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LOCAL BUCKLING IN CHANNEL COLUMNS

by

John T. DeWolf*

INTRODUCTION

In cold-formed sections the width-thickness ratios of the elements are often large so that local plate buckling occurs prior to attainment of the maximum member strength. The postbuckling strength of these elements is increasingly utilized in design, yielding substantial economics in many cases.

Many thin-walled open shapes loaded compressively are subject to torsional-flexural buckling as well as flexural buckling. Examples are channels, hat sections, angle sections and I-sections with unequal flanges. In these the centroid and the shear center do not coincide so that when the axial load is applied through the centroid, resulting in a uniform compressive stress distribution, buckling may involve twisting as well as translation.

This paper concerns itself with uniformly compressed channel columns subject to local buckling. As the load is increased, failure occurs in one of the following patterns: (1) for low slenderness ratios, local plate buckling occurs followed by the development of postbuckling strength, with failure occurring when the compressive strength of the component plates is reached; (2) for moderate slenderness ratios, local plate buckling occurs followed by the development of postbuckling strength,

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with failure precipitated by either overall flexural column buckling about the axis of non-symmetry or by torsional-flexural buckling with translation perpendicular to the axis of symmetry and twisting; (3) for large slenderness ratios, failure occurs by overall flexural buckling or by torsional-flexural buckling with no prior local plate buckling. Whether flexural buckling or torsional-flexural buckling occurs depends on the relative dimensions of the cross-section.

Flexural buckling and torsional-flexural buckling have been extensively studied, considering both elastic and inelastic behavior. Plate buckling has received much attention, including postbuckling behavior. Though the effect of local plate buckling on flexural buckling has been studied, with a recent investigation by DeWolf, Pekoz, and Winter ⁽⁴⁾ concerning cold-formed sections, the subject of the interaction of local buckling with torsional-flexural buckling has been neglected. For the design of such sections, the American Iron and Steel Institute Specification ⁽⁶⁾ utilizes a method similar to its procedure for predicting the flexural buckling load of locally buckled sections. Winter ⁽⁹⁾ states that the method is conservative because the effects of local buckling, which are interrelated to torsional-flexural buckling, are in effect counted for separately.

This paper presents a method in which the effects of local buckling are directly included in the overall column behavior, yielding a conceptually simple procedure for predicting the maximum column strength. Though the results are presented for channel columns, with and without lips, the approach is general and can be utilized for all thin-walled sections.

ANALYTICAL APPROACH

The differential equations for the elastic behavior of a concentrically compressed column are:

$$EI_y u'''' + P(u'' + y_o \phi'') = 0 \quad (1)$$

$$EI_x v'''' + P(v'' - x_o \phi'') = 0 \quad (2)$$

$$EC_w \phi'''' - (GJ - r_o^2 P) \phi'' - P x_o v'' + P y_o u'' = 0 \quad (3)$$

where u , v , and ϕ represent the two translations and the rotation of the member with respect to its shear center, P is the axial load, E the modulus of elasticity, G the shear modulus, I_x and I_y the moments of inertia, x_o and y_o the distances between the shear center and centroid, r_o the polar radius of gyration, J the torsional constant and C_w the warping constant.

For channels with symmetry about the x axis, y_o is zero. The possible buckling loads are then:

$$P_{cr} = \frac{\pi^2 EI_y}{\lambda^2} \quad (4)$$

$$P_{cr} = \frac{1}{2\beta} \left[(P_\phi + P_x) - \sqrt{(P_\phi + P_x)^2 - 4\beta P_\phi P_x} \right] \quad (5)$$

where λ is the effective column length and:

$$\beta = 1 - \left(\frac{x_o}{r_o}\right)^2 \quad (6)$$

$$P_x = \frac{\pi^2 EI_x}{\lambda^2} \quad (7)$$

$$P_\phi = \frac{1}{r_o^2} \left(GJ + \frac{EC_w \pi^2}{\lambda^2} \right) \quad (8)$$

Eq. 4 is the flexural buckling load about the y axis and Eq. 5 is the lesser of the two torsional-flexural buckling loads. The smaller value from these two equations determines the column strength.

