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On the Computation of the Cross-section Properties of Arbitrary Thin-walled Structures

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Abstract

In this paper, a generalized computational algorithm based on the line chain and tree models is developed for the cross section properties of arbitrarily configuration struts without closed loops. The two C++ programs for such models are developed. However, the two models cannot apply to struts with any cross-section possessing closed loops. Therefore, the further investigation should be completed.

1. Introduction

Cold-formed, thin-walled struts are widely used to support piping, conduit for building construction and other applications like storage shelving. Such thin-walled struts possess complex configurations to satisfy industrial needs, and the corresponding sections are open. Under external loading, the flexural, torsional and torsional-flexural buckling of thin-walled struts with open cross-sections may occur. The warping constant is a key to determine the torsional and torsional-flexural buckling. Cooper B-Line Inc. (in Highland, Illinois, USA) manufactures a range of cold-formed components for the support and restraint of electronic wiring, supports of pipes and similar support systems. Even though struts have standardized cross sections in B-Line systems, the stress and buckling designs are still not available in textbooks or industry standards such as AISI Cold-Formed Steel Design Manual (1996). Therefore, a generalized computational algorithm will be developed to solve such a problem.

In earlier investigations, one considered the effect of imperfections in materials, unavoidable eccentricity of loading as a hypothetical initial curvature of the thin-walled structures. Based on such assumptions, Ayrton and Perry (1886) gave an investigation of thin walled structures for safe loading. The Perry's formula applies strictly when the failure is by bending alone. Timoshenko (1945) developed a theoretical model for the elastic instability of struts having thin-walled open sections. In that theoretical model, the torque applied on the thin-walled structures

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consists of two parts resulting from purely torsional and warping stresses. In 1948, Baker and Roderick extended the development of the Timoshenko's model. The warping and buckling stability was investigated, and such an investigation can be found in Timoshenko et al. (1967) and Mikkola (1967) as well. With extensive uses of thin walled structures, the investigation on the stability for such structures becomes popular. Loughlan (1993) investigated the weakening effect of local buckling on the compressive load, and Chapman and Buhagiar (1993) used the Young's buckling equation to design against torsional buckling. In 1997, Wang considered the shear lag to investigate lateral buckling of thin walled structures through the spline finite member element method (see also, Wang, 1999; Wang and Li, 1999). In addition, Pi et al. (1999) investigated lateral buckling strengths of cold-formed Z-section beams, and Kesti and Davies (1999) discussed local and distorsional buckling of the thin-walled short columns. All the aforementioned investigations are based on some typical cross-sections of thin-walled structures. With modern industrial needs, the combination of common struts generates very complex cross-sections used in industry. The computation of cross-section properties is very difficult but such a computation is very important. Therefore, a generalized algorithm will be developed to obtain the cross section properties, including shear center and warping constant. The single line-chain and line-tree models for the line configurations of the cross-sections of struts are considered, as shown in Figure 1.

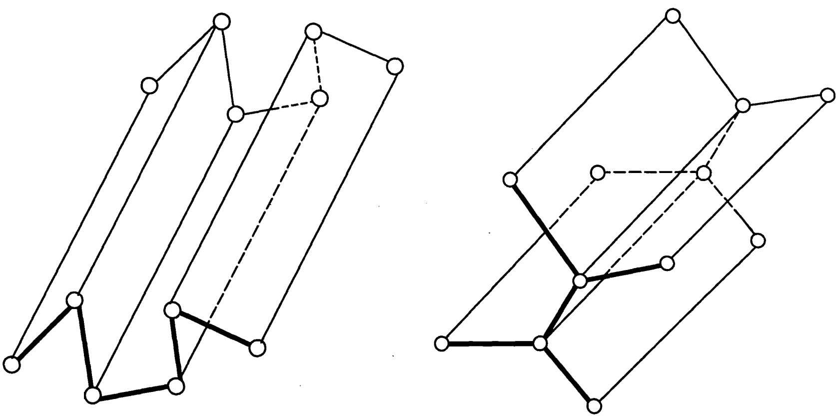

Figure 1 Struts with cross sections possessing line configuration: line-chain model (left) and line-tree model (right). (The circular symbols denote nodes and two close nodes are connected through a straight line.)

