International Specialty Conference on Cold-Formed Steel Structures

(1971) - 1st International Specialty Conference on Cold-Formed Steel Structures

Aug 20th, 12:00 AM

Orthotropic Folded Plate Structures by Extended Finite Strip Method

Ghulam Husain Siddiqi

C. V. Girija Vallabhan

ORTHOTROPIC FOLDED PLATE STRUCTURES BY EXTENDED FINITE STRIP METHOD

by

Ghulam Husain Siddiqi[1] and C. V. Girija Vallabhan[2]

Introduction

Steel folded plate structures due to structural fabrication are mostly orthotropic in material properties and are supported on flexible end beams and integrally built-in columns. A survey of literature on the steel folded plate structures (1,2,3,4) reveals hardly if any detailed methodological approach for analysis of such structures. Even though the finite element and other similar techniques can be used for the analysis of prismatic folded plates (5,6,7), the extended finite strip method has advantages over these methods in having smaller number of unknowns and requiring smaller number of input data on electronic computer for obtaining comparatively accurate end results. In addition, the method can consider flexible end beams in lieu of conventional rigid end diaphragms and integrally built-in columns.

The method presented here is developed by extending the finite strip method (6) to incorporate displacement of a folded plate structure along its transverse edges in addition to its deformation considered by the finite strip method. Thus, prismatic plate and shell structures that can be analyzed may have following features:

a) the transverse edges subjected to displacement in addition to the longitudinal edges,

b) the transverse edges supported on flexible end beams or ribs,

c) the whole structure built-in integrally with columns, or with other types of supports,

d) isotropic or orthotropic material and sectional properties, varying from strip to strip,

e) a detail, like north light window provided within the structure, and

f) a cantilevered canopy built-in within the structure.

The method has still many other potentialities (8). However, in this paper, the loads and the structure are assumed to be symmetric with respect to the center line of the structure.

The method uses the division of the structure into rectangular strips along the longitudinal axis. The displacement pattern of these strips is modelled by assumed suitable functions given later in the paper, and the stiffness matrix equation of the structure is developed by using the Ritz Method (9). A study of the convergence criteria of the displacement and stress functions reveals convergence to stable values.

A folded plate steel structure made with orthotropic properties, supported on elastic end beams, and supported by integrally built-in columns is analyzed and the results of the analysis are presented.

EXTENDED FINITE STRIP METHOD

The Extended Finite Strip Method requires division of a prismatic plate structure into a number of strip elements. Fig. 1 (a) shows a flat plate structure divided into strips, the variation of section and material properties from strip to strip and the various types of loads that can be applied. The length of the structure and a strip in y-direction is denoted by a, and the width and the thickness of a strip by b and t respectively. The bounding lines of a strip are called 'nodal lines'. Fig. 1(b) depicts a folded plate structure in naturally orthotropic properties carried on integrally built-in columns, supported by flexible end beams in lieu of rigid end diaphragms, with a canopy extension on one side, and having a north light window feature. Fig. 2 shows the elemental and global frames of reference and u, v and w displacements along these coordinates.

A strip in a folded plate action is subjected to both membrane and bending actions. For linear elastic materials and small deflections, these two actions are independent and, therefore, considered separately. A displaced shape of a strip in either action can always be split up into two parts: the symmetrically displaced shape and the antisymmetrically displaced shape. For the sake of brevity symmetrically displaced shape in membrane and bending actions are considered here in this paper.

Membrane Action

Figure 3 shows the symmetrically displaced shape of membrane action. The displacement functions u and v are individually expressed as a sum of products of a polynomial and a series of base functions as

$$u = \sum_n p_{1n}(x)\ \Psi_n(y)$$
$$v = \sum_n p_{2n}(x)\ \Omega_n(y), \text{ where } n = -1,1,3,5,.. \qquad (1)$$

where $p_{in}(x)$, $i = 1, 2$ are polynomials which describe the variation of the displacement in the x-direction and are expressed in terms of the amplitudes of u and v displacements along nodal lines or the so-called undetermined parameters

$$p_{1n}(x) = (1 - \frac{x}{b})\ u_{ni} + \frac{x}{b}\ u_{nj}$$
$$\text{and} \quad p_{2n}(x) = (1 - \frac{x}{b})\ v_{ni} + \frac{x}{b}\ v_{nj}\ , \qquad (2)$$

$\Psi_n(y)$ and $\Omega_n(y)$ are linearly independent but nonorthogonal base functions which span the domain in y-direction and a priori satisfy the boundary conditions along x-edge. The base functions $\Psi_n(y)$, $n = -1,1,3,5,..,$ are 1, $\sin\frac{\pi y}{a}$, $\sin\frac{3\pi y}{a}$, $\sin\frac{5\pi y}{a}$, . . . while the base functions $\Omega_n(y)$, $n = -1,1,3,5,..$ are $1 - \frac{2y}{a}$, $\cos\frac{\pi y}{a}$, $\cos\frac{3\pi y}{a}$, $\cos\frac{5\pi y}{a}$, . . . It may be noted that the displacements u and v at nodal line contributed by base function $n = -1$ are displacements without deformation.

Bending Action

Figure 4 shows the symmetrically displaced shape of bending action. The displacement function w is expressed as product of a polynomial and the series of base function as

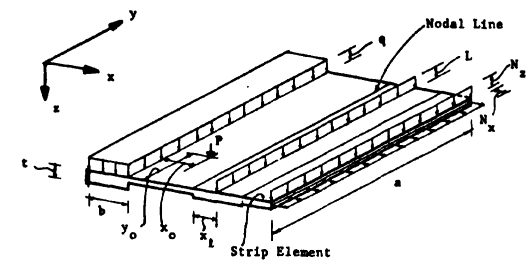
$$w = \sum_n p_{3n}(x)\ \Psi_n(y), \quad n = -1,1,3,5,. . . \qquad (3)$$

where $p_{3n}(x)$ are polynomials which describe the variation of w-displacement in the x-direction in terms of the amplitudes of w and θ displacements along nodal lines i and j of a strip.

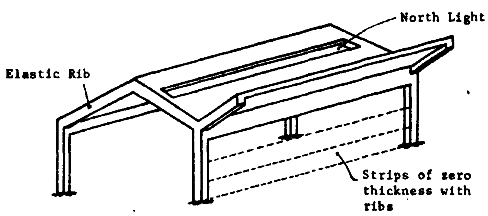
$$p_{3n}(x) = (1 - \frac{3x^2}{b^2} + \frac{2x^3}{b^3})\ w_{ni} + (x - \frac{2x^2}{b} + \frac{x^3}{b^2})\ \theta_{ni} + (\frac{3x^2}{b^2} - \frac{2x^3}{b^3})\ w_{nj} + (\frac{x^3}{b^2} - \frac{x^2}{b})\ \theta_{nj} \qquad (4)$$

[1]Assistant Professor of Civil Engineering, N.E.D. Govt. Engineering College, Karachi, Pakistan.

[2]Assistant Professor, Department of Civil Engineering, Texas Tech University, Lubbock, Texas.