No rigorous theory exists for the determination of the torsional-flexural buckling loads in the inelastic range. Chajes, Fang, and Winter (1) discuss attempts to include inelastic behavior and in an extensive experimental and analytical investigation into the torsional-flexural buckling of cold-formed members, found that replacing E with E_t , the tangent modulus, and G with $G(E_t/E)$ yielded good results. For the determination of the inelastic buckling stress, the authors used the following approximate expression:

$$E_t = CE \left[\frac{\sigma}{\sigma_y} \left(1 - \frac{\sigma}{\sigma_y} \right) \right] \quad (9)$$

where σ is the stress, σ_y the yield stress and C a constant depending on the material. This equation is the basis of the CRC column formula (5) and was used satisfactorily in the previously mentioned work of this author. Chajes, Fang, and Winter used $C = 4.5$ for their material with a proportional limit equal to $2/3\sigma_y$.

The proposed method for considering the effects of local buckling in the above predictions of column buckling is now discussed. Treating the postbuckling behavior of plates in a rigorous theoretical manner is extremely difficult. The complication of large deflections combined with inelastic behavior in the later stages of postbuckling makes a widely applicable solution untractable. Solutions which do exist are limited to specific sections or are limited to the earlier stages of postbuckling.

Fortunately, a much simpler, generally applicable means exists for determining the load on a plate in all stages of its postbuckling range. This is the effective width approach, developed from test data. In various forms it has been used successfully in all types of design. The most widely used form in the United States is that developed by Winter (8) for stiffened elements, flat elements with both edges parallel to the direction of stress stiffened by a web, flange, or stiffener which supplies sufficient rigidity to prevent out-of-plane distortion at the edges. His effective width expression is:

$$\frac{b}{w} = \sqrt{\frac{\sigma_{cr}}{\sigma_{max}}} \left(1.0 - 0.22 \sqrt{\frac{\sigma_{cr}}{\sigma_{max}}} \right) \quad (10)$$

where b is the effective width, w the full width, σ_{max} the maximum or edge stress, and σ_{cr} the classical plate buckling stress given by Bryan's formula:

$$\sigma_{cr} = k \frac{\pi^2 E}{12(1-\mu^2) \left(\frac{w}{t}\right)^2} \quad (11)$$

μ is Poisson's ratio, t the plate thickness, and k a factor which accounts for the actual edge conditions, ranging from 4.00 to 6.97 for stiffened elements. The width of the plate is replaced with an effective, smaller width assuming the plate is loaded with a uniform stress equal to σ_{max} as shown in Fig. 1a. The equivalent load on the buckled plate is $bt\sigma_{max}$. The width at ultimate strength is obtained by replacing σ_{max} with σ_y .

Since the out-of-plane deformations can be excessive, the postbuckling range has not been fully utilized in the design of unstiffened elements, flat elements with one edge stiffened and the other free. However, in

some cases the deformations are not great and use of the postbuckling range can lead to substantial economy.⁽⁴⁾ In such cases Winter's effective width expression can be utilized for unstiffened elements with appropriate changes in k , from 0.425 to 1.277. The effective width is shown in Fig. 2 a.

