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Direct Strength Method of Design for Shear of Cold-formed Channels Based on a Shear Signature Curve

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Abstract

Thin-walled sections in compression and/or bending may undergo one of the three modes of local, distortional or overall (Euler) buckling, or combinations of these. The Semi-Analytical Finite Strip Method (SAFSM) developed by YK Cheung has been widely used in computer software (THIN-WALL, CUFSM) to develop the signature curve of the buckling stress versus buckling half-wavelength for a thin-walled section under compression or bending to allow identification of these modes. The minimum points on the signature curve are now used in the Direct Strength Method (DSM) of design of cold-formed sections in the American Specification and Australian/New Zealand Standard for cold-formed steel structures.

Plank and Wittrick (1974) included shear in the SAFSM theory for calculating the stiffness and stability matrices by using complex mathematics. The complex mathematics is needed to allow for the phase shifts in the buckling modes (eigenvectors) for sections under shear.

This paper briefly summarises the theory then applies it to the buckling of channel sections in pure shear. Signature curves for shear are developed for channel sections and compared with classical solutions, and those produced by the Spline Finite Strip Method (SFSM) previously published by the authors. A proposed Direct Strength Method (DSM) of design for shear is explained in the paper.

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Introduction

Folded plate and finite strip theories for the buckling analysis of thin-walled sections and stiffened panels in compression have been developed since the mid-1960s. Two basic approaches were adopted. These are the exact solutions of Wittrick (1968), and Williams and Wittrick (1969), and the approximate solutions of Przemieniecki (1972) and Plank and Wittrick (1974) based on the finite strip method of analysis developed by YK Cheung (1976). The exact methods only apply to uniform stress such as uniform compression and not bending. The first paper of Wittrick (1968) shows three distinct buckling modes for a stiffened panel in pure compression being called overall, torsional and local. A paper by Williams (1974) using the exact method calls the three distinct modes of a lipped channel strut as local, flange and flexural/flexural-torsional. Hancock (1978) applied the finite strip method developed by Plank and Wittrick (now called the Semi-Analytical Finite Strip Method (SAFSM)) to beams and identified local, distortional and lateral-torsional modes. The SAFSM has the advantage that it includes strips in bending and so could study beams as well as compression members. The paper by Hancock (1978) clearly identified the signature curve for a beam being the buckling stress versus the buckle half-wavelength for a single half-wavelength. Recent developments have included the Constrained Finite Strip Method (cFSM) (Adany and Schafer, 2006) which has allowed the buckling mode decomposition into pure local, distortional and overall modes. In the earlier papers, the modes tended to be a combination of the basic modes although normally dominated by one at a particular half-wavelength.

The case of sections in pure shear has not been studied using the SAFSM although the methodology was available in the Plank and Wittrick (1974) paper. Recently the Spline Finite Strip Method (SFSM) of buckling analysis developed by Lau and Hancock (1986) was used to study the elastic buckling of thin-walled channel sections in pure shear (Pham and Hancock, 2009). However, the SFSM does not allow the signature curve for shear to be isolated because it computes the minimum buckling stress irrespective of the number of half-waves over the length. In the Plank and Wittrick paper, a complex finite strip method was developed to allow for the case of shear as well as compression and bending. The buckling modes require the deformations to be described with complex terms to allow for the phase shifts along the member.

The Direct Strength Method (DSM) of design of cold-formed sections (Schafer and Peköz, 1998) and recently incorporated in the North American Specification (AISI, 2007) and Australian New/Zealand Standard AS/NZS 4600:2005 (Standards Australia, 2005) for cold-formed steel structures provides design for compression and bending based on the signature curves for compression and bending. Proposals for shear by Pham and Hancock (2012) require the shear local buckling load V_{cr}. In the Pham and Hancock paper, the SFSM values have been used. However, it is suggested that the values from the signature curve in pure shear will provide a simpler and more reliable method of determining V_{cr}.

Plate Buckling Deformations

The plate flexural deformations (w) of a strip can be described by:

$$w = Re\big(f_1(y).X_1(x)\big) \tag{1}$$

where the x-axis is in the longitudinal direction in the plane of the strip, the y-axis is in the transverse direction in the plane of the strip, and w is in the z-direction perpendicular to the strip as shown in Fig. 1. The term Re denotes the real part of the complex function.

