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## Critical Elastic Shear Buckling Stress Hand Solution for C- and Z-Sections Including Cross-Section Connectivity

Kevin Aswegan<sup>1</sup>  
Cristopher D. Moen<sup>2</sup>

### Abstract

This paper presents an approximate solution for the critical elastic buckling stress of cold-formed steel C- and Z-section members including cross-section connectivity. The elastic buckling solution is developed to support the extension of the Direct Strength Method to the shear ultimate limit state, where the cross-sectional critical elastic shear buckling stress (load) is employed to predict shear capacity. The shear buckling stress and buckled half-wavelength are calculated with a classical energy solution for a thin plate with edges rotationally restrained. Rotational stiffness expressions in the AISI S100-07 specification, originally derived for distortional buckling of C- and Z-sections, are used with the energy solution to calculate the rotational restraint provided to the web-flange juncture by the flanges. The approach is validated with thin shell finite element eigen-buckling analysis.

### Introduction

The American Iron and Steel Institute's (AISI) North American Specification (AISI-S100 2007) calculates the shear strength of a thin-walled cold-formed steel member by treating the primary shear carrying cross-sectional element, for example the web of a C-section, as a simply-supported plate in shear (Figure 1). It is assumed that the flanges are unstressed by shear. The critical elastic buckling stress,  $\tau_{cr}$ , is approximated with a plate buckling coefficient,

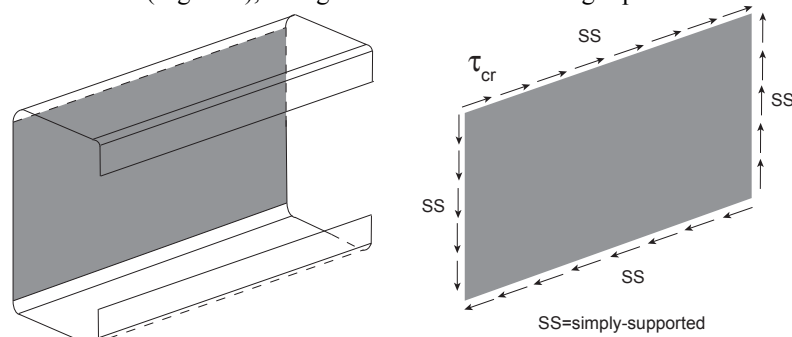
$$\tau_{cr} = k_v \frac{\pi^2 E t^2}{12(1-\nu^2)b^2}, \quad (1)$$

where  $E$  is the modulus of elasticity,  $\nu$  is Poisson's ratio,  $b$  is the plate width, and  $t$  is the plate thickness. For a cold-formed steel member with unreinforced

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webs,  $k_v=5.34$ , resulting from the elastic buckling solution for an infinitely long simply-supported plate in pure shear (Southwell and Skan 1924). The buckling stress is input into an empirically derived design expression (AISI-S100 2007, Section C3.2.1) to calculate the ultimate strength of the member in shear. The design approach is simple and convenient, however the beneficial contribution provided by adjacent connected cross-section elements, for example the flanges of a C-section (Figure 1), is neglected in the shear strength prediction.



**Figure 1. C-section web is treated as a simply-supported plate in design (SS=simply supported)**

Recent studies have demonstrated that  $\tau_{cr}$  calculated for a cold-formed steel member including cross-section connectivity can be up to 40% higher than that predicted by the classical solution considering just the isolated web (Pham and Hancock 2009). Furthermore, the critical elastic shear buckling load of a member,  $V_{cr}$ , which can be calculated from  $\tau_{cr}$ , has been confirmed to be a viable parameter for predicting the strength of cold-formed steel C-section members in shear and combined bending and shear with a Direct Strength approach (Pham and Hancock 2013).

The goal of this research is to develop a simplified method for predicting the critical elastic buckling stress (load) of a cold-formed steel member with the web loaded in a state of pure shear, including cross-section connectivity. A classical energy solution (Bulson 1970) is employed that calculates the critical elastic shear buckling stress of an infinitely long plate loaded in pure shear and with transverse rotational restraint. The rotational restraint provided by the flanges to the web-flange juncture is calculated with existing equations in AISI-S100-07 originally derived for predicting the critical elastic distortional buckling stress (Desmond et al. 1981; Schafer 2002). Details of the classical shear buckling solution are provided in the next section. The solution is validated with thin-shell finite element eigen-buckling analyses considering industry standard cold-formed steel C-sections.

