Oct 14th, 12:00 AM

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Nineteenth International Specialty Conference on Cold-Formed Steel Structures
St. Louis, Missouri, U.S.A, October 14 & 15 2008

Computed Flexural Buckling Stress for Cold-Formed Stainless Steel Columns

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Abstract

For the design of cold-formed stainless steel compression members, the ASCE Standard Specification can be used to determine the design axial strength. Due to the nonlinear stress strain behavior of the material, the design of stainless steel compression member is more complex than those of carbon steels. Instead of using the modulus of elasticity (E_o), the non-linear tangent modulus (E_t) were used for the design of cold-formed stainless steel columns. In this case, iterative procedures are needed to calculate the column buckling stress. Consequently, a simplified approach is developed to compute the column flexural buckling stress while without iterative process. In this simplified formulation, mathematical operation was utilized for numerical approximations. It is shown that the column strengths computed by the simplified formulas had good agreement with those determined by the ASCE Standard Specification. The simplified formulas are proposed to calculate the flexural buckling stress of cold-formed stainless steel columns. This paper presents the development of the proposed formulas for the design of stainless steel columns.

Key Words: Cold-Formed, Stainless Steel, Column, Specification, Tangent Modulus, Flexural Buckling, Approximation

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Introduction

Cold-Formed stainless steel compression members are widely used in architectural and structural applications, e.g., roof trusses, arched trusses and columns. These stainless steel structures are sometimes the preferred choice due to their superior corrosion resistance, attractive appearance, ease of maintenance and high strength. In the United States, ASCE Standard Specification, SEI/ASCE 8-02 (ASCE, 2002), can be used for the design of cold-formed stainless steel compression members. Because of the difference in mechanical behavior as shown in Fig. 1, the design of stainless steel columns is more complicated than those of carbon steels (ASCE, 1991). Stainless steels also have gradually yielding type of stress-strain curves with relatively low proportional limits (Johnson et al., 1969; Yu, 2000). Due to the nonlinear stress-strain behavior, the design of such compression members has long been followed by using the tangent modulus theory (Johnston, 1976; Galambos, 1968).

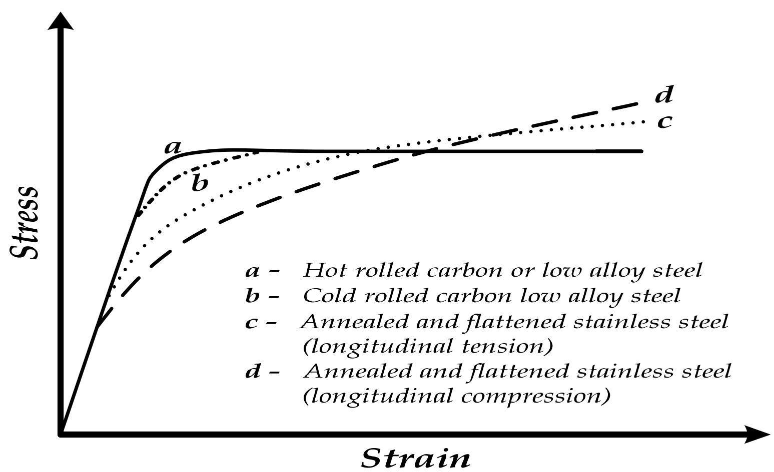

Fig. 1 Stress-Strain Curves of Carbon and Stainless Steels

Tangent modulus is used to account for the inelastic buckling of stainless steel compression components. It can be determined by using the modified Ramberg-Osgood equation (Ramberg et al., 1943; Hill, 1944) for specified types of stainless steels. Because of the nonlinear nature of tangent modulus, the column buckling stress is determined through an iterative process until the satisfied tolerance is reached. Previous research studies discussed different methods to deal with the nonlinear calculations (Rasmussen et al., 2000;

Rasmussen et al. 1997). This type of calculation is often tedious and time-consuming as compared with that of hot-rolled steel column design.

This paper presents the development of the simplified formulas for determining the flexural buckling stress of stainless steel column without successive iterations. Mathematical operations used to generate the simplified equations are discussed and the proposed design formulas are summarized herein. The proposed design formulas can be alternatively used for the design of austenitic type of cold-formed stainless steel columns subjected to flexural buckling. It is shown that the proposed design formulas can provide a quick and good solution as compared with the ASCE Standard solutions.