The complex function $f_1(y)$ is the transverse variation given by:

$$f_1(y) = \alpha_{1F} + \alpha_{2F}.\left(\frac{y}{b}\right) + \alpha_{3F}.\left(\frac{y}{b}\right)^2 + \alpha_{4F}F.\left(\frac{y}{b}\right)^3 \tag{2}$$

where the 4 polynomial coefficients α_{iF} each consist of a real part α_{iFR} and a complex part α_{iFI}. There are therefore 8 unknown coefficients in Eq. 2. The term b is the width of the strip.

The complex function $X_1(x)$ is the longitudinal variation given by:

$$X_1(x) = cos\left(\frac{m\pi x}{L}\right) + i\ sin\left(\frac{m\pi x}{L}\right) \tag{3}$$

where L is the length of the strip and m is the number of buckle half-waves along the length of the strip.

The problem essentially has double the number of freedoms of a normal SAFSM analysis as a result of the real and complex components of the polynomial coefficients although the matrices involved remain the same size and have real and complex parts.

The plate membrane deformations (u, v) in the (x,y) directions respectively can be described by:

$$v = Re\big(f_v(y).X_1(x)\big) \tag{4}$$

$$u = Re\big(f_u(y).i.X_1(x)\big) \tag{5}$$

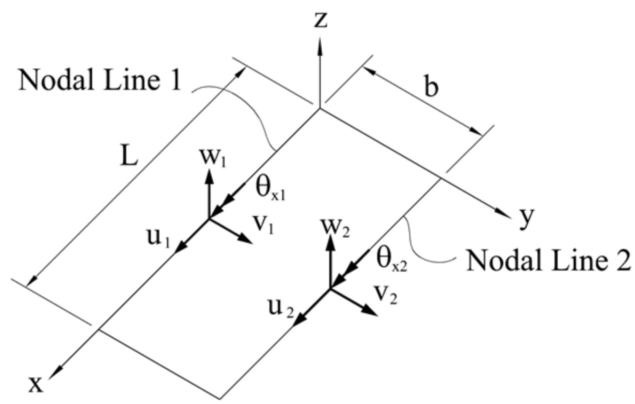

Figure 1. Strip Axes and Nodal Line Deformations

The complex functions $f_u(y)$ and $f_v(y)$ are the transverse variations given by:

$$f_v(y) = \alpha_{1M} + \alpha_{2M} . \left(\frac{y}{b}\right) \tag{6}$$

$$f_u(y) = \alpha_{3M} + \alpha_{4M} . \left(\frac{y}{b}\right) \tag{7}$$

where the 4 polynomial coefficients α_{iM} each consist of a real part α_{iMR} and a complex part α_{iMI}. There are therefore 8 unknown coefficients in Eqs. 6 and 7.

The nodal line flexural deformations $\{\delta_F\} = (w_1, \theta_{x1}, w_2, \theta_{x2})^T$ in Fig. 1 have real and complex components and can be related to the polynomial coefficients in (2) above by:

$$\{\delta_F\} = [C_F]\{\alpha_F\} \tag{8}$$

where $\{\alpha_F\} = (\alpha_{1F} \quad \alpha_{2F} \quad \alpha_{3F} \quad \alpha_{4F})^T$

Similarly, the nodal line membrane deformations $\{\delta_M\} = (u_1, v_1, u_2, v_2)^T$ in Fig. 1 have real and complex components and can be related to the polynomial coefficients in (6) and (7) above by:

$$\{\delta_M\} = [C_M]\{\alpha_M\} \tag{9}$$

where $\{\alpha_M\} = (\alpha_{1M} \quad \alpha_{2M} \quad \alpha_{3M} \quad \alpha_{4M})^T$

The matrices $[C_F]$ and $[C_M]$ are real. Their inverses $[C_F]^{-1}$, $[C_M]^{-1}$, which are required to compute the strip stiffness and stability matrices, are given in Appendices 1 and 2 of Hancock and Pham (2011) respectively.

Strain energy and potential energy

In order to compute the stiffness and stability matrices of the strip according to conventional finite strip theory (Cheung, 1976), it is necessary to define the strain energy in the strip under deformation and the potential energy of the membrane forces.

