Maximum Load Design of Cold Formed Steel Shapes

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SUMMARY

The advantages of maximum load capacity design for constant material, as compared with the more common minimum weight design for prescribed load capacity, are explained. Equivalance of the two approaches is proved under very general conditions. Application is illustrated on design of channels and lipped channels of cold formed steel under the Canadian or similar Standards.

1. INTRODUCTION

The subject of this paper is the optimization of structural cross-sections, based on load carrying capacity. It is worthwhile to have a practical way to find a set of optimal member profiles (of cold formed steel channels, for example) given a building code, a design standard, and also the conditions of structural geometry and loading. This type of problem has led to the much studied "minimum weight" optimization of structural members [8,2]. Developments in mathematical optimization methods [4] have resulted in a remarkable progress in this and other approaches to structural optimization [3,6].

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However, minimum weight design is not always appropriate. Several aspects of the production of cold formed steel shapes make minimum weight design unmanageable. The producer obtains the raw material by selection from a discrete set of sheet steel roll sizes with particular widths and thicknesses. Extra width can be trimmed off - involving some cost and waste, contrary to the minimum weight assumption, while the thickness must remain constant. Furthermore, the unit cost by weight increases as thickness decreases. Therefore, one cannot make the simplifying assumption that minimum cost occurs for minimum weight. Even without these complexities, current minimum weight formulations involve necessarily complicated nonlinear characteristics, both in the objective function and in the constraints, which lead to time-consuming and tedious solution procedures.

Maximum Load Design is an alternative to minimum weight design. Rather than minimizing the cost of a product subject to a specified demand (load), the producer chooses to maximize the market acceptability (load carrying capacity) of the product for a specified raw material of *known* size and cost. One simply maximizes the load carrying capacity of a parent sheet, for the structural conditions specified. If cost is proportional to weight, or even just a monotonically increasing function of weight, the two approaches will lead to the same result. This equivalence is proved in Appendix 1.

Minimum weight and maximum load design are not equivalent in other respects. Maximum load design employs constant volume as a constraint; volume is a convenient geometrical quantity like the state variables of the problem. It is represented by a simple constraint surface in state space. Minimum weight design, in contrast, follow a constant load surface which in the same space represents a more cumbersome constraint and requires generally more computing effort.

Moreover, maximum load design leads to a more conspicuous solution, indicating immediately which is the active factor that limits the load carrying capacity of a member. This indication is of decisive importance. Optimization to a fixed set of rules is often not the most productive approach to a problem. As already shown by Alexander the Great (at Gordia), it may be more fertile to ask: how can the rules of the game (i.e., the code and design standard provisions) be changed such that routine optimization leads to a better optimum? Conventional optimization is not the end, but merely a part of the total optimization process.

The sections which follow describe the rudiments of maximum load design, and its application to two simple problems: single span, unlipped and lipped channel beams under uniform loading.

When the cost relationship to thickness, say, is known (i.e., when a price list for parent sheets is available), the method can be used to determine cost optimal, rather than weight optimal, section geometry.

When cost is proportional to weight, the optimal relationships can be expressed compactly as in Figures 8 to 10. By virtue of a dimensionless representation, the figures throw light on many aspects of structural optimality and behaviour of cold formed steel beams. These aspects are discussed in the concluding section.

2. MAXIMUM LOAD DESIGN

2.1 General Formulation

Maximum load design reflects the viewpoint of a producer, whose task it is to transform a raw material into a specific finished product. The producer tries to make the best possible use of material. That is, for a given level of cost or weight of material, he maximizes the utility of the finished product. In the structural context, utility may be measured by the facility of the product to carry a load. The raw material may often be steel sheet or plate of various strengths. Hence the rationale of maximum load deisgn. From the set of beam profiles which can be produced at equal cost, the optimal is the one which will support the maximum load intensity.

For any particular thickness and width the unit weight of a parent sheet and its progeny is constant. Also, within a class of like shapes it may be reasonable to assume that the unit cost is also constant. For example, for two channel beams formed from the same size of sheet, with two corner bends each, the costs are likely to be the same. This is very close to the truth if the beams are produced in large numbers, possibly as standard sizes for inventory. It is clear that the two beam shapes will differ only in their aspect ratios. And it is equally clear that the optimal aspect ratio is the one for which the maximum load can be carried by the section under the specified loading type, span length, and so on.

