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SPATIAL BUCKLING BEHAVIOR OF HIGH-RISE RACK FRAMES

Lip H. Teh¹, Gregory J. Hancock² and Murray J. Clarke³

SUMMARY

The paper investigates the global elastic buckling behavior of a high-rise adjustable pallet rack frame composed of thin-walled open sections. It is concluded that the simple element used in many commercial programs for 3D frame analysis is not sufficiently refined for accurate linear buckling analyses of high-rise rack frames.

1 Introduction

There are two principal types of racking systems, i.e. the adjustable pallet racking system and the drive-in racking system. This paper is concerned with adjustable pallet racks only, which may be up to 40 metres (130 feet) high. An example of a single row of a high-rise adjustable pallet rack is shown in Fig. 1. This row of seven bays is termed a single-sided frame in this paper. Only tension braces are shown in Fig. 1 since the flat-plate compression braces are assumed ineffective.

Space, stability and usage considerations often result in two adjustable pallet racks being placed back-to-back, with backties (or rack spacers) connecting the adjacent uprights. Such an arrangement, illustrated in Fig. 2, is called a double-sided frame in the present paper. The backties in a spine tower also serve as anchorage for the down-aisle braces, as depicted in Fig. 2. In a single-sided frame, the backties are simply used to support the braces. The braced bay normally has horizontal braces at the beam levels, and this braced bay is often called a 'spine tower' in a high-rise rack structure. Nomenclature of the components of a spine tower in a double-sided frame is shown in Fig. 3.

The frames shown in Fig. 1 and Fig. 2 represent only those portions of storage racks which are assumed to share a spine tower in the structural analysis of the rack sub-structure. In reality, a series of such frames are placed end-to-end in a continuous fashion in the longitudinal direction to form a racking system up to 200 metres (650 feet) long. (The seven-bay frames depicted in Figs. 1 and 2 are the sub-structures of a racking system that has one spine tower for every eight bays.) Furthermore, for space and operational efficiency, a number of parallel frames are grouped together, with each group typically consisting of four or five double-sided frames on the inside and two single-sided frames on the outside. The space between two double-sided frames or between a double-sided frame and a single-sided frame, termed the aisle, is sized for efficient placement and retrieval of pallet goods. The

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top of the racks are commonly connected to each other by horizontal trusses in order to provide additional frame stability. A cross-aisle view example of such a group is illustrated in Fig. 4.

Steel storage rack structures usually have relatively slender compression members compared with other types of steel frameworks. Furthermore, these structures are often designed in such a way that the ultimate load is relatively close to the elastic buckling load [1, 2]. Thus the assessment of structural stability (member or frame buckling) plays an important role in the design of steel rack structures. In practice, design checks against member buckling are specified in the form of interaction equations which compare the nominal member (axial and moment) capacities with the design member forces determined from either first-order or second-order elastic analyses. In the absence of distortional buckling, the nominal member effective lengths for column buckling about orthogonal axes and twisting.

In recent years, there has been a trend towards the use of rational frame buckling analysis at the expense of the conventional method of determining the effective lengths of a member from its end-restraint stiffnesses and the frame bracing conditions. The use of rational buckling analysis, although not without problems [3], obviates many difficulties involved in the proper application of the conventional method [4-6]. However, due to the size and the topology of a typical steel storage rack structure, full three-dimensional buckling analysis is rarely if ever carried out for the purpose of determining the effective length of the compression members. The steel storage racking standards AS 4084 [7], RMI Specification [8] and FEM 10.2.02 [9] allow the use of independent 2D (planar) buckling analyses in the downaisle and in the cross-aisle directions. Although some simplified procedures may be used to simulate the transfer of horizontal forces among inter-connected parallel frames, and to account for other aspects of 3D behavior in 2D models [10], in general both the elastic buckling loads (down-aisle and cross-aisle) predicted with independent 2D buckling analyses are higher than the 3D buckling load since 3D buckling interaction modes are suppressed in the 2D models.