In addition to using Eq. 10 to determine the load on the plate, DeWolf, Pekoz and Winter have shown it is also possible to use it to obtain the element's contribution towards the member's stiffness. The stiffness of a column, both flexural and torsional, is related to the actual stress distribution, and thus for each element the effective area, equal to bt , should be distributed in a manner similar to the actual stress distribution. While the actual stress distribution is non-linear and unknown, it was shown that a linear distribution is adequate for cold-formed columns subject to flexural buckling. Thus the area should be distributed as shown in Figs. 1 b and 2 b for the stiffened and unstiffened elements, respectively. If the effective area is equal to or greater than one-half of the full area, the effective thickness is to be decreased linearly from the actual thickness at the point of maximum stress, the supported edge or edges, to the appropriately calculated thickness at the point of minimum stress, the center for stiffened elements and the free edge for unstiffened elements. If the effective area is less than one-half of the full area, its thickness should decrease linearly from a value of less than the actual thickness at the supported edge, or edges, to zero at the center for stiffened elements and at the free edge for unstiffened elements. The assumed stress on the effective area is then uniform and equal to the edge stress σ_{max} . The effective sections for plain and lipped channels are shown in Fig. 3.

The lips are assumed fully effective.

The load on the column is then:

$$P = \sigma_{\max} A_{\text{eff}} \quad (12)$$

where A_{eff} is the area of the effective column section. The average stress on the full, unreduced cross-section is then:

$$\sigma_{\text{avg}} = \frac{P}{A} \quad (13)$$

where A is the area of the full, unreduced cross-section.

Thus to include the effects of local buckling in columns subject to flexural buckling, it was shown that it is only necessary to reduce the column section to an effective one using Eq. 10. It is logical that the same procedure can be used for sections subject to torsional-flexural buckling. Once the section is reduced to an effective one, I_x , I_y , J , and C_w are calculated taking into account the variable thickness for the locally buckled elements. The calculation of I_x and I_y is straightforward. J is the summation of $1/3 w t^3$ for sections composed of rectangular elements. For the locally buckled variable thickness elements it is necessary to integrate this over the element length. Chilver ⁽²⁾ has shown the importance of including the effect of variable thickness for calculating C_w in sections containing elements of differing thickness and gives a procedure for doing so. This is also presented by the American Iron and Steel Institute ⁽⁷⁾ and should be used for the effective section.

Once the effective section properties are calculated, the flexural and torsional-flexural buckling loads are obtained using Eqs. 4 and 5. If σ_{\max} is above the proportional limit, the values of P_{cr} from Eqs. 4 and 5 are multiplied by (E_t/E) , where E_t is given by Eq. 9 with σ equal

σ_{\max} .

Thus for a given column length, it is necessary to assume a value of σ_{\max} , calculate the effective section properties using Eq. 10, and then find P_{cr} from Eqs. 4 and 5, modifying if necessary for inelastic behavior. If either of the values of P_{cr} are not equal to $\sigma_{\max} A_{\text{eff}}$, a new estimation of σ_{\max} should be made and the process repeated.

The procedure for determining the flexural buckling load of locally buckled columns is analogous to that proposed by DeWolf, Pekoz and Winter.

EXAMPLE COLUMN CURVES

Two types of steel sections are treated using the proposed method of analysis; plain channels, designated by PC, and channels with lips, designated by LC. A material is assumed which has a yield stress of 45 ksi. and a proportional limit equal to $2/3 \sigma_y$.

For each type, four sets of dimensions are considered, shown in Table 1. Two contain flange width-thickness ratios equal to two-thirds those of the web and two have flange ratios equal to the web ratio. Specimens PC-1, PC-3, LC-1 and LC-3 have width-thickness ratios approximately 15 percent greater than the ratios at which local plate buckling occurs at yield and the remaining specimens have ratios approximately 30 percent greater. The lips are dimensioned according to the AISI Specification and are assumed fully effective at all stresses.

Values for k are chosen according to graphs presented by Chilver (3) and are given in Table 2. Note that in the plain channels k for the web is smaller than 4.00, the minimum value for stiffened elements. Since buckling in the elements occurs simultaneously, with all plates buckling into the same number of half-waves, it is not correct to consider each

element separately. Thus for the relative width-thickness ratios given, the flanges do not sufficiently stiffen the web so that it behaves as a stiffened element. Similarly, k is below 4.00 for the flanges in sections LC-1 and LC-2. The elastic local buckling stresses determined from Eq.11 are also given in Table 2.