### Classical shear buckling solution including rotational restraint

The shear buckling coefficient,  $k_v$ , including edge rotational restraint can be approximated as (Bulson 1970)

$$k_v = \frac{1}{\sin 2\phi} \left[ \frac{1}{\left(\frac{\lambda}{b}\right)^2 \cos^2 \phi} + C_1 \left(\frac{\lambda}{b}\right)^2 \cos^2 \phi + C_2 (1 + 2 \sin^2 \phi) \right], \quad (2)$$

where  $\lambda$  is the buckled half-wavelength (Figure 2), and  $\phi$  is the half-wave angle of inclination in radians

$$\phi = \arccos \left( \sqrt{C_3 + \sqrt{C_3^2 + C_4}} \right). \quad (3)$$

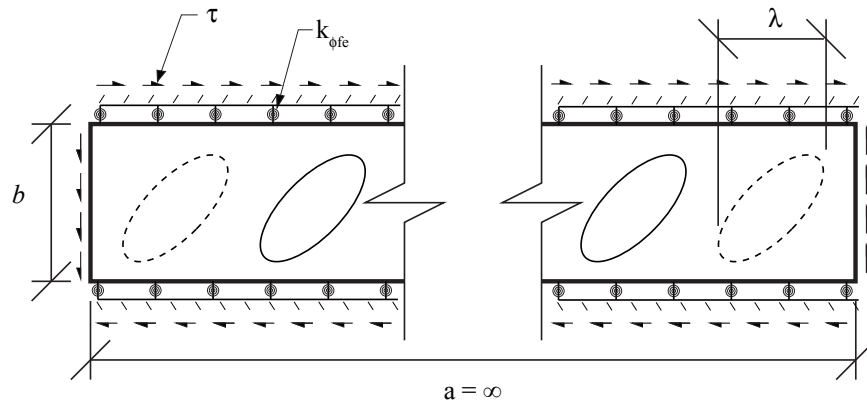


Figure 2. Shear buckling with rotationally restrained edges – loading, boundary conditions, and notation

The coefficients  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$  are

$$C_1 = \frac{\left(\frac{1}{8} - \frac{1}{\pi^2}\right) \varepsilon^2 + \left(\frac{1}{2} - \frac{2}{\pi^2}\right) \varepsilon + \frac{1}{2}}{\left(\frac{\pi^2}{120} + \frac{1}{8} - \frac{2}{\pi^2}\right) \varepsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right) \varepsilon + \frac{1}{2}} \quad (4)$$

$$C_2 = 2 \frac{\left(\frac{5}{24} - \frac{2}{\pi^2}\right)\varepsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\varepsilon + \frac{1}{2}}{\left(\frac{\pi^2}{120} + \frac{1}{8} - \frac{2}{\pi^2}\right)\varepsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\varepsilon + \frac{1}{2}} \quad (5)$$

$$C_3 = \frac{\frac{3}{2}C_2 - \frac{2}{\left(\frac{\lambda}{b}\right)^2}}{4C_2 + C_1\left(\frac{\lambda}{b}\right)^2} \quad (6)$$

$$C_4 = \frac{\frac{3}{\left(\frac{\lambda}{b}\right)^2}}{4C_2 + C_1\left(\frac{\lambda}{b}\right)^2}. \quad (7)$$

The coefficient of restraint,  $\varepsilon$ , is a normalized rotational stiffness parameter

$$\varepsilon = \frac{k_{\phi e} b}{D}, \quad (8)$$

where  $k_{\phi e}$  is the transverse rotational restraint (stiffness) per unit length, i.e., force·length/rad/length, along the edges of the plate. The plate flexural rigidity is  $D=Et^3/[12(1-\nu^2)]$ .