A major consideration in assessing the stability of rack uprights is the fact that they are usually composed of singly-symmetric open sections and may undergo flexuraltorsional buckling. The orientation of the axis of symmetry is usually in the plane of the upright frames so that flexural-torsional buckling is associated with down-aisle buckling deflections. The flexural-torsional buckling stress of an upright section can therefore be estimated based on the rational flexural buckling stress determined from the 2D down-aisle buckling analysis and the torsional buckling stress, using a flexural-torsional buckling formula specified in Clause 3.4.3 of AS/NZS 4600 [11] and Clause C3.4 of the AISI Specification [12]. However, the torsional buckling stress of a member cannot be determined rationally from 2D buckling analysis. The Australian steel storage racking code AS 4084 [7] and the RMI Specification [8] recommend the use of a torsional effective length factor of 0.8 (referred to the upright length between adjacent truss-bracing points) provided that the connection details between the upright columns and the braces are such that twisting of the upright column is prevented at the brace points. However, the use of a torsional effective length factor of 0.8 is inaccurate and may be unconservative for certain upright columns, even if connection details satisfy the forementioned requirement. Furthermore, the *down-aisle sway* (i.e. flexural) buckling load of a rack frame predicted with 2D buckling analysis may be significantly higher than that predicted with 3D buckling analysis, because the flexibility of the bracing system cannot be captured fully in the planar model, especially for a single-sided frame. It should also be noted that the flexural-torsional buckling formula [11-13] is strictly valid for a column which has the same effective length for flexural and torsional buckling modes, but this is not usually the case with the upright columns of a high-rise pallet rack frame. In order to avoid these shortcomings, 3D buckling analysis becomes necessary to assess the stability of a pallet rack frame.

The objective of this paper is to perform 3D linear buckling analyses of a high-rise adjustable pallet rack frame using beam elements with varying degrees of refinement in order to investigate the buckling behavior and the implications of using beam elements available in commercial structural analysis software packages. The computer program used to analyse the rack models in this paper is largely based on the research results previously reported by the authors [14-17], with some significant enhancements being incorporated into the software to account for section singly-symmetry and thin-wall torsion. Only the double-sided frame depicted in Fig. 2 is considered in this paper.

2 Topology of racks and member properties

The geometry, the section properties and the pallet loads assumed in the present work are representative of high-rise pallet racks used in the paper industry. Each frame is 27 metres (88.6 feet) high, comprising 13 storeys (called 12 levels in storage rack terminology as the bottom storey is not loaded) ranging from 1125 mm (3.7 ft) to 2800 mm (9.2 ft) in depth to accommodate the varying sizes of the pallets. The width of each bay is 2900 mm (9.5 ft) as measured between the column centres, resulting in a total frame length of 20.3 metres (66.6 ft) since each frame consists of seven bays. Each upright frame is 1100 mm (3.6 ft) wide (measured between the column centroids), and the distance between two upright frames in the double-sided frame is 450 mm (1.5 ft). Table 1 lists the beam levels and the backtie elevations.

Each shelf in the second and third storeys carries a pair of 1-tonne (2204 lbs) pallets, while that in the fourth through the eleventh storey carries 1.5-tonne (3307 lbs) pallets, and that in the top two storeys carries 2-tonne (4409 lbs) pallets. Such an arrangement, in which the heavier loads are stored in the higher shelves, is used in practice in order to minimise the beam spacings in the lower sections and hence the "down-aisle" effective length factors of the more heavily loaded members. Table 2 lists the cross-section area A, the second moments of area about the major and the minor axes I_z and I_y , and the torsion constant J of the beams, which are sized in accordance with the pallet loads that have to be carried by the beams. Note that the top beam is not loaded with pallets. The beams are of tubular section and hence resist torsion mainly by St. Venant torsion (no warping torsion).

