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## **Direct Strength Method for Lipped Channel Columns and Beams Affected by Local-Plate/Distortional Interaction**

Nuno Silvestre<sup>1</sup>, Pedro B. Dinis<sup>1</sup> and Dinar Camotim<sup>2</sup>

### **Abstract**

This paper reports the results of an investigation on the use of the Direct Strength Method (DSM) to estimate the ultimate strength of lipped channel cold-formed steel columns and beams affected by interaction phenomena involving local-plate and distortional buckling modes. Initially, one briefly presents the DSM approaches to safety check columns and beams against local-plate and distortional failures, and some attention is also devoted to a recently proposed extension aimed at accounting for the above buckling mode interaction. Next, one describes the results of a parametric study, carried out in code ABAQUS, to determine the “exact” ultimate strengths of *108 columns* and *90 beams* displaying various cross-section dimensions and lengths, all selected to ensure the occurrence of relevant mode interaction effects. Then, these ultimate strength data are compared with the estimates provided by the existing DSM equations and, on the basis of this comparison, one identifies some features that must necessarily be included in a DSM approach that properly accounts for local-plate/distortional interaction.

### **Introduction**

The Direct Strength Method (DSM) was originally proposed by Schafer & Peköz (1999) about six years ago and has been continuously improved ever

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since, mainly due to the efforts of Schafer (2002, 2003a,b). Moreover, note the very recent inclusion of the DSM in the NAS and AS/NZS cold-formed steel design specifications – it already appears in the current (new) versions of these codes (AISI 2004, SA-SNZ 2005). The method provides an elegant, efficient and consistent approach to estimate the ultimate strength of cold-formed steel columns and beams (i) exhibiting global (flexural, torsional or flexural-torsional), local-plate or distortional collapses, or (ii) failing in mechanisms that involve interaction between local-plate and global buckling modes. Indeed, the most recent DSM version prescribes the need to perform two independent safety checks, regardless of the member critical buckling mode nature: (i) one against a pure distortional collapse and (ii) another against failure in a pure local-plate mode (laterally braced members) or due to local-plate/global mode interaction. In the latter case, the DSM is a very efficient alternative to the classical “effective width method”.

However, as pointed out by Schafer (2002, 2003b, 2005), further research is needed before the DSM approach can be successfully applied to members (i) under compression and bending (Duong & Hancock 2004, Rasmussen & Hossain 2004) or (ii) affected by interaction phenomena involving distortional buckling modes (Yang & Hancock 2004, Kwon *et al.* 2005). Since it has been recently shown that coupling between local-plate and distortional buckling modes may strongly influence the post-buckling behavior and ultimate strength of commonly used lipped channel cross-sections (Dinis *et al.* 2005a,b), it would be obviously convenient to have this mode interaction phenomenon also covered by the DSM.

The aim of this work is to contribute towards extending the current DSM domain of application, by making it possible to predict ultimate strengths of lipped channel columns and beams affected by local-plate/distortional buckling mode interaction. To achieve this goal, one begins by carrying out an extensive parametric study involving the evaluation of the elastic-plastic failure loads/moments of lipped channel columns/beams with distinct cross-section dimensions, lengths and yield stresses, and containing critical-mode small-amplitude initial geometrical imperfections – the member geometries were carefully selected to ensure the occurrence of local-plate/distortional interaction. All second-order elastic-plastic analyses were carried out in the finite element code ABAQUS (HKS 2002), adopting 4-node shell elements to discretize the members. These ultimate strength values then provide a “data

bank” enabling the proposal and validation of preliminary recommendations on the use of a DSM approach to estimate the ultimate strength of lipped channel columns and beams against local-plate/distortional mode interaction.

### The Direct Strength Method (DSM)

Compared with the classical “*effective width*” approach”, the DSM exhibits three major innovative features, all due to the fact that the cross-section is treated as a *whole*: (i) wall-restraint effects are always taken into account, (ii) no effective width calculations are needed and (iii) strength estimates are provided for member *distortional* collapses. Moreover, the DSM provides a rational and systematic approach to design thin-walled members with arbitrary cross-sections, loadings or failure modes – of course, a given application must be properly calibrated and validated (comparison with experimental and/or numerical results). Finally, note that the DSM and effective width approaches share one basic assumption: the member ultimate strength can be accurately predicted solely on the basis of its elastic buckling and yield stresses.

The current DSM approach adopts “Winter-type” design curves, which were calibrated against a large number of experimental and/or numerical results (Schafer 2003a). It was shown that, when a given member fails in pure local-plate or distortional modes, safe and accurate ultimate strength estimates can be obtained on the basis of elastic buckling and yield stress values only. Thus, the DSM stipulates that the *nominal strengths*, against *local-plate* and *distortional* failure, of laterally braced columns ( $P_{nl}$  and  $P_{nd}$ ) and beams ( $M_{nl}$  and  $M_{nd}$ ) are yielded by the expressions (Schafer 2002, Hancock *et al.* 2001)

$$P_{nl} = P_y \quad \text{if} \quad \lambda_l \leq 0.776$$

$$P_{nl} = P_y \left( \frac{P_{crl}}{P_y} \right)^{0.4} \left[ 1 - 0.15 \left( \frac{P_{crl}}{P_y} \right)^{0.4} \right] \quad \text{if} \quad \lambda_l > 0.776 \quad , \quad (1)$$

$$P_{nd} = P_y \quad \text{if} \quad \lambda_d \leq 0.561$$

$$P_{nd} = P_y \left( \frac{P_{crd}}{P_y} \right)^{0.6} \left[ 1 - 0.25 \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right] \quad \text{if} \quad \lambda_d > 0.561 \quad , \quad (2)$$

$$M_{nl} = M_y \quad \text{if} \quad \lambda_l \leq 0.776$$

$$M_{nl} = M_y \left( \frac{M_{crd}}{M_y} \right)^{0.4} \left[ 1 - 0.15 \left( \frac{M_{crd}}{M_y} \right)^{0.4} \right] \quad \text{if} \quad \lambda_l > 0.776 \quad , \quad (3)$$

$$M_{nd} = M_y \quad \text{if} \quad \lambda_d \leq 0.673$$

$$M_{nd} = M_y \left( \frac{M_{crd}}{M_y} \right)^{0.5} \left[ 1 - 0.22 \left( \frac{M_{crd}}{M_y} \right)^{0.5} \right] \quad \text{if} \quad \lambda_d > 0.673 \quad , \quad (4)$$

where (i) one has  $\lambda_l = (P_y/P_{crd})^{0.5}$  or  $\lambda_l = (M_y/M_{crd})^{0.5}$  and  $\lambda_d = (P_y/P_{crd})^{0.5}$  or  $\lambda_d = (M_y/M_{crd})^{0.5}$ , (ii)  $P_y$  and  $M_y$  are the squash load and plastic moment, (iii)  $P_{crd}$  ( $M_{crd}$ ) and  $P_{crd}$  ( $M_{crd}$ ) are the *local-plate* and *distortional* critical buckling loads (moments). In order to capture the local-plate/global interaction (in laterally unbraced members), the DSM approach replaces  $P_y$  by  $P_{ne}$  in eqs. (1) and  $M_y$  by  $M_{ne}$  in eqs. (3), where  $P_{ne}$  and  $M_{ne}$  are the column/beam buckling strengths associated with *global* failure (Hancock *et al.* 2001). Note that it is important to predict accurately the column (beam) distortional failure load (moment), since (i) the distortional post-critical strength is considerably lower and more imperfection-sensitive than its local-plate counterpart (*e.g.*, Dinis & Camotim 2004) and (ii) there is clear (numerical) evidence that columns and beams buckling in local-plate modes often exhibit distortional failure mechanisms (Schafer & Peköz 1999, Dinis & Camotim 2004).

### Parametric Study: Scope, Numerical Analysis and Results

**Scope.** In order to be able to carry out a rather large parametric study on the ultimate strength of lipped channel columns and beams affected by local-plate/distortional interaction, their geometries had to be carefully selected: it was necessary to find sets of cross-section dimensions and lengths making it possible to “control” the closeness between the column/beam local-plate and distortional critical buckling stresses ( $\sigma_{crd}$  and  $\sigma_{crd}$  – in beams,  $\sigma$  is the uniform flange applied stress). This goal was achieved through a trial-and-error approach to find 12 “basic cross-section shapes” (6 for the columns and 6 for the beams) with commonly used dimensions and ensuring that  $\sigma_{crd}$  and  $\sigma_{crd}$  coincide. The search led to the following columns and beams:

- (i) Three *slender* columns ( $1.4 \leq \lambda_d \leq 2.6$ ),  
 (i<sub>1</sub>)  $b_w=100, b_f=50, b_s=5$  and  $t=1.0\text{mm}, L=270\text{mm}$ .  
 (i<sub>2</sub>)  $b_w=120, b_f=80, b_s=10$  and  $t=1.3\text{mm}, L=550\text{mm}$ .  
 (i<sub>3</sub>)  $b_w=95, b_f=80, b_s=10$  and  $t=0.95\text{mm}, L=600\text{mm}$ .  
 Three *stockier* columns ( $0.6 \leq \lambda_d \leq 1.4$ ),  
 (i<sub>4</sub>)  $b_w=180, b_f=100, b_s=20$  and  $t=3.4\text{mm}, L=650\text{mm}$ .  
 (i<sub>5</sub>)  $b_w=110, b_f=78, b_s=30$  and  $t=2.8\text{mm}, L=800\text{mm}$ .  
 (i<sub>6</sub>)  $b_w=100, b_f=100, b_s=26$  and  $t=2.0\text{mm}, L=950\text{mm}$ .
- (ii) Three *slender* beams ( $1.4 \leq \lambda_d \leq 2.6$ ),  
 (ii<sub>1</sub>)  $b_w=180, b_f=70, b_s=15$  and  $t=1.1\text{mm}, L=750\text{mm}$ .  
 (ii<sub>2</sub>)  $b_w=400, b_f=150, b_s=26$  and  $t=2.0\text{mm}, L=1400\text{mm}$ .  
 (ii<sub>3</sub>)  $b_w=390, b_f=100, b_s=12$  and  $t=1.4\text{mm}, L=750\text{mm}$ .  
 Three *stockier* beams ( $0.6 \leq \lambda_d \leq 1.4$ ),  
 (ii<sub>4</sub>)  $b_w=120, b_f=75, b_s=24$  and  $t=1.8\text{mm}, L=770\text{mm}$ .  
 (ii<sub>5</sub>)  $b_w=160, b_f=80, b_s=23$  and  $t=1.7\text{mm}, L=820\text{mm}$ .  
 (ii<sub>6</sub>)  $b_w=80, b_f=50, b_s=10$  and  $t=0.8\text{mm}, L=450\text{mm}$ .

Note that all these columns/beams satisfy the cross-section dimension requirements (“pre-qualified columns”) adopted in the DSM approach.

- (iii) Subsequently, the closeness between  $\sigma_{crl}$  and  $\sigma_{crd}$  was slightly altered, by just changing a single basic cross-section dimension: flange width  $b_f$ , web width  $b_w$ , or stiffener width  $b_s$ . This procedure made it possible to identify various members with (iii<sub>1</sub>) cross-section dimensions generated from the basic shapes and (iii<sub>2</sub>) very close (but not necessarily equal)  $\sigma_{crl}$  and  $\sigma_{crd}$  values – they are all such that  $0.85 \leq \sigma_{crl} / \sigma_{crd} \leq 1.20$ .

The member lengths considered always correspond to single distortional half-waves associated with the buckling stresses  $\sigma_{crd}$  and were obtained through finite strip analyses. The steel behavior is characterized by  $E=210\text{ GPa}$  (Young’s modulus),  $\nu=0.3$  (Poisson’s ratio) and  $f_y=250\text{-}350\text{-}550\text{ MPa}$  (columns), and  $f_y=250\text{-}350\text{-}450\text{ MPa}$  (beams) – these yield stresses meet the DSM limit for “pre-qualified columns and beams”. Finally, it is worth (i) noting that no residual stresses have been accounted for and (ii) addressing the criterion adopted to select the initial geometrical imperfections included in the non-linear analyses that provide the column and beam ultimate strengths:

- (i) Regardless of their critical stress ratios  $\sigma_{crl} / \sigma_{crd}$ , all the columns and beams analyzed contained initial geometrical imperfections with a single-wave distortional buckling mode shape, having an amplitude

(mid-span compressed flange-stiffener corner displacement) equal to 10% of the wall thickness  $t$  and involving either *outward* (columns) or *inward* (beams) flange-stiffener assembly motions – recent studies, involving lipped channel columns and beams with  $\sigma_{cr1} = \sigma_{crd}$ , showed that these imperfection shapes are the *most detrimental* ones, since they correspond to the lowest post-buckling strength and collapse loads and moments (Dinis *et al.* 2005, Martins 2006).

- (ii) The *slender* columns ( $i_1$ - $i_3$ ) (not the beams) with  $\sigma_{cr1}/\sigma_{crd} < 1.0$  (i.e., that buckle in *local-plate* modes with several half-waves) were also analyzed in the presence of *critical-mode* initial imperfections, again with amplitude  $0.1t$  – now the mid-web flexural displacement at mid-span.

A total of (i) 66 *slender* and 45 *stocky* columns, and (ii) 45 *slender* and 45 *stockier* beams were analyzed, corresponding to all possible combinations of 16 (column) and 15 (beam) different cross-section shapes and 3 yield stress values. All the cross-section dimensions ( $b_w, b_f, b_s, t$ ), length values ( $L$ ), yield stresses ( $f_y$ ) and initial imperfection shapes ( $D, LP$ ), considered in this work, as well as the corresponding critical stresses ( $\sigma_{cr1}, \sigma_{crd}$ ), are given in tables 1A-C (columns) and 2A-C (beams) and will be addressed further ahead.

**Numerical analysis.** This subsection deals with the *numerical* evaluation of the “*exact*” column and beam ultimate strengths, which are subsequently used to assess the merits of the DSM approach described before. These ultimate strengths were obtained by means of *finite element* analyses (FEA) carried out in the code ABAQUS (HKS 2002), discretizing the members into *shell* elements. As far as the performance of these FEA is concerned, the following aspects deserve to be mentioned here (Dinis & Camotim 2006):

- (I) *Discretization.* The member mid-surfaces were discretized into S4 finite elements (ABAQUS nomenclature: isoparametric 4-node shell elements with the shear stiffness yielded by a *full* integration rule), which were found to be the most adequate to carry out this task. One considered 20-30 elements along the cross-section mid-line (width of about 10 mm) and previous convergence/accuracy studies showed that the element length-to-width ratio should be comprised between 1 and 2.
- (II) *Support Conditions.* All member end sections are locally and globally pinned and can warp freely. Concerning the first aspect, these support conditions were modeled by imposing null transverse membrane and flexural displacements at all end section nodes – in order to preclude

the occurrence of a spurious longitudinal rigid-body motion, the axial displacement was prevented at one mid-span cross-section node.

- (III) Column Loading. Compressive forces, equivalent to a uniform normal stress distribution, are applied at the nodes of the column end-sections. Since the reference value of the *load parameter*  $p$  is  $tN/mm$  ( $t$  – wall thickness), which corresponds to a  $1\text{ MPa}$  uniform stress distribution, the value of  $p$  yielded by ABAQUS is numerically equal to the *average stress* acting on the column (expressed in  $\text{MPa}$ ).
- (IV) Beam Loading. Compressive and tensile forces  $p = \sigma t$ , equivalent to the linearly varying normal stress distribution due to a bending moment, were applied at the nodes of the beam end-sections. Since the reference value of the *load parameter*  $p$  corresponds to a  $1\text{ MPa}$  flange stress, the value of  $p$  yielded by ABAQUS is numerically equal to the *average stress* acting on the beam flanges (expressed in  $\text{MPa}$ ).
- (V) Material Modeling. The member (carbon steel) material behavior, deemed isotropic and homogeneous, was modeled through (i) linear elastic (bifurcation analysis) and (ii) elastic/perfectly-plastic (post-buckling analysis) stress-strain laws. In the latter case, the well-known Prandtl-Reuss model ( $J_2$ -flow theory), combining Von Mises’s yield criterion with the associated flow rule, was adopted. These stress-strain laws are readily available in the ABAQUS material behavior library and one just needs to provide the values of  $E$ ,  $\nu$  and  $f_y$ .
- (VI) Initial Imperfections. All initial geometrical imperfections, defined earlier (buckling mode shapes with amplitude  $0.1t$ ) are included in the analyses through a specific ABAQUS command. In columns/beams that buckle in local-plate modes ( $\sigma_{cr1} < \sigma_{crd}$ ), the single-wave “distortional” imperfection is, effectively, the column higher-order buckling mode most resembling it, which means that it is not possible to guarantee the “purity” of the distortional shape – *e.g.*, the small participation of a multiple half-wave local-plate mode is virtually undetectable.

**Numerical results.** The numerical results included in tables 1A-C (columns) and 2A-C (beams) consist of (i) local-plate and distortional bifurcation stresses and (ii) average stresses at collapse ( $\sigma_c$ ). In order to better convey the meaning of these results, they are illustrated in figure 1(a), which shows the post-buckling equilibrium paths ( $\sigma/\sigma_{cr}$  vs.  $v/t$ ) of columns with (i)  $\sigma_{cr1} = \sigma_{crd}$  ( $\equiv \sigma_{cr}$ ), (ii) identical outward distortional imperfections and (iii) four distinct



yield stress values:  $f_y/\sigma_{cr} \approx 1.2, 2.0, 3.5, 5.5$ . It is worth noting that the onset of yielding, always taking place in the stiffener free ends (see figure 1(b<sub>1</sub>)), occurs at the equilibrium points A and may or may not trigger the column failure – it depends on the  $f_y/\sigma_{cr}$  ratio. Indeed, for large enough  $f_y/\sigma_{cr}$  values failure occurs at a limit point B, following (i) a “snap-through” phenomenon and (ii) the yielding of the column central regions located around the web-flange corners – see figure 1(b<sub>2</sub>) (Dinis *et al.* 2005a,b). As for figure 1(c), it shows the corresponding (predominantly distortional) failure mechanism.

