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THE EFFECT OF END SUPPORTS ON THE BEHAVIOR OF BRACED GIRTS AND PURLINS

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SUMMARY

Results from a theoretical model are presented for the design of channel and Z-section girts and purlins with torsionally elastic end supports. Allowable loads are computed on the basis of strength and serviceability criteria, taking into account the various end and intermediate support conditions.

INTRODUCTION

Although cold-formed steel members provide substantial savings due to their high strength-to-weight ratio, their cross-sectional configuration gives rise to behavioral phenomena which are not encountered in the more familiar symmetrical sections. Of great concern is the tendency of cold-formed sections to bend and twist under most conditions of loading. The amount of lateral and rotational displacement depends not only on the cross-sectional characteristics of these members but also on the degree of bracing provided both along the span and at the supports. The most common use of cold-formed steel members is in the wall and roof systems as girts and purlins. They are usually attached along one flange to the wall or roof panels and are supported along their span by sag rods and at the ends they are supported by clip angles. Thus, some degree of restraint is always present. Quantitative evaluation of the various restraints is an essential and critical part of the design process not only of the members themselves but also of the connections which must be designed to provide the strength and stiffness required for an economical and safe structure. Present design methods(1) tend to be conservative since they are based on several assumptions which often neglect the bracing contribution of the wall or roofing materials as well as that of the sag rods.

The present paper deals with the effect of elastic end supports on the behavior of braced girts and purlins. More specifically, the paper examines how supports, such as those provided by clip angles, affect the behavior of braced channel and Z-sections. While the main discussion is based on the results from a theoretical model developed for the analysis of such members, some experimental results are also presented.

REVIEW OF RELATED LITERATURE

While the effectiveness of shear diaphragms as restraint agents against lateral and rotational displacement of girts and purlins has been examined by various researchers(2, 9, 15, 12, 10), the effectiveness of torsionally elastic supports, such as those provided by clip angles, has not been examined either theoretically or experimentally. In contrast, however, the effect of torsionally elastic supports on the buckling behavior of symmetric sections has been investigated by several researchers.

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Flint[3] was one of the earliest researchers to account for the elastic restraints in design calculations. Flint's work, however, did not include the effects of warping. While the effect of warping is negligible for the majority of hot rolled sections, light gage sections derive most of their resistance to torsional deformations from the warping action. The importance of including warping in the calculation of the buckling load of beams was realized by later researchers such as Hartman[4], Taylor and Ojalvo[13] and Nethercot and Rockey[5,6,7].

The first systematic study on the effect of torsionally elastic end connections on the lateral buckling load of prismatic I-beams was carried out by Trahair[14]. Using energy methods, Trahair developed expressions for the critical load of I-beams under various combinations of end restraints.

The behaviour of unsymmetrical sections with torsionally elastic end supports, however, has not been investigated. Research in this area has concentrated mainly on examining means of preventing the lateral and rotational movement of such members. By reducing their tendency for rotation, it has become possible to design unsymmetrical sections on the basis of failure by yielding in bending about the strong axis or by lateral buckling between lateral supports. Furthermore, the development of theoretical models for the analysis of unsymmetrical sections have assumed no lateral or rotational displacement of the supports.

THEORETICAL ANALYSIS

The theoretical analysis used in the present paper was based on the principle of minimum potential energy. The total potential energy of a beam continuously braced along one flange by a shear resisting diaphragm and elastically restrained at midspan as well as at the supports was first developed[8]. A set of differential equations and force equilibrium boundary conditions were subsequently formulated using the calculus of variations. These equations were then replaced by a set of algebraic expressions using the finite difference method[11]. Approximate solutions for stress and displacement were obtained by solving the derived algebraic expressions with the aid of a CDC Dual Cyber Computer System 170/750.

The developed theoretical model was based on the following assumptions:

1) The beams are simply supported with respect to bending;
2) the load is uniformly distributed and is applied in a direction parallel to the web;
3) the end supports provide equal elastic torsional restraint to the beams;
4) a discrete elastic lateral support is located at midspan; and
5) the shear diaphragm provides continuous lateral and torsional restraint to the beams.

