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ABSTRACT

This paper proposed an approach of evaluating the elastic buckling loads for unbraced cold formed steel storage racks under variable loading. In the case of variable loading, the conventional assumption of proportional loading is abandoned, and the magnitude of each individual load can vary independently. Therefore, different load patterns may cause the storage racks to buckle at different levels of critical loads. Among those critical loads, the ones associated with the minimum and the maximum magnitudes of total loads define the lower and upper bounds of the elastic critical loads, respectively. The load patterns associated with the minimum and maximum critical loads are the most critical and most favourable load patterns for the elastic buckling of the structure. In light of the use of storey-based buckling concepts to characterize the lateral sway buckling of framed structures, the problems of determining the lower and upper bounds of critical loads and their associated load patterns for multi-storey cold formed steel storage racks are presented as a minimization and maximization problem subject to elastic stability constraints and are solved by a linear programming method. The proposed approach has realistically taken into account the volatility of magnitudes and patterns of loads applied to the storage racks; therefore, it can be applied to the design of the storage racks.

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Introduction

The analysis and design of cold-formed steel storage racks is complex because of the significant perforations in the columns, the semi-rigid behaviour of the beam-to-column connections and column bases, and the susceptibility to local buckling and torsional-flexural buckling of cold-formed steel members. In addition, the nature of randomly applied loads both in magnitudes and locations is often one of the primary contributing factors causing structural failures. So far there are no design guidelines and tools available to assess the integrity of the structures under variable loadings.

In general, three types of methods available for stability analysis of unbraced steel framed structures, i.e. the system buckling method, the effective-length method, and the storey-based buckling method (Majid 1972, Livesley et al. 1987, Chen et al. 1987, AISC 2001). Although the system buckling method is considered the most accurate of these methods, it is considered to be impractical as it involves solving the minimum positive eigenvalue from either a highly nonlinear or transcendental equation. In the effective-length based methods, the alignment chart method is the most widely used method in practice for designing a frame, However, the alignment chart methods adopts certain assumptions which may result in inaccuracy of the estimated column strengths in the case that the assumptions are violated. The storey-based buckling method (Yura 1971, LeMessurier 1977), which is an alternative of the effective-length method in design practice that does not adopt the simplifications associated with the effective-length method, is efficient and provides accurate results. The method is based on the fact that lateral sway instability of an unbraced frame is a storey phenomenon involving the interaction of lateral sway resistance of each column in the same storey and the total gravity load in columns in that storey. In this study, a storey-based buckling method developed by Xu and Liu (2002) will be adopted and extended to facilitate the stability analysis of multi-storey cold formed steel storage racks subjected to variable loading.

In cold formed steel storage racks, beam end connectors are used to make beam-to-column connections. The semi-rigid nature of the connection is primarily due to the distortion of the column walls, tearing of the column perforation, and distortion of the beam end connector. The stability of the storage rack depends significantly on the behavior of the connection, thus it is imperative to incorporate the semi-rigid behaviour of the connection into the stability analysis of the storage racks.

Lateral stiffness of axially loaded semi-rigid column

For an axially loaded semi-rigid column as shown in Fig.1, let $EI_{c,ij}/L_{c,ij}$ be the flexural stiffness of the column axial load, and let $R_{l,ij}$ and $R_{u,ij}$ be the rotational restraining stiffness provided by the immediately connected beams at the lower and upper joints in the i^{th} storey and j^{th} column. The effect of beam-to-column end rotational restraints can be characterized by the end-fixity factors (Monforton and Wu, 1963) and can be expressed as follows:

$$r_{l,ij} = \frac{1}{1 + 3EI_{c,ij} / R_{l,ij} L_{c,ij}} \; ; \quad r_{u,ij} = \frac{1}{1 + 3EI_{c,ij} / R_{u,ij} L_{c,ij}} \tag{1a, 1b}$$

where $r_{l,ij}$ and $r_{u,ij}$ are the end-fixity factors for the lower and upper ends of the column, respectively.

The end-fixity factors in Eqns. (1) define the stiffness of each end connection relative to the attachment member. For flexible, i.e. pinned connections, the rotational stiffness of the connection is idealized as zero; thus, the value of the corresponding end-fixity factor is zero. For fully restrained or so-called rigid connections, the end-fixity factor is unity, because the connection rotational stiffness is taken to be infinite. A semi-rigid connection has a value of end-fixity factor between zero and unity.