The flexural strain energy U_F is given by:

$$U_F = \frac{1}{2}\int_0^L \int_0^b (-M_x \frac{\partial^2 w}{\partial x^2} - M_y \frac{\partial^2 w}{\partial y^2} + 2M_{xy} \frac{\partial^2 w}{\partial x \partial y})\, dy\, dx \qquad (10)$$

where M_x, M_y, and M_{xy} are the bending moments per unit length in the x, y directions and M_{xy} is the twisting moment per unit length.

The membrane strain energy U_M is given by:

$$U_M = \frac{1}{2}\int_0^L \int_0^b (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy})\, dy\, dx \qquad (11)$$

where σ_x, σ_y, and τ_{xy} are the membrane normal stresses in the x, y directions, τ_{xy} is the membrane shear stress, ε_x, ε_y, are the membrane normal strains per unit length in the x, y directions and τ_{xy} is the membrane shear strain.

The flexural potential energy of the membrane forces V_F is given by:

$$V_F = -\frac{1}{2}\int_0^L \int_0^b (\sigma_x \left(\frac{\partial w}{\partial x}\right)^2 + \sigma_y \left(\frac{\partial w}{\partial y}\right)^2 + 2\tau_{xy} \left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right))\, t dy\, dx \qquad (12)$$

where σ_x, σ_y, and τ_{xy} are the membrane normal and shear stresses with the signs given in Fig. 2, and t is the plate thickness.

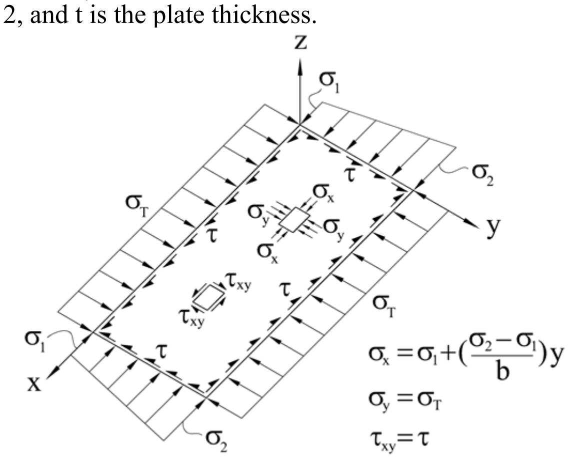

Figure 2. Membrane Stresses

The membrane potential energy of the membrane forces V_M is given by:

$$V_M = -\frac{1}{2}\int_0^L \int_0^b (\sigma_x \left(\frac{\partial u}{\partial x}\right)^2 + \sigma_x \left(\frac{\partial v}{\partial x}\right)^2)\, t dy\, dx \tag{13}$$

As stated in Plank and Wittrick (1974), it is believed that there are no membrane instabilities associated with transverse stress (σ_T) and shear stress (τ) so that there are no terms in Eq. 13 associated with these.

Plate theory

The plate flexural and membrane theory is that used by Cheung (1976) and is summarized in Hancock and Pham (2011).

Stiffness and stability matrices

For equilibrium, the theorem of minimum total potential for the flexural energy is:

$$\frac{\partial(U_F+V_F)}{\partial\{\delta_F\}} = 0 \tag{14}$$

Substitution for U_F from (10) and V_F from (12) using deformations w from (1) and using plate theory results in:

$$([k_F] - \lambda_F[g_F])\,\{\delta_F\} = 0 \tag{15}$$

The flexural stiffness matrix $[k_F]$ is real and the flexural stability matrix $[g_F]$ is real if the shear stress τ is zero. However, the stability matrix $[g_F]$ is complex Hermitian if the shear stress is non-zero. The term λ_F is the load factor. The solution to (15) is an eigenvalue problem which requires eigenvalue routines for Hermitian matrices if the shear stress is non-zero. The eigenvalues λ_F of a Hermitian matrix corresponding to the buckling load factors are real. The corresponding eigenvectors $\{\delta_F\}$ which are the buckling modes are complex if the shear stress is non-zero. The matrices $[k_F]$ and $[g_F]$ are given in Appendix 1 of Hancock and Pham (2011).

For equilibrium, the theorem of minimum total potential for the membrane energy is:

$$\frac{\partial(U_M+V_M)}{\partial\{\delta_M\}} = 0 \tag{16}$$

Substitution for U_M from (11) and V_M from (13) using deformations u from (5), deformations v from (4) and including the membrane theory results in:

$$([k_M] - \lambda_M\,[g_M])\,\{\delta_M\} = 0 \tag{17}$$

The membrane stiffness matrix [k_M] is real and the membrane stability matrix [g_M] is real. The term λ_M is the load factor. The solution to (17) is an eigenvalue problem. The corresponding eigenvectors {δ_M} are the buckling modes. The matrices [k_M] and [g_M] are given in Appendix 2 of Hancock and Pham (2011).

