Response of Frames Having Thin-walled Columns

Srinivasan Sridharan

M. A. Ali

Seventh International Specialty Conference on Cold-Formed Steel Structures
St. Louis, Missouri, U.S.A., November 13-14, 1984

RESPONSE OF FRAMES HAVING THIN-WALLED COLUMNS†

By S. Sridharan* and M. A. Ali**

INTRODUCTION

The local buckling and the assessment of the associated loss of stiffness are of primary concern in the design of thin-walled columns. In particular, the occurrence of local buckling can and often does undermine the capacity of the structure to resist overall buckling so that the latter soon follows the occurrence of the former. The result has been shown to be a catastrophic snap-through type of collapse which in many practical cases, begins in the elastic range of material behavior (7,8,10,13). Initial imperfections of such magnitudes as are unavoidable in practice can cause serious reductions in the maximum collapse load attainable by a perfect column. The imperfection-sensitivity is particularly severe for components so proportioned as to have their local and overall critical stresses to be equal. Even if the overall buckling strength far exceeds the local one, the collapse load is only of the same order of magnitude as the latter (10).

The foregoing discussion pertains to quasi-static loading. In recent studies, Sridharan and Benito (10) have dealt with the case of suddenly applied axial compression. For imperfect columns having nearly equal values of critical stresses ($\sigma_{c3} \simeq \sigma_{c1}$) and subjected to suddenly applied axial compression, there occur further reductions in the maximum load that can be carried by the column from that corresponding to quasi-static loading. This by itself is not disturbing, but the column develops huge oscillations (though bounded in nature) for loads considerably smaller than the dynamic buckling load. Such a behavior, in view of the uneven distribution of axial stresses caused by local buckling, may very well precipitate localized plastic collapse, at loads which may be a small fraction of the theoretical local critical load. These observations raise the important question of the safety of cold-formed steel structures subject to dynamic loads.

Earlier analytical approaches to the problem of interaction of modes suffered from one or more of the following limitations:

(i) The interaction was considered to be made up of exclusively of two modes, viz. a certain local mode (often called the 'primary') and an overall mode. However, it will be shown in the present paper that at least two local modes must be considered especially for the practically important case of doubly symmetric cross-sections.

(ii) The amplitude of the local mode was considered to be constant along the length of the column for a given load level. In the present paper, it will be shown that the amplitude of the local modes must be given freedom to vary, especially if the end-conditions other than simply supported are considered. This was realized quite early by Koiter but his analytical solution appears to be restricted to the

* Associate Professor, Washington University, St. Louis, MO 63130

** Graduate Student, Washington University, St. Louis, MO 63130

† The work reported herein was in part supported by NSF (Grant No. CEE-8318423).

particular case of stiffened panels (5,6). In this paper, amplitude modulation is taken care of by associating a "slowly varying" function with the local modes which are automatically determined by the finite element procedure.

(iii) No attempt was made to generate a general scheme which can describe the behavior of a member that forms part of a frame-work, whose ends are in general, elastically supported against deflections and elastically restrained against rotations. This limitation is removed in the present approach by describing the overall behavior in terms of a finite element model.

In these respects, the present study may be considered an advance of the state-of-the-art.

THEORY

General:

The method of approach consists of the following steps:

(i) The solution of the following field problems for each of the column members of the structural system, viz.

(a) The eigenvalue problems for the relevant modes of buckling (which are the same as the local modes of vibration for uniformly compressed members) and

(b) the respective second order displacement fields in the individual modes as well as the mixed second order fields which are essential for a description of the post-local-buckling stiffness of the member.

The solution of these field problems are accomplished using the finite-strip method. Note that the solution of the fields in (b) above, involves but a set of simultaneous equations.

(ii) The incorporation of the foregoing solutions in a nonlinear stiffness analysis involving degrees of freedom to depict the overall behavior and modulation of local amplitudes in the axial direction, to arrive at a set of nonlinear algebraic equations for the static problem and a set of ordinary but nonlinear differential equations in time domain for the dynamic problem. A one-dimensional finite-element procedure is developed for this purpose.

and (iii) The solution of these equations using a Newton-Raphson iterative procedure for the static problem or the same in conjunction with Newmark's 'β' method to produce the dynamic response.

Local Buckling Deformation:

The local buckling deformations are modelled using the "classical assumptions" introduced by Benthem (1). These are (ref. Fig. 1):

(i) At a corner or a plate junction where two plates meet at an angle w, the normal displacement for each plate = 0.

(ii) N_y, the normal stress resultant in the transverse direction = 0 for each plate meeting at a corner. The buckling mode of the column can then be obtained taking for each strip,

$$w_i = \tilde{w}_i(y) \text{ Sin} \left(\frac{m\pi x}{\ell}\right)$$

$$u_i = v_i = 0 \qquad 1(a\text{-}b)$$

where 'm' is the number of half-waves of buckling, 'i' identifies a certain local mode and $\tilde{w}_i(y)$ is an appropriate function of 'y', the transverse coordinate. For $m >> 1$, the local buckling mode and critical stress are not dictated by end conditions. The second order displacement field associated with local mode takes the form:

$$u_{ii} = \tilde{u}_{ii}(y) \text{ Sin} \left(\frac{2m\pi x}{\ell}\right)$$

$$v_{ii} = \tilde{v}_{ii,o}(y) + \tilde{v}_{ii,2}(y) \text{ Cos} \left(\frac{2m\pi x}{\ell}\right)$$

$$w_{ii} = 0 \qquad 2(a\text{-}c)$$

The meaning of these displacement functions is seen from a description of displacements in a single mode local buckling problem.

$$u = u_{ii}\xi_i^2$$

$$v = v_{ii}\xi_i^2$$

$$w = w_i\xi_i \qquad \text{(no sum on 'i')} \qquad 3(a\text{-}c)$$

where 'ξ_i' is a scaling factor. The eigenmodes and the respective second order fields are determined with ease using the finite-strip method developed in Ref. 9 and 11.

The Appearance of a Secondary Local Mode:

The local buckling deformation of the column would be considerably modified as it bends due to lateral excitation or overall buckling under axial compression. Consider for example, the I-section column undergoing local buckling in an anti-symmetric mode (Fig. 2) and overall buckling/bending about the weaker axis (the web). As a result, the displacements in the compression zone get accentuated while those in the tension zone get significantly reduced. The additional displacements which arise as a result of interaction can be shown to take on the character of a local buckling mode (henceforth called 'secondary') which has the same wavelength as the primary mode. This will be demonstrated in the sequel.

Letting ξ_1 and ξ_3 be the scaling factors of the primary local and overall buckling modes respectively and the subscripts 1 and 3 refer to the associated displacement fields, the middle surface strains take the form:

$$\varepsilon_x = \varepsilon_{x_0} + \varepsilon_{x_1}\xi_1 + \varepsilon_{x_3}\xi_3 + \varepsilon_{x_{11}}\xi_1^2 + \varepsilon_{x_{13}}\xi_1\xi_3 + \varepsilon_{x_{33}}\xi_3^2$$

$$\varepsilon_y = \varepsilon_{y_0} + \varepsilon_{y_1}\xi_1 + \varepsilon_{y_3}\xi_3 + \varepsilon_{y_{11}}\xi_1^2 + \varepsilon_{y_{13}}\xi_1\xi_3 + \varepsilon_{y_{33}}\xi_3^2$$

$$\gamma_{xy} = \gamma_{xy_1}\xi_1 + \gamma_{xy_3}\xi_3 + \gamma_{xy_{11}}\xi_1^2 + \gamma_{xy_{13}}\xi_1\xi_3 + \gamma_{xy_{33}}\xi_3^2 \qquad 4(a\text{-}c)$$

wherein the terms having subscripts 13 are the mixed second order terms arising out of interaction of modes 1 and 3. The terms ε_{x_0} and ε_{y_0} are the prebuckling strains.

