) algebraically, the substitution of the resulting equation in constraints g_1 and g_2 would produce a very involved expression. Instead, satisfaction of the condition represented by Eq. (30) was accomplished by a process wherein the independent variables were limited by upper and lower bounds within a certain percentage of the current values (105% and 80%) at each iteration and at the same time recalculating the value b. The calculation of the b was accomplished by appending a subroutine which is called by the main program GGP. The structure of the computer program and the algorithm itself is such that this is possible only before calling the subroutines which form and solve the GP^m problem. Before calling these subroutines the lower and upper bounds are also changed and coefficient matrix of GGP which is required to solve a GP^m problem is also modified. This solution approach is presented in the form of a flow chart in Figure 4.

Taking a convergence tolerance of .001 on the value of objective function, the final weight converges to 8.511 lb/ft in ten iterations. Figures 5 and 6a show, in normalized form, the optimum values of the independent variables x_1 and x_2 and objective function at every GP^m iteration.

The results of the computation showed that the values x_3 and x_4 did not change after the first iteration. Also, the values of b in successive iterations changed only slowly. In the 7th through 10th iteration the value of b remained practically constant and it could be assumed that the algorithm had converged to a local minimum. From Figure 5, it should be clear that x_1^m (value of x_1 at the optimum point) each time reaches its lower bound until the fifth GP^m iteration is reached. From the last iteration it can be observed that the total top flange width is effective. The execution time taken to obtain the above results using a WATFIV/compiler of IBM 370/168 was 1.3 seconds.

4.2 Channel Section

A minimum weight channel section is to be designed to carry a uniformly distributed load of 180 lb/ft (2.65×10^3N/m) over three spans of 25 ft. 7.6 m length each. The appropriate design loads, calculated using the equations given in Reference 1 , are M_p = 136.755 kips, V = 2.7 kips, and a support reaction of 5.4 kips. The yield stress, F_y, is assumed to be 45 ksi.

The objective function for this problem is given by Equation 20. The constraints, which were outlined in section 2, are given in their detailed form in Table 3, except for g_4 and g_{12}, which are too lengthy to give here and which can be found in Ref. 14. The transformations from the equations of the previous section to the ones presented subsequently involve the introduction of expressions for I, $\overline{y}_p$, and for artificial variables x_5, x_6, x_7 and x_8. An expression similar to Eq. (31), which relates the b and other variables as in the case of hat section problem, is obtained for this problem also. The same logic presented in Fig. 3 is used for this purpose.

As in the hat section problem, the equality relationships due to the effective width equation is approximately preserved by constricting the bounds. The procedure adopted here to solve this numerical problem using the GGP is also very similar to the one used in the hat section problem.

The following are the dimensions of the initially acceptable design section: x_1 = 1.128 in, (28.7 mm) x_2 = 5.5 in, (139.7 mm) x_3 = 8.5 in, (216 mm) x_4 = .119 in. (3.02 mm). The values of the remaining variables of the chosen initial feasible point are $x_5 = 2047.2$, $x_6 = 92.46$, $x_7 = 673.6$, $x_8 = 8.381$.

The weight of the section, W (Equation 19), from a starting value of 8.802 converges to 6.156 in four GP^m iterations. The results of these calculations are presented in Figures 5, 6a and 6b.

5. CONCLUDING REMARKS

For the range of depth to thickness ratio and flange width to thickness ratio, thickness of the section remained practically a constant. For these chosen examples, the GGP algorithm converges. Thus it was found possible to incorporate equality constraints by use of an approximation and by constricting the ranges of the independent variables.

It must be noted that for a given design only some design (especially allowable stress) constraints are valid and these need to be chosen before formulating the optimization problem. It is, however, possible to incorporate all the constraints at once, and using some subroutines to pick the needed constraints at any given time. These subroutines would be very valuable tools to obtain an optimum cold formed steel cross section.