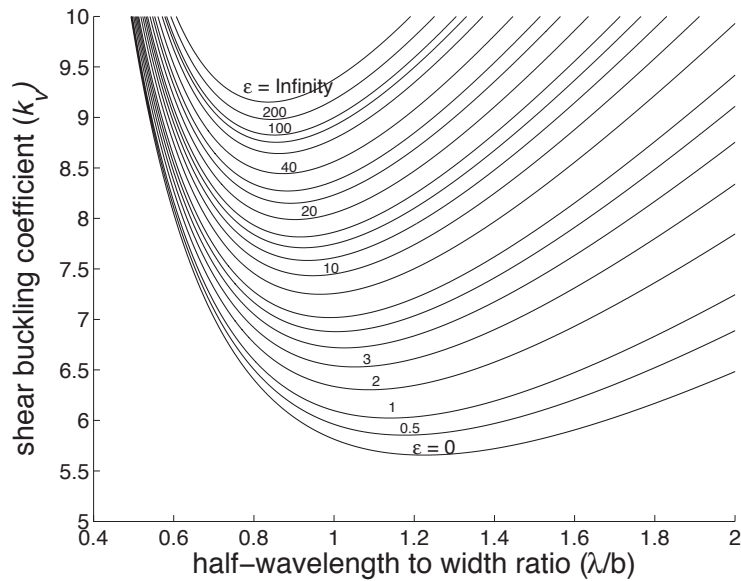
In Figure 3,  $k_v$  calculated with Eq. (2) is plotted as a function of  $\lambda/b$  for several different coefficients of restraint. The buckling coefficient and half-wavelength for each value of  $\varepsilon$  are defined by the curve minima. The bottommost curve in Figure 3 represents a plate with simply-supported edges, while the uppermost curve is for a plate with rotational restraint converging to fixed-fixed edges. Each  $k_v$  value making up the curves in Figure 3 is obtained with an iterative numerical solution because  $\phi$ ,  $\lambda$ , and  $k_v$  are all functions of  $\varepsilon$ .

The shear buckling coefficient  $k_v$  and corresponding  $\lambda/b$  are provided in Table 1 and in Figure 4 for a range of  $\varepsilon$ . When  $\varepsilon=0$ ,  $k_v=5.66$  which is 5.9% higher than the exact solution of  $k_v=5.34$  from Southwell and Skan (1924). As  $\varepsilon$  increases to 5000,  $k_v$  converges to 9.15, which is 1.9% higher than the exact solution for shear buckling of a plate with fixed-fixed edges of  $k_v=8.98$  (Bulson 1970). The ratio  $\lambda/b$  decreases with increasing  $\varepsilon$  which means that the shear buckling half-

wavelength ( $\lambda$ ) decreases with increasing edge rotational restraint for a constant plate width ( $b$ ).

**Table 1. Shear buckling coefficient  $k_v$  for various magnitudes of rotational restraint  $\varepsilon$**

$\varepsilon$	$\lambda/b$	$k_v$	$\phi$ (rad)
0	1.225	5.657	0.615
0.5	1.176	5.854	0.621
1	1.140	6.024	0.625
2	1.088	6.304	0.631
3	1.052	6.530	0.635
4	1.026	6.718	0.638
5	1.005	6.879	0.641
6	0.989	7.018	0.642
8	0.965	7.249	0.645
10	0.947	7.433	0.647
12	0.934	7.584	0.648
14	0.924	7.710	0.649
16	0.915	7.817	0.650
20	0.903	7.989	0.651
25	0.892	8.150	0.651
30	0.884	8.272	0.652
40	0.873	8.443	0.653
60	0.862	8.642	0.653
80	0.856	8.754	0.653
100	0.852	8.825	0.653
200	0.845	8.980	0.653
$\infty$	0.836	9.152	0.653



**Figure 3. Shear buckling garland curve (infinite number of half-waves) for different magnitudes of rotational restraint  $\varepsilon$**

