



Missouri University of Science and Technology
Scholars' Mine

International Specialty Conference on Cold-Formed Steel Structures

(1971) - 1st International Specialty Conference on Cold-Formed Steel Structures

Aug 20th, 12:00 AM

A Review of the Effective Width Formula

N. C. Lind

M. K. Ravindra

J. Power

Follow this and additional works at: <https://scholarsmine.mst.edu/isccss>

 Part of the [Structural Engineering Commons](#)

Recommended Citation

Lind, N. C.; Ravindra, M. K.; and Power, J., "A Review of the Effective Width Formula" (1971). *International Specialty Conference on Cold-Formed Steel Structures*. 2.

<https://scholarsmine.mst.edu/isccss/1iccfss/1iccfss-session2/2>

This Article - Conference proceedings is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in International Specialty Conference on Cold-Formed Steel Structures by an authorized administrator of Scholars' Mine. This work is protected by U. S. Copyright Law. Unauthorized use including reproduction for redistribution requires the permission of the copyright holder. For more information, please contact scholarsmine@mst.edu.

by

Niels C. Lind*, A.M. ASCE, Mayasandra K. Ravindra** and John Power***

INTRODUCTION

The purpose of this paper is to present a comprehensive review of the effective width expression used in light gage steel design. This study was made by the authors in connection with a review of the Canadian Standard S-136 for design of light gage steel structural members in buildings.

If a long thin plate is longitudinally compressed, such as the top flange of a light gage hat section in simple bending, it will buckle in a regular wave-like manner when the axial stress reaches a critical value, but it will not collapse if the material is elastic-plastic. The buckled plate can resist loads much larger than critical, with moderate additional deformation; with increasing load central strips of the plate deflect more, but they hardly participate in carrying the increase in load. The compressive stresses are continually redistributed so that stresses are increasing at the edges while remaining nearly constant over a central zone. The width of this zone depends on several factors of the problem in a complicated way.

The effective width concept for the design of such plates in compression was first introduced by von Karman [1] in 1932. The stress distribution across the width can be replaced by an equivalent distribution that is uniform over a portion, called the "effective width", of the plate. While such a substitution is always possible in a problem of this kind, it is useful only if it simplifies the design calculations. By an approximate analysis of classical elegance, he showed that the effective width, b , if less than the total width w , at full axial load capacity should be nearly independent of the total width and of the applied stress; further, that it should depend only on geometry and a material constant in the following simple way:

$$b = Bt\sqrt{E/f_y} \tag{1}$$

where t is the plate thickness, E is Young's Modulus and f_y is the yield strength of the material. According to the theory B is a constant which von Karman determined to be approximately equal to $\pi(3 - 3\nu^2)^{-1/2}$ where ν is Poisson's ratio. B may be called the "normalized effective width (at stress f_y)". Here the capital letter denotes a length that has been normalized with respect to a stress f (yield stress in Eq. 1):

$$B = (b/t)\sqrt{f/E} \tag{2}$$

The flat width is normalized in the same way:

$$W = (w/t)\sqrt{f/E} \tag{3}$$

Few and relatively crude experimental data were available when this theory was proposed. The results seemed to confirm the theory, particularly for relatively thin plates, with a tendency towards over-estimating the strength in the region of transition to thick plates. The theory has since gained general acceptance in aircraft structural design.

After conducting a series of tests on light gage steel beams, Winter [2] in 1948 proposed an alternative to von Karman's theoretical notion that B be constant if less than W . Winter's expression was used in the first light gage steel standard specification (AISI), and has since been copied widely and modified only slightly. In this somewhat complicated expression the original simplicity of von Karman's equation was sacrificed in favour of a more gradual transition to the thick plate region. Winter plotted B versus the reciprocal of W (Fig. 1). If there is a negative correlation between these quantities (as the experimental points would seem to suggest), a straight regression line in this graph will automatically give a (hyperbolic) relationship between B and W curved like the one given in the AISI specification.