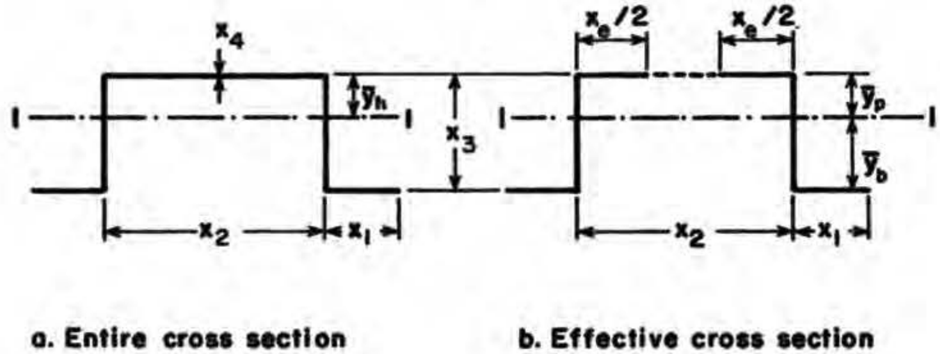


FIGURE 1 Hat section geometry

Note:
$x_e = b$

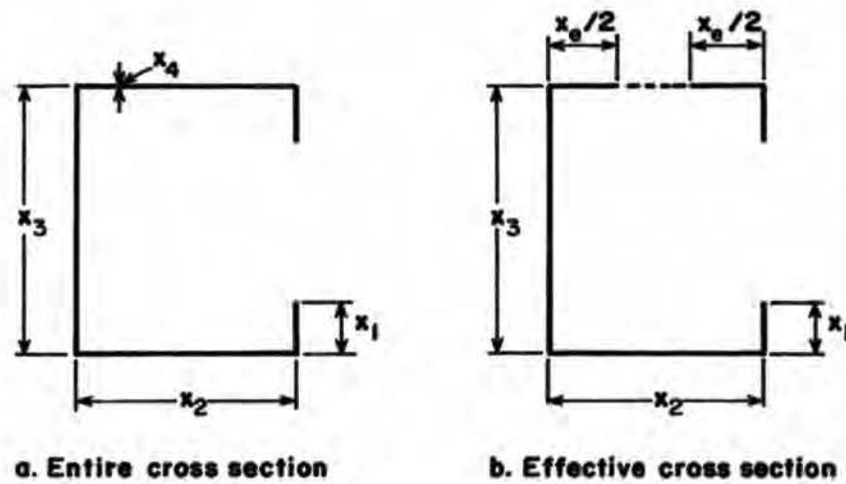


FIGURE 2 Channel section geometry

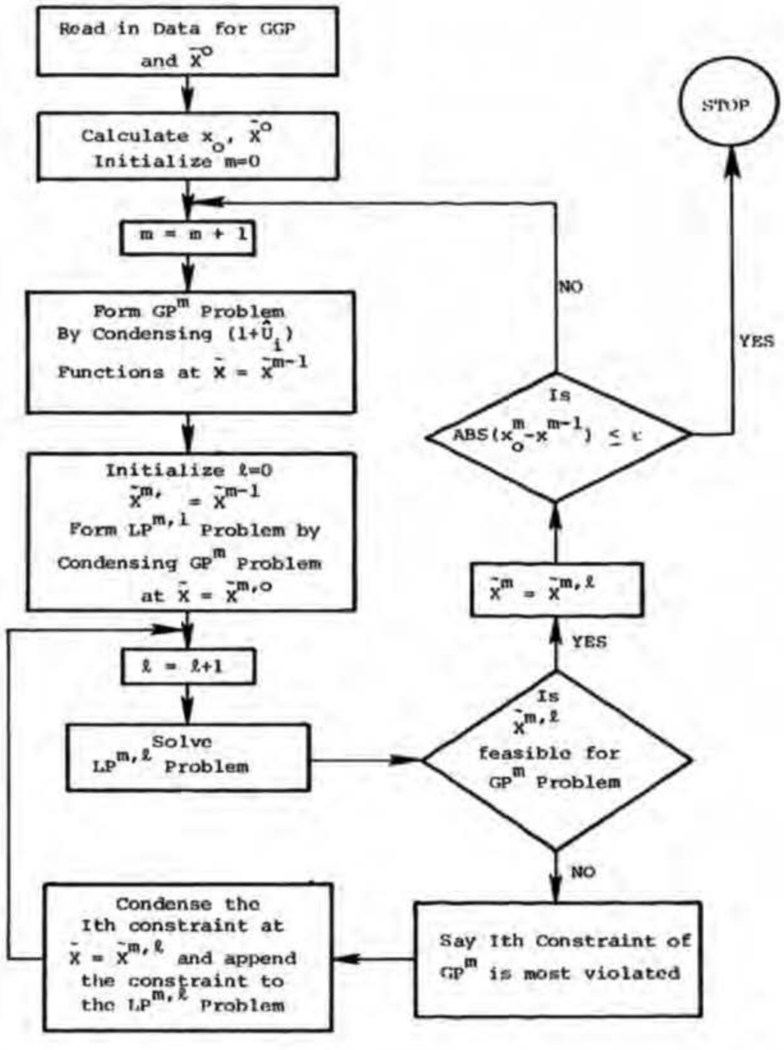


FIGURE 3 Flow chart for Generalized Geometric Programming algorithm