In addition, it was assumed that:

1) The beams are initially straight and have a constant cross section;

2) the cross-sectional configuration does not change after loading;
3) axial deformations are neglected;
4) the internal stresses do not exceed the proportional limit; and
5) the compression elements do not fail by local buckling.

With the limits provided by the foregoing assumptions, the total potential energy, Π, of the elastic system was formulated as a function of the displacement functions and their derivatives. The principle of minimum potential energy implies that the total potential energy of a system does not change when the system passes from its configuration of equilibrium to an infinitesimally near adjacent configuration. Mathematically, this may be expressed as:

$$\delta\Pi = 0 \qquad \text{.... (1)}$$

Equation (1) may be interpreted mathematically as the condition that Π assumes a minimum value. The condition that Π is a minimum furnished a set of Eulerian differential equations which determined the displacement functions along with a set of boundary conditions. These differential equations were then replaced by simultaneous algebraic expressions using the finite difference method.

In general, if n discrete points are chosen along the span of a member, a differential equation can be described by n simultaneous algebraic equations. The simultaneous equations may be expressed in matrix form as:

$$[K]\,\{r\} = \{F\} \qquad \text{.... (2)}$$

where the matrix [K] represents the relationship between displacements and force vectors; {r} is the vector of the lateral and rotational displacements; and {F} is the vector of the lateral and torsional components of the applied external forces.

When Eq.(2) is solved for the displacement vector {r}, an approximate value for the stress at any point then may be obtained from the following expression:[9]

$$\sigma = \left(\frac{M_w}{I_x} + \frac{EI_{xy}}{I_x}u''\right)y - Ex\,u'' - E\,\omega\,\phi'' \qquad \text{.... (3)}$$

u'' and ϕ'' in Eq.(3), are derivatives of the lateral and rotational displacements respectively; M_w is the bending moment; I_x and I_{xy} are moments of inertia; E is the modulus of elasticity; ω is the sectorial co-ordinate; and x and y are the co-ordinates of the point where the stress is to be calculated.

Since the number of algebraic equations developed through the finite difference method to describe the behavior of the system is very large, a practical solution is only possible through the use of a computer.

RESULTS AND DISCUSSION

The strength of the developed analytical model can best be illustrated through an example. Two cross-sections were chosen for present illustrative purposes: a channel and a Z-section. It was assumed here that both sections were formed from plates having identical physical and mechanical properties. The cross-sectional properties of the two sections chosen are given in Table 1. These sections were assumed to be members of a roof or wall system supporting light-gage corrugated panels acting as a shear diaphragm and subjected to uplift (suction) loading. The type of loading was purposely chosen in order to place the unsupported flange in compression while the tension flange was elastically restrained by the panels.

There are two possible restraints that a diaphragm may provide:

1) Restraint of the attached flange against displacement in the direction of the corrugations; and

2) restraint of the member against rotation.

The degree of lateral restraint present is a function of the shear rigidity, Q, of the diaphragm while the degree of rotational restraint is a function of its rotational stiffness, F. Quantitative evaluation of these restraints is quite difficult and requires physical testing. It is a very common practice to neglect the presence of these restraints in design calculations, especially when the bracing is provided to the tension flange only. Specification requirements in this case necessitate the use of additional bracing attached to the compression flange.[1]

In the present paper, two cases of restraint conditions were examined:

1) Both lateral and rotational restraints along the span
(F = 150 lb-in./rad/in; Q = 150 k)
(F = 430 kN-mm./rad/mm; Q = 627 kN), and

2) only lateral restraint along the span
(F = 0; Q = 150 k)
(F = 0; Q = 627 kN).

The chosen values of F and Q, though arbitrary, represent realistic restraint conditions commonly encountered in metal roof and wall systems.

The parametric variations in the selected illustrations also include the presence/absence of a discrete restraint at midspan, such as the one provided by a sag rod, and the degree of rotational restraint at the ends.

The following discussion centers on the effect of torsionally elastic end supports on the allowable load of channel and Z-sections used as girts or purlins that are either restrained or totally unrestrained at midspan.

