Oct 22nd, 12:00 AM

Optimization of Long Span Truss Purlins

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The worth of a variety of optimization techniques that have been steadily advancing in development ultimately must be judged by their performance in the marketplace. Evidence of such value has been slow in coming and the engineer quite rightly has become skeptical of mathematical sophistication that does not harvest sufficient benefit to overcome by considerable margin that which he can achieve by intuition alone. Gains in system efficiency achieved by such mathematical means quite often are more than counterbalanced by the labor spent in achieving them.

This does not mean that there are no design situations where a handsome payoff from the use of a suitable optimization tool might be possible. There could, in fact, be certain factors present in a design situation which might indicate that a mathematical optimization tool should be employed at the outset as the prime designer, and that human effort would be more effectively employed in setting the problem up for reduction by mathematical means rather than by attempting to evolve the design by traditional heuristic methods.

That this even is possible is because the particular mathematical formulation of the optimization problem that is to be solved actually represents a formal statement of the design

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situation itself. That is, in solving the mathematical optimization problem we seek somehow to assign values to a set of variables so that a particular combination of those variables achieves a "best value." This also describes the problem that is attacked by a structural designer.

Although that simple statement by itself represents a valid search (i.e., optimization) situation, solvable by a certain class of mathematical techniques for finding unconstrained optima, normally the search must be carried out within a bounded region, representing a constrained search situation. The optimization problem then in its most general form can be formally stated as:

$$\text{Optimize} \quad F(x_1, x_2, \ldots x_n) \tag{1}$$

where the search is subject to the set of constraints:

$$\begin{aligned} g_1(x_1, x_2, \ldots x_n) &\leq 0 \\ g_2(x_1, x_2, \ldots x_n) &\leq 0 \\ &\vdots \\ g_m(x_1, x_2, \ldots x_n) &\leq 0 \end{aligned} \tag{2}$$

As might be expected, the class of mathematical search techniques that solves this extended problem definition requires a higher level of sophistication than if the set of constraints (2) does not exist.*

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* So high, in fact, that one often must resort to guess and verify, (i.e., trial design followed by analysis of the configuration followed by redesign, etc.), which describes that process known as the "traditional approach" to design.

The significance of this representation of a bounded search situation for structural designers is in the recognition that it can represent the totality of effort involved in evolving a structural configuration. There have been no restrictions placed on the content of (1) and (2), the measure of the merit of a given search position (1) could just as well reflect production costs as well as weight, and the set of constraints (2) can just as well contain the entire set of restrictions as they appear, say, in the Specification for the Design of Cold-Formed Steel Structural Members (Ref. 2).

Further, in the generic form of (1) and (2) there is no implied limit on either the number of variables involved in the search, or the number of constraints defining the bounds of the search space. In fact, the size of the set of parameters contained in (1) has not been confined, and it could just as well be considered to contain all the design parameters (variables) defining a large assemblage of structural components as well as those defining the configuration of any one component.

Problem definition, however, does not necessarily guarantee solution. Even though the forms (1) and (2) can be employed to define completely a design situation, the state of the art of the mathematical mechanisms available to actually search the solution is not yet so refined that any situation can be solved merely because it can be so stated. This difficulty, of course, also confronts the designer attempting to optimize using more traditional avenues, especially when using the medium of cold-formed steel to efficiently serve a certain function. This is particularly apparent when flexural-torsional buckling is possible, a situation where one is happy to achieve at least one combination of parameters that is admissible, much less that combination which is the best possible. Yet the fact that even this difficult problem can be included in the framework of (1) and (2), combined with the realization that there are improving mathematical techniques to solve directly (1) and (2), should provide some incentive for designers of cold-formed shapes to continually monitor the progress of research effort aimed in this direction.

Assuming that there is an acceptably reliable mathematical algorithm suitable for solving the problem at hand, some of the factors that would favor a decision for the use of optimization for the design of a structural system might include these:

1. The system topology and geometry is invarient for a wide variety of uses, though the values for the descriptors (design parameters) are not. (This is another way of saying it is a standard component that is slightly varied to cover a variety of load carrying conditions.)

2. The number of design parameters is too large to be adequately searched by human endeavor.

3. The system's merit is judged in a highly competitive environment.

4. The design that is generated by the optimization algorithm is complete in all respects.

5. The computing cost involved in solving the problem is not likely to be inordinately high.

Obviously these are not rigid or even independent criteria, and can be used only in a subjective sense to guide a decision as whether "to optimize" or "not to optimize".* For example, if marketing studies indicate that the sales of a certain high volume building systems product were likely to be quite impressive, then even relatively large computing expenses could be tolerated if production costs saved thereby still produced a favorable cost-benefit ratio. This also relates to point three, which points out that competition may well be incentive enough to employ extraordinary means to improve the merit of the system.

