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OPTIMUM DESIGN OF COLD-FORMED STEEL Z-SHAPE PURLINS USING A GENETIC ALGORITHM

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Abstract

In this paper, a genetic algorithm is applied to optimize the dimensions of cold-formed Z-shape purlins continuous over two spans under gravity load. The optimization criterion is to maximize the load resistance per cross-section and the design variables are chosen from the discrete values based on the manufacturing requirements. Purlins are designed in accordance with Eurocode 3, Part 1.3. In addition, the modified Eurocode 3 method, in which the elastic local buckling stress and distortional buckling stress calculated using Finite Strip Method (FSM) are integrated into the design process, is used to determine the effective section properties. The results are compared with those obtained using Eurocode 3 method.

Introduction

Many folds along the flange and web, and the use of multiple lip stiffeners in the cross-section of cold-formed purlins make the section very resistant to local buckling and less prone to twist under uplift wind load and gravity load. The multiple choices of the cross-section raise the question of the optimal shape. In this paper, a Genetic Algorithm (GA) is used to optimize the dimensions of the Z-shape purlins continuous over two spans under gravity load.

GA is a general-purpose, derivative-free, stochastic search algorithm (Cogan, 2001 and Mitchell, 1998) and starts by randomly choosing an initial population that consists of candidate solutions to the problem at hand. Each individual in the population is characterized by a fixed length binary bit string, which is called chromosome. These chromosomes are evaluated by means of a fitness function. Combining the fittest individuals from the previous population, a new generation of chromosomes is created. Evolutionary operators such as selection, crossover, and

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mutation are used to create this new population. Besides, Elitism, which is a method that copies the best chromosome or a few better chromosomes to the new population, might be incorporated into the algorithm to avoid losing the best individual. This process continues until the specified level of fitness is reached.

In this paper, purlins are designed in accordance with Eurocode 3, Part 1.3 (ENV, 1996). In addition, the modified Eurocode 3 method, in which the elastic local buckling stress and distortional buckling stress calculated using Finite Strip Method (FSM) are integrated into the process, is used to determine the effective section properties. The results based on these two design methods are compared.

GA-Based Design

The Z-shape purlin is assumed to be continuous over two spans and under the gravity load. The purlin is connected to a sheeting at the wider flange and the dimensions of the cross-section are shown in Figure 1 where b_1 is the width of top (wider) flange; b_2 is the width of the bottom flange; c is the depth of the lip; h is the height of the cross-section; t is the thickness of the cross-section and L is the span of the purlin. The width of top flange is assumed to be 6 mm (0.24 in.) wider than that of the bottom flange.

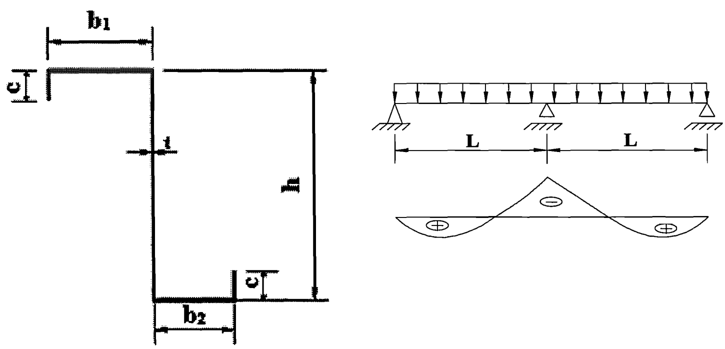


Figure 1 Dimensions of Z-shape purlin continuous over two spans under gravity load

In the optimization process, the width of the top flange, b_1, and the ratio of c to b_2 are chosen as the design variables. The possible values for the design variables are given in Table 1.