(a) PLATE STRUCTURES WITH VARYING THICKNESS AND DIFFERENT LOADS.



(b) FOLDED PLATE STRUCTURE WITH ELASTIC END BEAMS AND NORTHLIGHT IN ISOTROPIC OR NATURALLY ORTHOTROPIC MATERIAL

FIGURE 1

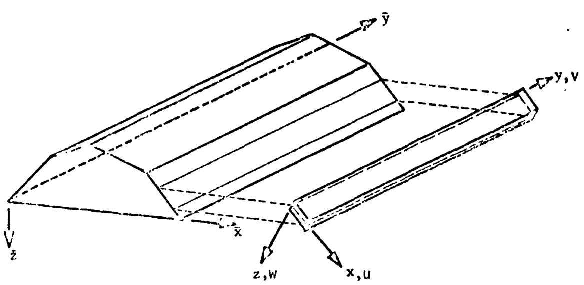


FIGURE 2. GLOBAL AND ELEMENTAL FRAMES OF REFRENCES

It may be noted that in modelling u, v, and w displacements by equations 1 and 3, the upper limit on n is fixed at 5. It was presumed that the contribution from higher base functions will be significantly small and this was confirmed by the results also. The stiffness matrix equations of membrane and bending actions are developed by the Ritz Method using the assumed displacement functions given in equations (1) and (3).

FORMULATION OF STIFFNESS MATRIX EQUATIONS

Membrane Action

Displacements: The displacement components u_n and v_n at any point within the strip corresponding to an nth base function when expressed in matrix notation are

$$\begin{Bmatrix} u_n \\ v_n \end{Bmatrix} = \begin{bmatrix} (1-\frac{x}{b})\Psi_n & 0 & (\frac{x}{b})\Psi_n & 0 \\ 0 & (1-\frac{x}{b})\Omega_n & 0 & (\frac{x}{b})\Omega_n \end{bmatrix} \begin{Bmatrix} u_{ni} \\ v_{ni} \\ u_{nj} \\ v_{nj} \end{Bmatrix} \qquad (6)$$

The column vector in the right hand member of Equation (6) is denoted by

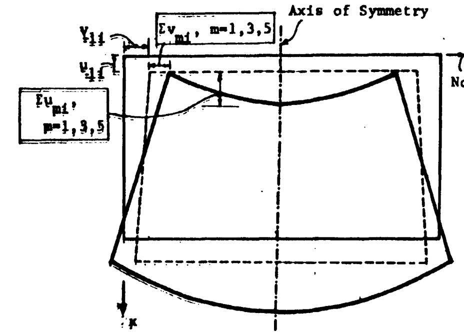


FIGURE 3. SYMMETRICALLY DISPLACED SHAPE OF MEMBRANE ACTION

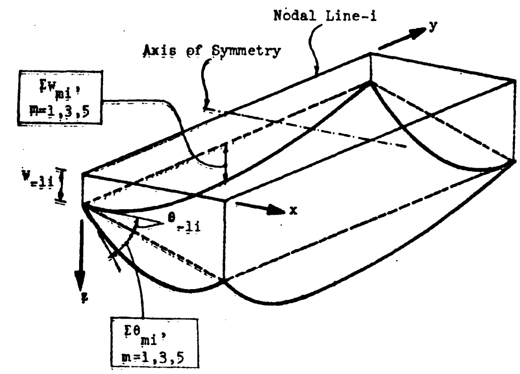


FIGURE 4. SYMMETRICALLY DISPLACED SHAPE OF BENDING ACTION

$\{\delta_n^m\}$. The superscript 'm' on the symbol, denotes its association with membrane action. The generalized displacement vector $\{\delta^m\}$ is defined as $[\delta_{-1}^m, \delta_1^m, \delta_3^m, \delta_5^m]^t$.

Forces: The membrane forces, i.e. forces acting in the xy-plane of a strip are the body forces distributed over the entire plane. The components of these forces in x- and y-directions are denoted by X and Y respectively. These components expressed in the chosen function space are

$$X = X_{-1}\Psi_{-1} + X_1\Psi_1 + X_3\Psi_3 + X_5\Psi_5$$
$$\text{and} \quad Y = Y_{-1}\Omega_{-1} + Y_1\Omega_1 + Y_3\Omega_3 + Y_5\Omega_5 \qquad (7)$$

where X_n and Y_n are amplitudes of these forces in the function space. It may be pointed out here that for uniformly distributed forces X_n and Y_n, n = 1, 3 and 5, are zero and X_{-1} and Y_{-1} are respectively equal to the magnitudes of X and Y.

Extremum of Potential Energy Functional: The potential energy functional Φ of the strip for the membrane action is

$$\Phi = U^m - V^m, \qquad (8)$$

where U^m is the strain energy stored in the strip and V^m is the potential energy of prescribed body forces due to membrane action. Differentiating Φ with respect to the undetermined parameters $\{\delta^m\}$ and equating the derivatives to zero, (9) the stiffness matrix equation is obtained

$$\frac{\partial\Phi}{\partial\{\delta^m\}} = [K^m]\{\delta^m\} - \{f^m\} = \{0\} \qquad (9)$$

The stiffness matrix $[K^m]$ and the generalized force vector $\{f^m\}$ evaluated from the Eq. (9) are given in Appendix III.

Bending Action

Displacements: The displacement w_n for a particular base function n expressed in matrix notation is

$$w_n = \Psi_n \, [f_1(x) \; f_2(x) \; f_3(x) \; f_4(x)] \begin{Bmatrix} w_{ni} \\ \theta_{ni} \\ w_{nj} \\ \theta_{nj} \end{Bmatrix} \qquad (10)$$

where $f_1(x) = (1 - \frac{3}{b^2} x^2 + \frac{2}{b^3} x^3)$, $f_2(x) = (x - \frac{2x^2}{b} + \frac{x^3}{b^2})$

$f_3(x) = (\frac{3x^2}{b^2} - \frac{2x^3}{b^3})$, and $f_4(x) = (-\frac{x^2}{b} + \frac{x^3}{b^2})$.

The column vector in the right hand member of Eq. (10) is denoted by $\{\delta_n^b\}$. The generalized displacement vector $\{\delta^b\}$ is defined as $[\delta_{-1}^b, \delta_1^b, \delta_3^b, \delta_5^b]^t$.

Loads: The transverse loads (parallel to z-axis) carried on the surface of a strip can be distributed, line and point loads. These loads are respectively denoted by symbols q, L, and P. The load L is located at distance x_ℓ from the origin of the elemental frame of reference while the load P is located at distances x_0 and y_0. All loads considered are symmetrical. These distributed loads are expressed in the chosen function space as

$$q = q_{-1}\Psi_{-1} + q_1\Psi_1 + q_3\Psi_3 + q_5\Psi_5 \qquad (11)$$

where q_{-1}, q_1, q_3 and q_5 are the amplitudes of q in the function sapce. The line loads in the function space are

$$L = L_{-1}\Psi_{-1} + L_1\Psi_1 + L_3\Psi_3 + L_5\Psi_5 \qquad (12)$$

where L_{-1}, L_1, L_3 and L_5 are the amplitudes of L in the function space. The concentrated load is

$$P = P_{-1}\Psi_{-1} = P_{-1} \qquad (13)$$

where P_{-1} is the amplitude of the load P in the function space. For uniformly distributed and uniform line loads, the amplitudes q_n and L_n, for n = 1, 3, 5 are equal to zero, and q_{-1} and L_{-1} are equal to q and L respectively.