Most of the factors pointed out above are especially relevant to the cold-formed steel industry. Building systems quite often employ cold-formed shapes closely engineered to satisfy a particular structural function for a high volume market. Also there are special factors involved in the design of cold-formed shapes that inhibit intuitive efforts towards optimization. Because cold-formed shapes more generally than not lack the symmetry necessary to preclude flexural-torsional buckling, design effort necessary to produce a class of shapes subjected to such behavior is rather involved. Finally, when considering the competitive posture of the industry, the capability for cold-forming to fit a particular function favors the incentive to spend some time in optimization by any medium.

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* While it can be argued that the designer if given enough resources can "optimize" a design, the term has come to describe certain mathematical techniques that have the potential of achieving the absolute best (i.e., "no fat") design possible.

An interesting and unusual opportunity to evaluate the economic worth of optimization per se to the cold-formed industry was provided recently when the Butler Manufacturing Company embarked on a program to improve the design efficiency of a class of long span purlins that they already had been marketing (Figs. 1, 2). The purlins are laterally braced only at a discrete number of points; in fact, one of the objectives of the project was to ascertain if bracing arrangements other than those being used could improve the efficiency of the design. Because of the unbraced situation the product does not fit into the class of joists governed by the specification of the Steel Joist Institute (Ref. 6).

Both chords resemble lipped hat sections, yet because only the top chord is acting as a beam-column, the purlin uses chords of different designs (Fig. 2). The interior web members consist of bent tubing welded to the chords.

The opportunity for evaluating the practical worth of a mathematical optimization tool was present because considerable engineering effort had already been expended in designing the class of components for the original marketing effort. The attempt to redesign the class of components to achieve better design efficiency was taking an inordinate amount of time because there simply were too many parameters involved to be manipulated efficiently in favorable directions by manual means, even with the aid of an inhouse computer. Figure 2 shows that there are twenty-two variables that completely define the system: seven for each of the chords, the diameter and thickness for each of

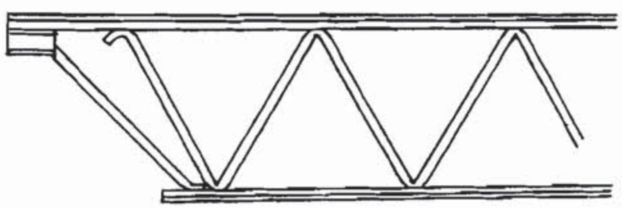
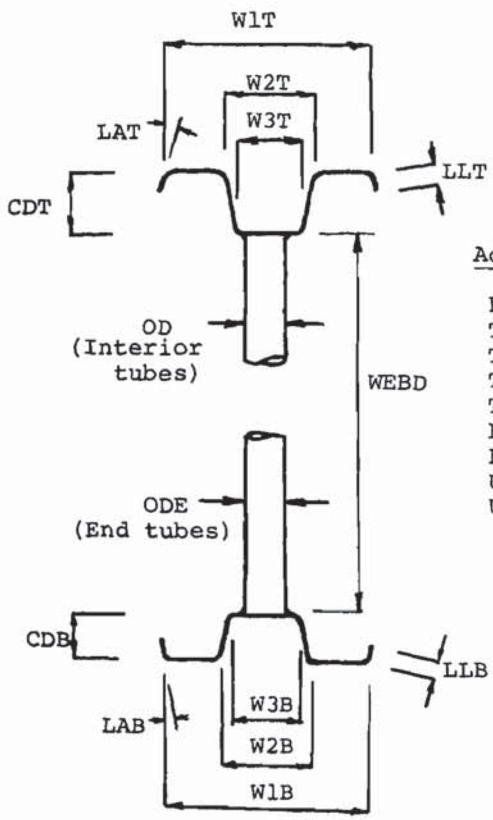


Figure 1. Butler Truss Purlin



Additional Parameters

- PS panel length
- TT top chord thickness
- TB bottom chord thickness
- TW interior tube thickness
- TWE end tube thickness
- FY chord strength
- FYW tube strength
- UNBLT unbraced length, top chord
- UNBLB unbraced length, bott.chord

Fig. 2 Design parameters used in optimization problem the interior and end web tubes, purlin depth, panel spacing, and spacing for top and bottom lateral bracing.

