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BUCKLING OF PLATES CONTAINING OPENINGS

by

Fong-Yen CHOW and Rangachari NARAYANAN*

SUMMARY

The buckling behaviour of simply supported or clamped square plates containing circular or square holes and subjected to uniaxial or biaxial compression or to shear loading is investigated. The parameters varied include the size of the hole and the eccentricity of its location with respect to the centre of the plate. The results of the parametric studies using an appropriate finite element formulation are presented and discussed. Experiments carried out to validate the numerical method are summarized.

INTRODUCTION

It is often necessary to provide openings in thin plated structures such as cold formed steel members, aeroplane fuselages, plate girders, box girders and ship plating, for affording access for services or inspection. The presence of a hole in a plate panel changes the stress distribution within the member, alters its elastic buckling and post buckling characteristics and generally reduces its ultimate load carrying capacity. The performance of a plate containing an opening is influenced by the nature of the applied stress (e.g. compressive, tensile, shear etc), besides the shape, size and location of the hole; as ease of access is usually the criterion for the deciding on the location for the hole, it is not always possible to locate it centrally.

Although plated structures containing holes are widely used, only a small amount of published information is available on the many facets of the problem. Early research on the subject has largely concentrated on the buckling behaviour of plates containing centrally located holes and subjected to pure shear or uniaxial or biaxial compression |1,4-6,12-18,20|.

Plates containing central holes, when subjected to pure shear sustained continuously diminishing buckling loads with increasing size of hole |14,15|. However, when subjected to uniaxial and biaxial compression, the buckling loads were sometimes found to increase, with increasing size of the hole, due to the redistribution of the membrane stress towards the edges of the plate; this increase was found to be significant for plates with clamped edges |1,12,18,20|. It must be noted that even where the buckling load increased with the increase in the hole size, there would be no corresponding increase in the ultimate or collapse load.

No systematic treatment of eccentrically placed openings is available in published literature.

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In view of the foregoing, a systematic series of studies on the many facets of the problem were undertaken at the University College, Cardiff. Experimental and analytical studies were carried out in parallel, with a view of searching for relatively simple techniques of predicting their buckling and ultimate strengths. Conclusions of studies on the elastic critical buckling of plates are presented in this paper; studies on ultimate loads are reported elsewhere |8-10|.

COMPUTATION OF THE ELASTIC CRITICAL LOADS

The finite element method was employed by Rockey, Anderson and Cheung |14| to examine the buckling of square plates having centrally placed circular holes when subjected to shear loading. Plates containing central circular holes and subjected to uniaxial compression were investigated by Pennington-Vann |12|; these studies were further extended by Shanmugam and Narayanan |18| and Azizian and Roberts |1| to include square and circular holes under biaxial and shear loadings. In these investigations, the inplane stresses were obtained from the in-plane stiffness matrices based on prescribed displacement functions. It has been shown by Chow |2| that such in-plane membrane elements, when used to analyse stress concentration problems, give results which converge slowly thus requiring the structure to be divided into a very large number of elements; the computing costs therefore rise to unacceptably high levels. However, this deficiency can be improved by using strain-based elements; when elements having an in-plane rotation as an additional degree of freedom were employed, more accurate results were obtained with much reduced number of elements, even in cases where the stresses varied rapidly with the distance from the edges of the holes.

The elastic critical load of a structural system is associated with its neutral equilibrium state, and can be obtained by considering the total potential energy of the system. This is a standard eigen-value problem for which the solution procedure has been fully documented |1,11,14|. Hence no detailed description of the finite element formulation or of the solution procedure is discussed here. A description of the elements used is, however, included to highlight their properties and limitations. The models used for computing are seen in Figs. 1 & 2.

Rectangular elements were used to analyse plates containing square holes; for plates with circular holes, triangular elements were used to model the boundaries.

The following strain functions are appropriate for a rectangular element having 12 degrees of freedom. (Besides translation (u and v), the in-plane rotation (i) is specified as a third degree of freedom at each node) :

$$\begin{aligned}
\varepsilon_x &= a_4 + a_7 y + (a_{11} y^2 + 2a_{12} x y^3) \\
\varepsilon_y &= a_5 + a_8 x - (a_{11} x^2 + 2a_{12} x^3 y) \qquad (1) \\
\gamma_{xy} &= 2a_6 + 2a_9 x + 2a_{10} y + (a_7 x + a_8 y)
\end{aligned}$$

The corresponding displacement functions are given by integrating equations (1) :

$$\begin{aligned}
u &= a_1 - a_3 y + a_4 x + a_6 y + a_7 xy + a_{10} y^2 + a_{11} xy^2 + a_{12} x^2 y^3 \\
v &= a_2 + a_3 x + a_5 y + a_6 x + a_8 xy + a_9 x^2 - a_{11} x^2 y - a_{12} x^3 y^2 \qquad (2) \\
i &= a_3 - \tfrac{1}{2} a_7 x + \tfrac{1}{2} a_8 y + a_9 x - a_{10} y - 2a_{11} xy - 3a_{12} x^2 y^2
\end{aligned}$$

where a_1, a_2 and a_3 are constant terms corresponding to rigid body displacements.