The backties in the spine tower, to which the 100×6 mm (3.94 $\times0.24$ in) flat-plate down-aisle braces are anchored, are composed of $75\times75\times3$ mm (2.95 $\times2.95\times0.12$ in) square hollow sections. Such backties are normally welded to plates bolted to the upright columns, and in the present study the backtie-to-upright connections are assumed to be rigid. However, the exact anchorage positions of the braces above/below the backties (see Fig. 5) are modelled in the buckling analyses, as evident in Fig. 3. The 112.5-mm (4.43 in) offset induces torsion in the backties remote from the spine tower (rack spacers only) are composed of $50\times50\times3$ mm (1.97 $\times1.97\times0.12$ in) square hollow sections.

As depicted in Figs. 1 and 2, the lower portion of each upright frame is reinforced in the cross-aisle direction with double bracing that extends up to the fifth storey, while the upper portion is single-braced only. The spacing between the doublebracing points in an upright column is 600 mm (2.0 ft). Two singly-symmetric coldformed sections of similar geometry but different thicknesses are commonly used for the upright columns in the lower and the upper portions, respectively. The crosssection properties of the two column sections and the cross-aisle braces assumed in the present study are listed in Table 3- the major z-axes of the column sections (the axes of symmetry as shown in Fig. 6) are in the plane of the upright frames. The braces are assumed to be pinned to the columns about the horizontal axis, so the corresponding second moment of area is listed as zero (the y and z subscripts are the local axes of the respective members, and are not global axes). It is also assumed that there is no torsional warping transmission between the columns and the braces, and that the braces are free to warp under torsion, so the warping constant C_w of the braces is also listed as zero. However, the brace-to-upright joints are assumed to be perfectly tightened as far as rotation about the vertical axis is concerned.

The last column of Table 3 lists the eccentricity of the shear-centre with respect to the centroid of each section. The shear-centres of the upright columns are located outside the upright frame, i.e. away from the cross-aisle braces.

The beam-to-column connections are assumed to be semi-rigid in bending about the horizontal axis as well as about the vertical axis. In general, the connection stiffness is greatly influenced by the wall thickness of the column section. In this paper, it is assumed that the average rotational (bending) stiffnesses of the connection about the horizontal axis in the lower and in the upper columns are 500 kN.m/rad (112.4 kips/rad) and 300 kN.m/rad (67.4 kips/rad), respectively. The corresponding values about the vertical axis are 200 kN.m/rad (45.0 kips/rad) and 120 kN.m/rad (27.0 kips/rad). The length of a beam-to-column connection is taken to be half the web width of the column, which is 60 mm (2.36 in) for the upper as well as the lower columns. As each bay is 2900 mm (9.5 ft) wide, the length of a shelf beam in the rack frames is equal to 2780 mm (9.1 ft). The connection element used in the present work was presented by Li et al. [18], which can model the bending, twisting, axial and shear stiffnesses of a connection. In this paper, the axial, the shear and the twist stiffnesses are assigned the values of 5000 kN/mm (342.6 k/ft), 10⁶ kN/mm (68522 k/ft) and 1000 kN.m/rad (68.5 k/ft), respectively. The axial, the shear and the torsional flexibilities of the beam-to-column connections are therefore effectively ignored in the present analyses owing to the artificially large stiffnesses relative to the sustained forces or torque. However, their effects on the buckling of the rack frames are insignificant compared with those due to the bending flexibility.

As the beam-to-column connections are relatively flexible, for the purpose of linear buckling analysis, it is assumed that the pallet loads are transferred to the columns as vertical loads only, which is indeed justified for the inner columns of the racking system. (As mentioned previously, the seven-bay frames represent the sub-structures of a racking system that has one spine tower for every eight bays.) It is also assumed that the gravity loads act through the column centroids, and are therefore eccentric with respect to the shear centres. In accordance with AS 4084 [7], an initial out-of-plumb of 0.2% in the down-aisle direction is applied to the seven-bay frames assuming no connector looseness. This initial out-of-plumb is simulated through the use of notional horizontal forces equal to 0.2% of the gravity loads. The transfer of horizontal loads between the aisle columns and the back columns is effected mainly through the horizontal braces which connect the pairs of pallet beams with each other in the spine tower (see Fig. 3). The horizontal braces are composed of $50 \times 50 \times 6$ mm ($1.97 \times 1.97 \times 0.24$ in) equal angle sections.