Column curves, stress versus slenderness ratio about the y axis where both quantities are based on the full, unreduced cross-section, are given in Figs. 4 and 5 for the plain channels and in Figs. 6 and 7 for the lipped channels. Four curves are given in each. The flexural buckling strength utilizing the effective section is designated as 'Effective Section: Flexural' and the torsional-flexural buckling strength using the effective section is designated as 'Effective Section: Torsional-Flexural'. The predicted stresses at buckling using the AISI Specification are also given. These are designated as 'AISI: Flexural' and 'AISI: Torsional-Flexural'.

For all sections, the flexural and torsional-flexural buckling curves overlap in the region of low slenderness ratios. In this region, column buckling does not occur, and the column strength is governed by the post-buckling strength of the component plates. In the region of moderate slenderness ratios, the stress, and thus column load, at which torsional-flexural buckling occurs are smaller than those at which flexural buckling occurs indicating failure in the former manner.

For the plain channels, the AISI values are considerably below those predicted by the proposed method, as much as 36 percent for section PC-3. This is primarily because the AISI Specification does not utilize the potential postbuckling strength for unstiffened elements. The previously

mentioned work by DeWolf, Pekoz and Winter has shown that it is possible to utilize the postbuckling strength of unstiffened elements with width-thickness ratios up to approximately 30 for steel columns with $\sigma_y = 41.9$ ksi. In addition, the AISI Specification is based on using k equal to 0.50 for unstiffened elements and 4.00 for stiffened elements. For those channels investigated k is much greater than the AISI value for the flanges and considerably smaller for the web. The combined effect is for increased strength over that predicted by the AISI Specification. Another factor that has a small effect in the region of moderate slenderness ratios is the difference in the stress at which the material becomes inelastic, $2/3 \sigma_y$ for the assumed material while the AISI procedure is based on $1/2 \sigma_y$.

For the channels with lips, both approaches yield very similar results. For sections LC-3 and LC-4 the two procedures give maximum stresses for both torsional-flexural buckling and flexural buckling which differ by no more than about 2 percent. For sections LC-1 and LC-2 the AISI predictions are as much as 6 percent greater. This is because the flange k factors are considerably smaller than the assumed AISI value of 4.00, while the web k factor is greater, with the former having a larger effect. Note that it is not correct to assume that the lips are fully effective. However, the reduction in effective area of the lip due to buckling is negligible.

The results clearly demonstrate the effect of using the postbuckling strength for unstiffened elements. The use of more accurate k values for the sections in this study indicates that present practice might be conservative in some cases and unconservative in others, though it appears that this can only be fully substantiated by tests. Additionally, the

proposed method considers directly reduction in stiffness due to local buckling as opposed to the AISI procedure. It is also not limited to the assumption that local buckling only effects columns subjected to compressive loads greater than half the stub column strength. The major disadvantage with the proposed method is of course the increased numerical effort involved.

CONCLUSIONS

A simple, logical analytical approach utilizing previous experimental and analytical results is presented for estimating the torsional-flexural buckling strength of singly symmetrical open thin-walled columns subject to local buckling. The cross-section is reduced when local buckling occurs, resulting in an effective section which can be used to model both the total load on the column and its stiffness, both flexural and torsional. Results are given for channel columns with and without lips. Though presently based on the CRC Column Curve approach, the method can be adapted to any other procedure for predicting torsional-flexural buckling loads.

Though what is presented here seems evident from previous research, experimental verification is thought advisable and would perhaps yield further insight into the torsional-flexural buckling strength of locally buckled columns. In addition, further simplification would be necessary for routine use since the procedure is iterative and the calculation of the warping constant is quite involved.