Thus not only could the amount of improvement that a mathematical optimization tool might attain be compared directly with what had already been achieved by considerable engineering effort and marketed successfully, but the development cost of obtaining the improvement could be determined as well. Just as important, the designs that were mathematically generated would be subjected to the most critical examination by both engineering and marketing professionals who are responsible for producing a competitive product.

It was decided to try to improve the existing design by applying an optimization technique that has been reported as being effective for the type of many variabled, highly nonlinear, and discontinuous type of mathematical programming problem that a cold-formed steel design situation represents (Ref. 4). Reviewing briefly, the nonlinear forms (1) and (2) are linearized by a Taylor series expansion producing the linear forms:

$$\text{Optimize} \quad [c_1^o \quad c_2^o \quad c_3^o \cdots c_n^o] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \qquad (3)$$

Subject to the constraints:

$$\begin{bmatrix} a_{11}^o & a_{12}^o & a_{13}^o \cdots a_{1n}^o \\ a_{21}^o & & \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ a_{m1}^o & & \cdots a_{mn}^o \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \leq - \begin{bmatrix} g_1^o \\ g_2^o \\ \cdot \\ \cdot \\ \cdot \\ g_m^o \end{bmatrix} + \begin{bmatrix} a_{11}^o & a_{12}^o & a_{13}^o \cdots a_{1n}^o \\ a_{21}^o & & \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ a_{m1}^o & a_{m1}^o & \cdots a_{mn}^o \end{bmatrix} \begin{bmatrix} x_1^o \\ x_2^o \\ \cdot \\ \cdot \\ \cdot \\ x_n^o \end{bmatrix} \quad (4)$$

where: c_j is $\partial F(x_1, x_2 \dots x_n)/\partial x_j$

a_{ij} is $\partial g_i(x_1, x_2 \dots x_n)/\partial x_j$

g_i is the set of (nonlinear) constraints

x_i the set of design parameters

x_i^o the expansion point for the series, which also can be viewed as a trial design which the algorithm is attempting to improve.

In less cumbersome form the problem can be represented as

$$\text{Optimize} \quad [C^o][X] \quad (5)$$

$$\text{Subject to} \quad [A^o][X] \leq -[G^o] + [A^o][X^o] \quad (6)$$

The superscript "o" indicates that the functional form of a matrix element is evaluated at the expansion point X^o (i.e., trial design).

A succession of linear programming problems (3 and 4) is then solved where the solution x_i to any one cycle is used for the trial design x_i^o for the succeeding cycle. The solution to the nonlinear problem is indicated when $x_j \approx x_{j-1}^o$. The technique differs primarily from that originally given in reference 5 in that constraints once evaluated at a given x_j^o are not accumulated from cycle to cycle. Because of the large number of discontinuities and mathematical nonconvexities inherent when the cold-formed specification is used for (2), iterative non-convergence and oscillation is the rule rather than the exception. In order to overcome these computational difficulties a convergence accelerator is also employed as follows:

Substitute $$x_j = u_j - x'_j \tag{7}$$

in (4) so that it becomes

$$-[A^o][X'] \leq -[G^o] + [A^o][X^o - U] \leq [B] \tag{8}$$

As described in detail in references 3 and 4, a certain manipulation is performed on u_j as the iteration proceeds until convergence is obtained. The strategy broadly is to reduce u_j slightly at intervals, but then to increase a certain subset of u_j and decrease another subset when the constriction produces a cyclic infeasibility, as seems to be inevitable with the particular transformation used (7).

A significant improvement to the method published in reference 4 was obtained here by suppressing the cycling indicated in (3) and (4) until one of a set of randomly generated vectors $[X^o]$ satisfied the constraint set (4).

Following the above, the Butler purlin was optimized in the following steps.

$$[X] = \begin{bmatrix} W1T \\ W2T \\ W3T \\ LLT \\ CDT \\ LAT \\ TT \\ W1B \\ W2B \\ W3B \\ LLB \\ CDB \\ LAB \\ TB \\ OD \\ TW \\ ODE \\ TWE \\ FYW \\ PS \\ WEBD \\ UNBLT \\ UNBLB \\ FY \end{bmatrix}$$

The matrix [X] appearing in (3) and (4) was first established with a particular ordering that must be adhered to throughout the problem. As shown to the left, all possible descriptors of the system that might ever be considered as a design parameter should be included when developing production software, even though it might be the present intention only to consider a lesser number for the design. In this manner the sensitivity matrix [A] that will be generated will be sufficiently general to handle the larger number of variables if that situation is ever desired and the significantly large expense of having to regenerate the entire sensitivity matrix in order to add new variables will be avoided.