When a beam is loaded through its shear center, the beam will deflect in the direction of the load and will bend about a plane perpendicular to the plane of load. In this case, the mode of failure would be either yielding, if the beam is sufficiently braced laterally, or lateral buckling, if the beam is laterally unbraced. However, when the load is applied in a plane which does not pass through the shear center, the beam will deflect and rotate continuously with increasing load. The amount of deflection and rotation of the beam depends on the bracing restraints imposed on the member. Thus, in this situation the beam's performance must be evaluated on stress (strength) as well as on serviceability criteria. While stress criteria are easily established, serviceability criteria are totally based on engineering judgement and acceptable practices.

In the present paper, the allowable loads were computed to satisfy the following strength and serviceability criteria:

Stress	=	0.60 Fy
Vertical Displacement	=	L/180
Lateral Displacement	=	L/180
Rotational Displacement:		
between Supports	=	15 degrees
at the Supports	=	2 degrees

where L is the span of the beam between supports and Fy is the yield stress of the section.

While the stress and vertical displacement limits are those recommended by the various design specifications, the limits for lateral and rotational displacements are rather arbitrary. They were chosen to provide a basis for evaluating the effect of the various restraints.

Assuming that the two sections chosen are sufficiently braced to allow bending about the strong axis only, the allowable uniformly distributed load, computed on the basis of a safety factor of 1.67, is 200 lb/ft (0.01 N/mm). The ability of the two sections to reach this desirable allowable load depends on:

1) The degree of lateral and rotational restraint along the span;
2) the degree of rotational restraint at the support;
3) the presence/absence of a discrete restraint at midspan; and
4) the cross-sectional characteristics of the member.

As shown in Fig. 1, the Z-section with no rotational restraint along the span, (F = 0), and with a shear rigidity of Q = 150 k (627 kN) present, reached 75% of the maximum allowable load. This, however, was possible only for relatively stiff rotational supports at the ends (K_{zr} > 38000 lb-in/rad.)(4293 kN-mm/rad). For the same restraint condition, the channel section reached only 20% of the maximum allowable load (Fig. 2). While strength criteria governed the ultimate allowable load of the Z-section, rotational displacement limits governed the behavior of the channel section.

When the rotational stiffness of the attached panels was taken into account (F = 150 lb-in/rad/in.), (430 kN-mm/rad/mm), the allowable load for the Z-section increased from 75% to 90% of the maximum allowable load (Fig. 4), while the allowable load for the channel section increased from 20% to 65% of the maximum allowable load (Fig. 2).

The above discussion was based on the assumption that the torsional stiffness of the end supports, K_{zr}, is sufficiently high to limit the rotation of the members at the supports to 2 degrees. The minimum torsional stiffness, $(K_{zr})_{min}$, required to reach this limit was found to be a function of the cross-sectional characteristics of the member as well as the support conditions along its span.

A continuous rotational restraint, F, along the span improved the load bearing capacity of both the channel and the Z-section without the need for an increase in the minimum torsional stiffness, $(K_{zr})_{min}$, of the supports (Figs. 1 and 2). The panels, in this case, provided the required support to resist the additional load.

The provision of a discrete restraint at midspan resulted in a redistribution of stresses in the section. In general, there are three components of stress at any point in the member: one component due to bending about the strong axis, one due to bending about the weak axis, and one due to twisting of the member. Thus, the allowable load of the member depends on the cumulative effect of all these components of stress. While the maximum stress in both the channel and the Z-section, which were unrestrained at midspan occurred at the web-to-flange juncture of the unbraced flange, (Figs. 3 and 4), the maximum stress shifted to the flange-to-lip juncture of the unbraced flange when a discrete support was applied at midspan. This behavioral phenomenon has also been observed experimentally[11]. In a research program carried out at the University of Toronto involving the testing of full size wall systems under suction it was shown that the effect of a discrete restraint at midspan, was to shift the location of maximum stress in the section (Figs. 5 and 6). The actual magnitude of the stress was a function of the cross-sectional characteristics of the member and the degree of lateral and rotational restraint provided along the span of the member. The theoretical results indicate that a discrete restraint at midspan has very little effect on the maximum stress in channel and Z-sections with high rotational restraint along their span (Figs. 3 and 4).