2. Line chain model

Consider a cross-section of thin walled struts with n-line segments, sketched in Figure 2. Such a cross-section configuration is positioned in the frame oxy, and each segment is modeled through a line and two nodes with the corresponding thickness of struts. The nodes of the line-chain cross-section are numbered from 0 to n. The node information includes the coordinates (x, y) of node location, and the second node of each segment stores its thickness. For instance, the i^{th} segment with thickness t_i has two nodes relative to the $(i-1)^{\text{th}}$ and i^{th} nodes. The i^{th} node is defined through (x_i, y_i, t_i). Note that the initial node is expressed by $(x_0, y_0, 0)$. Such a definition is very convenient for computer program. The frame OXY is relative to the principal moments of inertia and the origin is at the centroid of the entire cross-section.

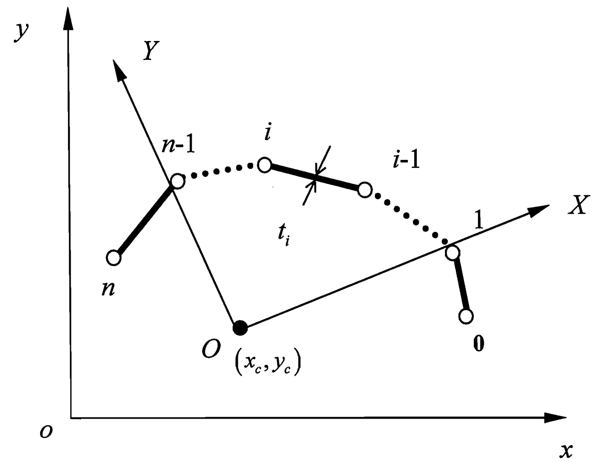

Figure 2 Line-chain model. The frame oxy is original, and the frame OXY is relative to the principal moments of inertia and its origin is at centroid.

In the coordinate (x, y), the cross section area is

$$A = \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} t_i l_i \tag{1}$$

and two components of centroid in the x and y-directions are given by

$$x_c = \frac{1}{A}\sum_{i=1}^{n} A_i \overline{x}_i, \text{ and } y_c = \frac{1}{A}\sum_{i=1}^{n} A_i \overline{y}_i; \tag{2}$$

where the length, center and coordinate differences of the i^{th} segment are

$$\left.\begin{array}{l} l_i = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}; \\ \overline{x}_i = \frac{1}{2}(x_i + x_{i-1}),\ \overline{y}_i = \frac{1}{2}(y_i + y_{i-1}); \\ d_{x_i} = x_i - x_{i-1}, d_{y_i} = y_i - y_{i-1}. \end{array}\right\} \tag{3}$$

The coordinate (x,y) is mapped into a new centroidal coordinate (x',y') through a map consisting of $x = x' + x_c$ and $y = y' + y_c$. The St. Venant torsion constant and moments of inertia of the i^{th} segment are:

$$J = \frac{1}{3}\sum_{i=1}^{n} l_i t_i^3 \quad \text{for } l_i >> t_i, \tag{4}$$

$$\left.\begin{aligned} I_{x'} &= \sum_{i=1}^{n}(\bar{y}_i^2 A_i + \frac{1}{12}d_{y_i}^2 A_i) - y_c^2 A, \\ I_{y'} &= \sum_{i=1}^{n-1}(\bar{x}_i^2 A_i + \frac{1}{12}d_{x_i}^2 A_i) - x_c^2 A, \\ I_{x'y'} &= \sum_{i=1}^{n-1}(\bar{x}_i \bar{y}_i A_i + \frac{1}{12}d_{x_i} d_{y_i} A_i) - x_c y_c A. \end{aligned}\right\} \tag{5}$$

The principal moments of inertia are computed

$$I_{\max}, I_{\min} = \frac{1}{2}(I_{x'} + I_{y'}) \pm \frac{1}{2}\sqrt{(I_{x'} - I_{y'})^2 + 4I_{x'y'}{}^2}, \tag{6}$$

and the angle between x and $I_{x(p)}$ is computed by

$$\theta_p = \begin{cases} \frac{1}{2}\arctan\left[2I_{x'y'}/\left(I_{y'} - I_{x'}\right)\right], & \text{if } I_{x'y'} \neq 0; \\ \pi/2, & \text{if } I_{x'y'} = 0 \text{ and } I_{x'} < I_{y'}; \\ 0, & \text{if } I_{x'y'} = 0 \text{ and } I_{x'} \geq I_{y'}. \end{cases} \tag{7}$$