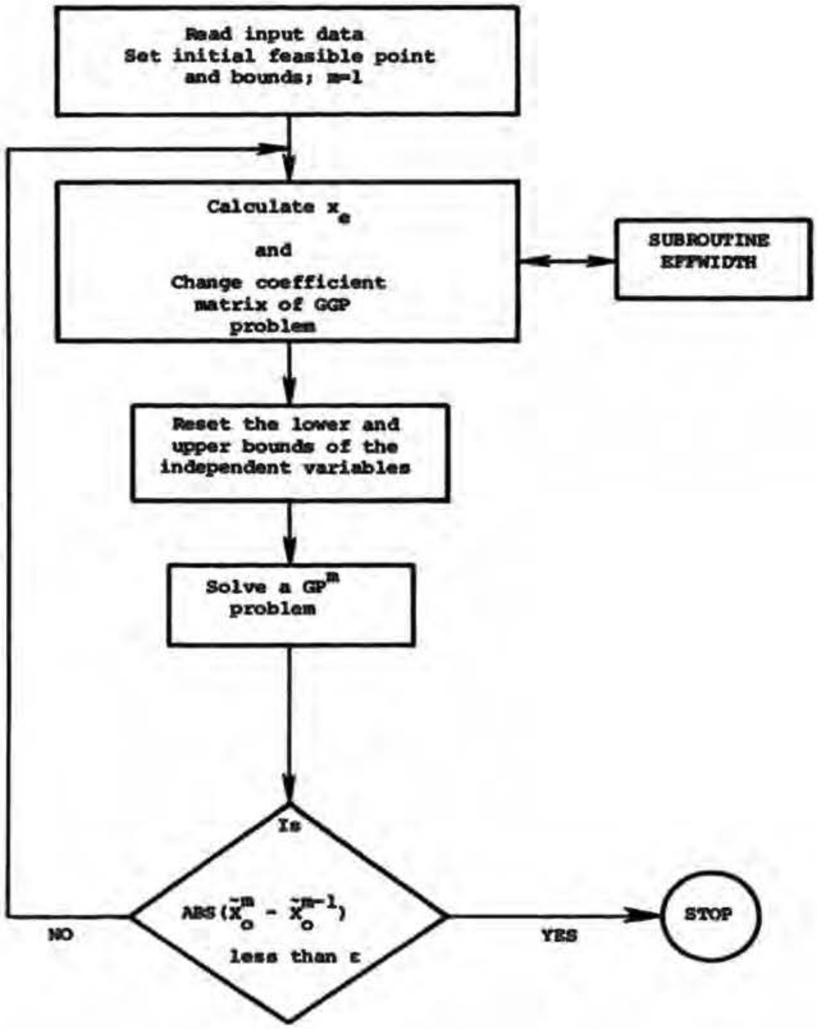


FIGURE 4 Flow chart for treatment of inequality constraints

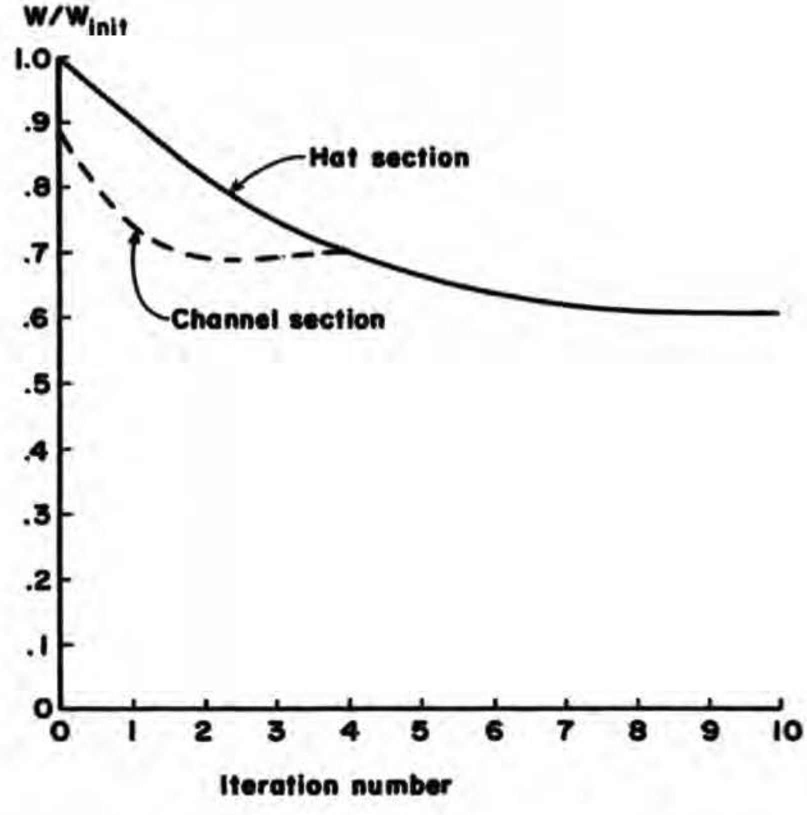


FIGURE 5 Convergence of objective functions

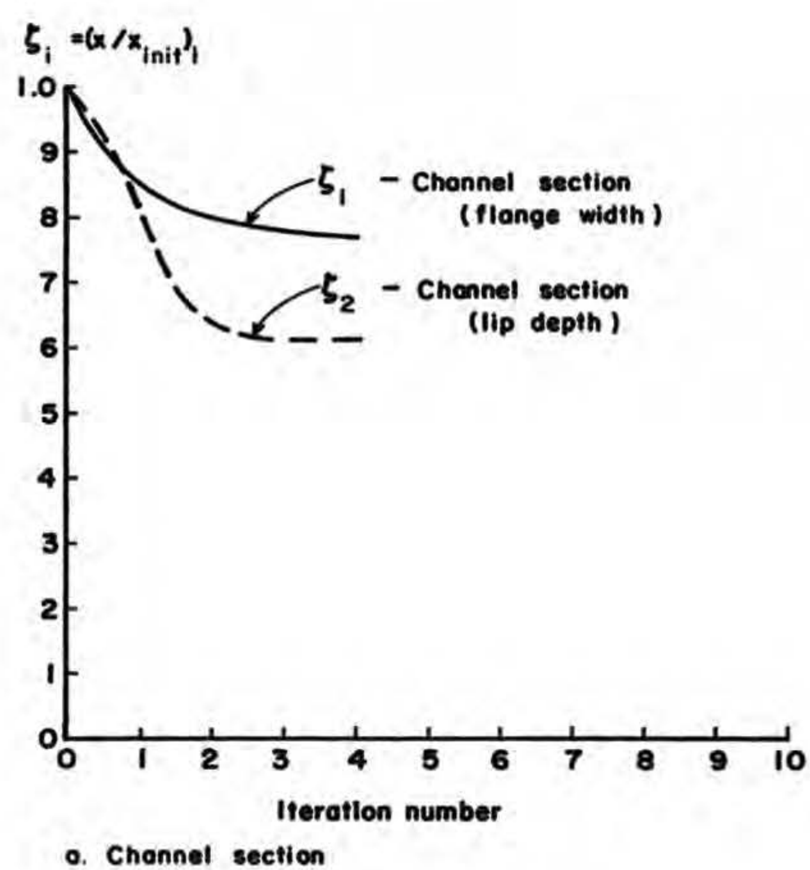