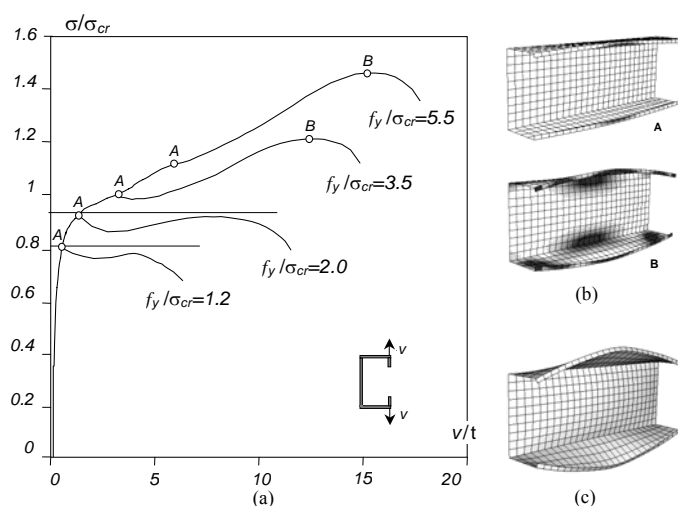


Fig. 1: (a) Post-buckling equilibrium paths, (b) plastic strain distributions and (c) failure mechanism

### Assessment of the DSM Estimates

The numerical and DSM results concerning the 108 columns and 90 beams analyzed, which are presented in tables 1A-C and 2A-C, make it possible to compare the “exact” ultimate strengths ( $\sigma_u$ ) with their DSM estimates ( $\sigma_{nd}$  and  $\sigma_{nl}$ ). The observation of these results prompts the following remarks:

- (i) The column  $\sigma_u$  values concerning the local-plate imperfections (their varying dimensions exhibit the superscript <sup>LP</sup> – tables 1A-B) are never below their distortional counterparts, thus confirming the assertion made earlier: the distortional imperfections are the *most detrimental* ones

Table 1A. Comparison between the “exact” ultimate strengths and their DSM estimates ( $\sigma_{nl}$ ,  $\sigma_{nd}$  and  $\sigma_{nld}$ ) for 42 (out of 108) columns

|                                  |                    | FEA            |                |                |               | DSM                    |                        |                        |                        |                         |                         |
|----------------------------------|--------------------|----------------|----------------|----------------|---------------|------------------------|------------------------|------------------------|------------------------|-------------------------|-------------------------|
| $b_f$                            | $f_y$              | $\sigma_{crl}$ | $\sigma_{crl}$ | $\sigma_u$     | $\sigma_{nl}$ | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$          | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$         | $\sigma_{nld}/\sigma_u$ |                         |
| $b_w=100, b_s=5, t=1.0, L=270mm$ | 55                 | 250            | 101            | 91             | 94            | 156                    | 1.66                   | 117                    | 1.24                   | 95                      | 1.01                    |
|                                  | 55                 | 350            | 101            | 91             | 110           | 193                    | 1.75                   | 138                    | 1.25                   | 106                     | 0.96                    |
|                                  | 55                 | 550            | 101            | 91             | 131           | 258                    | 1.97                   | 171                    | 1.31                   | 121                     | 0.92                    |
|                                  | 52.5               | 250            | 101            | 96             | 97            | 156                    | 1.61                   | 121                    | 1.25                   | 97                      | 1.00                    |
|                                  | 52.5               | 350            | 101            | 96             | 114           | 194                    | 1.70                   | 143                    | 1.25                   | 108                     | 0.95                    |
|                                  | 52.5               | 550            | 101            | 96             | 137           | 258                    | 1.88                   | 176                    | 1.28                   | 124                     | 0.91                    |
|                                  | 50                 | 250            | 102            | 102            | 102           | 156                    | 1.53                   | 125                    | 1.23                   | 99                      | 0.97                    |
|                                  | 50                 | 350            | 102            | 102            | 120           | 194                    | 1.62                   | 147                    | 1.23                   | 110                     | 0.92                    |
|                                  | 50                 | 550            | 102            | 102            | 147           | 259                    | 1.76                   | 182                    | 1.24                   | 127                     | 0.86                    |
|                                  | 47.5               | 250            | 102            | 108            | 107           | 157                    | 1.47                   | 128                    | 1.20                   | 101                     | 0.94                    |
|                                  | 47.5               | 350            | 102            | 108            | 127           | 194                    | 1.53                   | 151                    | 1.19                   | 113                     | 0.89                    |
|                                  | 47.5               | 550            | 102            | 108            | 156           | 259                    | 1.66                   | 187                    | 1.20                   | 130                     | 0.83                    |
|                                  | 45                 | 250            | 103            | 113            | 115           | 157                    | 1.37                   | 131                    | 1.14                   | 103                     | 0.90                    |
|                                  | 45                 | 350            | 103            | 113            | 136           | 195                    | 1.43                   | 155                    | 1.14                   | 115                     | 0.85                    |
|                                  | 45                 | 550            | 103            | 113            | 168           | 260                    | 1.55                   | 193                    | 1.15                   | 132                     | 0.79                    |
|                                  | 47.5 <sup>LP</sup> | 250            | 102            | 108            | 118           | 157                    | 1.33                   | 128                    | 1.08                   | 101                     | 0.86                    |
|                                  | 47.5 <sup>LP</sup> | 350            | 102            | 108            | 127           | 194                    | 1.53                   | 151                    | 1.19                   | 113                     | 0.89                    |
|                                  | 47.5 <sup>LP</sup> | 550            | 102            | 108            | 157           | 259                    | 1.65                   | 187                    | 1.19                   | 130                     | 0.83                    |
|                                  | 45 <sup>LP</sup>   | 250            | 103            | 113            | 128           | 157                    | 1.23                   | 131                    | 1.02                   | 103                     | 0.80                    |
|                                  | 45 <sup>LP</sup>   | 350            | 103            | 113            | 142           | 195                    | 1.37                   | 155                    | 1.09                   | 115                     | 0.81                    |
| 45 <sup>LP</sup>                 | 550                | 103            | 113            | 168            | 260           | 1.55                   | 193                    | 1.15                   | 132                    | 0.79                    |                         |
| $b_w=80, b_s=10, t=1.3, L=550mm$ | $b_w$              | $f_y$          | $\sigma_{crl}$ | $\sigma_{crl}$ | $\sigma_u$    | $\sigma_{nl}$          | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$          | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$          | $\sigma_{nld}/\sigma_u$ |
|                                  | 130                | 250            | 100            | 110            | 105           | 155                    | 1.48                   | 129                    | 1.23                   | 101                     | 0.96                    |
|                                  | 130                | 350            | 100            | 110            | 107           | 193                    | 1.80                   | 153                    | 1.43                   | 113                     | 1.06                    |
|                                  | 130                | 550            | 100            | 110            | 123           | 257                    | 2.09                   | 189                    | 1.54                   | 130                     | 1.06                    |
|                                  | 125                | 250            | 107            | 113            | 107           | 159                    | 1.49                   | 131                    | 1.22                   | 104                     | 0.97                    |
|                                  | 125                | 350            | 107            | 113            | 109           | 198                    | 1.82                   | 155                    | 1.42                   | 116                     | 1.06                    |
|                                  | 125                | 550            | 107            | 113            | 123           | 264                    | 2.15                   | 192                    | 1.56                   | 134                     | 1.09                    |
|                                  | 120                | 250            | 115            | 115            | 109           | 163                    | 1.50                   | 133                    | 1.22                   | 108                     | 0.99                    |
|                                  | 120                | 350            | 115            | 115            | 111           | 203                    | 1.83                   | 157                    | 1.41                   | 120                     | 1.08                    |
|                                  | 120                | 550            | 115            | 115            | 124           | 271                    | 2.19                   | 194                    | 1.56                   | 139                     | 1.12                    |
|                                  | 115                | 250            | 125            | 118            | 112           | 168                    | 1.50                   | 134                    | 1.20                   | 111                     | 0.99                    |
|                                  | 115                | 350            | 125            | 118            | 114           | 209                    | 1.83                   | 159                    | 1.39                   | 124                     | 1.09                    |
|                                  | 115                | 550            | 125            | 118            | 122           | 278                    | 2.28                   | 197                    | 1.61                   | 143                     | 1.17                    |
|                                  | 110                | 250            | 135            | 121            | 114           | 172                    | 1.51                   | 136                    | 1.19                   | 115                     | 1.01                    |
|                                  | 110                | 350            | 135            | 121            | 116           | 214                    | 1.84                   | 161                    | 1.39                   | 129                     | 1.11                    |
|                                  | 110                | 550            | 135            | 121            | 121           | 287                    | 2.37                   | 199                    | 1.64                   | 149                     | 1.23                    |
|                                  | 100                | 250            | 157            | 127            | 119           | 182                    | 1.53                   | 139                    | 1.17                   | 123                     | 1.03                    |
|                                  | 100                | 350            | 157            | 127            | 122           | 226                    | 1.85                   | 164                    | 1.34                   | 138                     | 1.13                    |
|                                  | 100                | 550            | 157            | 127            | 126           | 303                    | 2.40                   | 204                    | 1.62                   | 159                     | 1.26                    |
|                                  | 125 <sup>LP</sup>  | 250            | 107            | 113            | 119           | 159                    | 1.34                   | 131                    | 1.10                   | 104                     | 0.87                    |
| 125 <sup>LP</sup>                | 350                | 107            | 113            | 120            | 198           | 1.65                   | 155                    | 1.29                   | 116                    | 0.97                    |                         |
| 125 <sup>LP</sup>                | 550                | 107            | 113            | 122            | 264           | 2.16                   | 192                    | 1.57                   | 134                    | 1.10                    |                         |

