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NON-UNIFORMLY COMPRESSED STIFFENED ELEMENTS

by

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INTRODUCTION

The local buckling of non-uniformly compressed stiffened elements in light-gage cold-formed steel members does not necessarily lead to immediate failure, but instead considerable postbuckling strength may be available depending upon the slenderness of the element and the stress gradient. This paper presents a procedure for calculating the buckling and postbuckling strength of such elements.

Considerable attention has been devoted to study the local and postbuckling behaviour of uniformly compressed stiffened and unstiffened elements which are frequently encountered in practice (5,12) and the current design codes already have presented effective width design procedures accounting for the postbuckling strength (1).

On the other hand no widely accepted design procedure is available for the thin elements subjected to non-uniform compression which is the state of stress in beam webs and eccentrically compressed column webs. Recently a few research publications have appeared, dealing with post-buckling effective width of non-uniformly compressed stiffened elements (3,4,6,10). Most of these equations have been derived empirically and have been compared with test results in which the stress gradient is either only compressive or only uniform bending but not both cases.

An effective width equation is proposed in this paper which is similar in format to that given by Winter for uniformly compressed stiffened elements, but is applicable for the entire range of stress

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gradient from uniform compression to uniform bending. The effective width equation is compared with results of non-uniformly compressed stiffened element webs in beams and eccentrically loaded columns. The proposed equation is also compared with the other equations available in the literature for limited range of compressive stress gradient. It is shown that the proposed equation satisfactorily models the post-buckling behaviour of stiffened elements over the entire range of compressive stress gradient.

EXPERIMENTAL PROGRAM

Four beams with two point load causing uniform moment region and four columns under eccentric compression were tested (7) in order to obtain stress gradient in the range of uniform compression to uniform bending. The cross section of the beam and column specimen are shown in Fig.1. The dimension of the specimen are given in Table 1. The specimens were designed to have the stiffened webs undergo elastic local buckling whereas flanges do not experience local buckling before yield stress is reached. The beams having 2400 mm simple span were subjected to two equal loads at 800 mm from the supports.

The schematic sketch of eccentric compression test frame is shown in Fig.2. The mean yield strength of the hot rolled steel sheets from which the specimens were fabricated was $220.3\ N/mm^2$ and an ultimate strength of $323.1\ N/mm^2$. The specimens were instrumented with dial gauges and strain gauges in order to determine the stress at the web flange junction at each loading stage and local buckling stress of the stiffened web, which are necessary for comparison with analytical method proposed. The stress gradient, α, ($\alpha = 1 - f_{min}/f_{max}$) and ultimate strength of all these tests are also given in Table 1. The stress at which the webs buckle locally were determined by the stress deviation method (5). The effective cross section of the specimen in the postbuckling range has been assumed to be as shown in Fig.3. The effective depth of the web in the postbuckling range (y_{1e}, y_{2e}) may be determined from experimental results using moment and axial force equilibrium equations. Since the equilibrium equations are non-linear in terms of the unknown values of effective depths

(y_{1e}, y_{2e}), Sequential Unconstrained Minimisation Techniques (SUMT) has been used, in which the RMS error between internal stress resultant and external forces/moments has been minimised to obtain the experimental values of effective depths at various load stages (7). These experimental values have been used for comparison with analytical method proposed later in this paper.

ANALYTICAL EQUATIONS

Local Buckling Stress: The elastic local buckling stress, f_{cr}, of a plate element may be calculated from the following equation

$$f_{cr} = \frac{k\pi^2 E}{12(1-\mu^2)(h/t)^2} \tag{1}$$

where k = local buckling coefficient, E is Young's modulus, μ = Poisson's ratio, h/t = flat height to thickness ratio of the element.

DIN 4114 (3) suggests the following equation for the local buckling coefficient of non-uniformly compressed stiffened element having simply supported longitudinal edges.

$$k = \frac{8.4}{2.1-\alpha} \quad \text{for } 0 \leqslant \alpha \leqslant 1.0 \qquad 2(a)$$

$$k = 10\,\alpha^2 - 13.736\,\alpha + 11.372 \text{ for } 1.0 < \alpha < 2.0 \qquad 2(b)$$

where $\alpha = 1 - f_{min}/f_{max}$; f_{min} = minimum edge stress

(positive if compressive); f_{max} = maximum edge compressive stress.