Table 1 Values for design variables

Design variables	Possible values
Width of the top flange b_1	from 40 mm (1.57 in.) to 100 mm (3.94 in.) with step 1 mm (0.04 in.)
Ratio of c to b_2	from .2 to .6 with step .01

In Eurocode 3, Part 1.3, the free flange is considered as a beam on an elastic foundation. When the purlin is continuous over two spans, it should satisfy the following criteria for cross-section resistance. For the restrained flange

$$M_{y,Sd}/W_{eff,y} \leq f_y/\gamma_M \tag{1}$$

and for the free flange

$$M_{y,Sd}/W_{eff,y} + M_{fz,Sd}/W_{fz} \leq f_y/\gamma_M \tag{2}$$

The stability of the free flange at the internal support should be checked using the following equation:

$$1/\chi \cdot (M_{y,Sd}/W_{eff,y}) + M_{fz,Sd}/W_{fz} \leq f_y/\gamma_{M1} \tag{3}$$

where $M_{y,Sd}$ is the in-plane bending moment; $W_{eff,y}$ is the effective section modulus of the cross-section for bending about y-y axis; M_{fz} is the bending moment in the free flange due to the lateral load; W_{fz} is the gross elastic section modulus of the free flange plus 1/6 of the web height, for bending about the z-z axis; χ is the reduction factor for flexural buckling of the free flange and γ_M, γ_{M1} are the partial safety factors.

Since the bending moments in the above formulas are functions of distributed load q, the objective of the optimization is to obtain the optimum dimensions that maximize the distributed load per cross section when the material reaches its yield strength, i.e.

$$q/A_g \rightarrow \max \tag{4}$$

subjected to the geometrical constraints, which are specified in Eurocode 3, and fabrication constraints as following

$$h/t \leq 500, \quad b_1/t \leq 60, \\ 200\,\text{mm} \leq b_1 + b_2 + 2 \cdot c + h \leq 625\,\text{mm} \tag{5}$$

where A_g is the area of the gross cross-section. Since in the further optimization process, the span of the purlin is set to a fixed value for each case, the span of the purlin is not included into the objective function.

Because a GA is directly used for solving an unconstrained optimization problem, the constrained optimization problem mentioned above should be transformed into an unconstrained problem by including a penalty function. In this analysis, a quadratic penalty function is used, and the corresponding unconstrained optimization problem becomes

$$\text{Maximize} \quad F = \begin{cases} q/A_g - KK \cdot n \cdot CC & \text{when } q/A_g > KK \cdot n \cdot CC \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where F is the fitness function and $CC = \sum c_i$ is the constraint violation function, in which c_i are the constraint violations given by

$$c_i = \begin{cases} 0 & \text{if } \alpha_i \leq 0 \\ \alpha_i^2 & \text{otherwise} \end{cases} \tag{7}$$

where α_i are the normalized constraints provided by

$$\alpha_1 = {}^{h/t}\!/_{500} - 1, \quad \alpha_2 = {}^{b_1/t}\!/_{60} - 1, \\ \alpha_3 = {}^{(b_1 + b_2 + 2 \cdot c + h)}\!/_{625} - 1, \quad \alpha_4 = 1 - {}^{(b_1 + b_2 + 2 \cdot c + h)}\!/_{200} \tag{8}$$

In addition, n is the coefficient that makes the values of q/A_g and CC at the same order to avoid one value dominating the other. In this analysis, the value of n is defined as $10^{L_f - 1 - L_c}$ so as to keep the order of CC one degree lower than that of q/A_g, in which L_f and L_c are the orders of q/A_g and CC, respectively. Moreover, $KK \geq 0$ is a coefficient and the solution of the penalty problem can be made arbitrarily close to the solution of the original problem by choosing KK sufficiently large (Bazaraa et al., 1993).

Figure 2 shows how the purlin design is integrated into the GA optimization process. GA-based design starts by randomly generating an initial population that is composed of candidate solutions to the problem. Each individual in the population is a binary string of fixed length. After decoding, these individuals that represent the dimensions of the purlins are sent to the purlin design program. The constraints are checked and if the constraints are violated, the penalty is applied. By combining the fittest individuals in the previous population, the new generation is created using such operators as selection, crossover and mutation. In order to keep the best individuals in each generation, the elitism may also be used. This process is continued until the specified stopping criteria are satisfied.