Table 1B. Comparison between the “exact” ultimate strengths and their DSM estimates ( $\sigma_{nl}$ ,  $\sigma_{nd}$  and  $\sigma_{nld}$ ) for 36 (out of 108) columns

|  |                    | FEA            |                |                |               | DSM                    |                        |                        |                        |                         |                         |
|--|--------------------|----------------|----------------|----------------|---------------|------------------------|------------------------|------------------------|------------------------|-------------------------|-------------------------|
| $b_s$                                    | $f_y$              | $\sigma_{crl}$ | $\sigma_{crl}$ | $\sigma_u$     | $\sigma_{nl}$ | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$          | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$         | $\sigma_{nld}/\sigma_u$ |                         |
| $b_w=95, b_s=80, \epsilon=0.95, L=600mm$ | 11                 | 250            | 92             | 100            | 94            | 150                    | 1.60                   | 123                    | 1.31                   | 95                      | 1.01                    |
|  | 11                 | 350            | 92             | 100            | 95            | 187                    | 1.97                   | 145                    | 1.53                   | 106                     | 1.12                    |
|  | 11                 | 550            | 92             | 100            | 99            | 249                    | 2.52                   | 180                    | 1.82                   | 122                     | 1.23                    |
|  | 10.5               | 250            | 91             | 96             | 91            | 150                    | 1.65                   | 121                    | 1.33                   | 94                      | 1.03                    |
|  | 10.5               | 350            | 91             | 96             | 92            | 187                    | 2.03                   | 142                    | 1.54                   | 104                     | 1.13                    |
|  | 10.5               | 550            | 91             | 96             | 97            | 249                    | 2.57                   | 176                    | 1.81                   | 120                     | 1.24                    |
|  | 10                 | 250            | 91             | 91             | 86            | 150                    | 1.74                   | 118                    | 1.37                   | 92                      | 1.07                    |
|  | 10                 | 350            | 91             | 91             | 87            | 187                    | 2.15                   | 139                    | 1.60                   | 103                     | 1.18                    |
|  | 10                 | 550            | 91             | 91             | 92            | 249                    | 2.71                   | 171                    | 1.86                   | 118                     | 1.28                    |
|  | 9.5                | 250            | 91             | 87             | 83            | 150                    | 1.81                   | 115                    | 1.39                   | 91                      | 1.10                    |
|  | 9.5                | 350            | 91             | 87             | 84            | 187                    | 2.23                   | 135                    | 1.61                   | 101                     | 1.20                    |
|  | 9.5                | 550            | 91             | 87             | 91            | 248                    | 2.73                   | 167                    | 1.84                   | 116                     | 1.27                    |
|  | 9                  | 250            | 91             | 83             | 78            | 150                    | 1.92                   | 112                    | 1.44                   | 89                      | 1.14                    |
|  | 9                  | 350            | 91             | 83             | 79            | 186                    | 2.35                   | 132                    | 1.67                   | 99                      | 1.25                    |
|  | 9                  | 550            | 91             | 83             | 84            | 248                    | 2.95                   | 162                    | 1.93                   | 114                     | 1.36                    |
|  | 10.5 <sup>LP</sup> | 250            | 91             | 96             | 109           | 150                    | 1.38                   | 121                    | 1.11                   | 94                      | 0.86                    |
|  | 10.5 <sup>LP</sup> | 350            | 91             | 96             | 109           | 187                    | 1.72                   | 142                    | 1.30                   | 104                     | 0.95                    |
|  | 10.5 <sup>LP</sup> | 550            | 91             | 96             | 109           | 249                    | 2.28                   | 176                    | 1.61                   | 120                     | 1.10                    |
| 11 <sup>LP</sup>                         | 250                | 92             | 100            | 114            | 150           | 1.32                   | 123                    | 1.08                   | 95                     | 0.83                    |                         |
| 11 <sup>LP</sup>                         | 350                | 92             | 100            | 114            | 187           | 1.64                   | 145                    | 1.27                   | 106                    | 0.93                    |                         |
| 11 <sup>LP</sup>                         | 550                | 92             | 100            | 114            | 249           | 2.18                   | 180                    | 1.58                   | 122                    | 1.07                    |                         |
| $b_w=180, b_s=20, \epsilon=3.4, L=650mm$ | $b_f$              | $f_y$          | $\sigma_{crl}$ | $\sigma_{crl}$ | $\sigma_u$    | $\sigma_{nl}$          | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$          | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$          | $\sigma_{nld}/\sigma_u$ |
|  | 90                 | 250            | 361            | 399            | 240           | 239                    | 1.00                   | 221                    | 0.92                   | 220                     | 0.92                    |
|  | 90                 | 350            | 361            | 399            | 298           | 301                    | 1.01                   | 276                    | 0.93                   | 256                     | 0.86                    |
|  | 90                 | 550            | 361            | 399            | 361           | 406                    | 1.12                   | 360                    | 1.00                   | 306                     | 0.85                    |
|  | 95                 | 250            | 358            | 377            | 231           | 239                    | 1.03                   | 218                    | 0.94                   | 217                     | 0.94                    |
|  | 95                 | 350            | 358            | 377            | 287           | 300                    | 1.05                   | 270                    | 0.94                   | 252                     | 0.88                    |
|  | 95                 | 550            | 358            | 377            | 341           | 405                    | 1.19                   | 351                    | 1.03                   | 300                     | 0.88                    |
|  | 100                | 250            | 355            | 355            | 222           | 238                    | 1.07                   | 213                    | 0.96                   | 213                     | 0.96                    |
|  | 100                | 350            | 355            | 355            | 276           | 299                    | 1.08                   | 264                    | 0.96                   | 247                     | 0.89                    |
|  | 100                | 550            | 355            | 355            | 323           | 404                    | 1.25                   | 342                    | 1.06                   | 294                     | 0.91                    |
|  | 105                | 250            | 353            | 338            | 217           | 238                    | 1.10                   | 210                    | 0.97                   | 210                     | 0.97                    |
|  | 105                | 350            | 353            | 338            | 267           | 298                    | 1.12                   | 259                    | 0.97                   | 243                     | 0.91                    |
|  | 105                | 550            | 353            | 338            | 307           | 403                    | 1.31                   | 334                    | 1.09                   | 289                     | 0.94                    |
|  | 110                | 250            | 350            | 321            | 211           | 237                    | 1.12                   | 206                    | 0.98                   | 206                     | 0.98                    |
|  | 110                | 350            | 350            | 321            | 256           | 298                    | 1.16                   | 253                    | 0.99                   | 239                     | 0.93                    |
|  | 110                | 550            | 350            | 321            | 292           | 402                    | 1.38                   | 326                    | 1.12                   | 284                     | 0.97                    |

(Silvestre *et al.* 2005a,b). Since the DSM cannot capture the initial imperfection effect, its estimates should preferably approximate well the  $\sigma_u$  values related to distortional imperfections. If this is the case, then the DSM underestimates  $\sigma_u$  for columns with local-plate imperfections.

Table 1C. Comparison between the “exact” ultimate strengths and their DSM estimates ( $\sigma_{nl}$ ,  $\sigma_{nd}$  and  $\sigma_{nld}$ ) for 36 (out of 108) columns and overall results

