Strength of Cold-formed Steel Box Columns

N. E. Shanmugam

S. P. Chiew

S. L. Lee

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N.E. Shanmugam[1], M.ASCE, S.P. Chiew[2], S.L. Lee[3], F.ASCE

ABSTRACT

Cold-formed steel box columns have two obvious modes of failure; they can reach the ultimate capacity either by overall column buckling or local buckling. This paper is concerned with a numerical method to predict the ultimate load-carrying capacity of cold-formed steel box columns subjected to axial force and unequal end moments. The method accounts for the effect of local plate buckling and initial imperfections upon the ultimate strength of columns. Moment-curvature-thrust relationships are developed by using piecewise linear stress-strain curves; they are incorporated into the column analysis in which the differential equation of bending is numerically integrated. Use of a suitable failure criterion and numerical procedure makes it possible to obtain column curves. For design purposes, column curves from which ultimate strength of column under axial or eccentric loading conditions can be easily obtained are presented.

INTRODUCTION

Thin-walled stiffened compression elements are commonly encountered in cold-formed steel box columns. When such columns are subjected to compressive loading, with the onset of buckling the growth of out of plane deflections in the plate elements results in changes in the stress pattern which in turn reduces the plate stiffness. This reduction causes the failure of the column at a load less than its classical Euler buckling load. The design of cold-formed steel box columns, therefore, requires the consideration of local plate buckling, overall column buckling and the interaction between the local and overall buckling. A close-form evaluation of ultimate strength of such columns is well nigh impossible and the use of numerical procedure, therefore, becomes necessary.

Numerous investigators (6,8,11,17) have studied the effect of local buckling on the strength of plate elements subjected to compressive loading. Rigorous methods using large deflection theory coupled with finite difference methods (12,14) and elasto-plastic finite

1. Senior Lecturer, Department of Civil Engineering, National University of Singapore, Kent Ridge, Singapore 0511.
2. Structural Engineer, T.H. Chuah and Associates (Pte) Ltd, 190 Middle Road, Fortune Centre, #11-04, Singapore 0718.
3. Professor and Head, Department of Civil Engineering, National University of Singapore, Kent Ridge, Singapore 0511.

element formulations (5,7) have been proposed for the ultimate strength analysis of plate elements. For design office use, however, simple methods using the effective width concept were presented by Von Karman (16) and Winter (18). Many of the design specifications (1,2,4) for cold-formed steel structural elements have accepted the effective width principle and it has been proved to yield satisfactory results.

A simple analytical method was presented recently by the authors (10,15) for predicting the strength of thin-walled welded steel box columns subjected to axial load and end moments. The method accounts for local buckling of component plates, welding residual stresses and initial column imperfections. The method is extended for the analysis of cold-formed steel box columns in the present study. Moment-curvature-thrust relationships are developed by using piecewise linear stress-strain curves. They are incorporated into the column analysis in which the differential equation of bending is numerically integrated. Use of a suitable failure criterion and numerical procedure makes it possible to obtain the ultimate strength. Results are presented in the form of column curves which can be used readily by designers.

THEORY

An exact analysis for ultimate strength of cold-formed steel box columns is complicated because of the local buckling of component plates and the non-linearity of the stress-strain curves. However, the solution can be greatly simplified by adopting approximate linearised stress-strain curves. Box columns can be treated as an assemblage of long plates supported along the longitudinal edges as shown in Figure 1. Local buckling of the component plates, which are under axial compression, is allowed for by applying an appropriate load-shortening curve, while those under tension are treated by assuming an elastic-perfectly plastic stress-strain curve.

Simplified piecewise linear stress-strain curves (Figure 2b) based on approximations of stress-strain curves by Moxham, Crisfield, Harding et al. and Little (3) were proposed by Shanmugam et al. (15). These curves represent unwelded plates having slenderness ratios equal to 80, 55, 40 and 30 or less, and initial imperfection of b/1000, 'b' being the width of the plate in a direction normal to the compressive loadings. These curves have been used to account for the local buckling of the component plates of box columns and the moment-curvatures-thrust (M-ϕ-P) relationships for column cross-sections were developed as explained in the following sections.