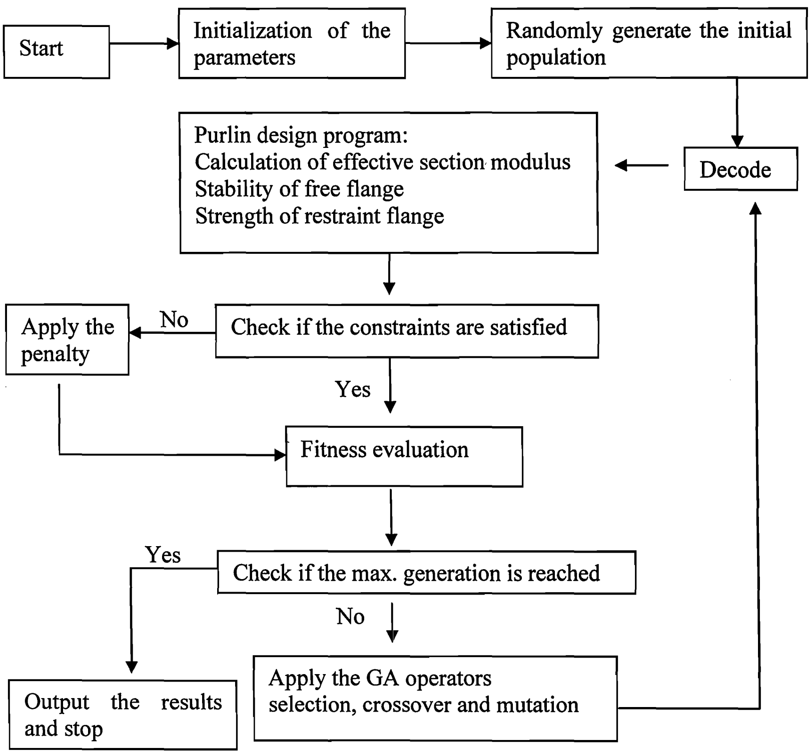


Figure 2 Integrating purlin design into GA optimization

Optimization Based on Eurocode 3 Method

The GA, which is based on a binary representation, two-point crossover, bit-flip mutation, and tournament selection with elitism, is used to optimize the Z-shape purlin with a height of 150 mm (5.90 in.), a thickness of 1.5 mm (0.06 in.) and a span of 4.5 m (14.76 ft.). The yield strength is 350 MPa (50.76 ksi) and the modulus of elasticity is 210 GPa (30457.92 ksi). After the parameter analysis, the population size is set to 30, the crossover rate to 0.8 and the mutation rate to 0.001. Using these parameters, 10 runs of optimization analysis are performed, in which one run is defined as the complete running of the GA. The optimization results are shown in Table 2. The optimum dimensions are chosen as those corresponding to the largest value of q/A_g in the 10 runs. In addition, Table 2 also lists the value of M_y/A_g where M_y is the moment that causes the first yield and is defined as $M_y = W_{eff,y} \cdot f_y/\gamma_M$.

Table 2 Optimization results for 10 runs

No. of run	b_1 (mm) $(in.)$	c (mm) $(in.)$	q/A_g $(N/mm/mm^2)$ $(lb/in./sq.in.)$	M_y/A_g $(N \cdot mm/mm^2)$ $(lb \cdot in/sq.in.)$	A_g (mm^2) (sq. in.)
1	53 (2.09)	25.38 (1.00)	0.003375 (12.433)	12515.32 (71461.22)	430.58 (0.667)
2	56 (2.20)	26.50 (1.04)	0.003370 (12.414)	12428.26 (70964.12)	442.61 (0.686)
3	53 (2.09)	23.97 (0.94)	0.003373 (12.426)	12551.13 (71665.70)	426.47 (0.661)
4	54 (2.12)	24.48 (0.96)	0.003375 (12.433)	12527.59 (71531.29)	430.88 (0.668)
5	54 (2.12)	25.44 (1.00)	0.003376 (12.436)	12503.55 (71394.02)	433.68 (0.672)
6	56 (2.20)	25.50 (1.00)	0.003369 (12.411)	12454.37 (71113.21)	439.69 (0.682)
7	53 (2.09)	23.97 (0.94)	0.003373 (12.426)	12551.13 (71665.70)	426.47 (0.661)
8	53 (2.09)	25.38 (1.00)	0.003375 (12.433)	12515.32 (71461.22)	430.58 (0.667)
9	**54 (2.12)**	**25.92 (1.02)**	**0.003376 (12.436)**	**12489.93 (71316.25)**	**435.08 (0.674)**
10	54 (2.12)	24.48 (0.96)	0.003375 (12.433)	12527.59 (71531.29)	430.88 (0.668)