The average in-plane rotation (i) is defined in the Cartesian system of coordinates by :

$$i = \tfrac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad (3)$$

The triangular element with a circular edge is used to model the circular boundary of the edge. The strain functions are given as follows

$$\begin{aligned} \varepsilon_x &= a_3 + a_7 y + a_8 x \\ \varepsilon_y &= a_5 + a_8 x + a_7 y \\ \gamma_{xy} &= 2a_6 + 2a_9(x+y) \end{aligned} \tag{4}$$

The corresponding displacement functions are

$$\begin{aligned} u &= a_1 - a_3 y + a_4 x + a_6 y + a_7 xy + a_8 \left(\frac{x^2}{2} - \frac{y^2}{2}\right) + a_9 y^2 \\ v &= a_2 + a_3 x + a_5 y + a_6 x + a_7\left(\frac{y^2}{2} - \frac{x^2}{2}\right) + a_8 xy + a_9 x^2 \\ i &= a_3 - a_7 x + a_8 y + a_9 (x-y) \end{aligned} \tag{5}$$

A word of caution regarding this triangular element : the shape function is prone to singularity in the transformation matrix when a right angled isosceles triangle with the sides parallel to the x and y axes is encountered; hence such a condition must be avoided when this shape function is used.

The computations given in this paper were carried out using the rectangular elements with 12 degrees of freedom and triangular bending elements with 9 degrees of freedom suggested by Zienkiewicz and Cheung |21| and Nath |11| respectively; these are reproduced in equations (6) and (7) below :

$$w = a_1 + a_2 x + a_3 y + a_4 xy + a_5 x^2 + a_6 y^2 + a_7 x^3 + a_8 x^2 y + a_9 xy^2 + a_{10} y^3 + a_{11} x^3 y + a_{12} xy^3 \tag{6}$$

$$w = a_1 + a_2 x + a_3 y + a_4 xy + a_5 x^2 + a_6 y^2 + a_7 x^3 + a_8 (x^2 y + xy^2) + a_9 y^3 \tag{7}$$

BUCKLING COEFFICIENTS

The elastic critical stress (σ_{cr}) under in-plane compressive loading applied along the two opposite edges of a square plate of thickness t, is given by

$$\sigma_{cr} = K \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{a}\right)^2 \tag{8}$$

where K = the buckling coefficient
a = side of the square plate

The reduced critical stress due to the introduction of the hole can be expressed by analogous equations corresponding to the uniaxial compression, biaxial compression and shear loading cases, in terms of the respective buckling coefficients of K_u , K_b and K_s as follows :

$$\tau'_{cr}, \sigma'_u, \sigma'_b = K_{()} \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{a}\right)^2 \tag{9}$$

As stated previously, the buckling analysis was extended to square plates with eccentrically located square and circular holes, when subjected to uniaxial loading, biaxial loading and uniform edge shear. Two boundary conditions, namely fixed or simply supported along the four edges, were considered.

Since the location of the hole was eccentric, no advantage could be taken of symmetry and the entire plate had to be split into finite elements; Fig. 1 shows a typical finite element idealisation for the full plate with a square cutout and Fig. 2 shows the corresponding one for a plate with a circular opening.

For simplicity, the studies were confined to holes located along the diagonal of the square plate only, although the method is general and applicable for holes located in any position of the plate. The eccentricity (e) is defined as the distance from the centroid of the hole to the centroid of the plate. The results of constant stress loading case are obtained for the plate simply supported or clamped along all four edges.

PLATES COMPRESSED UNIAXIALLY

Figures 3 and 4 give sets of curves corresponding to clamped and simply supported cases respectively for plates containing eccentrically located square cutouts. The results show the variation of K_u with eccentricity for hole sizes ranging from 0.1a to 0.5a. The buckling coefficients are seen to be unaffected by the hole location for holes smaller than 0.2a; for larger holes, it is observed that the buckling coefficient decreases with the increase of eccentricity; the drop in K_u is more pronounced for larger holes.

A similar set of results for the plate with circular holes is given in Figs. 5 and 6 for clamped and simply supported plates respectively. The pattern is similar to the one observed in plates with square holes, except that the drop in the buckling coefficient is less pronounced for simply supported plates.

PLATES COMPRESSED BIAXIALLY

Figures 7 and 8 show the results for the clamped plate and simply supported plate respectively and follow a pattern similar to those observed in Figs. 3 and 4; however the biaxial loading has a more severe effect in reducing the buckling coefficients for the clamped plates compared with the uniaxial loading case, when hole sizes were larger than 0.3a.

The results corresponding to the circular holes are shown plotted in Figs. 9 and 10, showing the same pattern in the drop in buckling coefficients.

PLATE WITH A HOLE IN THE TENSION DIAGONAL SUBJECTED TO SHEAR

Shear loading causes a diagonal compression and a diagonal tension. Figures 11 and 12 show the change in the buckling coefficients with the eccentricity of the hole for a clamped plate and a simply supported plate respectively when the hole is located in the tension diagonal. The results show that, for very small holes, the buckling coefficients are unaffected; for larger holes, they reduce with the increase of eccentricity.

A similar set of results were obtained for plates with circular holes (not shown here).