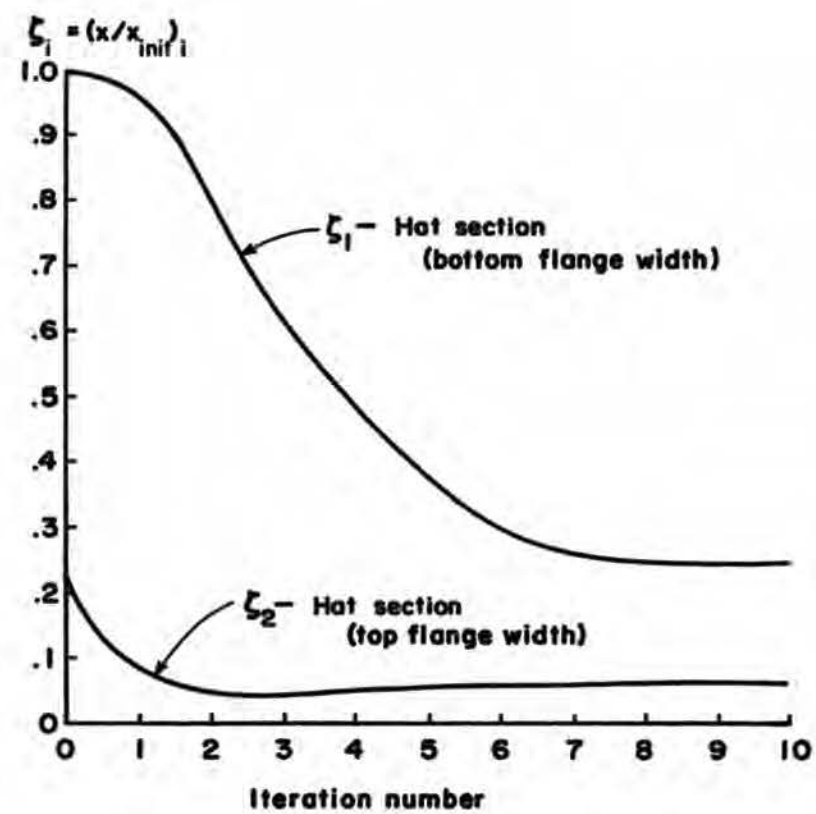


FIGURE 6a Convergence of design variables



b. Hat section

FIGURE 6b Convergence of design variables

TABLE 1 - DESIGN CONSTRAINTS

Constraint Condition	Hat Section	Channel Section
M_P (Pos. Bending Moment)	Flange Bending Stresses Bottom and Top	Top Flange Bending and Web Bending Stresses
M_N (Neg. Bending Moment)	Flange Bending Stress Bottom and Top	-
V (Shear)	YES	YES
Web Crippling	-	YES
Lip Stiffener	-	YES
Artificial Variables	-	YES
Limits on Member sizes	YES	YES
Combined Bending and Shear-Web	-	YES