Extremum of Potential Energy Functional: The potential energy functional Φ of the strip for symmetrically displaced shape in bending action is

$$\Phi = U^b - V^b \qquad (14)$$

where U^b is the strain energy of the strip and V^b is the potential energy of the prescribed loads in 'bending action'. Differentiating this Φ with respect to the undetermined parameters $\{\delta^b\}$ and equating the derivatives to zero the stiffness matrix equation is obtained.

$$\frac{\partial \Phi}{\partial \{\delta^b\}} = [K^b]\{\delta^b\} - \{f^b\} = \{0\} \qquad (15)$$

The stiffness matrix $[K^b]$ and the generalized force vector $\{f^b\}$ evaluated from Eq. (15) are given in Appendix III.

Combined Stiffness Matrix Equation: The membrane and bending stiffness matrix equations of the symmetrically displaced shape are superposed, to obtain the combined stiffness matrix equation of the strip.

$$\begin{bmatrix} K^m & 0 \\ 0 & K^b \end{bmatrix} \begin{Bmatrix} \delta^m \\ \delta^b \end{Bmatrix} = \begin{Bmatrix} f^m \\ f^b \end{Bmatrix} \qquad (16)$$

The generalized displacement vector in Eq. (16) is rearranged such that

$$\{\delta\} = [u_{-1i} \; v_{-1i} \; w_{-1i} \; {}_{-1i} \cdots\cdots\cdots u_{5i} \; v_{5i} \; w_{5i} \; {}_{5i} \; u_{-1j} \; v_{-1j} \; w_{-1j} \; {}_{-1j} \cdots\cdots\cdots u_{5j} \; v_{5j} \; w_{5j} \; {}_{5j}]^t \qquad (19)$$

so that Eq. (16) is now

$$[K]\{\delta\} = \{f\} \qquad (18)$$

End Beam Element: Conventional folded plate and cylindrical shell structures are designed and constructed with rigid end diaphragms along the x-edges. To permit analysis of structures with flexible stiffeners along the edges beam elements which are subjected to axial and bending deformations in xz-plane are considered in this study. The end beam elements are attached to the soffit of the strip along both x-edges. (See Fig. 5a.) They serve two purposes: elastic end beams in lieu of rigid end diaphragm and integrally built-in columns. To simulate the conditions of integrally built-in columns the thickness of strip elements with elastic end beams in particular location is equated to zero. This is shown in Fig. 1(b).

Figure 5(b) shows the displaced state of the end beam in terms of strip-displacements along x-edge. The displacements of middle surface of the end beam in xz-plane at a nodal line i are denoted by u_i^r, w_i^r and θ_i^r. The beam displacements at nodal line i and j in terms of plate displacements along x-edges are

$$\begin{Bmatrix} u_i^r \\ w_i^r \\ \theta_i^r \\ u_j^r \\ w_j^r \\ \theta_j^r \end{Bmatrix} = \begin{bmatrix} 1 & 0 & r & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_{-1i} \\ w_{-1i} \\ \theta_{-1i} \\ u_{-1j} \\ w_{-1j} \\ \theta_{-1j} \end{Bmatrix} \qquad (19)$$

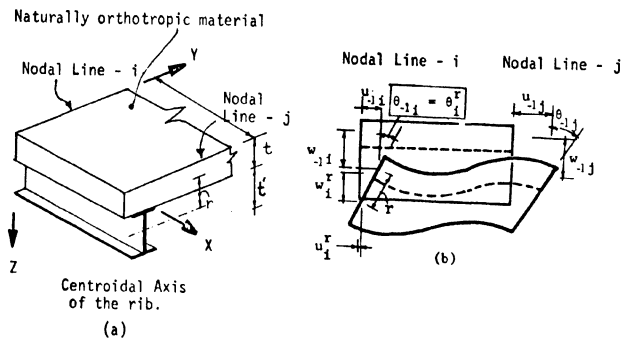


FIGURE 5. END BEAM ATTACHED TO THE SOFFIT

It may be pointed out here that for r = 0, the centroidal axis of the end beam coincides with the middle surface of the strip or the end beam becomes a separate body which is not integrally built with the strip. The deformation pattern of a end beam established in Eq. (19) excludes its rotation about x-axis.

The stiffness matrix $[K^r]$ of the end beam element shown in Appendix III, is superposed on the stiffness matrix [K] of the strip, resulting in the combined stiffness matrix [K'] of the strip and the end beams.

Transformation to Global Coordinates

The $\bar{y}$ and y-axis in the global and elemental coordinates remain parallel to each other so that no transformation of v- and θ- displacements from elemental to global coordinates or vice versa is required. The transformation of coordinates takes place in the xz-plane alone. Figure 6 shows elemental frame of reference rotated in a positive direction through an angle α with respect to the global frame. The stiffness matrix equation transformed to the global coordinates, is written as

$$[\bar{K}]\{\bar{\delta}\} = \{\bar{f}\} \qquad (20)$$

where $[\bar{K}]$, $\{\bar{\delta}\}$ and $\{\bar{f}\}$ are the global stiffness matrix, and the global displacement and force vectors respectively.

The overall stiffness matrix of a structure composed of all the strips is assembled using Eq. (20) for each strip by superposing them at their common nodal lines. This matrix is banded with one-half band width equal to thirty-two, and hence is, stored in a rectangular array thirty-two locations wide.

Nodal Line Forces

The line forces N along nodal lines of uniform intensity are applied in the xz-plane along the nodal lines. These forces have two components N_x and N_z referred to the global coordinates. These components expressed in the corresponding function space are

$$N_x = (N_x)_{-1}\Psi_{-1} + N_{x1}\Psi_1 + N_{x3}\Psi_3 + N_{x5}\Psi_5$$
$$N_z = (N_z)_{-1}\Psi_{-1} + N_{z1}\Psi_1 + N_{z3}\Psi_3 + N_{z5}\Psi_5 \quad (21)$$

where N_{xn} and N_{zn} are amplitudes of N_x and N_z in the function space. N_{xn} and N_{zn}, n = 1, 3, 5 are equal to zero when N_x and N_z have uniform intensity; and since $\Psi_{-1} = 1$ for this case, $(N_x)_{-1} = N_x$ and $(N_z)_{-1} = N_z$.

The generalized global force vector $\{\bar{f}^n\}$ due to these nodal line forces is obtained from the potential energy of these forces and the values are given in Appendix III.

Solution by Gauss Elimination

The overall stiffness matrix equation is solved for the global generalized displacements by Gauss Elimination procedure using half band width storage.