PLATE WITH A HOLE IN COMPRESSION DIAGONAL, SUBJECTED TO SHEAR

The results for shear loading of plates with holes located in the compression diagonal are shown in Figs. 13 and 14 for a plate with a square cutout. The results obtained were very different from those in the previous case. Although for small holes, the buckling coefficient is unaffected by their location, it increases with the increase of eccentricity for larger holes. Plates with circular holes show a similar pattern as for those with square holes (not shown here).

EXPERIMENTAL INVESTIGATIONS

Previous studies on the buckling and post-buckling behaviour of (unperforated) plates carried out at Cardiff |2,7| and elsewhere |3,19| had established that small scale models could be employed without any significant loss of accuracy to simulate the buckling and post buckling characteristics of slender plates; hence scaled-down models were employed to carry out a large number of experiments and obtain test data. The resulting economy in fabrication and testing costs made it possible to employ at least two models for each test, thus checking the repeatability of the results.

Series 1 tests consisted of 23 tests on square plates containing centrally placed and eccentrically placed square or circular openings and subjected to uniaxial compression. Series 2 tests were on 23 similar plates, but subjected to biaxial compression. Series 3 tests consisted of 38 tests on plates containing centrally placed holes or eccentrically placed openings and subjected to shear. All the tests were on the constant strain condition,because of the difficulty in testing in the constant stress state; the parameters varied were the plate slenderness (a/t), the hole size (a'/a or d/a) and its eccentricity (e/a). In the case of the shear plates, the holes located in the compression diagonal as well as in tension diagonal were tested.

Uniaxial or biaxial compression tests were carried out in a specially fabricated test rig seen in Fig. 15. The rig is desk-mounted and hand operated and is capable of applying incremental loads using very small load increments. The out-of-plane displacements were measured, using a ripple scanner. From the measured values of deflection at various locations corresponding to the applied loads, the buckling load was derived using the Inflection Point Method |8,9|.

Series 3 tests were carried out by bolting the plates into specially fabricated pairs of boundary members, which were pinned together so that they formed a mechanism. A photograph of the test assembly is shown in Fig. 16. All the specimens were tested with their edges clamped and using "deflection control". The assembly was mounted on a 600kN Avery Denison machine so that a tensile force could be applied in the direction of a diagonal; this enabled a uniform shear force to be transferred from the boundary members and distributed along the edges of the web.

The test results were compared with the corresponding values obtained by using the finite element formulation described in the foregoing pages; in a large majority of cases the agreement was within 5% to 10% |2,8,9,10|.

CONCLUDING REMARKS

The elastic critical buckling behaviour of square plates containing holes and subjected to uniaxial or biaxial compression or to shear loading has been investigated. The parameters varied included two types of holes (square and circular), edge support conditions (simplysupported and clamped), the size of the hole and its location. Parallel studies were carried out using experimental and computational methods.

It was found that small diameter holes did not influence the buckling coefficients, irrespective of their location. When the holes were larger than 0.2a, and these values reduced with the increase in eccentricity of the hole location. However for plates under uniform shear loading, two opposing results were obtained for holes located in the compression and tension diagonals; in the former case, the eccentricity reduced the buckling coefficient and in the latter, increased it.

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APPENDIX 2 NOTATIONS

a	side of a square plate
$a_1, a_2 ..$	constants
e	eccentricity of the centre of the hole from the centre of the plate
E	modulus of elasticity
i	in-plane rotation (see equation (3))
K_b, K_s, K_u	buckling coefficients for biaxial, shear and uniaxial loading
u, v, w	displacements in the x,y,z directions
$\varepsilon_x, \varepsilon_y, \gamma_{xy}$	strain functions (equations (1) and (4))
ν	Poisson's ratio
σ'_b	biaxial compressive critical stress for perforated plate
σ_{cr}	elastic critical stress
σ'_u	uniaxial compressive critical stress for perforated plate
τ'_{cr}	shear critical stress for perforated plate