Current Design Specification

The ASCE Standard Specification (ASCE 2002) provides the design requirements to determine the flexural buckling strength for concentrically loaded cold-formed stainless steel compression members. It specifies that the flexural buckling stress, F_n, shall be determined as follows:

$$F_n = \frac{\pi^2 E_t}{(KL/r)^2} \leq F_y \tag{1}$$

in which KL/r is the slenderness ratio and F_y is the specified yield strength as given in Table 1 obtained from ASCE specification for austenitic type stainless steels.

Table 1 ASCE Specified F_y for Austenitic Type Stainless Steels

Types of Stress	F_y, MPa			
	Types 201, 301, 304, 316			
	Annealed	1/16Hard	1/4 Hard	1/2Hard
Longitudinal Tension	206.9	310.3	517.1	758.5
Transverse Tension	206.9	310.3	517.1	758.5
Transverse Compression	206.9	310.3	620.6	827.4
Longitudinal Compression	193.1	282.7	344.8	448.2

1 ksi = 6.895 MPa

The tangent modulus, E_t, in compression corresponding to buckling stress, F_n, can be determined by using the modified Ramberg-Osgood equation [1] as follows:

$$E_t = \frac{E_o F_y}{F_y + 0.002 n E_o (F_n / F_y)^{n-1}} \quad (2)$$

in which E_o is the initial modulus of elasticity and n is the coefficient used for determining tangent modulus of specified type of stainless steel. Table 2 gives values of E_o and n for austenitic type stainless steels as specified in the ASCE Standard.

Because of the correlation between the buckling stress and tangent modulus in Eq. (2), an assumed buckling stress F_n is needed to determine the value of E_t. Then, this calculated value of E_t is substituted into Eq. (1) to determine the buckling stress, F_n. Since the calculated buckling stress is seldom equal to the first assumed buckling stress, further successive iterations are required to obtain the true buckling stress. Though the process is tedious and time-consuming, this buckling stress can be achieved when the satisfied convergence of iteration is reached.

Table 2 Specified E_o and n Values for Austenitic Type Stainless Steels

Types of Stress	Types 201, 301, 304, 316					
	Annealed and 1/16 Hard		1/4Hard		1/2Hard	
	E_o(MPa)	n	E_o(MPa)	n	E_o(MPa)	n
Longitudinal Tension	193100	8.31	186200	4.58	186200	4.21
Transverse Tension	193100	7.78	193100	5.38	193100	6.71
Transverse Compression	193100	8.63	193100	4.76	193100	4.54
Longitudinal Compression	193100	4.10	186200	4.58	186200	4.22

1 ksi = 6.895 MPa

Development of Mathematical Formulation

A simplified approach was developed to determine the flexural buckling stress without using iterative process. The tangent modulus value obtained from the modified Ramberg-Osgood equation was used to generate the simplified design equation. Numerical approximation by using Taylor series expansion is applied to simplify the calculations.

Linearization Model

A typical flexural buckling stress curve for type 304 cold-formed stainless steel column is shown in Fig. 2. Now by applying logarithm operation to the flexural buckling stress curves, i.e., log(F_n), it was found that a portion of the nonlinear buckling stress curve can be approximately expressed by a line segment between two points at A and B as shown in Fig. 3. Then this linear portion of the curve can be defined by these two specified points at A(C_0, logF_y) and B(C_1, logF_1) as follows:

$$\frac{\log F_1 - \log F_y}{C_1 - C_0} = \frac{\log F_n - \log F_y}{C - C_0} \tag{3}$$

in which C = KL/r = slenderness ratio, and C_0 and C_1 are two specified slenderness ratios with their corresponding buckling stresses at F_y and F_1, respectively.