Convergence Characteristics of the Extended Finite Strip Method

The convergence characteristics of the displacements and stress resultants obtained from this method are given Ref. (8). It is shown that the results converge rapidly to a stable value.

Before discussing the results of an orthotropic steel folded plate structure, the stress-strain relations in technically orthotropic plate strip element are discussed.

ORTHOTROPIC STEEL PLATES

In structural steel design, the orthotropic material property is often used, when the plate is stiffened by isotropic elements of different sizes, welded to it in mutually perpendicular directions. Such a plate is called "technically orthotropic plate". The force deformation relations used are similar to those of the naturally orthotropic plates. The elastic constants are obtained from some assumed deformation criteria. A technically orthotropic plate made from skin element and rib elements of different sizes in x- and y-directions, is shown in Fig. 7. The moments of inertia of the ribs per unit length are I_x and I_y respectively. The dimensions a_1, a_2, h_1, h_2, d_1 and d_2 are shown in Fig. 7.

The elastic constants of such a plate for bending and membrane actions are obtained seperately.

Elastic Constants for Bending Action

Huber (9) has obtained the elastic constants, i.e., the equivalent flexural rigidities D_x, D_y, D_1 and D_{xy} using the "principle of elastic equivalence".

$$D_x = \frac{EI_x}{a_2}$$

$$D_y = \frac{EI_y}{a_1}$$

$$D_{xy} = \frac{G}{3}\left(\frac{h_1 d_1^3}{a_1} + \frac{h_2 d_2^3}{a_2}\right) \quad (22)$$

$$\text{and} \quad D_1 = \nu_x D_y = \nu_y D_x$$

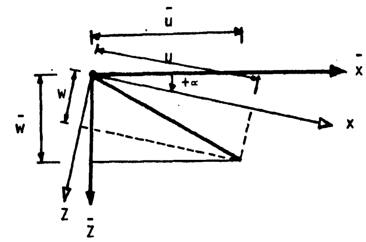


FIGURE 6. TRANSFORMATION OF COORDINATES

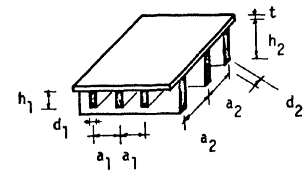


FIGURE 7. A TECHNICALLY ORTHOTROPIC PLATE

where E and G are the Young's and shear moduli of elasticity of steel, ν_x and ν_y are the Poisson's ratios in x- and y-directions respectively.

Elastic Constants for Membrane Action

Using similar principles and assuming the corresponding strains in the skin element and the ribs are equal, and that shearing action is resisted by skin alone, the following relations for equivalent elastic constants are developed.

The stress resultants per unit length in isotropic skin element are:

$$(T_x)_1 = \sigma_x t = \frac{Et}{1-\nu^2}(\epsilon_x + \nu\epsilon_y)$$

$$(T_y)_1 = \sigma_y t = \frac{Et}{1-\nu^2}(\epsilon_y + \nu\epsilon_x) \quad (23)$$

$$(T_{xy})_1 = \tau_{xy} t = Gt\,\gamma_{xy}$$

where E, G and ν are the material properties of the isotropic steel. For the same strains in the rib, the stress resultants per unit length, are

$$(T_x)_2 = E\,\epsilon_x A_x$$

$$(T_y)_2 = E\,\epsilon_y A_y \quad (24)$$

$$(T_{xy})_2 = 0$$

where A_x and A_y are the cross sectional areas per unit length of the ribs in x- and y-directions respectively. The total stress resultants per unit length of an element are related to the strains by the following equation.

$$\begin{Bmatrix} T_x \\ T_y \\ T_{xy} \end{Bmatrix} = t \begin{bmatrix} \left(\frac{E}{1-\nu^2} + \frac{EA_x}{t}\right) & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \left(\frac{E}{1-\nu^2} + \frac{EA_y}{t}\right) & 0 \\ 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$= t \begin{bmatrix} \bar{E}_x & \bar{E}_1 & 0 \\ \bar{E}_1 & \bar{E}_y & 0 \\ 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (25)$$

where the elastic constants $\bar{E}_x$, $\bar{E}_y$, $\bar{E}_1$ and G for the membrane action of a technically orthotropic plate are as defined in Eq. (25).

Orthotropic Steel Folded Plate Structure

A single span folded plate structure made from technically orthotropic steel plates, supported by elastic end beams and integrally built-in columns, is shown in Fig. 8. The material properties are given below.

$$\bar{E}_x = 33350.0 \text{ ksi}$$
$$\bar{E}_y = 40600.0 \text{ ksi}$$
$$\bar{E}_1 = 9560.0 \text{ ksi}$$
$$G = 11500.0 \text{ ksi}$$
$$D_x = 2900.0 \text{ k-in}$$
$$D_y = 8000.0 \text{ k-in}$$
$$D_{xy} = 32.6 \text{ k-in}$$
$$D_1 = 0.0 \quad \text{(assumed)}$$

Displacements of the structure along the mid and end sections in transverse direction are plotted in Fig. 9. The stress resultants are produced in Tables I, II and III. It can be seen that the values of M_{xy} are considerably small.

Conclusions

The following conclusions may be drawn from application of the extended finite strip method to folded plate structures.

1) Prismatic plate structures of isotropic or orthotropic material

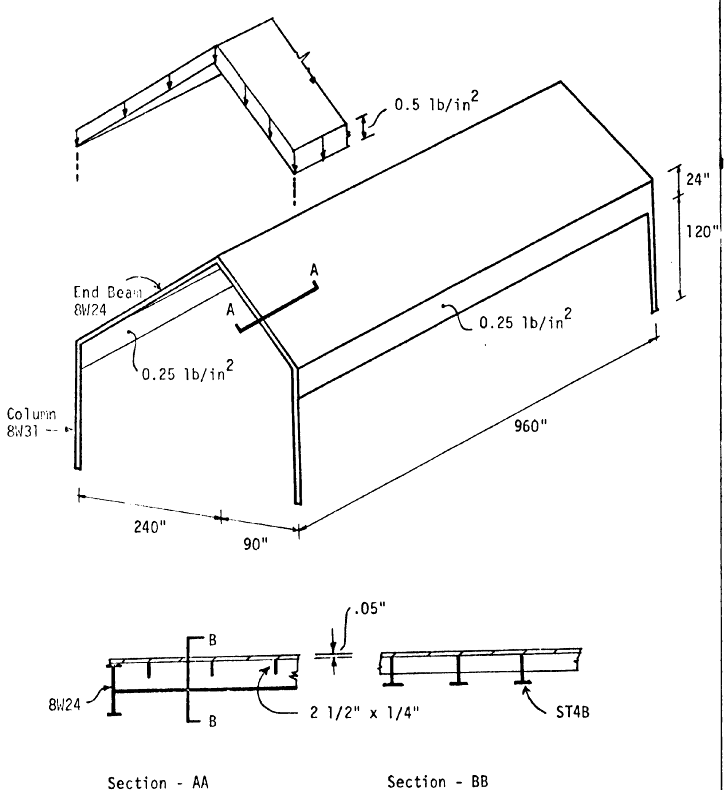


FIGURE 8. A FOLDED PLATE STRUCTURE IN TECHNICALLY ORTHOTROPIC MATERIAL

which are free to rotate about their transverse edges can be analyzed by the method.