For a general formulation, it is necessary to define several terms. An *incident*, j, is any design condition that must be checked. An incident may be a *limit state*, such as a flange yielding or local buckling, or it may be an "arbitrary" code provision dictated by fabrication requirements, etc. A *design function*, D_j, is defined as the variable chosen to represent the behaviour of the structural section for incident j (e.g., calculated compressive bending stress). A *code function*, C_j is a prescribed limit set by the appropriate standard (e.g., allowable stress), which must not be exceeded by the design function D_j. D_j and C_j are defined for j = 1, 2, ..., n incidents. A *satisfactory design* is one which for all incidents meets the following condition:

$$D_j \leq C_j \qquad j = 1, 2, \ldots, n \tag{1}$$

In general, both design function and code function depend on applied load (P), geometry of the structural member (L_i, $i = 1, 2, \ldots, m$), and material properties (Y):

$$D_j = D_j(P, L_i, Y) \tag{2}$$

$$C_j = C_j(P, L_i, Y) \tag{3}$$

The applied load (P) may be a pure load, a load intensity, a load factor or even any monotonic function of the load parameter. The explicit forms of these functions are to be found by pertinent structural analysis and in an appropriate design code, respectively. Specific examples are given in Appendix 3. It must be noted that they do not follow from the optimization analysis.

An *analysis function*, A_j, is defined by

$$A_j = D_j/P \tag{4}$$

Finally, the *incident capacity function*, P_j, is defined as:

$$P_j = C_j/A_j \tag{5}$$

In general, the incident capacity function depends on applied load, geometry and material as do C_j and D_j:

$$P_j = P_j(P, L_i, Y) \tag{6}$$

From equations (1), (4) and (5) a satisfactory design may be redefined as one which has,

$$P \leq P_j \qquad j = 1, 2, \ldots, n \tag{7}$$

The compact statement of maximum load design is therefore,

$$\begin{aligned} &\text{Maximize: } P\,, \\ &\text{Subject to: } P \leq P_j \end{aligned} \tag{8}$$

In many structural problems, the incident capacity is independent of applied load. That is,

$$P_j = P_j(L_i, Y) \tag{9}$$

Such systems are called *linear* and are simple to optimize. The more general systems, equation (6), are called *non-linear* since the load capacity may vary with the applied load. Non-linear capacity functions P_j are often insensitive to a change in applied load P within a limited range. A linear solution may therefore provide a good initial point for the iterative solution of a non-linear system. Figure 1 depicts typical linear (solid lines) and non-linear (dashed lines) characteristics for the respective system types.

For either kind of system, in equation (8), P is limited by each P_j. Thus,

$$P_{max} = \text{Min}\{P_j;\ j = 1, 2, \ldots, n\} \ . \tag{10}$$

that is, the maximum feasible load is equal to the minimum capacity, for specified geometric and material parameters (L_i, Y).

2.2 Selection of Variables

The optimization procedure is facilitated by the use of independent dimensionless parameters.

Figure 2 illustrates a parent sheet of a typical cold formed steel structural member of length L, width a and a thickness t. In the formed member, the span will be L, the thickness t, and the sum of the widths of the component elements will be a. There are two independent dimensionless geometric sheet parameters, for example, r and s given by

$$r = L^2/at \tag{11}$$

$$s = a/t \ . \tag{12}$$

These *generalized slenderness ratios*, related to sheet length and width respectively, play an important part in this study, governing overall and local behaviour of a member, respectively.

The remaining parameters necessary to determine uniquely the geometric properties of a member are those of its cross sectional shape. These are also chosen in dimensionless form and are denoted by:

$$q_k, \; k = 1, 2, \ldots m$$

for m independent dimensionless *shape parameters*. For example, if corner radii are deemed negligible, in the case of unlipped channel sections, there exists only one independent variable (e.g., the aspect ratio of flange width to web depth).

