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INTRODUCTION

The influence of local buckling on the response of thin-walled cold-formed beams subjected to impact loading was considered in a previous paper (10)[4]. A mathematical model which accounted for the nonlinearity introduced into the response due to the dependence of beam stiffness on applied load was developed and compared with a series of impact tests. The purpose of this paper is to compare results obtained from this mathematical model with results from a linear elastic analysis of simply supported beams in which the effects of local buckling are neglected. Such a comparison is of interest to evaluate how much effect local buckling of portions of the beam cross section has on the overall dynamic response. Note that no attempt will be made herein to establish design recommendations for the case of impact loading of thin-walled cold-formed beams. The writers believe, however, that these results may be used for this purpose.

DYNAMIC RESPONSE

Typical response curves showing the variation of the internal moment as a function of time obtained from a linear elastic analysis and from the mathematical model developed previously (10) are shown in Fig. 1 for various values of the time duration of a triangular load pulse (2). These curves are for a simply supported thin-walled beam with a hat-shaped cross section subjected to a uniform load. The maximum value of the load with time is equal to four times the static load required to produce local buckling of the section. The time duration of the load has been nondimensionalized with respect to the fundamental period of vibration of the beam calculated by neglecting local buckling, $\beta = t_d/t_o$. The time axis has been nondimensionalized with respect to the time duration of the load pulse, $\tau = t/t_d$, and the moment with respect to the local buckling moment of the cross section. Note that the results obtained from both methods agree quite well.

Since design criteria for impact loading are usually based on maximum response, maximum stress or maximum deflection, it is usually not necessary for design purposes to consider the complete dynamic response of the specimen as a function of time. In order to simplify the design procedure, therefore, response spectra are usually developed (1, 3). These spectra are presented in a graphical form and show for a particular dynamic system and excitation the maximum values of some significant response (deflection, stress, velocity, etc.). A response spectra plot for the maximum edge stress in the top flange obtained for the same beam as in Fig. 1 is shown in Fig. 2. For the linear analysis the influence of local buckling was neglected in both the dynamic analysis for determining the internal moments and in the stress calculation using these moments. For a low value of load, $\alpha = 1$, very little local buckling occurs and the two methods of analysis give almost identical results. As the load magnitude increases, however, $\alpha = 4, 6$, the stresses obtained from

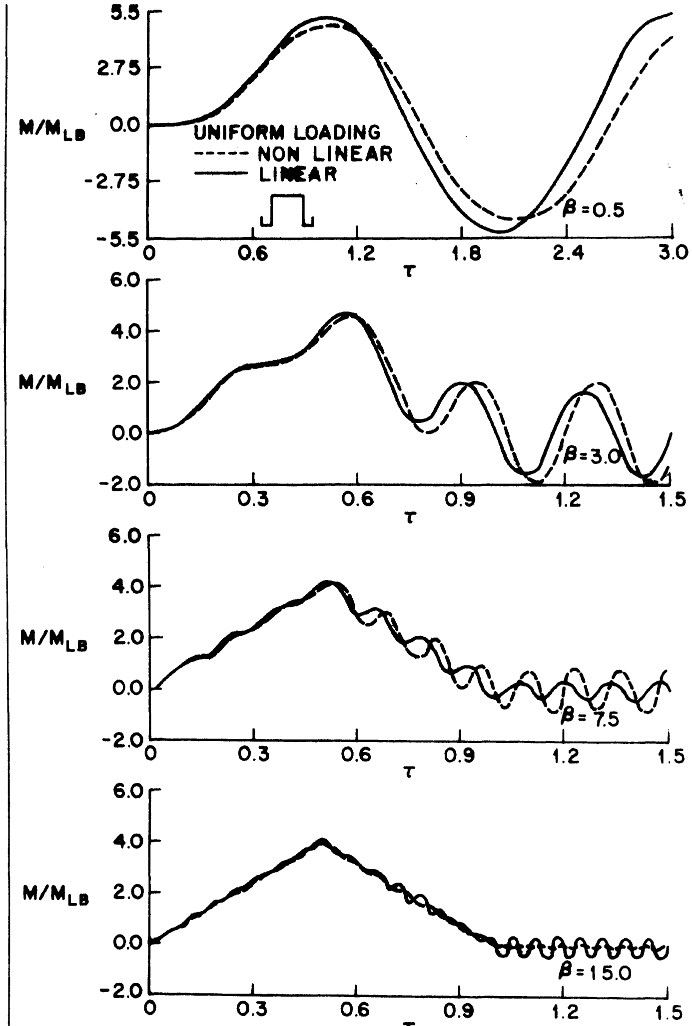

FIG. 1 - Beam Response-Moment (Triangular Pulse - $\alpha = 4.0$)

the linear analysis differ appreciably from those obtained taking local buckling into account.

The results in Figs. 1, 2 indicate that local buckling does not significantly influence the internal moments produced by impact loading but obviously must be considered in determining the stress produced by these moments. Results presented in Fig. 3 illustrate this trend. Spectra for both the maximum top flange and bottom flange stress are presented in Fig. 3. Note that the top flange stress obtained from the linear analysis is considerably less than that obtained from the nonlinear analysis. If however, the linear analysis is only used to determine the internal moments and then the stress computed using the concept of effective width (4), the results agree very closely with the nonlinear analysis. Since the maximum bottom flange stress is not influenced by local buckling to the same degree as the top flange stress, results obtained from the three methods of analysis agree more closely.

In dealing with nonlinear structures of the type considered above, it is generally not possible to develop response spectra which are applicable to a broad class of problems as is the case with linear structures (7). Since the degree of nonlinearity in the beam response is influenced by the geometric proportions of the cross section, it would be necessary to use a nonlinear dynamic analysis (10) or to develop response spectra similar to those in Figs.

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[4]Numerals in parentheses refer to corresponding items in Appendix I - References.

