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Lateral Buckling of Prismatic Members About an Imposed Axis of Rotation

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LATERAL BUCKLING OF PRISMATIC MEMBERS ABOUT AN IMPOSED AXIS OF ROTATION

by Leopold K. Sokol*

ABSTRACT

This paper deals with the lateral buckling of prismatic members whose position of the axis of rotation is imposed by the conditions of lateral support. As a function of the type of connection with the lateral support, the rotation may be free or hindered (elastic end restraint). This may concern, for instance, the case of a purlin stabilized by a roof structure or of a column stabilized by an external cladding or by another type of efficient continuous bracing.

Usually, the check of this type of instability is complex and leads to sophisticated and tedious computations. In order to avoid this inconvenience, approached solutions are often used. One of the most frequently used consists in calculating the compressed member as being fictitiously separated, submitted to lateral buckling in an elastic medium. The stiffness of this fictitious member is taken equal to its stiffness in lateral bending.

The present paper proposes an improved behaviour model of members submitted to lateral buckling through which the stiffness of the fictitious member is determined by taking into account the torsional stiffness of the whole transverse section of the profile.

1. INTRODUCTION

In Reference 1, adopting the model defined in Refs. 2 and 3, the free flange of purlin is dealt with as a fictitious separate member, in elastic medium, where:

- the stiffness of elastic medium depends on the transverse deformability of the cross-section of the purlin, on the flexibility of the steel sheeting and on the local deformability at the fasteners sheeting - purlin;
- the stiffness of the fictitious member is considered as being equal to its lateral bending stiffness with no taking account of the participation of the remaining section.

In the present paper we propose an improved behaviour model of members submitted to the lateral buckling, through which the stiffness of the fictitious member is determined by taking into account the torsional stiffness of the whole section of the profile.

To alleviate the drawing-up, some justifications of the formulas get developed in the Appendix.

2.1. Usual simplified model with flexural stiffness of the member

In the framework of this model:

- the member rests laterally on a cylindrical longitudinal elastic hinge,
- the free compressed flange is considered as being separate (not connected to the remaining part of the section) and its stability is checked in the same way as for a member on elastic soil where only its flexural stiffness intervenes in the equation of equilibrium of the member at the moment the torsional buckling process occurs.

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2. THEORETICAL BEHAVIOUR MODEL

This means that the torsional buckling of the member about an imposed axis get reduced to the lateral buckling of the flange. As an example, let us take a Z-steel member laterally supported at the level of the upper flange.

The deflected shape of the member is shown in Figure 1. For better intelligibility, the case is shown where the hinge is perfectly free. In the contrary case, the effect of the transverse deformation of the section will be added by superposition.

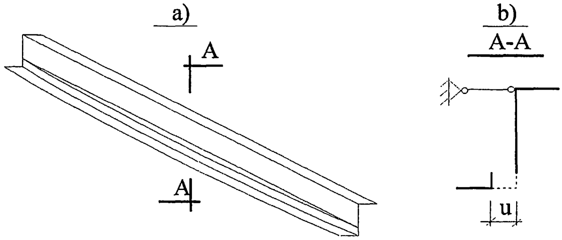

Figure 1- Deflected shape of the freely supported member under buckling of separated free flange

It is obvious that the stability of the thus modelled member is underestimated, as only a part of the stiffness of the section is taken into account, namely that relative to the lateral bending of the separate flange, considered as being not integrated into the section as a whole.

2.2 Proposed improved model, with torsional stiffness of the whole member

Let recall that we take into consideration the torsional buckling of a member laterally supported on a cylindrical elastic longitudinal hinge. The deflected shape of the member is shown in Figure **2**. In case of zero stiffness of the hinge, the section of the member remains transversally undeformed (Fig. 2.b), according to the law of the non-uniform torsion. In case of elastic restraint, the member section would in addition undergo a transverse deformation, which should be added by superposition.

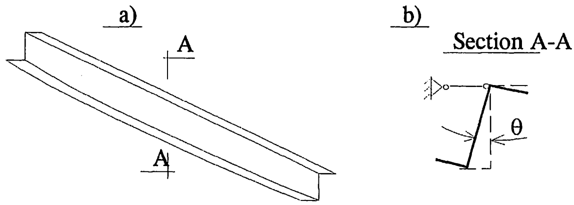

Figure 2- Deflected shape of the freely supported member under torsional buckling

The basic difference between the behaviour modes shown in figures 1 and 2 consists in the fact that:

- according to the first mode, the free flange is fictitiously separated and only its stiffness in lateral bending cooperates to the equilibrium;
- in the second one, as this flange is locked with the rest of the member, it is the stiffness of the whole section and namely the torsional stiffness at non uniform torsion that cooperates to the equilibrium.

At this point, let us recall that both types of stiffness are tightly linked, but the torsional one is a more general notion. This topic will be developed in the chapters hereafter and in the Appendix.

3. - WRITING DOWN THE EQUATIONS OF THE VARIOUS CASES OF APPLICATION OF THE PROPOSED MODÈL

3.1. Case of a purlin restrained by steel sheeting

3.1.1. Description of the behaviour model

Let consider a purlin restrained by steel sheeting attached on the upper flange; one or several sag-bars may possibly be included in the structure. The stability of the free (lower) flange has to be checked in the compression zones, i.e. on the supports under a downward load (Fig. 3.a) and in the span under an upward load (Fig. 3.b).

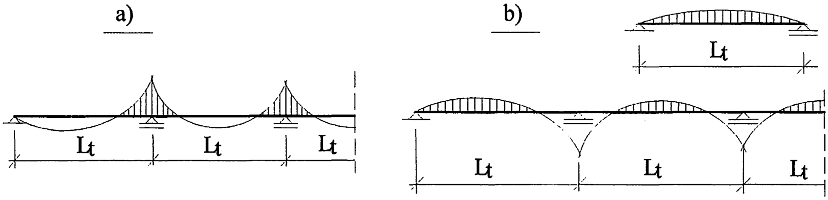

Figure 3- Moment diagram under downward load (a) and upward load (b)

In order to facilitate the analysis, it is assumed that at the ends of the purlin segment under consideration (limited either by sag-bars or by vertical supports) an inflexion point appears when the buckling of the free flange occurs resulting in a hinge (Ref. 4). This assumption is safe as, on account of its continuity, the flange is partially restrained and consequently, its stability is better.

Let remark that this simplification may lead under certain conditions to "paradoxical" results such as those where the addition of a sag bar will allow to obtain the increased buckling length. It is obvious that in such cases, that is the more beneficial value that shall be adopted.

In the model under consideration, the free member composed by the bottom flange and an adjacent part of the web get dealt with as a fictitious member in elastic medium. The stiffness of this medium depends on the lateral displacement under a fictitious linear load F applied at the level of the centroid of the member (Fig. 4). The member is considered as jointly integrated in the rest of the section, i.e. the lateral displacement is calculated by taking into account the torsional stiffness of the whole section of the purlin.