|                                    |       | FEA            |                |                |               |                        | DSM                    |                        |                        |                         |                         |
|------------------------------------|-------|----------------|----------------|----------------|---------------|------------------------|------------------------|------------------------|------------------------|-------------------------|-------------------------|
| $b_w$                              | $f_y$ | $\sigma_{crl}$ | $\sigma_{cfd}$ | $\sigma_u$     | $\sigma_{nl}$ | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$          | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$         | $\sigma_{nld}/\sigma_u$ |                         |
| $b_w=78, b_y=30, t=2.8, L=800mm$   | 100   | 250            | 736            | 656            | 249           | 250                    | 1.00                   | 247                    | 0.99                   | 247                     | 0.99                    |
|                                    | 100   | 350            | 736            | 656            | 345           | 350                    | 1.01                   | 324                    | 0.94                   | 324                     | 0.94                    |
|                                    | 100   | 550            | 736            | 656            | 508           | 514                    | 1.01                   | 442                    | 0.87                   | 442                     | 0.87                    |
|                                    | 105   | 250            | 680            | 641            | 249           | 250                    | 1.00                   | 246                    | 0.99                   | 246                     | 0.99                    |
|                                    | 105   | 350            | 680            | 641            | 344           | 350                    | 1.02                   | 322                    | 0.94                   | 322                     | 0.94                    |
|                                    | 105   | 550            | 680            | 641            | 503           | 501                    | 1.00                   | 438                    | 0.87                   | 429                     | 0.85                    |
|                                    | 110   | 250            | 630            | 625            | 248           | 250                    | 1.01                   | 246                    | 0.99                   | 246                     | 0.99                    |
|                                    | 110   | 350            | 630            | 625            | 344           | 350                    | 1.02                   | 320                    | 0.93                   | 320                     | 0.93                    |
|                                    | 110   | 550            | 630            | 625            | 499           | 489                    | 0.98                   | 434                    | 0.87                   | 416                     | 0.83                    |
|                                    | 115   | 250            | 581            | 611            | 248           | 250                    | 1.01                   | 245                    | 0.99                   | 245                     | 0.99                    |
|                                    | 115   | 350            | 581            | 611            | 342           | 350                    | 1.02                   | 318                    | 0.93                   | 318                     | 0.93                    |
|                                    | 115   | 550            | 581            | 611            | 493           | 476                    | 0.97                   | 430                    | 0.87                   | 403                     | 0.82                    |
|                                    | 120   | 250            | 538            | 596            | 248           | 250                    | 1.01                   | 244                    | 0.98                   | 244                     | 0.98                    |
|                                    | 120   | 350            | 538            | 596            | 341           | 342                    | 1.00                   | 316                    | 0.93                   | 316                     | 0.93                    |
| 120                                | 550   | 538            | 596            | 489            | 464           | 0.95                   | 426                    | 0.87                   | 391                    | 0.80                    |                         |
| $b_w=100, b_y=100, t=2.0, L=950mm$ | $b_s$ | $f_y$          | $\sigma_{crl}$ | $\sigma_{cfd}$ | $\sigma_u$    | $\sigma_{nl}$          | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$          | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$          | $\sigma_{nld}/\sigma_u$ |
|                                    | 22    | 250            | 317            | 285            | 226           | 230                    | 1.02                   | 197                    | 0.87                   | 195                     | 0.86                    |
|                                    | 22    | 350            | 317            | 285            | 262           | 288                    | 1.10                   | 241                    | 0.92                   | 224                     | 0.85                    |
|                                    | 22    | 550            | 317            | 285            | 276           | 388                    | 1.41                   | 308                    | 1.12                   | 265                     | 0.96                    |
|                                    | 24    | 250            | 317            | 299            | 227           | 230                    | 1.01                   | 201                    | 0.89                   | 198                     | 0.87                    |
|                                    | 24    | 350            | 317            | 299            | 270           | 288                    | 1.07                   | 246                    | 0.91                   | 227                     | 0.84                    |
|                                    | 24    | 550            | 317            | 299            | 287           | 388                    | 1.35                   | 315                    | 1.10                   | 268                     | 0.93                    |
|                                    | 26    | 250            | 317            | 314            | 230           | 229                    | 1.00                   | 205                    | 0.89                   | 200                     | 0.87                    |
|                                    | 26    | 350            | 317            | 314            | 279           | 288                    | 1.03                   | 251                    | 0.90                   | 230                     | 0.82                    |
|                                    | 26    | 550            | 317            | 314            | 300           | 388                    | 1.29                   | 323                    | 1.08                   | 273                     | 0.91                    |
|                                    | 28    | 250            | 316            | 331            | 232           | 229                    | 0.99                   | 208                    | 0.90                   | 202                     | 0.87                    |
|                                    | 28    | 350            | 316            | 331            | 288           | 288                    | 1.00                   | 257                    | 0.89                   | 233                     | 0.81                    |
|                                    | 28    | 550            | 316            | 331            | 316           | 388                    | 1.23                   | 331                    | 1.05                   | 277                     | 0.88                    |
|                                    | 30    | 250            | 315            | 350            | 234           | 229                    | 0.98                   | 212                    | 0.91                   | 205                     | 0.88                    |
| 30                                 | 350   | 315            | 350            | 297            | 287           | 0.97                   | 263                    | 0.89                   | 237                    | 0.80                    |                         |
| 30                                 | 550   | 315            | 350            | 337            | 387           | 1.15                   | 339                    | 1.01                   | 281                    | 0.83                    |                         |
|                                    |       |                |                |                |               | <b>Av.</b>             | <b>1.52</b>            | <b>Av.</b>             | <b>1.20</b>            | <b>Av.</b>              | <b>0.97</b>             |
|                                    |       |                |                |                |               | <b>Sd.</b>             | <b>0.482</b>           | <b>Sd.</b>             | <b>0.268</b>           | <b>Sd.</b>              | <b>0.129</b>            |

- (ii) The ratios between the predicted and “exact” *column* ultimate strength values  $\sigma_{nl}/\sigma_u$  and  $\sigma_{nd}/\sigma_u$  are often much higher than 1.0. In fact, the DSM provisions for local-plate and distortional failure yield estimates 52% and 20% higher than the “exact” values (in average). Moreover, the  $\sigma_{nl}/\sigma_u$  and  $\sigma_{nd}/\sigma_u$  values are also very scattered (standard deviations of 0.48 and 0.27) – i.e.,  $\sigma_{nl}$  and  $\sigma_{nd}$  considerably overestimate  $\sigma_u$  in *columns* affected by local-plate/distortional mode interaction.

Table 2A. Comparison between the “exact” ultimate strengths and their DSM estimates ( $\sigma_{nl}$ ,  $\sigma_{nd}$  and  $\sigma_{nld}$ ) for 30 (out of 90) beams

|  |       | FEA   |                |                |            | DSM           |                        |               |                        |                |                         |
|--|-------|-------|----------------|----------------|------------|---------------|------------------------|---------------|------------------------|----------------|-------------------------|
|  | $b_f$ | $f_y$ | $\sigma_{crl}$ | $\sigma_{crl}$ | $\sigma_u$ | $\sigma_{nl}$ | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$ | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$ | $\sigma_{nld}/\sigma_u$ |
| $b_y=120, b_s=24, t=1.8, L=770\text{mm}$ | 55    | 250   | 898            | 889            | 253        | 250           | 0.99                   | 250           | 0.99                   | 250            | 0.99                    |
|  | 55    | 350   | 898            | 889            | 344        | 350           | 1.02                   | 350           | 1.02                   | 350            | 1.02                    |
|  | 55    | 450   | 898            | 889            | 419        | 450           | 1.07                   | 437           | 1.04                   | 437            | 1.04                    |
|  | 65    | 250   | 692            | 677            | 249        | 250           | 1.00                   | 250           | 1.00                   | 250            | 1.00                    |
|  | 65    | 350   | 692            | 677            | 327        | 350           | 1.07                   | 338           | 1.03                   | 338            | 1.03                    |
|  | 65    | 450   | 692            | 677            | 393        | 439           | 1.12                   | 403           | 1.03                   | 403            | 1.01                    |
|  | 75    | 250   | 535            | 542            | 239        | 250           | 1.05                   | 249           | 1.04                   | 249            | 1.04                    |
|  | 75    | 350   | 535            | 542            | 311        | 341           | 1.10                   | 316           | 1.02                   | 316            | 1.00                    |
|  | 75    | 450   | 535            | 542            | 370        | 405           | 1.09                   | 375           | 1.01                   | 357            | 0.94                    |
|  | 85    | 250   | 423            | 454            | 230        | 250           | 1.09                   | 237           | 1.03                   | 237            | 1.03                    |
|  | 85    | 350   | 423            | 454            | 296        | 316           | 1.07                   | 299           | 1.01                   | 284            | 0.94                    |
|  | 85    | 450   | 423            | 454            | 347        | 375           | 1.08                   | 352           | 1.02                   | 318            | 0.90                    |
|  | 95    | 250   | 341            | 389            | 223        | 235           | 1.05                   | 226           | 1.01                   | 219            | 0.97                    |
|  | 95    | 350   | 341            | 389            | 282        | 295           | 1.05                   | 283           | 1.00                   | 256            | 0.90                    |
| 95                                       | 450   | 341   | 389            | 304            | 349        | 1.15          | 333                    | 1.09          | 285                    | 0.93           |                         |
| $b_y=160, b_s=80, t=1.7, L=820\text{mm}$ | $b_s$ | $f_y$ | $\sigma_{crl}$ | $\sigma_{crl}$ | $\sigma_u$ | $\sigma_{nl}$ | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$ | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$ | $\sigma_{nld}/\sigma_u$ |
|  | 19    | 250   | 410            | 345            | 226        | 249           | 1.10                   | 218           | 0.96                   | 218            | 0.96                    |
|  | 19    | 350   | 410            | 345            | 281        | 313           | 1.11                   | 272           | 0.97                   | 264            | 0.90                    |
|  | 19    | 450   | 410            | 345            | 315        | 371           | 1.18                   | 318           | 1.01                   | 294            | 0.89                    |
|  | 21    | 250   | 410            | 368            | 227        | 249           | 1.10                   | 222           | 0.98                   | 222            | 0.98                    |
|  | 21    | 350   | 410            | 368            | 285        | 313           | 1.10                   | 278           | 0.98                   | 268            | 0.91                    |
|  | 21    | 450   | 410            | 368            | 323        | 371           | 1.15                   | 326           | 1.01                   | 299            | 0.89                    |
|  | 23    | 250   | 409            | 394            | 228        | 249           | 1.09                   | 227           | 1.00                   | 227            | 0.99                    |
|  | 23    | 350   | 409            | 394            | 289        | 313           | 1.08                   | 285           | 0.99                   | 272            | 0.92                    |
|  | 23    | 450   | 409            | 394            | 331        | 371           | 1.12                   | 334           | 1.01                   | 304            | 0.89                    |
|  | 25    | 250   | 408            | 423            | 229        | 249           | 1.09                   | 232           | 1.01                   | 232            | 1.01                    |
|  | 25    | 350   | 408            | 423            | 292        | 313           | 1.07                   | 292           | 1.00                   | 276            | 0.93                    |
|  | 25    | 450   | 408            | 423            | 338        | 370           | 1.10                   | 343           | 1.02                   | 309            | 0.90                    |
|  | 27    | 250   | 406            | 455            | 230        | 248           | 1.08                   | 237           | 1.03                   | 237            | 1.03                    |
|  | 27    | 350   | 406            | 455            | 296        | 312           | 1.05                   | 299           | 1.01                   | 281            | 0.94                    |
|  | 27    | 450   | 406            | 455            | 345        | 370           | 1.07                   | 352           | 1.02                   | 314            | 0.90                    |