Some of the terms which appear in Eqn. 4 are given below in terms of displacements as they are needed in our demonstration.

$$\varepsilon_{x_1} = \varepsilon_{y_1} = \gamma_{xy_1} = 0$$

$$\varepsilon_{y_3} = \gamma_{xy_3} = 0$$

$$\varepsilon_{x_3} = -Z\, W_{3,xx}$$

$$\varepsilon_{x_{13}} = u_{13,x}$$

$$\varepsilon_{y_{13}} = v_{13,y}$$

$$\gamma_{xy_{13}} = u_{13,y} + v_{13,x} \qquad 5(a\text{-}f)$$

In these expressions W represents lateral displacements of the centroidal axis of the column caused by overall buckling. (u_{13}, v_{13}, w_{13}) represents the mixed second order displacement field. 'Z' represents the distance of the given point from the axis of bending. Note that first order inplane strains associated with local buckling are zero in view of the approximations stated earlier; also no strains other than purely axial are caused by overall bending. For simplicity, the normal displacements of plate elements which arise because of overall bending are assumed to make no contribution to the inplane axial strain term. The ordered contributions to stress resultants (N_x, N_y and N_{xy}) can be obtained from Hooke's law, neglecting both the strains and stresses in the transverse direction due to overall bending.

The governing differential equations of the mixed second order field can now be obtained by substituting the foregoing expressions in the von Karman plate equations and equating the coefficients of $\xi_1\xi_3$ on both sides of the equations.

These take the form:

$$u_{13,xx} + \frac{1}{2}(1-\nu)u_{13,yy} + \frac{1}{2}(1+\nu)v_{13,xy} = 0$$

$$v_{13,yy} + \frac{1}{2}(1-\nu)v_{13,xx} + \frac{1}{2}(1+\nu)u_{13,xy} = 0$$

$$D\nabla^4 w_{13} - N_{x_0} w_{13,xx} = N_{x_3} w_{1,xx} \qquad 6(a-c)$$

In these equations there occurs a decoupling of w_{13} on the one hand and u_{13}, v_{13} on the other. The Eqn. 6(a-b) admit a solution $u_{13} = v_{13} = 0$. Eqn. 6(c) may be written in the form:

$$D\nabla^4 w_{13} + \sigma_0 t w_{13,xx} = -Et\left(\frac{m^2\pi^2}{\ell^2}\right) Z\, W_{3,xx}\tilde{w}_1(y)\ \mathrm{Sin}\left(\frac{m\pi x}{\ell}\right) \qquad 7$$

in which σ_o represents the magnitude of the applied compressive stress. Note that the term $Z\tilde{w}_1(y)$ is an even function of y if $\tilde{w}_1(y)$ is an odd function and vice versa. In doubly symmetric cross-sections, the local modes would be either an odd or even function of y, 'y' being measured from the axis of bending. Taking the case of the I-section, this product would take the form shown in Fig. 2c. Eqn. 7 then is a differential equation of a plate subjected to a lateral load which varies nearly sinusoidally in the longitudinal direction and as an even (odd) function of 'y' if $\tilde{w}_1(y)$ is an odd (even) function of y. It is apparent that this load triggers a secondary local mode whose wavelength is the same as the primary mode and this is consistent with taking $u_{13} = v_{13} = 0$. Fig. 4(a-b) shows typical primary local modes of doubly symmetric sections and the corresponding secondary local mode triggered by bending about an axis of symmetry. As σ_o approaches the critical value associated with the secondary mode (σ_{c2}) the response in terms of this mode as seen from Eqn. 7 can be significant.

Any analysis which seeks to extract the mixed second order field in the manner suggested above, breaks down as $\sigma_o \to \sigma_{c2}$, as then the contribution of w_{13} grows without limit. This is the familiar singularity problem of interactive buckling. The problem arises because the secondary local mode appears as a second order field. The solution lies in recognizing the secondary mode from the outset as an additional participating mode in the interaction with full complement of higher order terms taken into account. (This is in contrast to an upper bound solution in Ref. 10 in which σ_o was set to zero in evaluating the mixed second order field). The amplitude of the secondary local mode must, however be modulated to account for the presence of the slowly varying function $W_{3,xx}$ in Eqn. 7. By a similar argument, it can be shown that the interaction of secondary local mode with overall bending will in turn cause the amplitude of the primary local mode to be modulated. It appears that the phenomenon of amplitude modulation was first recognized by Koiter and the present analysis is but a generalization of this concept of "amplitude modulation" using the notion of a "slowly varying function".

Mixed Second Order Fields:

Our interactive buckling analysis, must then necessarily involve two fundamental local modes and their respective second order fields. To this, we must add a third one, the mixed second order field which arises by the interaction of the two local modes. Since the two local modes are of the same wave-length, it can be shown that the displacement functions which describe this field must be given by expressions similar to those in Eqn. 2(a-c). On the other hand the mixed second order field which arises by the interaction of overall bending with any one of the local modes as shown above is essentially given by the other mode and therefore need not be separately calculated. It is readily seen that the present analysis is the most complete so far attempted for interactive buckling of prismatic plate structures.

Finite Element Formulation:

The local buckling deformation of an element of length ℓ_e of the column may be written in the form:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ w_1 \end{Bmatrix} \xi_{1i} f_i + \begin{Bmatrix} 0 \\ 0 \\ w_2 \end{Bmatrix} \xi_{2i} f_i + \begin{Bmatrix} u_{11} \\ v_{11} \\ 0 \end{Bmatrix} \xi_{1i}\xi_{1j} f_i f_j +$$

$$+ \begin{Bmatrix} u_{22} \\ v_{22} \\ 0 \end{Bmatrix} \xi_{2i}\xi_{2j} f_i f_j + \begin{Bmatrix} u_{12} \\ v_{12} \\ 0 \end{Bmatrix} \xi_{1i}\xi_{2j} f_i f_j \qquad i,j=1,2 \qquad 8$$

In Eqn. 8, the subscripts 1 and 2 identify the primary and secondary local modes respectively; f_i are linear functions of x. With origin taken at one end of the element

$$f_1 = 1 - \eta \;; \quad f_2 = \eta \;; \quad \text{with} \quad \eta = x/l_e \qquad 9\text{(a-b)}$$

As already mentioned, the functions w_1, w_2 ..... v_{12} are evaluated using the finite-strip technique. However in Eqn. 8, the constant amplitudes are replaced by a "slowly varying" function in the sense of Ref. 5. ξ_{i_1}, ξ_{i_2} are the nodal amplitudes of the i-th mode and are the degrees of freedom of the structure depicting the local deformation.

The axial and lateral displacements of the column are represented in the form:

$$\begin{Bmatrix} U \\ W \end{Bmatrix} = \begin{Bmatrix} U_i \\ W_i \end{Bmatrix} \phi_i \qquad (i=1,4) \qquad 10$$

where $\phi_1 = (1-3\eta^2+2\eta^3)$; $\phi_2 = (\eta-2\eta^2+\eta^3)/\ell_e$; $\phi_3 = 3\eta^2-2\eta^3$; $\phi_4 = (-\eta^2+\eta^3)/\ell_e$

11(a-d)

U_1------W_4 are the nodal degrees of freedom corresponding to the overall behavior of the column. The need for a higher order polynomial representation of axial displacements in geometrically nonlinear problems has been emphasized elsewhere (2). The choice of a cubic polynomial for U represents a compromise between rigor and simplicity. Thus there are 12 degrees of freedom for each column element. The contribution to longitudinal displacement at any point of U and W, is obtained by invoking the plane sections hypothesis.