ACKNOWLEDGEMENT

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APPENDIX I - REFERENCES

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APPENDIX II - NOTATION

- A = full, unreduced area of cross-section
- A_{eff} = sum of effective areas of those elements which are partially effective and full areas of other elements
- b = effective width of flat plate element
- C = material stress-strain constant
- C_w = warping constant
- D = channel depth
- E = modulus of elasticity
- E_t = tangent modulus
- G = modulus of elasticity in shear
- I_x, I_y = moments of inertia with respect to x, y axis
- J = St. Venant torsional constant
- k = edge support coefficient for plate buckling
- k_f, k_w = edge support coefficients for flange, web
- L = length of lip in lipped channels
- P = column axial load
- P_{cr} = flexural or torsional-flexural column buckling load
- r_o = polar radius of gyration about shear center
- T = plate thickness in channel sections
- t = plate thickness
- u, v = displacements in x, y directions
- W = channel flange width
- w = flat width of element exclusive of fillets

- x, y = principal coordinates for channel cross-sections
- x_o, y_o = distances between the shear center and centroid in the principal directions
- β = constant given by Eq. 6
- λ = effective column length
- μ = Poisson's ratio
- σ = stress
- σ_{cr} = plate buckling stress
- σ_{max} = maximum stress in plate element; also uniform stress on effective section
- σ_y = yield stress
- ϕ = rotation of cross-section with respect to shear center

TABLE 1
DIMENSIONS OF SECTIONS

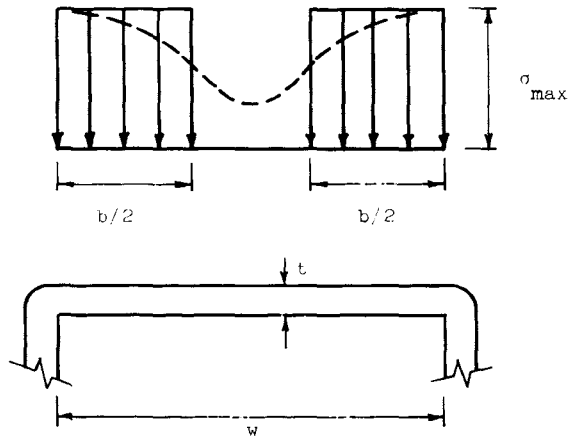
Speciman	W* (in)	D (in)	L (in)	T (in)
Plain Channels				
PC-1	1.55	2.34		0.06
PC-2	1.75	2.64		0.06
PC-3	1.62	1.65		0.06
PC-4	1.82	1.85		0.06
Lipped Channels				
LC-1	2.62	3.91	0.57	0.06
LC-2	2.96	4.41	0.60	0.06
LC-3	3.46	3.46	0.64	0.06
LC-4	3.91	3.91	0.66	0.06

* Shown in Fig. 3.

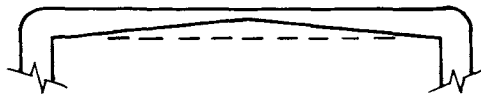
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TABLE 2
 k VALUES AND LOCAL
 BUCKLING STRESS

Speciman	K_{flange}	K_{web}	σ_{cr} (ksi)
Plain Channels			
PC-1	0.83	1.85	35.6
PC-2	0.83	1.85	27.7
PC-3	0.89	0.89	36.3
PC-4	0.89	0.89	28.4
Lipped Channels			
LC-1	2.33	5.25	35.3
LC-2	2.33	5.25	27.4
LC-3	4.10	4.10	35.2
LC-4	4.10	4.10	27.2

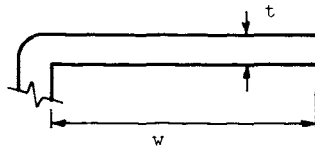
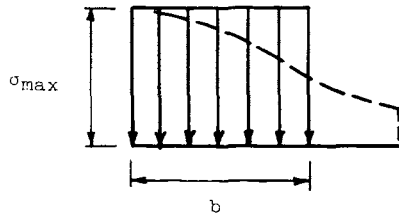