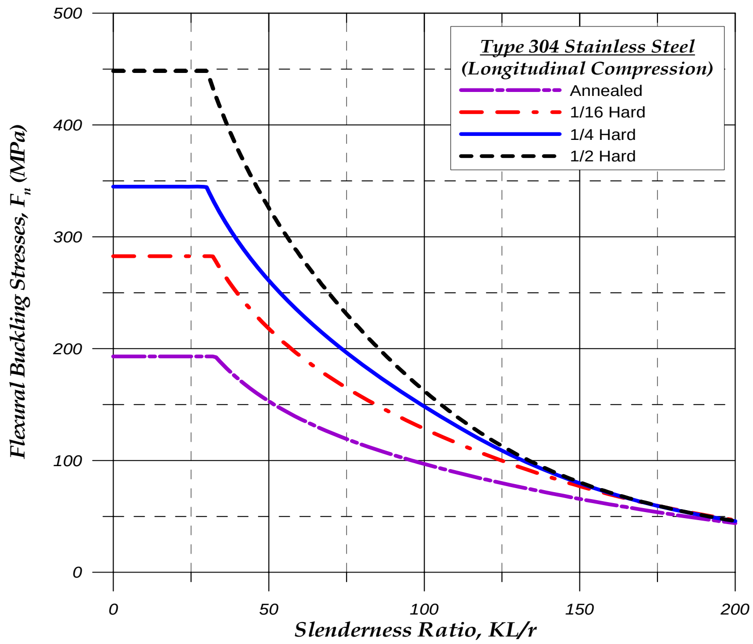

Fig. 2 Flexural Buckling Stresses For Type 304 Stainless Steel Columns

Equation (3) can be rearranged in terms of exponential expression as

$$F_n = F_1^{\frac{C-C_0}{C_1-C_0}} \times F_y^{\frac{C_1-C}{C_1-C_0}} \qquad (4)$$

The slenderness ratio of C_0 can be determined when F_n is equal to F_y, i.e.,

$$C_0 = KL/r = \pi\sqrt{\frac{E_y}{F_y}} \qquad (5)$$

where E_y is the tangent modulus at yield strength level and is equal to

$$E_y = \frac{E_o}{1 + 0.002n\dfrac{E_o}{F_y}} \qquad (6)$$

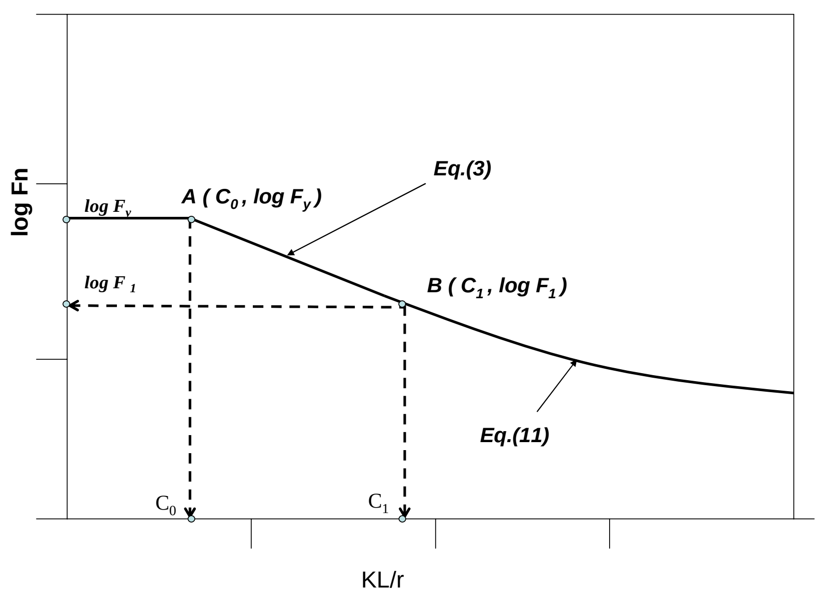

Fig. 3 Simplified Flexural Buckling Stress Curve

The buckling stress F_1 defined in Fig. 3 can be obtained from Eq. (2) by rearranging E_t and F_n and replacing F_n by F_1 as follows:

$$F_1 = \left[\left(\frac{E_o - E_t}{E_t}\right)\frac{F_y}{0.002nE_o}\right]^{\frac{1}{n-1}} \times F_y \quad (7)$$

Let $\alpha = \frac{E_o - E_t}{E_t} = \frac{E_o}{E_t} - 1$ (8)