Since 1946 there has been considerable development in experimental mechanics, in results available, in computing technology, in statistical inference, in reliability theory and in decision theory. Faced with the scatter of Fig. 1, the best that could be done 25 years ago was to have a competent engineer place a "conservative" curve through

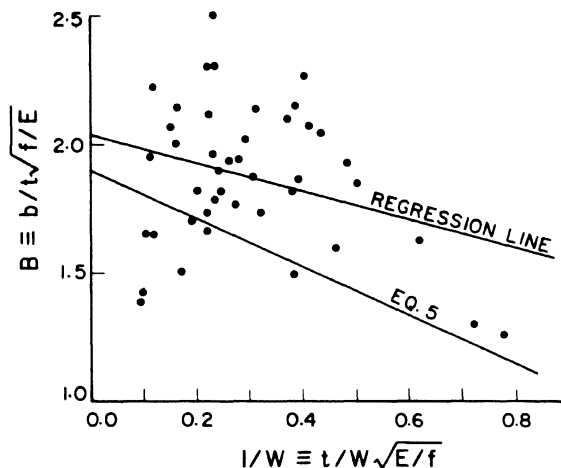


Fig. 1

the data. This procedure is not necessary any more - nor is it sufficient.

In the following, we shall re-examine the effective width, first from a scientific viewpoint to draw inference and, second, from an engineering viewpoint to select a "best" effective width rule for rational design. Each viewpoint has its own merit, and both are relevant in the proper choice of design specification parameters. We are, of course, primarily interested in making the best design decisions possible on the basis of available data, but the quality of the evidence for the chosen design rule cannot be disregarded; its evaluation is a scientific question.

We now proceed to make a statistical study of several plausible hypotheses regarding the effective width, including those advanced by von Karman and Winter. The results favour the simpler hypotheses. Subsequently we develop a new effective width formula (Eq. 17) suitable for design use, based on second moment reliability analysis.

WINTER'S TESTS AND FORMULA

Von Karman's investigations were concerned only with the determination of the ultimate strength of such plates. In practical design it is also necessary to determine the effective widths at smaller loads as well, for example, in the analysis of deflection at service loads.

* Professor of Civil Engineering, University of Waterloo, Waterloo, Ontario, Canada.

** Post-Doctoral Fellow, Dept. of Civil and Environmental Engineering, Washington University, St. Louis, Mo.

*** Research Assistant, Department of Civil Engineering, University of Waterloo, Waterloo, Ontario, Canada.

For this purpose, Winter [2] conducted a series of tests on cold formed light gauge members of hat section and other profiles. The effective width was calculated for various loads from measurements of the positions of the neutral axis.

As a very useful generalization of Eq. 1, Winter expressed the effective width of the plate at stress f as

$$b = Bt\sqrt{E/f}, \quad (4)$$

where f is the calculated maximum longitudinal stress in the plate, occurring along the lines of support.

Winter reasoned that the coefficient B should depend primarily on the non-dimensional parameter W , (Eq. 3). However, the experimentally determined coefficients were plotted (Fig. 1) against the parameter $1/W$, possibly to effect a smoother transition to fully effective plates. For each test specimen B was found at yield load, at one half and at two thirds of the yield load. There is considerable scatter in the test results, apparently due to high sensitivity of the method to minor experimental deviations, such as errors in the determination of the location of neutral axis.

Based on the test results, Winter proposed a straight line relationship between B and $1/W$. This produces a "reasonable and somewhat conservative but simple formula" for the effective width. In the notation of this paper

$$b = 1.9t\sqrt{E/f} [1 - 0.475(t/W)\sqrt{E/f}]. \quad (5)$$

Since b cannot exceed w , Eq. (5) indicates that a compression plate is fully effective for values of w/t smaller than $0.95\sqrt{E/f}$.

THE EXPERIMENTAL DATA

As a first step in this study all available relevant data known to the author after a literature search was screened for inclusion or exclusion as data for the analysis.

The earliest tests to check von Karman's theory seem to be those made by Sechler [3]. His results have not been included as data in this study because they exhibit much more experimental scatter than the rest of the literature. All results of Winter's tests published in 1948 [2] have been included. From his earlier (1947) paper [4], the tests on built-up lipped I-beams were included. The series A tests were excluded because of high scatter.