(a) Element with Stress Distribution

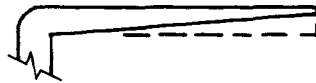


(b) Distribution to Represent Stiffness

FIG. 1 EFFECTIVE WIDTH REPRESENTATION
FOR STIFFENED ELEMENTS

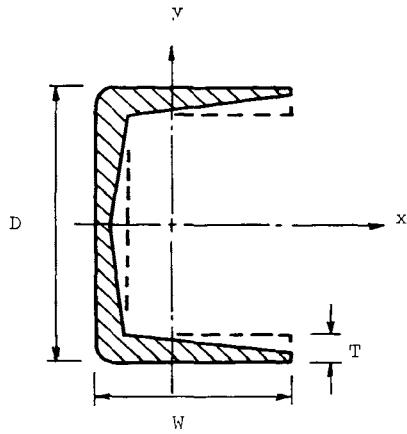


(a) Element with Stress Distribution

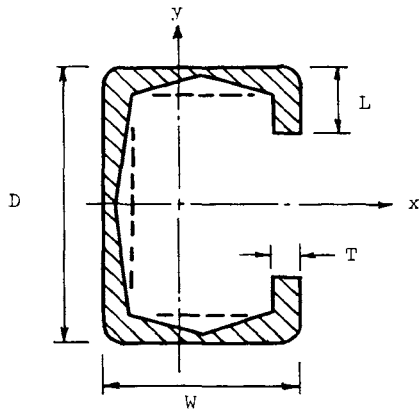


(b) Distribution to Represent Stiffness

FIG. 2 EFFECTIVE WIDTH REPRESENTATION
FOR UNSTIFFENED ELEMENTS



(a) Plain Channel



(b) Lipped Channel

FIG. 3 EFFECTIVE COLUMN SECTIONS

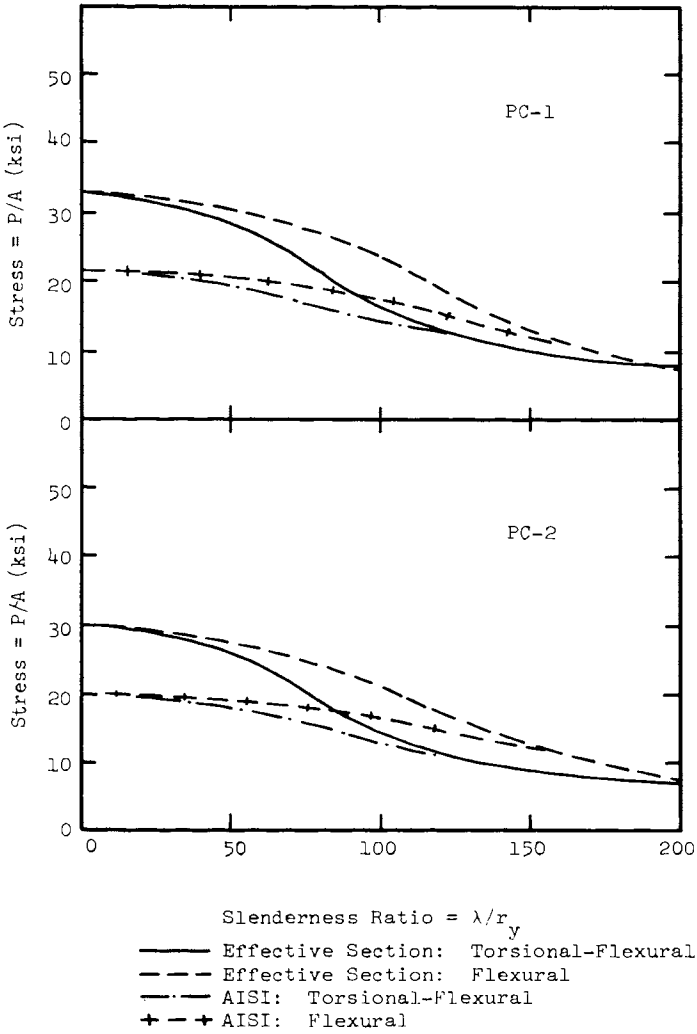