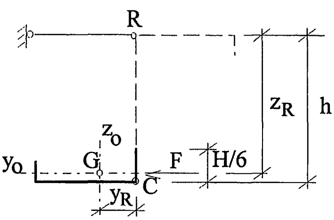

Figure 4- Transverse section of the fictitious member (lower flange of the purlin)

On the figure 4:

- h = depth of the purlin,
- R = imposed axis of rotation,
- G = centroid of the fictitious member,
- F = fictitious transverse linear load, applied at the level of the centroid axis y_0,
- y_0 , z_0 = neutral axes of the fictitious member,
- y_R , z_R = coordinates of the center of rotation R in the system of axis y_0, z_0.

The fictitious member is submitted to a normal variable load in form of a uniformly distributed normal stress in the cross section. The scheme of the stresses is shown in figure 5: the external load in 5.a, and the internal normal force generated by the external load in 5.b.

In a general case, each one of the external loads P, q_0 or q_1, may be of the opposite orientation. The maximal compressive force (N_{max}) has to be determined in any actual case as a function of the external loads. As an example, for the case presented in figure 5: $N_{max} = P+0,5(q_o+q_l)L$.

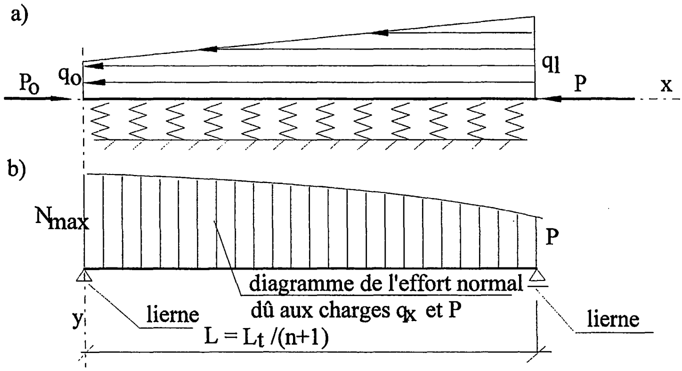

Figure 5- Static scheme of a segment of purlin between two sag bars

The state of deformation of the transverse section of the purlin in instability situation is shown in figure 6. Both the side support and the elastic restraint on the sheeting are continuous. Let notice that this behaviour model may be applied to any section of light purlins such as "Z" or "C", and to their derived products "Zeta" or "Sigma".

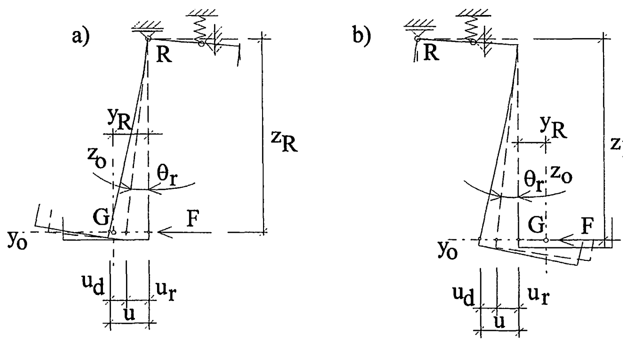

Figure 6- Deformed shape of the transverse section of the purlin in instability state.

The orientation of the force F depends on the application direction of the load (upward or downward) (Ref. 1). The displacement "u" depends on the stiffness of fastening of the purlin in the steel decking and of transverse deformation of the section. This stiffness may be found (either by computation or by tests) by determining the components u_r and u_d of the displacement under the load F, at the level of the centroid of the flange (Fig. 4).

Thus the following is obtained: $\beta_r=\frac{F}{u_r}$, $\beta_d=\frac{F}{u_d}$, $\beta=\frac{F}{u}$ (1)

where:
- u_r ensues from the rotation θ_r of the transversally not deformed section,
- u_d results from the transverse deformation of the section,
- u is the complete displacement,

- β_r is the stiffness of the elastic hinge (of the connection with the steel-decking),
- β_d is the stiffness of the transverse deformation of the section,
- β is the global stiffness.

As it is known that $u = u_r + u_d$, the following relation is obtained:
$$\frac{1}{\beta}=\frac{1}{\beta_r}+\frac{1}{\beta_d} \tag{2}$$

The components u_r and u_d remain in constant proportion with respect to the total value "u", while the load F is variable. In fact, by taking count of (1), we have:

$$\frac{u_r}{u}=\left(\frac{1}{\beta_r}\right)/\left(\frac{1}{\beta}\right)=\frac{\beta}{\beta_r}=R_r \tag{3.a}$$

$$\frac{u_d}{u}=\left(\frac{1}{\beta_d}\right)/\left(\frac{1}{\beta}\right)=\frac{\beta}{\beta_d}=R_d \tag{3.b}$$

3.1.2. Energy of deformation of the system

The energy of deformation of the system may be computed in relation to the two displacement components u_r and u_d. With taking into account (3.a), the external deformation energy when assuming linear conditions corresponding to the displacement u_r, becomes:

$$E_{er}=\frac{\beta R_r}{2}\int_0^L u^2 dx \tag{4}$$

With taking into account (3.b), the internal energy of transverse deformation of the linear section of the member corresponding to the displacement u_d, becomes:

$$E_{ed}=\frac{\beta R_d}{2}\int_0^L u^2 dx \tag{5}$$

The sum of both energies (4) and (5) is:
$$E_e=E_{er}+E_{ed}=\frac{\beta}{2}\int_0^L u^2 dx \tag{6}$$

The internal energy generated by the non-uniform torsion of the section is equivalent to:

$$E_i=E_{ir}+E_{id} \tag{7}$$

where: E_{ir} is the energy resulting from the torsion of the whole transversally not deformed section, concomitant with the horizontal displacement u_r of the flange,
E_{id} is the energy resulting from the torsion of the lower part of the section, concomitant with the horizontal displacement u_d of the flange.

- The energy due to the torsion of the global section is equal to (see Ref. 5):

$$E_{ir}=\frac{EI_{\omega R}}{2}\int_0^L\left(\frac{d^2\theta_r}{dx^2}\right)^2 dx+\frac{GI_s}{2}\int_0^L\left(\frac{d\theta_r}{dx}\right)^2 dx \tag{8}$$

where: E, G = Young's and shear modulus respectively,
θ_r = angle of torsion of the section around the point R,
$I_{\omega R}$ = warping constant with respect to the imposed center of rotation R (Ref. 6),
I_s = St. Venant torsional constant.

By converting the angle of rotation θ_r in displacement u_r (figure 6): $\theta_r=\dfrac{u_r}{z_R}$ (9)

in (9) and with taking account of (3.a), we obtain:

$$E_{ir} = \frac{EI_{\omega R} R_r^2}{2z_R^2} \int_0^L \left(\frac{d^2u}{dx^2} \right)^2 dx + \frac{GI_s R_r^2}{2z_R^2} \int_0^L \left(\frac{du}{dx} \right)^2 dx \qquad (10)$$

- The energy related to the horizontal displacement u_d of the flange is determined by taking into account the rotation of the lower part of the section that consists of the flange and a part of the web, see figure 8. For more simplicity, only the effect of the displacement u_d is shown in this figure whilst knowing that it has to be added to that of the displacement u_r, see Reference 6.