TABLE 2 NUMERICALLY EVALUATED CONSTRAINTS - HAT SECTION

Constraint g_i								Constraint Description
	$X_1X_3^{-1}X_4^{-1}$	$X_3^{-1}X_4^{-1}$	$X_1X_eX_3^{-2}$	$X_eX_3^{-1}$	$X_1X_3^{-1}$	$X_3^{-2}X_4^{-1}$	$X_eX_3^{-3}X_4^{-1}$	
g_1	33.75	16.875	-6	-2	-4			Top flange stress due to M_p (Eq. (1))
g_2			-6	-2	-4	16.875	16.875	Bottom flange stress due to M_p (Eq. (2))
	$X_1X_3^{-3}X_4^{-1}$	$X_3^{-2}X_4^{-1}X_5^{-1}$	$X_1X_2X_3^{-2}$	$X_2X_3^{-1}$	$X_1X_3^{-1}$	$X_3^{-2}X_4^{-1}$	$X_2X_3^{-2}X_4^{-1}X_5^{-1}$	
g_3	-	195	-6	-2	-4	-	195	Bottom flange stress due to M_H (Eq. (6))
g_4	16.25	-	-6	-2	-4	8.125	-	Top flange stress due to M_H (Eq. (5))
	$X_1X_4^{-1}$	X_5	$X_3^{-1}X_4$	$X_3X_4^{-3}$				
g_5	0.02175	0.03295	-	-	-	-	-	Limiting Comp. Stress (Eq. (7))
g_6	0.04392	-	-	-	-	-	-	Limiting range of X_1/X_4 ratio
g_7	-	-	86.488	-	-	-	-	Limiting ratio of X_3/X_4 (Eq. (9))

g_8	-	-	-	.0012185	-	-	-	Constraint on allow. F_v (Eq. (12))
	$x_e x_1 x_2^{-2} x_4^3$	$x_e x_2^{-2} x_3 x_4^3$	$x_2^{-2} x_3^2 x_4^3$	$x_1 x_2^{-2} x_3 x_4^3$	$x_1 x_3^{-1}$			
g_9	433.2	144.4	72.2	286.9	-2	-	-	Constraint Limiting the use of Eff. Width Eq

TABLE 3 NUMERICALLY EVALUATED CONSTRAINTS - CHANNEL SECTION

Constraint g_i	Coefficients a							Constraint description
	$x_2^{-1}x_4^{-1}x_6x_7^{-1}$	$x_2^{-1}x_6^{2}x_7^{-1}$	$x_1x_2^{-1}$	$x_2^{-1}x_3$	$x_ex_2^{-1}$	$x_3^{2}x_6^{-1}$	$x_2x_3x_6^{-1}$	
g_1	5.06	1	-2	-1	-1	-	-	Top flange stress due to Mp (Eq (1))
g_2	-	-	-	-	-	0.5	2	Artificial variable x_6 (Eq (16))
	$x_1^{-1}x_3^{-2}x_7$	$x_1x_3^{-1}$	$x_1^{2}x_3^{-2}$	$x_1^{-1}x_2$	$x_1^{-1}x_3$	x_4^{-2}	-	
g_3	1	1	-0.6667	-1	-.3333	-	-	Related to Artificial Variable x_7 (Eq (17))
g_5	-	-	-	-	-	$2.64795\text{x}10^{-3}$	-	Permissible Shear Stress (Eq (10))
	$x_3^{-1}x_4^{-1}$	$x_3^{-1}x_4$	$x_3x_4^{-1}$	$x_1^{-1}x_4$	$x_2^{2}x_4^{-2}x_5^{-1}$	x_5^{-1}	-	
g_6	.15	1.0	-	-	-	-	-	Permissible Shear Stress (Eq (11))
g_7	-	-	$1.2115\text{x}10^{-2}$	-	-	-		Limiting ratio of Web depth to thickness(Eq 15)
g_8	-	-	-	4.3	-	-	-	Constraint on the depth of tip stiffner (Eq (15))
g_9	-	-	-	-	1.0	-88.889	-	Artificial Variable x_5 (Eq (15))