FIG. 4 COLUMN CURVES FOR PC-1
AND PC-2

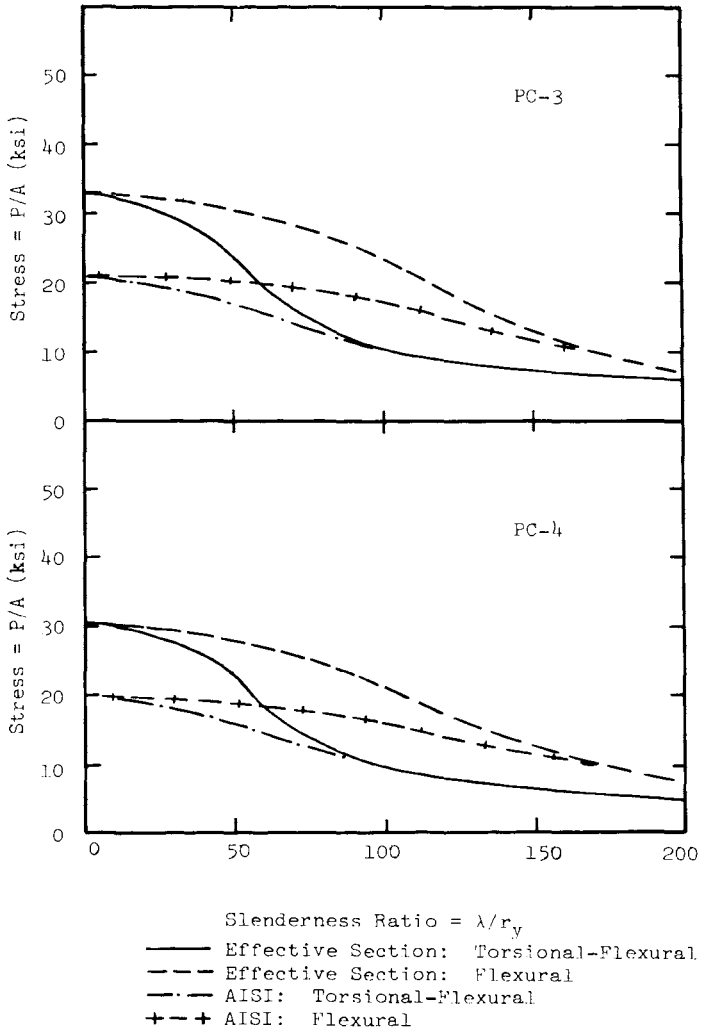


FIG. 5 COLUMN CURVES FOR PC-3 AND

PC-4

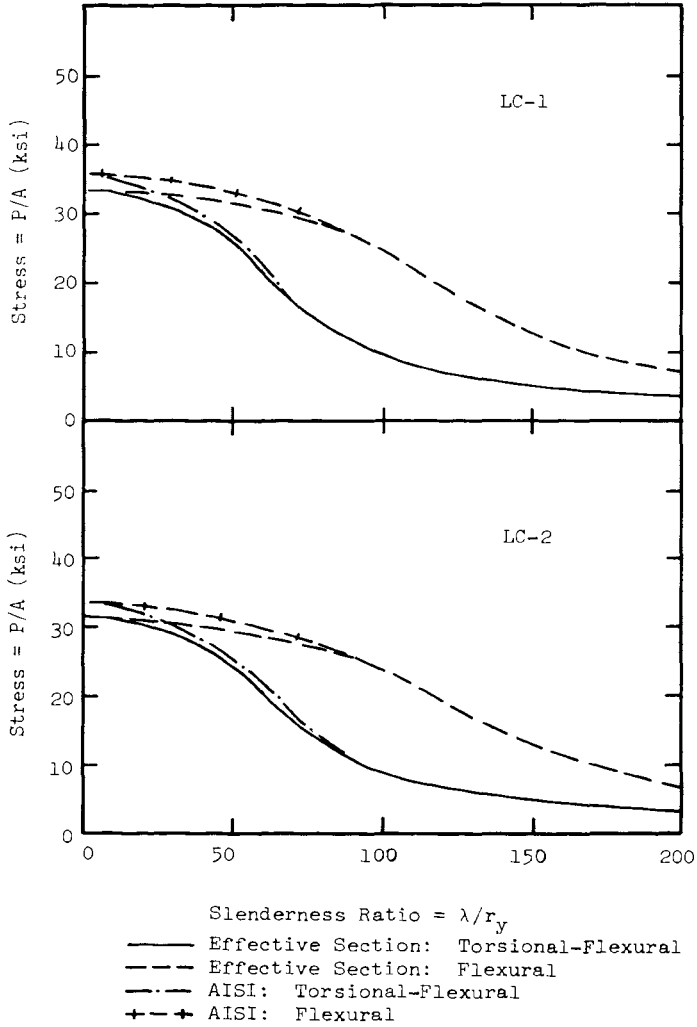


FIG. 6 COLUMN CURVES FOR LC-1
AND LC-2

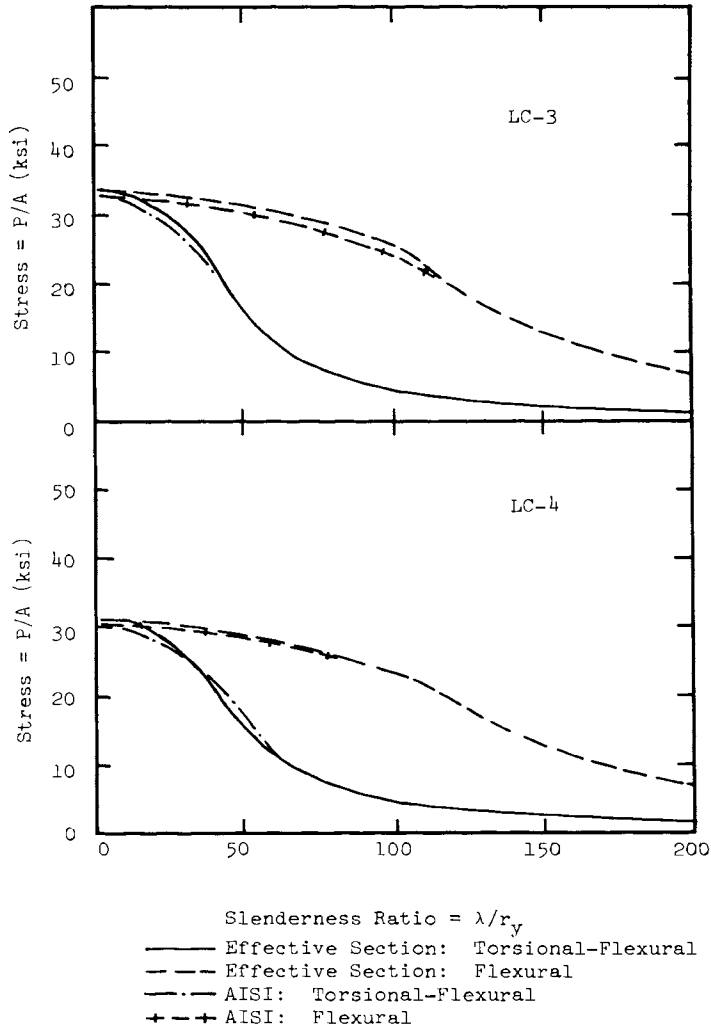


FIG. 7 COLUMN CURVES FOR LC-3

AND LC-4