Set the maximum principal axes of the moments of inertia $I_{\max}$ and $I_{\min}$ to be X and Y axes, respectively, and the origin is at the centroid of the entire cross-section. To compute shear center in the X-direction, the corresponding coordinate (x,y) transforms to the new frame (X,Y) for all nodes

$$\begin{pmatrix} X_i \\ Y_i \end{pmatrix} = \begin{pmatrix} x_i - x_c & y_i - y_c \\ y_i - y_c & -(x_i - x_c) \end{pmatrix} \begin{pmatrix} \cos\theta_p \\ \sin\theta_p \end{pmatrix}. \tag{8}$$

Based on shear flow distributions in the cross section, the shear center in the X-direction is computed by

$$X_s = \begin{cases} 0, & \text{if } I_{\max} = 0, \\ -\frac{1}{I_{\max}}\sum_{i=1}^{n}\delta_{X(i)} d_{X(i)} l_i \left[V_{X(i-1)} + \frac{1}{6}A_i(Y_i + 2Y_{i-1})\right], & \text{if } I_{\max} \neq 0; \end{cases} \tag{9}$$

where

$$d_{X(i)} = \frac{1}{l_i}\left|X_{i-1}Y_i - X_iY_{i-1}\right|,\ V_{X(i)} = V_{X(i-1)} + \frac{A_i}{2}(Y_i + Y_{i-1}),\ V_0 = 0;$$
$$\delta_{X(i)} = \begin{cases} 1, & \text{if } Y_{i-1}(X_i - X_{i-1}) < X_{i-1}(Y_i - Y_{i-1}), \\ -1, & \text{if } Y_{i-1}(X_i - X_{i-1}) > X_{i-1}(Y_i - Y_{i-1}), \\ 0, & \text{if } Y_{i-1}(X_i - X_{i-1}) = X_{i-1}(Y_i - Y_{i-1}). \end{cases} \tag{10}$$

In a similar fashion, the shear center in the Y-axis is:

$$Y_s = \begin{cases} 0, & \text{if } I_{\min} = 0, \\ -\frac{1}{I_{\min}}\sum_{i=1}^{n} \delta_{Y(i)} d_{Y(i)} l_i \left[V_{Y(i-1)} - \frac{1}{6}A_i(X_i + 2X_{i-1}) \right], & \text{if } I_{\min} \neq 0; \end{cases} \tag{11}$$

where

$$d_{Y(i)} = \frac{1}{l_i}\left|Y_iX_{i-1} - Y_{i-1}X_i\right|, V_{Y(i)} = V_{Y(i-1)} - \frac{A_i}{2}(X_i + X_{i-1}),\ V_{Y(0)} = 0;$$
$$\delta_{Y(i)} = \begin{cases} 1, & \text{if } X_{i-1}(Y_i - Y_{i-1}) < Y_{i-1}(X_i - X_{i-1}), \\ -1, & \text{if } X_{i-1}(Y_i - Y_{i-1}) > Y_{i-1}(X_i - X_{i-1}), \\ 0, & \text{if } X_{i-1}(Y_i - Y_{i-1}) = Y_{i-1}(X_i - X_{i-1}). \end{cases} \tag{12}$$

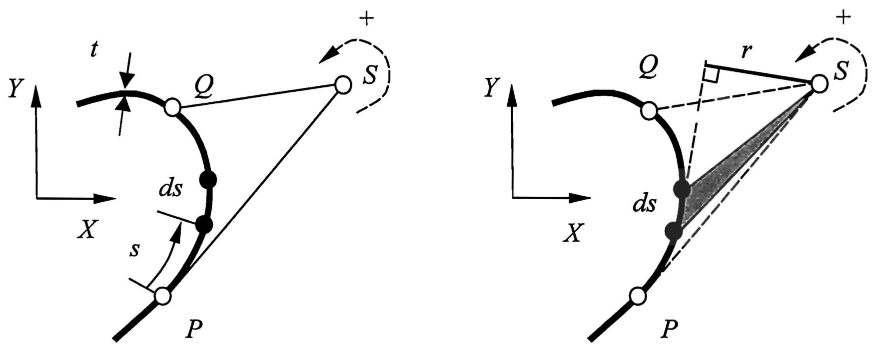

Figure 3 Struts with an arbitrary cross section at rotation center at point S: basic configuration (left) and increment of areas wept by rotation (right).