Upon the introduction of the end-fixity factors, the lateral stiffness of an axially loaded column in an unbraced frame can be expressed as (Xu and Liu, 2002)

$$S_{ij} = \beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij}) \frac{12EI_{c,ij}}{L_{c,ij}^3} \tag{2}$$

$\beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij})$ is the modification factor of the lateral stiffness that takes into account the effects of both axial force and column end rotational restraints. The expression of the modification factor $\beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij})$ in terms of the end-fixity factors can be expressed as

$$\phi_{ij} = \sqrt{\frac{P_{ij} L_{c,ij}^2}{EI_{c,ij}}} = \pi \sqrt{P_{ij} / P_{e,ij}} \tag{3}$$

in which P_{ij} is the column axial load and $P_{e,ij}$ is the Euler buckling load for the column with pinned connections.

$\beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij})$ is the modification factor of the lateral stiffness that takes into account the effects of both axial force and column end rotational restraints. The expression of the modification factor $\beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij})$ in terms of the end-fixity factors can be expressed as (Xu, and Liu, 2002)

$$\beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij}) = \frac{\phi_{ij}^3}{12} \frac{a_1\phi_{ij}\cos\phi_{ij} + a_2\sin\phi_{ij}}{18r_{l,ij}r_{u,ij} - a_3\cos\phi_{ij} + a_4\phi_{ij}\sin\phi_{ij}} \tag{4}$$

where

$$a_1 = 3[r_{l,ij}(1 - r_{u,ij}) + r_{u,ij}(1 - r_{l,ij})] \tag{5a}$$

$$a_2 = 9r_{l,ij}r_{u,ij} - (1 - r_{l,ij})(1 - r_{u,ij})\phi_{ij}^2 \tag{5b}$$

$$a_3 = 18r_{l,ij}r_{u,ij} + [3r_{l,ij}(1 - r_{u,ij}) + 3r_{u,ij}(1 - r_{l,ij})]\phi_{ij}^2 \tag{5c}$$

$$a_4 = -9r_{l,ij}r_{u,ij} + 3r_{l,ij}(1 - r_{u,ij}) + 3r_{u,ij}(1 - r_{l,ij}) + (1 - r_{u,ij})(1 - r_{l,ij})\phi_{ij}^2 \tag{5d}$$

Storey-based stability equation

The concept of storey-based buckling indicates that lateral stability of an unbraced frame is a storey phenomenon involving the interaction of lateral stiffness among columns in the same storey. It presents that the columns with larger stiffnesses are able to provide lateral support for the weaker columns in the same storey to maintain the lateral stability of the storey while the columns with insufficient stiffnesses rely on such lateral support to sustain the applied gravity load. The interaction of lateral stiffness among columns can be signified by the column lateral stiffness modification factor defined in Eqn (4). A column with $\beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij})$ value greater than zero indicates that the column can provide lateral support to other columns to maintain the stability of the storey. A column with the value of $\beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij})$ value less or equal to zero signifies the column relies on the lateral restraint provided by other columns in the same storey in order to sustain the applied load.

Rewriting Eqn (2) for a column ij in a multi-storey unbraced frame,

$$S_{ij} = \beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij}) \frac{12EI_{c,ij}}{L_{c,ij}^3} \tag{6}$$

where subscripts i and j are the indices of storey and column; E is Young's modulus; and $I_{c,ij}$ and $L_{c,ij}$ are the moment of inertia and the length of the column, respectively.