An expression for the total potential energy can now be obtained in terms of local degrees of freedom for each element. This provides the basis for generating the tangential stiffness matrix at any given point on the equilibrium path for the prediction of increments in global degrees of freedom for a given increment in the load level. Newton-Raphson iterations are then performed to converge to the solution of desired accuracy. To solve the dynamic problem which arises when the structure is subjected to ground motion or time-dependent axial or lateral load, an expression for kinetic energy is set up for each element. Retaining only the lowest order terms, this expression takes the form of a quadratic function of the degrees of freedom. This together with the potential energy function provides the basis for generating the Lagrangian equations of motion for the structure in terms of global degrees of freedom which are solved using an implicit procedure such as Newmark's 'β' method in conjunction with Newton-Raphson iterations.

EXAMPLES AND DISCUSSION

In this section, examples are presented with the following objectives in view:

(1) (a) To compare the maximum loads that can be carried by I-section columns as given by the present technique with those given by the upper bound solution of ref. 10.

(b) To compare the results of the present technique for square box columns with those of Koiter (4).

The columns are simply supported at their ends. The amplitude modulation was neglected for comparison with Ref. (4) and (10); the overall buckling displacements were assumed in the form of Eqn. 1a and 2a supplemented by suitably prescribed end rotations introduced to ensure the concentric application of load all along the equilibrium path.

(c) To compare the present results with a calculation based on the effective width approach.

(2) To illustrate the reduction in the capacity of thin-walled columns of framed structures to carry axial loads in presence of small lateral loads, and

(3) To assess the effect of suddenly applied lateral loads which may be induced by wind, earthquakes or moving loads on the connecting beams.

Comparison of I-Section Column Results:

Fig. 3 shows the cross-sectional dimensions of an I-section column; two different lengths are considered, viz. ℓ = 2000t (case (a), $\sigma_{c_3}/\sigma_{c_1}$ = 1.06) and 1200t (case (b), $\sigma_{c_3}/\sigma_{c_1}$ = 2.90) where t is the thickness of flange and σ_{c_1} and σ_{c_3} are respectively the primary local and overall critical stresses. Fig. 5a shows the variation of maximum load carried by the column (a) expressed in a dimensionless form as σ_u/σ_{c_1} with overall imperfections. The effect of introducing a given imperfection in the local mode equal to 0.2t is also illustrated in the same figure. These results are compared with the approximate solution of Ref. 10. The latter values are generally higher which is indicative of the destabilizing influence of the secondary local mode duly considered in the present theory. In Fig. 5b, the same results are plotted for the column (b). Here the differences are more pronounced, which reflects the increasing destabilization as the load approaches the secondary local critical load which is about 20% higher than the primary local one. The upper bound solution completely neglects this aspect and predictably gives higher estimate of the collapse load.

Comparison with Koiter's Results of Square Box Columns:

Fig. 6a shows the variations of σ_u/σ_{c_1} with $\sigma_{c_3}/\sigma_{c_1}$ for two different levels of local imperfection amplitudes ($w_1^{(0)} = \xi_1^{(0)} \cdot t$), viz. t/80 and t/20 as obtained by the present theory and as obtained by Koiter (4). In Fig. 6b similar results are plotted for a given value of $w_1^{(0)} = t/40$ and two different values of overall imperfection amplitude, viz. 1 and 8% of the radius of gyration r of the cross-section. In the range considered, the Koiter's theory gives results which exceed those of the present theory by a maximum of 4%. Koiter used a mechanical model to which he built in the degrees of freedom corresponding to local buckling and postbuckling deformation in an approximate manner. A major approximation in his approach is the complete neglect of the mixed second order field which is given in our theory by u_{12}, v_{12} - an approximation which overestimates the coefficient of the term $\xi_1^2\xi_2^2$ in the energy function which in turn overestimates the stiffness of the structure. This field, if taken into account, gives an additional freedom to the plate elements to 'pull in' in their own plane. The results of the present theory must therefore be deemed more accurate.

Comparison of the Present Theory with Effective Width Approach:

There are a number of empirical methods proposed for the design of columns liable to fail by interactive buckling. These often produce results which are at sharp variance from each other. (For a good review, see Ref. 12.) Of these the one based on Cornell University test results (3) has the virtue of lucidity which more than compensates the disadvantage of its being an iterative procedure. This procedure appears to correlate well with Cornell test data on hinged-hinged columns. In experiments, the specimens are apt to be near-perfect and end conditions (which are assumed to be perfectly hinged in calculations) do end up having partial fixity in them, especially for short specimens. Imperfection-sensitivity and end conditions being key factors in interactive buckling, a method which provides results in good agreement with experiments may turn out to be unconservative for practical use.

This indeed appears to be the case for I-section columns detailed in Table 1. For the case of $\sigma_{c_3}/\sigma_{c_1} = 1.06$, the Cornell University method predicts a value of $\sigma_u = 13.9$ ksi while the present theory would indicate 11.3 ksi for imperfection levels of 0.1t and $\ell/1000$ in the primary local and overall modes respectively. This means an error of 23% on the unconservative side. The error margin increases to about 30% for a shorter column with $\sigma_{c_3}/\sigma_{c_1} = 2.9$. A careful review of the currently advocated design methods in the light of theoretical results appears necessary to evolve a safe and rational design approach.

Response of a Model of a Single Bay Portal Frame:

The single bay portal frame shown in Fig. 7a is made up of thin-walled I-section columns connected by a rigid beam. The cross-section of the column is indicated in Fig. 3 with the flanges having alternative widths of 80t [case (a)] and 60t [case (b)]. In the analysis the column was divided into five elements which appears adequate for the variations of the displacements depicted. In Fig. 7b the maximum axial load carried by each column P_u (expressed as a fraction of the primary local critical load P_u/P_c) is plotted against the lateral load expressed as a percentage of P_c for the case (a). Three different lengths are considered, viz. 2000t, 1200t, and 800t. These correspond to ratios P_o/P_c (P_o being the value of P for buckling of the frame) respectively of 1.06, 2.90 and 3.90. The columns are assumed to have local imperfections in the primary mode with an amplitude of 0.1t. As may be seen there occur serious reductions (of the order of 50%) in the axial load carrying capacity of the columns under lateral loads which are about 0.5% of the axial load P_c for the case of P_o/P_c = 1.06. For other cases too, the erosion in the capacity of the columns, if certainly smaller, are too significant to be ignored. Similar trend is noticed for the column with B/t = 60 whose characteristics for two different lengths (ℓ = 1080t, P_o/P_c = 0.98 and ℓ = 720t, P_o/P_c = 2.15) are shown in Fig. 7c. It must be noted that lateral loads of the order of 1% can reduce the axial load carrying capacity to well below the loads calculated by the Cornell University method.

Fig. 8(a-c) shows typical variations of the amplitude of the primary and secondary local modes at a load close to collapse for the column (a) with 1 = 2000t. As may be expected the sign of the secondary local mode changes with the sign of the bending moment on the cross-section.

Response of the Frame Under Suddenly Applied Lateral Loads:

Under a sudden application of lateral load, it is seen that the capacity of the column to carry axial load is reduced from that corresponding to static application of load. For a given lateral load, there is a certain level of axial load below which the response of the frame is one of bounded oscillations and above which the response is one of escalating oscillations and this load is designated as the dynamic buckling load (P_d) of the structure. The build up of oscillations is illustrated for a frame shown in Fig. 7a with ℓ = 2000t. The frame carries two masses one at each column-beam junction and equal to P/gt_1. (t_1 is a dimensionless constant so that t, the thickness of flange = t_1 in.) Once again, the column was assumed to have initial imperfections in the primary mode given by $\xi_1^{(0)}$ = 0.1. Fig. 9a illustrates the nature of oscillations for two loads, one about 10% less than P_d and the other about 10% more. For the former, the frame vibrates in the overall mode essentially about its static equilibrium position; the amplitude of the local mode, however varies gradually but this variation is periodic; also the extrema of this variation synchronize with the extrema of overall oscillations. The frequency of local vibrations is an order of magnitude greater (not illustrated) than that of the overall vibrations. When the load exceeds P_d, the overall deflections build up monotonically, typical of divergence phenomenon in conservative systems.

