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# Analysis of Conventionally Framed Hip Roofs Using Cold-formed Steel Members

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# Analysis of Conventionally Framed Hip Roofs Using Cold-Formed Steel Members

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**Abstract**

Cold-formed steel is continuing to increase in popularity in the residential construction market and is gaining an increasing market share compared with other construction materials. Conventionally framed hip roof construction uses rafters, ridge members, hip members and ceiling joists but does not include any supplemental interior supports or collar ties between rafters. Conventional framing has traditionally been used in light framed timber structures and more recently in roofs framed with cold-formed steel members. Based upon a review of building codes, specifications and standards, it was determined that analysis and design of rafters and hip members for conventional hip roof framing does not typically consider axial forces in rafters or hip members. This paper investigates the behavior of conventionally framed cold-formed steel roof framing members using elastic, finite element analysis methods. The roof system as a whole was considered in the analysis, including the contribution of the roof sheathing and ceiling joists. An analysis of members which does not consider any strength contribution from sheathing was conducted and shows that depending on the slope of the roof, axial forces developed in these members can be significant and should not be ignored. Alternatively, when roof sheathing is considered to act in combination with other framing members, the axial forces in the rafter and hip members are reduced and bending moment in the hip members are significantly reduced. This study indicates that conventional roof framing behavior can be predicted more accurately by considering the entire roof system, including sheathing, rather than as individual members.

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## Introduction

Cold-formed steel is continuing to increase in popularity in the residential construction market and is gaining an increasing market share compared with other construction materials. Conventionally framed hip roof construction uses rafters, ridge members, hip members and ceiling joists without any supplemental interior supports or collar ties between rafters. Conventional framing has traditionally been used in light framed timber structures and more recently in roofs framed with cold-formed steel members.

The objective of this preliminary study was to investigate a more rational design methodology for conventionally framed gable and hip roofs. This paper provides the results of an analysis of conventionally framed hip roofs of various slopes. Details of a three dimensional structural analysis that was conducted to investigate the interaction of plywood sheathing with a conventionally framed hip roof are presented. The main goal was to determine what contribution the sheathing makes in the distribution of forces to other members in the framing system.

## Current Design Methodology

The current method of analysis for conventionally framed roof rafters, whether of wood or cold-formed steel, is to ignore compressive forces and design the rafter for bending only (AFPA 2001, AISI 2003). However, a simple two dimensional analysis shows that a compression force is developed in the rafters.

The current conventional method of analysis for unsupported roof hip framing involves designing the members as individual pieces, neglecting any contribution of the sheathing other than to provide restraint for the rafters against lateral-torsional buckling. Side rafters act as simple beams spanning from the ridge member to exterior bearing walls and ceiling joists provide out-of-plane restraint for the bearing wall. This type of analysis ignores the axial forces, which exist in the simple truss system of sloped rafters and ceiling joists. Simple relationships from statics (equilibrium, method of joints) show that a sloped rafter member with gravity loads will develop an axial force. It should also be noted that the lower the slope of the rafter, the greater the axial compression. For an unsupported hip roof system, the rafters and hips must act as beam-columns, with both moment and axial compression. A supported hip roof system, on the other hand, would provide some type of vertical support for the ridge and hip members. This supporting member would then be required to transfer the gravity loads down to the foundation.

**3D Analysis of Hip Roof Framing System**

*Analysis Procedure*

An unsupported hip roof system does not provide direct support, via either columns or bracing, for the hip and ridge members.  Because of this, the structural support for these members is provided by the rafters that frame into them, the ridge member being supported by the side rafters, and the hip rafters being supported by the jack rafters.  Sheathed and unsheathed models for 3:12 and 10:12 roof slopes were analyzed using RISA-3D (RISA 2005) analysis and design software.

*Model Development*

All hip roof systems were analyzed assuming a 32 ft. x 60 ft. (9.75m x 19.5m) rectangular building with a 2 ft. (0.61 m) roof overhang around the perimeter of the roof.  The horizontal rafter span was 16 ft. (4.88 m) from the centerline of the ridge to the outside face of the wall.  The rafters were spaced at 24-inches on center (0.61 m), the most common spacing for roof rafters.  The ceiling joists were also spaced at 24-inches on center and were connected to the rafter at the top of the wall.  Roof slopes of 3:12 and 10:12 were analyzed.