The present work does not simulate the shear flexibility of the upright frames caused by the looseness of the cross-aisle bracing connections, which may significantly reduce the cross-aisle buckling load of the frames. However, in general even the reduced cross-aisle buckling load of a single-sided frame is higher than the flexuraltorsional buckling load of the critical upright column, and in most cases, also higher than the down-aisle buckling load of the corresponding multi-bay frames. Due to the topology of the frames considered in this paper as depicted in Figs. 2 and 4, crossaisle buckling of the frames is effectively circumvented and the issue concerning the frame shear flexibility becomes irrelevant.

It may also be noted that due to the ground bracing in the spine tower, the rigidity of the base-column connections is an insignificant factor as far as the frame buckling loads are concerned. Accordingly, the linear buckling analysis results presented in this paper are equally valid whether the base-column connections are pinned or rigid.

For the purpose of the present work, no load factors (which vary from standard to standard) are used in the linear buckling analyses, and no initial out-of-plumb in the cross-aisle direction is assumed in any model of the frames. The issue concerning the number of (cubic) elements per member used to model a steel frame is largely immaterial to the buckling analyses of high-rise storage rack frames. This is due to two reasons. Firstly, for a high-rise storage rack frame, the flexural effective length extends over more than one storey (column member). Secondly, the truss bracing of an upright frame and the presence of backties (rack spacers) necessitate the use of several elements per storey column.

3 Linear buckling analyses

Four 3D linear buckling analyses are performed, starting from the least accurate model to the most accurate one. Such progressive analyses may give valuable insights into the behavior of the steel rack structure under different conditions, and also illustrate the pitfalls of using simpler elements available in most commercial structural analysis programs.

In the first model, the upright columns are assumed to twist uniformly. This assumption means that torsion in the upright columns is resisted solely by the uniform (St. Venant) torsion, with the torsion constant normally denoted as J. The spatial beam-column element used in this model is a direct extension of the planar element used for 2D frame analysis [19] with the addition of the linear (uniform) torsion stiffness [20], so the shear-centre eccentricity of the cross-section is ignored in this model. In addition, this element is unable to detect torsional buckling of a column [16]. Such an element is used in many commercial structural analysis programs claimed to have a 3D frame analysis capability. The predicted elastic buckling load factor using this simple element is 2.14, and the associated buckling mode is shown in Fig. 7. The low St. Venant torsional rigidity means that torsion of the uprights occurs with motion in opposed directions of the front and back upright frames.

For the buckling analysis of the second model, the simple element used in the first model is augmented with the Wagner effect term [21, 22], rendering it comparable to some beam-column elements proposed in the literature [23]. As with the first model, the buckling of the upright columns is alternate flexural-torsional without down-aisle sway as depicted in Fig. 8. However, the present buckling load factor of 0.45 is considerably lower due to the Wagner effect. Such a phenomenon is in fact possible if the columns are composed of open sections that are weak in torsion. The possibility of the upright frames of a double-sided frame to buckle in opposite directions is mentioned in the European rack design code [9].

The elements used in the first and the second models ignore torsional warping rigidity in the upright columns and therefore produce low buckling load factors. In the third model, a (two-noded) spatial beam-column element with 7 degrees of freedom per node is used to model the upright columns. The seven degrees of freedom at each node comprise three translational degrees of freedom, three rotational degrees of freedom plus a warping degree of freedom. The warping degree of freedom is included to account for the fact that torsion in an upright column is not uniform, and that the cross-section tends to displace longitudinally (warp) under torsion. Such an element is comparable to that presented by Conci & Gattass [24]. In this paper, it is assumed that torsional warping at the column bases is fully restrained, and that torsional warping is continuous throughout the upright columns. Due to torsional warping rigidity, the buckling mode of the frame changes from alternate flexural-torsional to overall sway at a much higher buckling load factor of 2.95. The sway buckling mode is shown in Fig. 9. It should be kept in

mind, however, that the shear-centre eccentricity of the mono-symmetric column sections is ignored in all the first three models.