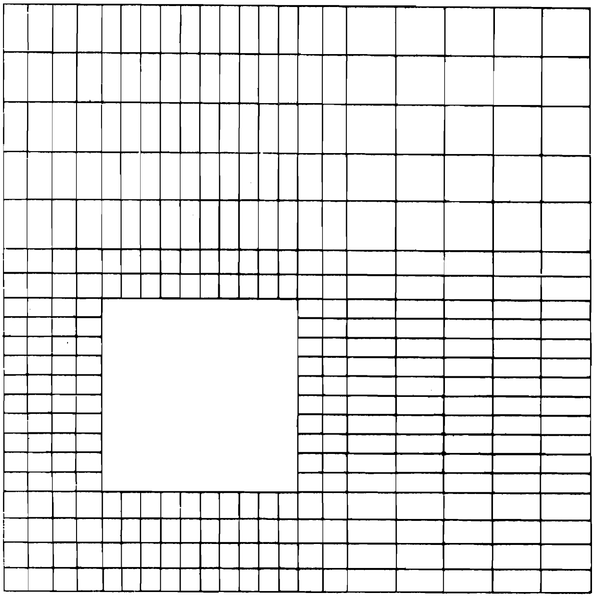

FIG. I SQUARE OPENING

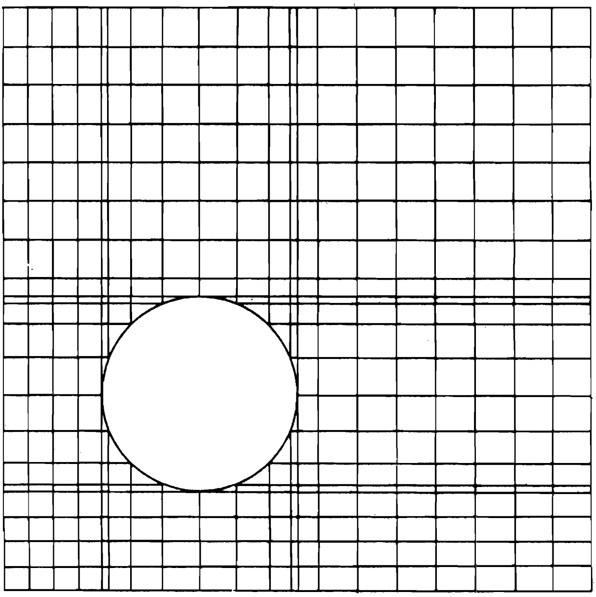

FIG. 2 CIRCULAR OPENING

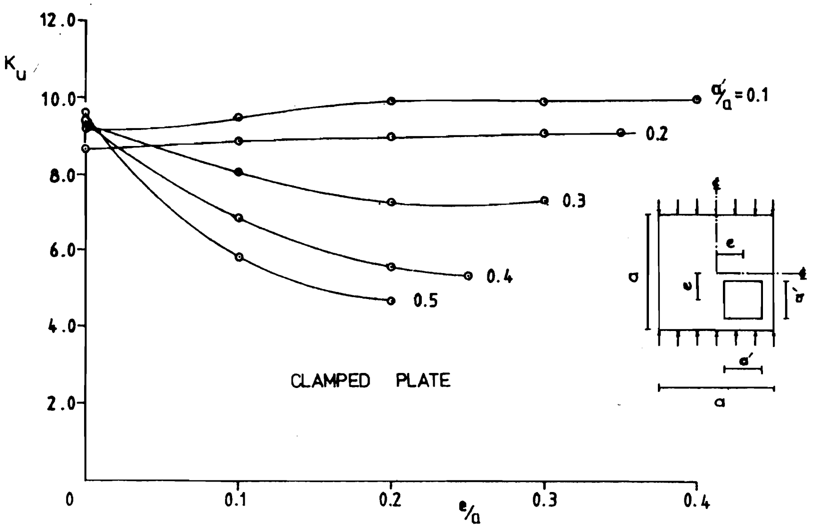

FIG. 3 CLAMPED PLATE WITH A SQUARE HOLE UNDER UNIAXIAL COMPRESSION

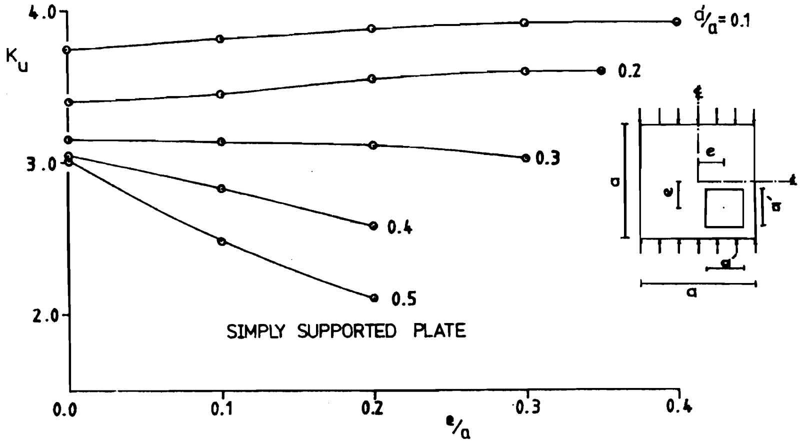

FIG. 4 SIMPLY SUPPORTED PLATE WITH A SQUARE HOLE UNDER UNIAXIAL COMPRESSION