From the paper by Dwight and Ractliffe [5] all the "as-rolled" specimens were included as opposed to welded specimens, because the residual stress in the weld zones render the analysis inapplicable as borne out conclusively also in the paper by Dwight and Moxham [6]. Welded sections will be treated separately.

Similarly, the tests by Johnson and Winter [7] on stainless steel have been excluded for separate treatment. These authors note apparent agreement between the conservative formula due to Winter and the mean curve for their test results; there is, however, a marked difference to the mean curves for the carbon steel data.

Chilver's column test results [8] were not included, although the effective width formula is used in column design as well. It was felt better to have a fairly accurate representation for beams alone, used as a conservative approximation in column design than to use the best fit to combined beam and column data, which would be unconservative for beam design.

Figure 2 shows the data; results for specimens with a

normalized flat width W less than 1.75 show that this width is fully effective, admitting some intrinsically one-sided experimental scatter. These results have therefore been left out of the subsequent analysis.

STATISTICAL INFERENCE FROM THE DATA

The data constitute a set of n "points", i.e., pairs B_m, W where B_m and W are the measured effective width and flat width, respectively, normalized with respect to stress as in Eqs. (2) and (3). We accept the notion that there exists an (unknown) unique normalized true effective width B that is a function of W only. This defines the experimental error ΔB :

$$\Delta B = B_m - B. \quad (6)$$

The experimental error is a random process over the domain of W ; it is the sum of a systematic experimental error (which we assume to be identically zero) and a random error. We further assume that the process ΔB is stationary and Gaussian; it is therefore completely characterized for our purpose by its variance. Thus, B_m is assumed Gaussian with mean $B = B(W)$ and standard deviation $\sigma_B = \text{const}$. The problem is to infer the value of σ_B and the function $B(W)$ from the data set.

A scientifically accepted approach to solve this kind of problem (statistical inference) proceeds by pairwise comparison of all elements in a set of hypotheses $\{H_i\} = \{[B_1(W), \sigma_1], [B_1(W), \sigma_2], \dots, [B_p(W), \sigma_p]\}$ where B_1, \dots, B_p are alternative hypothetical functions of W , and $\sigma_1, \dots, \sigma_p$ are associated values of σ_B . The method of comparison is based on the likelihood (= hypothetical probability) of observations of the data assuming the truth of the hypotheses. Many methods to solve typical problems of this kind are available in the literature [9].

The problem therefore reduces to the selection of a suitable set of hypotheses $\{H_i\}$ to choose from. There is no standard approach to this selection; it is unfortunately a matter of insight, intuition, philosophy and taste. Some of the desirable qualities of a representation are: accuracy, stability, efficiency and plausibility. Obviously, the more accuracy (i.e., lower σ) in a representation, the better. For example, if we represent B by a polynomial in W , σ can be reduced by increasing the order of the polynomial until equal to the number, n' , of different abscissas W in the data set; orders higher than $n' - 1$ are unnecessary and are rejected on philosophical grounds following the principle of "Occam's razor". A polynomial of order $n' - 1$ is rejected by considerations of stability; the perfect fit of which it is capable is likely to be upset by addition of one more data point. Stability is thus one aspect of efficiency in use, ease of calculation is one of many others. For example, a lengthy polynomial might be preferable to a conceptually simple, "exotic" function that is hard to evaluate. Nevertheless, the exotic function might be preferred if it is justified by theory over the less plausible polynomial that is just the outcome of a curve-fitting process.

Based on these considerations, a reasonable procedure is to generate the hypotheses H_i in order of decreasing simplicity. Some subjective aspects remain, but first attention should clearly be given to the two families of polynomial structure:

$$\{H_1\} = \{B = \sum_0^i a_j W^j\}, \quad (7)$$

$$\{H_1\} = \{B = \sum_0^i a_j W^{-j}\}. \quad (8)$$

In Eq. (7), B is a polynomial function of W:

$$B = P(W). \quad (9)$$

Mathematically, the family $W = P(B)$ is just as simple as Eq. (7), but it seems to reverse the role of cause and effect, and it is therefore rejected. Winter's relationship, Eq. (5) belongs to the family

$$B = P(1/W) \quad (10)$$

of polynomials with argument $1/W$; we consider this family for comparison with $B = P(W)$. Finally, we note that $1/B = P(1/W)$ would seem just as reasonable, but it is left out of consideration together with $1/W = P(B)$, etc.