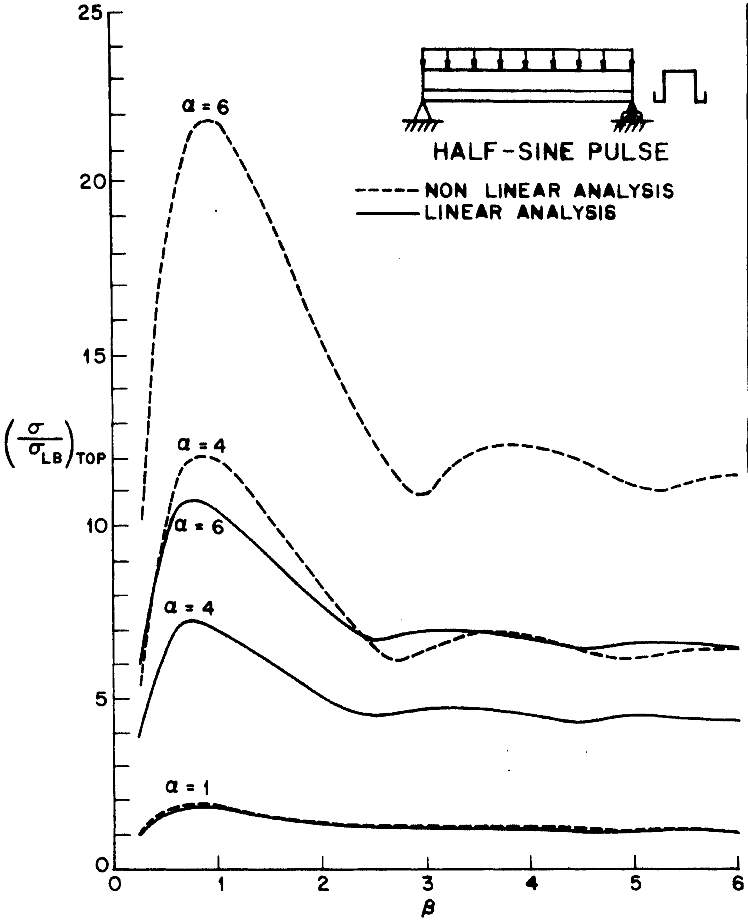

FIG. 2 - Response Spectra for Stress

2, 3 for the wide variety of cold-formed sections which may be manufactured (9). The advantages of using a linear analysis to determine the internal forces are obvious. Existing linear response spectra or simple impact factor formulas could be used to obtain the internal forces. Using these forces, the stresses could be calculated in the same manner as for static loading (9). The numerical accuracy of such a procedure is evaluated in this paper.

PROBLEM STATEMENT

The dynamic response of thin-walled beams is influenced by the following parameters: type of loading - uniform, concentrated; load magnitude; time variation of applied load; and the degree of nonlinearity of the specimen. In practical problems of impact the magnitude and time variation of the load are not well defined. Simplifying assumptions regarding these quantities are usually required. The dynamic loading used for this study is shown in Fig. 4. The degree of nonlinearity in a thin-walled beam is a function of the length of the beam over which the internal moments exceed the local buckling moment. Since this length is related to moment gradient the four loading conditions in Fig. 4a were selected. The time variation of the applied loads in Fig. 4b correspond to those used in previous studies of dynamic response.

The degree of nonlinearity of a thin-walled beam is influenced by the type of cross section. Although a wide variety of cold-formed sections are available only the cross sections shown in Fig. 5 were considered. The nonlinearity for these sections may be characterized by the relationship between the applied moment and the beam stiffness or moment of inertia. Relationships of this type are shown in Fig. 6 (10). These results are presented in nondimensional form by relating the stiffness to the original moment of inertia prior to local buckling, I_o and the internal moment to the moment required to produce local buckling, M_{LB} .

The curve for section A in Fig. 6 is linear for negative values of the moment and nonlinear for positive moment. The curve for section B is nonlinear for both positive and negative bending and is symmetric due to the symmetry