Then, Eq. (7) becomes

$$F_1 = \left(\frac{\alpha F_y}{0.002nE_0}\right)^{\frac{1}{n-1}} \times F_y \quad (9)$$

The value of F_1 can be considered as the proportional limit, which varies with respect to the type of stainless steels. The tangent modulus E_t can be expressed in terms of α, i.e.,

$$E_t = \frac{E_o}{(1+\alpha)} \quad (10)$$

Substitution of Eq. (10) into Eq. (1) yields the following general expression for F_n:

$$F_n = \frac{\pi^2 E_t}{(KL/r)^2} = \frac{\pi^2 E_o}{C^2(1+\alpha)} \quad (11)$$

By using Eq. (11), the limiting slenderness ratio of C_1 can be determined for the buckling stress at $F_n = F_1$ as follows:

$$C_1 = \pi\sqrt{\frac{E_o}{F_1(1+\alpha)}} \quad (12)$$

Approximation of α

Once the α value is known, the buckling stress F_1 in Eq. (9) and the limiting slenderness ratio C_1 in Eq. (12) can be calculated for specified type of cold-formed stainless steels. The determination of buckling stress F_n becomes easy and without iterative calculations as presented in Eq. (4). As a result, the parameter α can be expressed as

$$\alpha = 0.002n\left(\frac{E_o}{F_y{}^n}\right)\left[\frac{\pi^2 E_o}{C^2(1+\alpha)}\right]^{n-1} \quad (13)$$

The above equation can also be rearranged to form a polynomial function, namely

$$f(\alpha) = \alpha(1+\alpha)^{n-1} = 0.002n\left(\frac{\pi^2}{C^2}\right)^{n-1}\left(\frac{E_o}{F_y}\right)^n \tag{14}$$

Equation (14) can be approximately expressed by using Taylor series expansion as

$$\alpha(1+\alpha)^{n-1} = \sum_{i=0}^{N}\frac{f^i(\alpha)}{i!}\alpha^i + \cdots \tag{15}$$

in which $f^i(\alpha)$ is the i^{th} derivative of the function $f(\alpha)$.

Higher degrees of derivatives in Eq. (15) are assumed to be neglected for common engineering practice. Then, for N = 2, Eq. (15) can be approximately expressed as

$$\alpha + (n-1)\alpha^2 = 0.002n\left(\frac{\pi^2}{C^2}\right)^{n-1}\left(\frac{E_o}{F_y}\right)^n \tag{16}$$

The above equation is a typical second order equation and, therefore, can be solved by the quadratic formula as follows:

$$\alpha = \frac{-1+\sqrt{1+4(n-1)0.002n\left(\frac{\pi^2}{C^2}\right)^{n-1}\left(\frac{E_o}{F_y}\right)^n}}{2(n-1)} \tag{17}$$

This α value is used for determining the elastic buckling stress in Eq. (11). To consider the inelastic buckling stress, the α value is determined by taking N = 3 in Eq. (15). To meet a satisfied convergence, the following limitation is recommended:

$$\frac{(n-1)(n-2)\alpha^3/2}{\alpha+(n-1)\alpha^2} \le 5\% \tag{18}$$

Assume that the maximum value of the parameter α determined from Eq. (18) is equal to β. It yields

$$\alpha_{max} = \beta = \frac{1+\sqrt{1+\frac{2(n-2)}{0.05(n-1)}}}{n-2}\times 0.05 \tag{19}$$

in which β is used to determine the buckling stress of F_1 in Eq. (9) and the limiting slenderness ratio of C_1 in Eq. (12) as shown in Fig. 3.

Proposed Design Formulas

Based on the above-mentioned simplified formulations, the following design provisions were proposed herein to determine the flexural buckling stress, F_n, for austenitic types of cold-formed stainless steel compression members.