2) The inclusion of end beam elements along the transverse edges makes the method versatile for analyzing structures a) supported on integrally built-in columns, b) carried on elastic ribs instead of conventional rigid end diaphragms and employing features such as north light window.

3) The number of unknowns involved in the method is small for equally

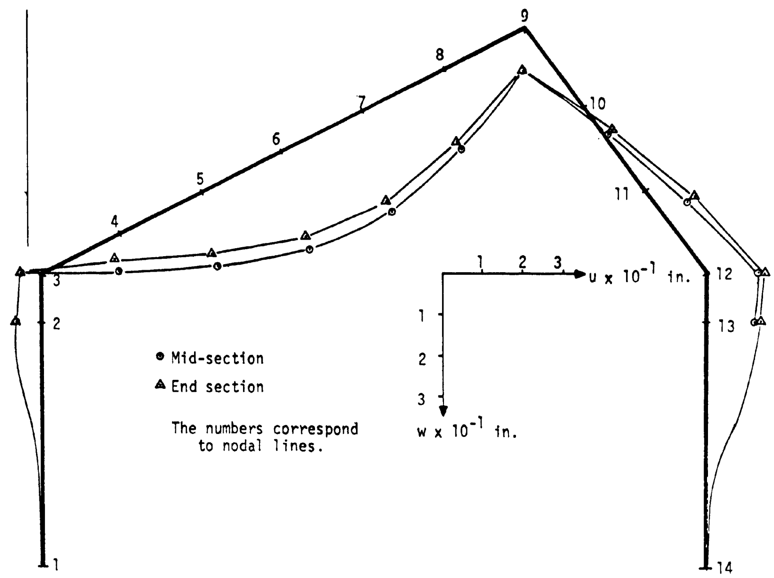


FIGURE 9. DEFORMED SHAPE OF THE FOLDED PLATE STRUCTURE

TABLE I. STRESS RESULTANTS (K/IN) AT MID POINTS OF THE STRIPS ALONG MID SECTION IN TRANSVERSE DIRECTION

Strip	Tx	Ty	Txy
2-3	0.0062	0.0642	-0.0316
3-4	-0.0310	0.0047	-0.0124
4-5	-0.0299	0.0165	-0.0476
5-6	-0.0575	-0.0152	-0.0359
6-7	-0.0732	-0.0208	-0.0073
7-8	-0.0626	-0.0105	0.0187
8-9	-0.0306	-0.0024	0.0246
9-10	-0.0513	0.0009	0.0301
10-11	0.0322	0.0173	0.0239
11-12	0.0521	-0.0040	0.0029
12-13	-0.0318	0.0455	0.0226

TABLE II. STRESS RESULTANT (K-IN/IN) AT MID POINTS OF THE STRIPS ALONG MID SECTION IN TRANSVERSE DIRECTION

Strip	Mx	My	Mxy
2-3	-0.1603	-0.0045	0.0009
3-4	-0.0615	0.2235	0.0011
4-5	0.0269	0.3553	0.0000
5-6	0.0705	0.3470	-0.0001
6-7	0.0915	0.3512	0.0001
7-8	0.0962	0.3498	-0.0002
8-9	0.0018	0.1358	-0.0017
9-10	-0.0416	0.0636	0.0017
10-11	-0.0143	0.2440	0.0002
11-12	-0.0589	0.1997	-0.0005
12-13	-0.1037	0.0458	-0.0009

accurate results when compared to the finite element method.

4) The contribution from the fifth and sixth base functions are significantly small and therefore exclusion of higher base funcitons is justified.

5) The convergence tests show that the values of displacements stabilize

with the division of half structure into 3 or 4 strips while the values of stress and moment resultants stabilize with this division into 6 to 8 strips.

6) In the case of plate structures in bending, for a ratio of strip width to length (b/a) of 0.05, the end results from 15 digit arithmetic may become inaccurate due to round off errors on the computer. However, the number of strips required also depends upon the boundary conditions along the transverse edges.

7) The method also permits the study of distribution of stresses due to support settlement.

8) Representation of loads especially the uniformly distributed load is simple because of the first base function used in the analysis.

TABLE III. STRESS RESULTANTS IN END BEAM ELEMENTS

Nodal Line	Strip	P (kips)	V (kips)	M_x (K-in)
1				73.69
	1-2	-4.487	-1.443	
2				-98.90
	2-3	-3.989	-1.400	
3				-130.90
	3-4	-3.092	2.288	
4				-27.20
	4-5	-1.273	1.574	
5				43.46
	5-6	6.079	0.792	
6				78.91
	6-7	1.625	0.019	
7				79.80
	7-8	1.394	-0.764	
8				45.00
	8-9	2.616	-1.397	
9				-18.22
	9-10	-2.378	-0.080	
10				-21.90
	10-11	-3.678	-0.547	
11				-48.78
	11-12	-4.721	-1.131	
12				-106.50
	12-13	-4.670	1.456	
13				-73.06
	13-14	-5.040	1.442	
14				-9.96

APPENDIX I

References

1. Baer, Oliver, A., "Steel Frame Folded Plate Roof," Journal of Structural Division, ASCE, Vol. 87, ST 5, June 1961.
2. "Folded Plate Roof Will Span 252 Feet," Engineering News Record, April 3, 1969.
3. Report of the Task Committee on Folded Plate Construction, Journal of the Structural Division, Proceedings of A.S.C.E., December 1963.
4. Nilson, Arthur H., "Folded Plate Structures of Light Gage Steel," Journal of the Structural Division, Proceedings of A.S.C.E., October 1961.
5. Goldberg, John E., and Leve, H. L., "Theory of Folded Plate Structures," International Association of Bridge and Structural Engineering, Publication, Vol. 17, 1957.
6. Cheung, Y. K., "Folded Plate Structures by Finite Strip Method," Journal of the Structural Division, ASCE, Vol. 95, ST12, December 1969.
7. Rockey, K. C., and Evans, H. R., "A Finite Element Solution for Folded Plate Structures," Proceedings International Conference on Space Structures, University of Surrey, 1966.
8. Siddiqi, Ghulam H., "Extended Finite Strip Method for Prismatic Plate and Shell Structures," Doctoral Dissertation, Department of Civil Engineering, Texas Tech University, Lubbock, Texas, 1971.
9. Crandall, Stephen. Engineering Analysis, McGraw-Hill Book Co., New York, 1956.
10. Troitsky, M. S. Orthotropic Bridges Theory and Design, The James F. Lincoln Arc Welding Foundation, Cleveland, Ohio, 1967.