Fig. 9b illustrates the variation of P_d/P_s, (P_s = maximum load carried under static conditions) with Q/P_c, the nondimensional lateral load for the frame. As may be seen a lateral load whose magnitude is given by 0.3% of P_c is enough to reduce the capacity of the column by about 18% of its static value. This loss of strength is too serious to be ignored and continues to be a subject of further study in Washington University.

CONCLUSIONS

1. A new analytical model has been developed for the study of interaction of overall buckling with two principal local modes of buckling for columns with doubly symmetric cross-sections. The procedure accounts for the problem of "wandering centroid" in a rational manner. The model also accounts for the amplitude modulation - a necessary feature in situations of rapidly changing bending moment. The version of the model in which dynamic effects are included is expected to prove a powerful tool in the study of seismic response of frames having thin-walled columns.

2. The Cornell University method produces unconservative design solutions for columns with unstiffened plate members and practical levels of initial imperfections. This calls for a thorough revision of the existing code provisions and further tests in which initial imperfections are measured.

3. The current methods of prediction are even more inadequate to deal with frames having thin-walled columns where serious reductions in the capacity of the columns can occur under small lateral loads.

4. Dynamic application of lateral load can produce reductions of the order of 20% in the maximum static axial load carrying capacity of frames.

Appendix.--References

1. Benthem, J.P., "The Reduction in Stiffness of Combinations of Rectangular Plates in Compression after Exceeding the Buckling Load", NLL-TR, S. 539, June 1959.

2. Dawe, D.J. in "Finite Elements for Thin Shells and Curved Members", Ed. D.G. Ashwell and R.H. Gallagher, Wiley, London, 1976, pp. 131-153.

3. Kalyanaraman, V., Pekoz, T. and Winter, G., "Performance of Unstiffened Compression Elements", Report No. 362, Dept. of Structural Engineering, Cornell University, 1978.

4. Koiter, W.T. and Neut, van der, A., "Interaction Between Local and Overall Buckling of Stiffened Compression Panels" in "Thinwalled Structures", Ed. J. Rhodes and A.C. Walker, Granada, 1980, pp. 61-85.

5. Koiter, W.T. and Pignataro, M., "An Alternative Approach to the Interaction Between Local and Overall Buckling in Stiffened Panels", IUTAM Symp., Cambridge, 1974. "Buckling of Structures", Ed. B. Budiansky, Springer Verlag, 1976, pp. 133-148.

6. Koiter, W.T. and Pignataro, M., "A General Theory for the Interaction Between Local and Overall Buckling of Stiffened Panels", Estratto Dal. Volume, Problemi altuali di meccanica teorica e applicata, Atti del Convegno Internazionale a ricordo di Modesto Panetti, Torino, 1977.

7. Neut, van der, A., "Mode Interaction in Stiffened Panels", IUTAM Symp., Cambridge, 1974, "Buckling of Structures", Ed. B. Budiansky, pp. 117-132, Springer-Verlag, Berlin, 1976.

8. Sridharan, S., "Doubly Symmetric Interactive Buckling of Plate Structures", International Journal of Solids and Structures, Vol. 19, No. 7, 1983, pp. 625-641.

9. Sridharan, S., "Finite Strip Post-Local-Buckling Analysis of Plate Structures Subject to Nonuniform Compression", Engineering Structures, 1982, Vol. 4, pp. 249-255.

10. Sridharan, S. and Benito, R., "Static and Dynamic Interactive Buckling", Journal of Engineering Mechanics, ASCE, Vol. 110, No. 1, pp. 49-65, Jan. 1984.

11. Sridharan, S. and Graves-Smith, T.R., "Postbuckling Analyses with Finite Strips", Journal of the Engineering Mechanics Division, ASCE, EM5, pp. 869-888, Oct. 1981.

12. Trestain, T.W.J., "A Review of Cold Formed Steel Column Design", Report No. 81109-1, CSCC Project CCC 817, Canadian Steel Construction Council, 1983.

13. Tvergaard, V., "Imperfection-Sensitivity of a Wide Integrally Stiffened Panel Under Compression", International Journal of Solids and Structures, 1973, Vol. 9, pp. 177-192.

Appendix.--Notation

- B : Width of flange of I-section
- D : Flexural rigidity of plate [$= Et^3/12(1-\nu^2)$] (Ref. Eqn. 7); also depth of web of an I-section column
- E : Young's Modulus
- N_x, N_y, N_{xy} : Membrane stress resultants in a plate element
- M : Mass
- P : Axial load on a column
- P_c : Critical value of P corresponding to local buckling
- P_d : Dynamic buckling load
- P_o : Overall buckling load of the frame
- P_u : Collapse load under static conditions
- Q : Lateral load on the frame
- g : Acceleration due to gravity
- m : Number of halfwaves of local buckling
- ℓ : Length of the column
- ℓ_e : Length of a column element
- r : Radius of gyration of square box column
- t : Thickness of flange of I-section or of the walls of a square box column
- u, v, w : Displacements contributed by local buckling
- Z : Distance of any point on the cross-sectional profile from the axis of overall bending of the column
- δ : Maximum lateral deflection at the beam level of the portal frame
- ξ_1, ξ_2, ξ_3 : Scaling factors respectively of primary local, secondary local and overall modes of buckling
- $\xi_1^{(0)}, \xi_2^{(0)}, \xi_3^{(0)}$: Imperfections in the respective modes of buckling divided by 't'
- σ_o : Applied axial stress
- $\sigma_{c_1}, \sigma_{c_2}, \sigma_{c_3}$: Critical values of σ_o corresponding to the primary local, secondary local and overall modes respectively
- σ_u : The average stress at collapse

Table 1

Properties of the I-Section Column*						$\frac{\sigma_u}{E} \times 10^3$	$\frac{\sigma_u}{E} \times 10^3$	
B/t	D/t	ℓ/t	$\frac{\sigma_{c_1}}{E} \times 10^3$	$\xi_1^{(0)}$	$\xi_3^{(0)}$	Cornell University Method	Present Theory	% error
80	80	2000	0.621	0	0	0.463	0.621	25.4 (safe)
80	80	2000	0.621	0.1	2.0	0.463	0.377	22.8 (unsafe)
80	80	1200	0.621	0	0	0.770	0.740	4.1 (unsafe)
80	80	1200	0.621	0.1	1.2	0.770	0.592	30.1 (unsafe)

yield stress = 50 ksi, E = 30,000 ksi

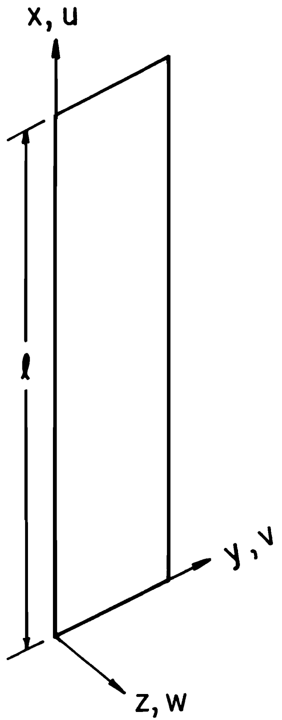

Fig. 1. Finite Strip Configuration

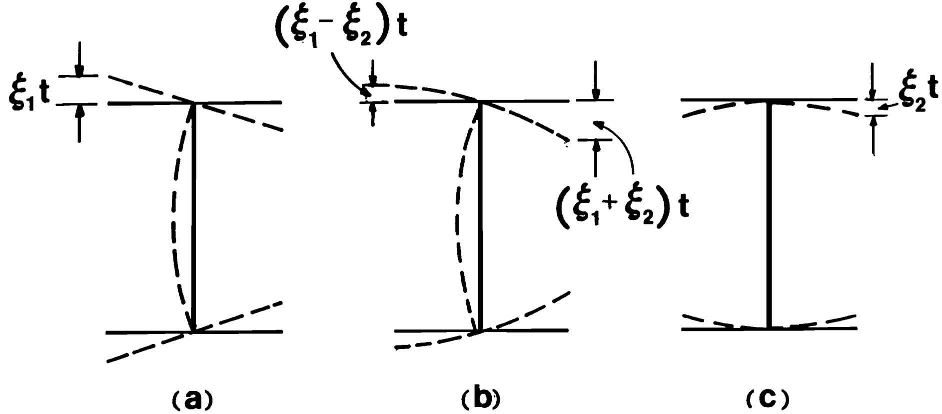

Fig. 2. (a) The primary local mode
(b) Total cross-sectional deformation
(c) The secondary local mode