First, compare the first two hypotheses in the set in Eq. (7), of the type in Eq. (9). This amounts to a test whether or not the slope a_1 is significantly different from zero in the relationship

$$B = a_0 + a_1 W. \quad (11)$$

The null hypothesis is that a_1 equal zero; the alternative hypothesis is that a_1 is different from zero.

A regression line fitted to the data has for a_1 the value -0.006. A t-test of significance on the coefficient shows that we cannot reject the null hypothesis that a_1 equals zero, even at a 10% level of significance. In other words, there is no reason to suggest, with the given data, that B is in any way dependent upon W.

Next, the null hypothesis is made that a_0 equal 1.90 as suggested by the approximate theory of von Karman. The alternate hypothesis is that a_0 differs from 1.90. The mean of the sample of B values is 1.880. We assume the means of all samples of B to be random variables, and normally distributed, and find that at a 10% level of significance we cannot reject the null hypothesis that a_0 equal 1.90. In other words, we are "90% confident" that the true value of a_0 lies between the limits $1.880 \pm .392$.

It should be noted that with a scatter of the B values as observed, approximately 18,000 samples would be required to establish if 1.90 was the exact true mean. If the mean of these 18,000 values were

equal to or less than 1.880 we would reject the null hypothesis that a_0 equals 1.90 at the 10% level of significance. The less stringent 5% level of significance commonly used would, of course, require a much greater amount of data.

Turning to the family of hypotheses in Eq. (10), it may be asserted that there is no significant dependence of B on $1/W$ using a similar test; as before, $B = 1.90$ cannot be rejected on the basis of the data.

Next, representatives of the two families, Eqs. (9) and (10) may be compared. Linear regression lines are, respectively

$$B = 1.905 - 0.006W \quad (11)$$

$$B = 1.94 - 0.19W^{-1} \quad (12)$$

while the mean of the data gives the value

$$B = 1.880 \quad (13)$$

as a member of both families.

The sample standard deviation of the test results from Eqs. (11), (12) and (13) is 0.241, 0.240 and 0.242, respectively. Evidently, there is no basis for asserting that one representation is more accurate than the other.

COMPARISON OF BIAS

An attractive method to select a suitable curve to represent experimental data has recently been presented by Dylewski [10].

If the laws of nature pertaining to a phenomenon are unknown, bias is almost sure to exist in the representation of data by an arbitrary curve. Bias is a systematic discrepancy between the fitted curve and the true equation governing the data. Bias can arise both from oversmoothing or from undersmoothing. Oversmoothing means that some deterministic variation has been regarded as random variation and discarded, while undersmoothing means that some random variation has been regarded as deterministic and has been retained. To illustrate this point, let the points in Fig. 3 represent a set of experimental data (12 points in total) to be fitted by a polynomial of degree n. Evidently, $n = 0$