Unlike the alignment chart method, which ignores the fact that columns in a storey of the frame will restrain each other in resisting buckling, the stiffness interaction among the columns is taken into account in storey-based buckling. The condition for multicolumn storey-based buckling in a lateral sway mode is that the lateral stiffness of the storey vanishes; therefore, the corresponding the stability equation for a single storey *i* which buckles in a lateral sway mode is given by (Xu and Liu, 2002)

$$S_i = \sum_{j=1}^{m} S_{ij} = \sum_{j=1}^{m} \beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij}) \frac{12EI_{c,ij}}{L_{c,ij}^3} = 0 \tag{7}$$

As lateral instability can occur in any storey of a multi-storey frame, the general stability equation of a multi-storey unbraced frame can then be expressed as

$$\prod_{i=1}^{n} S_i = \prod_{i=1}^{n} \sum_{j=1}^{m} \beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij}) \frac{12EI_{c,ij}}{L_{c,ij}^3} = 0 \tag{8}$$

Characterized by the vanishing of the lateral stiffness of the storey, Eqn (8) implies that if any one of the stories fails to maintain its lateral stability, the frame becomes unstable laterally.

However, it is impractical to evaluate the frame stability with use of Eqn (8) because the product of the storey stiffness S_i shown in Eqn (8) yields to a nonlinear equation which is compounded with the transcendental relationship between $\beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij})$ and ϕ_{ij} defined in Eqn (4). To overcome such numerical difficulty, Xu and Liu (2002) applied the first-order Taylor series approximation of Eqn (4) in the investigation of lateral stability of single-storey unbraced steel frames. Subsequently, procedures of decomposing of a multi-storey unbraced frame into a series of single-storey frames are proposed (Liu and Xu, 2005).

Upon applying the first-order Taylor series approximation to Eqn (4), the column lateral stiffness modification factor of column *ij* can be simplified as follows

$$\beta_{ij}(\phi_{ij}, r_{l,ij}, r_{u,ij}) = \beta_{0,ij}(r_{l,ij}, r_{u,ij}) - \beta_{1,ij}(r_{l,ij}, r_{u,ij})\phi_{ij}^2 \tag{9}$$

where $\beta_{0,ij}$ and $\beta_{1,ij}$ are given by

$$\beta_{0,ij}(r_{l,ij}, r_{u,ij}) = \frac{r_{l,ij} + r_{u,ij} + r_{l,ij}r_{u,ij}}{4 - r_{l,ij}r_{u,ij}} \tag{10a}$$

$$\beta_{1,ij}(r_{l,ij}, r_{u,ij}) = \frac{8(5 + r_{u,ij}^2) - (34 - r_{u,ij})r_{l,ij}r_{u,ij} + (8 + r_{u,ij} + 3r_{u,ij}^2)r_{l,ij}^2}{30(4 - r_{l,ij}r_{u,ij})^2} \tag{10b}$$

Thus, the lateral stiffness defined in Eqn (6) can be simplified as

$$S_{ij} = 12\left(\frac{EI_{c,ij}}{L_{c,ij}^3}\beta_{0,ij} - \frac{P_{ij}}{L_{c,ij}}\beta_{1,ij}\right) \quad (11)$$

Stability of multi-storey unbraced frames subjected to variable loading

The lateral stability of single-storey unbraced frames subjected to variable loading was first investigated by Xu (2002). Based on the concept of storey-based buckling, the problem of determining critical elastic buckling loads of the frames under non-proportional loading is expressed as a pair of minimization and maximization problems with stability constraints. The study finds out that in the case of variable loading, the difference between the maximum and minimum elastic buckling loads' associated lateral instability of the single-storey unbraced frames can be as high as 20% in some cases. As the beam-to-column connections were considered either as purely pinned or fully rigid in the study, further investigation was carried out on the single-storey unbraced semi-rigid frames (Xu, 2002). It was discovered that the difference between the minimum and maximum buckling loads is insignificant for frames whose connection rigidities are approximately the same and evenly distributed among the columns and beams. However, the difference can still be substantial in some cases with lean-on columns, but it is not as significant as in the case where connections are simplified as ideally pinned or fully rigid.