For folded plate assemblies including thin-walled sections such as channels, (15) and (17) must be transformed to a global co-ordinate system to assemble the stiffness [K] and stability [G] matrices of the folded plate assembly or section. Since the flexural displacements w given by (1) and the membrane displacements v given by (4) use the same longitudinal displacement function $X_1(x)$, they are conformable resulting in convergence to classical solutions for thin-walled sections.

The stiffness equations for a folded plate system are given by:

$$([K] - \lambda_E [G])\{\delta\} = \{0\} \quad (18)$$

where λ_E is the load factor for the whole system. The system stability matrix [G] is Hermitian if the shear stress is non-zero. Hence eigenvalue routines for Hermitian matrices are required to solve (18).

Eigenvalue routines for Hermitian matrices

Sturm sequence property

From the theory of equations (Turnbull, 1946), the leading principal minors of [C] - λ[I] (where [I] is a unit matrix) form a Sturm sequence. The leading principal minor of order r is given by det([C_r] – λ[I_r]) where [C_r] is the leading principal sub-matrix of order r of [C]. The first term of the Sturm sequence is the leading principal minor of order r = 0 and is defined to be unity.
The number of eigenvalues greater than λ is equal to the number of agreements in sign between consecutive members of the Sturm sequence from r = 0 to r = n where n is the dimension of the matrix [C]. This property is very useful in isolating the range of λ in which a particular eigenvalue is located. The eigenvalue corresponding to a particular mode number can be isolated by bisection between values of λ which bound the eigenvalue.

Direct computation of sign count of ([A] - λ[B])

Peters and Wilkinson (1969) have shown that the sign of det([A_r] – λ[B_r]) is the same as that of det([C_r] – λ[I_r]). Consequently, it is possible to apply the Sturm sequence directly to ([A] - λ[B]) without the need to transform to the standard eigenvalue problem det([C]-λ[I]) = 0.

For the finite strip buckling analysis given by (18), [G] is chosen as [A] and [K] is chosen as [B] so that the computed eigenvalues λ of ([A] - λ[B]) are the reciprocals of the load factors λ_E. Now the determinant of a Hermitian matrix is real so the Sturm sequence count is not affected by [G] being Hermitian. This allows the Sturm sequence property to be used to compute the eigenvalues of (18).

Eigenvector calculation

Wilkinson (1958) has produced a method for computing the eigenvector {δ} of (18) by solving the equations at the value of λ_E for a unit right hand side vector {1} replacing {0} in (18). The components of {1} are given in Eq. (19). The process is usually repeated once to purify the eigenvector with the unit vector {1} replaced by {δ} from the first iteration. This method has been used in the calculations in this paper. It is to be noted that since [G] is complex Hermitian, {δ} is complex reflecting the phase shifts in the eigenmode along the member. An interesting discovery is that the phase position of the mode along the member can be shifted by adjusting the real and imaginary components of the unit vector {1}. This is perfectly valid since the eigenvector position along the member is not predetermined.

Computer program bfinst7.cpp

A computer program bfinst7.cpp has been written in Visual Studio C++ to assemble the stiffness and stability matrices (15), (17) and (18) and to solve for the eigenvalues using the Sturm sequence property described above, and to compute the corresponding eigenvectors. The program stores the real [K] matrix and the real [G] and complex [GI] components of the stability matrices in order to extract the eigenvalues and eigenvectors. In the calculation of the eigenvectors the unit vector {1} is represented for all terms by:

$$\{1\} = \{f + (1 - f)i\} \tag{19}$$

where f gives the fraction of the real and complex components in the unit vector.

Solutions to plates and sections in shear

Plate simply supported on both longitudinal edges

The solution for a plate simply supported along both longitudinal edges is compared with the classical solution of Timoshenko and Gere (1961) Item 9.7 Buckling of rectangular plates under the action of shearing stresses. This comparison has also been performed by Plank and Wittrick (1974) to validate the accuracy of the method. The equation for the elastic buckling of a rectangular plate is given (Timoshenko and Gere, 1961) as:

$$\tau_{cr} = \frac{k_v \pi^2 D}{t\, b^2} \qquad (20)$$

where D is the plate flexural rigidity, b is the width of the plate which may consist of multiple strips and k_v is the plate buckling coefficient in shear.