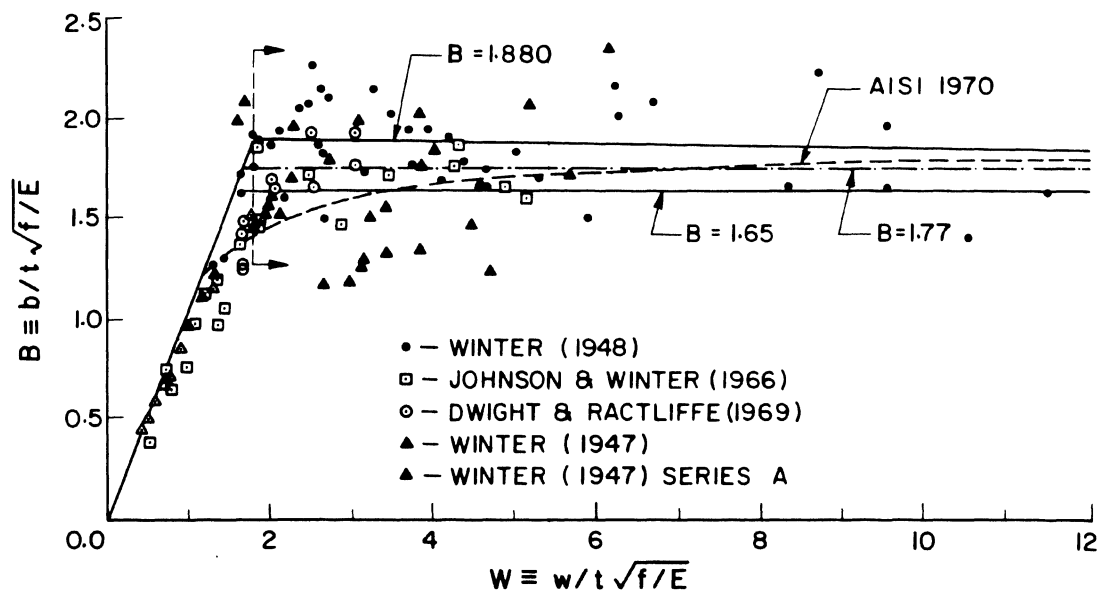


Fig. 2

and $n = 11$ would oversmooth and undersmooth the data, respectively. The least-squares method of curve-fitting readily provides the "best fit" polynomial of any degree n to a set of points, but it does not give any guide to the selection of n .

Dylewski defined the bias of a fitted curve with respect to a data set as the ratio of the sums of squares of the deviations from the curve fitted over all the points divided by the sum of two such sums calculated for two curves of the same type, each fitted to half the data points. Dylewski suggested that minimum bias indicates a curve that is smoothed correctly. We accept this as a convention.

Table 1 shows the bias calculated for the curves of best fit. Unfortunately, bias is not defined for other than such curves (e.g., $B = 1.90$ in each of the two families of Eqs. (9) and (10). The constant value of B has less bias than the regression line in both families; it is concluded that the linear expressions do not smooth the data correctly.

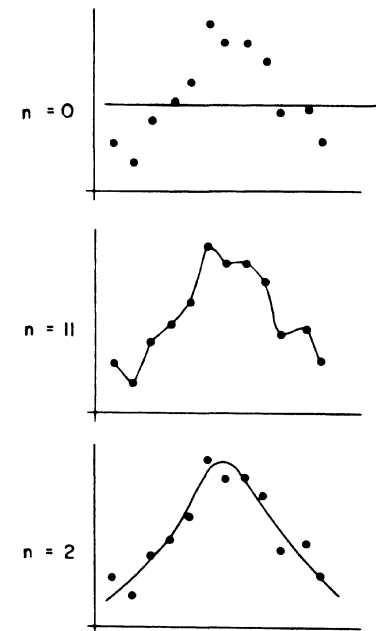


FIG. 3

TABLE 1

B	Bias
1.880	1.01
$1.905 - 0.006W$	1.08
$1.552 + 0.152W - 0.014W^2$	1.01
$1.721 + 0.034W + 0.010W^2 - 0.001W^3$	1.06

1.880	1.01
$1.936 - 0.190/W$	1.07
$1.075 - 8.64/W - 21.060/W^2$	1.02
$1.320 + 5.854/W - 16.60/W^2 + 13.34/W^3$	1.02

Concluding that the attempts to identify a statistically significant dependence of B on W in the data have been unsuccessful, we proceed to select a constant value of B suitable for design. Two possible approaches are followed here; the results are then compared and a selection made on the basis of judgement.