The point D is obtained by the intersection of the tangent at the lower end of the deformed web with its initial position. It can be estimated with a satisfactory accuracy that for a given load F, the whole part of section under this point is submitted to a uniform rotation θ_d (Fig. 7). The depth of this part is:

$$z_D = (2/3) z_R \qquad (11)$$

The energy corresponding to this deformation may thus be calculated in the same way as that of the torsion θ_d of this part of the section. The calculation has to be effected with taking count of the fact that the rotation θ_d get done in addition to θ_r and that the rotation θ_d around the point D is obtained as a sum of the rotations θ_{pr} around the point R and θ_{pc} around the point C (Fig. 7):

$$E_{id} = \int_0^L \left\{ \frac{EI_{\omega pR}}{2} \left[\left(\frac{d^2\theta_r}{dx^2} + \frac{d^2\theta_{pr}}{dx^2} \right)^2 - \left(\frac{d^2\theta_r}{dx^2} \right)^2 \right] + \frac{EI_{\omega pC}}{2} \left(\frac{d^2\theta_{pc}}{dx^2} \right)^2 \right\} dx +$$

$$\int_0^L \frac{GI_{sp}}{2} \left[\left(\frac{d\theta_r}{dx} + \frac{d\theta_{pr}}{dx} \right)^2 - \left(\frac{d\theta_r}{dx} \right)^2 + \left(\frac{d\theta_{pc}}{dx} \right)^2 \right] dx \qquad (12)$$

where: $I_{\omega pR}$ = warping constant of the lower part of the section, with respect to R,

$I_{\omega pC}$ = warping constant of the lower part of the section, with respect to C,

I_{sp} = St. Venant torsional constant of the lower part of the section,

θ_r = angle of rotation of the whole section around the point R,

θ_{pr}, θ_{pc} = angles of rotation of the lower part of the section around the point R and C respectively

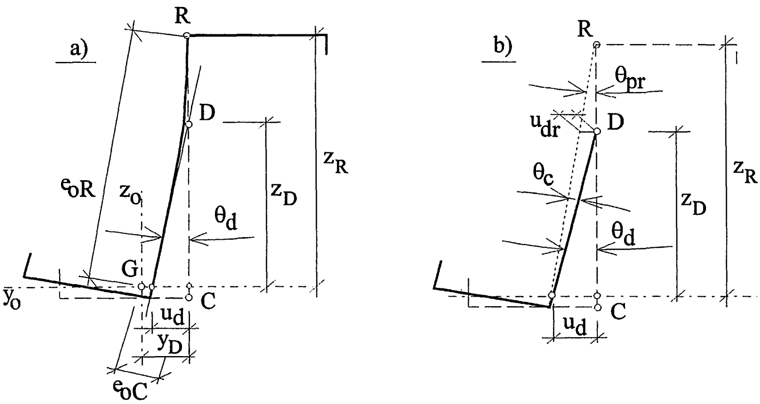

Figure 7- Torsion of the lower part of the section, corresponding to the displacement u_d

By substituting (9) and with:

$$\theta_d = \frac{u_d}{z_D}, \qquad \theta_{pr} = \frac{u_d}{z_R}, \qquad \theta_{pc} = \frac{u_{dr}}{z_D}, \qquad u_{dr} = u_d \frac{z_R - z_D}{z_R} \qquad (13)$$

in (12) and with taking into account the equations (3.a) and (3.b) we obtain:

$$E_{id} = \frac{ER_d}{2z_R^2}\left[I_{\omega pR}(1+R_r) + 0{,}25 I_{\omega pC} R_d\right]\int_0^L \left(\frac{d^2u}{dx^2}\right)^2 dx + \frac{GI_{sp}}{2z_R^2} R_d(1+R_r+0{,}25R_d)\int_0^L \left(\frac{du}{dx}\right)^2 dx \qquad \text{.........................} \quad (14)$$

By introducing (10) and (14) in (7) we obtain:

$$E_i = \frac{EI_m}{2}\int_0^L \left(\frac{d^2u}{dx^2}\right)^2 dx + \frac{GS_m}{2}\int_0^L \left(\frac{du}{dx}\right)^2 dx \qquad (15)$$

where:
$$I_m = \frac{I_{\omega R} R_r^2 + I_{\omega pR} R_d (1+R_r) + 0{,}25 I_{\omega pC} R_d^2}{z_R^2} \qquad (16)$$

$$S_m = \frac{I_s R_r^2 + I_{sp} R_d (1+R_r+0{,}25R_d)}{z_R^2} \qquad (17)$$

The first term of the equation (15) stands for the energy of deformation in non-uniform torsion, which results in differential elongation of the longitudinal fibres of the member, leading to the warping of the section. The product EI_m expresses the stiffness corresponding to this deformation. Let us call it ***efficient stiffness***. The inertia I_m , which we shall call ***efficient inertia***, replaces that of the axial bending in the whole process of proposed calculation, including the definition of the gyration radius and as a consequence, the slenderness.

The second term of the equation (15) represents the energy of deformation in uniform (St. Venant) torsion of the member.

The potential energy of the external static loads acting on the fictitious member consists of that due to the load at the end and of that due to the distributed load on the length of the member (Fig. 5). Let recall that according to the simplifying assumption stated in 3.1.1, the load is considered as being uniformly distributed on the section of the fictitious member, as a normal stress.

- The potential energy of the load at the end P is:

$$E_P = E_{Pr} + E_{Pd} \qquad \textbf{(18)}$$

E_{Pr} is the potential energy related to the displacement of the force P due to the rotation θ_r of the section of the fictitious member (Fig. 6):

$$E_{Pr} = \frac{\delta_P P}{2A_m}\int_0^L \left[\int_{A_m} \rho_R \left(\frac{d\theta_r}{dx}\right)^2 d\rho_R\right] dx \qquad (19)$$

where: δ_P = a parameter indicating the direction of the load P at the end of the member:
+1 indicates the positive direction (x-axis orientation),
-1 indicates the negative direction (opposite orientation of the x-axis),
ρ_R = radius connecting the pole R and a given point of the section,
A_m = section area of the fictitious member.