MOMENT-CURVATURE-THRUST RELATIONSHIPS

It becomes imperative to formulate the moment-curvature-thrust relationships for individual cross-sections in order to determine the equilibrium curves defining the domain of stable equilibrium of moment, thrust and column length. The M-ϕ-P relationship is computed numerically using the method similar to that adopted by Nishino et al. (13). The following assumptions are made in the analysis which follows:

(i) the material is homogeneous and isotropic in both the elastic and plastic states,

(ii) elastic-perfectly plastic stress-strain relation (Figure 2a) is assumed for flange plate under tension and the local buckling of the flange plate under compression is allowed by applying an appropriate stress-strain curve from the set of curves given in Figure 2b. The portion of the web plate under compression is treated in a similar manner to that of the compression flange by applying the stress-strain curve with the assumption that 'b' is equal to the depth of the compression zone,

(iii) the local buckling of the plate due to shear is ignored,

(iv) the strain distribution is linear across the depth of the cross-section (Figure 3),

(v) the residual stresses in each component plate are assumed to be in self-equilibrium and distributed in the form shown in Figure 4,

(vi) the effect of strain reversal is negligible, and

(vii) the deflections are small so that curvature can be expressed by the second derivatives of deflections.

Consider the cross-section as shown in Figure 3(a). The neutral axis is located at a distance R from the concave extreme fibre of the cross section. For purpose of obtaining the M-ϕ-P relationship, the cross section is discretised into 'n' elements. The strain at any element 'i' can be expressed in the non-dimensional form

$$\frac{\varepsilon_i}{\varepsilon_y} = \frac{\varepsilon_c}{\varepsilon_y} + \phi\frac{2y_i}{d} + \frac{\varepsilon_{ri}}{\varepsilon_y} \qquad (1)$$

in which ε_i = total strain at element i, positive if in tension

ε_c = strain at the centroid of the section

ϕ = curvature non-dimensionalised by the curvature at initial yielding for bending, $\Phi = 2\varepsilon_y/d$

y_i = distance of the centre of element i from the centroidal axis

ε_{ri} = residual strain at element i

ε_y = yield strain

d = depth of cross section

The corresponding stress σ_i is obtained by making use of the appropriate stress-strain curves given in Figure 2b, σ_i being positive if in tension.

The axial force and bending moment carried by the cross-section can be easily obtained by using the equilibrium equations

$$p = - \sum_{i=1}^{n} \frac{1}{A} \frac{\sigma_i}{\sigma_y} \Delta A_i \tag{2(a)}$$

$$m = \frac{1}{z} \sum_{i=1}^{n} \frac{\sigma_i}{\sigma_y} y_i \Delta A_i \tag{2(b)}$$

where $p = P/P_y$

$m = M/M_y$

n = total number of elements

A = area of cross-section

z = plastic modulus of section

ΔA_i = area of element i

P_y = squash load

M_y = plastic moment = $\sigma_y z$

The moment developed about the centroidal axis of a cross-section can be determined numerically for any given value of axial thrust and curvature ϕ. The computations involve the determination of the correct position of the neutral axis. Successive values of R can be interpolated and the correct value is determined such that the net compressive force from the resulting strain distribution defined by R and ϕ obtained from Eq. 2(a) together with Eq. 1 matched the given axial thrust. The resulting moment can be calculated from Eq. 2(b) and the procedure repeated for other values of ϕ. Typical M-ϕ-P relationships obtained are given in Figure 5. The variation of $m = M/M_y$ with respect to $\phi = \Phi/\Phi_y$ for $p = P/P_y = 0.3$, is plotted in Figure 5 for plate slenderness, b/t = 40, 55 and 80.

ULTIMATE STRENGTH OF COLUMNS

Box columns subjected to eccentric load can be treated by considering the cantilever column under the action of axial force P, transverse shear force Q and bending moment M at the free end as shown in Figure 6. Initial imperfection is assumed to be represented by initial curvature which is constant throughout the length of the column. The equilibrium condition and the curvature-displacement relation are given, respectively in the non-dimensional form as

$$m = m_f - \frac{Ar}{z} (pw + qx) \tag{3}$$

$$\frac{d^2w}{dx^2} = (\phi + \phi_i) \frac{2r}{d} \tag{4}$$

in which $m_f = M_f/M_y$

$q = Q/P_y\sqrt{\varepsilon_y}$

$x = x\sqrt{\varepsilon_y}/r$, $w = W/r$ and $\phi_i = \Phi_i/\Phi_y$.

The deflected shape of a column for given values of m_f, p and q and prescribed values of ϕ_i, can be obtained by integrating Eq. 4 in view of Eq. 3 and the moment-curvature-thrust relationship for a particular cross-section developed earlier. In order to simplify the integration of Eq. 4 a numerical procedure was adopted. The method is explained in reference 15 and hence it is not repeated herein.