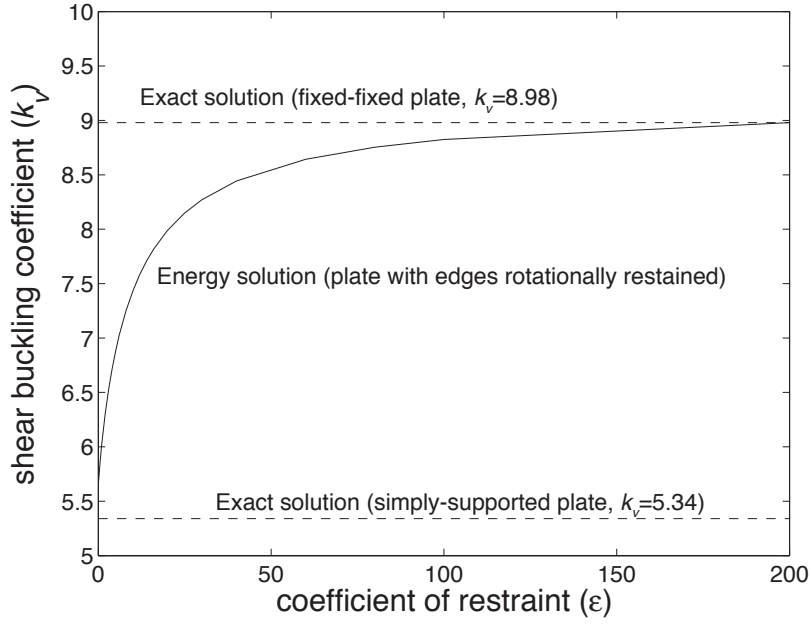


Figure 4. The shear buckling coefficient  $k_v$  versus rotational restraint  $\varepsilon$

#### Calculating edge rotational restraint for C- and Z-Sections

The coefficient of restraint,  $\varepsilon$ , in Eq. (4) is defined for C- and Z-sections by the amount of rotational stiffness,  $k_{\phi fe}$ , the lip-stiffened flanges provide to the web-flange junction, i.e.,

$$k_{\phi fe} = \left(\frac{\pi}{\lambda}\right)^4 \left( EI_{xf}(x_o - h_x)^2 + EC_{wff} - E \frac{I_{xyf}^2}{I_{yf}}(x_o - h_x)^2 \right) + \left(\frac{\pi}{\lambda}\right)^2 GJ_f, \quad (9)$$

where  $I_{xf}$  is the  $x$ -axis (strong axis) flange moment of inertia;  $x_o$  is the  $x$  distance from the centerline flange/web junction to the centroid of the flange;  $h_x$  is the  $x$  distance from the flange centroid to the flange shear center;  $C_{wff}$  is the flange warping torsion constant;  $I_{xyf}$  is the product of the flange moments of inertia;  $I_{yf}$  is the centroidal  $y$ -axis (weak axis) flange moment of inertia,  $G$  is the shear modulus; and  $J_f$  is the St. Venant torsion constant of the compression flange and lip stiffener. Table C-C3.1.4(b)-1 in the AISI-S100-07 commentary aids in calculating the geometric flange properties for use in Eq. (9).

### Prediction validation with a finite element parameter study

The values of  $k_v$  were calculated for the 99 Steel Stud Manufacturers Association C-sections (SSMA 2011) using thin shell finite element eigen-buckling analysis with the boundary conditions and applied loading shown in Figure 5; see Naik (2010) for full details and the finite element  $k_v$  magnitudes. The member length ( $L$ ) to web height ( $H$ ) ratio was kept constant at 8:1 in the FE models to accommodate multiple shear buckling half-waves along the length.

The finite element eigen-buckling results and energy solution predictions are compared in Figure 6, and the FE-to-energy solution  $k_v$  mean and coefficient of variation (COV) are 0.90 and 0.03 respectively. The energy solution has an unconservative bias (i.e., the energy solution is higher than the finite element solution) which can be corrected by multiplying Eq. (2) by 0.90, resulting in a  $k_v$  mean and COV of 1.0 and 0.03 respectively.

A complication of the Eq. (2) energy solution is that an iterative numerical solution is required. If  $\lambda/b=0.85$  is assumed for any value of  $\varepsilon$ , then Eq. (2) can be solved by hand without iteration and without a loss of prediction accuracy demonstrated by the FE-to-energy solution mean and COV of 1.0 and 0.02. The COV is improved because the energy solution with  $\lambda/b=0.85$  is more accurate for small  $\varepsilon$ , see Figure 6. Work is ongoing to validate the proposed approach for calculating the critical elastic buckling stress of Z-section members.