All members were modeled with simple pin-pin connections.  The hips and ridge members were modeled as a built-up section consisting of a C-shape and a track section (Figure 1).  The hips and ridge were assumed continuous for their full length.  The ceiling joists were supported at mid-span by an interior bearing wall and were assumed continuous over the interior support.  Table 1 lists the different member sizes used for analysis of all roof models.  The member designations used in the table are in accordance with the *Standard for Cold-Formed Steel Framing – General Provisions* (AISI, 2004)

The perimeter walls were assumed to provide vertical support for gravity load from the roof rafters and ceiling joists.  They also were assumed to have sufficient in-plane stiffness to provide lateral restraint for the roof and ceiling members in the plane of the wall.  The walls were not assumed to provide any lateral support in the out-of–plane direction at the top of the wall for the roof and ceiling framing members.  The wall itself was not modeled for this analysis; however, the continuous top track of the wall was included in the model.  This member provided a tie in what would be the plane of the wall, just as in an actual structure.

The loading applied to all roof framing models were dead and snow loads. For this study, a 21 psf roof snow load was assumed as it represents a practical maximum for many portions of the United States. Wind loads were not considered for this study. Figure 2 shows the method used for load application to the rafters. A description of the loads used for this study is given as follows:

Roof Dead Load:     7 psf (335 Pa)
Ceiling Dead Load:   5 psf (240 Pa)
Roof Snow Load:    21 psf (1006 Pa)

*Unsheathed Models*

The first two models (3:12 and 10:12 roof slopes) were analyzed without including structural sheathing. As previously stated, it is common practice to neglect any contribution of roof sheathing when designing the framing members except to prevent lateral-torsional buckling of the rafter. This is usually done to simplify analysis and design, and is not assumed to add undue conservatism.

*Sheathed Models*

Wood structural panels, typically used as sheathing and attached directly to the top of the rafter, were modeled as plate elements in RISA-3D. The individual plates were applied as 1/2-inch (12.7mm) thick 4 ft. x 8 ft. (1.22 m x 2.44 m) sheets, which is the actual size of panels used for roof sheathing. The full rectangular plates were initially submeshed into 2 ft. x 1 ft. (610mm x 305 mm) rectangular subplates but then later reduced to 1 ft. x 1 ft. (305mm x 305mm) subplates.

The plates themselves were attached at their corners. They were connected to provide continuity, thereby transferring all shear forces, axial forces, and bending moments between adjacent plates. The plates were directly attached to the members, and were considered to be attached at the centerline of the rafters. This prevented any unintentional composite action between the sheathing and the rafters, even though the plates were also connected to provide continuity at all joints along the rafters. The connections to the hip and ridge members, however, did require a different approach.

It was determined the plates could not be attached directly to the hips or the ridge member because RISA-3D was treating the connection between the plates on either side of the ridge or hip as fixed. This configuration caused the rafters to act as if they were continuous over the support and thereby producing moment in the rafter at the supports. The solution to this problem was to use very

short but very stiff beams to connect the plate to the hips and ridge at the same locations where the rafters also connected to the hip and ridge members. These stiff beams were connected to the plates to provide continuity, while being pin connected to the hip and ridge members. This configuration approximated a pinned connection from the plate to the hip or ridge, allowing a transfer of shear and axial force without producing moment in the rafter. These additional stiff beam members were 0.02 ft. (6 mm) long (RISA-3D's smallest increment of noticeable change between nodes), and connected in-plane with the plates (Figure 3). This was done to prevent coplanar errors within the solution.

## RISA-3D Analysis Results

The primary focus of this study was the effect sheathing has on forces for typical rafters and hip members. A typical rafter in these models is defined as one that frames into the ridge member on one end and is supported at the other on one of the 60-foot long walls. This rafter in combination with the ceiling joist and the rafter on the opposite side of the ridge form a truss type framing system as shown in Figure 4. Forces in the typical rafter include axial compression, bending, and shear. The hip members are supported at the building corners and span to the ridge at 45 degrees to the rafters. The hip members support the jack rafters, both along the end and the side walls.

### *Unsheathed Model Results*

The results from the 3:12 and 10:12 unsheathed models are summarized in Tables 2 and 3, and provide axial force and bending moments for six of the typical hip roof framing members as shown in Figures 5 and 6.