With the help of the numerical integration technique the relation between m and x can be obtained for a set of p, q and ϕ_i and various assumed values of m_f. The m-x relationships thus obtained are plotted as shown in Figure 7 and they are referred to as equilibrium curves. Applying Horne's (9) stability criterion the envelope of the equilibrium curves can be constructed. The envelope is the boundary of the stable equilibrium domain of the cantilever column of certain length subjected to the combined action of end moment, axial thrust and shear. The variation of the moment capacity along the length of the column for a particular value of axial thrust and shear is given by the envelopes in Figure 7.

It is more useful to present the ultimate strength of box columns in the form of column curves. The equations developed for cantilever column can be extended to the treatment of simply supported box columns subjected to unequal end moments with $-1 \leqslant \kappa \leqslant 1$ as shown in Figure 8. The simply supported column may be treated as two cantilever columns of lengths x_1 and x_2 with fixed ends at point 0. The part of the colum to the right of point 0 corresponds to the cantilever column of Figure 6. With $Q = M(1 - \kappa)/L$ whilst the part to left of 0 corresponds to a column under a transverse load Q opposite in sense to that shown in Figure 6 and hence care should be exercised in using the appropriate envelopes.

A typical envelope constructed for the whole length of a column with particular values of ϕ_i and q and various magnitudes of p is shown in Figure 9. The curves are plotted on the m-λ plane in which λ is normalised slenderness ratio given by

$$\lambda = \frac{1}{\pi}\frac{L}{r}\sqrt{\varepsilon_y} \qquad (5)$$

The envelopes on the left correspond to the cantilever column of length x_1 and on the right to the cantilever column of length x_2. For given values of p and q, at the limit of stability, the point (m, λ_1) and $(\kappa m, \lambda_2)$ must be on the envelopes corresponding to p and q. The critical values of λ, and x_2 are not known a priori, however, they must satisfy the following equations:

$$\lambda_1 + \lambda_2 = \lambda \qquad (6)$$

$$q = \frac{z}{Ar} \frac{m(1 - \kappa)}{\pi\lambda} \tag{7}$$

A value of λ is first assumed along with the prescribed values of κ, m and p. q is determined from Eq. 7 and the values of λ_1 and λ_2 and hence λ are obtained from the envelopes corresponding to the set of value of p and q. This process is repeated until the assumed value of λ matches the computed one. A family of column curves (p-λ curves) for various values of m and κ can be constructed. The steps involved in the construction of column curves are illustrated in Figure 10.

RESULTS AND DISCUSSION

The proposed method was applied to the analysis of the behaviour of box columns having different plate slenderness ratios. The parameters were so chosen that they show the influence of end moment, the moment ratio and column slenderness. Column curves which were generated from the results are presented in Figures 11-13. In each of these figures curves are given for moment ratios equal to 1.0, 0.0 and -1.0, respectively. κ equal to one represents the case of a column under equal end moments. Results are presented for end moment (M/M_y) equal to 0.0, 0.1 and 0.3 and plate slenderness ratio of 30, 40, 55 and 80. All these curves were obtained assuming initial curvature, ϕ_i, equal to 0.1 and fixed value of residual stress level shown in Figure 4.

The results for κ = 0.0 and -1.0 clearly show the effect of unequal end moments upon the strength of axially loaded box columns. For all values of m the column strength increases with the decrease in κ values. For example consider column curves in Figures 11(a) and 13(a) in which κ = 1.0 and -1.0 respectively. For λ = 0.6 and m = 0.3 the strength of column for κ equal to 1.0 is 0.56 and the corresponding value for κ equal to -1.0 is 0.72 an increase of about 29 percent. It is also observed that the columns under single curvature bending (κ = 0.0, 1.0) are normally weaker than those under double curvature bending (κ = -1.0). This effect is found to be more significant in the case of intermediate column range. Similar observations can be made for all values of κ in the case of columns with other slenderness values also.

It is obvious from the figures that the column strength drops significantly as the plate slenderness increases. For axially loaded short columns this reduction is approximately 20 percent when the slenderness ratio of the component plate is increased from 30 to 55. The figures also show that the column strength is independent of λ in the lower range of column slenderness, this range increasing with plate slenderness. At low b/t ratio the interactive buckling failure occurs for the whole range of λ, whereas for larger b/t ratios the failure is mainly due to local buckling upto a certain value of λ. The interactive buckling becomes predominant thereafter.