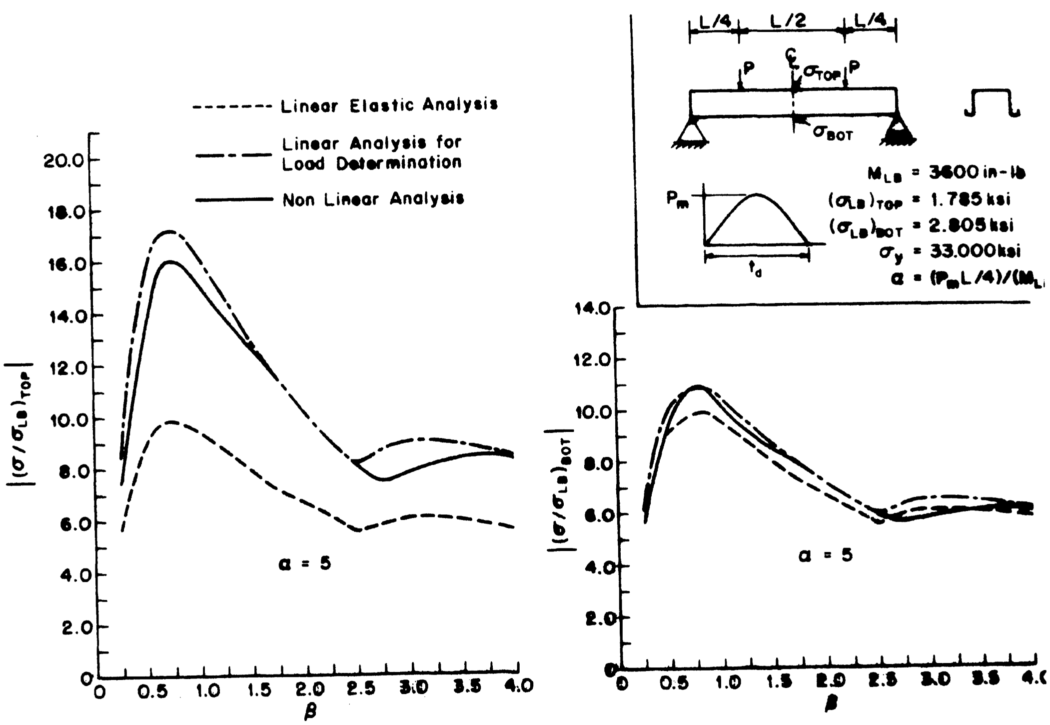

FIG. 3 - Comparison of Linear and Nonlinear Response Spectra

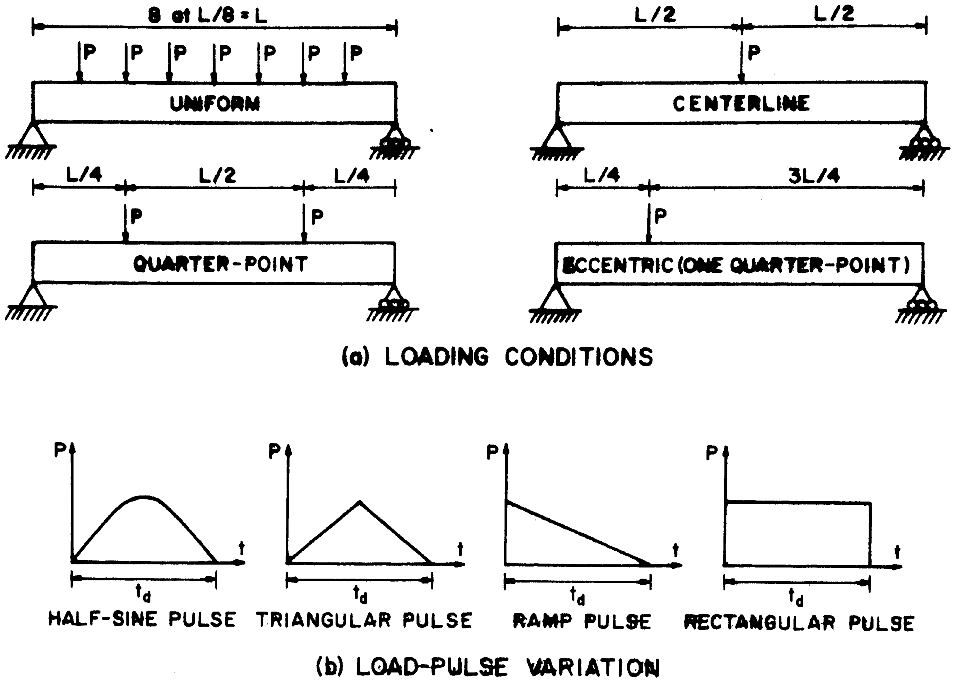

FIG. 4 - Dynamic Loading

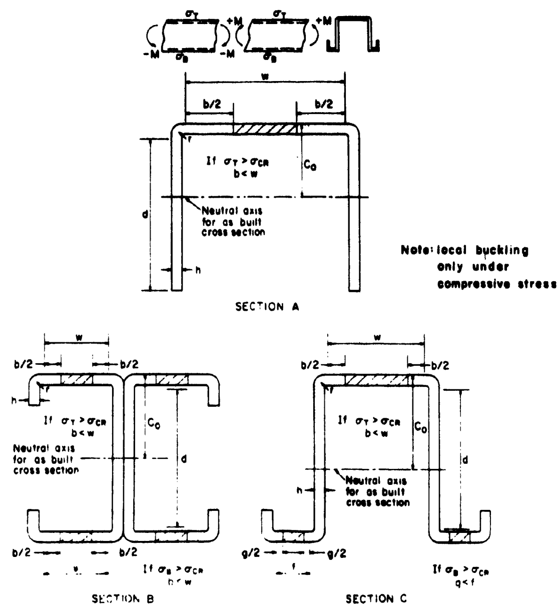

FIG. 5 - Cross Sections Investigated

of the cross section about the axis of bending. The curve for Section C is also nonlinear for positive and negative bending but is not symmetric since the buckling behavior of the top and bottom flanges is different. The three types of stiffness curves shown, therefore, cover the entire range of possibilities which could occur for any cold-formed beam regardless of the shape of the cross section.

Although the curves in Fig. 6 cover all the possible forms of stiffness variation, the amount of nonlinearity is influenced by the actual dimensions of the cross section. In order to determine this influence, I/I_o vs. M/M_{LB} curves were developed for a wide range of cross sectional dimensions (5). The cross sectional dimensions expressed in terms of the width to thickness ratios of the various elements for the three sections in Fig. 5 considered in these calculations were: Section A-$50 \leq w/h \leq 250$, $30 \leq d/h \leq 150$; Section B-$30 \leq w/h \leq 60$, $30 \leq d/h \leq 150$; Section C-$50 \leq w/h \leq 200$, $90 \leq d/h \leq 150$, $f/h = 40$. These curves

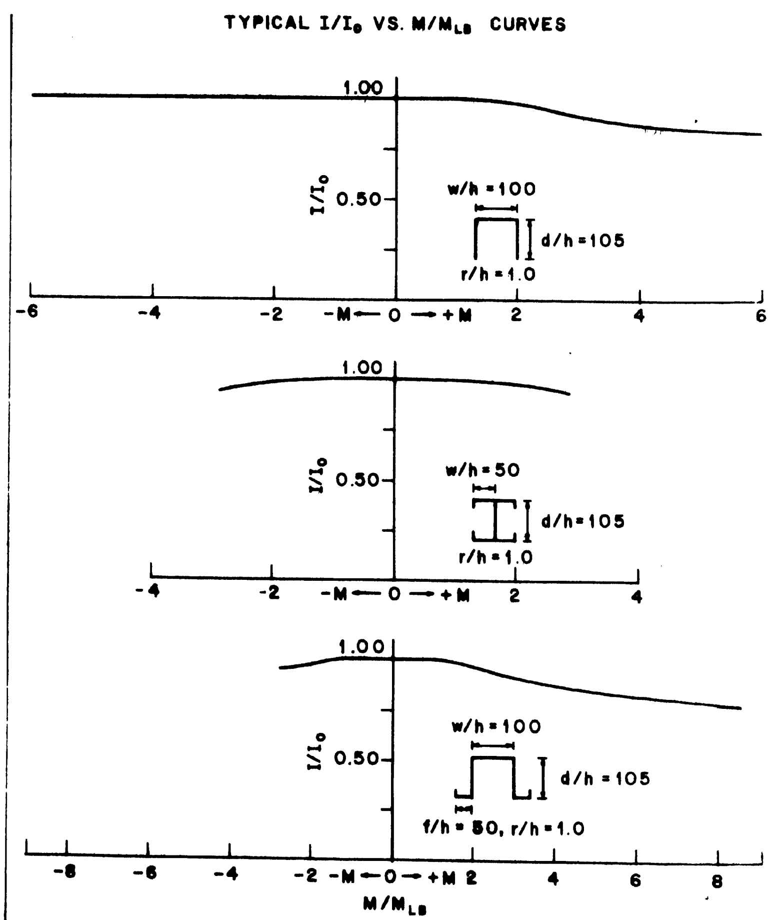

FIG. 6 - Typical I/I_o vs. M/M_{LB} Plot

were based on the 1962 edition of the AISI Design Specification (4) since the major portion of this study was completed prior to the release of the latest edition of this specification (6). The influence of the new specification on the results obtained will be discussed in a later section. All the curves for a particular cross section followed the same trend. The primary influence of the cross-sectional dimensions was the value of M/M_{LB} at which yielding of the section occurred. The curves were terminated at this point since initial yield is considered failure for cold-formed beams (4). For each cross section in Fig. 5 the I/I_o curve with the greatest nonlinearity or the largest reduction in stiffness for a particular moment was used in the study. The results presented in the next section, therefore, represent the maximum differences between a linear and nonlinear dynamic analysis for the ranges of cross sectional dimensions considered.

NUMERICAL RESULTS

Comparisons were made between the two methods of analysis for a wide range of the parameters mentioned above. Only a limited number of these results are included herein. All the results are, however, available elsewhere (5).

Internal Moment

The influence of the various parameters on the internal moments is shown in Figs. 7-10. The ratio of the maximum moments calculated from the linear and nonlinear analyses are plotted as a function of the nondimensional load duration β. Presented in this form, the percentage difference between the two moments can be readily obtained from these curves.

The influence of pulse shape is shown in Fig. 7. Note that the largest deviation between the two analyses occurs for the rectangular pulse for $\beta \simeq 0.33$. The magnitude of this deviation decreases for the other pulse shapes. In most cases the moment calculated from a linear analysis is greater than the moment from the nonlinear analysis, $M_{NL}/M_L < 1$.