With the Butler purlin, even though it was the present intention to use a particular higher grade steel for both the web tubing and the cold-formed chords, the parameters FYW and FY were included in the parameter list in the anticipation that conditions could possibly change such that other grades might at some time become economically attractive. In fact, if the yield strengths were permitted to float,* solving the optimization problem would produce that particular combination of strengths that were optimal, provided a suitable cost function were made available.

The column matrix [G] is then assembled merely by listing all the inequalities that govern the design of the purlin, and for cold-formed shapes most of these are taken directly from the AISI specification (Ref. 2). Each member of g_i is a machine readable statement that defines the left side of the design restriction given in the form:

$$g_i \leq 0 \qquad (9)$$

For example, the design of the top chord is governed by AISI specification section 3.7.3 relating axial and bending stresses in singly symmetric shapes having a Q factor less than unity and which is subject to torsional-flexural buckling. The first element of the matrix [G] is actually the constraint drawn directly from section 3.7.3 which restricts interaction of bending and axial stresses at the center of the critical panel due to dead plus live load, appearing as:

$$g_1 = f_a/F_{a1} + f_b C_m/F_{b1}(1-f_a/F_e') - 1 \qquad (10)$$

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* In an optimization context, "floating" a variable means permitting its value to be chosen by the algorithm. In an opposite sense, "freezing" a variable means setting both range limits at an identical value so that in effect the designer determines the value.

(The notation conforms to that appearing in the AISI specification.)

Even though (10) appears as if it might be coded in machine readable form (e.g., FORTRAN, etc.) easily enough, each term actually represents a considerable amount of coding necessary to define the term before it is used in g_1. F_{a1}, for example, must be defined as any of the alternate axial buckling formulas that appear in the specification. One can easily grasp the size of the statement(s) required to do this when considering that each of those formulas must be given in terms of the design variables as listed in [X].

Also, the actual axial stress f_a not only is a function of the seven design parameters associated with the top chord, but also the seven associated with the bottom chord because the axial force in the top chord is obtained using the moment arm between the two centroids of the chords, both of which are functions of the chord parameters. Of course f_a is a function of WEBD as well.

Similarly large machine readable expressions are used to define the other terms in (10) so that the evaluation of the expression which represents just g_1 represents a sizeable amount of coding.

The second element of [G], which is g_2, is a statement defining the limit of interaction as

$$f_a/F_{a0} + f_b/F_{b1} - 1 \leq 0 \qquad (11)$$

Both g_1 and g_2 are active in the optimization process whenever, as specified by AISI, $f_a/F_{a2} \geq 0.15$. If the inequality shown is not satisfied, the elements g_1 and g_2 are set to zero, which effectively eliminates them from being able to govern the course of the design. In this case the alternative form given in 3.7.2 is then activated as g_3, simply by not equating it to zero.

In addition to interaction at the positive moment region of the panel, that at the negative moment regions (i.e., panel points) is listed as several more elements in [G]. The list propagates even more when duplicate constraints are included at both points for wind loading as a factor.

The complete list of constraints that was used as elements for [G] in optimizing the Butler purlin is described below.

g_1: dead+live, interaction, pos. mom. top chord, amplification factor, $f_a/F_{a1} \geq 0.15$

g_2: dead+live, interaction, neg. mom. top chord, no amplification factor, $f_a/F_{a1} \geq 0.15$

g_3: dead+live, interaction, neg. mom. top chord, $f_a/F_{a1} \leq 0.15$

g_4: dead+live, interaction, pos. mom. top chord, $f_a/F_{a1} \leq 0.15$

g_5: dead+wind, interaction, pos. mom. top chord, amplification factor, $f_a/F_{a1} \geq 0.15$ and no uplift

g_6: dead+wind, interaction, neg. mom. top chord, no amplification factor, $f_a/F_{a1} \geq 0.15$ and no uplift

g_7: dead+wind, interaction, neg. mom. top chord, no amplification factor, $f_a/F_{a1} \leq 0.15$ and no uplift

g_8: dead+wind, interaction, neg. mom. top chord, no amplification factor, $f_a/F_{a1} \leq 0.15$ and no uplift

g_9: dead+live, tension, bottom chord

g_{10}: dead+wind, compression, bottom chord

g_{11}: limit on axial comp. stress, dead+live, top chord, from lateral buckling of purlin as a whole between top braces,

$$.36\pi^2 EC_b/F_y < L^2 S_{xc}/dI_y <= 1.8\pi^2 EC_b/F_y$$

g_{12}: limit on axial comp. stress, dead+wind, bottom chord, from lateral buckling of purlin between bottom braces,

$$.36\pi^2 EC_b/F_y < L^2 S_{xc}/dI_y <= 1.8\pi^2 EC_b/F_y$$

g_{13}: allowable axial stress from lateral buckling of entire purlin, top chord in comp.