CONCLUSIONS

A simple analytical method to the analysis of cold-formed steel box columns pinned at their ends and subjected to axial load and unequal end moments has been presented. The method is capable of predicting the load-carrying capacity of these columns which experience local buckling of component plates and may have initial column imperfections. Results presented in the form of column curves clearly show the adverse effect of the local buckling of component plates. The ultimate capacity of these columns is reduced significantly for larger b/t ratios of component plates. The columns subjected to double curvature bending have been found to be stronger than those under single curvature bending. The column curves provide the means for estimating the ultimate strength of cold-formed steel box columns subjected to loads with unequal eccentricity at the ends. They should be useful design tools and intermediate values can be obtained by interpolation.

APPENDIX I - REFERENCES

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APPENDIX II - NOTATION

The following symbols are used in this paper:

A = area of column cross-section,

b = width of a compressed plate element

d = depth of column cross-section,

L = length of simply-supported column,

M = moment at any section along the column,

M_f = fixed end moment of cantilever column,

M_y = plastic moment and equals to $\sigma_y z$

m = M/M_y

n = total number of elements in cross-section,

P = axial force,

P_y = squash load and equals to $\sigma_y A$

p = P/P_y

Q = transverse shear force,

r = radius of gyration,

W = transverse deflection,

X = distance along the length of cantilever column,

X_1 = length of left cantilever column,

X_2 = length of right cantilever column,

y_i = distance of the centre of element i from the centroidal axis,

z = plastic section modulus,

ΔA_i = area of element i,

ε = normal strain,

ε_c = strain at the centroidal axis of section,

ε_i = total strain at element i,

ε_{ri} = residual strain at element i,

ε_y = strain at yield point,

κ = moment ratio,

σ = normal stress,

σ_{rc} = compressive residual stress,

σ_y = yield stress,

Φ = curvature caused by bending,

Φ_i = initial curvature,

Φ_y = curvature at initial yielding for pure bending moment and equals to $(2\varepsilon_y/d)$,

ϕ = Φ/Φ_y

ϕ_i = Φ_i/Φ_y

λ = nondimensionalised slenderness ratio of simply-supported column,

λ_1 = $(X_1/\pi r)\sqrt{\varepsilon_y}$,

λ_2 = $(X_2/\pi r)\sqrt{\varepsilon_y}$.