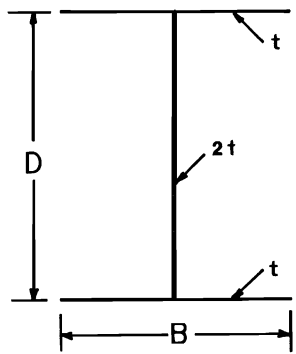

Fig. 3. I-Section of column
(a) B = D = 80t
(b) B = 60t; D = 80t

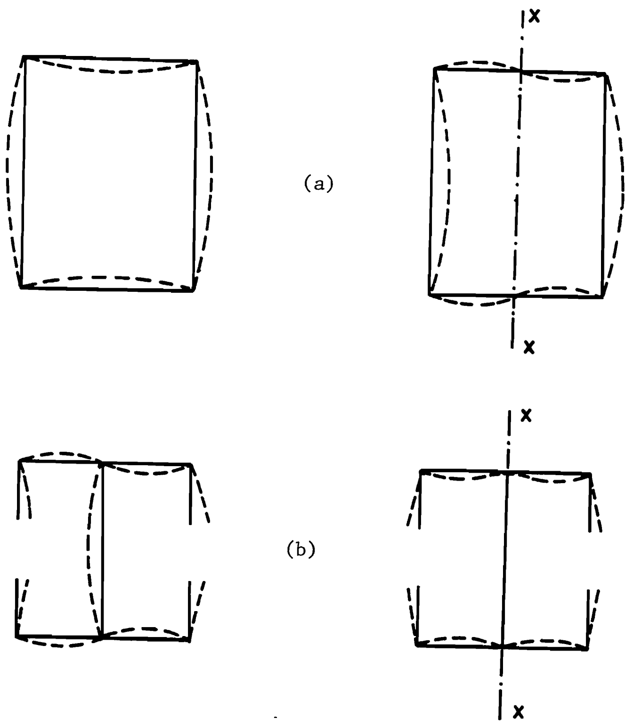

Fig. 4(a-c).

Primary and secondary local modes of a rectangular box column and I-section column with lipped flange.

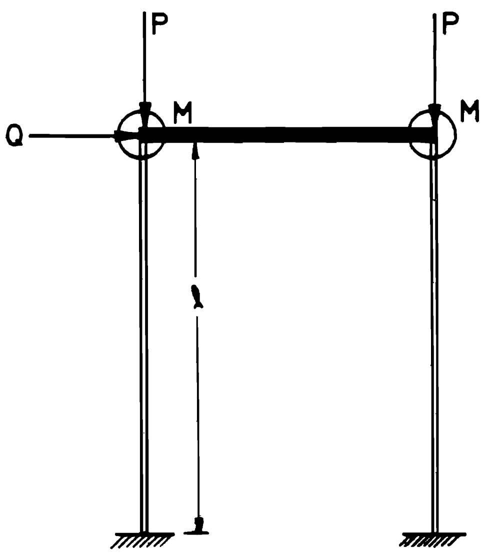

Fig. 7(a).

Frame made up of two I-section columns fixed at their bases and connected by a rigid beam.

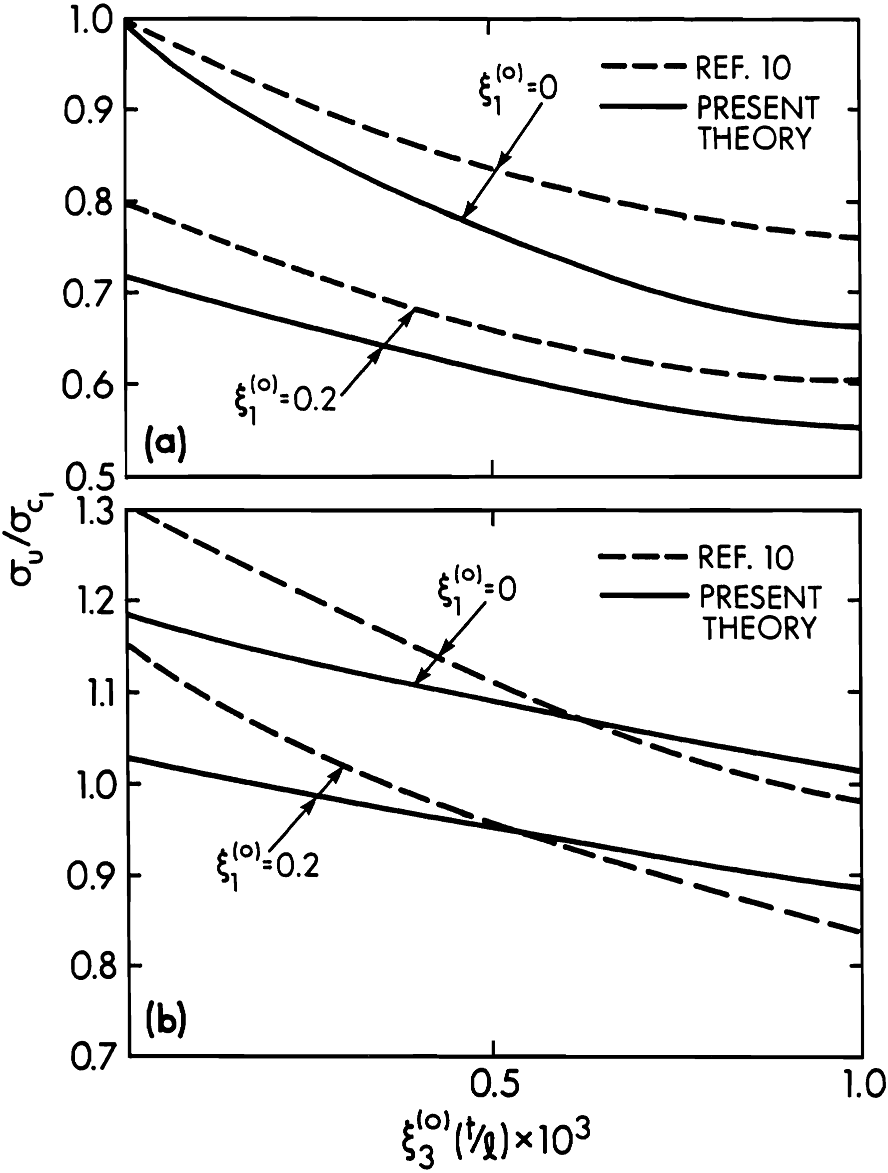

Fig. 5(a-b). The variation of maximum load with overall imperfections for given values of local imperfections for columns (a) and (b) respectively.