Fig. 8 shows the influence of load location on the internal moments. The maximum deviation between the two methods occurs for quarter-point loading. This is to be expected since the length of the beam over which local buckling has occurred is greatest for quarter-point loading. The greatest increase in moment for the nonlinear analysis over the linear analysis is less than 5%.

Fig. 9 illustrates the effect of increasing load magnitude on the nonlinearity of the problem. As the load magnitude increases local buckling also increases resulting in an increasing difference between the two methods of analysis. The maximum increase in the moment based on the nonlinear analysis is less than 10% for this case.

The influence of the dimensions of the cross section on the method of analysis is shown in Fig. 10 for Section C. For this cross section, local buckling or the degree of nonlinearity is directly related to the width-to-thickness ratio, w/h, of the top flange. As this ratio increases the difference between the two methods of analysis also increases. Note that the maximum increase in the moment based on the nonlinear analysis over the linear analysis is only approximately 2% for $w/h = 250$.

The results presented in Figs. 7-10 indicate that, in general, the internal moment calculated using a linear elastic analysis is larger than the internal moment obtained from a nonlinear analysis. Using the linear analysis would, therefore, be conservative. For those cases in which the nonlinear moment exceeds the linear value, the deviation is less than 10% in all cases considered. For the practical situations in which impact loading must be considered this deviation is usually less than the accuracy to which the externally applied impact loading is known.

Deflections

In certain practical applications the value of the deflections produced by the dynamic loads are also of interest. Using the nonlinear analysis, values of the maximum deflections were calculated for the same conditions of dynamic loading as discussed for the moment calculations. Since the deflections are directly related to the beam stiffness it is obvious that results obtained from a linear analysis using the beam stiffness neglecting local buckling would differ considerably from the nonlinear results. For the case of static loading, deflection calculations using a uniform beam stiffness based on the effective cross section after local buckling at the point of maximum moment were found to be reasonably accurate (8). Using the same reasoning for dynamic loading, the following procedure was used to calculate deflections. The maximum deflection using a linear dynamic analysis and the original beam stiffness neglecting local buckling effects was first calculated. Using the maximum dynamic moment obtained from the nonlinear analysis the reduced