For doubly symmetric sections, closed cross sections, and any other sections which can be shown not to be subjected to torsional or torsional-flexural buckling, the flexural buckling stress, F_n, *shall be determined as follows:*

For $KL/r \le C_1$:

$$F_n = F_y^{\lambda_o} F_1^{\lambda_1} \le F_y \tag{20}$$

For $KL/r > C_1$:

$$F_n = \frac{\pi^2 E_o}{\left(\frac{KL}{r}\right)^2 (1+\alpha)} \tag{21}$$

where:

$$\lambda_o = \frac{C_1 - KL/r}{C_1 - C_o} \tag{22}$$

$$\lambda_1 = 1 - \lambda_o \tag{23}$$

$$C_o = \pi \sqrt{\frac{E_y}{F_y}} \tag{24}$$

$$C_1 = \pi \sqrt{\frac{E_o}{F_1(1+\beta)}} \tag{25}$$

$$E_y = \frac{E_o}{1 + 0.002n \frac{E_o}{F_y}} \tag{26}$$

$$F_1 = F_y \left(\frac{\beta F_y}{0.002nE_o} \right)^{\frac{1}{n-1}} \tag{27}$$

$$\alpha = \frac{-1+\sqrt{1+4(n-1)0.002n\left[\frac{\pi^2}{(KL/r)^2}\right]^{n-1}\left(\frac{E_o}{F_y}\right)^n}}{2(n-1)} \tag{28}$$

$$\beta = \frac{0.05+\sqrt{0.0025+\frac{0.1(n-2)}{(n-1)}}}{n-2} \tag{29}$$

Comparisons of Results

Comparisons are made between the predicted buckling stresses computed from the ASCE Standard design equations and the proposed design formulas. This paper summarizes the result of comparison. Type 304 stainless steel columns are used to compare the predicted flexural buckling stresses. The specified material properties used to determine the buckling stress for ASCE Standard are given in Table 2. The design parameters for the same materials determined from the proposed design equations are listed in Table 3. For this type of stainless steel, the computed buckling stresses, $F_{n,ASCE}$ and $F_{n,prop}$, and the ratios of $F_{n,prop}/F_{n,ASCE}$ with respect to the slenderness ratios, *KL/r*, in longitudinal compression are given in Table 4. In this table, $F_{n,ASCE}$ and $F_{n,prop}$ are predicted flexural buckling stresses determined from the ASCE Standard and proposed design equations, respectively. This comparison is also illustrated in Fig. 4. It is shown that the proposed design equations, without having iterative calculations, can predict good results as compared with the ASCE Standard results.

Conclusions

The buckling stress of cold-formed stainless steel compression members is determined on the basis of the tangent modulus theory because of the nonlinear stress strain behavior of the materials. The determination of flexural buckling stress needs iterative process which is often tedious and time-consuming for a typical column design. In order to simplify the design calculation, mathematical approximations are utilized to calculate flexural buckling stress which needs non-iterative process. This paper discusses the reasoning behind for the development of the simplified formulas. Comparisons are made between the predicted column flexural buckling stresses determined from the ASCE design formulas and the proposed design equations. It is shown that the flexural buckling stresses determined by the proposed design equations are in good agreement with those calculated by the ASCE design formulas.

Table 3　Computed Parameters Used in the Proposed Design Formulas

Type of Stress(304S.S.)		β	C_0	C_1	F_1 (MPa)
Longitudinal Compression	Annealed	0.1500	32.8	176.6	53.12
	1/16 Hard	0.1500	32.0	137.3	87.94
	1/4 Hard	0.1252	29.9	115.0	123.48
	1/2 Hard	0.1429	30.2	98.4	165.91
Transverse Compression	Annealed	0.0526	23.2	136.1	97.72
	1/16 Hard	0.0526	22.9	108.2	154.55
	1/4 Hard	0.1179	27.8	80.5	259.62
	1/2 Hard	0.1270	27.2	67.3	373.58