The optimization results are verified by investigating the load resistance of the same purlin with the variation of the width of the flange and the depth of the lip. The width of the wider flange is varied from 40 mm (1.57 in.) to 100 mm (3.97 in.) with a step of 1 mm (0.039 in.) and the ratio of the depth of the lip to the width of the shorter flange is varied from .2 to .6 with a step of .01. The dimensions corresponds to the maximum value of q/A_g are chosen as the optimums during this variation and are compared with those obtained using GA in Table 3.

Table 3 Comparison of optimization results by calculation to that by GA

Calculation methods	b_1 (mm) (in.)	b_2 (mm) (in.)	$\alpha = c/b_2$	c (mm) (in.)	q/A_g $(N/mm/mm^2)$
Opt-Cal	**54 (2.12)**	48 (1.89)	**0.54**	25.92 (1.02)	**0.003376 (12.436)**
Opt-GA	**54 (2.12)**	48 (1.89)	**0.54**	25.92 (1.02)	**0.003376 (12.436)**

In the table, the 'Opt-Cal' represents the calculation based on parameter variations and 'Opt-GA' represents the calculation based on GA. The maximum value of q/A_g for Opt-GA is the same as that of Opt-Cal. Thus, the computer source code for optimization and the selected parameters for GA are verified and can be used for further analysis.

Integrating the Modified Eurocode 3 Method into Optimization

The modified Eurocode 3 method integrates elastic local and distortional buckling stresses calculated using the FSM analysis into the design procedure based on Eurocode 3. In this process, the reduction factor, ρ, is calculated according to the elastic local buckling stress from the FSM analysis. The effective width of the lip, flange and web are all calculated based on this value. Similarly, the reduction factor, χ, for reduced thickness due to the partially effective of edge or intermediate stiffener is calculated using the distortional buckling stress from the FSM analysis. By doing so, the interaction between the lip, the flange and the web are integrated due to the treatment of the section as a whole.

The elastic local and distortional buckling stresses are calculated using the computer program CUFSM, which is developed by Schafer (Schafer, 2002) and can be freely downloaded from the website. However, there might exist an indistinct buckling mode for some sections, i.e. there is no obvious minimum in the buckling curve for the local buckling mode or the distortional buckling mode. Thus, in either of these cases the design procedure is based on Eurocode 3.

The elastic buckling stresses calculated using Finite Strip Method (FSM) are shown in Table 4 and are compared to those calculated using the EC3 method (EC 3) in Table 5. The comparison is based on the ratio of EC 3 to FSM with a web height of 100 mm (3.94 in.) and a thickness of 2 mm (0.08 in.). The elastic local buckling stress for the flange, lip and web are all compared to the same local buckling stress in FSM.