In the absence of a continuous rotational restraint along the span of the members (F = 0), the allowable load for the Z-section increased from 75% to 98% of the maximum allowable load when a discrete restraint was applied at midspan (Figs. 1 and 8). Similarly, for the channel section, there was an increase in the allowable load from 20% to 60% of the maximum allowable load (Figs. 2 and 7). The addition of discrete restraint at midspan resulted in an increase in the required minimum torsional stiffness, $(K_{zr})_{min}$, of the end supports from 40000 lb-in/rad (4519 kN-mm/rad) to 50000 lb-in/rad (5649 kN-mm/rad). For the channel section, the increase in the allowable load also necessitated an increase in the minimum torsional stiffness, $(K_{zr})_{min}$, from 20000 lb-in/rad (2259 kN-mm/rad) to 45000 lb-in/rad (5084 kN-mm/rad).

With a continuous rotational restraint, F, along the span, in addition to the discrete restraint at midspan, the allowable load on the channel section increased from 60% to 65% of the maximum allowable load (Fig. 8). The continuous rotational restraint in this case resulted in a reduction of the minimum torsional stiffness, $(K_{zr})_{min}$, at the supports from 45000 lb-in/rad (5084 kN-mm/rad) to 35000 lb-in/rad (3954 kN-mm/rad).

CONCLUSIONS

The design of cold-formed channel and Z-sections based on strength and serviceability criteria provide a more realistic representation of member behavior. Since such members usually form an integral part of wall or roof systems, their design must account for the various end and intermediate restraint conditions present.

In the present paper, results from an analytical model were presented to show that the allowable load of open sections is a function of the cross-sectional properties of the sections as well as a function of the various restraint conditions present.

The analytical model described here was developed for evaluating the performance characteristics of elastically restrained open sections. Work is currently underway at the University of Texas at Austin to test full size wall systems in order to obtain experimental data on the effect of rotationally elastic end supports on the behavior of channel and Z-section girts. This data will be used to evaluate further the analytical model.

APPENDIX--NOTATION

a = Dimension of a 90° lip stiffener

b = Flange width

d = Depth of Section

E = Elastic Modulus of Elasticity

F = Rotational restraint provided by the diaphragm

$[F]$ = Force vector

F_y = Yield stress

I_x = Moment of inertia with respect to the x-axis

I_y = Moment of inertia with respect to the y-axis

I_{xy} = Moment of inertia with respect to the x- and y-azes

K = St. Venant's torsional constant

$[K]$ = Stiffness matrix

K_{zr} = Torsional stiffness of end supports

L = Span of member between supports

M_w = Bending moment

APPENDIX--NOTATION (Cont'd)

Q = Shear rigidity of the shear diaphragm

{r} = Displacement vector

t = Thickness of the section

u" = Derivative of the lateral displacement of the sections

W = Uniformly distributed load

x = Co-ordinate with respect to the centroid

y = Co-ordinate with respect to the centroid

β" = Derivative of the rotational displacement

Γ = Warping constant

Π = Potential energy

σ = Longitudinal stress

ϕ = Rotational displacement

ω = Sectorial co-ordinate

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Table 1. Sectional Properties of Members

Property	Channel Section	Z Section
t (in.)	0.1	0.1
a (in.)	1.0	1.0
b (in.)	2.5	2.5
d (in.)	6.0	6.0
I_x (in^4)	7.567	7.567
I_y (in^4)	1.319	2.292
I_{xy} (in^4)	0	-3.115
K (in^4)	0.0043	0.0043
Γ (in^6)	11.687	13.683

1 in. = 25.4 mm.