To compute the warping constant, the sectorial area for an arbitrary cross section is introduced and through the sectorial area, the warping constant is computed as in Cook and Young (1999), i.e.,

$$\omega = \int_0^s r\,ds, \text{ and } C_w = t\int_A \omega^2 ds, \tag{13}$$

where s is the local coordinate along the middle line of the thin walled cross section, and t is thickness for the thin-walled cross section. A distance r is from the rotation point P (i.e., shear center for struts) to the tangential direction of the differential element ds, as shown in Figure 3.

For the line chain model, we have

$$\omega = \sum_{i=1}^{n} \omega_i = \sum_{i=1}^{n} \int_{s_{i-1}}^{s} r_i ds, \; C_w = \sum_{i=1}^{n} t_i \int_{s_{i-1}}^{s_i} \omega_i^2 ds \;\text{ for the } i^{\text{th}} \text{ segment;} \tag{14}$$

Therefore, the warping constant is computed by

$$C_w = \sum_{i=1}^{n} A_i \left(w_{i-1}^2 + w_{i-1} d_{si} l_i \delta_{si} + \frac{1}{3} d_{si}^2 l_i^2 - 2 w_{av} w_{i-1} - w_{av} d_{si} l_i \delta_{si} + w_{av}^2 \right) \tag{15}$$

where

$$w_i = w_{i-1} + d_{si} d_i \delta_{si}, \; w_0 = 0; \; w_{a(i)} = \frac{1}{2}(w_{a(i-1)} + w_i), \; w_{a0} = 0; \; w_{av} = \frac{1}{A} \sum_{i=1}^{n} A_i w_{a(i)}; \tag{16}$$

and

$$\left.\begin{array}{l} X_{s(i)} = X_i - X_s, \; Y_{s(i)} = Y_i - Y_s; \; d_{si} = \dfrac{1}{l_i} \left| X_{s(i-1)} Y_{s(i)} - X_{s(i)} Y_{s(i-1)} \right|; \\ \delta_{si} = \begin{cases} 1, & \text{if } Y_{s(i-1)}(X_i - X_{i-1}) < X_{s(i-1)}(Y_i - Y_{i-1}), \\ -1, & \text{if } Y_{s(i-1)}(X_i - X_{i-1}) > X_{s(i-1)}(Y_i - Y_{i-1}), \\ 0, & \text{if } Y_{s(i-1)}(X_i - X_{i-1}) = X_{s(i-1)}(Y_i - Y_{i-1}). \end{cases} \end{array}\right\} \tag{17}$$

Consider the B22 strut of B-Line systems (1999) as a sampled problem. The cross-section model of struts and input parameters are given. For the B22 strut, the computational model is given in Figure 4. The (x, y) coordinates are an initial frame selected arbitrarily. A root (initial) node is selected from the beginning or end of the chain, and all nodes of the cross-section are numbered. From node numbers, the input data including coordinates and line thickness is tabulated in Table 1. The point (x_c, y_c) is the location of centroid in the (x, y)-coordinate frame, which is as the origin of the (X, Y) coordinate frame relative to the moment of inertia principals.

Running the corresponding C++ program gives all cross section properties for the B22 strut: Area is $A = 0.57070 \text{ in}^2$. St. Venant Constant is $J = 0.00199 \text{ in}^4$. The location of centroid is $(x_c, y_c) = (0.68115, 0.0000)$ (unit: in). The origin is mapped into the centroid through a translation map $x = x' + x_c$ and $y = y' + y_c$, and a new coordinate frame is (x', y') with the origin at the centroid. The moments of inertia for the (x', y') coordinates are $I_{x'} = 0.24426 \text{ in}^4$ and $I_{y'} = 0.19919 \text{ in}^4$, respectively, and the product of inertia to x'-y' plane is $I_{x'y'} = 0.0 \text{ in}^4$. Further, the corresponding maximum and minimum principal moments of inertia are computed, and the principal axes of the inertia moment are assigned to X and Y-axes respectively. Therefore, $I_{pX} = 0.24426 \text{ in}^4$, $I_{pY} = 0.19919 \text{ in}^4$ and the rotation angle from x'-axis to X-axis is $\theta_p = 0$. Through such a rotation angle, the coordinate (x', y') is converted to the (X, Y)-frame. The location of shear center is computed, (X_s, Y_s) = $(-1.5806,\ 0.0000)$ (unit: in.). The warping constant for the cross-section is C_w = 0.15575 in^4. In a similar manner, The cross-section properties for other struts B11, B12, B24, B52, B54 in B-line (1999) are computed, and the results are tabulated in Table 2.