Table 4 Elastic buckling stresses obtained using FSM

Width of flange (mm) $(in.)$	$c/b_2 = 0.2$ (MPa) (ksi)		$c/b_2 = 0.4$ (MPa) (ksi)		$c/b_2 = 0.6$ (MPa) (ksi)	
	local	**dist.**	**local**	**dist.**	**local**	**dist.**
34 (1.34)	--	835.00 (121.10)	2093.10 (303.57)	1263.90 (183.31)	2055.00 (298.04)	1438.80 (208.67)
44 (1.73)	--	657.80 (95.40)	1751.80 (254.07)	1091.00 (158.23)	1458.50 (211.53)	1340.80 (194.46)
54 (2.12)	1214.60 (176.16)	549.42 (79.68)	1244.50 (180.49)	982.36 (142.47)	1084.20 (157.24)	1258.80 (182.57)
64 (2.52)	893.07 (129.52)	480.33 (69.66)	902.26 (130.86)	895.11 (129.82)	860.12 (124.74)	1223.00 (177.37)
74 (2.91)	676.79 (98.16)	432.23 (62.69)	684.97 (99.34)	835.45 (121.17)	693.64 (100.60)	1213.00 (175.92)
84 (3.31)	529.81 (76.84)	388.53 (56.35)	538.66 (78.12)	788.94 (114.42)	555.51 (80.57)	1223.00 (177.37)
94 (3.70)	425.06 (61.65)	358.14 (51.94)	434.68 (63.04)	751.75 (109.03)	448.49 (65.04)	--

Table 4 shows that when the ratio of c to b_2 is .2 and the width of the bottom flange is varied from 34 mm (1.34 in.) to 44 mm (1.73 in.), the distortional buckling mode is the critical failure mode and there is no local mode occurred. When the width of the flange is varied from 34 mm (1.34 in.) to 64 mm (2.52 in.), the web buckling is the critical local mode. Thus, the increasing the ratio of c to b_2 from .2 to .4 does not improve the elastic local buckling stress and if this ratio is further increased .6, lip buckling becomes the critical local mode. When the width of the flange exceeds 64 mm (2.52 in.), the local mode starts to transform from web buckling to flange buckling. As far as the distortional buckling stress is concerned, the decrease of the depth of the lip and the increase of the width of the flange decrease its value. If the depth of the lip and the width of the flange are not large enough, distortional buckling is the critical failure mode. However, when the width of the flange is increased too much, the flange buckling will become critical. In addition, when the ratio of c to b_2 is .6 and the width of the flange is 94 mm (3.70 in.), the distortional buckling is not distinct.

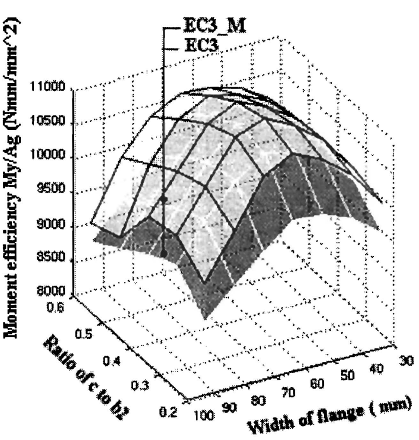
Table 5 Comparison of elastic buckling stress of Eurocode 3 with FSM ($EC3/FSM$)

Width of flange (mm) $(in.)$	$c/b_2=0.2$				$c/b_2=0.4$				$c/b_2=0.6$			
	flange	lip	web	dist.	flange	lip	web	dist.	flange	lip	web	dist.
34 (1.34)	--	--	--	0.49	1.36	1.32	0.78	0.69	1.38	0.78	0.81	0.88
44 (1.73)	--	--	--	0.53	0.94	0.91	0.92	0.68	1.13	0.64	1.14	0.79
54 (2.12)	0.89	3.11	1.16	0.55	0.87	0.84	1.29	0.65	0.99	0.56	1.51	0.71
64 (2.52)	0.85	2.92	1.51	0.56	0.84	0.81	1.75	0.65	0.88	0.50	1.88	0.60
74 (2.91)	0.83	2.82	1.89	0.57	0.82	0.79	2.20	0.60	0.81	0.46	2.24	0.51
84 (3.31)	0.82	2.75	2.29	0.58	0.80	0.77	2.64	0.56	0.78	0.45	2.66	0.44
94 (3.70)	0.81	2.70	2.73	0.59	0.79	0.76	3.10	0.52	0.77	0.44	3.16	--