Dimensionless forms of applied load and material strength are easily found and are denoted by a bar over the previous notations. For example, the following are used in the section on applications:

$$\bar{P} = P/F_y L \tag{13}$$

$$\bar{Y} = g_y \tag{14}$$

in which P is load per unit length, F_y is the material yield stress, L is span length and g_y is *characteristic function of the stress*, a convenient dimensionless material property used in the Canadian Standard CSA S136-1974 ($g_y = \sqrt{E/F_y}$). The optimization statement, equation (8), may now be written,

Maximize: P

Subject to the implicit relationship:

$$\bar{P} \leq \bar{P}_{j(r,s,\bar{y})}(q_k, \bar{P}); \; j = 1, 2, \ldots, n; \; k = 1, 2, \ldots, m . \tag{15}$$

In a wide sense, therefore, optimization is the determination of the relationships between the shape parameters q_k and sheet slenderness ratios r and s, which maximize a feasible dimensionless load, $\bar{P}$.

2.3 Special Cases

The simplicity of maximum load design allows great flexibility in solving problems of various types. It all depends on how the producer envisages the role of his product. The simplest problem type is one for which a product is "made to order". Specified are the length, width and thickness of sheet, material properties, type of loading and shape classification (e.g., channel beam). The optimal solution for this problem is found by simply finding the set of shape parameters, q_k, which has the largest governing load capacity $\bar{P}_j$:

$$\underset{q_k}{\text{Max}} \left\{ \underset{j}{\text{Min.}} \left\{ \bar{P}_j (q_k) \right\} \right\} \tag{16}$$

wherein the only variables manipulated are the shape parameters q_k.

A variation of this problem is constant weight optimal design. An example is given in the section on applications. From a set of optimal solutions of the type mentioned above, it is easy to obtain curves of optimal geometry for constant weight (for any specific value of span length L, the cross sectional area is constant for r constant). Weight-optimal design is then accomplished by selecting for constant r the appropriate values of s and hence q_k which maximize the dimensionless load $\bar{P}$ overall.

If unit cost varies with a sheet parameter (e.g., thickness and/or yield point), maximum load design can be employed in stages (such as for different typical thicknesses), to yield the least cost optimum. Contours of optimal geometry for different constant cost levels may then be constructed by employing the known cost relationship. The final step in the optimization procedure is to select the cost-optimal design in the same manner as for weight-optimal design, just described, except by dealing with curves of constant cost rather than curves of constant r.

A further variation is optimization with respect to a slate of span lengths L with corresponding frequencies of demand. Such a slate might approximate the distribution of lengths that a producer expects to encounter. Optimality may then be measured by a weighted average unit load, based on the frequencies for each span length. Weight is easily assessed by summing the lengths (times frequencies) for constant cross sectional area (at). The optimal relationships are then those which maximize this new measure of load for constant cost levels.

If the choice of material is also free to be made (although it is typical to take the material as given) the optimization can be changed to include this as well. It is likely that the unit cost is different for each material strength. From weight optimal design, for example, a section for the required load capacity can be obtained for each material strength value. Then one simply picks the cheapest. An alternative might be to include a factor for each material strength in the weights of the cross sections.

3. APPLICATIONS: COLD FORMED STEEL BEAMS

This section discusses two applications: plain and lipped simply supported channel beams. The former is representative of a linear system, the latter of a nonlinear system, since it involves an effective width calculation for the compression flange. Both beams are subjected to a uniformly distributed load (Figure 3) acting through the shear center.

As explained and defined in the previous section, the sheet slenderness ratios r and s are used. The dimensionless load is taken to be $\bar{P} = P/F_yL$ in which F_y is the material yield stress, P is the load intensity and L is the span length. The only shape parameter used in both applications is the aspect ratio, $q = w/h$ in which w is the flange width and h the depth of section.