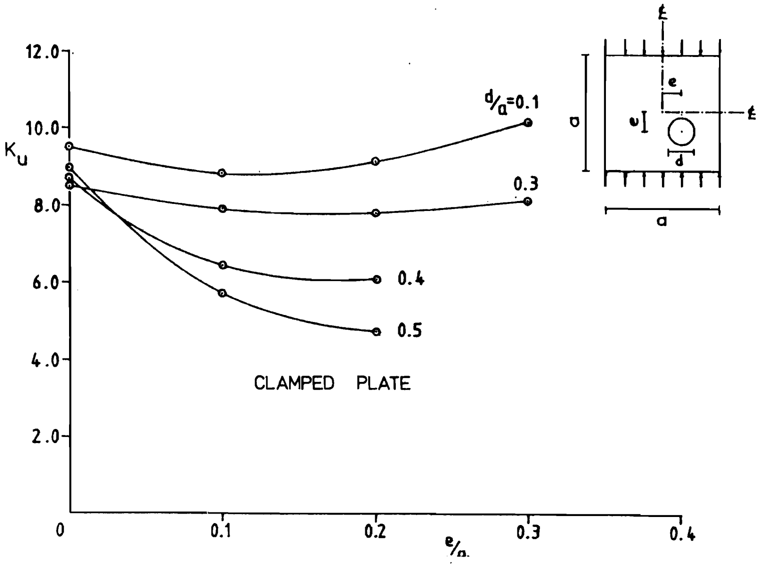

FIG. 5 CLAMPED PLATE WITH A CIRCULAR HOLE UNDER UNIAXIAL COMPRESSION

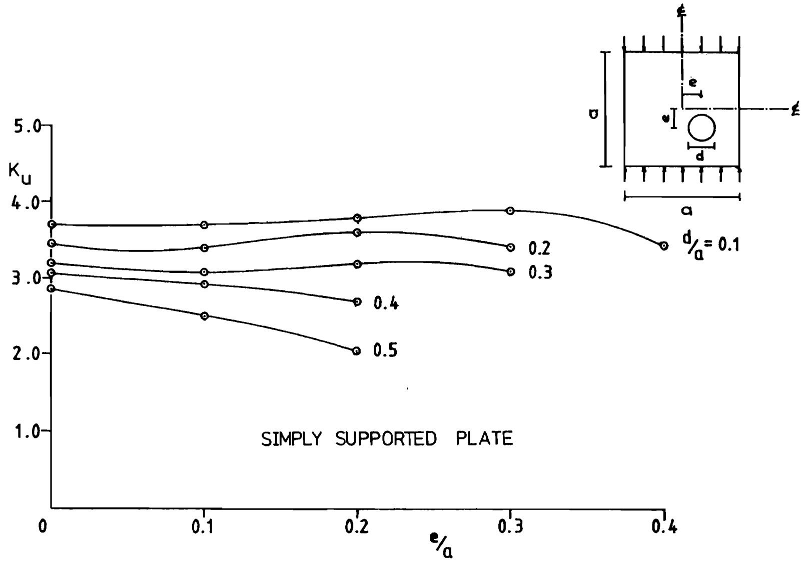

FIG. 6 SIMPLY SUPPORTED PLATE WITH A CIRCULAR HOLE UNDER UNIAXIAL COMPRESSION

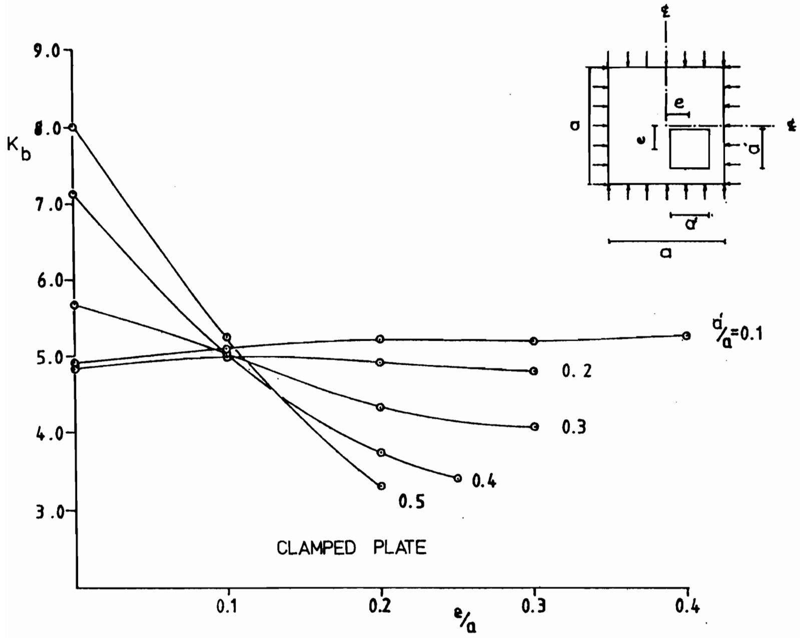

FIG. 7 CLAMPED PLATE WITH A SQUARE HOLE UNDER BIAXIAL COMPRESSION