Yu (13) has presented the following equation for the local buckling coefficient of non-uniformly compressed simply supported plate elements.

$$k = 4 + 2\alpha + 2\alpha^3 \tag{3}$$

Through a regression analysis the following equations have been derived for the local buckling coefficient of nonuniformly compressed stiffened elements with pinned and rigidly fixed longitudinal boundaries (7).

$$k_{pinned} = 4 + 3.314\alpha - 2.344\alpha^2 + 2.83\alpha^3 \tag{4a}$$

$$k_{fixed} = 6.97 + 2.702\alpha + 0.968\alpha^2 + 2.92\alpha^3 \tag{4b}$$

All these equations compare reasonably well with theoretical values (9) the maximum difference being less than 3%.

Postbuckling Effective Depth: Dewolf (2) has presented an empirical equation for the effective depth at failure of the web subjected to non-uniform compression (Fig.4), which may be written as follows:

$$\frac{y_{1e}}{h} = 0.7\sqrt{f_{cr}/f_y}/\alpha$$

$$\frac{y_{2e}}{h} = (\alpha - 1)/\alpha$$

where y_{1e} = effective depth of web in compression zone; y_{2e} = effective depth of web in tension zone; h = flat depth of the web plate and f_{cr}, f_y are local buckling and yield stress (N/mm^2), respectively. This equation has been compared with tests in which stiffened elements experience bending stress over the depth (compressive and tensile stress).

Laboube and Yu (6) have presented equations for the effective depth of beam webs at ultimate load which were derived by a regression analysis of large number of test results. These equations may be written in the following form

$$\frac{y_{1e}}{h} = 0.376\sqrt{f_{cr}/\gamma_1\gamma_2 f_y} \leqslant 1.0 \tag{5a}$$

$$\frac{y_{2e}}{h} = (\alpha - 1)/\alpha \qquad \geqslant 0 \tag{5b}$$

where

(a) for webs with adjacent stiffened flanges:

$$\gamma_1 = 1.037 - 0.000476\,(h/t)\sqrt{f_y} \tag{5c}$$

when $(w/t)_{lim} \leqslant (w/t) \leqslant 2(w/t)_{lim}$

$$\gamma_2 = 1.074 - 0.0735 \frac{(w/t)}{(w/t)_{lim}} \tag{5d}$$

when $(w/t) \leqslant (w/t)_{lim}$

$$\gamma_2 = 1.0 \tag{5e}$$

$$(w/t)_{lim} = \sqrt{449 / f_y} \tag{5f}$$

(b) for webs with adjacent unstiffened flanges

$$\gamma_1 = 0.800 \tag{5g}$$

when $(w/t) \geqslant (w/t)_{lim}$

$$\gamma_2 = 1.024 - 0.024 \frac{(w/t)}{(w/t)_{lim}} \tag{5h}$$

when $(w/t) \leqslant (w/t)_{lim}$

$$\gamma_2 = 1.0 \tag{5j}$$

$$(w/t)_{lim} = \sqrt{166 / f_y} \tag{5k}$$

Effective depth equation, Eq. 5 has also been calibrated only with beam web test where the stress gradient is large enough to cause both compressive and tensile stresses within the stiffened web ($\alpha > 1.0$).