- (iii) The ratios between the predicted and “exact” *beam* ultimate strength values  $\sigma_{nl}/\sigma_u$  and  $\sigma_{nd}/\sigma_u$  are also often quite higher than 1.0 – however, the differences are smaller than for the *columns*. Indeed, the DSM local-plate and distortional strength estimates are now, 28% and 15% higher than the “exact” values (again in average) – the  $\sigma_{nl}/\sigma_u$  and  $\sigma_{nd}/\sigma_u$  standard deviations are also less pronounced: 0.24 and 0.17. Even then,  $\sigma_{nl}$  and  $\sigma_{nd}$  overestimate  $\sigma_u$  by a fair amount in beams affected by local-plate/distortional mode interaction.

Table 2B. Comparison between the “exact” ultimate strengths and their DSM estimates ( $\sigma_{nl}$ ,  $\sigma_{nd}$  and  $\sigma_{nld}$ ) for 30 (out of 90) beams

|  |       | FEA            |                |                |               | DSM                    |                        |                        |                        |                         |                         |
|--|-------|----------------|----------------|----------------|---------------|------------------------|------------------------|------------------------|------------------------|-------------------------|-------------------------|
| $b_w$                                    | $f_y$ | $\sigma_{cr1}$ | $\sigma_{crd}$ | $\sigma_u$     | $\sigma_{nl}$ | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$          | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$         | $\sigma_{nld}/\sigma_u$ |                         |
| $b_w=50, b_s=10, t=0.8, L=450\text{mm}$  | 60    | 250            | 241            | 256            | 203           | 210                    | 1.03                   | 197                    | 0.97                   | 178                     | 0.86                    |
|  | 60    | 350            | 241            | 256            | 222           | 262                    | 1.18                   | 243                    | 1.09                   | 206                     | 0.91                    |
|  | 60    | 450            | 241            | 256            | 228           | 309                    | 1.36                   | 283                    | 1.24                   | 228                     | 0.99                    |
|  | 70    | 250            | 239            | 242            | 200           | 209                    | 1.05                   | 193                    | 0.96                   | 176                     | 0.86                    |
|  | 70    | 350            | 239            | 242            | 215           | 262                    | 1.22                   | 238                    | 1.11                   | 203                     | 0.92                    |
|  | 70    | 450            | 239            | 242            | 223           | 309                    | 1.38                   | 277                    | 1.24                   | 224                     | 0.99                    |
|  | 80    | 250            | 237            | 231            | 197           | 209                    | 1.06                   | 189                    | 0.96                   | 173                     | 0.86                    |
|  | 80    | 350            | 237            | 231            | 223           | 261                    | 1.17                   | 233                    | 1.05                   | 199                     | 0.87                    |
|  | 80    | 450            | 237            | 231            | 229           | 308                    | 1.34                   | 271                    | 1.19                   | 221                     | 0.94                    |
|  | 90    | 250            | 234            | 220            | 193           | 208                    | 1.08                   | 186                    | 0.96                   | 171                     | 0.86                    |
|  | 90    | 350            | 234            | 220            | 210           | 260                    | 1.24                   | 229                    | 1.09                   | 196                     | 0.91                    |
|  | 90    | 450            | 234            | 220            | 215           | 307                    | 1.43                   | 266                    | 1.24                   | 217                     | 0.98                    |
|  | 100   | 250            | 231            | 211            | 190           | 207                    | 1.09                   | 183                    | 0.96                   | 168                     | 0.86                    |
|  | 100   | 350            | 231            | 211            | 203           | 259                    | 1.27                   | 225                    | 1.11                   | 193                     | 0.92                    |
| 100                                      | 450   | 231            | 211            | 210            | 305           | 1.45                   | 262                    | 1.25                   | 213                    | 0.99                    |                         |
| $b_w=180, b_s=15, t=1.1, L=750\text{mm}$ | $b_t$ | $f_y$          | $\sigma_{cr1}$ | $\sigma_{crd}$ | $\sigma_u$    | $\sigma_{nl}$          | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$          | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$          | $\sigma_{nld}/\sigma_u$ |
|  | 60    | 250            | 210            | 252            | 207           | 201                    | 0.97                   | 195                    | 0.94                   | 170                     | 0.82                    |
|  | 60    | 350            | 210            | 252            | 240           | 251                    | 1.04                   | 241                    | 1.01                   | 196                     | 0.82                    |
|  | 60    | 450            | 210            | 252            | 252           | 295                    | 1.17                   | 281                    | 1.12                   | 217                     | 0.86                    |
|  | 65    | 250            | 204            | 221            | 196           | 198                    | 1.01                   | 186                    | 0.95                   | 163                     | 0.82                    |
|  | 65    | 350            | 204            | 221            | 219           | 248                    | 1.13                   | 229                    | 1.05                   | 187                     | 0.85                    |
|  | 65    | 450            | 204            | 221            | 223           | 292                    | 1.31                   | 267                    | 1.20                   | 207                     | 0.92                    |
|  | 70    | 250            | 194            | 196            | 184           | 195                    | 1.06                   | 178                    | 0.97                   | 156                     | 0.83                    |
|  | 70    | 350            | 194            | 196            | 195           | 244                    | 1.25                   | 219                    | 1.12                   | 179                     | 0.90                    |
|  | 70    | 450            | 194            | 196            | 200           | 287                    | 1.44                   | 254                    | 1.27                   | 198                     | 0.97                    |
|  | 75    | 250            | 182            | 176            | 173           | 191                    | 1.10                   | 171                    | 0.99                   | 148                     | 0.84                    |
|  | 75    | 350            | 182            | 176            | 180           | 238                    | 1.32                   | 210                    | 1.16                   | 170                     | 0.92                    |
|  | 75    | 450            | 182            | 176            | 185           | 281                    | 1.52                   | 243                    | 1.31                   | 187                     | 0.99                    |
|  | 80    | 250            | 167            | 160            | 154           | 185                    | 1.20                   | 165                    | 1.07                   | 141                     | 0.89                    |
|  | 80    | 350            | 167            | 160            | 160           | 231                    | 1.44                   | 201                    | 1.26                   | 161                     | 0.98                    |
|  | 80    | 450            | 167            | 160            | 164           | 272                    | 1.66                   | 233                    | 1.42                   | 177                     | 1.06                    |

Finally, the variation of  $\sigma_{nd}/\sigma_u$  with the distortional slenderness  $\lambda_d$  is shown in figures 2(a) (columns) and 2(b) (beams). It is clear that the DSM distortional failure expressions yield fairly accurate and mostly safe ultimate strength estimates for the stockier *columns* and *beams* ( $\lambda_d \leq 1.2$ ). However, these same expressions perform poorly for moderate-to-slender columns and beams ( $\lambda_d \geq 1.2$ ), *i.e.*, their predictions are inaccurate and mostly unsafe – moreover, the error grows with  $\lambda_d$ . This means that the presence of local-plate buckling