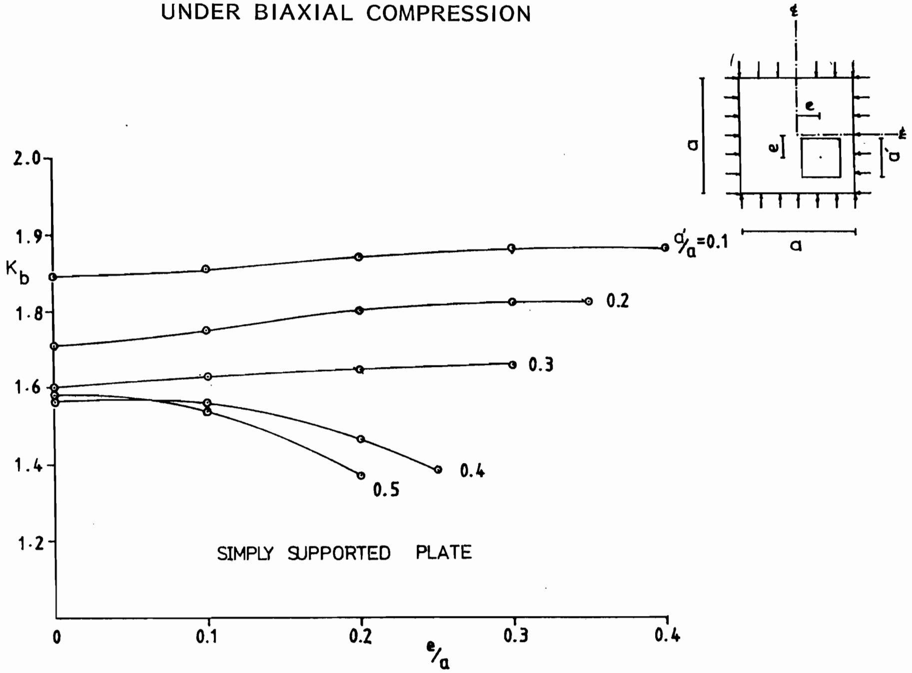

FIG. 8 SIMPLY SUPPORTED PLATE WITH A SQUARE HOLE UNDER BIAXIAL COMPRESSION

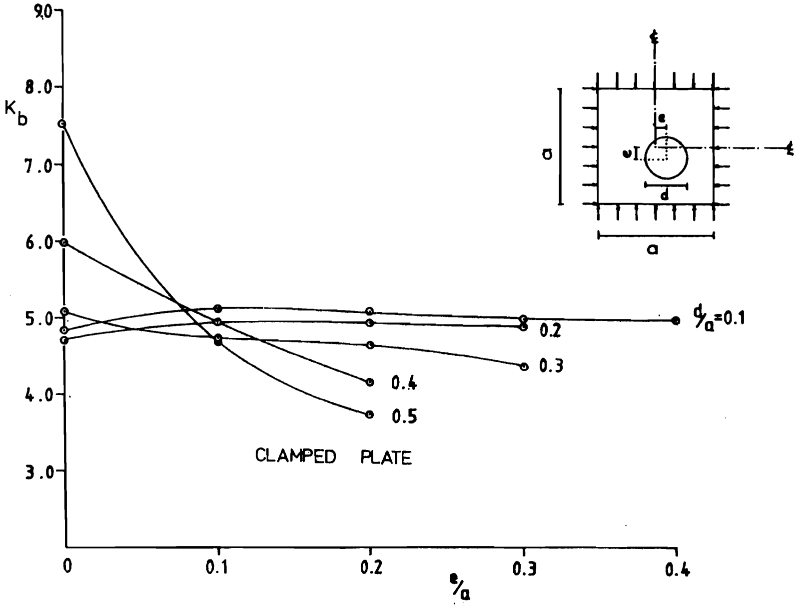

FIG. 9 CLAMPED PLATE WITH A CIRCULAR HOLE UNDER BIAXIAL COMPRESSION

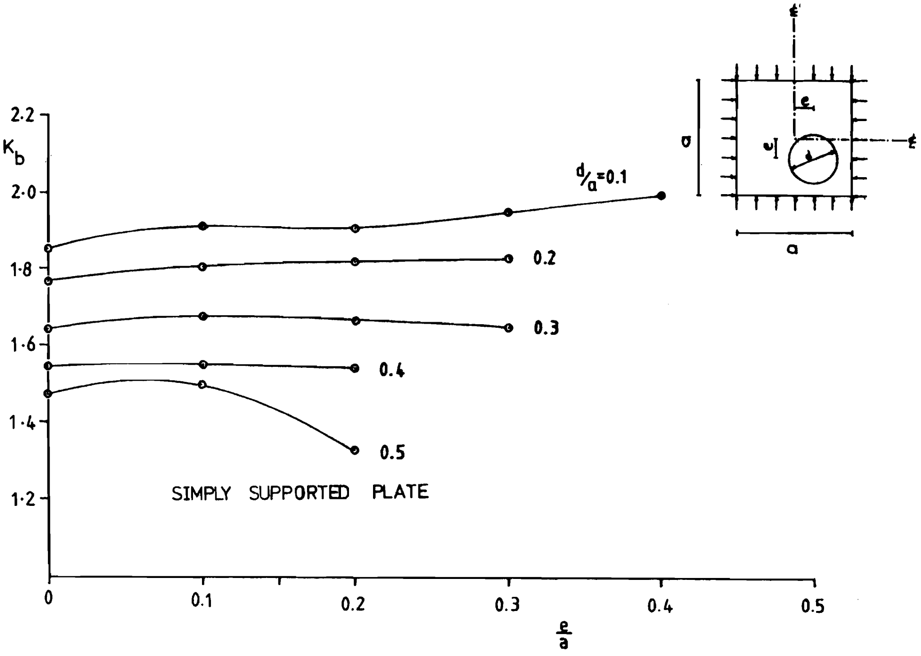