Following the procedures of decomposing of a multi-storey unbraced frame into a series of single-storey frames proposed by Liu and Xu (2005), the lateral stability of the multi-storey unbraced frame subjected to variable loading can be formulated as a pair of problems of seeking the maximum and minimum buckling loads of the frames, in which the maximization problem can be stated as follows:

Maximize:

$$Z = \sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij} \quad (12a)$$

Subject to:

$$S_k = 12\sum_{j=1}^{m}\left(\frac{EI_{kj}}{L_{kj}^3}\beta_{0,kj} - \frac{\beta_{1,kj}}{L_{kj}}\sum_{i=k}^{n} P_{ij}\right) \geq 0 \quad (12b)$$

$$0 \leq P_{ij} \leq P_{u,ij} = \frac{\pi^2 EI_{ij}}{K_{ij}^2 L_{ij}^2} \quad (12c)$$

(k=1,2…n; i=1,2…n; j=1,2…m)

where n is the number of the storey of the frame, and m is the number of columns in one storey. P_{ij} is the applied load associated with column ij and is the variable of the maximization/minimization problem. ϕ_{ij} is the applied load ratio and is defined in Eqn (3). Z is the objective function corresponding to the maximum elastic buckling loads of the frame and is the sum of the variables P_{ij}. It is noticed that Eqns (12) are a linear programming problem which can be solved by an applicable algorithm such as the simplex method.

Eqn (12b) represents the storey-based stability condition for the k-th storey of the frame. In the case that the lateral stiffness of storey k, S_k, is greater than zero, the storey is lateral stable; otherwise, the storey becomes laterally unstable if S_k=0. Eqn (12c) is a side constraint imposed on each applied load to ensure that the magnitude of the applied load will not exceed the buckling load associated with non-sway buckling of the individual column, in which the column effective length factor associated with non-sway buckling can obtained from (Xu, 2003)

$$K_{ij}^2 = \frac{[(\pi^2 + (6-\pi^2)r_{u,ij}]\times[\pi^2 + (6-\pi^2)r_{l,ij}]}{[\pi^2 + (12-\pi^2)r_{u,ij}]\times[\pi^2 + (12-\pi^2)r_{l,ij}]} \tag{13}$$

where $r_{l,ij}$ and $r_{u,ij}$ are the end-fixity factors provided by the beams connected at the lower and upper ends of the column, respectively.

The problem of seeking the minimum buckling loads of a multi-storey unbraced frame subjected to variable loading can be stated as,

$$\text{Minimum } Z = \min\left\{ Z_l = \sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij} \mid l = 1, 2, 3\ldots n \right\} \tag{14}$$

where Z_l (l = 1, 2, 3…n) is obtained from the minimization problem,

$$\text{Minimize: } Z_l = \sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij} \tag{15a}$$

$$\text{Subject to: } S_l = 12\sum_{j=1}^{m}\left(\frac{EI_{lj}}{L_{lj}^3}\beta_{0,kj} - \frac{\beta_{1,lj}}{L_{lj}}\sum_{i=l}^{n} P_{ij}\right) = 0 \tag{15b}$$

$$S_k = 12\sum_{j=1}^{m}\left(\frac{EI_{kj}}{L_{kj}^3}\beta_{0,kj} - \frac{\beta_{1,kj}}{L_{kj}}\sum_{i=k}^{n} P_{ij}\right) \ge 0 \tag{15c}$$

$$0 \le P_{ij} \le P_{u,ij} \tag{15d}$$

$$(k = 1, 2\ldots n;\ k \ne l;\ i = 1,2\ldots \mathrm{n};\ j = 1, 2\ldots m)$$

It is noticed that both the formulation and procedure of seeking the minimum buckling load are different from those of the maximum buckling loads. First, an equality constraint, Eqn (15b), is imposed in the minimization problem to ensure that the minimum value of the loads obtained from Eqns (15) will result in lateral instability at least in one storey, say storey l in this case. Second, the linear programming problem shown as Eqns (15) needs to be solved by n times with $l = 1, 2, 3, \ldots, n$, and the minimum buckling load obtained from Eqn (14) is the minimum of the minimum buckling loads associated with the instability of each storey.

Numerical examples

Cold formed steel storage racks with three different member end connection rigidities are investigated in this section to understand how the connection behaviour affects the magnitudes of the maximum and minimum buckling loads in the case of variable loading. For the 3-storey and 3-bay cold formed steel storage racks shown in Figs. 2 and 3, the material and cross-sectional properties of the cold formed steel column and beam are: Young's modulus: E=203N/mm^2(MPa); Column: box section with I_c=0.7492×10^{6}mm^4, area A=774.24mm^2 and thickness t=2.54mm; and Beam: shelf-beam with I_b=2.4738×10^{6}mm^4, area A=904.38mm^2 and thickness t=3.2mm. Three different member end connection rigidities are investigated. In Case 1, both column base and beam-to-column connections are rigidly connected. Cases 2 and 3 have semi-rigid connections for both column base and beam-to-column connections with corresponding end-fixity factors being 0.8 and 0.5, respectively.