Table 2C. Comparison between the “exact” ultimate strengths and their DSM estimates ( $\sigma_{nl}$ ,  $\sigma_{nd}$  and  $\sigma_{nld}$ ) for 30 (out of 90) beams and overall results

|                                     |       | FEA   |                |                | DSM        |               |                        |               |                        |                |                         |
|-------------------------------------|-------|-------|----------------|----------------|------------|---------------|------------------------|---------------|------------------------|----------------|-------------------------|
|                                     | $b_s$ | $f_y$ | $\sigma_{crl}$ | $\sigma_{crl}$ | $\sigma_u$ | $\sigma_{nl}$ | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$ | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$ | $\sigma_{nld}/\sigma_u$ |
| $b_w=400, b_s=150, t=2.0, L=1400mm$ | 22    | 250   | 131            | 115            | 126        | 171           | 1.36                   | 144           | 1.14                   | 119            | 0.91                    |
|                                     | 22    | 350   | 131            | 115            | 130        | 212           | 1.63                   | 175           | 1.35                   | 135            | 1.01                    |
|                                     | 22    | 450   | 131            | 115            | 133        | 250           | 1.88                   | 202           | 1.52                   | 148            | 1.08                    |
|                                     | 24    | 250   | 132            | 123            | 131        | 171           | 1.30                   | 148           | 1.13                   | 121            | 0.90                    |
|                                     | 24    | 350   | 132            | 123            | 134        | 213           | 1.59                   | 180           | 1.35                   | 138            | 1.00                    |
|                                     | 24    | 450   | 132            | 123            | 138        | 250           | 1.81                   | 208           | 1.51                   | 152            | 1.07                    |
|                                     | 26    | 250   | 132            | 132            | 137        | 171           | 1.25                   | 152           | 1.11                   | 123            | 0.88                    |
|                                     | 26    | 350   | 132            | 132            | 140        | 213           | 1.52                   | 186           | 1.33                   | 141            | 0.99                    |
|                                     | 26    | 450   | 132            | 132            | 143        | 250           | 1.75                   | 214           | 1.50                   | 155            | 1.07                    |
|                                     | 28    | 250   | 132            | 142            | 142        | 171           | 1.21                   | 157           | 1.11                   | 126            | 0.88                    |
|                                     | 28    | 350   | 132            | 142            | 146        | 213           | 1.46                   | 192           | 1.31                   | 144            | 0.98                    |
|                                     | 28    | 450   | 132            | 142            | 149        | 250           | 1.68                   | 221           | 1.49                   | 158            | 1.05                    |
|                                     | 30    | 250   | 132            | 163            | 147        | 171           | 1.16                   | 166           | 1.13                   | 131            | 0.89                    |
|                                     | 30    | 350   | 132            | 163            | 153        | 213           | 1.39                   | 203           | 1.33                   | 149            | 0.98                    |
|                                     | 30    | 450   | 132            | 163            | 156        | 250           | 1.60                   | 235           | 1.51                   | 164            | 1.06                    |
| $b_w=100, b_s=12, t=1.4, L=750mm$   | $b_w$ | $f_y$ | $\sigma_{crl}$ | $\sigma_{crl}$ | $\sigma_u$ | $\sigma_{nl}$ | $\sigma_{nl}/\sigma_u$ | $\sigma_{nd}$ | $\sigma_{nd}/\sigma_u$ | $\sigma_{nld}$ | $\sigma_{nld}/\sigma_u$ |
|                                     | 350   | 250   | 91.1           | 83.9           | 104        | 150           | 1.44                   | 126           | 1.22                   | 96             | 0.90                    |
|                                     | 350   | 350   | 91.1           | 83.9           | 113        | 186           | 1.65                   | 153           | 1.35                   | 109            | 0.94                    |
|                                     | 350   | 450   | 91.1           | 83.9           | 121        | 219           | 1.81                   | 176           | 1.45                   | 120            | 0.97                    |
|                                     | 370   | 250   | 82.7           | 80.0           | 102        | 145           | 1.42                   | 124           | 1.21                   | 92             | 0.88                    |
|                                     | 370   | 350   | 82.7           | 80.0           | 111        | 180           | 1.62                   | 150           | 1.35                   | 104            | 0.92                    |
|                                     | 370   | 450   | 82.7           | 80.0           | 120        | 211           | 1.76                   | 172           | 1.43                   | 114            | 0.94                    |
|                                     | 390   | 250   | 75.3           | 76.5           | 101        | 140           | 1.39                   | 121           | 1.20                   | 88             | 0.86                    |
|                                     | 390   | 350   | 75.3           | 76.5           | 110        | 174           | 1.58                   | 147           | 1.33                   | 99             | 0.90                    |
|                                     | 390   | 450   | 75.3           | 76.5           | 118        | 204           | 1.73                   | 169           | 1.43                   | 109            | 0.92                    |
|                                     | 410   | 250   | 68.1           | 72.3           | 99.1       | 135           | 1.37                   | 119           | 1.20                   | 84             | 0.84                    |
|                                     | 410   | 350   | 68.1           | 72.3           | 109        | 168           | 1.54                   | 143           | 1.31                   | 95             | 0.86                    |
|                                     | 410   | 450   | 68.1           | 72.3           | 117        | 197           | 1.68                   | 164           | 1.41                   | 103            | 0.88                    |
|                                     | 430   | 250   | 61.9           | 68.3           | 97.5       | 131           | 1.34                   | 116           | 1.19                   | 80             | 0.82                    |
|                                     | 430   | 350   | 61.9           | 68.3           | 107        | 162           | 1.51                   | 140           | 1.30                   | 90             | 0.84                    |
| 430                                 | 450   | 61.9  | 68.3           | 115            | 190        | 1.65          | 160                    | 1.39          | 98                     | 0.86           |                         |
|                                     |       |       |                |                |            | <b>Av.</b>    | <b>1.28</b>            | <b>Av.</b>    | <b>1.15</b>            | <b>Av.</b>     | <b>0.95</b>             |
|                                     |       |       |                |                |            | <b>Sd.</b>    | <b>0.241</b>           | <b>Sd.</b>    | <b>0.167</b>           | <b>Sd.</b>     | <b>0.069</b>            |

effects leads to a substantial erosion of the column or beam ultimate strength associated with the distortional failure. In addition, this erosion grows as the yield-to-critical distortional stress ratio  $f_y/\sigma_{crl}$  increases. Therefore, the influence of the local-plate/distortional mode interaction phenomenon on the ultimate strength of columns or beams (distortional failure) must be taken into account whenever their slenderness value  $\lambda_d$  is moderate-to-high.

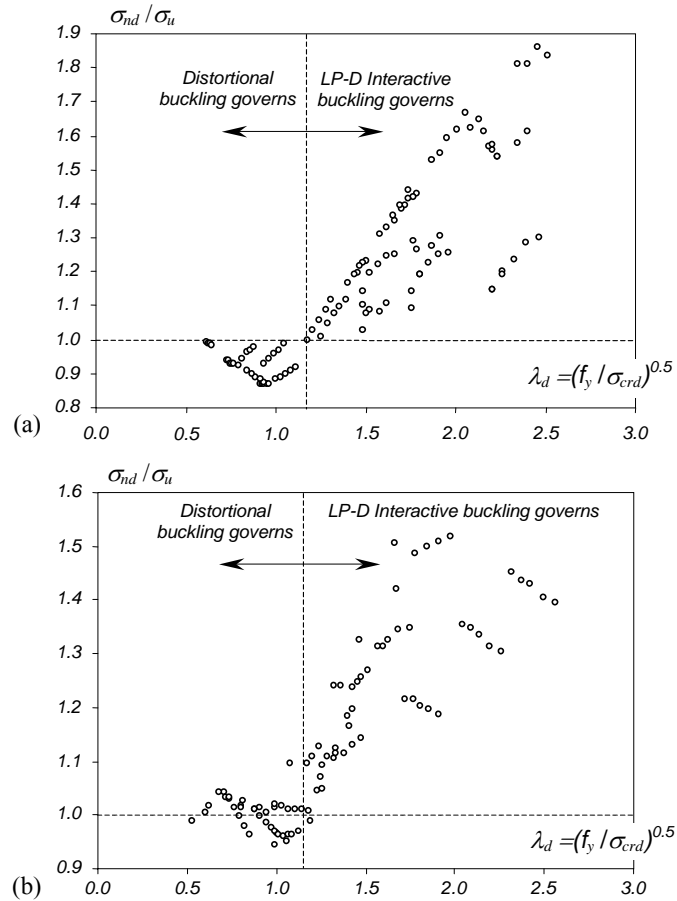


Fig. 2: Variation of  $\sigma_{nd} / \sigma_u$  with  $\lambda_d$  for (a) columns and (b) beams

### DSM for Local-Plate/Distortional Interaction

Following a strategy similar to the one adopted to develop a DSM approach to estimate the ultimate strength of columns and beams exhibiting a local-plate/global interactive buckling behavior, it becomes possible to propose



expressions that are applicable to columns and beams experiencing local-plate/distortional mode interaction effects. For the columns, this can be done by replacing either (i)  $P_y$  by  $P_{nd}$  in eqs. (1) or (ii)  $P_y$  by  $P_{nl}$  in eqs. (2) –  $P_{nd}$  and  $P_{nl}$  are the distortional and local-plate buckling strengths given by eqs. (1) and (2). Then, one obtains ultimate load estimates  $P_{nld}$  and  $P_{ndl}$ , respectively. Yang & Hancock (2004) have recently adopted the first approach, which is schematically presented in the flowchart of figure 3(a) – the “role” of the overall strength  $P_{ne}$  is now played by the distortional strength  $P_{nd}$ . Finally, note that the approach just outlined involves the knowledge of *accurate* local-plate and distortional buckling loads ( $P_{crf}$ ,  $P_{crd}$ ), which can be readily determined through finite element, finite strip or generalised beam theory (GBT) analyses. The same methodology can also be applied to the beams, as illustrated in the flowchart of figure 3(b) – one then obtains a  $M_{nld}$  value, which estimates the corresponding beam ultimate strength.