Usami (10) has presented the following semi-empirical equation for the postbuckling behaviour of non-uniformly compressed stiffened elements:

$$\frac{f_{av}}{f_{max}} = (1 - 0.5\alpha) \sqrt{\frac{f_{cr}}{f_{max}}} \left((1+0.1\alpha) - (0.22 + 0.05\alpha) \sqrt{\frac{f_{cr}}{f_{max}}}\right) \tag{6}$$

Usami has also proposed the following effective depth equations for the postbuckling range of non-uniformly compressed elements.

$$\frac{y_{1e}}{h} = 0.5 \sqrt{\frac{f_{cro}}{f_{max}}} \left(1 - 0.22 \sqrt{\frac{f_{cro}}{f_{max}}}\right) \tag{7a}$$

$$\frac{y_{2e}}{h} = \frac{y_{1e}}{h}(1 + 0.42\ \alpha) \qquad \text{for } \alpha \leqslant 1.0 \tag{7b}$$

$$\frac{y_{2e}}{h} = 1.42\ \frac{y_{1e}}{h} + \frac{\alpha - 1}{\alpha} \qquad \text{for } \alpha \geqslant 1.0 \tag{7c}$$

where y_{1e}, y_{2e} = effective widths in compression and tension zone (Fig.3); f_{cro} = local buckling stress under uniform compression (α=0). These equations have been empirically derived and have been compared with eccentrically loaded rectangular box section test results. It is not very clear as to how the effective depth equation (Eq.7) is derived from the postbuckling behaviour equation (Eq.6).

ECCS Committee recommendation (4) for the postbuckling effective depth may be written in the following form.

for α < 1.0

$$\frac{y_{1e}}{h} = \frac{1}{(2+0.5\alpha)\ \sqrt{f_{cr}/f_y}}\ (1-0.22\ \sqrt{f_{cr}/f_y}) \tag{8a}$$

$$\frac{y_{2e}}{h} = (1 + 0.5\alpha)\ \frac{y_{1e}}{h} \tag{8b}$$

for α > 1.0

$$\frac{y_{1e}}{h} = \frac{0.4}{\alpha}\ \sqrt{f_{cr}/f_y}\ (1 - 0.22\ \sqrt{f_{cr}/f_y}) \tag{8c}$$

$$\frac{y_{2e}}{h} = 1.5\ \frac{y_{1e}}{h} + \frac{\alpha - 1}{\alpha} \tag{8d}$$

The value of f_{av}/f_{max} calculated from experimental values discussed earlier when compared with Eq.6 indicated good correlation validating equation for the postbuckling range of both beam and eccentrically loaded column webs. Consequently the effective depth equation for non-uniformly compressed webs can be analytically derived using Eq.6.

The effective depths (y_{1e}, y_{2e}), were derived for various values of maximum edge stress, f_{max}/f_{cr} by equating the internal stress resultants to the corresponding external forces and moments obtained from Eq.6 (7).

These equilibrium equations, which are non-linear in terms of the unknown effective depths, were solved using SUMT. On the basis of this exercise, the following effective width equations for non-uniformly compressed stiffened elements have been derived through regression analysis (7).

$$\frac{y_{1e} + y_{2e}}{h} = \sqrt{f_{cr}/f_{max}}/(C_o - C_1\sqrt{f_{cr}/f_{max}}) \tag{9a}$$

The C_o and C_1 are functions of stress gradient factor, α, as shown in Fig.5. It can be seen that there is a sudden change in the values of the two variables, C_o, C_1, at $\alpha = 1.5$. This is due to the fact that the non-linear equations of equilibrium, from which they are derived, have two real solutions at $\alpha = 1.5$, both satisfying the equilibrium equations. The y_{1e} and y_{2e} are given by the following equations (7).

$$\frac{y_{2e}}{h} = \frac{(y_{1e} + y_{2e})/h}{(2.0 - 0.65\alpha)} \qquad \text{for } \alpha < 1.5 \tag{9b}$$

$$\frac{y_{2e}}{h} = 0.58\ (\alpha - 1)\ \frac{(y_{1e} + y_{2e})}{h} \qquad \text{for } \alpha \geqslant 1.5 \tag{9c}$$

$$\frac{y_{1e}}{h} = \frac{y_{1e} + y_{2e}}{h} - \frac{y_{2e}}{h} \tag{9d}$$

In Fig.6 the equations for ratio of depth of compression zone y_{1e} to total depth for non-uniformly compressed stiffened elements, suggested by different authors (Eq.4,5,7,8) are compared with the effective depth equation proposed in this paper (Eq.9) for various values of α and f_{max}/f_{cr}. Eqs. 4 and 5 presented in refs.2 and 6 respectively are valid only for elements with bending stress (α>1.0) and hence the corresponding curves appear only in such cases. For the case of uniform compression ($\alpha = 0$) Eqs.7, 8 and 9 give essentially the same value (Winter's effective width value) and hence only one curve is drawn for $\alpha = 0$.