Following the procedures described in previous section, the critical buckling loads associated with the three-storey and three-bay storage racks subjected to variable loading can be obtained from solving the maximization and minimization problems stated in Eqns (12), (14) and (15). For the foregoing three cases, the maximum and minimum frame buckling loads, together with their relative differences, are presented in Tables 1 to 3. For each case, the magnitudes of each variable load P_{ij} associated with the maximum and minimum frame buckling loads are presented so that the load patterns associated with the critical buckling loads can be obtained. Also presented in the tables are influential column attributes, such as the end-fixity factors, coefficient $\beta_{0,ij}$ of the column lateral stiffness modification factor, the effective length factor and the buckling load associated with non-sway buckling. It is demonstrated in the tables that an increase of column end-fixity factors would result in a decrease of the column effective length factor, which consequently leads to an increase of the magnitude of column buckling load in non-sway mode. It is also observed that columns with larger values of end-fixity factors would yield to a larger value of coefficient $\beta_{0,ij}$ which indicates the larger lateral stiffness against lateral instability.

For Case 1 in which both column base and beam-to-column connections are rigidly connected, it is observed from Table 1 that the maximum frame buckling load, 3235.98 kN, is achieved when lateral instability takes place in both the first and second stories of the rack (S_1=S_2=0, S_3≥0). The minimum frame buckling loads associated with lateral instability of the first and second stories are 3184.62 kN and 2851.51 kN, respectively. Therefore, the relative difference between the maximum and minimum frame buckling loads is 13.5%, which is significant. It is also observed from Table 1 that load pattern associated with the maximum and minimum frame buckling loads are different. The load pattern corresponding to the maximum frame buckling loads tends to place the loads only on the exterior columns which are more laterally flexible than the interior ones as characterized by the smaller value of coefficient $\beta_{0,ij}$. Contrasting to that, the load pattern associated with the minimum frame buckling loads applies the loads only on the interior columns which are laterally stiffer than the exterior ones.

In Case 2 the column base and beam-to-column connections are semi-rigidly connected with the corresponding end-fixity factor being 0.8. The presence of semi-rigid connections yields a more flexible frame which is evidenced by decreasing the coefficient $\beta_{0,ij}$ compared to that of Case 1. Consequently, the maximum frame buckling load of Case 2 reduces to 2606.85 kN, and the corresponding minimum frame buckling load decreases to 2508.45 kN, which yields the relative difference between the maximum and minimum frame buckling loads to be only 3.9%. The load patterns associated with the maximum and minimum frame buckling loads are similar to that of Case 1. Also found in Case 2 is that there are two different load patterns associated with the maximum frame buckling load of 2606.85 kN, in which one of them involves the lateral instability of the first storey only, and the other causes the lateral instability of both of the first and second storey.

As the connection rigidity is further decreased in Case 3 (r=0.5), the storage rack becomes more flexible, and the magnitudes of the maximum and minimum frame buckling loads are found to be identical and reduced to 1586.55 kN. In other words, there is no difference between the maximum and minimum frame buckling loads in terms of both magnitude and load patterns for Case 3. It is also found that lateral instability occurs in the first storey alone, and it is impossible to have the instability taking place in the second storey without the first storey failing. This can perhaps be explained by the considerable differences in the magnitudes of the coefficients $\beta_{0,ij}$ between the first and second storey columns.

Conclusions

An approach of evaluating the critical buckling loads for unbraced cold formed steel storage racks under variable loading is proposed in this paper. In the case of variable loading, the conventional assumption of proportional loading is abandoned, and the magnitude of each individual load can vary independently. Having the general stability equation developed, the concept of storey-based buckling is employed to formulate the problem of determining critical frame buckling loads to be a pair of minimization and maximization problems with stability constraints and solved by a linear programming method.