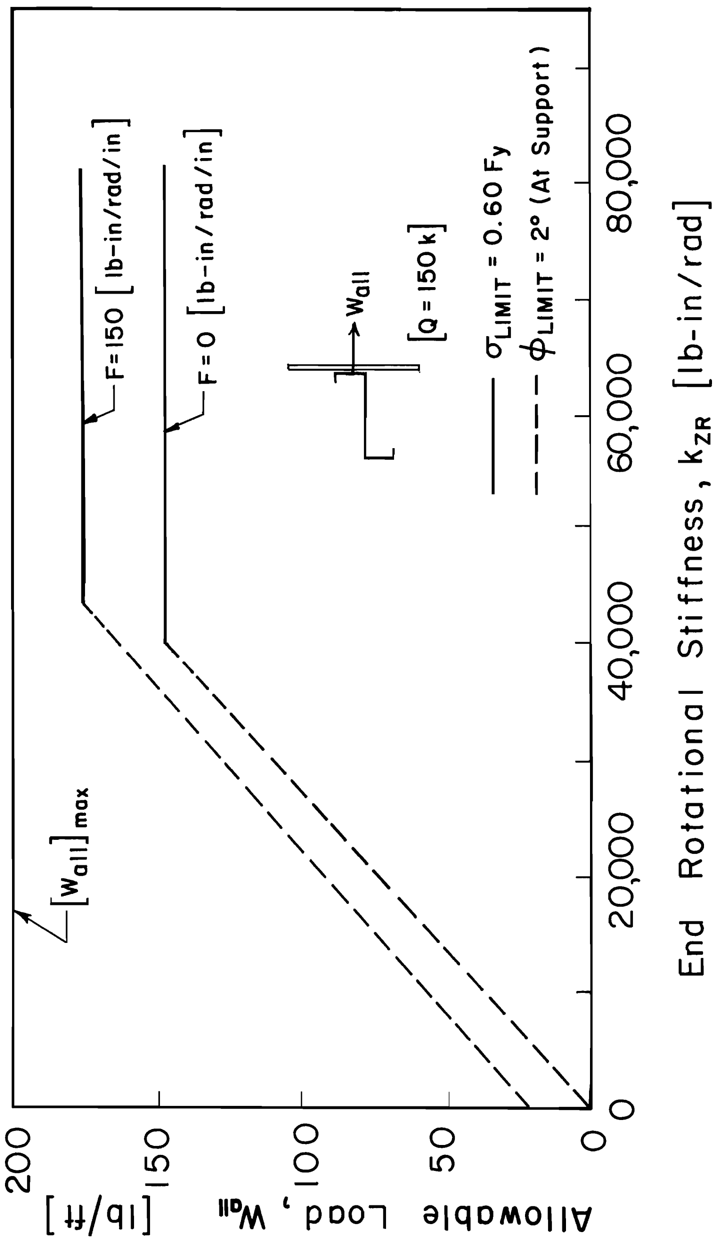

Fig. 1: Effect of Torsionally Elastic End Supports on the Behavior of a Z-Section Beam