For 3:12 roof slope, there were significant axial forces developed within all structural members investigated. The typical rafter M42 had an axial compression force of 2.60 kips (11.57 kN) at 1.9 ft. (0.58 meters) from the cantilevered (lower) end. Its maximum moment was 31.99 kip-inches (3.61 kN-m) at 10.63 ft. (3.24 m) from the cantilevered end. The axial compressive force to be used for design is at the section of maximum moment. Because the maximum axial force in the member decreases from the wall support to the ridge connection, the maximum moment does not occur at the same cross-section as the maximum axial force. The axial compression force at the point of maximum moment was 2.43 kips (10.81 kN). The hip member had significant bending of 192.48 kip-inches (21.75 kN-m) combined with an axial compressive force of 6.62 kips (29.43 kN).

The 10:12 model results also showed significant axial force present in all structural members investigated.  As expected, the axial force present in the typical rafter (M113) was reduced by more than 40% from 2.60 kips (11.57 kN) to 1.57 kips (6.97  kN) due to the increase in slope The hip had significant bending moment combined with axial compression but these forces were reduced by about 25% in bending and over 50% in axial compression from the 3:12 model.  The typical rafter, M113, had a maximum axial compression of 1.56 kips (6.97 kN) at 2.7 ft. (823 mm) past the cantilevered end.  The maximum moment was 33.71 kip-inches (3.81 kN-m) at 13.42 ft. (4.09 m) from the cantilevered end, with a concurrent axial force of 1.10 kips (4.88 kN).  The typical hip member, M35, had a bending moment of 147.18 kip-inches (16.63 kN-m) at 17.2 ft. (5.24 m) from its cantilevered end, with an axial force of 1.93 kips (8.56 kN).

*Sheathed Model Results*

The results from the 3:12 and 10:12 sheathed models are summarized in Tables 3 and 4, and provide axial force and bending data for six of the typical framing members referred to in Figures 5 and 6.  The typical rafter and hip member were of primary concern in this analysis; however, other members are included to show the effect sheathing has on the entire roof system.

For the 3:12 roof slope, bending moments in the rafters (typical and jack) were relatively consistent with the unsheathed models, however, axial compression was usually significantly reduced.  The maximum moment in the hips was only 5.58 kip-inches (0.63 kN-m) combined with an axial compression force of 2.94 kips (13.06 kN).

The results for the 10:12 roof slope were similar to the unsheathed model.  Moment in the rafters did not change appreciably between the 3:12 and 10:12 models; however, axial compression was reduced significantly as expected. The moment and axial compression for the hip member was reduced by over 70%.

**Comparison of Unsheathed and Sheathed Roof Framing Systems**

Table 5 summarizes the results between the sheathed and unsheathed models.  The results indicate the inclusion of sheathing greatly reduces the bending moment in the hip and ridge members, while also reducing the axial compression.

Comparison of the unsheathed and sheathed roof systems shows that the roof system acts more as a stiffened shell, rather than as individual members acting independently.  Rafters and the hip members share the axial forces with

the sheathing.  In general, the sheathing acts to distribute axial forces more evenly between the rafters and hips.  Both roof slopes showed over 50% reduction in axial compression in the side rafters at the point of maximum bending due to the contribution of the structural sheathing.  There was also a 35-40% reduction in the maximum axial compression in the side rafters.

## Conclusions and Recommendations

Current design methodologies for both wood and cold-formed steel roofs framed with rafters do not consider any contribution of the roof sheathing. It is apparent from the results of this preliminary study that there was an interaction between the wood structural panel sheathing and the cold-formed steel framing members of the hip roof system.  By the inclusion of sheathing in the analysis of the roof framing system, axial forces were distributed between the sheathing, rafters and hip members.  The result was a lower axial force present in the rafters and lower axial and bending forces in the hip members. The main advantage for considering sheathing to act together with the framing members is that the rafters and hip members can be designed as smaller members when comparing forces and moments to the unsheathed model results.

The results of this pilot study indicate that current design methodologies for conventionally framed roofs are not in agreement with a more rigorous elastic analysis.  These results would also suggest that the roof sheathing is contributing substantially to the structural integrity of conventionally framed roofs.  It is recommended that further study be conducted with additional models and loading configurations in an attempt to establish a more accurate design methodology for conventionally framed cold-formed steel roof systems.

## Acknowledgments

## References

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American Iron and Steel Institute (AISI) (2003), *Commentary on the Standard for Cold-Formed Steel Framing – Prescriptive Method for One and Two Family Dwellings*, AISI, Washington, D.C., December.

American Iron and Steel Institute (AISI) (2004), *Standard for Cold-Formed Steel Framing – General Provisions,* 2004 Edition, Washington, D.C.