	$x_5^{.1667}$	$x_1x_4^{-1}$	$x_3x_8^{-1}$	$x_4x_8^{-1}$	$x_3x_4^{-1}$	$x_3x_4^{-1}$	$x_3^{-1}x_4$	
g_{10}	5.6	-2	-	-	-	-	-	Minimum deith of the Lip Stiffner
g_{11}	-	-	1.0	-1.0	-	-	-	Artificial Variable x_8 (Eq. (18))
g_{13}	-	-	-	-	2.3676	.00495	-177.77	Web Crippling Stress (Eq. (13))

$$g_4 = 1.3333x_1^2x_2^{-1}x_3^{-1} + .6667x_1x_3^{-1} + x_1^{-1}x_3 + .6667x_1^{-1}x_2^{-1}x_3^2$$
$$+ .3333x_1^{-2}x_3^2 + .0833x_1^{-2}x_2^{-1}x_3^3 + .6667x_ex_1x_2^{-1}x_3^{-1}$$
$$+ x_1^{-1}x_2^{-1}x_3x_e + x_1^{-2}x_3x_e + .3333x_1^{-2}x_2^{-1}x_3^2x_e - 1.3333x_1x_2^{-1}$$
$$- 2.3384\cdot10^{-3}x_1^{-2}x_2x_3x_4^{-3} - 4.6768\cdot10^{-3}x_1^{-1}x_2x_4^{-3}$$
$$- 4.6768\cdot10^{-3}x_1^{-2}x_2^2x_4^{-3} - x_2^{-1}x_e \leq 1$$

$$g_{12} = 1.315\cdot10^{-4}x_2^{-1}x_3^2x_4^{-3}x_7^{-1}x_8^2 + 2.630\cdot10^{-4}x_3x_4^{-3}x_7^{-1}x_8^2$$
$$+ 2.630\cdot10^{-4}x_1x_2^{-1}x_3x_4^{-3}x_7^{-1}x_8^2 + x_2^{-1}x_6^2x_7^{-1}$$
$$- 2.630\cdot10^{-4}x_1x_2^{-1}x_4^{-2}x_7^{-1}x_8^2 - 1.315\cdot10^{-4}x_4^{-2}x_7^{-1}x_8^2$$
$$- 1.315\cdot10^{-4}x_2^{-1}x_3x_4^{-2}x_7^{-1}x_8^2 - 1.315\cdot10^{-4}x_ex_2^{-1}x_4^{-2}x_7^{-1}x_8^2$$
$$- 2x_1x_2^{-1} - x_2^{-1}x_3 - x_2^{-1}x_e \leq 1$$

APPENDIX I - REFERENCES

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APPENDIX II NOTATION

a_{kj}	-	Exponent of the independent variables x_j
b	-	Effective width of the compression flange
c_k,	-	Coefficient of the monomial U_k
F_c	-	Allowable bending compressive stress of the unstiffened element
F_v	-	Average shear stress
F_y	-	Yield stress
f	-	Actual bending compressive stress considering the effective width equation
g_i	-	Sigomial which is expressed as the difference between two posynomials U_i and V_i
I	-	Moment of inertia of the cross section
I_e	-	Moment of inertia of the effective cross section
k	-	Web depth-to-thickness ration
M_N	-	Negative Bending Moment
M_P	-	Positive Bending Moment
R_B	-	Support Reaction
U_i	-	Posynomial
u_r	-	Monomial
V	-	Shear force
x_j, x_j^{UB}, x_j^{LB}	-	j^{th} independent variable and its upper bound and lower bounds
$\overline{Y}_b$	-	Distance between the controid and the bottom flange of the effective section.

$\bar{y}_N$	-	Distance between the centroid and the top flange of the entire section,
$\bar{y}_{Nb}$	-	Distance between the centroid and the bottom flange of the entire section,
$\bar{y}_P$	-	Distance between the centroid and the top flange of the effective cross section,
ρ_i	-	Ratio of initial value of design variable to the value calculated in a particular GGP iteration.