FIRST ALTERNATIVE

The first alternative is to select the value that gives the same number of data points below the line as the curve in the operating standard. It may be argued [11] that the sample reliability is then unaltered, and that it may reasonably be taken as an indication that the present unknown reliability of the design standard would be maintained. This is strictly correct only if the data can be considered as a simulation of a random sample of the values of W for the designs that will be built according to the standard. It is believed that this assumption is practically fulfilled because the majority data points were selected as practical shapes with an empirical curve in mind (rather than a set of shapes designed to put a theory to a more crucial test), and because each of many specimen shapes, through the dependence of W on stress, gives rise to several points covering a range of W . This gives (see Fig. 2) for design

$$B = 1.65. \quad (14)$$

An F-test (and Hoel's test) applied to the variances indicates no difference at the 5% level of significance between any new design rule satisfying Eq. (14) and the formula currently in use. The comparison can be made on two bases. First (considering the actual data points to be in error), relative to the formula value $B = 1.65$, the mean error is -0.147 and the standard error is 0.147 . Using the same viewpoint for the AISI formula, mean and error are -0.154 and 0.157 , respectively.

Alternatively (considering the data as the correct values which the formula imperfectly manages to represent), the mean errors are 0.115 (0.116) and the standard errors 0.115 (0.128), for the value $B = 1.65$ (for the AISI formula), respectively. All these tests indicate that Eq. (14) is at least as accurate, if not better, than the old rule.

SECOND ALTERNATIVE

It has recently become possible to select safety margins by rational analysis on the basis of a well-defined small set of propositions regarding the nature of loads, strengths, structural behaviour and design objectives [12]. A standard design format has been proposed to the International Standards Organisation, recommending the use of a set of partial safety factors typically of the form $(1 + CV)$ where C is a constant and V is the coefficient of variation, hereinafter called the dispersion, of the uncertain quantity being considered. Cornell [13] has proposed a first-order second moment reliability analysis that considers all uncertainty separated into five mutually independent random factors: material strength M , load T , structural analysis, P , strength of materials analysis E , and fabrication F . If these quantities are provided with safety factors $(1 + CV_M)$, $(1 + CV_T)$, ..., $(1 + CV_P)$ where C is a common constant, it has been shown [12] that the reliability can be made practically independent of the dispersions V_i over a wide range of variation.

Since the dispersions can be estimated fairly accurately, it is possible to estimate the coefficient C implied in an existing code from the total safety margin. The appropriate safety factor for another technology, with a different value for one or more of the dispersions, is therefore easily calculated. In the following, the method will be used to calculate the safety factor on the effective width that would yield approximately the same reliability as fully effective sections.

The first step is to determine the change in stress in a member

when the normalized effective width ratio B is increased by a given small percentage. This analysis for bending is quite elementary. It is easily shown that the stress in a partially effective compression flange is reduced approximately by the same percentage, while the stress in the tension flange is nearly unaffected. If tension governs, neither initial cost nor safety are affected appreciably by a change in B ; such flexural members can therefore be left out of the discussions. It follows that the safety factor $(1 + CV_B)$ to be applied to B is identical with the factor to be applied to the strength for a member using the effective width concept in the analysis.

Table 2 shows estimated values of the dispersion for the various uncertainties in two contexts, viz. design when dead load is dominating, and the tests that gave the results plotted in Fig. 2. Some of these estimates are based on extensive data (e.g., material strength), while others are quite subjective.

The values of C implied in the dispersions are easily calculated from dead load design. The nominal safety factor 1.67 is the product of five partial safety factors, and C is a solution of the fifth order equation

$$1.67 = (1 + V_T C)(1 + V_P C)(1 + V_M C)(1 + V_F C)(1 + V_E C) \quad (15).$$

TABLE 2
DISPERSIONS (COEFFICIENTS OF VARIATION)

Case	Conventional Dead Load Design			Laboratory Beam Tests		
	Estimated Minimum	Best Estimate	Estimated Maximum	Estimated Minimum	Best Estimate	Estimated Maximum
Load, T	5	10	15	0	2	5
Force Analysis, P	5	10	20	0	2	5
Material Strength, M	7	10	13	7	8	13
Fabrication, F	5	10	15	3	6	10
Stress Analysis, E	3	7	10	3	6	10
Implied C	2.14	1.15	0.74	-	-	-
Observed Dispersion of Sample, %				12.9	12.9	12.9
Dispersion of Effective Width Analysis, Calculated (Eq. 16), %				10	5	-