To demonstrate the efficiency of the proposed approach and to understand how the connection behaviour affects the maximum and minimum buckling loads in the case of variable loading, a 3-storey and 3-bay cold formed steel storage rack with three different member end connection rigidities are investigated. The investigation found that the difference between the maximum and minimum frame buckling loads can be significant as much as 13.5% for the storage rack with rigid connections. However, this difference becomes less significant when the semi-rigid connection behaviour was accounted for.

The minimum and maximum frame loads obtained from variable loading represent the lower and upper bounds of the buckling loads of the structure, which characterize the stability capacities of the frame under extreme loading conditions. It is noted that the minimum frame buckling load is the load of interest because loading patterns can not normally be controlled in a warehouse environment. The stability of cold formed steel storage racks subjected to variable loading takes into consideration of the volatility of live loads during the life span of the structures, which provides more appropriate results that may not available with the use of conventional stability analysis with assumption of proportional loading. Therefore, the proposed variable loading approach is recommended for design practice.

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Table 1. Case 1 (r = 1.0): results of the storage rack shown in Figure 2

Col. ij	$r_{l,ij}$	$r_{u,ij}$	$\beta_{0,ij}$	K_{ij}	$P_{u,ij}$ (kN)	$S_1 = 0, S_2 \geq 0, S_3 \geq 0$ Max. (kN)	Min. (kN)	$S_1 \geq 0, S_2 = 0, S_3 \geq 0$ Max. (kN)	Min. (kN)
11	1.0	0.682	0.712	0.565	2025	124.20	0	124.20	0
12	1.0	0.808	0.82	0.539	2224	0	271.17	0	0
13	1.0	0.808	0.82	0.539	2224	0	61.94	0	0
14	1.0	0.682	0.712	0.565	2025	163.62	0	163.62	0
21	0.682	0.805	0.590	0.610	1936	1936	0	1936	0
22	0.808	0.892	0.738	0.563	2039	0	2039	0	2009.46
23	0.808	0.892	0.738	0.563	2039	0	688.55	0	0.1
24	0.682	0.805	0.590	0.610	1936	337.39	0	337.39	0
31	0.805	0.805	0.674	0.583	1902	337.39	0	337.39	0
32	0.892	0.892	0.805	0.545	2176	0	61.98	0	841.84
33	0.892	0.892	0.805	0.545	2176	0	61.98	0	0.1
34	0.805	0.805	0.674	0.583	1902	337.39	0	337.39	0
Critical frame buckling loads $\sum P_{ij}$ =						3235.98 (S_2=0)	3184.62	3235.98 (S_1=0)	2851.51
Difference max. & min. loads						1.6%		13.5%	

Table 2. Case 2 (r = 0.8): results of the storage rack shown in Figure 3

Col. ij	$r_{l,ij}$	$r_{u,ij}$	$\beta_{0,ij}$	K_{ij}	$P_{u,ij}$ (kN)	$S_1 = 0, S_2 \geq 0, S_3 \geq 0$ Max. (kN)	Min. (kN)	$S_1 \geq 0, S_2 = 0, S_3 \geq 0$ Max. (kN)	Min. (kN)
11	0.8	0.593	0.529	0.631	1623	1623	0	5.27	0
12	0.8	0.739	0.625	0.599	1801	0	83.78	0	0
13	0.8	0.593	0.625	0.599	1801	0	1801	0	0
14	0.8	0.739	0.529	0.631	1623	196.77	0	6.63	0
21	0.593	0.734	0.494	0.418	3699	196.77	0	648.88	0
22	0.739	0.846	0.655	0.346	5399	0	0.12	0	0
23	0.739	0.846	0.655	0.346	5399	0	0.12	0	1.81
24	0.593	0.734	0.494	0.418	3699	0	0	648.73	0
31	0.734	0.734	0.579	0.378	4524	0	0	648.73	0
32	0.846	0.846	0.733	0.319	6351	0	645.9	0	0
33	0.846	0.846	0.733	0.319	6351	0	28.13	0	2506.64
34	0.734	0.734	0.579	0.378	4524	196.77	0	648.99	0
Critical frame buckling loads $\sum P_{ij}$ =						2606.85	2562.66	2606.85 (S_1=0)	2508.45
Difference max. & min. loads						1.7%		3.9%	