FIG. 10 SIMPLY SUPPORTED PLATE WITH A CIRCULAR HOLE UNDER BIAXIAL COMPRESSION

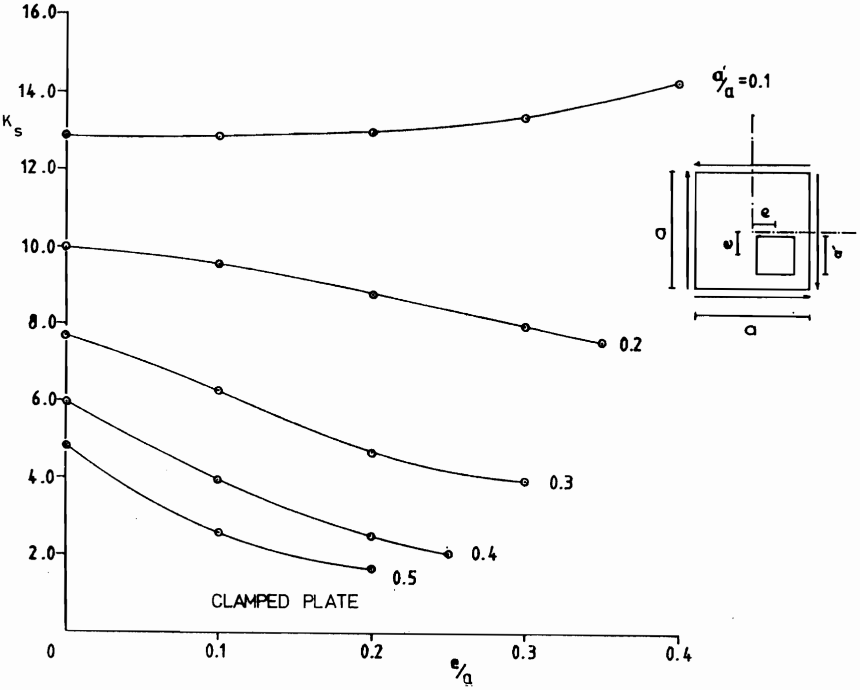

FIG. 11 CLAMPED PLATE IN SHEAR-SQUARE HOLE IN TENSION DIAGONAL

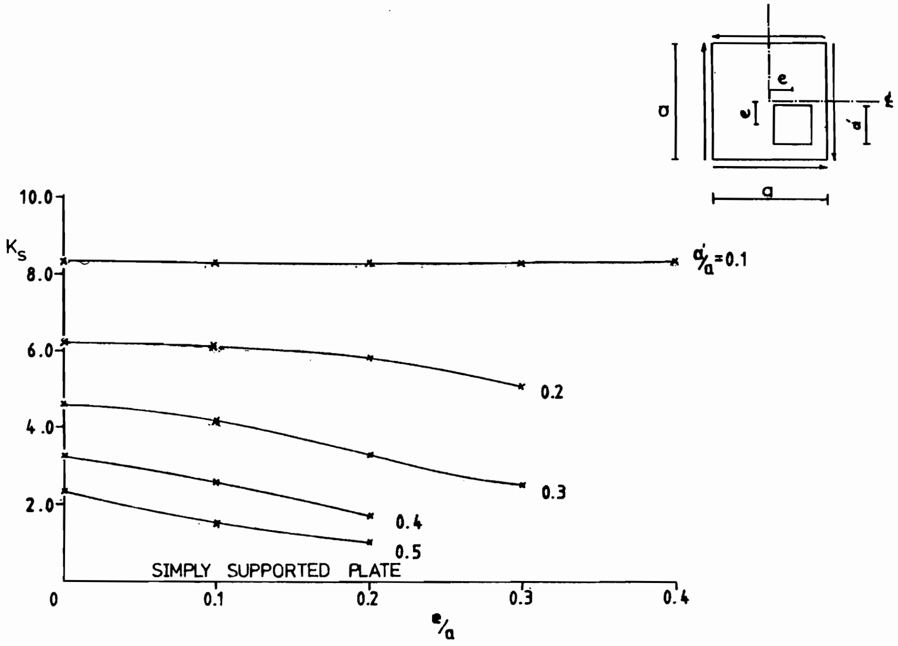

FIG. 12 SIMPLY SUPPORTED PLATE IN SHEAR-SQUARE HOLE IN TENSION DIAGONAL

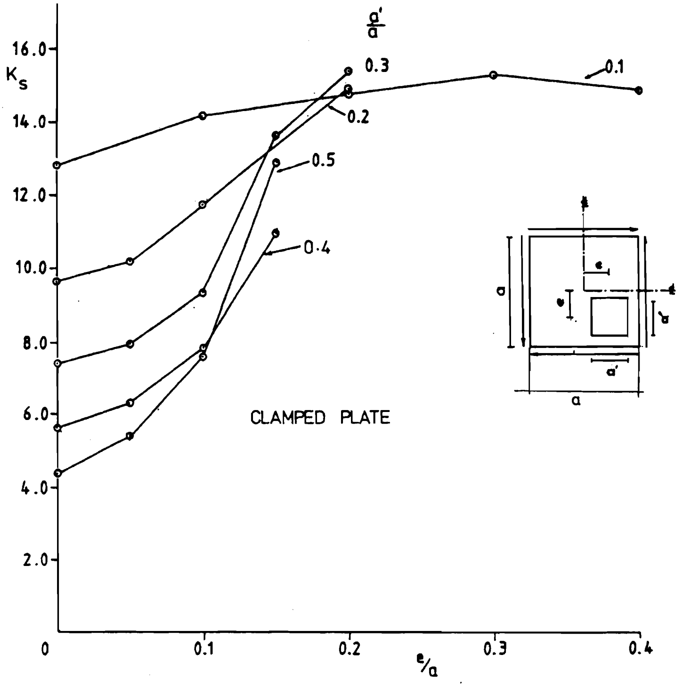