Table 2 shows the resulting C -values for two extreme cases, and the value $C = 1.15$ as resulting from the best estimate of dispersion. From the dispersions for the beam tests, assuming an additional stochastic factor, independent of the factors listed, it is possible to calculate the dispersion of B that would yield the dispersion observed in the data ($V_{DATA} = 0.241/1.88 = 12.9\%$) by using

$$(V_D^2 + V_P^2 + V_M^2 + V_F^2 + V_E^2) + V_B^2 \approx V_{DATA}^2 \quad (16)$$

The best estimate gives $V_B = 5\%$; an extreme value is 10% calculated from the estimated minima. The value $V_B = 12.9\%$ is an absolute upper limit if sampling uncertainty is neglected. Using $C = 1.15$ and $V_B = 5\%$, gives the safety factor to be applied to B as $(1 + 1.15(0.05)) = 1.06$; this value is listed with others obtained in a similar way in Table 3.

Comparing with the other partial safety factors it is concluded that the concept of effective width ratio as a function only of $(w/t)/\sqrt{E/E}$ for the element is not an oversimplification. Rather, it seems to be in harmony with the general level of uncertainty in structural design, at least insofar as flexure is concerned and reflected in the test data.

TABLE 3
NORMALIZED EFFECTIVE WIDTHS AND CORRESPONDING SAFETY FACTORS

		$C = 0.74$	$C = 1.15$	$C = 2.14$
$V_B = 5\%$	B.	1.81	1.77	1.70
	F.S.	1.04	1.06	1.11
$V_B = 10\%$	B.	1.76	1.68	1.55
	F.S.	1.07	1.12	1.21

While the calculation of V_B is quite uncertain, it is clear that the safety factor on B should not be less than 1.04 nor greater than 1.21. The value 1.06 resulting from the best estimates of dispersion would be quite conservative in the sense that it seems to compare rationally with the safety margins on all other factors combined as reflected in the overall safety factor 1.67. The result is therefore the normalized effective width limit

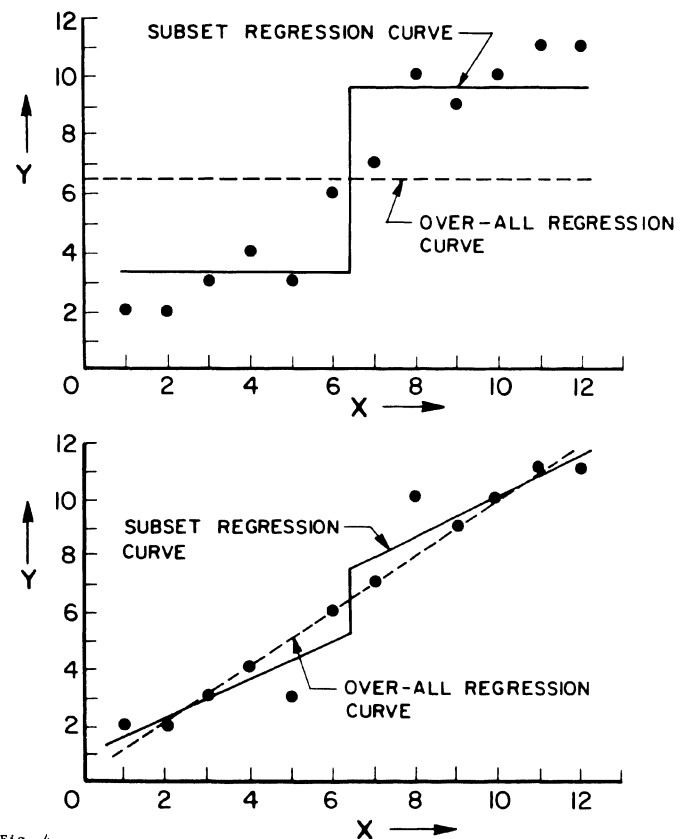
$$B = 1.77. \quad (17).$$


Fig. 4

CONCLUSIONS

1. The concept of normalized effective width ($B = (b/t)\sqrt{E/E}$) of an element of a cross-section in bending as a function only of normalized flat width ($W = (w/t)\sqrt{E/E}$) due to Winter, is an appropriate simplification of actual behaviour in harmony with the general level of uncertainty in structural design.
2. If an element of a section in flexure is only partially effective, the simplest hypothesis is that the normalized effective width is independent of the normalized flat width. This hypothesis has minimum bias, and it cannot be rejected on the basis of the data. Moreover, there is not sufficient data to reject von Karman's approximate theoretical value of the normalized effective width ($B = 1.90$). It is therefore recommended for purposes of design that the normalized effective width be

taken equal to the normalized flat width or a given constant B (containing an appropriate safety factor), whichever is less.