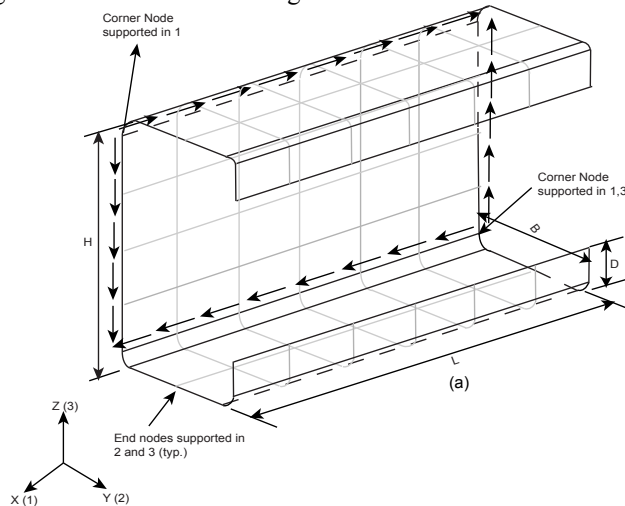


Figure 5. Finite element model boundary conditions and loading (Naik 2010)



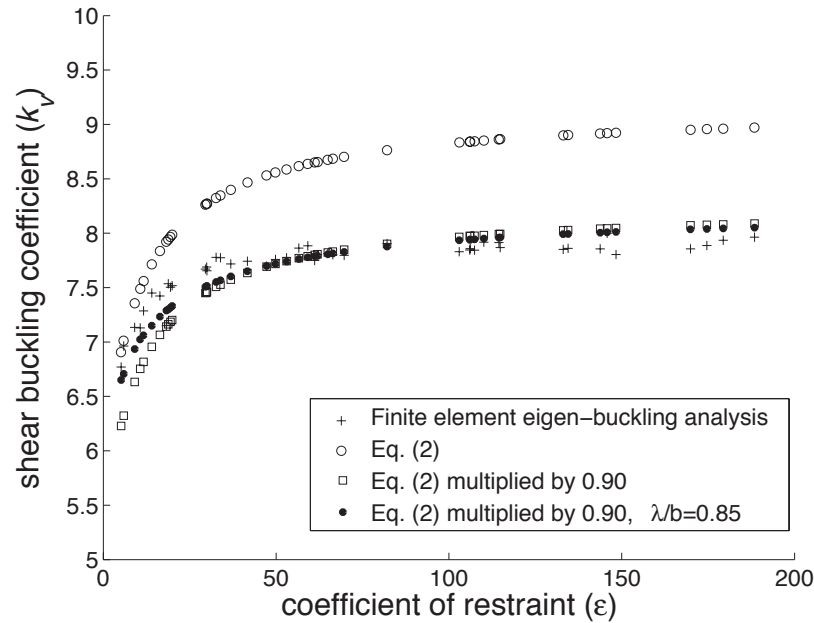


Figure 6. Shear buckling coefficient  $k_v$ , calculated with finite element eigen-buckling and Eq. (2) for SSMA C-sections

## CONCLUSIONS

Approximate engineering expressions are developed for calculating the critical elastic shear buckling stress of C- and Z-section webs including cross-section connectivity. The approach merges a classical energy solution for shear buckling of an edge-restrained infinitely long plate with an existing hand solution for the rotational stiffness provided to the web-flange junctures by a C- or Z-section flange and stiffening lip. The approximate method is accurate for the C-sections considered, however the solution requires iteration because the shear buckling half-wavelength is a function of edge rotational restraint. A viable solution without iteration can be achieved for C-sections if the shear buckling half-wavelength is assumed to be 85% of the plate (web) width regardless of the rotational restraint magnitude. Finite element modeling work is ongoing to validate the energy solution for Z-sections.

## ACKNOWLEDGMENTS

Prof. R.H. Plaut of Virginia Tech identified and brought to our attention the energy solution presented in this paper.

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