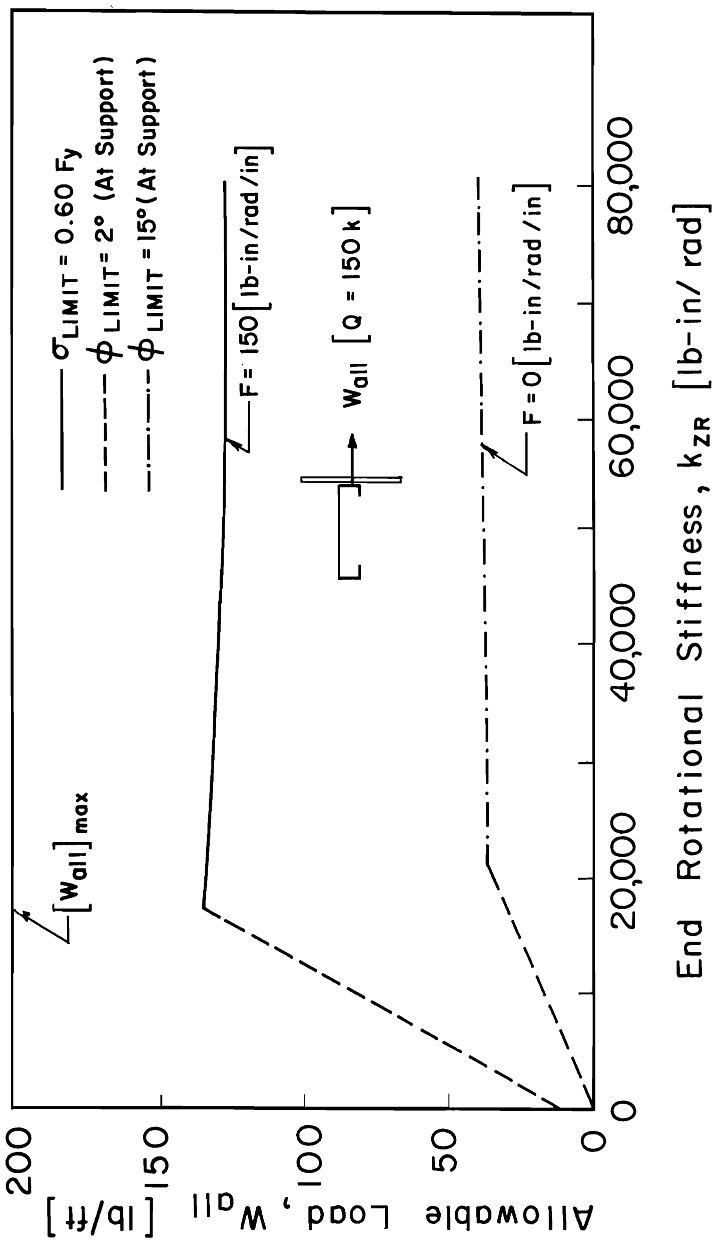

Fig. 2: Effect of Torsionally Elastic End Supports on the Behavior of a Channel Section Beam

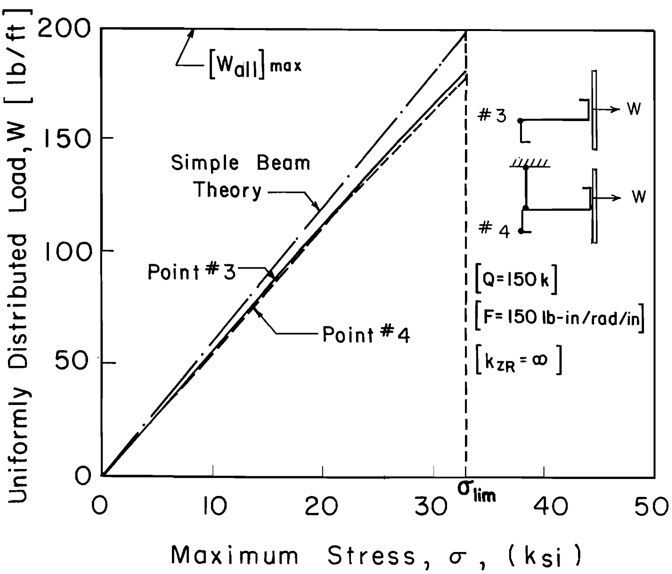

Fig. 3: Effect of a Discrete Restraint at Midspan of a Z-Section on the Maximum Stress

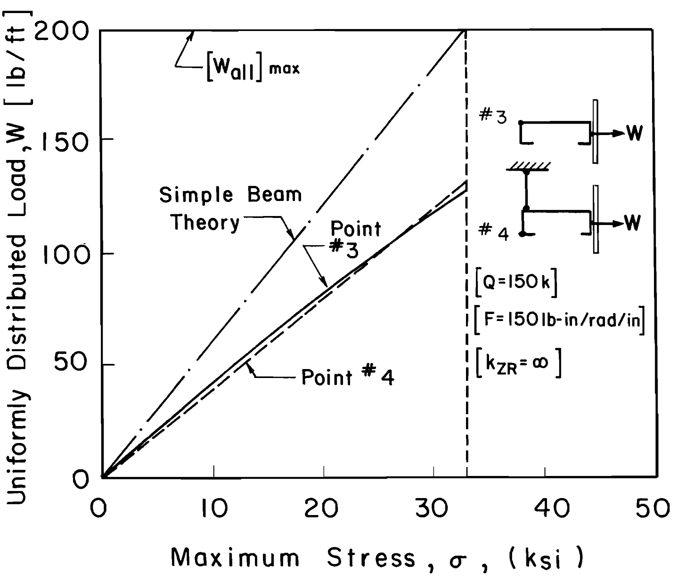

Fig. 4: Effect of a Discrete Restraint at Midspan of a Channel Section on the Maximum Stress