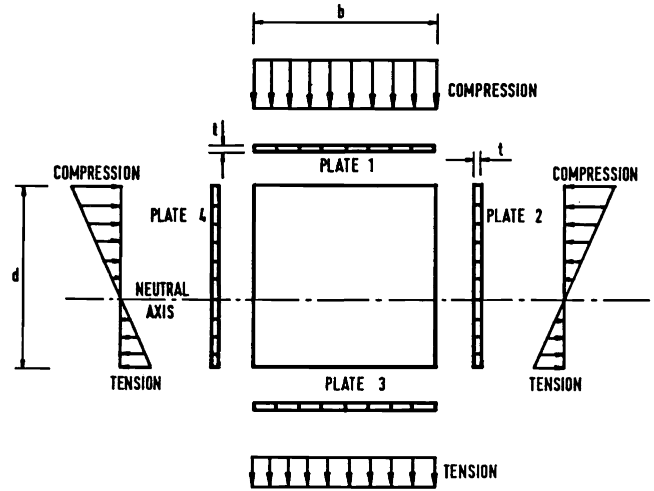


FIG 1 - Loading on Component Plates in a Box Column

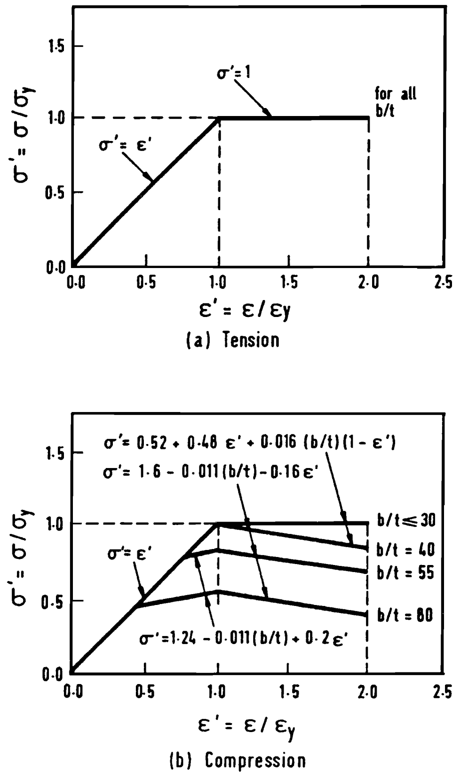


FIG 2 - Simplified Piecewise Linear Stress-Strain Curves

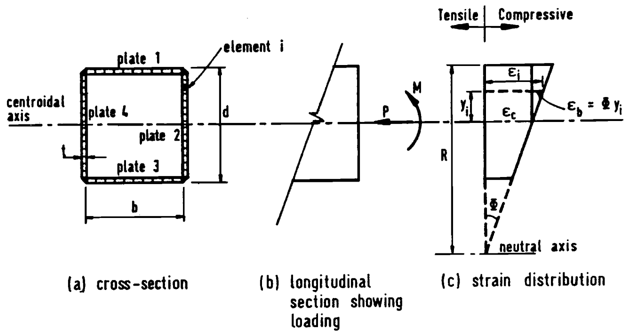


FIG 3 - Thin-Walled Box Column

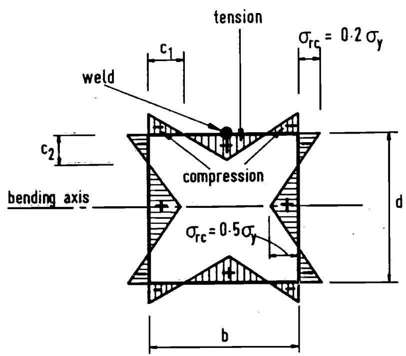


FIG 4 - Residual Stress Distribution

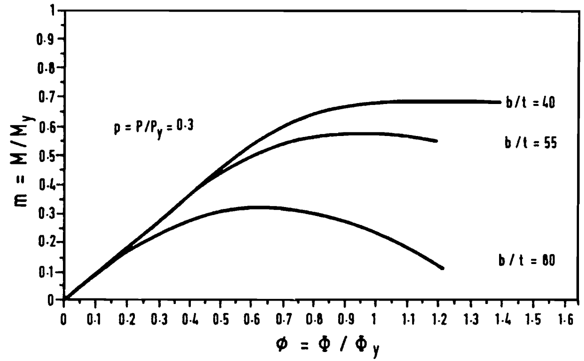


FIG 5 - Typical M-ϕ-P Curves

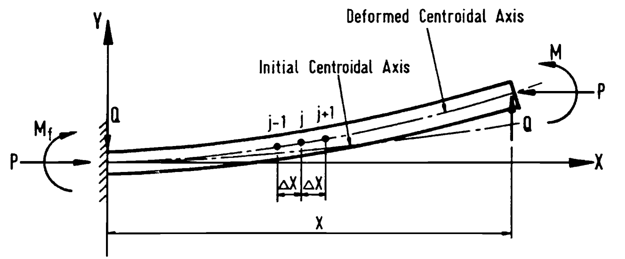


FIG 6 - Cantilever Column with Initial Curvature

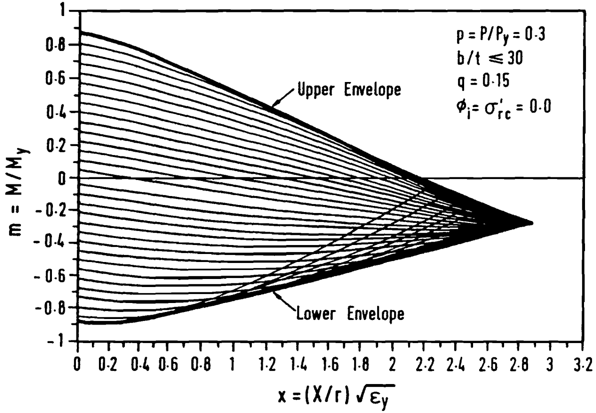
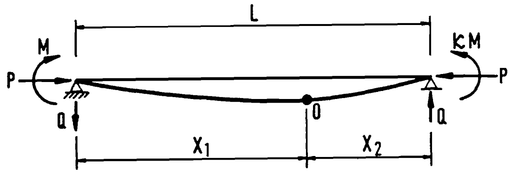
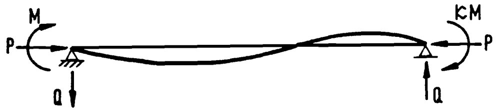


FIG 7 - Equilibrium Curves and Envelopes



(a) Single-Curvature Bending (K < 1·0)



(b) Double-Curvature Bending (K < -1·0)