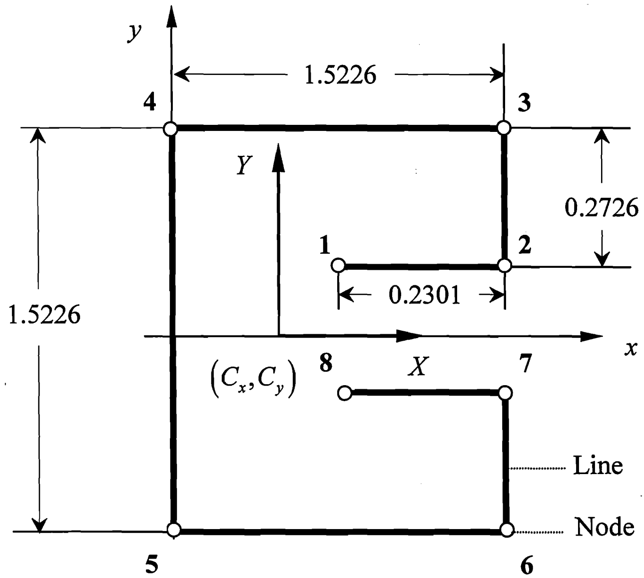

Figure 4 Th cross section of strut B22 of B-Line systems (uniform thickness: 0.1024 inch).

Table 1 Input data for the cross-section properties of the B22 of B-Line systems (unit: inch)

Node number	*x*(in.)	*y*(in.)	*t*(in.)
1	1.2925	0.4887	0.0000
2	1.5226	0.4887	0.1024
3	1.5226	0.7613	0.1024
4	0.0000	0.7613	0.1024
5	0.0000	- 0.7613	0.1024
6	1.5226	- 0.7613	0.1024

Table 2 Cross Section Constants for B11, B12, B22, B24, B52, B54 in B-Line (1999)

	B11	B12	B22	B24	B52	B54
A (in^2)	0.90350	0.73710	0.57070	0.42926	0.40430	0.30771
x_c (in)	1.47555	1.07630	0.68115	0.70234	0.29513	0.31297
y_c (in)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
J (in^4)	0.00316	0.00258	0.00199	0.00080	0.00141	0.00057
$I_{x'}$ (in^4)	0.43714	0.34070	0.24426	0.18866	0.14782	0.11563
$I_{y'}$ (in^4)	1.14843	0.55222	0.19919	0.15656	0.03427	0.02833
$I_{x'y'}$ (in^4)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
I_{pX} (in^4)	1.14843	0.55222	0.24426	0.18866	0.14782	0.11563
I_{pY} (in^4)	0.43714	0.34070	0.19919	0.15656	0.03427	0.02833
θ_p (rad)	1.57080	1.57080	0.00000	0.00000	0.00000	0.00000
X_s (in)	0.00000	0.00000	-1.58064	-1.63316	-0.75200	-0.79818
Y_s (in)	3.21213	2.39783	0.00000	0.00000	0.00000	0.00000
C_w (in^6)	0.84662	0.41800	0.15575	0.13584	0.02788	0.02568

3. Line tree model

Consider a line tree model of the open cross-section without any close loop, as shown in Figure 5. The coordinates (x,y) and (X,Y) are set up as in the line chain model. In the line tree model, it is very important that the concept of the parent-children nodes should be introduced. For a cross section with the line tree model, any single path can be traced back to the root node. For each path, if a node is directly from another certain node, the certain node is called the parent one and the node itself is termed the child one. The parent node may have a lot of children, but each child node has only one parent. Consider the i^{th} line segment having the $(i-1)^{\text{th}}$ and i^{th} nodes. To compute the cross-section properties, the parent-children hereditary system is established herein, as shown in Figure 4. The $(i-1)^{th}$ node is one of children nodes of the $(i-2)^{\text{th}}$ node. In other wards, the $(i-2)^{\text{th}}$ node is the parent of the $(i-1)^{th}$ node. For the $(i-1)^{th}$ node, it has many children nodes numbered by $(i,j,k,\cdots)$. Therefore, the $(i-1)^{th}$ node is the parent of such nodes, and this node is stored in the information archive of those nodes as the parent information through expressions $p_i, p_j, p_k, \cdots$. Similarly, the i^{th} node has children nodes $(i_1, i_2, \cdots, i_r, \cdots, i_m)$, and the node is the parent of the nodes $(i_1, i_2, \cdots, i_r, \cdots, i_m)$, where m is the total number of the direct children nodes of the i^{th} node. Through this tree structure, any node has only one parent except for starting node (or root node), but may have many children except for the end node of each branch.