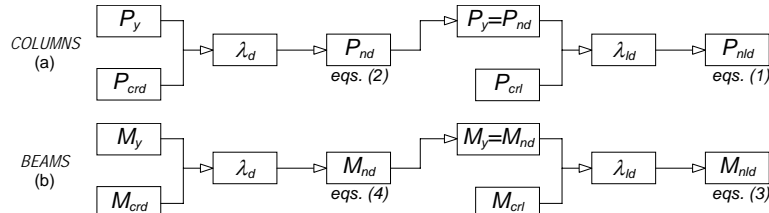


Fig. 3: Flowcharts concerning the application of the DSM to (a) columns ( $P_{nld}$ ) and (b) beams ( $M_{nld}$ ) under local-plate/distortional mode interaction

After comparing the ultimate strength estimates provided by their DSM approach with the results of a series of experimental tests involving lipped channel columns with “v-shaped” web and flange intermediate stiffeners (Yang 2004), which provided clear evidence of an adverse local-plate/distortional interaction, Yang & Hancock (2004) concluded that (i) the above estimates were safe and reasonably accurate (differences in the 10-20% range), and also that (ii) further investigation was required concerning the design of columns with nearly coincident local-plate and distortional buckling stresses. On the other hand, Silvestre *et al.* (2005), in the context of simply supported “plain” lipped channel columns experiencing local-plate/distortional mode interaction, compared the two aforementioned DSM approaches ( $P_{nld}$  and  $P_{ndl}$  – LD and DL approaches) and concluded that they lead to very similar ultimate strength estimates. In view of these facts,

it was decided to adopt the “LD approach” and employ it to estimate the ultimate strength of the columns and beams addressed in this work.

### Assessment of DSM Estimates for LP/D Interaction

Besides the predictions yielded by the individual local-plate and distortional DSM failure expressions, tables 1A-C and 2A-C also include DSM  $\sigma_{nld}$  estimates. Their observation leads to the following remarks:

- (i) Although the column  $\sigma_{nld}$  estimates are reasonably accurate in average (mean of  $\sigma_{nld}/\sigma_u$  equal to 0.97), there are well scattered: the  $\sigma_{nld}/\sigma_u$  standard deviation is 0.13. Among the whole set of  $\sigma_{nld}$  estimates, 1 is exact, 38 are safe and accurate ( $\sigma_{nld}/\sigma_u \geq 0.9$ ), 36 are excessively safe ( $0.79 \leq \sigma_{nld}/\sigma_u < 0.90$ ), 16 are slightly unsafe ( $\sigma_{nld}/\sigma_u \leq 1.10$ ) and 17 are very unsafe ( $1.10 < \sigma_{nld}/\sigma_u \leq 1.36$ ).
- (ii) The beam  $\sigma_{nld}$  estimates are also reasonably accurate in average (mean of  $\sigma_{nld}/\sigma_u$  equal to 0.95). However, unlike in the columns, the scatter is now quite low:  $\sigma_{nld}/\sigma_u$  standard deviation of 0.069. Out of the whole set of  $\sigma_{nld}$  estimates, 3 are exact, 44 are safe and accurate ( $\sigma_{nld}/\sigma_u \geq 0.9$ ), 28 are too safe ( $0.82 \leq \sigma_{nld}/\sigma_u < 0.90$ ) and 15 are accurate but slightly unsafe ( $\sigma_{nld}/\sigma_u \leq 1.10$ ).

The variation of the stress ratios  $\sigma_u/f_y$  and  $\sigma_{nld}/f_y$  with the *distortional slenderness*  $\lambda_d = (f_y/\sigma_{crd})^{0.5}$  are shown in figures 4(a) (columns) and 4(b) (beams). Also included are the “Winter-type” curves defined by eqs. (1)-(2) (columns) and (3)-(4) (beams), which provide the DSM *local-plate* and *distortional* ultimate strength estimates. From the joint observation of all these results, the following comments can be drawn:

- (i) The proposed DSM predictions (black dots) always (i<sub>1</sub>) lie well below both the local-plate and distortional curves, for the *slender* members ( $\lambda_d > 1.2$ ), and (i<sub>2</sub>) are located near the distortional curve, for the *stockier* members ( $\lambda_d < 1.2$ ). This means that, at least for the critical stress ratio range considered ( $0.90 \leq \sigma_{crd}/\sigma_{crl} \leq 1.10$ ), the local-plate/distortional mode interaction always causes a substantial strength erosion in the *slender* members (w.r.t. the individual local-plate and distortional values).
- (ii) Regardless of the member distortional slenderness values, the black dots (ii<sub>1</sub>) always remain quite “aligned” and (ii<sub>2</sub>) lie in a fairly close vicinity of the “exact” ultimate strength values (white dots), in spite of their “vertical dispersion” – they lie mostly below (particularly in the beams).

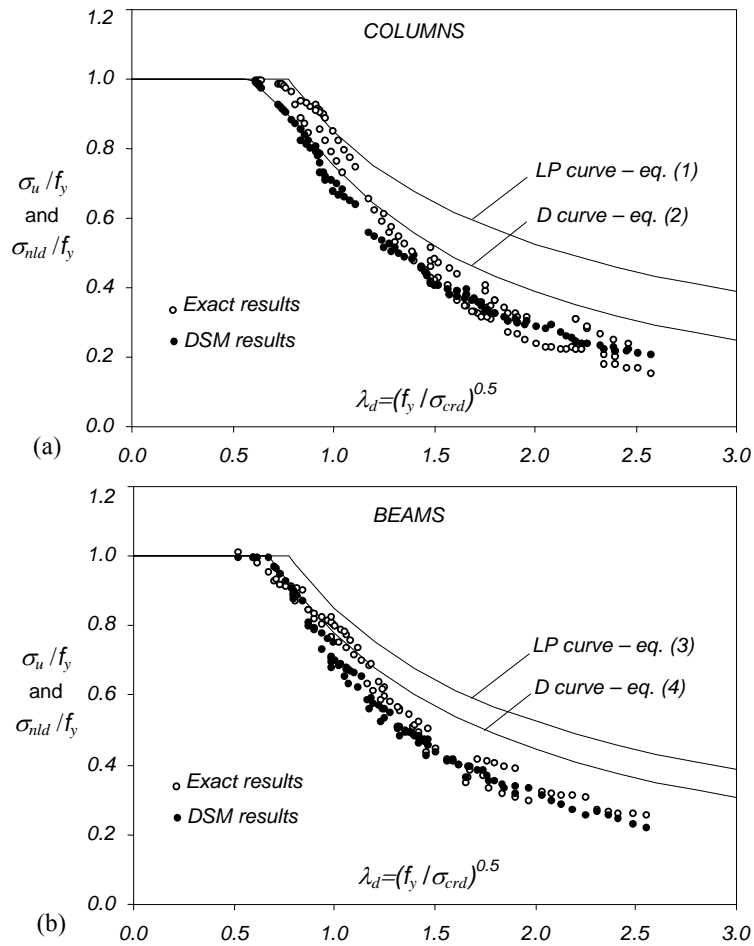


Fig. 4: Variation of  $\sigma_{nld}/f_y$  and  $\sigma_u/f_y$  with  $\lambda_d$  for (a) columns and (b) beams

### Design Recommendations

All the members analyzed (a total of 198) display distortional slenderness values falling inside the range for which the individual local-plate and distortional DSM curves were experimentally calibrated. If the column or beam

$\sigma_{crit}$  and  $\sigma_{crd}$  values are less than 15% apart, it seems fair to say that:

- (i) In members with a low slenderness ( $\lambda_d \leq 1.2$ ), the  $\sigma_{nld}$  values accurately predict the ultimate strength  $\sigma_u$  under local-plate/distortional mode interaction – thus, the current DSM provisions for distortional failure can be satisfactorily employed.
- (ii) In members with a moderate-to-high slenderness ( $\lambda_d \geq 1.2$ ), the current DSM provisions yield unsatisfactory results, while the  $\sigma_{nld}$  approach yields mostly accurate predictions of the ultimate strength  $\sigma_u$  under local-plate/distortional mode interaction – however, for wide-flange columns with high yield stresses,  $\sigma_{nld}$  may provide unsafe ultimate strength estimates (see Silvestre *et al.* 2005 for more details).
- (iii) Regardless of the member slenderness, the  $\sigma_{nld}$  values provide fairly accurate predictions of the ultimate strength of member prone to local-plate/distortional mode interaction (with the exception mentioned in the previous item) – such an achievement cannot be reached through the use of the current DSM (local-plate and distortional) provisions.

### Conclusion

Results of an ongoing investigation on the use of the Direct Strength Method to estimate the ultimate strength of lipped channel columns and beams affected by local-plate/distortional interaction were presented and discussed. On the basis of the results of a FEM-based parametric study involving 108 columns and 90 beams, it was possible (i) to obtain numerical evidence of the severe member strength erosion caused by the local-plate/distortional interactive and (ii) to reveal the inability of the DSM individual (local-plate and distortional) expressions to predict such erosion in arbitrary lipped channel columns and beams. However, it was shown that a DSM approach based on those individual expressions (recently proposed by Yang & Hancock 2004 for columns) predicts quite well the strength reduction caused to lipped channel columns and beams by local-plate/distortional mode interaction – even so, it was also possible to identify a number of features that must be included in a more elaborate DSM approach, specifically developed to take into account this type of interactive behavior. Finally, the paper closed with some design recommendations about the application of the DSM approach dealt with here to lipped channel members exhibiting nearly coincident local-plate and distortional buckling stresses.

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