Table 3. Case 3 (r = 0.5): results of the storage rack shown in Figure 3

Col. ij	$r_{l,ij}$	$r_{u,ij}$	$\beta_{0,ij}$	K_{ij}	$P_{u,ij}$ (kN)	$S_1 = 0, S_2 \geq 0, S_3 \geq 0$ Max. (kN)	Min. (kN)	$S_1 \geq 0, S_2 = 0, S_3 \geq 0$ Max. (kN)	Min. (kN)
11	0.5	0.432	0.304	0.743	1171	0	0	N/A	N/A
12	0.5	0.592	0.375	0.703	1308	0	0		
13	0.5	0.592	0.375	0.703	1308	0	0		
14	0.5	0.432	0.304	0.743	1171	0	0		
21	0.682	0.805	0.59	0.610	1465	0	0	N/A	N/A
22	0.808	0.892	0.738	0.563	1774	0	0		
23	0.808	0.892	0.738	0.563	1774	0	0		
24	0.682	0.805	0.59	0.610	1465	0	0		
31	0.805	0.805	0.674	0.583	1614	0	0	N/A	N/A
32	0.892	0.892	0.805	0.545	1928	794.71	794.71		
33	0.892	0.892	0.805	0.545	1928	791.84	791.84		
34	0.805	0.805	0.674	0.583	1614	0	0		
Critical frame buckling loads $\sum P_{ij}$ =						1586.55	1586.55	N/A	N/A
Difference max. & min. loads						0%		N/A	

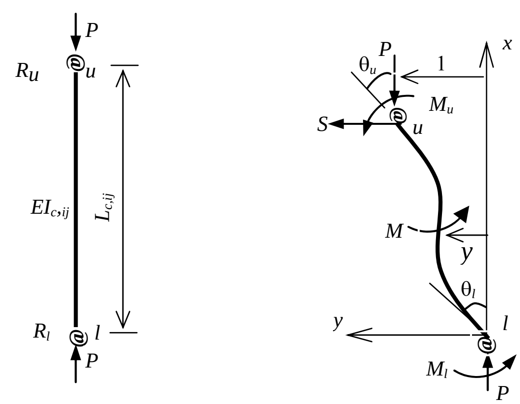

Fig. 1: An axially loaded semi-rigid column

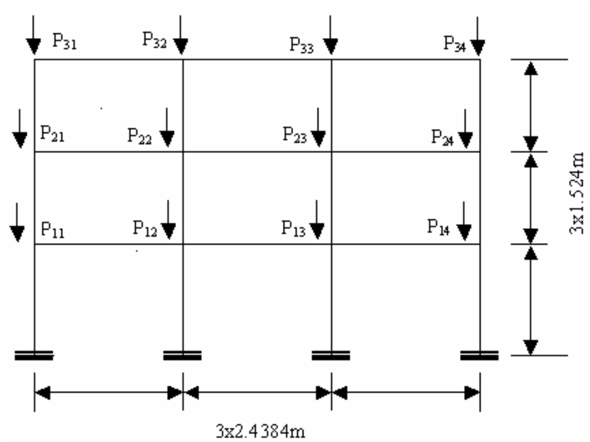

Fig. 2: 3-bay 3-storey storage rack (Case 1)

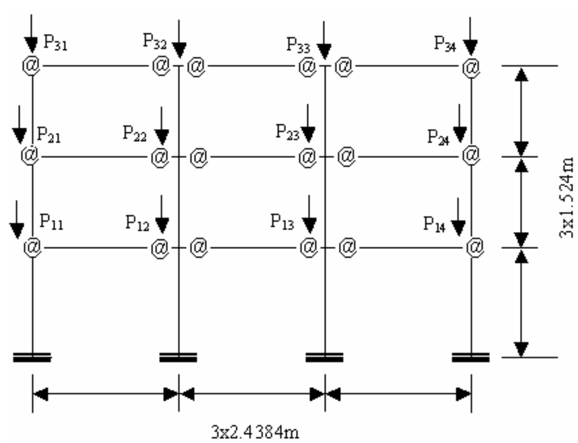

Fig. 3: 3-bay 3-storey storage rack (Case 2 and 3)