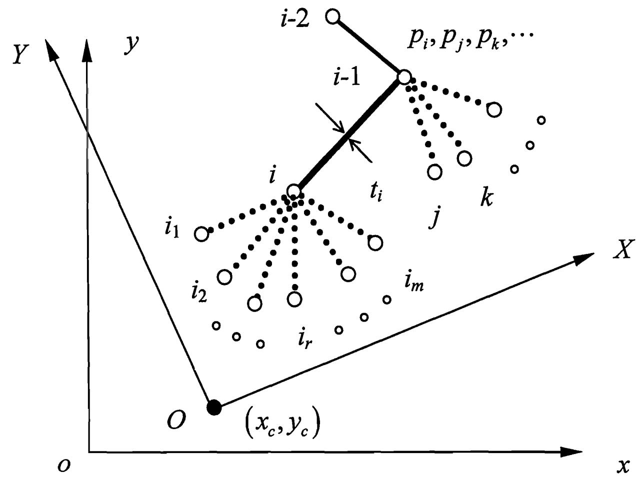

Figure 5 The parent-children tree structure for the line-tree model. The frame oxy is original, and the frame OXY is relative to the principal moments of inertia and its origin is at centroid.

For the i^{th} segment, all formulas for the cross-section properties are derived. In the coordinate (x, y), the cross section area, the St. Venant torsion constant, moments of inertia and the corresponding principal inertia are computed through equations (1), (2), (4)-(7), respectively. However, the length, center and coordinate differences of the i^{th} segment are computed by

$$\left.\begin{aligned} & l_i = \sqrt{(x_i - x_{p_i})^2 + (y_i - y_{p_i})^2}; \\ & \bar{x}_i = \frac{1}{2}\left(x_i + x_{p_i}\right), \bar{y}_i = \frac{1}{2}\left(y_i + y_{p_i}\right); \\ & d_{x_i} = x_i - x_{p_i}, d_{y_i} = y_i - y_{p_i}; \end{aligned}\right\} \tag{18}$$

where $\left(x_{p_i}, y_{p_i}\right)$ are the coordinate values for the parent node of the i^{th} node. In a similar manner, set the maximum principal axes of the inertia moment $I_{\max}$ and $I_{\min}$ to be X and Y axes, respectively, and the origin is at the centroid of the entire cross section. To compute shear center in the frame OXY, the corresponding coordinates are mapped into the new frame (X,Y) for all nodes through equation (8), and the parent node $\left(x_{p_i}, y_{p_i}\right)$ is mapped to $\left(X_{p_i}, Y_{p_i}\right)$ as well. For the i^{th} node having the child nodes i_r $(1 \le r \le m)$ where m is the total number of the direct children nodes of the i^{th} node, the function S_{X_i} is computed

$$S_{X_i} = \begin{cases} 0, & \text{if } I_{\max} = 0, \\ -\dfrac{1}{I_{\max}} \displaystyle\sum_{r=1}^{m} \delta_{X(i_r)} d_{X(i_r)} l_{i_r} \left[V_{X(i_r)} + \frac{1}{6} A_{i_r} (Y_i + 2Y_{i_r}) \right], & \text{if } I_{\max} \neq 0; \end{cases} \tag{19}$$

where

$$d_{X(i_r)} = \frac{1}{l_{i_r}} \left| X_{i_r} Y_i - X_i Y_{i_r} \right|, V_{X(i)} = \sum_{r=1}^{m} V_{X(i_r)} + \frac{1}{2} A_{i_r} \left(Y_i + Y_{i_r} \right);$$
$$\delta_{X(i_r)} = \begin{cases} 1, & \text{if } Y_{i_r}(X_i - X_{i_r}) < X_{i_r}(Y_i - Y_{i_r}), \\ -1, & \text{if } Y_{i_r}(X_i - X_{i_r}) > X_{i_r}(Y_i - Y_{i_r}), \\ 0, & \text{if } Y_{i_r}(X_i - X_{i_r}) = X_{i_r}(Y_i - Y_{i_r}). \end{cases} \tag{20}$$