FIG. 13 CLAMPED PLATE IN SHEAR - A SQUARE HOLE IN COMPRESSION DIAGONAL

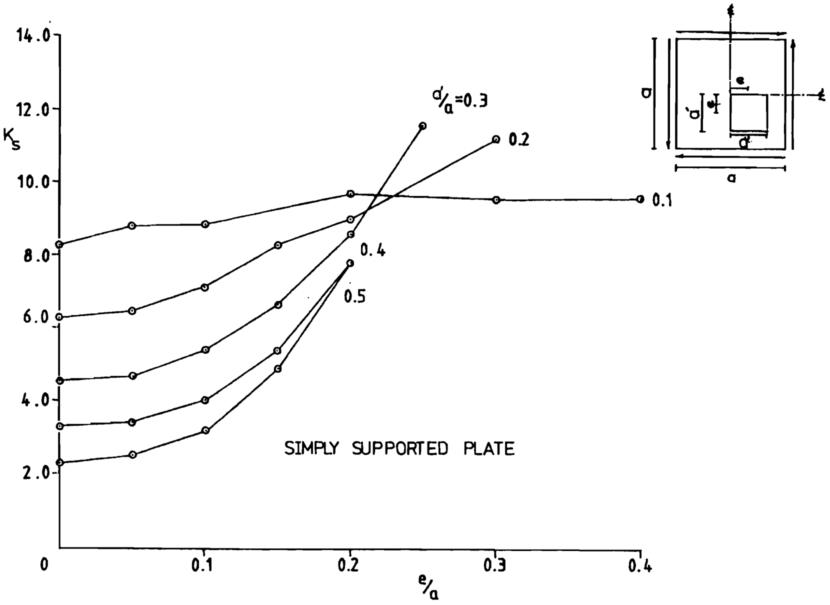

FIG. 14 SIMPLY SUPPORTED PLATE IN SHEAR SQUARE HOLE IN COMPRESSION DIAGONAL

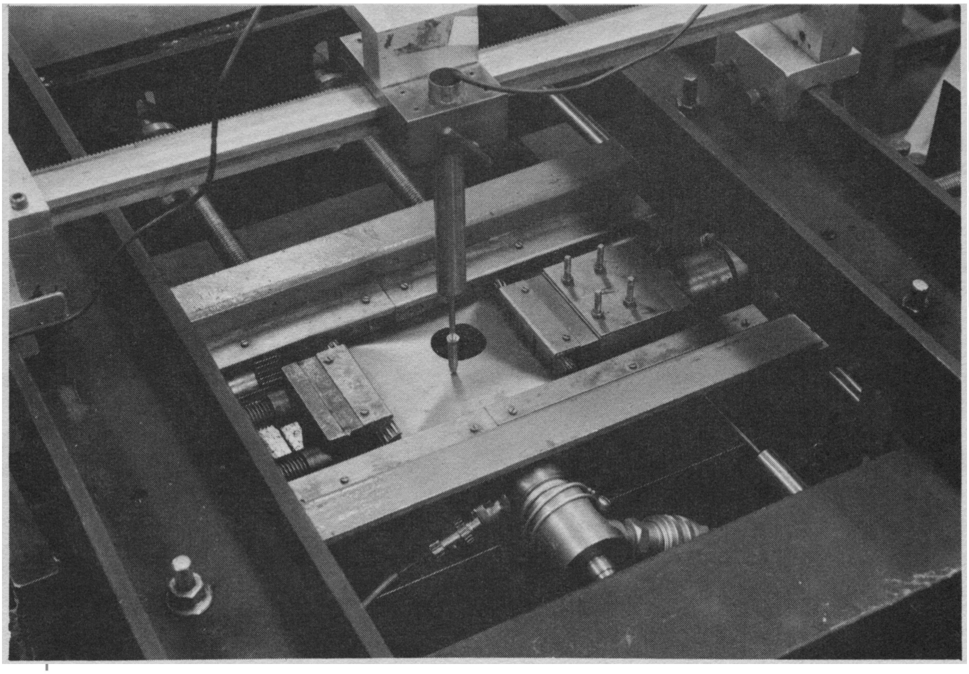

FIG. 15 TEST RIG FOR UNIAXIAL AND BIAXIAL LOADING

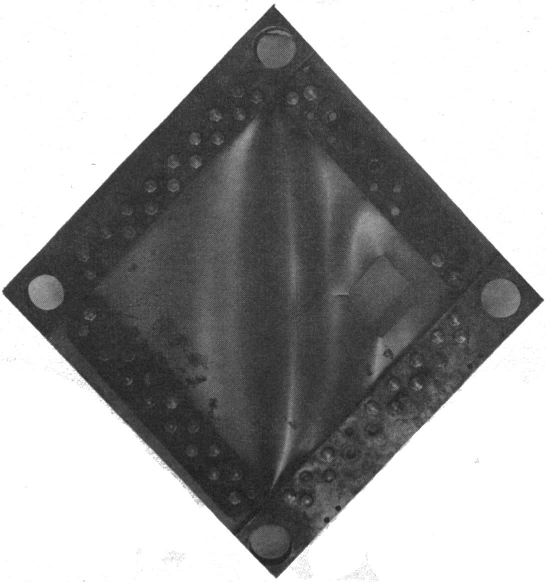

FIG. 16 BRACKET FOR SHEAR LOADING