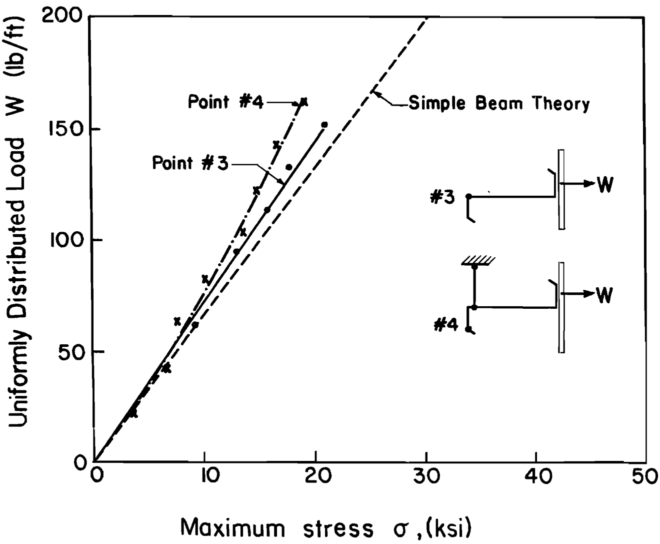

Fig. 5: Effect of a Discrete Restraint on the Maximum Stress of a Z-section(11)

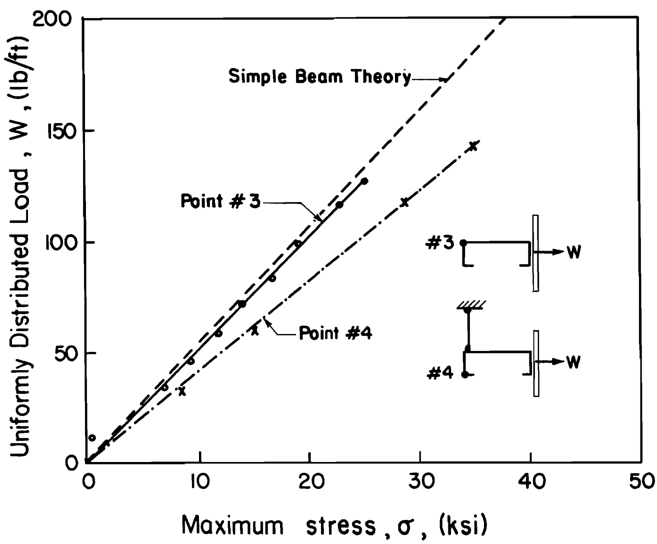

Fig. 6: Effect of a Discrete Restraint on the Maximum Stress of a Channel Section(11)