Based on the recursive function, the functions $V_{X(i)}$ and S_{X_i} are computed and $X_s = S_{X_0}$. Similarly, we have

$$S_{Y_i} = \begin{cases} 0, & \text{if } I_{\min} = 0, \\ -\dfrac{1}{I_{\min}} \displaystyle\sum_{r=1}^{m} \delta_{Y(i_r)} d_{Y(i_r)} l_{i_r} \left[V_{Y(i_r)} - \frac{1}{6} A_{i_r} (2X_{i_r} + X_i) \right], & \text{if } I_{\min} \neq 0; \end{cases} \tag{21}$$

where

$$d_{Y(i_r)} = \frac{1}{l_{i_r}} \left| Y_i X_{i_r} - Y_{i_r} X_i \right|, V_{Y(i)} = \sum_{r=1}^{m} V_{Y(i_r)} - \frac{A_{i_r}}{2} \left(X_i + X_{i_r} \right);$$
$$\delta_{Y(i_r)} = \begin{cases} 1, & \text{if } X_{i_r}(Y_i - Y_{i_r}) < Y_{i_r}(X_i - X_{i_r}), \\ -1, & \text{if } X_{i_r}(Y_i - Y_{i_r}) > Y_{i_r}(X_i - X_{i_r}), \\ 0, & \text{if } X_{i_r}(Y_i - Y_{i_r}) = Y_{i_r}(X_i - X_{i_r}). \end{cases} \tag{22}$$

$Y_s = S_{Y_0}$ is obtained through the function $V_{Y(i)}$ and S_{Y_i} computed by the recursive function.
The warping constant is

$$C_w = \sum_{i=1}^{n} A_i \left(w_{p_i}^2 + w_{p_i} d_{si} l_i \delta_{si} + \frac{1}{3} d_{si}^2 l_i^2 - 2 w_{av} w_{p_i} - w_{av} d_{si} l_i \delta_{si} + w_{av}^2 \right); \tag{23}$$

where

$$w_i = w_{p_i} + d_{si} d_i \delta_{si}, \; w_0 = 0; \; w_{a(i)} = \frac{1}{2}(w_{a(p_i)} + w_i), \; w_{a0} = 0; \; w_{av} = \frac{1}{A} \sum_{i=1}^{n} A_i w_{a(i)}; \tag{24}$$

and

$$\left.\begin{aligned} &X_{s(i)} = X_i - X_s,\ Y_{s(i)} = Y_i - Y_s; \\ &X_{s(p_i)} = X_{p_i} - X_s, Y_{s(p_i)} = Y_{p_i} - Y_s; \\ &d_{si} = \frac{1}{l_i}\left|X_{s(p_i)}Y_{s(i)} - X_{s(i)}Y_{s(p_i)}\right|; \\ &\delta_{si} = \begin{cases} 1, & \text{if } Y_{s(p_i)}(X_i - X_{p_i}) < X_{s(p_i)}(Y_i - Y_{p_i}), \\ -1, & \text{if } Y_{s(p_i)}(X_i - X_{p_i}) > X_{s(p_i)}(Y_i - Y_{p_i}), \\ 0, & \text{if } Y_{s(p_i)}(X_i - X_{p_i}) = X_{s(p_i)}(Y_i - Y_{p_i}). \end{cases} \end{aligned}\right\} \tag{25}$$

Note that the function w_i is computed through the recursive function.

The implementation of the tree model is much more complicated than the line model. This model needs a standard data structure of tree. It is implemented by using the dynamic array plus the computation of recursive functions. Through the theoretical development, a C++ program will be developed for the line tree model. As in the line chain model, a root node should be selected first. But in this model, any node or even a casual point (for example, middle point) in a line can be selected as a root node, and then number the nodes naturally. Furthermore, a certain sequence is followed for the sake of simplicity.