It can be seen that there is considerable variation between the values of effective depth, y_{1e}/h, given by the different equations. Equations from refs. 2 and 6 (Eqs.4 and 5) generally give a higher value, whereas refs.4 and 10 (Eqs.7 and 8) generally give a lower value. The proposed equation (Eq.9) is in between the value, being closer to that given by Eqs.4 and 5 at lower stresses and closer to that given by Eqs. 7 and 8 at higher stresses.

In. Fig.7 the effective depth to total depth ratio $(\frac{y_{1e} + y_{2e}}{h})$ for various values of α and f_{max}/f_{cr} as given by different equations are compared. The test values are also plotted in the same figure. It can be seen that the proposed effective depth equation compares best with test results for over a range of α values.

The failure moments or axial loads calculated on the basis of effective depth equation proposed in this paper are compared with test values in Table 2. It is seen that the comparison is generally good except for exceptional case. The mean error is 6 percent and the standard deviation is 10 percent.

DESIGN PROCEDURE

Step 1: Assume a value of α which may be based on the full cross section of beams or full cross section and eccentricity of load in columns.

Step 2: Calculate the buckling stress from Eqs.4 and hence f_{cr}/f_y.

Step 3: Calculate the effective depth of webs from Eqs.9.

Step 4: Calculate the neutral axis location of the resulting effective section and from this the new value of α, α_{new}.

Step 5: Repeat steps 1 through 4 until assumed α converges to α_{new}.

Step 6: Calculate the strength of effective section.

SUMMARY AND CONCLUSIONS

An effective depth equation for non-uniformly compressed stiffened plates has been derived on the basis of regression analysis, which is suitable for the entire possible range of stress gradient (α). This equation has been compared with other equations available in the literature and with tests conducted using beams and beam columns by the authors and the other tests reported in the literature. It is seen that the strength predicted by the proposed effective depth method compares well with tests.

APPENDIX I - REFERENCES

1. "AISI - Specification for Design of Cold-Formed Steel Structural Members", American Iron and Steel Institute, Washington, D.C., 1980.

2. Dewolf, J.T. and Gladding, C., "Web Buckling in Beams", Proceedings of Fourth International Conference on Research and Developments in Cold-Formed Steel Structures, University of Missouri, Rolla, MO, June 1-2, 1978, pp 191-218.

3. DIN 414, Blatt 2, "Stahlbau, Stabilitatsfalle (Knickung, Kippang, Beulung)", Berechnungsgrundlagen, Richtlinien, Berlin, Germany, Feb. 1953.

4. ECCS Committee TC7, "European Recommendations for the Design of Profiled Sheeting", Working Group TWG 7.1, Apr. 1983.

5. Kalyanaraman, V. Pekoz , T. and Winter, G., "Unstiffened Compression Elements" Journal of Structural Division, ASCE, Vol.103, No.ST9, Sep. 1977, pp 1982-1848.

6. Laboube, R.A. and Yu Wei-wen, "Bending Strength of Cold-Formed Steel Beams", Journal of Structural Division, ASCE, Vol.108, No.ST7, July 1982, pp 1589-1604.

7. Ramakrishna, P., "Local and Post Buckling Strength of Nonuniformly Compressed Stiffened Elements", M.S. Thesis, Dept. of Civil Engineering, IIT, Madras, 1983.

8. Rhodes, J. and Harvey, J.M., "Effect of Eccentricity of Load or Compression on Buckling and Postbuckling Behaviour of Plates", International Journal of Mechanical Science, Vol.13, No.10, Oct. 1971, pp 867-879.