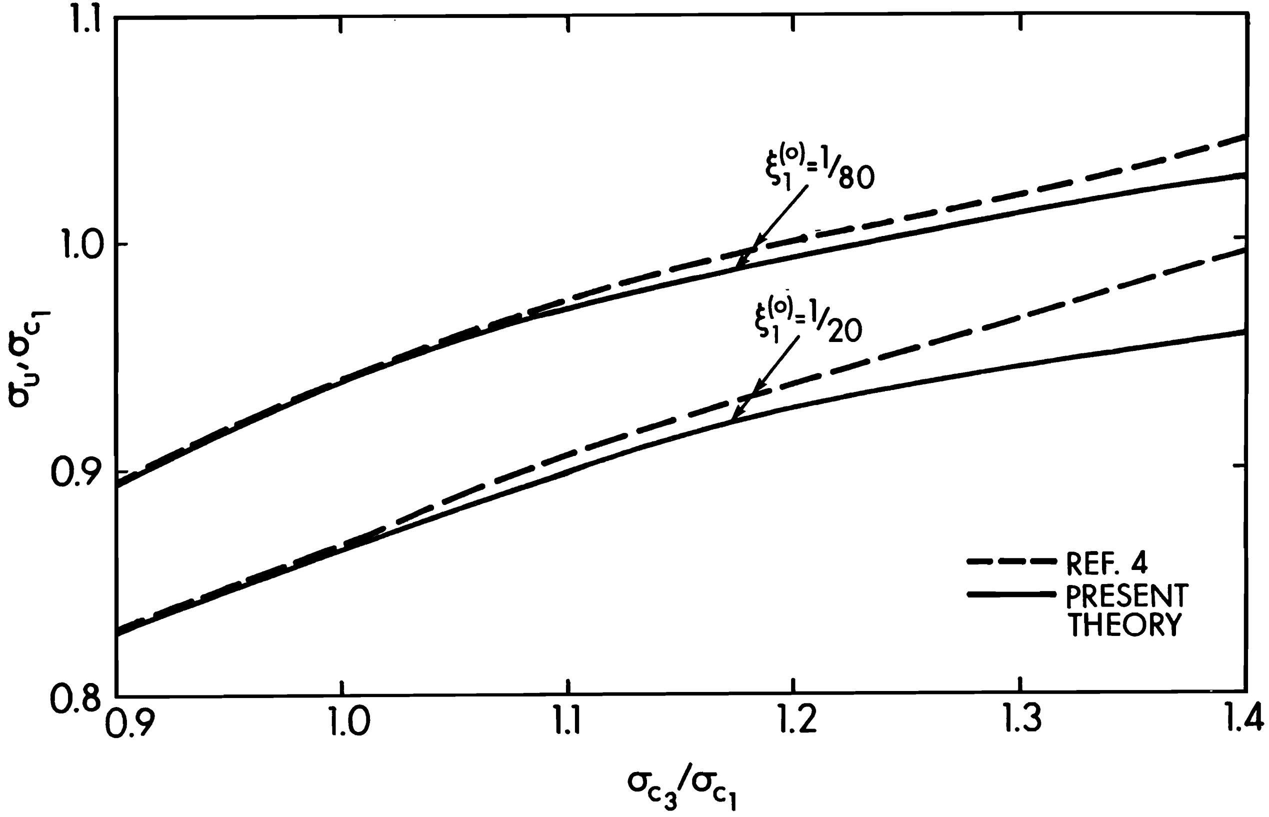

Fig. 6(a). Maximum load capacity of square box column vs. the ratio of overall to local critical stresses as given by Koiter and present theory. Note only local imperfections are considered ($\xi_3^{(0)}=0$).

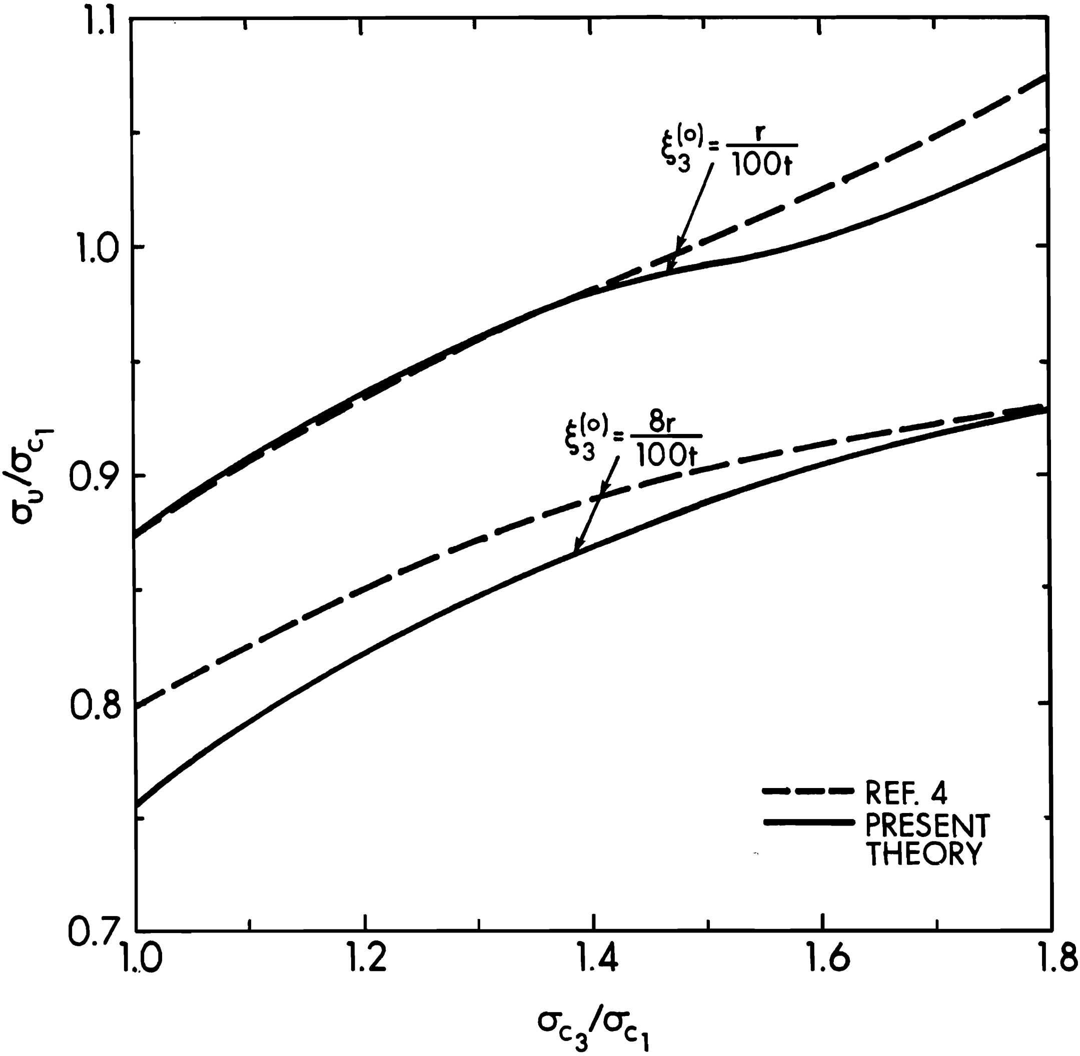

Fig. 6(b). Maximum capacity of a square box column vs. ratio of overall to local critical stresses for combined imperfections. $\xi_1^{(0)}=1/40$.