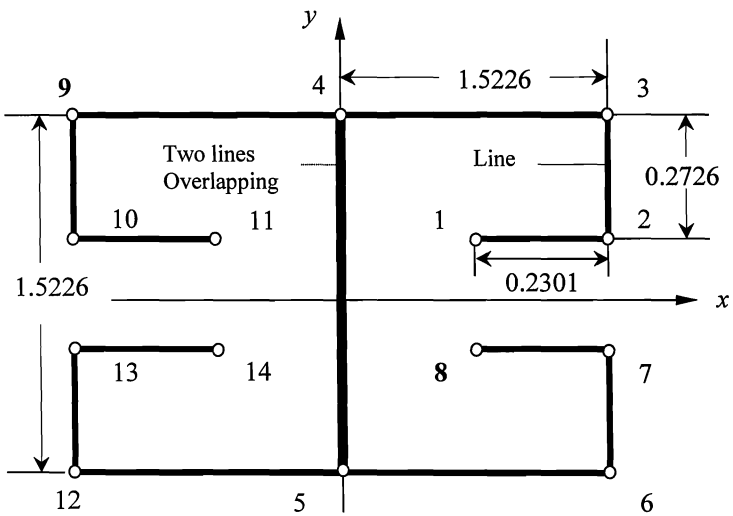

Figure 6 The cross section of the strut B22A of B-Line systems (thickness: 0.1024 inch).

Table 3 Input data for the cross-section properties of the B22A of B-Line systems (unit: inch)

Node number	x(in.)	y(in.)	t(in.)	p(parent index)
1	1.3437	0.4887	0.0000	0
2	1.5738	0.4887	0.1024	1
3	1.5738	0.7613	0.1024	2
4	0.0000	0.7613	0.1024	3
5	0.0000	- 0.7613	0.2048	4
6	1.5738	- 0.7613	0.1024	5
7	1.5738	- 0.4887	0.1024	6
8	1.3437	- 0.4887	0.1024	7
9	-1.5738	0.7613	0.1024	4
10	-1.5738	0.4887	0.1024	9
11	-1.3437	0.4887	0.1024	10
12	-1.5738	-0.7613	0.1024	5
13	-1.5738	-0.4887	0.1024	12
14	-1.3437	-0.4887	0.1024	13

Consider the B22A strut of B-Line systems as an example, and node-1 is selected as the root node, as shown in Figure 6. All the other nodes are numbered. For node-13, it can traced back to the root node-1 through a path: node 13 $\Rightarrow$ 12 $\Rightarrow$ 5 $\Rightarrow$ 4 $\Rightarrow$ 3 $\Rightarrow$ 2 $\Rightarrow$ 1. Node 12 is the parent of node-13, and node-5 is the parent of node-12, etc. However, node-5 has two children: node-6 and node-12. Except for the root node, each node has only one parent. Once the root node is assigned, the parent indices of the other nodes will be determined accordingly. The parent node index will be as a parameter in the node data information. Therefore, in the C++ program, the data structure of a node is changed from (x, y, t) in the line chain model to (x, y, t, p) for the line tree model, where p denotes the index of the parent node of the node. The input data for computation of the cross section is in Table 3. Running the C++ program for the B22A gives the corresponding cross section properties in Table 4. The other struts in B-line systems relative to the line tree model are also computed, and the corresponding results are given in Table 4.

4. Conclusions

In this paper, the line chain and tree general models are developed for the cross section properties of arbitrarily configuration struts of B-Line systems without closed loops. Based on the two models, two C++ programs are developed. From the cross-section properties, the buckling stability of struts under centroidally loading can be investigated However, the two models cannot apply to struts with cross-section possessing closed loops.

Table 4 Cross Section Constants for B11A, B12A, B22A, B24A, B52A, B54A in B-Line (1999)

	B11A	B12A	B22A	B24A	B52A	B54A
A (in^2)	1.82796	1.49512	1.16236	0.86968	0.83366	0.62658
x_c (in)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
y_c (in)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
J (in^4)	0.00966	0.00850	0.00733	0.00292	0.00618	0.00247
$I_{x'}$ (in^4)	0.88644	0.69355	0.50067	0.38403	0.30877	0.23798
$I_{y'}$ (in^4)	6.50808	2.97754	1.00975	0.78254	0.16587	0.13189
$I_{x'y'}$ (in^4)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
I_{px} (in^4)	6.50808	2.97754	1.00975	0.78254	0.30877	0.23798
I_{py} (in^4)	0.88644	0.69355	0.50067	0.38403	0.16587	0.13189
θ_p (rad)	-1.57080	-1.57080	-1.57080	-1.57080	0.00000	0.00000
X_s (in)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Y_s (in)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C_w (in^4)	4.51388	2.13922	0.76568	0.63374	0.13975	0.11846

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