$$L^2 S_{xc}/dI_y >= 1.8\pi^2 EC_b/F_y$$

g_{14}: allowable axial stress from lateral buckling of entire purlin, bott. chord in compression,

$$L^2 S_{xc}/dI_y = 1.8\pi^2 EC_b/F_y$$

g_{15}: deflection limitation, span/180 or as given

g_{16}: top chord lip stiffness ($1.83t^4\sqrt{(w/t)^2 - 4000/F_y}$)

g_{17}: top chord lip stiffness ($9.2t^4$)

g_{18}: bott. chord lip stiffness (see g_{16})

g_{19}: bott. chord lip stiffness (see g_{17})

g_{20}: max. w/t of top chord lip

g_{21}: max. w/t of bott. chord lip

g_{22}: max. w/t of chord web

g_{23}: clear dist. between flanges, top chord

g_{24}: shear stress, top chord, dead+live

g_{25}: slenderness ratio, top chord, vert. axis

g_{26}: slenderness ratio, top chord, horiz. axis

g_{27}: slenderness ratio, bott. chord, vert. axis

g_{28}: slenderness ratio, bott. chord, horiz. axis

g_{29}: slenderness ratio, end tubes

g_{30}: slenderness ratio, interior tubes

g_{31}: tension stress, interior tubes, dead+live

g_{32}: comp. stress, interior tubes, dead+live

g_{33}: tension stress, interior tubes, dead+wind

g_{34}: comp. stress, interior tubes, dead+wind

g_{35}: tension stress, end tubes, dead+live

g_{36}: comp. stress, end tubes, dead+wind

g_{37} to g_{60}: lower limits on design parameters

g_{61}: pos. mom. interaction, top chord, uplift

g_{62}: neg. mom. interaction, top chord, uplift

The above are not necessarily given in any logical order, but rather as they actually were assembled and appear in the program. Constraints 61 and 62 for example were added to the end of the list well into the development of the program when it was noticed they inadvertantly had been omitted.

After the set of constraints that govern the design were assembled as the matrix [G], each was algebraically differentiated with respect to all twenty-four variables in order to define the sensitivity matrix [A]. Even though many of the elements a_{ij} are easily differentiated (e.g., constraints 37 to 60), there still are enough very large nonlinear forms involved that the task is well beyond human capability in view of the fact that each differential expression must virtually be without blemish in order for the solution to progress satisfactorily. Because it contains an easily used differentiation capability, the formula manipulation language PL/1-FORMAC (Ref. 7) was used to generate directly the computer program arithmetic statements

(PL/1) that define each element a_{ij}. The magnitude of the task can be inferred from the size of the resulting object code which indicates that the statements defining the sensitivity matrix alone [A] for this structural system occupies in excess of 300,000 bytes of storage.

The only matrix remaining to be synthesized in order to complete the formulation of the problem ready for solution (7 and 8) is [U], representing upper limit values for the design parameters. For the first cycle of the solution these are permanent values established as range bounds for the design parameters which will not be violated during the progress of the solution. In subsequent cycles these limits may gravitate downward at different rates, perhaps even occasionally retreating back upwards as manipulated by the algorithm which has been devised to force convergence for involved problems of this type and which is described more completely in references 3 and 4.

The absolute range limits are preset at values that are practical for the series of purlins being optimized. (The lower limits are contained in constraints 37 to 60.) These are used by the program as default limits unless overridden by the data used for any particular case, as explained further below.

Actual usage of the optimization program by engineers of the Butler Manufacturing Company is quite simple. A free form language has been devised to convey problem data to the program so that the system can be used easily either in a batch mode or from remote terminals located near their desks. Basic problem data such as span and loading is given in free form following the punched or typed keyword DATA, as:

```
DATA   SPAN   30   LIVE   30   DEAD   3   WIND   -25
```

Limits on variable ranges are similarly given after the keywords UPPER LIMITS, LOWER LIMITS, or USE. The last sets an identical upper limit and lower limit for the parameter so specified. An example of usage of range limit commands is:

```
LOWER LIMITS   W1T   5.5   TT   .08
UPPER LIMITS   W1T   5.0   TT   .10
USE   OD   1.25   TW   .12
```

Any parameter which does not appear in the USE command is floated automatically. However, it is not practical to float all 24 parameters since computational expense correspondingly can be quite high. Rather, it has been found more practical to float five or six variables at a time, a not particularly restrictive situation since a solution usually is obtained easily in this case. Successive trials floating different sets of variables commonly permit an entire purlin to be optimized. For example, as a first try the optimal web depth might be obtained by floating web parameters and only a few parameters in each chord. Subsequent to this the depth might be frozen at the value so obtained with a USE command and each chord in turn then optimized by floating the pertinent parameters. Other subsets may be floated as easily.