9. Tomoshenko, S.P. and Gere, J.M., "Theory of Elastic Stability", McGraw Hill Book Co. Inc., N.Y., 1961.

10. Usami, J., "Postbuckling of Plates in Compression and Bending", Journal of Structural Division, ASCE, Vol.108, No.ST3, Mar. 1982, pp 591-609.

11. Walker, A.C., "Flat Rectangular Plates subjected to Linearly Varying Edge Compression Loading", Thin Walled Structures, eds. A.H. Chiluer, Chatto and Windus, London, 1967, pp 208-247.

12. Winter, G., "Cold-Formed Light Gage Steel Construction", Journal of Structural Division, ASCE, Vol.85, No.ST9, Nov. 1959.

13. Yu, W.W., "Cold-Formed Steel Structures", McGraw-Hill Book Co., Inc., New York, N.Y. 1973.

APPENDIX II - NOTATIONS

f_{av} ; average compressive stress;

f_{cr} ; local buckling stress;

f_{cro} ; local buckling stress under uniform compression;

f_{max} ; maximum edge stress in web;

f_y ; yield strength of the material (N/mm^2);

h ; flat depth of web;

k ; local buckling coefficient;

t ; thickness of element;

w/t ; flat width to thickness ratio of flange elements;

y_{1e} ; effective depth of web adjacent to maximum compression edge;

y_{2e} ; effective depth of web adjacent to minimum compression or tension edge;

α ; Compressive stress gradient factor.

TABLE 1 : SPECIMEN DIMENSIONS* AND TEST RESULTS

Specimen	H in mm	B in mm	T_1 in mm	T_2 in mm	D in mm	α	Experimental Ultimate Load in kN	Experimental Ultimate Moment in kN-m
COLUMNS								
C1	150.0	52.0	1.00	-	-	0.00	58.9	-
C2	150.0	52.0	1.00	-	-	0.16	63.8	-
C3	210.0	52.0	1.00	-	-	0.77	52.2	-
C4	210.0	52.0	1.00	-	-	0.80	44.5	-
BEAMS								
B1	219.0	44.0	0.90	0.0	19.5	2.00	-	4.32
B2	216.0	42.5	0.95	0.95	21.4	1.92	-	5.10
B3	220.1	44.0	0.99	1.99	19.7	1.76	-	5.15
B4	216.0	43.5	0.95	2.99	21.4	1.48	-	4.59

*See Fig.1

TABLE 2 : COMPARISON OF TEST WITH STRENGTH BASED ON AUTHORS' EFFECTIVE WIDTH

(a) Columns

Specimen	Fully effective section P_y in kN	P_u, theory in kN	P_u, expt. in kN	P_u, theory / P_u, expt.
C1	111.0	69.0	58.9	1.17
C2	102.2	64.8	63.8	1.02
C3	84.6	46.7	52.2	0.89
C4	82.5	45.6	44.5	1.03

(b) Beams

Specimen	Fully effective section M_y in N-m	M_u, theory in N-m	M_u, test in N-m	M_u theory / M_u test
Author's tests:				
B1	5277	4288	4316	0.99
B2	7408	5313	5100	1.04
B3	8662	6361	5150	1.22
B4	8598	5788	4591	1.26
DeWolf's Tests:				
A1	30942	26127	24408	1.07
A2	47223	35164	34578	1.02
A3	66452	44049	44183	1.00
A4	88628	52874	52432	1.01
B1	23309	20331	16950	1.20
B2	45106	30922	32431	0.95
B3	72783	41545	40680	1.02
B4	107038	47497	43957	1.08