RISA-3D Software (RISA 2005), RISA-3D Version 5.15, RISA Technologies, Foothill Ranch, CA.

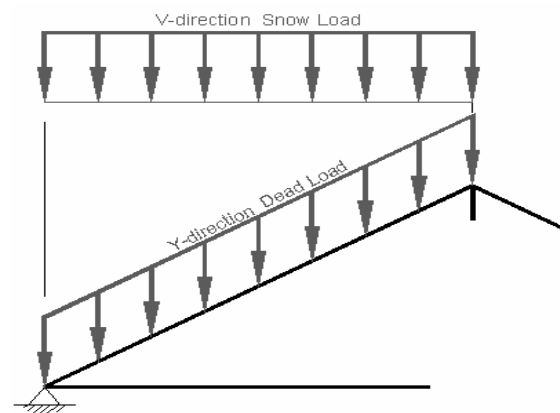Figure 1: Typical hip and ridge member configuration

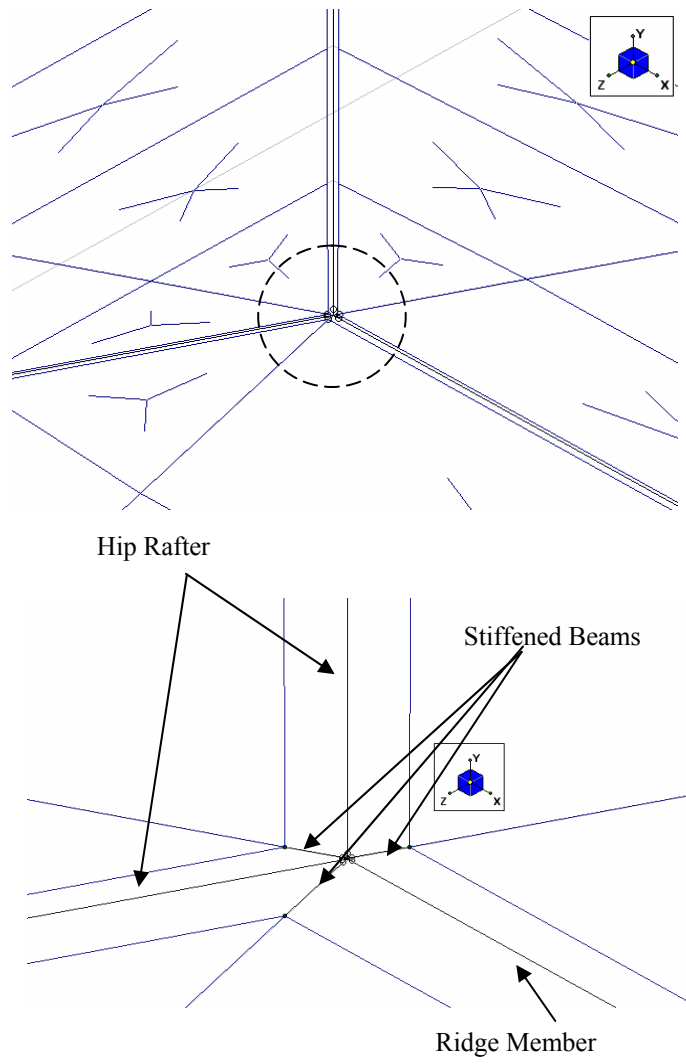

Figure 2: Dead and snow load application on rafters

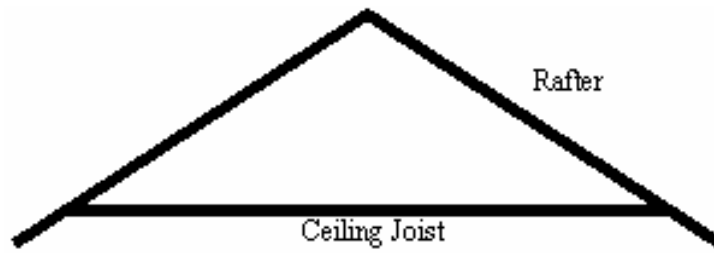Figure 3: Clarification of plate to hip assembly connection