APPENDIX II

Notations

A	= Area of rib; amplitude of displacement.
a	= Length of a structure and a strip in y-direction.
b	= Width of a strip element.
D_x, D_y, D_{xy}, D_1	= Plate rigidities.
E_x, E_y, E_1	= Young's Moduli of elasticity.
$\bar{E}_x$, $\bar{E}_y$, $\bar{E}_1$	= Elastic constants of an orthotropic body.
{f}	= Column vector of generalized forces referred to elemental coordinates.
{$\bar{f}$}	= Column vector of generalized forces referred to global coordinates.
G	= Shear Modulus.
I_x, I_y	= Moments of inertia of rib about x- and y-centroidal axis.
k_{ij}, [K]	= Stiffness influence coefficients; matrix of these coefficients.
L	= Distributed line loads applied transversely on a strip.
L_m	= Amplitudes of L-loads in the function space.
M_x, M_y M_{xy}	= Moment resultants in x- and y-directions and twisting moment.
N, N_x, N_z	= Nodal line force and its components.
$(N_x)_m$, $(N_z)_m$	= Amplitudes of N-forces in the function space.
P_i	= Point load at point-i.
{P}	= Column vector of P-loads.
q, q_m	= Distributed loads; amplitudes of these loads in the function space.
r	= Centroidal distance between a strip and end beam.
T_x, T_y, T_{xy}	= Stress resultants in x- and y-directions and in xy-plane.
t	= Thickness of a strip element.
t'	= Depth of a beam element.
U	= Strain energy.
V	=Potential energy of loads.
u, v, w	= Displacements of a point on the middle surface in x-, y- and z-directions respectively.
$\bar{u}$, $\bar{v}$, $\bar{w}$	= Displacements of a point on the middle surface in $\bar{x}$-, $\bar{y}$- and $\bar{z}$-directions respectively.
u', v', w'	= Displacements of a point in the plate in x-, y- and z-directions.
X, Y; X_m, Y_m	= Body forces in x- and y-directions; amplitudes in the function space.
x, y, z	= Coordinates of elemental frame of reference.
$\bar{x}$, $\bar{y}$, $\bar{z}$	= Coordinates of global frame of reference.
α	= Rotation about Y axis.
ε_x, ε_y, ε_{xy}	= Strains in x- and y-directions and shear strain in xy-plan .
σ_x, σ_y, σ_{xy}	= Stresses in x- and y-directions and shear stress in xy-plane.
ν_x, ν_y	= Poisson's ratio in x- and y-directions.
θ	= Rotation about y-axis.
Ψ_m, Ω_m	= Base functions of y.
ϕ	= Potential energy functional

APPENDIX III

STIFFNESS AND FORCE MATRICES

The Stiffness matrix $[K^m]$ and the force vector $\{f^m\}$ are subdivided into matrices of size eight as

$$[K^m] = \begin{bmatrix} K_{11}^m & K_{12}^m \\ (K_{12}^m)^t & K_{22}^m \end{bmatrix}$$

and $\{f^m\} = \begin{Bmatrix} f_1^m \\ f_2^m \end{Bmatrix}$.

The Stiffness matrix $[K^b]$ is similarly subdivided

$$[K^b] = \begin{bmatrix} K_{11}^b & K_{12}^b \\ (K_{12}^b)^t & K_{22}^b \end{bmatrix} .$$

The lower quadrant of $[K_{11}^b]$ is denoted by $[K_{\mu m}^b]$. The elements in this quadrant are functions of μ_m and repeat themselves in the upper left and lower right quadrants of $[K_{22}^b]$. All these matrices, the force vectors, and the end beam stiffness matrix $[K^r]$ are produced on the following pages.

$[K_{11}^m]$:

	$(a/b)\bar{E}_x$	$\bar{E}_1$	$-(a/b)\bar{E}_x$	$\bar{E}_1$	$(2a/\pi b)\bar{E}_x$	$\bar{E}_1$	$-(2a/\pi b)\bar{E}_x$	$\bar{E}_1$
		$(4b/3a)\bar{E}_y + (a/3b)G$	$-\bar{E}_1$	$(2b/3a)\bar{E}_y - (a/3b)G$	$(2/\pi)(\bar{E}_1 - G)$	$(4b/3a)\bar{E}_y + (4a/\pi^2 b)G$	$-(2/\pi)(\bar{E}_1 + G)$	$(2b/3a)\bar{E}_y - (4a/\pi^2 b)G$
			$(a/b)\bar{E}_x$	$-\bar{E}_1$	$-(2a/\pi b)\bar{E}_x$	$-\bar{E}_1$	$(2a/\pi b)\bar{E}_x$	$-\bar{E}_1$
				$(4b/3a)\bar{E}_y + (a/3b)G$	$(2/\pi)(\bar{E}_1 + G)$	$(2b/3a)\bar{E}_y - (4a/\pi^2 b)G$	$-(2/\pi)(\bar{E}_1 - G)$	$(4b/3a)\bar{E}_y + (4a/\pi^2 b)G$
(t)					$(a/2b)\bar{E}_x + (\pi^2 b/6a)G$	$(\pi\nu_x/4)\bar{E}_y - (\pi/4)G$	$-(a/2b)\bar{E}_x + (\pi^2 b/12a)G$	$(\pi\nu_x/4)\bar{E}_y + (\pi/4)G$
						$(\pi^2 b/6a)\bar{E}_y + (a/2b)G$	$-(\pi\nu_x/4)\bar{E}_y - (\pi/4)G$	$(\pi^2 b/12a)\bar{E}_y - (a/2b)G$
		Symmetric					$(a/2b)\bar{E}_x + (\pi^2 b/6a)G$	$-(\pi\nu_x/4)\bar{E}_y + (\pi/4)G$
								$(\pi^2 b/6a)\bar{E}_y$ [illegible]

$[K_{12}^m]$: and $\{f_1^m\}$:

									$\{f_1^m\}$
	$(2a/3\pi b)\bar{E}_x$	$\bar{E}_1$	$-(2a/3\pi b)\bar{E}_x$	$\bar{E}_1$	$(2a/5\pi b)\bar{E}_x$	$\bar{E}_1$	$-(2a/5\pi b)\bar{E}_x$	$\bar{E}_1$	$\frac{ab}{2}(X_{-1})$
	$(2/3\pi)(\bar{E}_1 - G)$	$(4b/3a)\bar{E}_y + (4a/9\pi^2 b)G$	$-(2/3\pi)(\bar{E}_1 + G)$	$(2b/3a)\bar{E}_y - (4a/9\pi^2 b)G$	$(2/5\pi)(\bar{E}_1 - G)$	$(4b/3a)\bar{E}_y + (4a/25\pi^2 b)G$	$-(2/5\pi)(\bar{E}_1 + G)$	$(2b/3a)\bar{E}_y - (4a/25\pi^2 b)G$	$\frac{ab}{6}(Y_{-1})$
	$-(2a/3\pi b)\bar{E}_x$	$-\bar{E}_1$	$(2a/3\pi b)\bar{E}_x$	$-\bar{E}_1$	$-(2a/5\pi b)\bar{E}_x$	$-\bar{E}_1$	$(2a/5\pi b)\bar{E}_x$	$-\bar{E}_1$	$\frac{ab}{2}(X_{-1})$
	$(2/3\pi)(\bar{E}_1 + G)$	$(2b/3a)Ey - (4a/9\pi^2 b)G$	$-(2/3\pi)(\bar{E}_1 - G)$	$(4b/3a)\bar{E}_y + (4a/9\pi^2 b)G$	$(2/5\pi)(\bar{E}_1 + G)$	$(2b/3a)\bar{E}_y - (4a/25\pi^2 b)G$	$-(2/5\pi)(\bar{E}_1 - G)$	$(4b/3a)\bar{E}_y + (4a/25\pi^2 b)G$	$\frac{ab}{6}(Y_{-1})$
(t)									$\frac{ab}{\pi}(X_{-1})$
				Remaining	Elements				$\frac{2ab}{\pi^2}(Y_{-1})$
				are Equal	to Zero				$\frac{ab}{\pi}(X_{-1})$
									$\frac{2ab}{\pi^2}(Y_{-1})$