In the fourth model, a spatial beam-column element that accounts for torsional warping and shear-centre eccentricity of the cross-section is used. This element has been verified against the classical flexural-torsional buckling solutions [22] of columns with singly-symmetric and asymmetric cross-sections which may be subjected to torsional warping, and is comparable to the element presented by Conci [25]. Inclusion of the shear-centre eccentricity of the upright sections reduces the buckling load factor of the double-sided frame from 2.95 to 2.48, and the buckling mode is no longer a sway one as shown in Fig. 10. The flexural-torsional buckling mode in this more accurate model is restricted to the outermost columns, especially those on the end of the frame away from the spine tower from Storey 5 through Storey 7, as evident in Fig. 10. It can be seen that the back columns are torsionally restrained by the rack spacers which connect the adjacent back columns, and are thus less susceptible to flexural-torsional buckling than the aisle columns.

4 Conclusions

This paper has presented a new study regarding the three-dimensional frame buckling behavior of high-rise adjustable pallet racks. The topology, the member sizes and the pallet loads of the model used in the present work are representative of those used in the paper industry.

It was demonstrated that there is a problem in using the beam-column elements available in many commercial structural analysis programs which neglect torsional buckling and the coupling between axial, flexural and torsional deformation modes at the element level. This neglect may result in significant overestimation of the elastic buckling load of a steel frame composed of open sections that are weak in torsion. Conversely, if torsional warping of the upright column sections is ignored in the analysis (as it is in most commercial structural analysis programs) so that torsion in the columns is assumed to be resisted solely by the uniform (St. Venant) torsion, then the predicted linear buckling load of a rack frame may be unrealistically low. For sections with significant warping rigidity, the resistance to non-uniform torsion afforded by warping restraints may increase the buckling load of a double-sided frame by more than five times as the buckling mode changes from alternate flexural-torsional to overall sway. However, it was also shown that the shear-centre eccentricities of the singly-symmetric sections cause the upright columns to buckle flexural-torsionally in a localised region without sway at a lower buckling load. The buckling modes predicted using beam-column elements of varying degrees of refinement are very different from each other.

5 References

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	Beam level (mm)	Backtie elevation (mm)
1	1125	1335
2	2550	2750
3	3975	4175
4	6150	6350
5	8350	8550
6	10500	10750
7	12700	13000
8	14850	15100
9	17100	17500
10	19250	19500
11	21475	21800
12	24200	24400
13	27000	26650

Table 1 Beam levels and backtie elevations

Table 2 Beam sizes

Storey	A (mm ²)	<i>I</i> _z (mm ⁴)	<i>I</i> _y (mm ⁴)	<i>J</i> (mm⁴)	
2-3	550	900,000	235,000	500	
4-10	650	1,500,000	280,000	550	
12-13	700	2,100,000	320,000	600	
Тор	500	600,000	200,000	400	

	A (mm ²)	<i>I</i> _z (mm ⁴)	<i>I</i> _y (mm ⁴)	J (mm ⁴)	<i>C</i> _w (mm ⁶)	zs (mm)
Lower column	800	1,300,000	700,000	1,500	2.5×10 ⁹	85.0
Upper column	600	1,000,000	500,000	700	2.0×10 ⁹	90.0
Channel brace	130	0	10,000	100	0	20.0

Table 3 Column and cross-aisle brace sizes

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Fig. 1 Single-sided frame



Fig. 2 Double-sided frame



Fig. 3 Components of a spine tower in a double-sided frame

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Fig. 4 Cross-aisle view of a group of four double-sided frames and two single-sided frames



Fig. 5 Backtie details, elevation



Fig. 6 Mono-symmetric column section



Fig. 7 Buckling of double-sided frame with simple elements (Model 1)



Fig. 8 Buckling of double-sided frame due to the Wagner effect (Model 2)



Fig. 9 Buckling of double-sided frame with warping torsion rigidity included (Model 3)



Fig. 10 Buckling of double-sided frame due to shear-centre eccentricity (Model 4)

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