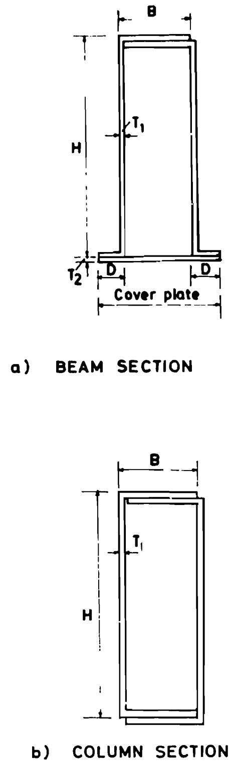
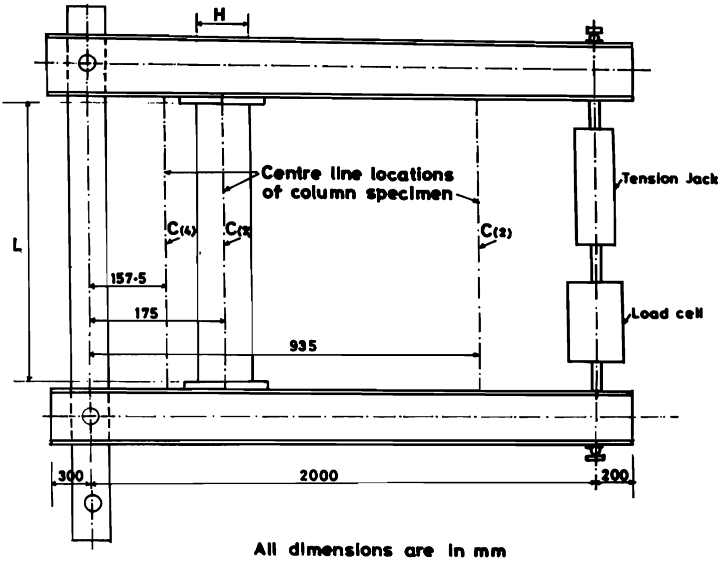


Fig.1. Test Specimen Sections



All dimensions are in mm

Fig. 2. Non-Uniform Compression Test Frame

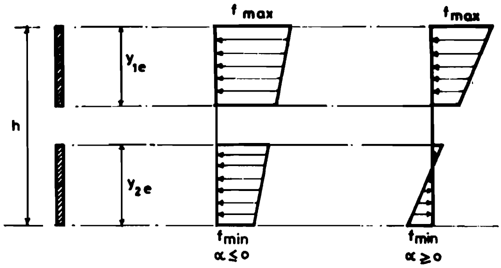


Fig.3. Effective Depth Under Non-Uniform Compression (ref.10)

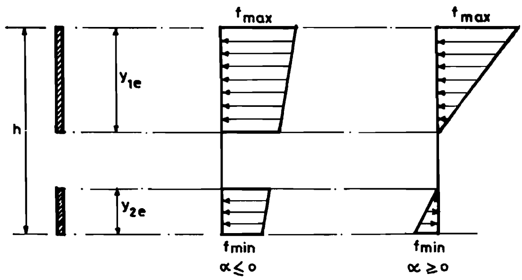


Fig.4. Effective Depth Under Non-Uniform Compression (ref.2)

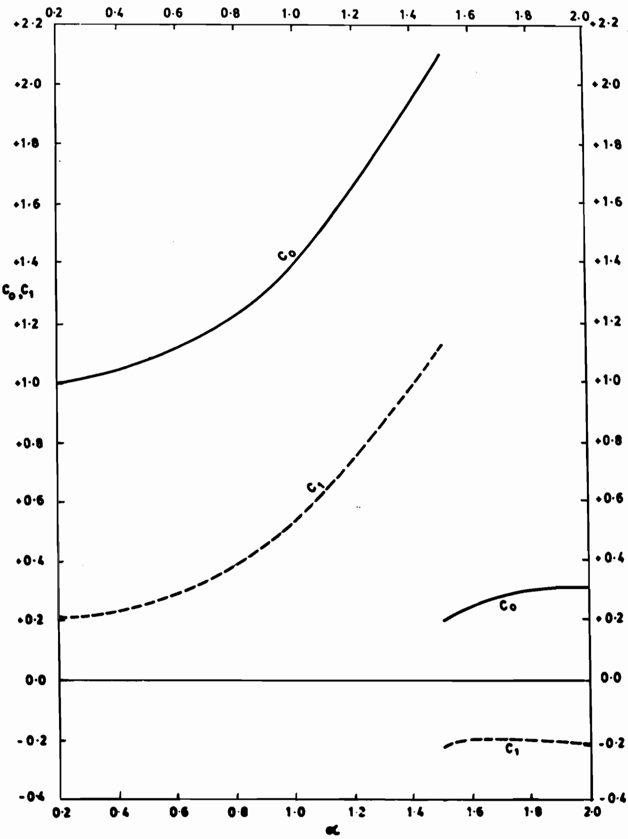


Fig.5 Coefficents C_o, C_1 in Effective Depth Equation (Eq-9)