FIG 8 - Simply-supported Columns

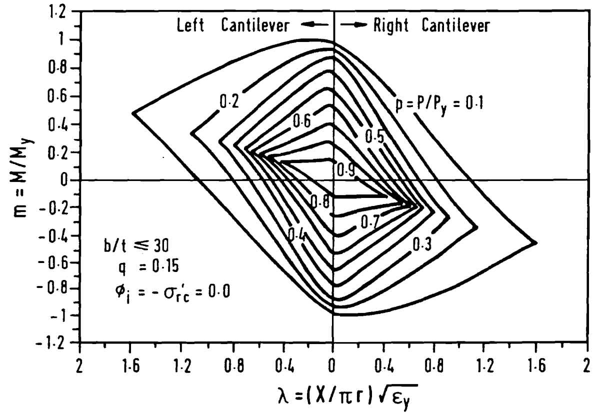


FIG 9 - Envelopes of Equilibrium Curves

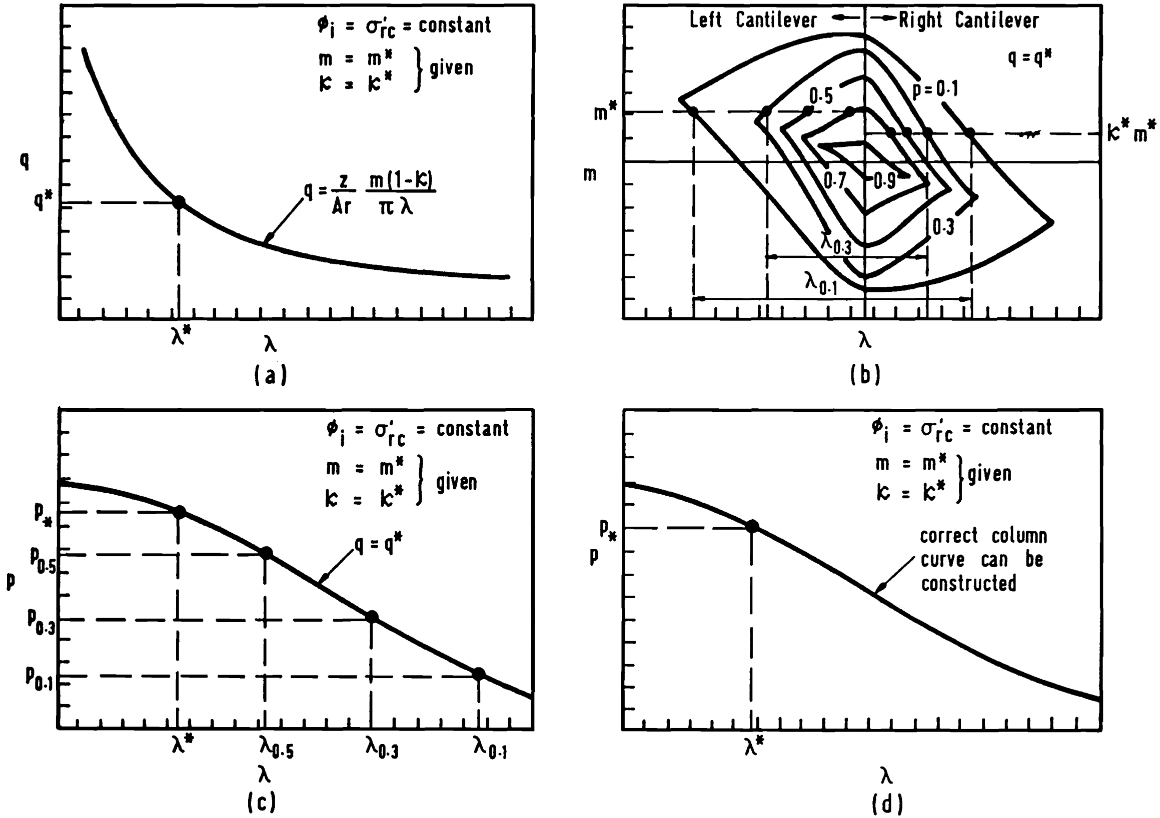