The technique described above has been quite effective. For example, the design arrived at for a particular member of the purlin series prior to its manufacture and marketing involved a weight of 251 lbs. When subjected to the optimization process described herein, a more efficient design was mathematically generated which demonstrated that the same loading over the same span could be carried by a purlin weighing less than 200 lbs. and fabricated from the same kinds of components. The more efficient design satisfies in all respects the provisions of the AISI Cold-Formed Steel Specification, including effects of flexural-torsional buckling behavior of both chords, a factor not previously included in the heavier design.

Even though the entire purlin series is being optimized as described, it is being done so for standard spans and loadings. However, quite often special loading situations arise in practice which cannot be considered for a standard series, such as the case where it is necessary to impose concentrated loads at various locations. Fortunately the problem formulation as indicated in (6) can more easily be used to review an existing design than to produce a new design. It is necessary only to specify all design parameters in a USE command as described above. Since no parameters are in this case floated, the sensitivity matrix [A] of the linear programming problem that would be solved to improve the design contains zeros in all elements and the process terminates immediately after one cycle, at which time a branch is made to a routine which prints the calculations obtained in the initial pass through the coding that defines [G]. In order to expedite a review process, a file of the standard designs has been established on secondary storage. The engineer executing the program in a review mode specifies the problem data as above, including the concentrated loads, but also gives a part number as data for the purlin to be reviewed. The program then inserts in [X^o] all values contained in the data record for that part and prints the design calculations immediately after having evaluated [G].

From the point of view of the engineers using the program to optimize products such as the Butler purlin, there are important though less obvious economical benefits to be gained other than mere improved design efficiency with respect to unit cost or weight. With this capability it now becomes possible to treat the total process of creating a complete set of optimum products. Any strategy for developing a set of similar products at optimum costs must take into consideration the following factors:

1. Material costs
2. Inventory costs
3. Manufacturing constraints
   a. Set-up labor
   b. Cost of tracking and shipping
   c. Tooling variables permitted or practicable
4. Cost of design and detailing
5. Volume of expected use

Normally the cost of materials represents a major portion of the total cost. It is therefore incumbent to have the lightest, most structurally efficient section possible. In this particular case the steel cost increases at a lower rate than that of the yield stress and the higher yield strengths produce a more favorable cost to weight ratio. However, this can be deceptive because both limited volume and inventory requirements can totally eliminate the advantage of optimum weight potential. When the volume of geometrically similar parts is low, sacrifices must be made in weight in order to balance the total manufacturing cost by standardizing and grouping the parts considered.

The process used to optimize the manufacturing costs of the total set of truss purlins can best be described as an iterative one where there is cycling through all the above until the total group of similar parts satisfies minimum manufactured cost objectives. First a group of similar purlins are completely optimized by the mathematical algorithm described herein without any limitations whatsoever. This solution is then considered the zero cost point. Then all known manufacturing constraints are fed into the problem via range limit restrictions and the process is repeated. If the resulting additional costs because of the added constraints appear to be too high, these manufacturing constraints themselves are closely examined for the cost of possible changes. After this all of the similar parts are subjected to an inventory cost analysis that accounts for such items as expected volume, inventory costs, manufacturing set-up labor costs and paper and accounting costs. In this way the actual penalty paid by retaining certain restrictive manufacturing constraints can be accurately assessed and better argument can be made for changing then if so indicated.

The analysis described above eventually reduces the total number of mathematically obtained optimum sections to a smaller set that is optimum for manufacturing. Typically a set of ten truss purlins of similar geometry but of different load carrying capabilities will ultimately be reduced to three or four parts to be marketed. Yet the total cost of production will be less than it would be if all ten parts were produced as mathematically generated.

Example

A typical use of the program to generate a least weight design is demonstrated here where it was desired to redesign an existing standard purlin forty feet long which previously had been developed for a dead load of 15 lbs/ft, a live load of 150 lbs/ft, and wind uplift of 125 lbs/ft. The primary objective in this study was to determine the optimal web depth, however the lip lengths and thicknesses of both chords were floated as well, along with the diameters and thickness of both the interior and end tube diagonals. Manufacturing restrictions favored certain values at this time for the other parameters and those were not floated for this particular study. For example, chord depths and lateral dimensions were held constant, and the lip angle was made identical to the web angle because of stacking requirements. This resulted in an optimization problem in which nine variables were to be floated.