Taking into account the equations (3.a) and (10) as well as the equality:

$$\int_{A_m} \rho_R \, d\rho_R = I_{oR} \qquad (20)$$

where: $I_{oR} = I_{oy} + I_{oz} + A_m e_{oR}^2$ = polar inertia moment of the member with respect to R,
e_{oR} = distance between the centroid G and the pole R,

I_{oy} and I_{oz} = axial moments of inertia with respect to the y and z axes respectively,

we get: $$E_{Pr} = \frac{\delta_P\, P\, I_{oR} R_r^2}{2A_m z_R^2} \int_0^L \left(\frac{du}{dx}\right)^2 dx \qquad (21)$$

E_{Pd} is the potential energy corresponding to the displacement of the force P due to the rotation θ_d of the section of the fictitious member (Fig. 4) around the point D (Fig. 7). By superposition of the rotations θ_r around the point R and θ_c around the point C and by proceeding in a similar way as in that used to calculate the internal energy E_{id}, we obtain:

$$E_{Pd} = \frac{\delta_P\, P}{2A_m} \int_0^L \left\{ \int_{A_m} \rho_R \left[\left(\frac{d\theta_r}{dx} + \frac{d\theta_{pr}}{dx} \right)^2 - \left(\frac{d\theta_r}{dx} \right)^2 \right] d\rho_R + \rho_C \left(\frac{d\theta_{pc}}{dx} \right)^2 d\rho_C \right\} dx \qquad (22)$$

By taking count of (3.a), (3.b), (13) and (22) as well as of the equality:

$$\int_{A_m} \rho_C \, d\rho_C = I_{oC} \qquad (23)$$

where: I_{oC} = polar moment of inertia of the fictitious member (Fig. (8)) with respect to the pole C,

we get: $$E_{Pd} = \frac{\delta_P\, P\, R_d}{2A_m z_R^2} \left[I_{oR}(1 + R_r) + 0{,}25 I_{oC} R_d \right] \int_0^L \left(\frac{du}{dx}\right)^2 dx \qquad (24)$$

By substituting (21) and (24) in (18) we obtain: $$E_P = \frac{\delta_P\, P\, C}{2} \int_0^L \left(\frac{du}{dx}\right)^2 dx \qquad (25)$$

where: $$C = \frac{A_f}{A_m} \qquad (26)$$

$$A_f = \frac{I_{oR} + 0{,}25 I_{oC} R_d^2}{z_R^2} \qquad (27)$$

- <u>The potential energy of the distributed load</u>: $q_x = \delta_1 q_1 \left[(1-\eta) x / L + \eta \right]$ (28)

where: $$\eta = \frac{\delta_0 q_0}{\delta_1 q_1} \qquad (29)$$

q_0, q_1 = absolute values of the load q_x at the origin and at the end respectively,
δ_0, δ_1 = parameters indicating the direction of the load q_x at the ends of the member: +1, -1 indicate respectively the positive and negative direction (x-axis orientation)

is equal to: $$E_q = E_{qr} + E_{qd} \qquad (30)$$

E_{qr} is the potential energy relative to the displacement of the load q_x, due to the rotation θ_r of the fictitious member (Fig. 4) around the point R:

$$E_{qr} = \frac{1}{2A_m} \int_0^L \left[\int_{A_m} \rho_R \left(\frac{d\theta_r}{dx}\right)^2 d\rho_R \right] \left(\int_x^L q_x dx \right) \left(\frac{du}{dx}\right)^2 dx \qquad (31)$$

Taking count of the equations (3.a), (3.b), (20) and (28), we get:

$$E_{qr} = \frac{\delta_1 q_1\, I_{oR} R_r^2}{2A_m z_R^2} \int_0^L \left[(\eta - 1)\frac{x^2}{2L} - \eta x + \frac{L}{2}(1+\eta) \right] \left(\frac{du}{dx}\right)^2 dx \qquad (32)$$

E_{qd} is the potential energy relative to the displacement of the load q_x, due to the rotation θ_d of the the fictitious member (Fig. 4) around the point D (Fig. 8). By proceeding in a similar way as in that used to calculate the load at the end, we obtain:

$$E_{qd} = \frac{\delta_1 q_1 R_d}{2A_m z_R^2}\left[I_{oR}(1+R_r) + 0{,}25 I_{oC} R_d\right]\int_0^L \left[(\eta-1)\frac{x^2}{2L} - \eta x + \frac{L}{2}(1+\eta)\right]\left(\frac{du}{dx}\right)^2 dx \quad (33)$$

By substituting (32) and (33) in (30) we have:

$$E_q = \frac{\delta_1 q_1 C}{2}\int_0^L \left[(\eta-1)\frac{x^2}{2L} - \eta x + \frac{L}{2}(1+\eta)\right]\left(\frac{du}{dx}\right)^2 dx \quad (34)$$

<u>The total energy of the deformed system</u> is: $E_{sys} = E_e + E_i + E_P + Eq$

Substituting in this formula (6), (15), (25) and (34) and replacing x by $\xi = \frac{x}{L}$, we obtain:

$$E_{sys} = \frac{1}{2}\int_0^1 \left\langle \frac{EI_m}{L^3}(u'')^2 + \left\{\frac{\delta_P P\, C}{L} + \frac{\delta_1 q_1 C}{2}\left[(\eta-1)\xi^2 - 2\eta\xi + \eta + 1\right] + GS_m\right\}(u')^2 + \beta L u^2 \right\rangle d\xi$$

.... (35)

where: $u' = \frac{du}{d\xi}$ $\quad u'' = \frac{d^2u}{d\xi^2}$

3.1.3. Equation of equilibrium of the system

The equation (35) represents a functional of type $J[u] = F(\xi, u, u', u'')d\xi$ that meets its minimum when its first variation is zero, expressed by the so-called Euler-Poisson relation:

$$F_u - \frac{d}{d\xi}F_{u'} + \frac{d^2}{d\xi^2}F_{u''} = 0 \quad (36)$$

where: $F_u = \frac{\partial F}{\partial u}$ $\quad F_{u'} = \frac{\partial F}{\partial u'}$ $\quad F_{u''} = \frac{\partial F}{\partial u''}$

By expressing the load P in form of: $\frac{q_1 L}{2}$ (37)

where the factor ψ has to be preliminarily determined as a function of the ratio $\psi = P / \left(\frac{q_1 L}{2}\right)$,

and by substituting (37) in (38), we get:

$$u^{IV} - \frac{GS_m L^2}{EI_m}u'' + \frac{\beta L^4}{EI_m}u - \frac{\delta_1 q_1 C L^3}{2EI_m}\left[\delta_P \delta_1 \psi + (\eta-1)\xi^2 - 2\eta\xi + \eta + 1\right]u'' - \frac{\delta_1 q_1 C L^3}{2EI_m}\left[(\eta-1)\xi - \eta\right]u' = 0 \quad (38)$$

By setting in these: $N_e = \frac{\pi^2 EI_m}{L^2}$ (39), $N_r = \frac{q_1 L}{2}$ (40), $\alpha_{ref} = \frac{N_e}{N_{ref}}$ (41)

$N_{ref} = N_r C$ (42), $K = \frac{\beta L^4}{\pi^4 EI_m}$ (43), $K_s = \frac{GS_m L^2}{\pi^2 EI_m}$ (44)

the following differential equation of equilibrium of the 4th degree results:

$$\alpha_{ref}\left(u^{IV} - \pi^2 K_s u'' + \pi^4 K u\right) - \pi^2\delta_l\left[(\eta-1)\xi^2 - 2\eta\xi + \delta_P\delta_l\psi + \eta + 1\right]u'' - 2\pi^2\delta_l\left[(\eta-1)\xi - \eta\right]u' = 0 \quad \text{............} \quad (45)$$