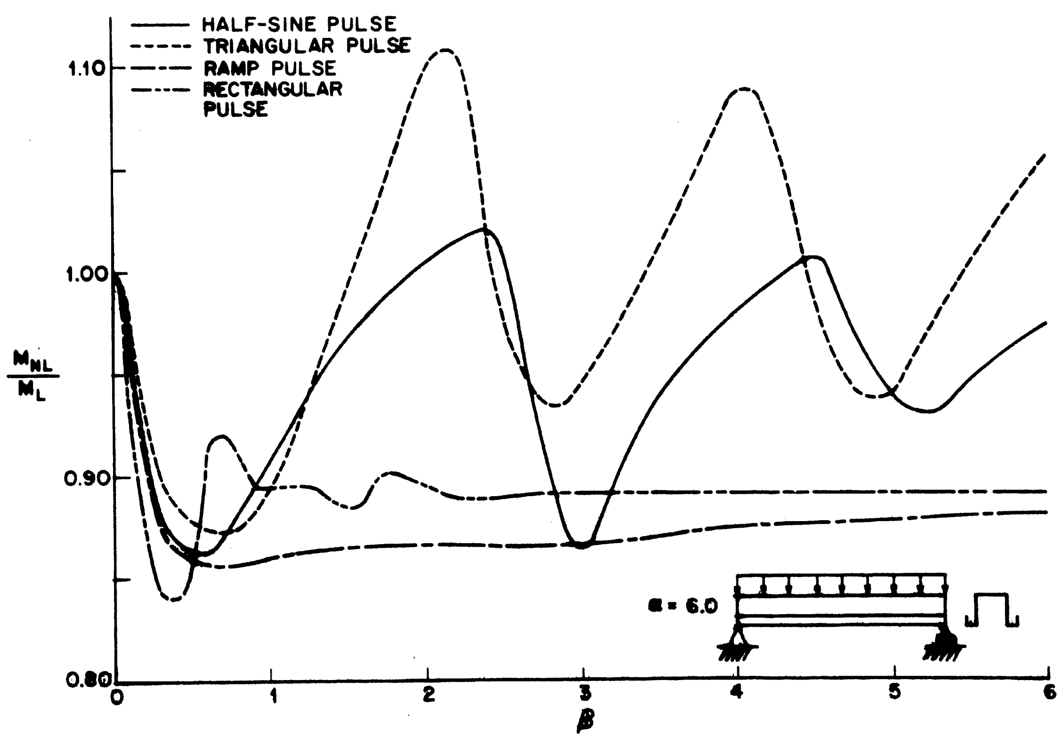

FIG. 7 - Influence of Pulse Shape on Response Spectra-Moment

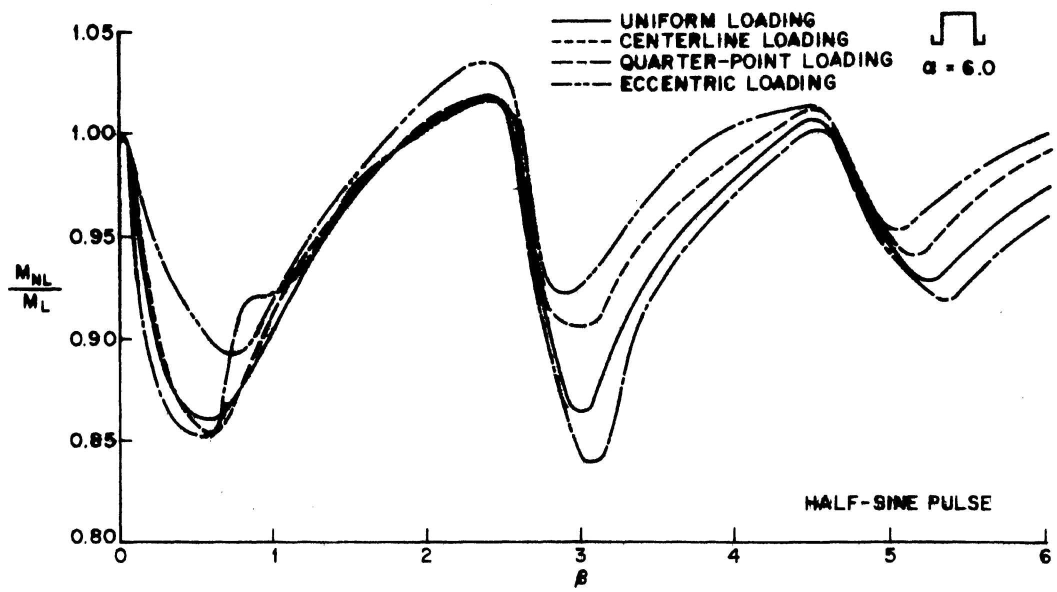

FIG. 8 - Influence of Load Location on Response Spectra-Moment

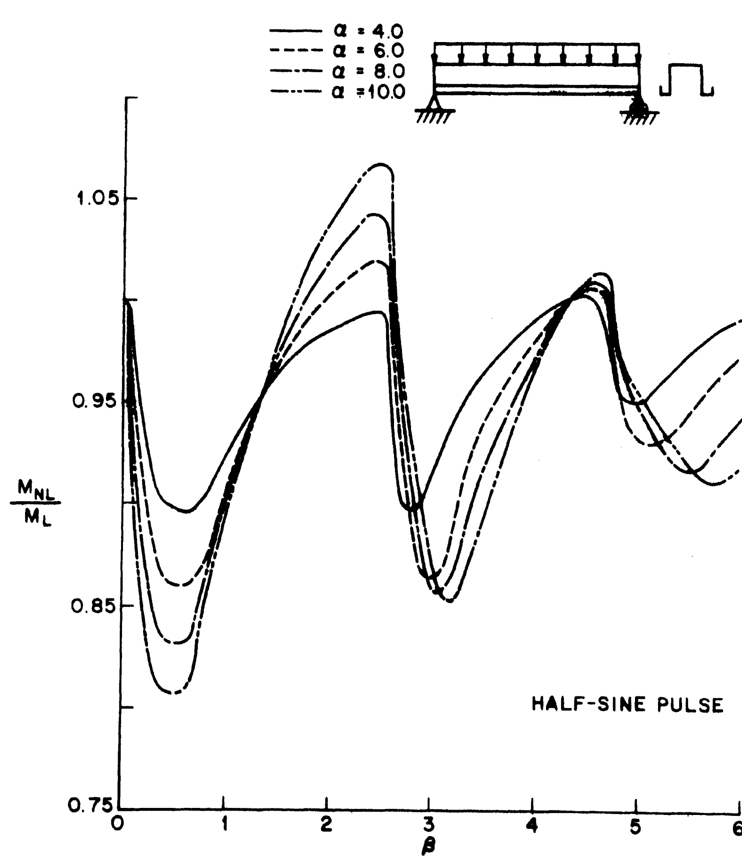

FIG. 9 - Influence of Load Magnitude on Response Spectra-Moment

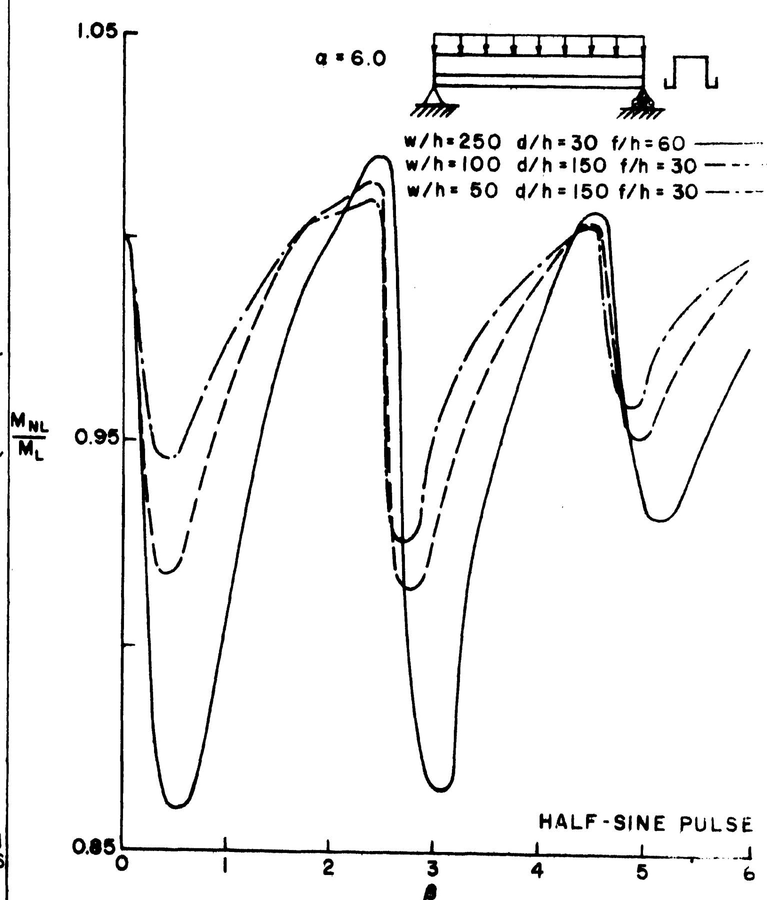

FIG. 10 - Influence of Beam Stiffness on Response Spectra-Moment

moment of inertia based on the effective width concept was calculated in accordance with established procedures (4, 9). The deflection obtained from the linear analysis was multiplied by the ratio of the original moment of inertia to the value obtained from the nonlinear analysis, $(I_o/I)_{max}$. This step replaced the original moment of inertia in the linear analysis with the minimum value which occurred under the dynamic loading.

Values of the ratio of the deflections obtained from the nonlinear analysis to values using the above procedure are given in Figs. 11-14 for the same range of dynamic loading parameters and section geometries considered for moment. The influence of the various parameters on the ratio of these deflections is similar to that observed for the ratio of internal moments. In general, the deflections calculated using the modified linear analysis are greater than those obtained from the nonlinear analysis. For those cases in which the nonlinear deflection exceeds the modified linear value, the deviation is less than 7%.

As mentioned previously, the above results were based on the 1962 edition of the AISI Specification. It is of interest, therefore, to consider the influence of the revised expressions for effective width in the 1968 edition of this specification on these results.

Comparing the two editions of the specification, it will be noted that the stress at which local buckling first occurs for a particular value of w/h has been increased in the 1968 edition. Also the effective width for a given stress level is greater according to the 1968 edition. Thus the beam stiffness based on the latest specification is increased above the corresponding value using the 1962 Specification. This increased stiffness tends to decrease the nonlinearity of the dynamic response and therefore the differences between the linear and nonlinear analyses considered previously should be less using the new specification.

A comparison of the internal moments using the 1968 Specification for the beam stiffness is given in Fig. 15. These results may be compared with those in Fig. 10 for the 1962 Specification. The value of α in Fig. 15 is based on the local buckling moment according to the new specification. Due to the higher value of this moment in the new specification the absolute value of the uniform load used to plot Fig. 15 corresponds to a load which is 83% greater than the load used to plot Fig. 10. Despite this substantial increase in load, values of the ratio of the internal moments in Fig.15 are only slightly different from these in Fig. 10. The general trends indicated in Figs. 7-15 and the accuracy of the linear analysis should therefore be similar for cold-formed beams designed according to the 1968 Specification.