FIG 10 - Construction of Column Curves

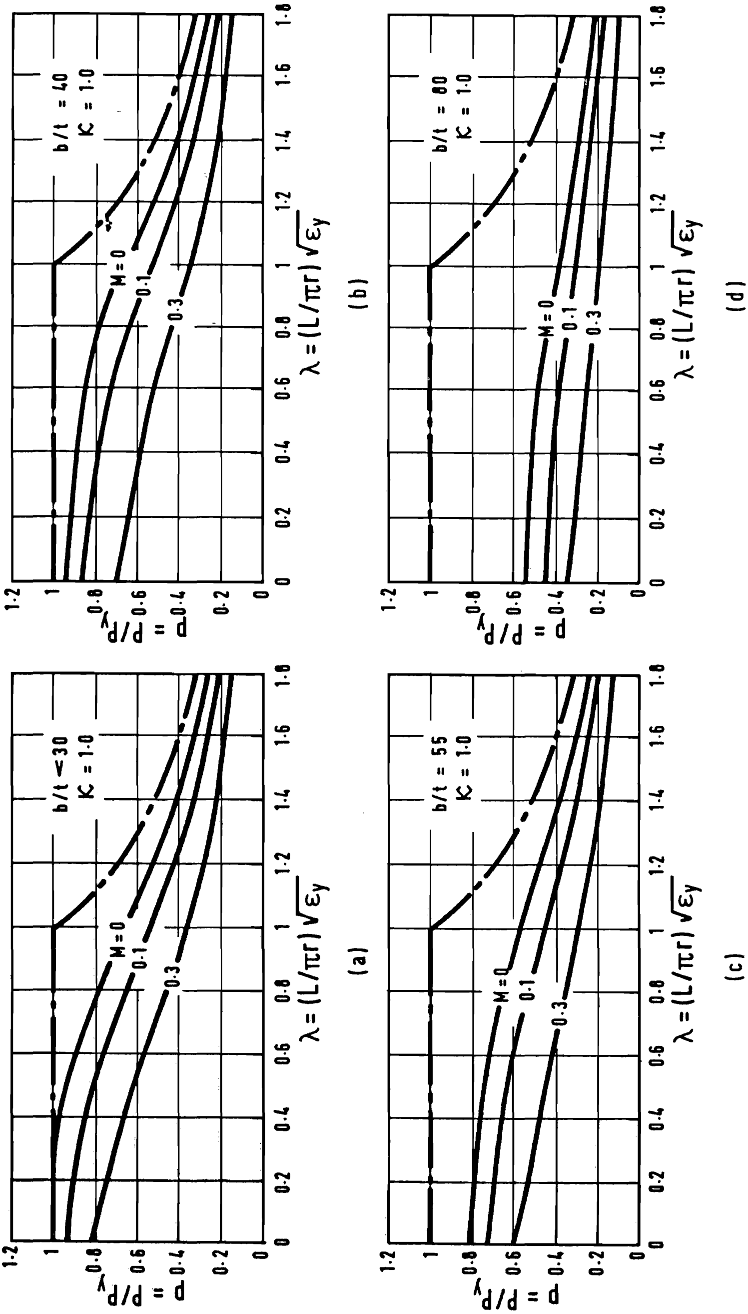


FIG 11 - Column Curves (κ = 1.0)