The analysis must be carried out for a range of half-wavelengths (L/m in Eq. 3) to find the minimum value of buckling stress and corresponding buckle half-wavelength. Timoshenko and Gere give this latter value as $1.5^{0.5}b = 1.225b$ and Plank and Wittrick have arrived at the value corresponding to the minimum point of 1.252b, slightly higher than Timoskenko and Gere. The minimum k_v at 1.252b from bfinst7.cpp using 8 strips is 5.3385 which is exactly the same as the Plank and Wittrick paper value as would be expected. The minimum value of k_v in Timoshenko and Gere is 5.35.

Lipped channel section in pure shear

In order to extend the study to lipped channel sections, a 200mm deep lipped channel with flange width 80mm, lip length 20mm and thickness 2mm as studied by Pham and Hancock (2009) has been used. These dimensions are all centreline and not overall. In Pham and Hancock (2009), three different shear stress distributions have been investigated. These are uniform shear in the web alone (called Cases A/B), uniform shear in the web and flanges (called Case C), and a shear stress equivalent to a shear flow as occurs in a channel section under a shear force parallel with the web through the shear centre (Case D as shown in Fig. 3).

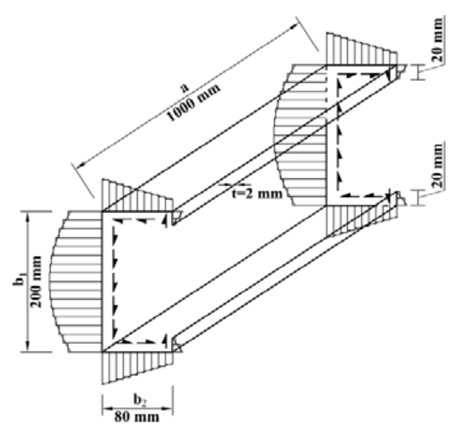

Figure 3. Shear Flow Distribution Assumed (Case D)

In this paper, only Case D is studied as it is the most representative of practice. The shear flow distribution is not in equilibrium longitudinally as this can only be achieved by way of a moment gradient in the section. However, it has been used in these studies to isolate the shear from the bending for the purpose of identifying pure shear buckling loads and modes. The finite strip buckling analysis allows the uniform shear stresses in each strip, as shown in Fig. 2, to be used to assemble the stability matrix [k_g] of each strip then the system stability matrix [G]. Fig. 3 demonstrates that the shear flow in each strip is assumed uniform. In the studies in this paper, the web is divided into eight equal width strips, the flanges into four each and the lips into one each making 18 strips and 19 nodal line with a total of 76 degrees of freedom each having real and complex components.

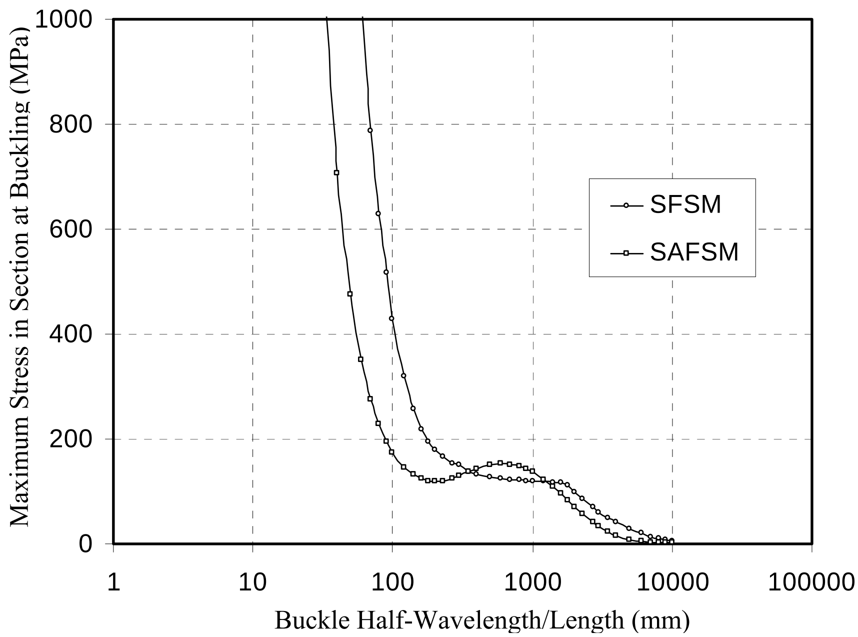

Figure 4. SAFSM and SFSM Curves of Buckling Stress versus Half Wavelength/Length for Plain Lipped Channel