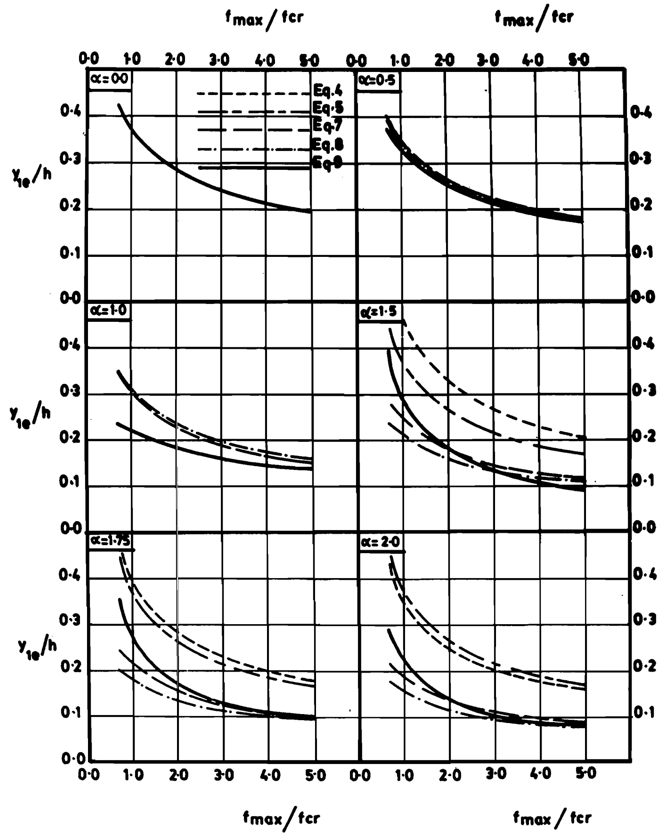


Fig.6 Comparison of Effective Depth Equations

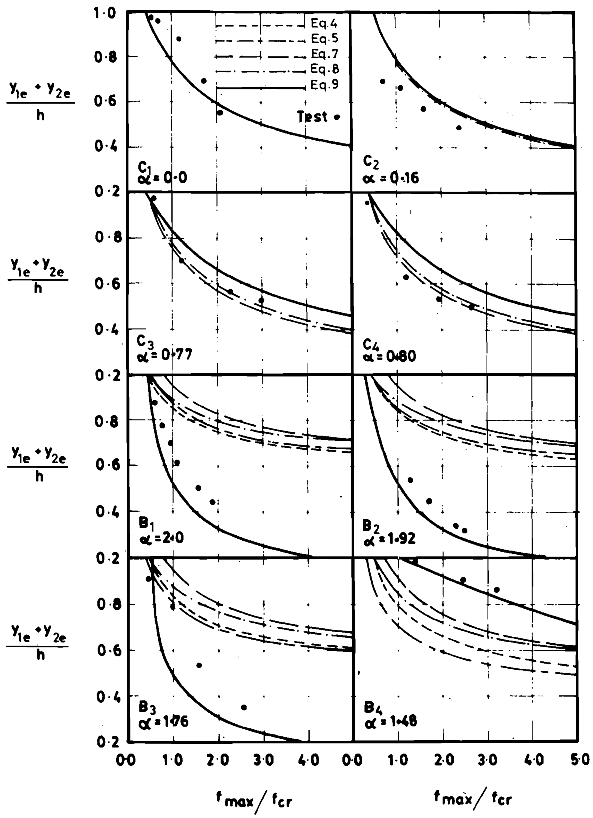


Fig.7 Comparison of Effective Depth Equations with Tests