The process required only thirteen cycles to converge to a design, shown in Fig. 3, which weighed 194.7 lbs. (compared to 250.9 lbs. for the existing design).

Extended design calculations (truncated here for brevity) for the generated design are also produced by the program. They show that the stress interaction relationship for the top chord is almost exactly the upper bound limit of unity (Fig. 3). The allowable axial stress is that obtained from the flexural-torsional buckling formula of the AISI specification and determined by positive flexure in the top chord.

It would be seen in a portion of the lengthy calculations that are not shown here that the bottom chord and the diagonals are slightly overdesigned because the thicknesses were optimized at values corresponding to lower limits that had been established because of welding requirements. Thus, it is evident that if this manufacturing limitation could somehow be relaxed, an even lighter design could be obtained which satisfies the AISI specification.

Conclusions

Because of the many parameters quite often necessary to define a cold-formed structural system, overdesign is perhaps inevitable when the result of human effort alone. This comparative study indicates that mathematical optimization appears to have evolved to a state that is difficult to ignore when the development of cold-formed products in a competitive environment is contemplated.

```
GENERATED DESIGN
    TOP CHORD-                  BOTTOM CHORD-               DIAGONALS-
      W1      =  5.500 IN        W1      =  5.500 IN         OD =  1.315 IN
      W2      =  2.500 IN        W2      =  2.500 IN         T  =  0.050 IN
      W3      =  2.000 IN        W3      =  2.000 IN         FY = 55.000 KSI
      LIP     =  0.500 IN        LIP     =  0.500 IN
      LIP ANG= 12.191 DEG.       LIP ANG= 13.558 DEG
      DEPTH   =  1.500 IN        DEPTH   =  1.250 IN (OUT - OUT)
      T       =  0.072 IN        T       =  0.050 IN
      UNBL    =120.000 IN        UNBL    =150.000 IN
      FY      = 55.000 KSI       FY      = 55.000 KSI

      PANEL POINT SPACING          = 30.0 IN
      NO. OF SPACES                =   15
      END DISTANCE                 = 12.5 IN

       TOTAL DEPTH                 = 34.21 IN
       CENTROID-CENTROID DIST.     = 33.07 IN
       WEB DEPTH-INSIDE CHORDS     = 31.46 IN


DESIGN CALCULATIONS
    DESIGN FORCES          DEAD + LIVE                  DEAD + WIND
    MAX PURLIN MOMENT      387.79 K-IN (PNL  9)    -258.53 K-IN (PNL  9)
    MAX AXIAL FORCE         11.73 KIPS                  7.82 KIPS
    NEG MOM TOP CHORD        1.04 K-IN                  0.69 K-IN
    POS MOM TOP CHORD        0.61 K-IN                  0.73 K-IN

    MAX AXIAL FORCE INT. DIAGONAL =  3.06 KIPS (DIAG NO  2)  TENSION
                                  =  4.00 KIPS (DIAG NO  2)  COMP

    MAX AXIAL FORCE END DIAGONAL  =  4.28 KIPS - TENSION
                                  =  2.85 KIPS - COMP

  TOP CHORD-

   SECTION PROPERTIES- FULL SECTION
    EL      LENGTH      Y        LY         LY2
     1      0.581     0.354    0.2058     0.0730
     2      2.161     0.036    0.0782     0.0028
     3      2.199     0.750    1.6492     1.2369
     4      1.581     1.464    2.3125     3.3866
     5      0.607     0.099    0.0600     0.0059
     6      0.608     0.099    0.0601     0.0059
     7      0.608     1.401    0.8513     1.1927
   TOTALS   8.344              5.21[illegible]    5.9039
             TOP)= 0.6254
```