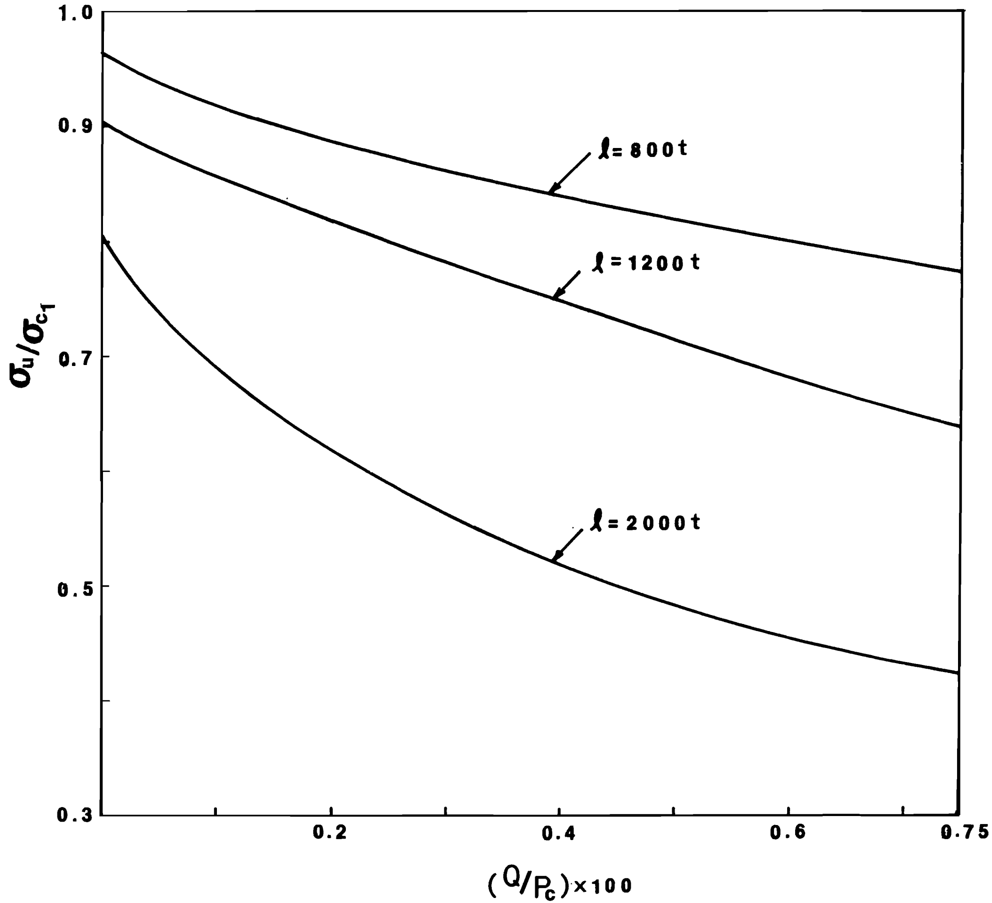

Fig. 7(b). Axial load carrying capacity $\{\sigma_u/\sigma_{c_1}\}$ of I-section columns of type (a) (Fig. 3) plotted against the lateral load carried by the frame (Q/P_c)

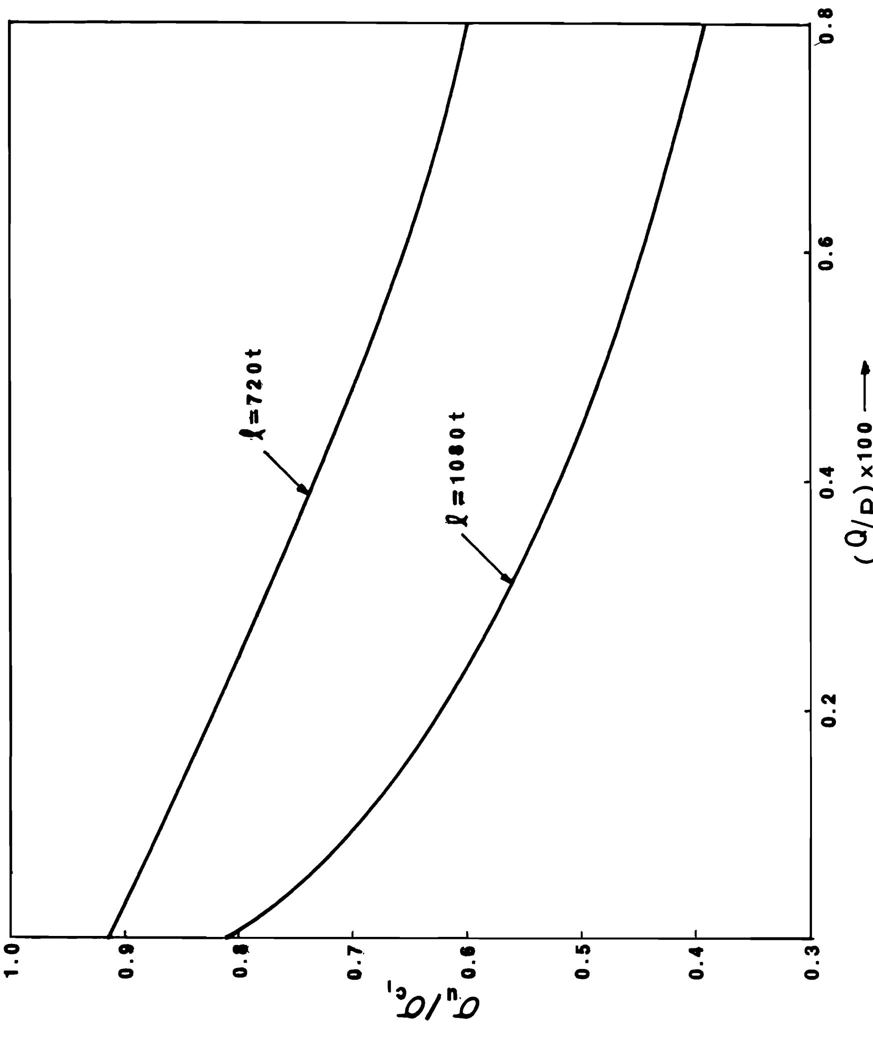

Fig. 7(c). Axial load carrying capacity (σ_u/σ_{c_1}) of I-section columns of type (b), Fig. 3 plotted against the lateral load carried by the frame (Q/P_c).

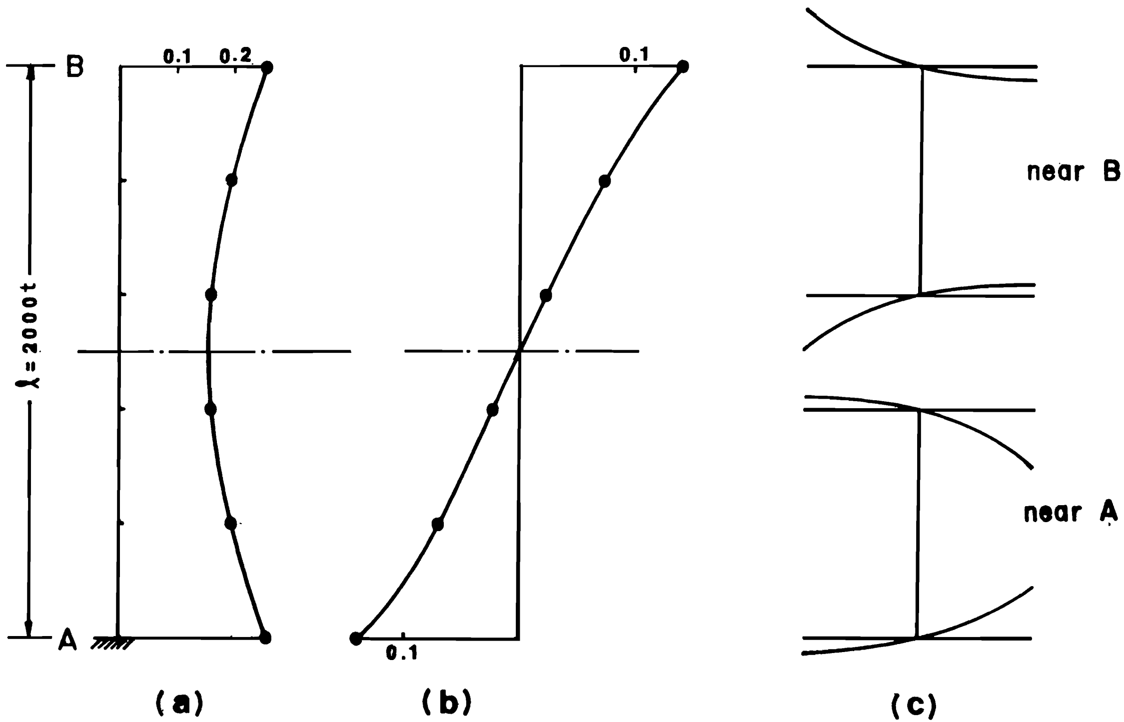

Fig. 8. (a) Variation of the amplitude of the primary local mode along the length of the column.
(b) Variation of the amplitude of the secondary local mode along the length of the column.
(c) Variation of cross-sectional deformations in the neighborhood of top and bottom.