3.1.4. Solution of the equation of equilibrium

In order to resolve the equation (45), one of various "direct" methods may be used that consist in minimizing a residue ε_i obtained with a previously adopted functionale basis Φ_i , whilst nevertheless respecting the limit conditions. We have chosen the Galerkin method (see Ref. 9), searching after the solution in the form:

$$u = \sum_{i=1}^{n} a_i \Phi_i \qquad (46)$$

with the functional basis:

$$\Phi_i = \sin \pi i \xi \qquad (47)$$

By substituting (46) in (45), the following residue is obtained:

$$\varepsilon_i = \sum_{i=1}^{n} \pi^4 a_i \Big\{\left[\alpha_{ref}\left(i^4 + i^2 K_s + K\right) + \delta_l i^2\left(\eta + \delta_P\delta_l\psi + 1\right)\right]\sin \pi i\xi + \\ \delta_l i^2(\eta-1)\xi^2 \sin \pi i\xi - 2\delta_l \eta i^2 \xi \sin \pi i\xi - \delta_l \frac{2i}{\pi}(\eta-1)\xi\cos \pi i\xi + \delta_l \frac{2i}{\pi}\eta\cos \pi i\xi\Big\} \qquad (48)$$

that generally is not nil. The minimization of the residues ε_i (i=1,n) is effected by writing down the conditions of orthogonality:

$$\int_0^1 \varepsilon_i \Phi_j d\xi = 0 \qquad \text{.....} \quad j = 1,n \qquad (49)$$

We thus obtain a system of homogeneous equations, whose determinant:

$$\begin{vmatrix} (A_{11} - \alpha_{ref}) & A_{12} & \cdots\cdots & A_{1n} \\ A_{21} & (A_{22} - \alpha_{ref}) & \cdots\cdots & A_{2n} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ A_{n1} & A_{n2} & \cdots\cdots & (A_{nn} - \alpha_{ref}) \end{vmatrix} \qquad (50)$$

has to be zero in order that a non-nil solution be possible. This leads us to search after the eigen-values α_{ref} of a matrix whose terms are:

- for i = j:

$$A_{ii} = -\delta_l \frac{2\pi^2(2 + \eta + 3\delta_l\delta_P\psi)i^2 - 3(1-\eta)}{6\pi^2\left(i^4 + K_s i^2 + K\right)}$$

- for i ≠ j:

$$A_{ij} = -\frac{4\delta_l}{\pi^2\left(j^4 + K_s j^2 + K\right)}\sum_{\substack{i=1 \\ i\neq j}}^{n} \frac{ij\left(i^2 + j^2\right)}{\left(i^2 - j^2\right)^2}\left[\eta - (-1)^{i+j}\right]$$

Let remark that the above obtained eigen-values α_{ref} do not correspond to the maximum (critical) force N_{max}, but to the reference one (equation (41)). To obtain the buckling length, we set:

$$\gamma = \frac{N_{ref}}{N_{max}} \qquad (51)$$

With the equalities (39), (41) and (50) being taken into account, the buckling length is:

$$L_f = L\sqrt{\alpha_{cr}} \quad (52), \qquad \alpha_{cr} = \gamma\alpha_{ref} \quad (53)$$

that corresponds to the critical force called Euler's force: $N_{max} = \dfrac{\pi^2 EI}{L_f^2} = N_{cr}$

3.1.5. Examples

For practical demonstration, we present hereafter the results by computer design of various cases of a purlin MULTIBEAM A300, a product of Profil du Futur – GROUPE USINOR (Fig. 9), under an upward load, as a simple 6000 mm long span, with one sag-bar and with no sag-bar, of various thickness and with different fastening conditions with the steel decking.

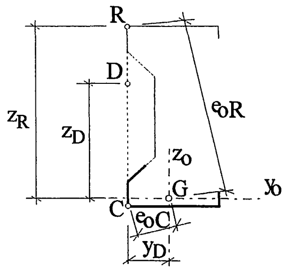

Figure 8- Transverse section of the purlin. Design geometrical parameters.

With reference to Fig. 5, the data are displayed in Table 1 (Ref. 5)

With 1 sag bar	With no sag bar
$q_0=0, \quad q_1=0.5$ (N/m)	$q_0=0, \quad q_1=0.5$ (N/m)
$\delta_0=1, \quad \delta_1=-1$	$\delta_0=0.5, \quad \delta_1=-1$
$P=0$ (N)	$P=0$ (N)
$\eta=\frac{\delta_0 q_0}{\delta_1 q_1}=0, \quad \psi=\frac{P}{\frac{q_1 L}{2}}=0$	$\eta=\frac{\delta_0 q_0}{\delta_1 q_1}=-1, \quad \psi=\frac{P}{\frac{q_1 L}{2}}=0$
L = 3000 mm (reference length = distance between sag bar and bearing)	L = 6000 mm (reference length = distance between bearings)

1 kN/m = 0.06852 kips/foot

1 kN = 4.45 kips

Table 1- Design data

The design procedure is as follows:

- the reference eigenvalue α_{ref} is calculated from the equation (50),
- the critical eigenvalue α_{cr} is calculated from the equation (53),
- the buckling length L_f is calculated from the equation (52).

For purposes of this calculation work, a computer program has been especially evolved. The results are displayed in Table 2.

T	Number of	β	Calculation method						Ratio
			proposed			traditional			
mm	sag bars	N/mm/mm	L_f/L	i_m (mm)	χ_ω	L_f/L	i_m (mm)	χ_ω	χ_ω/χ_m
1.8	1	0.0051	0.763	26.7	0.586	0.755	24.5	0.529	1.11
		0	0.801	27.8	0.581	0.802	24.5	0.483	1.20
	0	0.0051	0.429	26.7	0.497	0.410	24.5	0.467	1.06
		0	0.675	27.8	0.247	0.694	24.5	0.185	1.34
3.2	1	0.0100	0.749	27.3	0.618	0.766	24.5	0.517	1.20
		0	0.787	27.8	0.594	0.802	24.5	0.483	1.23
	0	0.0100	0.415	27.3	0.540	0.400	24.5	0.486	1.11
		0	0.628	27.8	0.281	0.694	24.5	0.185	1.52

Table 2- Calculation results

1 mm = 0.0394 in.

In the table 2: $1\ kN/m^2 = 0.0186$ kips / square foot

$i_m = (I_m/A_m)^{0.5}$ = *efficient* radius of gyration of the free fictitious member,
t = thickness of the transverse section of the purlin ,
β = lateral spring stiffness of the connection between purlin and sheeting (N/mm/mm),
L = reference length (distance between the side bearings),
L_f = buckling length,
χ_ω = buckling coefficient obtained with taking count of the member torsional stiffness (according to the proposed approach), calculated in compliance with Ref. 3,
χ_m = buckling coefficient obtained in conformity with the traditional approach (without taking count of the member torsional stiffness).

The calculation according to the traditional approach (Refs. 3 to 6) has been effected as a comparison. The values in the last column indicate the ratios between the buckling coefficients obtained according to these two different approaches. Although this comparison is not exhaustive, the following general conclusions may be inferred from:

- the torsional stiffness significantly collaborate to the stability strength,
- the gain in performance rises with the increase of the section thickness, due to the greater increase of the St. Venant torsional constant,
- the proposed calculation model is especially interesting in the case when the steel decking offer a simple side bearing only, without impending the rotation (lateral stiffness $\beta = 0$).