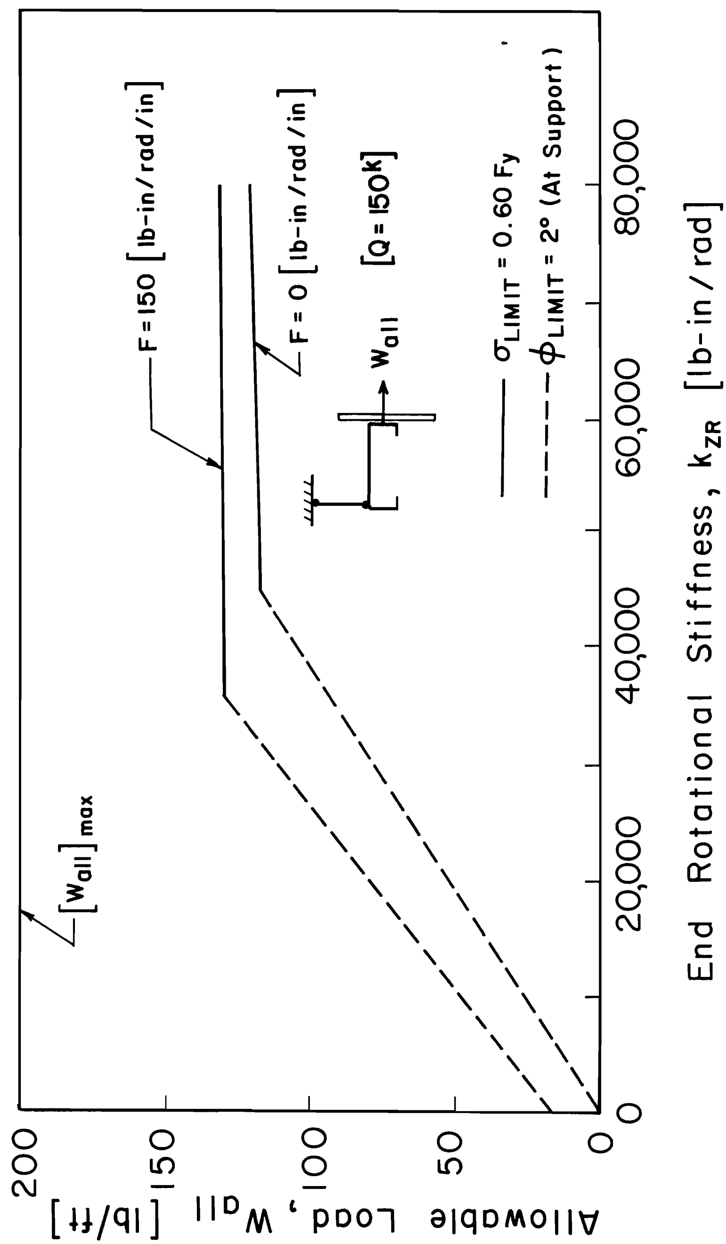

Fig. 7: Effect of Torsionally Elastic End Supports on the Behavior of a Channel Section Beam Restrained at Midspan

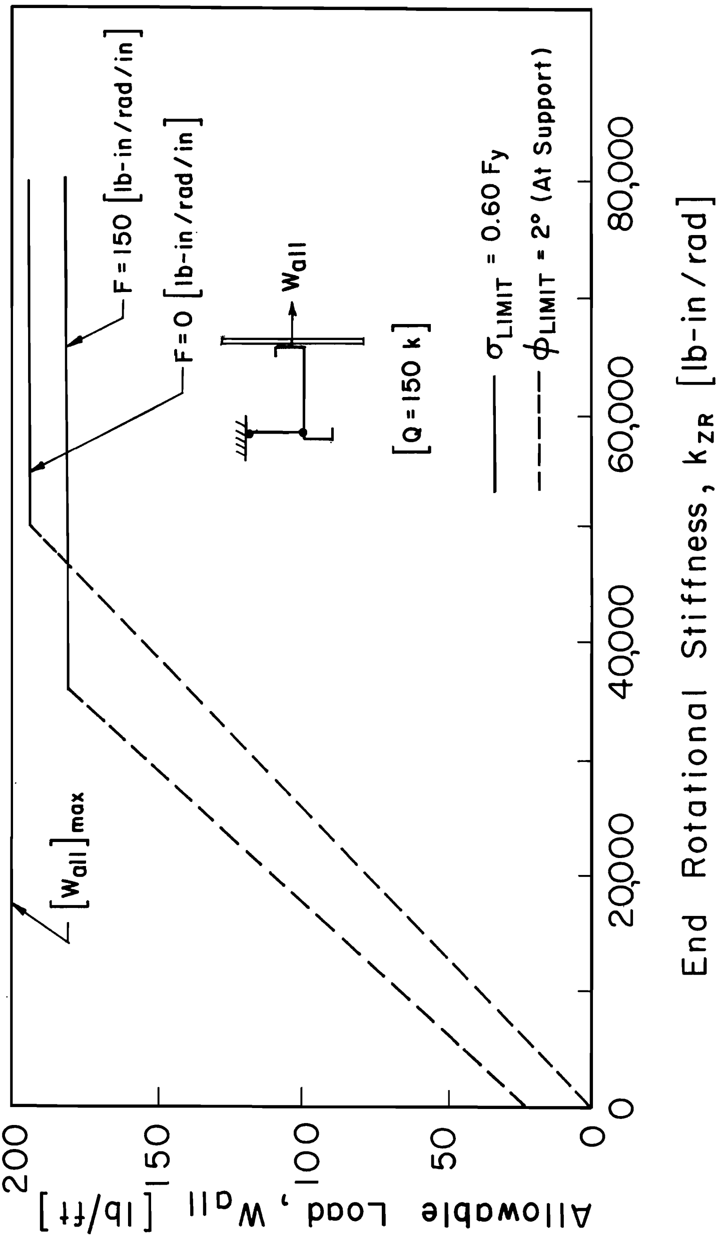

Fig. 8: Effect of Torsionally Elastic End Supports on the Behavior of a Z-Section Beam Restrained at Midspan