3. The limiting value of B equal to 1.65 will provide approximately the same reliability as the AISI formula, in operation now.
4. A limiting value of B equal to 1.77 will provide approximately the same level of reliability as in current conventional flexural design in steel for buildings.
5. Since the use of the effective width formula is not confined to flexure of cold formed sections, but includes axial compression and welded sections, it is recommended that the conservative value $B = 1.65$ be used in design codes until sufficient data warrants a higher value.
6. Experimental scatter prevents the detection of any dependence of B on W. Modern experimental techniques could reduce this scatter somewhat but possible returns are limited, and the expense of further tests in flexure would not seem warranted from an engineering viewpoint. In contrast, the experimental basis for axial compression seems insufficient and might profitably be extended.

Acknowledgement

The work reported in this paper was supported financially by the National Research Council of Canada.

REFERENCES

1. von Karman, T., Sechler, E.E. and Donnell, L.H., "The Strength of Thin Plates in Compression", Transactions of the American Society of Mechanical Engineers, Vol. 54, pp. 53, 1932.
2. Winter, G., "Performance of Thin Steel Compression Flanges", Prelimin. Publication, Third Congress, International Association for Bridge and Structural Engineering, p. 137, Liège, 1948.
3. Sechler, E.E., "The Ultimate Strength of Thin Flat Sheet in Compression", Publication No. 27, Guggenheim Aeronautics Lab., California Inst. of Technology, Pasadena, 1933.
4. Winter, G., "Strength of Thin Steel Compression Flanges", Transactions American Soc. of Civil Engrs., Vol. 112, pp. 527-554, 1947.
5. Dwight, J.B., Ractliffe, A.T., "The Strength of Thin Plates in Compression", in "Thin Walled Steel Structures", Ed. by K.C. Rocke and H.V. Hill, Lockwood & Co., London, 1969.
6. Dwight, J.B. and Moxham, K.E., "Welded Steel Plates in Compression", The Structural Engineer, V. 47, No. 2, Feb. 1969, pp. 49-66.
7. Johnson, A.L. and Winter, G., "Behaviour of Stainless Steel Columns and Beams", Journal of the Structural Division, American Society of Civil Engineers, Vol. 92, No. ST5, pp. 97-118, Oct. 1966.
8. Chilver, A.H., "The Stability and Strength of Thin-Walled Steel Struts", The Engineer, pp. 180-183, August, 1953.
9. Benjamin, J.R., Cornell, C.A., "Probability, Statistics and Decision for Civil Engineers", McGraw-Hill, 1970.
10. Dylewski, T.J., "Criteria for Selecting Curves for Fitting to Data", AIAA Journal, pp. 1411-1415, August 1970, V. 8, No. 8.
11. Ravindra, M.K. and Lind, N.C., "Optimization of a Structural Code", Proc. Conf. on Applications of Statistics and Probability to Soil and Structural Engineering, University of Hong Kong, Sept. 1971 (to be published).
12. Lind, N.C., "Consistent Partial Safety Factors", Proc. ASCE, J. Struct. Div. (to be published), V. 97, 1971.

APPENDIX - NOTATION

b	effective width
B	normalized effective width; Eq. 1
B_M	measured normalized effective width; Eq. 6
E	Young's Modulus
E	strength of materials analysis
f_y	yield strength of the material
F	fabrication
M	material strength
P	structural analysis
t	plate thickness
T	load
V	coefficient of variation
w	total width
W	normalized flat width; Eq. 3
Δ_B	experimental error; Eq. 6
ν	Poisson's ratio
σ_B	standard deviation of B_M .