3.2. Case of a column under eccentric compression

3.2.1. Description of the system

Let consider a laterally restrained and simply supported column, under a compression force P and a bending moment M constant over the whole length. In case of variable forces and a possible elastic restraint on the side bearing, a more complex analysis would be necessary such as that used in chapter 3.1. It is assumed that the end bearing supports hinder the rotation in the plane perpendicular to the longitudinal axis, but they don't hinder the end sections from warping. Let take, as an example, the profile shown on figure 9.

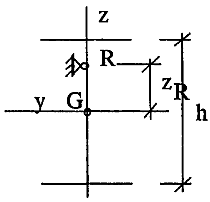

Figure 9- Laterally restrained column

On this: R = imposed axis of rotation,
G = centroid of the profile,
z_R = coordinate of the point of rotation R.

3.2.2. Energy of deformation of the system

Two effects are superposed:

- that of the force P, applied over the whole section,
- that of the moment M, represented by the couple $P_m = M/h$, where:
  - h = distance between the axes of the flanges (considered as fictitious members),
  - the compression force P_m is applied on the free fictitious member,
  - the tension force P_m is neglected in the energy balance (that is safe simplification).

Internal energy due to the non-uniform torsion of the section

By introducing the relationship: $\theta = \dfrac{u}{z_R}$ (54)

this energy may be written : $E_i = \dfrac{EI_{\omega R}}{2z_R^2}\int_0^L \left(\dfrac{d^2u}{dx^2}\right)^2 dx + \dfrac{GI_s}{2z_R^2}\int_0^L \left(\dfrac{du}{dx}\right)^2 dx$ (55)

where: $E, G, I_{\omega R}$ and I_s are defined in chapter 3.1.2,

z_R = component about the z axis of the distance between the centroid G and the pole R,

u = side displacement of the centroid of the section.

Potential energy of the load

The potential energy relative to the displacement of the forces P and P_m, due to the rotation θ_r of the section is:

$$E_P = -\frac{1}{2z_R^2}\left(\frac{P\,I_{oR}}{A} + \frac{P_m\,I_{moR}}{A_m}\right)\int_0^L \left(\frac{du}{dx}\right)^2 dx \qquad (56)$$

where: A, A_m = area of the total section and of the fictitious member, respectively,

I_{oR} = polar inertia moment of the total section with respect to the pole R,

I_{moR} = polar inertia moment of the fictitious member with respect to R.

After substituting:

$$\left.\begin{aligned} P_m/P &= \psi_P \\ A_m/A &= \psi_A \\ I_{oe} &= I_{oR} + I_{moR}\,(\psi_P/\psi_A) \end{aligned}\right\} \qquad (57)$$

into (56), we get:

$$E_P = -\frac{P\,I_{oe}}{2Az_R^2}\int_0^L \left(\frac{du}{dx}\right)^2 dx \qquad (58)$$

After replacing the variable x by: $\xi = x/L$, the total energy of the system may be written:

$$E_{sys} = \frac{1}{2Lz_R^2}\int_0^1 \left[\frac{EI_{\omega R}}{L^2}(u'')^2 + \left(GI_s - \frac{P\,I_{oe}}{A}\right)(u')^2\right]d\xi \qquad (59)$$

where: $u' = \dfrac{du}{d\xi}$ $\quad u'' = \dfrac{d^2u}{d\xi^2}$

We may remark that by substituting: $\beta = 0$, $q_l = 0$, R_r=1, R_d=0 directly in (33) and (35) we would obtain the same result.

3.2.3. Equation of equilibrium of the system

The equation (59) represents a functional of type: $J[u] = F(u', u'')d\xi$ that reaches its minimum

value when:

$$-\frac{d}{d\xi}F_{u'} + \frac{d^2}{d\xi^2}F_{u''} = 0 \qquad (60)$$

where: $F_{u'} = \dfrac{\partial F}{\partial u'}$ $\qquad F_{u''} = \dfrac{\partial F}{\partial u''}$

By substituting (59) in this, we obtain the following differential equation of equilibrium:

$$u^{IV} + k^2L^2u'' = 0 \qquad \textbf{(61)}$$

where:

$$k^2 = \frac{P\,I_{oR}}{EI_{\omega R}A} - GI_s \qquad (62)$$

The general solution of (62) is: $u = C_1 \sin kL\xi + C_2 \cos kL\xi + C_3\,\xi + C_4$ (63)

For the limit conditions (end hinges): u(0)= u(1)= u"(0)= u"(1)= 0, we find:

$$C_2 = C_3 = C_4 = 0 \text{ and } k = \frac{\pi}{L} \tag{64}$$

By taking into account (62), the critical force is obtained by the formula:

$$P_{cr} = \frac{A}{I_{oe}}\left(\frac{\pi^2 EI_{\omega R}}{L^2} + GI_s\right) \tag{65}$$

Particular cases:

- For M = 0 (axial compression), with account of the equalities (63) being taken, we obtain:

$$P_{cr} = \frac{A}{I_{oR}}\left(\frac{\pi^2 EI_{\omega R}}{L^2} + GI_s\right) \tag{66}$$

We note that in a case of buckling by torsion around the shear centre, the equation (66) becomes identical to that well known of the strength of materials (Refs. 1 and 5).

- For P = 0 (bending without axial force), the only terms corresponding to the bending are to be maintained in the equation (62) and as a consequence, the critical force is:

$$P_{cr} = \frac{A_m}{I_{moR}}\left(\frac{\pi^2 EI_{\omega R}}{L^2} + GI_s\right) \tag{67}$$

Let remark that although developed by the example of a doubly symmetrical profile, this formulation should be made generally applicable to non-symmetrical sections.

Comparative example: proposed and traditional calculations

Let check an IPE 300 vertical column under the axial load P, with a laterally restrained flange.

Section properties (Fig. 9):

$f_y = 240$ N/mm^2, $A = 5380$ mm^2, $I_z = 6{,}04$ E+6 mm^4, $I_y = 8{,}356$ E+7 mm^4, $I_s = 1{,}947$ E+5 mm^4
$t_s = 10{,}7$ mm (flange thickness), $t_a = 7{,}1$ mm (web thickness)
$h = 300 - t_s = 289{,}3$ mm (distance between neutral fibres of the flanges),
$z_R = h/2 = 144{,}65$ mm,
$I_{oR} = I_z + I_y + A^* z_R^2 = 2{,}022$ E+8 mm^4 , $I_{\omega R} = I_m{}^* h^2 = 2{,}528$ E+11 mm^6

Calculation according to the proposed approach:

Starting from the equation (72) we have: $P_{cr} = \frac{A}{I_{oR}}\left(\frac{\pi^2 EI_{\omega R}}{L^2} + GI_s\right) = 1{,}963 \text{ E+6 N}$

$\bar{\lambda} = (f_y A / P_{cr})^{0,5} = 0{,}811$, $\varphi = 0{,}5\left[1 + 0{,}21(\bar{\lambda} - 0{,}.2) + \bar{\lambda}^2\right] = 0{,}893$

Column buckling factor: $\chi = \dfrac{1}{\varphi + \left(\varphi^2 - \bar{\lambda}^2\right)^{0,5}} = 0{,}789$

Stress: $\sigma = \dfrac{P}{\chi A}\gamma_{M1} = 1{,}267\dfrac{P}{A}\gamma_{M1}$

where: γ_{M1} = partial safety factor (Ref. 1)

Calculation according to traditional approaches:

The free member is considered as separated, loaded by force $P_m = 0.5$ P.
Two modes of calculation are made: mode 1, with the free member consisting of the flange and a half-web and mode 2, with the free member consisting of the only flange. The results are given in the table 3.