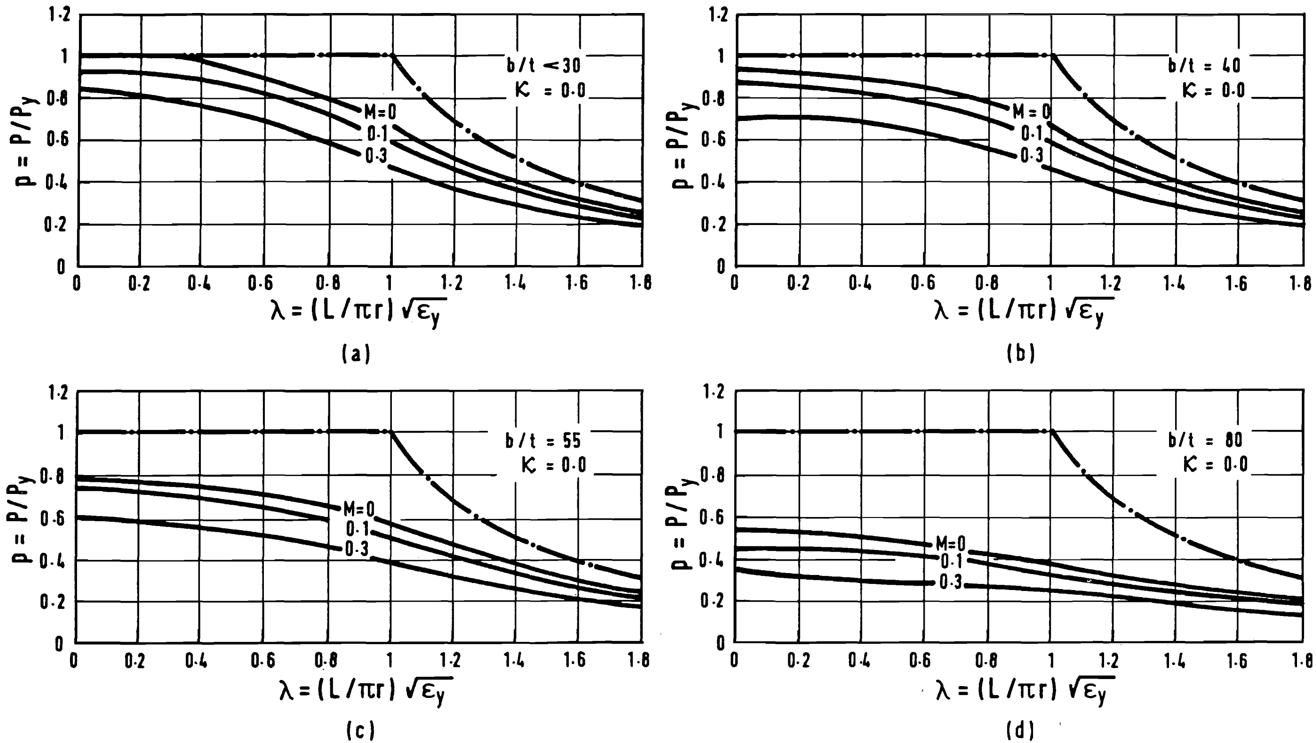


FIG 12 - Column Curves (κ = 0.0)

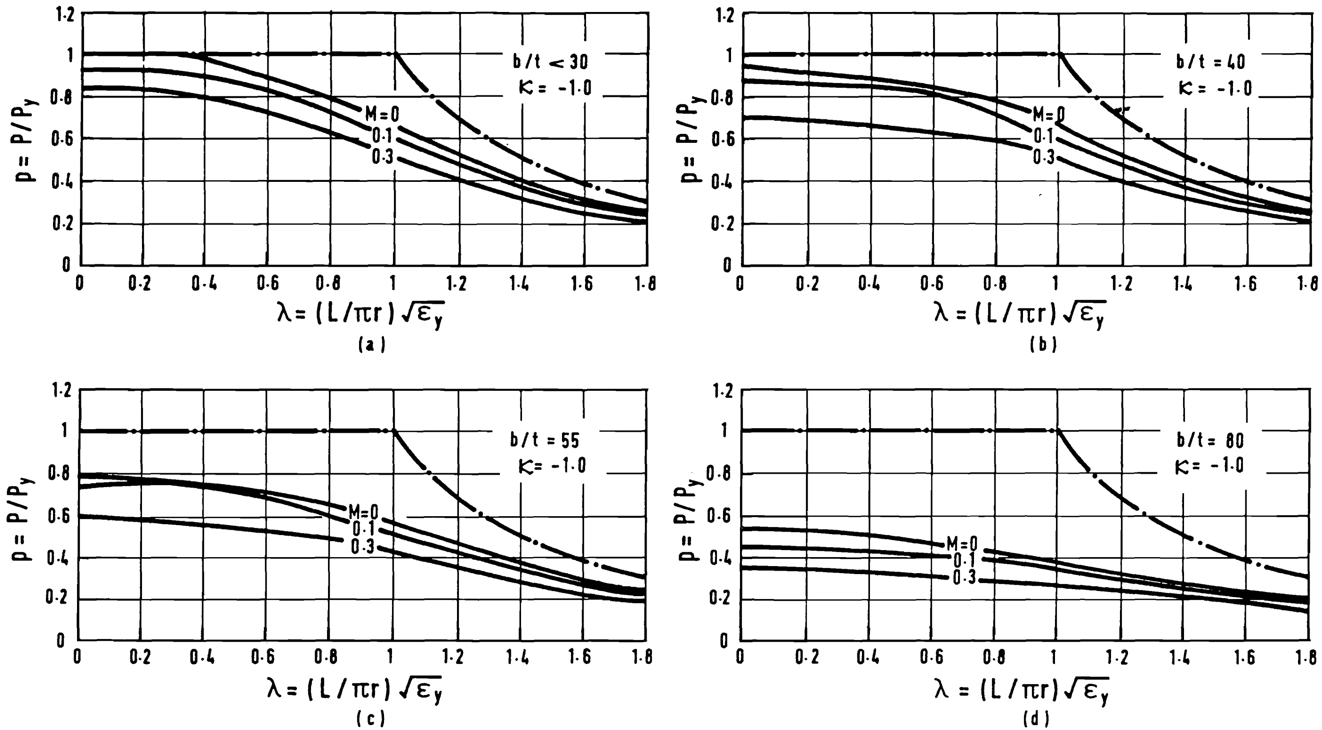


FIG 13 - Column Curves ($\kappa = -1.0$)