1 ksi = 6.895 MPa

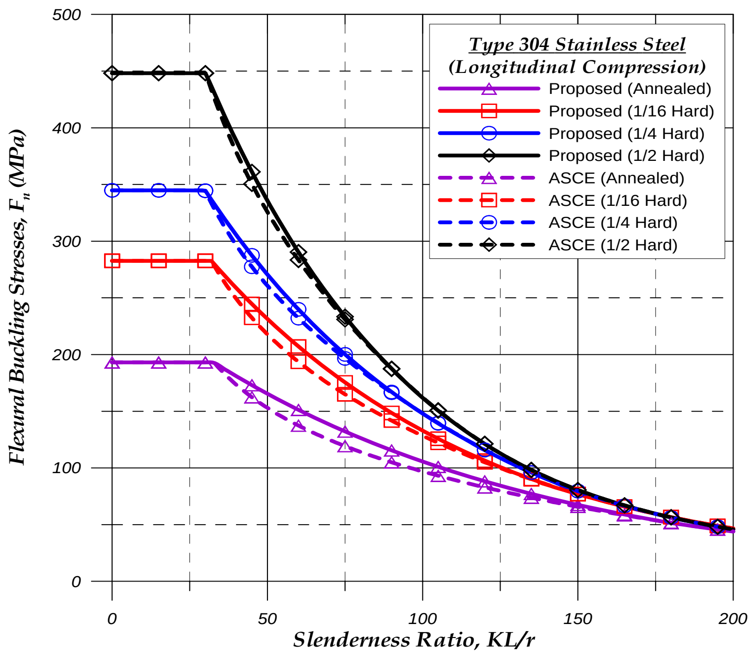

Fig. 4　Comparisons of Computed Buckling Stress Curve

Table 4 Comparisons of Computed Buckling Stresses for Type 304 Stainless Steel Columns in Longitudinal Compression

	Annealed			1/16 Hard			1/4 Hard			1/2 Hard		
KL/r	$F_{n,ASCE}$ (MPa)	$F_{n,prop}$ (MPa)	$\frac{F_{n,prop}}{F_{n,ASCE}}$	$F_{n,ASCE}$ (MPa)	$F_{n,prop}$ (MPa)	$\frac{F_{n,prop}}{F_{n,ASCE}}$	$F_{n,ASCE}$ (MPa)	$F_{n,prop}$ (MPa)	$\frac{F_{n,prop}}{F_{n,ASCE}}$	$F_{n,ASCE}$ (MPa)	$F_{n,prop}$ (MPa)	$\frac{F_{n,prop}}{F_{n,ASCE}}$
20	193.1	193.1	1.00	282.7	282.7	1.00	344.4	344.5	1.00	448.2	448.2	1.00
40	173.4	181.0	1.04	249.1	258.6	1.04	296.3	305.4	1.03	378.5	388.4	1.03
60	137.4	151.2	1.10	193.7	207.2	1.07	232.1	239.9	1.03	283.4	290.3	1.02
80	114.2	126.4	1.11	156.7	166.0	1.06	186.0	188.4	1.01	215.6	217.0	1.01
100	96.9	105.6	1.09	128.4	133.0	1.04	148.2	148.0	1.00	162.2	161.5	1.00
120	82.9	88.3	1.07	105.0	106.5	1.01	116.0	115.7	1.00	121.3	121.3	1.00
140	71.0	73.8	1.04	85.5	85.1	1.00	90.1	90.1	1.00	91.8	91.8	1.00
160	60.7	61.7	1.02	69.4	69.4	1.00	70.6	70.6	1.00	71.1	71.1	1.00
180	51.7	51.5	1.00	56.7	56.6	1.00	56.3	56.3	1.00	56.5	56.5	1.00
200	44.0	43.9	1.00	46.7	46.7	1.00	45.8	45.8	1.00	45.8	45.8	1.00
		AVG =	1.05			1.02			1.01			1.01
		COV =	0.041			0.027			0.013			0.011

1 ksi = 6.895 MPa

Appendix. – References

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Appendix. - Notation

The following symbols are used in this paper.

C = Slenderness ratio, KL/r

C_o = Specified slenderness ratio at $F_n = F_y$

C_1 = Limiting slenderness ratio at $F_n = F_1$

E_o = Initial modulus of elasticity

E_t = Tangent modulus

E_y = The tangent modulus at yield strength level

F_y = Specified yield strength

F_1 = Specified buckling stress with respect to C_1

F_n = Nominal buckling stress

$F_{n,ASCE}$ =Nominal buckling stress determined from ASCE Standard Specification

$F_{n,prop}$ = Nominal buckling stress determined from the proposed design formulas

K = Effective length factor
L = Unbraced length of member
n = Coefficient used for determining the tangent modulus
r = Radius of gyration
α = $\frac{E_o}{E_t} - 1$
β = Constant
λ_o = Parameter used for determining buckling stress
λ_1 = 1-λ_o