Fig. 3 Generated design for example

```
TOP CHORD-
   REQD LIP I (ELS 1.5) = 0.00062 IN4
   ACTUAL LIP I          = 0.00091 IN4

   ALLOWABLE STRESS- AXIAL
    L**2(SXC)/(D(IYC)) =120.00**2( 19.09)/(34.21(  1.595))=  5026.0
      ALLOWABLE AXIAL STRESS AT CG OF TOP CHORD FROM UNBRACED FLEXURE
       OF PURLIN = 28.931 KSI (3.3(A))
      MAX SLENDERNESS RATIO =  73.8 <= 200
      FC ON LIP =   33.000 KSI (ASSUMED AT CNTR OF EL 1)
      QS =   33.000/33.000 =   1.0000
         THEREFORE Q = QA*QS = 1.0000
      CC/SQRT(Q)=  102.90
         THEREFORE FA1 =  21.323 KSI

      TORSIONAL-FLEXURAL BUCKLING STRESS-

         CW =        23.840        J =  0.0010850
         RO =1.9046478             BETA=   0.8241952
         SIGEX=  53.520            SIGT=9.40939
         SIGTFO=     50.826
         ALLOWABLE STRESS (3.6.1.3) FA2 = .522*Q*FY-FY**2/(7.67*SIGTFO)
                                        = 20.950 KSI

         CHECK SECTION 3.7.1-
           FAO =  28.710 (3.6.1.1)
           KL/R ABT AXIS OF BENDING =  60.345
           NEG MOM D+L COMP FB = MC/I(REDUCED SECTION)=   1.036 *  0.875/0.20715 =  4.374
           POS MOM D+L COMP FB = MC/I(REDUCED SECTION)=  0.610* 0.625/0.20715= 1.841
           FB1 =  33.000 KSI
           FA/FA1 + CM*FB/(1-FA/F'E)(FB)       (POS FLEXURE)
           = 19.413/20.950+0.866* 1.841/(1-19.413/57.879)(33.000)=0.99929
           FA/FAO+FB/FB1  (NEG  FLEXURE) = 19.413/28.710+ 4.374/33.000=0.80871 <= 1

BOTTOM CHORD-

   REQD LIP I (ELS 1.5) = 0.00024 IN4
   ACTUAL LIP I          = 0.00065 IN4

    L**2(SXC)/(D(IYC)) =150.00**2( 28.23)/(34.21(  1.242))=  9564.4
      OF PURLIN = 22.831 KSI (3.3(A))
    ALLOWABLE AXIAL COMP STRESS- DEAD + WIND
     MAX SLENDERNESS RATIO =  85.1 <= 200
     QA =1.0000
     FC ON LIP =    33.000 KSI
     QS =   33.000/33.000 =    1.0000
        THEREFORE Q = QA*QS = 1.0000
     CC/SQRT(Q)=  102.90
        THEREFORE FA1 =  18.873 KSI

     TORSIONAL-FLEXURAL BUCKLING STRESS-

        CW =        1?.972          J =  0.0003401
        RO =        1.9502          BETA=   0.8812253
        SIGEX=  40.170          SIGT= 118.96759
        SIGTFO=      38.04593
        ALLOWABLE STRESS (3.6.1.3) FA2 = .522*Q*FY-FY**2/(7.67*SIGTFO)
                                       = 18.344 KSI

        ACTUAL COMP          (WIND) = 19
        ACTUAL T~            ~AD+LI~
        COM~
```

Fig. 3 (cont.)

Notation

a_{ij}	algebraic form of sensitivity matrix, $\partial g_i(X)/\partial x_j$
a_{ij}^o	sensitivity matrix evaluated at $[X^o]$
C_b	moment gradient bending coefficient
c_j	algebraic form of $\partial F(X)/\partial x_j$
c_j^o	c_j evaluated at a trial design $[X^o]$
C_m	end moment coefficient for interaction
d	total depth of purlin
E	elastic modulus
f_a	axial compression stress in top chord
F_{a0}	allowable compression stress under concentric loading from AISI 3.6.1.1 for L=0
F_{a1}	allowable axial compression stress in top chord
F_{a2}	allowable compression stress under concentric loading from AISI 3.6.1.1
F_{b1}	maximum bending stress in compression
f_b	bending stress in top chord
F_e'	$12\pi^2E/23(KL_b/r_b)^2$ (see AISI spec.)
$F(x_j)$	a nonlinear cost function of the design parameters
F_y	yield stress
$g_i(x_j)$	a set of generally nonlinear constraints which govern the design
g_i^o	g_i evaluated at a trial design $[X^o]$
I_y	I of compression chord about vertical axis
L	unbraced length for lateral buckling of entire purlin
Q	stress and/or area factor to modify allowable axial stress
S_{xc}	section modulus of purlin

t	thickness of lip-stiffened element
u_j	upper limits for x_j
w	flat width of lip-stiffened element
x_j	design parameters
x'_j	parameters obtained by a linear transformation

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SUMMARY

OPTIMIZATION OF LONG SPAN TRUSS PURLINS

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An evaluation of mathematical optimization techniques to the cold-formed steel industry is made by using it to improve the design efficiency of a product already being successfully marketed.