The results and conclusions presented above were based on the analysis of simply supported beams subjected to well defined externally applied impact loads. Practical structures usually consist of several interconnected beams which act as a unit in resisting applied loads. A complete analysis of the influence of the parameters considered above on such structures is beyond the scope of this investigation. Based on the results presented herein, however, calculation of the internal moments, stresses and deflections using a linear elastic dynamic analysis modified in accordance with the procedures described above should lead to sufficiently accurate results for design purposes. For those cases in which extreme accuracy is required, it may be necessary to perform a nonlinear dynamic analysis which takes into account the variation with time of the stiffness of the members due to local buckling.

ILLUSTRATIVE EXAMPLE

The case of a beam subjected to a falling weight is one example of an impact loading problem of practical interest. Consider a ten foot long simply supported cold-formed beam with a hat-shaped cross section. Referring to Fig. 5, Section C, the dimensions of the cross section are given as

$$w = 6 \text{ in.}, d = 5.4 \text{ in.}, f = 2.4 \text{ in.}, r = 0.06 \text{ in.}, h = 0.06 \text{ in.}$$

and the depth of the bottom flange stiffening lip is 0.565 in. Using these properties, the moment of inertia, I_o, is 7.132 in^4. The lowest natural frequency of the beam, ω_o, is 156 radians/sec. and the fundamental natural period, t_o, is 0.0641 sec.

The dynamic force applied by the weight which strikes the beam at the centerline is assumed to be represented by the triangular pulse in Fig. 4. The magnitude and duration of the pulse are arbitrarily specified as 1.139 kips

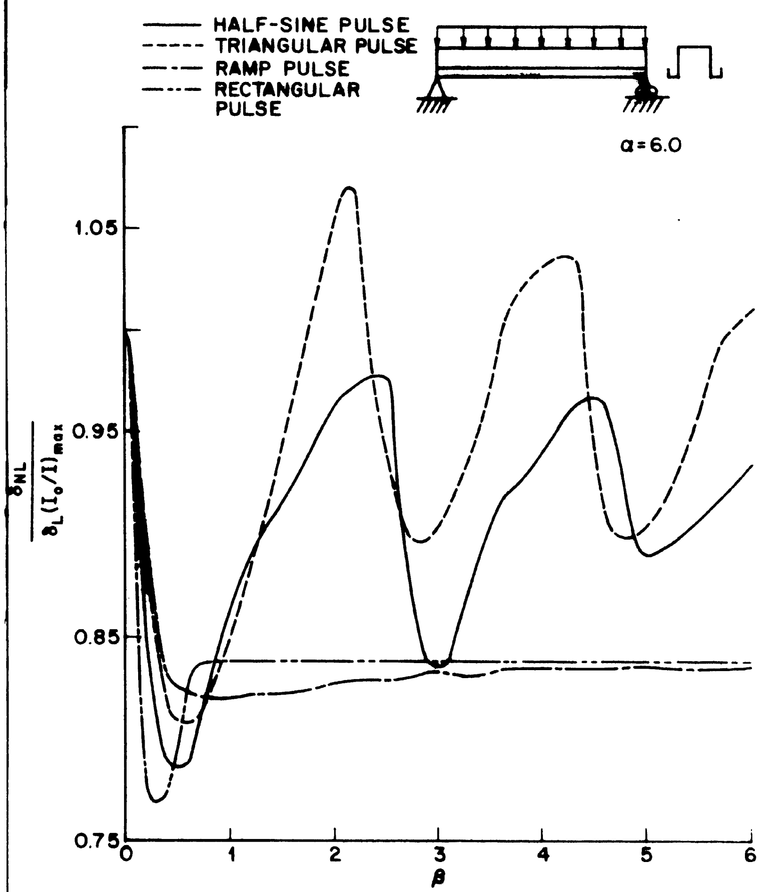

Fig. 11 Influence of Pulse Shape on Response Spectra - Deflection

and 0.077 sec., respectively. In a practical problem, methods are available for calculating this load magnitude and time duration using the weight and height of drop. Using the above values, the dimensionless parameters associated with the externally applied load are $\alpha = 4.0$ and $\beta = 1.2$.

Using a linear elastic dynamic analysis based on normal mode superposition techniques (1), the maximum internal moment and the maximum deflection produced by this load are 36.21 kip-in. and 0.28 in. respectively. The original moment of inertia, I_o, was used to obtain these results.

Using the 1962 edition of the AISI Specification (4), the local buckling stress for this beam is 2.663 ksi, and the local buckling moment, M_{LB}, based on the original section modulus is 6.823 kip-in. The ratio of the maximum dynamic moment to the local buckling moment therefore is $M/M_{LB} = 36.21/6.823 = 5.31$. Using this value of M/M_{LB} and $w/h = 100$, $d/h = 90$, and $f/h = 40$, a value of $\sigma/\sigma_y = 0.61$ is obtained for the maximum edge stress in the compression flange. For $\sigma_y = 33$ ksi, the maximum compressive stress produced by this impact load is therefore 20.0 ksi. Calculating the reduced moment of