$[K^m_{11}]$: $\{f^m_1\}$:

	$(a/2b)\bar{E}_x + (9\pi^2 b/6a)G$	$(3\pi\nu_x/4)\bar{E}_y - (3\pi/4)G$	$-(a/2b)\bar{E}_x + (9\pi^2 b/12a)G$	$(3\pi\nu_x/4)\bar{E}_y + (3\pi/4)G$					$\frac{ab}{3\pi}(X_{-1})$
		$(9\pi^2 b/6a)\bar{E}_y + (a/2b)G$	$-(3\pi\nu_x/4)\bar{E}_y - (3\pi/4)G$	$(9\pi^2 b/12a)\bar{E}_y - (a/2b)G$		Remaining Elements are Equal to Zero			$\frac{2ab}{9\pi^2}(Y_{-1})$
			$(a/2b)\bar{E}_x + (9\pi^2 b/6a)G$	$-(3\pi\nu_x/4)\bar{E}_y + (3\pi/4)G$					$\frac{ab}{3\pi}(X_{-1})$
				$(9\pi^2 b/6a)\bar{E}_y + (a/2b)G$					$\frac{2ab}{9\pi^2}(Y_{-1})$
(t)					$(a/2b)\bar{E}_x + (25\pi^2 b/6a)G$	$(5\pi\nu_x/4)\bar{E}_y - (5\pi/4)G$	$-(a/2b)\bar{E}_x + (25\pi^2 b/12a)G$	$(5\pi\nu_x/4)\bar{E}_y + (5\pi/4)G$	$\frac{ab}{5\pi}(X_1)$
		Symmetric				$(25\pi^2 b/6a)\bar{E}_y + (a/2b)G$	$-(5\pi\nu_x/4)\bar{E}_y - (5\pi/4)G$	$(25\pi^2 b/12a)\bar{E}_y - (a/2b)G$	$\frac{2ab}{25\pi^2}(Y_{-1})$
							$(a/2b)\bar{E}_x + (25\pi^2 b/6a)G$	$-(5\pi\nu_x/4)\bar{E}_y + (5\pi/4)G$	$\frac{ab}{5\pi}(X_1)$
								$(25\pi^2 b/6a)\bar{E}_y + (a/2b)G$	$\frac{2ab}{25\pi^2}(Y_1)$

$[K^b_{11}]$:

$(12a/b^3)D_x$	$(6a/b^2)D_x$	$-(12a/b^3)D_x$	$(6a/b^2)D_x$	$(24a/\pi b^3)D_x + (12\pi/5ab)D_1$	$(12a/\pi b^2)D_x + (\pi/5a)D_1$	$-(24a/\pi b^3)D_x - (12\pi/5ab)D_1$	$(12a/\pi b^2)D_x + (\pi/5a)D_1$
	$(4a/b)D_x$	$-(6a/b^2)D_x$	$(2a/b)D_x$	$(12a/\pi b^2)D_x + (11\pi/5a)D_1$	$(8a/\pi b)D_x + (4b\pi/15a)D_1$	$-(12a/\pi b^2)D_x - (\pi/5a)D_1$	$(4a/\pi b)D_x - (b\pi/15a)D_1$
		$(12a/b^3)D_x$	$-(6a/b^2)D_x$	$-(24a/\pi b^3)D_x - (12\pi/5ab)D_1$	$-(12a/\pi b^2)D_x - (\pi/5a)D_1$	$(24a/\pi b^3)D_x + (12\pi/5ab)D_1$	$-(12a/\pi b^2)D_x - (\pi/5a)D_1$
			$(4a/b)D_x$	$(12a/\pi b^2)D_x + (\pi/5a)D_1$	$(4a/\pi b)D_x - (b\pi/15a)D_1$	$-(12a/\pi b^2)D_x - (11\pi/5a)D_1$	$(8a/\pi b)D_x + (4\pi b/15a)D_1$
				$(6a/b^3)D_x + (6\mu_1^2/5ab)D_1 + (13\mu_1^4 b/70a^3)D_y + (12\mu_1^2/5ab)D_{xy}$	$(3a/b^2)D_x + (3\mu_1^2/5a)D_1 + (11\mu_1^4 b^2/420a^3)D_y + (\mu_1^2/5a)D_{xy}$	$-(6a/b^3)D_x - (6\mu_1^2/5ab)D_1 + (9\mu_1^4 b/140a^3)D_y - (12\mu_1^2/5ab)D_{xy}$	$(3a/b^2)D_x + (\mu_1^2/10a)D_1 - (13\mu_1^4 b^2/840a^3)D_y - (\mu_1^2/5a)D_{xy}$
		$\mu_1 = \pi$			$(2a/b)D_x + (2\mu_1^2 b/15a)D_1 + (\mu_1^4 b^3/210a^3)D_y + (4\mu_1^2 b/15a)D_{xy}$	$-(3a/b^2)D_x - (\mu_1^2/10a)D_1 + (13\mu_1^4 b^2/840a^3)D_y - (\mu_1^2/5a)D_{xy}$	$+(a/b)D_x - (\mu_1^2 b/30a)D_1 - (\mu_1^4 b^3/280a^3)D_y - (\mu_1^2 b/15a)D_{xy}$
		Symmetric			$K^b_{\mu_1}$	$(6a/b^3)D_x + (6\mu_1^2/5ab)D_1 + (13\mu_1^4 b/70a^3)D_y + (12\mu_1^2/5ab)D_{xy}$	$-(3a/b^2)D_x - (3\mu_1^2/5a)D_1 - (11\mu_1^4 b^2/420a^3)D_y - (\mu_1^2/5a)D_{xy}$
							$(2a/b)D_x + (2\mu_1^2 b/15a)D_1 + (\mu_1^4 b^3/210a^3)D_y + (4\mu_1^2 b/15a)D_{xy}$

$[K^b_{12}]$:

$(8a/\pi b^3)D_x + (36\pi/5ab)D_1$	$(4a/\pi b^2)D_x + (3\pi/5a)D_1$	$-(8a/\pi b^3)D_x - (36\pi/5ab)D_1$	$(4a/\pi b^2)D_x + (3\pi/5a)D_1$	$(24a/5\pi b^3)D_x + (12\pi/ab)D_1$	$(12a/5\pi b^2)D_x + (\pi/a)D_1$	$-(24a/5\pi b^3)D_x - (12\pi/ab)D_1$	$(12a/5\pi b^2)D_x + (\pi/a)D_1$
$(4a/\pi b^2)D_x + (33\pi/5a)D_1$	$(8a/3\pi b)D_x + (4b\pi/5a)D_1$	$-(4a/\pi b^2)D_x - (3\pi/5a)D_1$	$(4a/3\pi b)D_x - (b\pi/5a)D_1$	$(12a/5\pi b^2)D_x + (11\pi/a)D_1$	$(8a/5\pi b)D_x + (4b\pi/3a)D_1$	$-(12a/5\pi b^2)D_x - (\pi/2a)D_1$	$(4a/5\pi b)D_x - (b\pi/3a)D_1$
$-(8a/\pi b^3)D_x - (36\pi/5ab)D_1$	$-(4a/\pi b^2)D_x - (3\pi/5a)D_1$	$(8a/\pi b^3)Dx + (36\pi/5ab)D_1$	$-(4a/\pi b^2)D_x - (3\pi/5a)D_1$	$(-24a/5\pi b^3)D_x - (12\pi/ab)D_1$	$-(12a/5\pi b^2)D_x - (\pi/2a)D_1$	$(24a/5\pi b^3)D_x + (12\pi/ab)D_1$	$-(12a/5\pi b^2)D_x - (\pi/a)D_1$
$(4a/\pi b^2)D_x + (3\pi/5a)D_1$	$(4a/3\pi)D_x - (b\pi/5a)D_1$	$-(4a/\pi b^2)D_x - (33\pi/5a)D_1$	$(8a/3b\pi)D_x + (4\pi b/5a)D_1$	$(12a/5\pi b^2)D_x + (\pi/a)D_1$	$(4a/5\pi b)D_x - (b\pi/3a)D_1$	$-(12a/5\pi b^2)D_x - (11\pi/a)D_1$	$(8a/5b\pi)D_x + (4\pi b/3a)D_1$
			Remaining Elements are Equal to Zero				

$[K^b_{22}]$:

		$K^b_{\mu_3}$					
		$\mu_3 = 3\pi$			Remaining Elements are Equal to Zero		
						$K^b_{\mu_5}$	
						$\mu_5 = 5\pi$	
	Symmetric						

$\{f^b\}$:

q-Load	L-Load	P-Load
$(ab/2)\ q_{-1}$	$[f_1(x_\ell)]aL_{-1}$	$[f_1(x_0)]P$
$(ab^2/12)\ q_{-1}$	$[f_2(x_\ell)]aL_{-1}$	$[f_2(x_0)]P$
$(ab/2)\ q_{-1}$	$[f_3(x_\ell)]aL_{-1}$	$[f_3(x_0)]P$
$-(ab^2/12)\ q_{-1}$	$[f_4(x_\ell)]aL_{-1}$	$[f_4(x_0)]P$
$(ab/\pi)\ q_{-1}$	$[f_1(x_\ell)](\frac{2a}{\pi})L_{-1}$	$[f_1(x_0)](\mathrm{Sin}\frac{\pi y_0}{a})P$
$(ab^2/6\pi)\ q_{-1}$	$[f_2(x_\ell)](\frac{2a}{\pi})L_{-1}$	$[f_2(x_0)](\mathrm{Sin}\frac{\pi y_0}{a})P$
$(ab/\pi)\ q_{-1}$	$[f_3(x_\ell)](\frac{2a}{\pi})L_{-1}$	$[f_3(x_0)](\mathrm{Sin}\frac{\pi y_0}{a})P$
$-(ab^2/6\pi)\ q_{-1}$	$[f_4(x_\ell)](\frac{2a}{\pi})L_{-1}$	$[f_4(x_0)](\mathrm{Sin}\frac{\pi y_0}{a})P$
$(ab/3\pi)\ q_{-1}$	$[f_1(x_\ell)](\frac{2a}{3\pi})L_{-1}$	$[f_1(x_0)](\mathrm{Sin}\frac{3\pi y_0}{a})P$
$(ab^2/18\pi)\ q_{-1}$	$[f_2(x_\ell)](\frac{2a}{3\pi})L_{-1}$	$[f_2(x_0)](\mathrm{Sin}\frac{3\pi y_0}{a})P$
$(ab/3\pi)\ q_{-1}$	$[f_3(x_\ell)](\frac{2a}{3\pi})L_{-1}$	$[f_3(x_0)](\mathrm{Sin}\frac{3\pi y_0}{a})P$
$-(ab^2/18\pi)\ q_{-1}$	$[f_4(x_\ell)](\frac{2a}{3\pi})L_{-1}$	$[f_4(x_0)](\mathrm{Sin}\frac{3\pi y_0}{a})P$
$(ab/5\pi)\ q_{-1}$	$[f_1(x_\ell)](\frac{2a}{5\pi})L_{-1}$	$[f_1(x_0)](\mathrm{Sin}\frac{5\pi y_0}{a})P$
$(ab^2/30\pi)\ q_{-1}$	$[f_2(x_\ell)](\frac{2a}{5\pi})L_{-1}$	$[f_2(x_0)](\mathrm{Sin}\frac{5\pi y_0}{a})P$
$(ab/5\pi)\ q_{-1}$	$[f_3(x_\ell)](\frac{2a}{5\pi})L_{-1}$	$[f_3(x_0)](\mathrm{Sin}\frac{5\pi y_0}{a})P$
$-(ab^2/30\pi)\ q_{-1}$	$[f_4(x_\ell)](\frac{2a}{5\pi})L_{-1}$	$[f_4(x_0)](\mathrm{Sin}\frac{5\pi y_0}{a})P$

$\{F^n\}$:

aN_x
0
aN_z
0
$(2a/\pi)N_x$
0
$(2a/\pi)N_z$
0
$(2a/3\pi)N_x$
0
$(2a/3\pi)N_z$
0
$(2a/5\pi)N_x$
0
$(2a/5\pi)N_z$
0

$[K^r]$:

	1	2–8	9	10	11–16	17	18–24	25	26	27–32
1	$E_x\Lambda/b$			$-rE_x\Lambda/b$		$-E_x\Lambda/b$			$rE_x\Lambda/b$	
2–8										
9			$12E_xI_y/b^3$	$6E_xI_y/b^2$				$-12E_xI_y/b^3$	$6E_xI_y/b^2$	
10				$r^2E_x\Lambda/b + 4E_xI_y/b$		$rE_x\Lambda/b$		$-6E_xI_y/b^2$	$-r^2E_x\Lambda/b + 2E_xI_y/b$	
11–15										
16						$E_x\Lambda/b$			$-rE_x\Lambda/b$	
17–24		Remaining elements are equal to zero								
25								$12E_xI_y/b^3$	$-6E_xI_y/b^2$	
26									$r^2E_x\Lambda/b + 4E_xI_y/b$	
27–32										