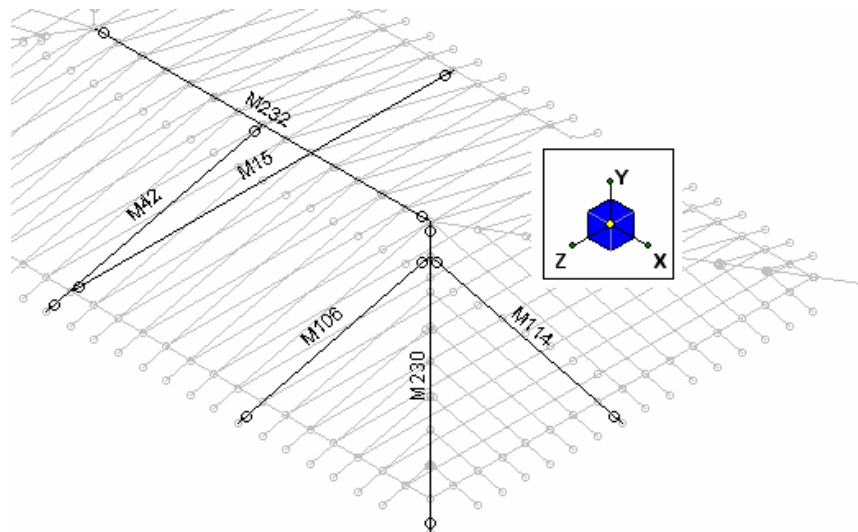Figure 4: Truss assembly formed from typical roof section



Figure 5: 3:12 members used for analysis

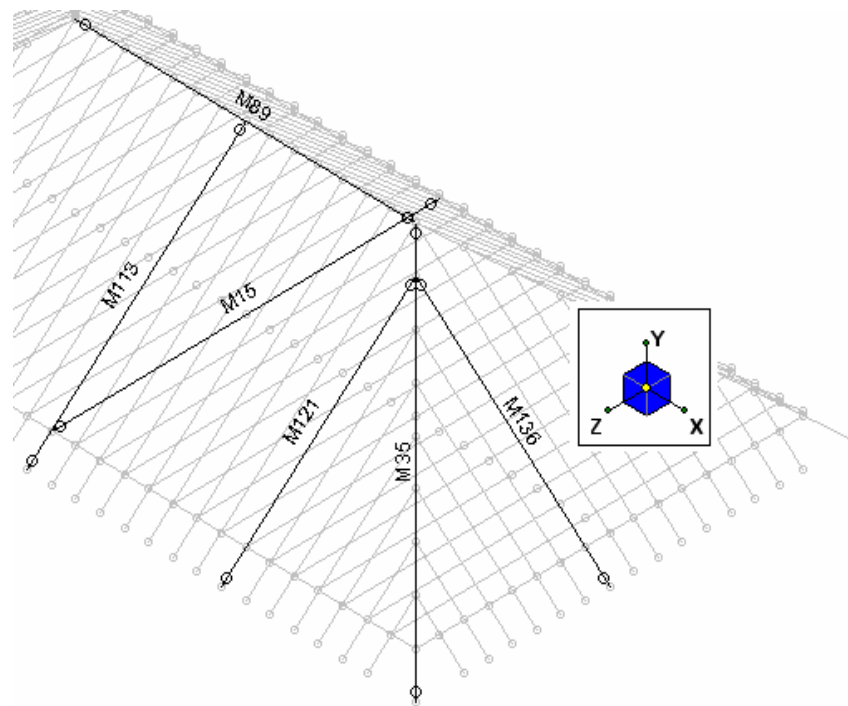Figure 6: 10:12 members used for analysis

Table 1: Typical member sizes

| Member Location | Member Size |
|---|---|
| Rafters | 1000S162-43 |
| Hip/Ridge Members | 1000S162-43 and 1000T150-43 |
| Ceiling Joists | 550S162-68 |
| Wall Tracks | 350T125-33 |

Table 2: Analysis results for 3:12 roof models

**3:12 Roof Slope: Unsheathed**

| Member Type | Member # | Max Moment | | Axial Force at Max Moment | | Max Axial Force | |
|---|---|---|---|---|---|---|---|
| | | kip-inches | kNm | kips | kN | kips | kN |
| Typical Rafter | M42 | 31.99 | 3.61 | 2.43 | 10.81 | 2.60 | 11.57 |
| Jack Rafter (side) | M106 | 23.79 | 2.69 | 1.65 | 7.33 | 1.79 | 7.97 |
| Jack Rafter (end) | M114 | 26.50 | 2.99 | 1.53 | 6.80 | 1.69 | 7.52 |
| Hip | M230 | 192.48 | 21.75 | 6.62 | 29.43 | 8.08 | 35.94 |
| Ridge | M232 | 34.76 | 3.93 | 9.10 | 40.45 | 9.10 | 40.46 |
| Ceiling Joist | M15 | -4.61 | -0.52 | -2.36 | -10.50 | -2.36 | -10.50 |