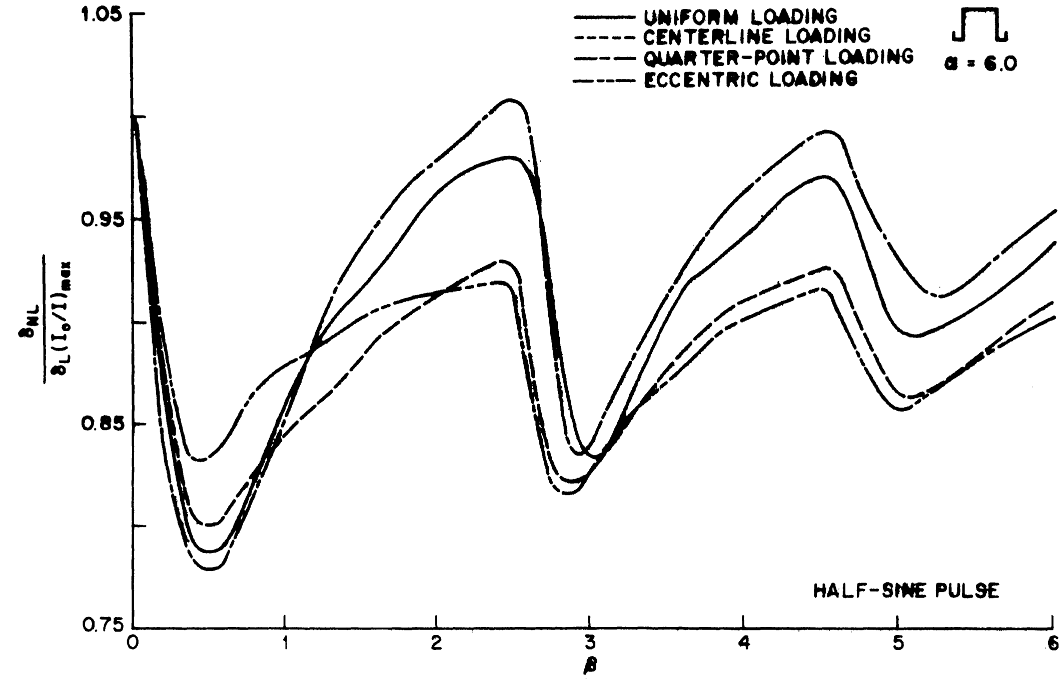

FIG. 12 - Influence of Load Location on Response Spectra-Deflection

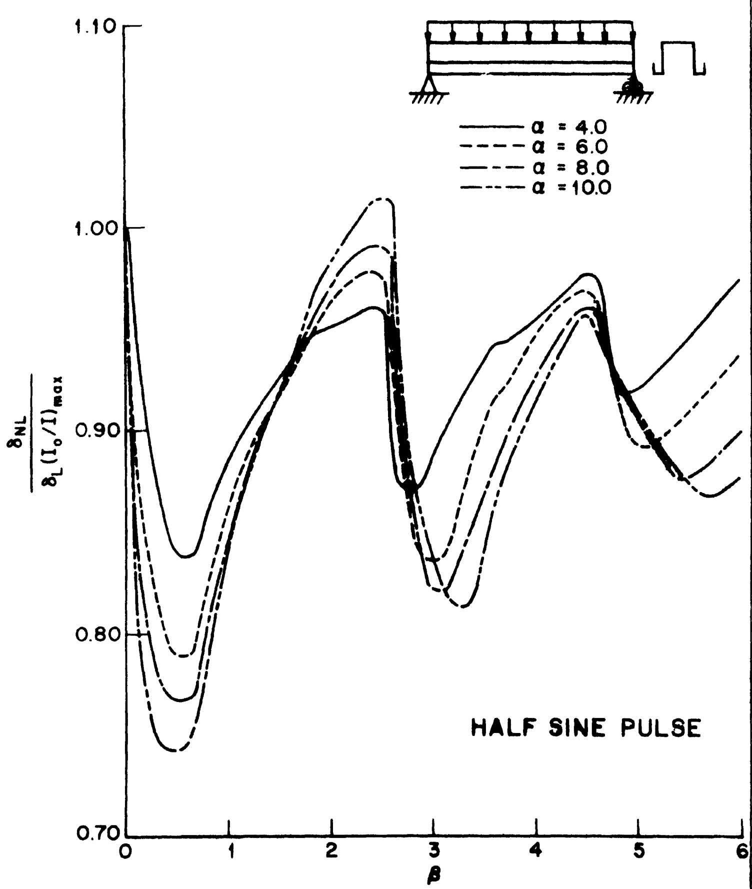

FIG. 13 - Influence of Load Magnitude on Response Spectra-Deflection

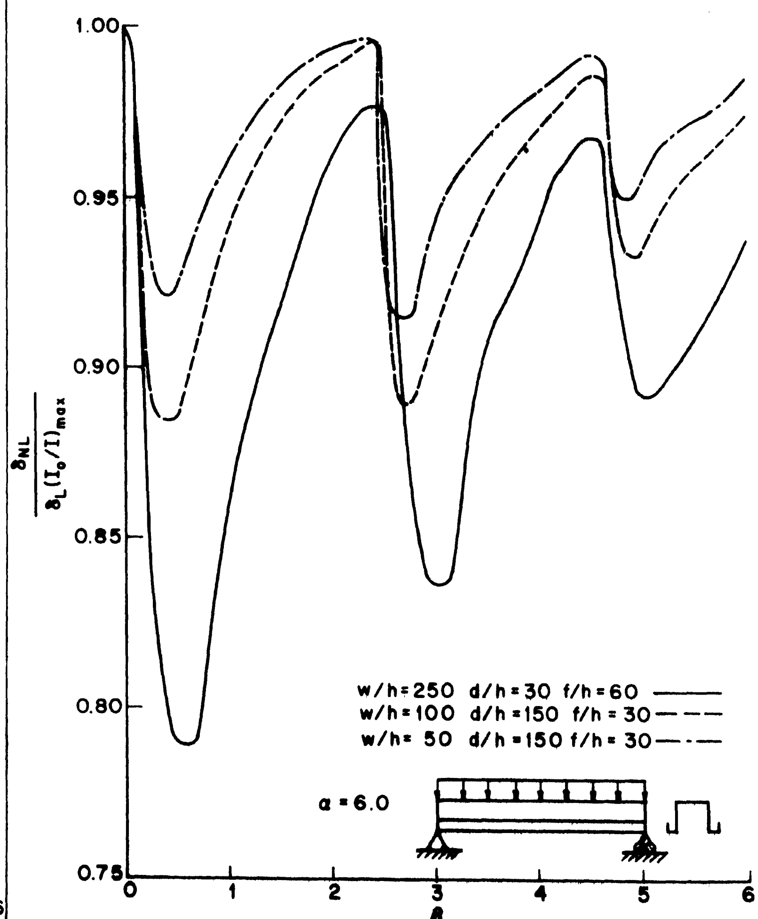

FIG. 14 - Influence of Beam Stiffness on Response Spectra-Deflection

inertia using the effective width corresponding to this stress level (4) gives $I = 5.95$ in^4. The ratio of this value to the original moment of inertia, I/I_o, is 0.84. Multiplying the deflection obtained above by the inverse of this value gives a final deflection of 0.336 in.

The exact value of the maximum stress and deflection for this problem, obtained from the nonlinear mathematical model (10) are 17.9 ksi and 0.309 in., respectively. Comparing these results with the approximate values given above indicates that the approximate solution based on the linear analysis overestimates the maximum stress by 11.7% and overestimates the maximum deflection by 8.7%.

Design aids could be provided to facilitate the computations mentioned above.

A typical example of such a design aid is illustrated in Fig. 16. This figure was prepared for a hat section and relates the top flange stress to the local buckling moment for a variety of cross section dimensions. In order to determine the maximum top flange stress produced by a particular shock loading using this figure, it is only necessary to determine the maximum moment in the structure using a linear dynamic analysis. Using this moment, the local buckling moment calculated using the local buckling stress and the original cross-sectional moment of inertia, the ratio of the maximum stress produced by this moment to the yield stress can be read directly from Fig. 16. Note that a design aid in this form eliminates the necessity of a trial and error calculation of the section modulus using the effective width concept (9). Similar design aids could also be prepared for other types of cross sections as well as different values of the yield stress. Simplifications could also be made in performing the linear dynamic analysis. Existing response spectra for linear systems (1) or impact factor formulas could be used to eliminate the need for a complete dynamic analysis.