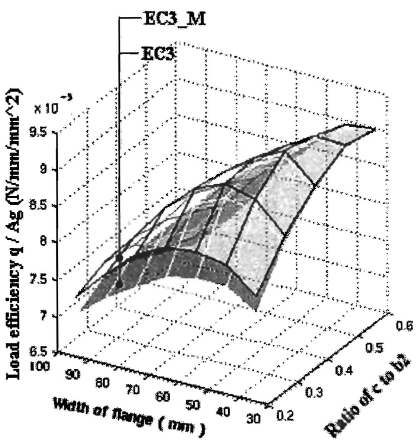
In Eurocode 3, the interactions between the elements are not considered. Thus, Table 5 shows that for most of the cases, the elastic buckling stresses calculated using EC 3 are lower than those obtained using FSM except for the elastic buckling stress of the web when the width of the flange is larger than 44 mm (1.73 in.). For these sections, the elastic buckling stresses of the web calculated using EC 3 are higher. This is due to the fact that the local failure mode in FSM is flange buckling and this same value is chosen as the elastic buckling stress for the web. For the same reason, the elastic buckling stress of the lip calculated using EC 3 is higher when the ratio of c to b_2 is .2.

Figure 3 illustrates the comparison of the moment efficiency, M_y/A_g, and load efficiency, q/A_g, calculated using Eurocode 3 to those obtained using the modified Eurocode 3 method. For both moment efficiency and load efficiency, the modified Eurocode 3 shows higher values.

Table 6 lists the comparison of optimization results obtained using GA based on Eurocode 3 (EC3) to those on modified Eurocode 3 method (EC3_M). Table 5 indicates that optimum dimensions for 'EC3_M' are different from those for 'EC3' but the difference for this case is not too large and the value of q/A_g for 'EC3_M' is about 0.85 % higher than that of 'EC3'.



(a) Moment efficiency



(b) Load efficiency

Figure 3 Comparison of Eurocode 3 method (EC3) with modified Eurocode 3 method (EC3_M)

Table 6 Comparison of optimization results

Design methods	b_1 (mm) $(in.)$	$\frac{c}{b_2}$	c (mm) $(in.)$	q/A_g $(N/mm/mm^2)$ $(lb/in./sq.in.)$	M_y/A_g $(N\cdot mm/mm^2)$ $(lb\cdot in/sq.in.)$
EC3	46 (1.81)	0.44	17.60 (0.693)	0.009161 (33.75)	9484.060 (54153.33)
EC3_M	48 (1.89)	0.42	17.64 (0.694)	0.009239 (34.03)	9682.177 (55284.57)

Conclusions

As demonstrated in this paper, the Genetic Algorithm (GA) can be used as an optimization tool to obtain the optimum dimensions of the Z-shape purlins under gravity load. This GA-based design method can also be applied to the optimization of other shapes of cold-formed steel purlins and other cold-formed steel members. In addition, the comparison of the modified Eurocode 3 method to the Eurocode 3 method indicates that the modified Eurocode 3 method shows higher values due to the inclusion of the interaction between the elements. However, there is no big difference in the optimum dimension in the given example. For other cross-sections, further analyses need to be carried out.

Appendix.—References

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Appendix.—Notations

A_g	Gross area of cross-section
b_1	Width of top flange
b_2	Width of bottom flange
CC	Sum of constraint violation function
c	Depth of the lip
c_i	Constraint violation function
F	Fitness function
f_y	Yield stress
h	Height of the cross-section
L	Span of the purlin

M_{fz}	Bending moment about z-z axis in the free flange due to lateral load
$M_{y,Sd}$	In-plane bending moment
q	Applied load
t	Thickness of the cross-section
$W_{eff,y}$	Effective section modulus for bending about y-y axis
W_{fz}	Gross section modulus of the free flange plus $1/6$ web height for bending about z-z axis
α_i	Normalized constraints
χ	Reduction factor for flexural buckling of the free flange
γ_M, γ_{M1}	Partial safety factor
ρ	Reduction factor to determine the effective width