**3:12 Roof Slope: Sheathed**

| Member Type | Member # | Max Moment | | Axial Force at Max Moment | | Max Axial Force | |
|---|---|---|---|---|---|---|---|
| | | kip-inches | kNm | kips | kN | kips | kN |
| Typical Rafter | M42 | 31.41 | 3.55 | 1.17 | 5.20 | 1.68 | 7.45 |
| Jack Rafter (side) | M106 | 23.83 | 2.69 | 1.25 | 5.55 | 2.32 | 10.33 |
| Jack Rafter (end) | M114 | 23.84 | 2.69 | -0.33 | -1.46 | -0.49 | -2.18 |
| Hip | M230 | 5.58 | 0.63 | 2.94 | 13.06 | 2.94 | 13.06 |
| Ridge | M232 | 1.07 | 0.12 | 1.05 | 4.68 | 1.20 | 5.36 |
| Ceiling Joist | M15 | -4.61 | -0.52 | -2.76 | -12.28 | -2.76 | -12.28 |

Table 3: Analysis results for 10:12 roof models

**10:12 Roof Slope: Unsheathed**

| Member Type | Member # | Max Moment | | Axial Force at Max Moment | | Max Axial Force | |
|---|---|---|---|---|---|---|---|
| | | kip-inches | kNm | kips | kN | kips | kN |
| Typical Rafter | M113 | 33.71 | 3.81 | 1.10 | 4.88 | 1.57 | 6.97 |
| Jack Rafter (side) | M121 | 25.16 | 2.84 | 2.39 | 10.61 | 2.80 | 12.44 |
| Jack Rafter (end) | M136 | 27.92 | 3.15 | 2.74 | 12.18 | 3.19 | 14.18 |
| Hip | M35 | 147.18 | 16.63 | 1.93 | 8.56 | 4.79 | 21.31 |
| Ridge | M89 | 7.79 | 0.88 | 2.22 | 9.88 | 2.22 | 9.88 |
| Ceiling Joist | M15 | -4.61 | -0.52 | -0.84 | -3.75 | -0.84 | -3.75 |

**10:12 Roof Slope: Sheathed**

| Member Type | Member # | Max Moment | | Axial Force at Max Moment | | Max Axial Force | |
|---|---|---|---|---|---|---|---|
| | | kip-inches | kNm | kips | kN | kips | kN |
| Typical Rafter | M113 | 33.45 | 3.78 | 0.47 | 2.09 | 0.89 | 3.96 |
| Jack Rafter (side) | M121 | 25.51 | 2.88 | 0.51 | 2.26 | 0.82 | 3.63 |
| Jack Rafter (end) | M136 | 24.75 | 2.80 | -0.04 | -0.18 | 0.23 | 1.02 |
| Hip | M35 | 1.90 | 0.21 | 0.53 | 2.36 | 1.17 | 5.19 |
| Ridge | M89 | 0.20 | 0.02 | 0.27 | 1.21 | 0.28 | 1.25 |
| Ceiling Joist | M15 | -4.61 | -0.52 | -0.85 | -3.79 | -0.85 | -3.79 |

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Table 5: Comparison of results between sheathed and unsheathed models

| Member Type | 3:12 Roof Slope | | | 10:12 Roof Slope | | |
|---|---|---|---|---|---|---|
| | % Reduction (moment) | % Reduction (axial at moment) | % Reduction (axial at max) | % Reduction (moment) | % Reduction (axial at moment) | % Reduction (axial at max) |
| Typical Rafter | 1.81% | 51.95% | 35.56% | 0.77% | 57.25% | 43.14% |
| Jack Rafter (side) | -0.17% | 24.33% | -29.63% | -1.41% | 78.75% | 70.83% |
| Jack Rafter (end) | 10.03% | 121.53% | 129.05% | 11.37% | 101.50% | 92.79% |
| Hip | 97.10% | 55.61% | 63.66% | 98.71% | 72.47% | 75.66% |
| Ridge | 96.93% | 88.43% | 86.76% | 97.45% | 87.75% | 87.39% |
| Ceiling Joist | 0.00% | -16.99% | -16.99% | 0.00% | -0.95% | -0.95% |

*Italics*: Member switched from maximum compressive value to maximum tension value