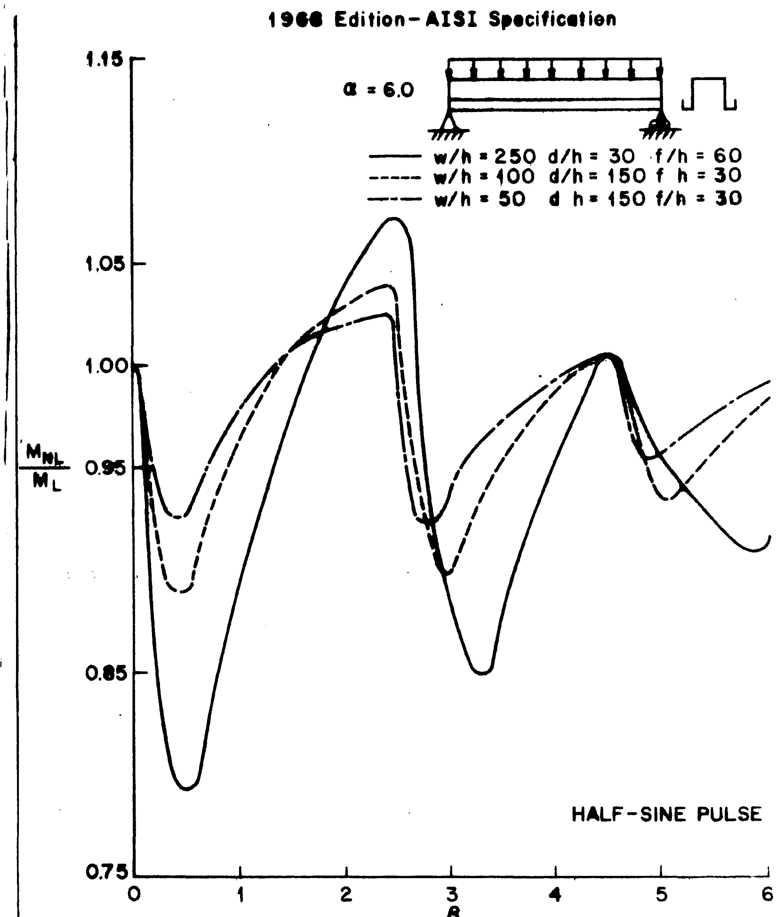

FIG. 15 - Influence of Beam Stiffness on Response Spectra-Moment (1968 Specification)

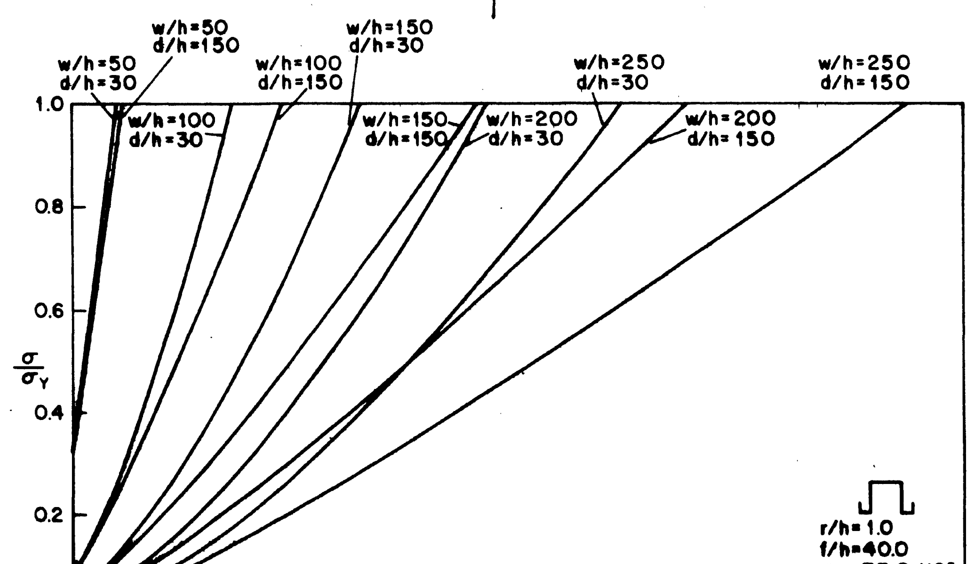

FIG. 16 - Sample Design Aid

SUMMARY AND CONCLUSIONS

The internal moments and deflections in thin-walled cold-formed beams subjected to impact loading were obtained using a linear and nonlinear dynamic analysis. Comparisions of the two methods of analysis indicated that the internal moments in these beams subjected to impact loading could be determined reasonably accurately using classical methods of linear vibration theory. These moments can then be used in connection with methods of static analysis which incorporate the effects of local buckling to obtain stresses and deflections.

ACKNOWLEDGMENTS

The results presented herein were taken from a masters thesis submitted by Arthur H. Osgood to the Department of Civil Engineering, Carnegie-Mellon University. Charles G. Culver served as the thesis advisor. This work represents the third phase of a research program on "Light Gage Cold-Formed Structural Elements Subjected to Time-Dependent Loading" sponsored by the American Iron and Steel Institute at Carnegie-Mellon University. The cooperation of W. G. Kirkland, Vice President, AISI, and the members of the AISI Task Group on the Influence of Dynamic Loading on Structural Behavior of Light Gage Steel, R. B. Matlock, Chairman, J. B. Scalzi and C. R. Clauer is gratefully acknowledged. A. L. Johnson and J. J. Healey of AISI provided assistance in reviewing the progress of this research and editing of the publications.

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APPENDIX II - NOTATION

The following notation is used in this paper:

b	=	effective width of flange of beam;
c_o	=	distance from the neutral axis for as built cross section to extreme top flange fiber of beam;
d	=	flat depth of beam - excludes fillets;
E	=	modulus of elasticity;
f	=	flat width of bottom flange of beam Section C;
g	=	effective width of bottom flange of beam Section C;
h	=	thickness of beam element;
I	=	moment of inertia;
I_o	=	I computed from the original cross section properties;
L	=	span length of beam;
M_L	=	internal moment obtained from linear analysis;
M_{LB}	=	local buckling moment;
M_{NL}	=	internal moment obtained from nonlinear analysis;
P	=	magnitude of applied load;
r	=	radius to centerline of fillet;
t	=	time;
t_d	=	time duration of load pulse;
t_o	=	fundamental natural period of beam;
w	=	flat width of top flange of beam;
α	=	maximum moment produced by external load divided by local buckling moment;
β	=	time duration of load pulse divided by natural period of beam;
δ_L	=	deflection obtained from linear analysis;
δ_{NL}	=	deflection obtained from nonlinear analysis;
σ	=	stress;
σ_{LB}	=	local buckling stress for beam;
σ_y	=	yield stress;
τ	=	nondimensional time; and
ω_o	=	natural frequency of the lowest mode of beam.