The SAFSM graph (symbolically squares) of buckling stress versus buckle half-wavelength (signature curve) is shown for the lipped channel for a range of buckle half-wavelengths from 30mm to 10000mm in Fig. 4 (also in Table A3-1 in Appendix 3 of Pham and Hancock (2011)). The graph reaches a minimum at approximately 200mm half-wavelength then rises and starts to drop at about

800mm. The rise in the curve is nowhere near as marked as for equivalent compression or bending curves for the same section (Hancock, 2007). The mode at 200mm, which is the width of the web, is shear local with local buckling also in the flanges as shown in Fig. 5. The buckling coefficient k_v corresponding to the minimum point is 6.583 based on the average stress in the web ($\tau_{av} = V/A_w$) computed from the shear load V on the section divided by the area of the web A_w. The average stress in the web is 0.8723 times the maximum stress at the centre of the web where 0.8723 is a constant related to the cross-section geometry.. The buckling coefficient k_v is significantly greater than 5.34 for a plate simply supported on its longitudinal edges due to the restraint from the flanges on the web.

The SFSM graph (symbolically circles) of buckling stress versus length (as opposed to half-wavelength for the SAFSM) was computed as in Pham and Hancock (2009). The SFSM analysis assumes no cross-section distortion at both ends of the section under analysis (Z = 0, L) and so this restraint increases the buckling stress above that of the SAFSM which is free to distort at the ends. For a section of length 200mm, the SFSM analysis gives a buckling coefficient k_v of 9.927 for the web which is very much higher than the SAFSM value of 6.583 above and greater than that for a simply supported square panel in shear at 9.34 due to the flange restraint. The SFSM values asymptote to a value very close to the minimum on the SAFSM curve at lengths between 1000mm and 1500mm. At these lengths, the end restraint effects become very small and so local buckling in multiple half-wavelengths in the SFSM matches closely with the SAFSM at 200mm as shown in Fig. 5.

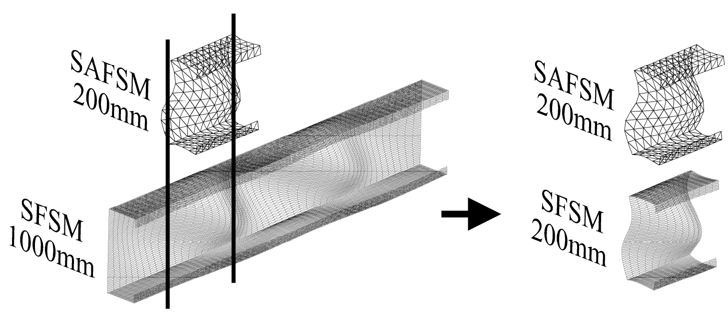

Figure 5. Comparison of Local Buckling Modes from SAFSM and SFSM

At lengths greater than approximately 1500mm, the SFSM curve falls and the mode switches to a type of flange-distortional mode where the buckling in the two opposite flanges is out of phase as shown in Fig. 6 at 2000 mm for the SFSM and 1600mm for the SAFSM. The web buckling stresses result in buckling coefficients at these lengths which are approximately equal at 5.509 and 5 265 for the SFSM and SAFSM respectively (see Table A3-1 in Appendix 3 of Hancock and Pham (2011)).