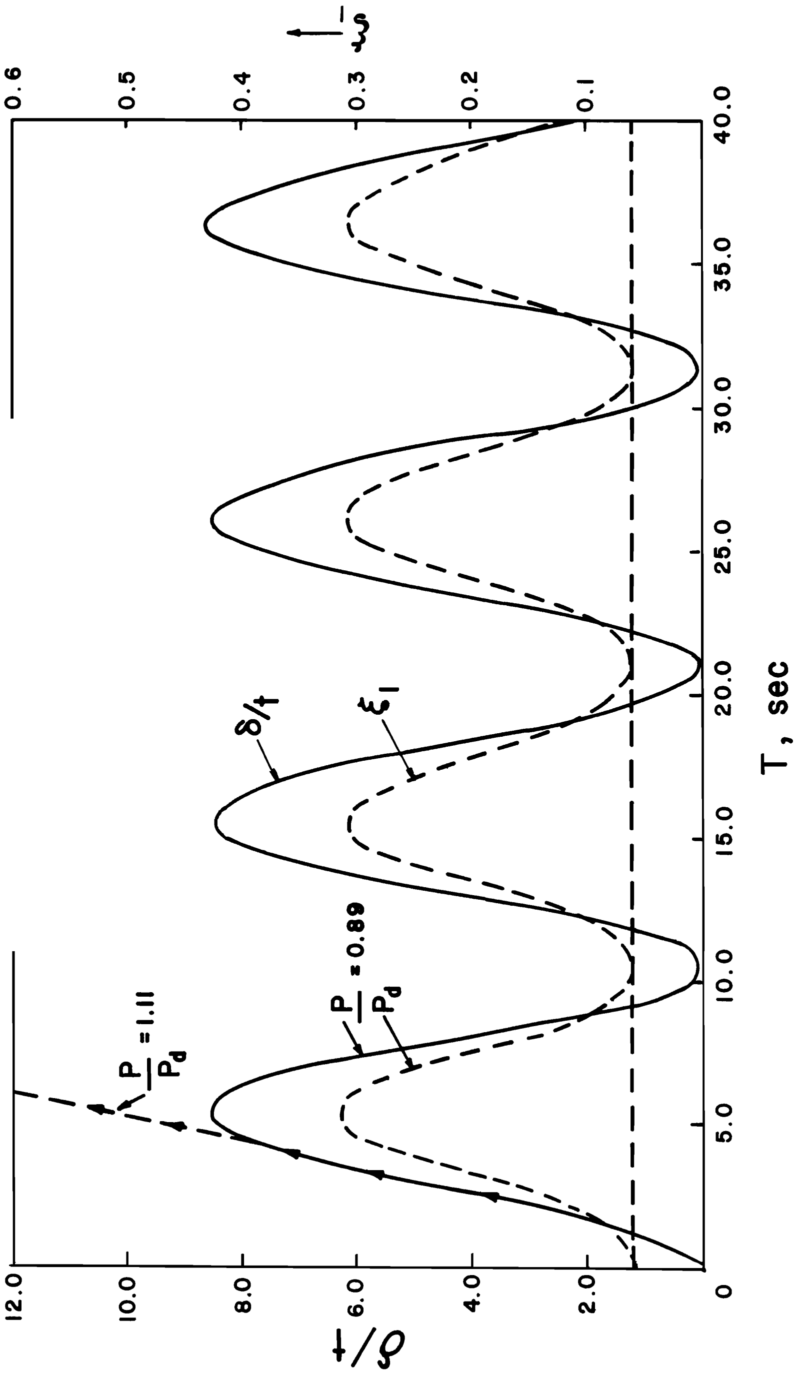

Fig. 9(a). Variation of overall deflections at the top of the frame and amplitude of the primary local mode under suddenly applied lateral load Q given by $(Q/P_c = 0.336 \times 10^{-2})$.

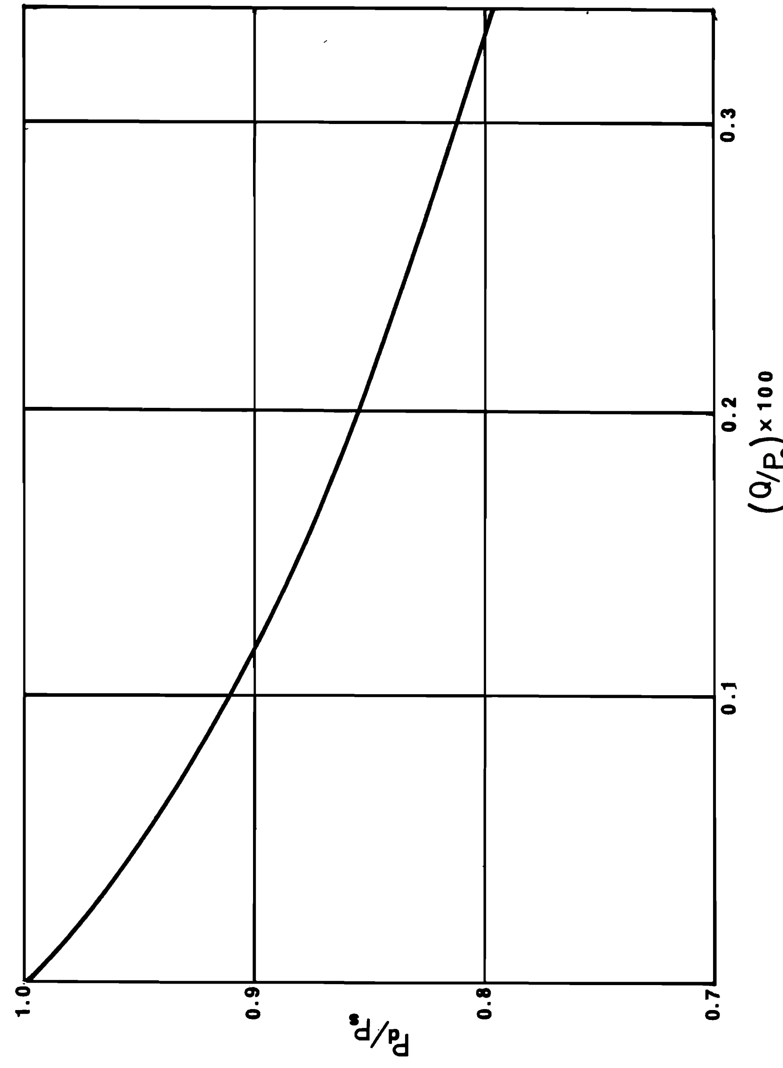

Fig. 9(b). The variation of P_d/P_S with (suddenly applied) lateral load Q. ($\xi_1^{(0)} = 0.1$)