Mode 1	Mode 2
$A_m = 0.5A = 0.5*5380 = 2690$ mm^2	$A_m = 10.7*150 = 1605$ mm^2 = 0.298A
$I_m = 0.5\ I_z = 3.02$ E+6 mm^4	$I_m = 10.7*150^3/12 = 3.01$ E+6 mm^4
$im=(Im/Am)^{0.5}=33.5$ mm	$im=(Im/Am)^{0.5}=43.2$ mm
$\lambda m=L/im=89.5$	$\lambda m=L/im=69.3$
$\bar{\lambda}m=\frac{\lambda m}{\pi}\left(\frac{fy}{E}\right)^{0.5}=0.963$	$\bar{\lambda}m=\frac{\lambda m}{\pi}\left(\frac{fy}{E}\right)^{0.5}=0.746$
$\varphi m=0.5\left[1+0.21\left(\bar{\lambda}m-0.2\right)+\bar{\lambda}_m^2\right]=1.044$	$\varphi m=0.5\left[1+0.21\left(\bar{\lambda}m-0.2\right)+\bar{\lambda}_m^2\right]=0.835$
$\chi m=\frac{1}{\varphi m+\left(\varphi_m^2-\bar{\lambda}_m^2\right)^{0.5}}=0.691$	$\chi m=\frac{1}{\varphi m+\left(\varphi_m^2-\bar{\lambda}_m^2\right)^{0.5}}=0.825$
Stress: $\sigma m=\frac{Pm}{\chi mAm}\gamma M1=1.447\frac{P}{A}\gamma M1$	Stress: $\sigma m=\frac{Pm}{\chi mAm}\gamma M1=2.034\frac{P}{A}\gamma M1$
With regard to the proposed calculation,	With regard to the proposed calculation,
we get: $\frac{\sigma m}{\sigma}=\frac{1.447}{1.267}=1.14$	we get: $\frac{\sigma m}{\sigma}=\frac{2.034}{1.267}=1.60$

1 mm^2 = 1.55*10^{-3} square inches
1 mm^4 = 2.40*10^{-6} inches^4
1 mm = 0.3937 inches

Table 3 - Calculation results according to the traditional approach

4. - CONCLUSION

The present study is aimed at developing a behaviour model of a member submitted to torsional buckling with the position of its axis of rotation being imposed by the connections with the other structural members.

The essential difference between this approach and the usually used methods consists in taking into account the stiffness in non-uniform torsion about the imposed axis of the whole transverse section of the member, instead of the only stiffness in lateral bending of the separated free flange.

We have applied this model to light purlins steadied by steel sheeting and to laterally restrained columns.

Numerical examples illustrate the advantages of the proposed approach by bringing out the not yet used resistance reserves generated by the simplified traditional calculation that takes into account the only bending stiffness of the free member considered as being separate.

APPENDIX – Calculation of sectorial properties

A.1. Sectorial section properties - General principles

The sectorial moment of inertia (warping constant) implied in the calculation of members submitted to non-uniform torsion is given by the following formula:

$$I_\omega = \int_A \omega^2 dA \qquad \text{(A.1)}$$

where: ω = warping coordinate (sectorial area), A = section area.

The warping coordinate ω is calculated with regard to the centre of rotation. When the member is free (as the case is for free torsion), the centre of rotation coincides with the shear centre. When the member is laterally supported by a fixed longitudinal cylindrical hinge, the centre of rotation coincides with this hinge (Ref. 8). In this case, a difference has to be made between two conditions of the connection with the hinge:

- preventing the elongation of the restrained fibre,
- unrestricting the elongation of the restrained fibre.

In the first case, for the calculation of the warping coordinate ω, the initial point starting from which the calculation of the initial warping coordinate is effected, is located at the imposed centre of rotation. It means that the warping coordinate is zero at this point, according to the assumption of null elongation of the restrained fibre. On the contrary, the shear force between the member and the support does not get annulled at this point since it is proportional to the sectorial static moment $S_\omega = \int_A \omega dA \neq 0$

This force has to be taken into account for the dimensioning of the connection between the member and the support (let take as example the case of the connection of a composite beam with a concrete slab).

In the second case, the resultant of the normal forces in the section due to the non-uniform torsion is zero. To meet this requirement, the warping coordinate has to be "normalized" according to the formula (Ref. 10):

$$\omega = \Omega - \frac{\int_0^L \Omega dA}{A}$$

To calculate the "not normalized" initial warping coordinate Ω, we may start from any (arbitrarily chosen) initial point. The convenient choice of this point may nevertheless facilitate the calculation. Let remark that in this case there is no shear stressing between the member and the support, since the relationship (28) implicitly ensures the nullity of the integral of the sectorial static moment on the section:

$$S_\omega = \int_A \omega dA = 0$$

A.2. - Displacement of the axis of rotation

Let consider any thin-walled member in the y - z system of reference (Fig. A.1). It is a matter of defining the relationship between the warping constants (sectorial inertia moments) $I_{\omega A}$ and $I_{\omega B}$, of the section when the centre of rotation get displaced from A to B.

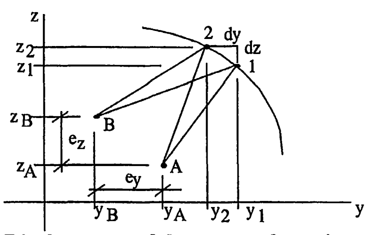

Figure A.1- Displacement of the centre of rotation of the section

The warping coordinates of the infinitesimal element i 1-2, calculated with regard to the centres of rotation A and B are:

$$d\omega_A = dy(z - z_A) - dz(y - y_A) \quad \text{(A.2)}$$

$$d\omega_B = dy(z - z_B) - dz(y - y_B) \quad \text{(A.3)}$$

from which:

$$d\omega_B = d\omega_A - dy * e_z - dz * e_y \quad \text{(A.4)}$$

e_y and e_z: see Figure A.1

By substituting (A.4) in (A.1) and knowing that $\omega_B = \oint d\omega$, we get:

$$I_{\omega B} = \int_A \left(\omega_A^2 + e_z^2 y^2 + e_y^2 z^2 - 2e_z \omega_A y + 2e_y \omega_A z - 2e_y e_z yz\right) dA \quad \text{(A.5)}$$

after having "normalized" the warping coordinates. If the point A coincides with the centroid of the section, we obtain:

$$I_{\omega B} = I_{\omega A} + e_z^2 I_z + e_y^2 I_y - 2e_z I_{\omega z} + 2e_y I_{\omega y} - 2e_y e_z I_{yz} \quad \text{(A.6)}$$