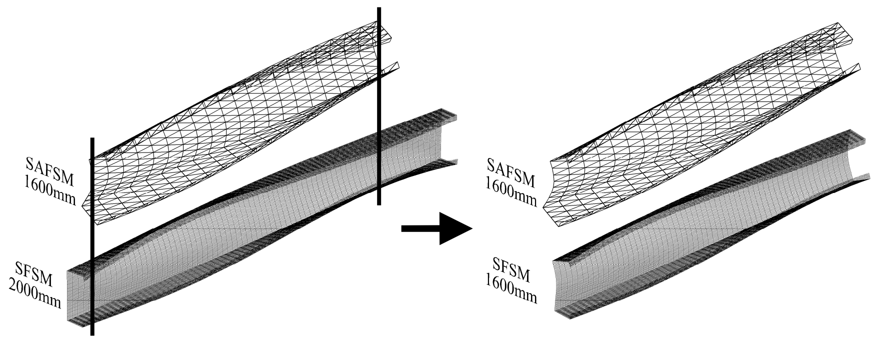

Figure 6. Shear Distortional Buckling Modes from SAFSM and SFSM Analyses

At long lengths such as 5000mm, the flexural-torsional mode occurs where the cross-section remains undistorted. This mode is somewhat artificial as it is difficult to see how mainly pure shear could occur at such lengths.

Direct Strength Method (DSM) design rules for pure shear

Proposed DSM design rules have recently been approved for the 2012 Edition of NAS S100 as follows:

Proposed DSM design rules in shear without Tension Field Action (TFA)

The equations in Section C3.2.1 of the North American Specification (AISI, 2007) which are expressed in terms of a nominal shear stress F_v have been changed to DSM format by replacing stresses by loads as follows:

$$\text{For } \lambda_v \le 0.815 \text{ : } V_v = V_y \tag{21}$$

$$\text{For : } 0.815 < \lambda_v \le 1.227 \text{ : } V_v = 0.815\sqrt{V_{cr}V_y} \tag{22}$$

$$\text{For : } \lambda_v > 1.227 \text{ : } V_v = V_{cr} \tag{23}$$

$$V_y = 0.6 A_w f_y \tag{24}$$

where V_y = yield load of web based on an average shear yield stress of 0.6f$_y$. V_{cr} = elastic shear buckling force of the section derived by integration of the shear stress distribution in Fig. 3 at buckling over the whole section for the minimum point in Fig. 4, $\lambda_v = \sqrt{V_y / V_{cr}}$.

Proposed DSM design rules in shear with Tension Field Action (TFA)

The DSM nominal shear capacity *(V_v)* including Tension Field Action (TFA) is proposed based on the local buckling *(M_{sl})* equation (Standards Australia, 2005) where M_{sl}, M_{ol} and M_y are replaced by V_v, V_{cr} and V_y respectively as in Eq. 25. The choice of this equation as a good fit to the test results implies the post-buckling strength in shear is similar to local buckling in bending and/or compression implicit in the DSM equations.

$$V_v = \left[1 - 0.15\left(\frac{V_{cr}}{V_y}\right)^{0.4}\right]\left(\frac{V_{cr}}{V_y}\right)^{0.4} V_y \tag{25}$$

where V_y is yield load of web given by Eq. 24, V_{cr} is the elastic shear buckling force of the section based on the SFSM where end restraint is included. Although Eq. 25 is empirical, it has proven successful for a range of section shapes for local buckling in compression and bending and it is most likely the case for shear. Further investigation for validation for different shapes is part of ongoing research.

Experimental justification for Equations 21-25 is given in Pham and Hancock (2012).

Conclusions

The Semi-Analytical Finite Strip Buckling Analysis (SAFSM) of thin-flat-walled structures under combined loading using a complex finite strip method, and developed by Plank and Wittrick, has been programmed in Visual Studio C++. The method includes the extraction of the eigenvalues and eigenvectors from the Hermitian matrices produced when thin-walled sections are subjected to shear in addition to compression and bending. The signature curve for a plain lipped channel in pure shear has been produced using the SAFSM. The curve shows a single minimum at local buckling mainly in the web at a half-wavelength approximately equal to the web depth. At longer lengths equal to about 8 times the web depth, a flange distortional mode occurs with the buckling in the flanges out-of-phase. At very long lengths, a flexural-torsional mode occurs. The signature curve has been compared with the spline finite strip buckling analysis (SFSM) where the ends are fixed against distortion. The SFSM asymptotes to the minimum buckling stress of the SAFSM at approximately 5 times the depth of the web. Reducing the size of the lip does not materially change the shape of the SAFSM curve. The proposed DSM design rules for shear both with and without Tension Field Action use the shear buckling load V_{cr} of the section. In the case without TFA, the minimum on the shear signature curve can be used. For the case with TFA where end restraints occur, the SFSM analysis at the appropriate length can be used.

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