If the point A does not coincide with the centroid G of the section, we may write by referring at first to the centroid G:

$$I_{\omega A} = I_{\omega G} + e_{zA}^2 I_z + e_{yA}^2 I_y - 2e_{zA} I_{\omega z} + 2e_{yA} I_{\omega y} - 2e_{yA} e_{zA} I_{yz}$$

$$I_{\omega B} = I_{\omega G} + e_{zB}^2 Iz + e_{yB}^2 I_y - 2e_{zB} I_{\omega z} + 2e_{yB} I_{\omega y} - 2e_{yB} e_{zB} I_{yz}$$

where: $I_{\omega G}$ = sectorial inertia with regard to the centroid G

$e_{yA} = y_C - y_A$, $e_{zA} = z_C - z_A$, $e_{yB} = y_C - y_B$, $e_{zB} = z_C - z_B$

The formula is obtained by defining the relationship between the sectorial inertias of the section $I_{\omega A}$ and $I_{\omega B}$, when the centre of rotation is displaced from A to B:

$$I_{\omega B} = I_{\omega A} + \left(e_{zB}^2 - e_{zA}^2\right) I_z + \left(e_{yB}^2 - e_{yA}^2\right) I_y - 2\left(e_{zB} - e_{zA}\right) I_{\omega z} + 2\left(e_{yB} - e_{yA}\right) I_{\omega y} - 2\left(e_{yB} e_{zB} - e_{yA} e_{zA}\right) I_{yz} \quad \text{(A.7.a)}$$

Should the point A be placed at the centroid, we have:

$$I_{\omega B} = I_{\omega A} + z_B^2 I_z + y_B^2 I_y - 2z_B I_{\omega z} + 2y_B I_{\omega y} - 2y_B z_B I_{yz} \quad \text{(A.7.b)}$$

When the centroid coincides with the shear centre, the formula is:

$$I_{\omega B} = I_{\omega A} + z_B^2 I_z + y_B^2 I_y - 2y_B z_B I_{yz} \quad \text{(A.7.c)}$$

For a symmetrical section, this is:

$$I_{\omega B} = I_{\omega A} + z_B^2 I_z + y_B^2 I_y \quad \text{(A.7.d)}$$

It ensues from the equations (A.10) that:

for a finite distance z_B, there is:

$$\frac{I_{\omega B}}{z_B^2} > I_z \quad \text{(A.8.a)}$$

and for an infinite distance z_B, we get:

$$\frac{I_{\omega B}}{z_B^2} = I_z \quad \text{(A.8.b)}$$

That means that the equations (A.7) check up the unequivocal limit condition: the stiffness in non-uniform torsion becomes equal to the bending one, when the distance of the point of rotation tends to infinity. It is interesting to remark that the left term of the equalities (A.8) intervenes in a similar way in the equations that define the torsional strain energy of the members (see, for instance, the equations (10), (14)), leading to the notion of *efficient inertia* (equation (16)).

The demonstration has thus been made that the sectorial inertia moment and the axial inertia moment are tightly related and that more especially:

- in the case where the centre of rotation is placed in finite distance, the stiffness in non-uniform torsion is higher than that in lateral bending;
- in the case where the centre of rotation is placed in infinite distance, the stiffness in non-uniform torsion is equal to that in lateral bending.

More generally, one may come to the conclusion that the stiffness to non-uniform torsion is a more general notion than that of the bending stiffness, as the latter is only a particular case of the first one.

As an example, for the section defined on figure A.2, the results displayed in the table A.1 show that when increasing the distance of the centre of rotation, the ratio $I_{\omega R} / z_R^2$ tends to I_z.

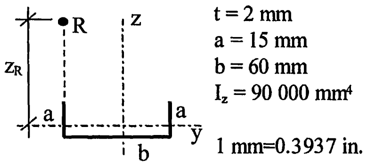

Figure A.2 Data relative to the section

z_R (mm)	$I_{\omega R} / z_R^{\,2}$ (mm^4)
50	98 775
100	93 544
500	90 574
1000	90 278
5000	90 054
10000	90 027
100000	90 003

Table A.1 Computational results

A.3. - Potential energy of the load in case of torsional buckling

Let consider a simply supported member on a longitudinal cylindrical hinge, submitted to compression by a pressure σ= Const. acting upon the end section; the resultant of this is P=σA. The rotation of this member due to the torsional buckling occurs about the imposed axis of rotation R (Fig. A.3).

The potential energy of the load is then: $$E_P = -\frac{1}{2}\frac{P}{A}\int_0^L\left[\int_A\left(\frac{du}{dx}\right)^2 t\,d\rho\right]dx \tag{A.9}$$

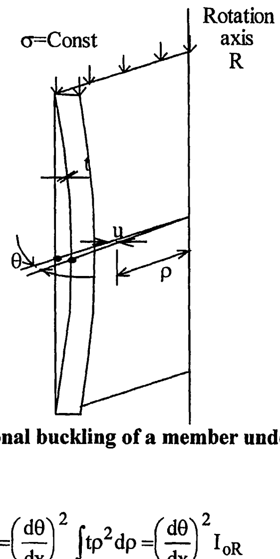

Figure A.3 - Torsional buckling of a member under constant pressure

where: $u = \rho\theta$ (A.10)

$$\int_A\left(\frac{du}{dx}\right)^2 t\,d\rho = \int_A\left(\frac{d\theta}{dx}\right)^2 t\rho^2 d\rho = \left(\frac{d\theta}{dx}\right)^2\int_A t\rho^2 d\rho = \left(\frac{d\theta}{dx}\right)^2 I_{OR} \tag{A.11}$$

$I_{OR} = I_y + I_z + A * z_R^2$ = moment of polar inertia, (A.12)

I_y, I_Z = axial inertia moments with regard to the y and z axes respectively,

z_R = distance between the centroid of the section and the centre of rotation R.

Should the rotation θ in (A.15) be interpreted in lateral displacement "u" of the centroid of the section according to the relationship $\theta = \frac{u}{z_R}$, we obtain:

$$E_P = -\frac{1}{2}\frac{PI_{oR}}{Az_R^2}\int_0^L\left(\frac{d\theta}{dx}\right)^2 dx \qquad \text{(A.13)}$$

Let note that by taking count of (A.12), we have: $\frac{I_{oR}}{Az_R^2} > 1$.

This means that the computational parameter C in the equation (26) may be taken equal to 1; this does not only simplify the calculation but also put on the safe side. Moreover, this simplification leads to the accurate solution for the case where the load P acts upon the member as a normal concentrated force at the centroid of the section.

Appendix. --References

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Appendix. -- Notation

Symbols:

E	Young's modulus
G	shear modulus
I	inertia moment
I_o	polar inertia moment
L	span length
M	moment
P	concentrated load
q	distributed load
β	elastic constant
χ	buckling coefficient
λ	slenderness
Θ	rotation
σ	stress
ω	warping coordinate (sectorial area)

Subscripts:

ω	warping (sectorial) property