Simplifications of the analysis include the (conservative) neglect of the St. Venant torsional resistance for the case of lateral-torsional buckling which is usual, for example, in the AISI specification. And for the lipped channel application, the moment of inertia about the minor (y) axis (Figure 3b) is not reduced in accordance with the effective width of the compression flange. The lip stiffener length is not treated as an optimization variable, since its merit is in the stiffening of the compression flange and not in its own mechanical contribution. This does not affect optimality as explained in Appendix 2. Corner radii, if treated as optimization variables, lead to a solution with large, impractical radii [8]. This study treats corners as perfect right-angles, leaving the actual value to be set by the producer. The corner radius to thickness ratio is expected to have little effect on the optimality of the other shape parameters. The material strength considered is F_y = 50ksi.

Code functions for the applications are obtained from the limit state design provisions of Canadian Standard CSA S136-1974 [7] for cold formed steel structural members. Analysis functions are chosen with respect to each code provision based on accepted procedures of structural analysis. The total deflection limit, which is not prescribed by the standard, is set arbitrarily at $\delta/L = 1/240$ for this study, δ being the limit of midspan deflection and L the span length. This is very conservative, since it is based on a total limit load rather than a service live load level. As will be seen, deflection is not a controlling incident, even for this conservative case. The pertinent code and analysis functions are listed in Appendix 3.

3.1 Optimization of Plain Channels

This application is concerned with the following design incidents: flange yielding and local buckling (j=1), web buckling in bending (j=2), lateral-torsional buckling (j=3), shear yielding and web buckling in shear (j=4), elastic deflection (j=5) and, optionally, web crippling (j=6).

Because the section properties of plain channels are not reduced according to stress level, the incident capacities are readily determined over a wide range for all design parameters.

3.2 Optimization of Lipped Channels

The design incidents for this part of the study are, flange yielding and web buckling in bending (j=1), lateral-torsional buckling (j=2), shear yielding and web buckling in shear (j=3), elastic deflection limited by flange yielding (j=4), and web crippling as an option (j=5). To assess the effective width of the compression flange and hence the effective section (a function of the stress level) the method of successive inverse interpolation [5] is employed.

3.3 Results

The incident relationships between dimensionless load ($\bar{P}$) and aspect ratio (q) for certain fixed values of the sheet parameters (r and s) are given in Figure 4. The dashed line represents the incident locus for web crippling, which may be active in certain ranges if the web of the beam is unreinforced. For plain channels (Figure 4) a design is likely to be governed by local buckling of the compression flange (j=1) or lateral buckling (j=3). For the most part, web crippling (j=6) can be kept inactive by providing the maximum bearing length, as in this study. It may also be noted that the thinner the parent sheet (the larger s), the more sensitive is the load capacity to the aspect ratio (q).

In contrast, for lipped channels (Figure 5) the incident curves often lie close together, depending on the values of the independent parameters. Lateral buckling (j=2) usually governs, but not by a great margin. Web crippling is of great importance in this case, and cannot be ignored in the practical middle range for q and s. However, load capacity ($\bar{P}$) appears to be relatively insensitive to aspect ratio (q). If web crippling can be eliminated, it is noted that the limit of w/t = 60 is active for large values of s, and seems to interrupt a potential increase in the maximum load capacity.

By allowing the aspect ratio (q) to vary, the governing incident capacity (eg., in Figure 4) can be maximized for fixed values of the sheet parameters. The resulting values of $\bar{P}_{max}$ and q_{opt} are optimal, therefore, for prescribed values of r and s. These relationships are given in Figure 6 and Figure 7 for plain and lipped channels respectively. Weight-optimal design may easily be accomplished by the use of this figure, since the contours of constant r represent constant weight curves for a given span length. For fixed cross sectional area (fixed r) optimality is achieved by allowing s to vary and maximizing the new load parameter $\bar{P}_{max}$ to give finally the optimal values P_{opt}, q_{opt}, s_{opt}, r_{opt}. This information is presented concisely in Figure 8 and Figure 9.

Figure 6 permits some observations concerning sensitivity of the optimum and the effects of web crippling. For plain channels without considering web crippling, the maximum point is very insensitive to the value of s. The introduction of the web crippling criterion, while having negligible effect on the value of the maximum, increases its sensitivity somewhat. For lipped channels (Figure 7) the optimum is always more sensitive to parameter values and, notably, the consideration of web crippling produces a remarkable relative drop in the optimal load capacity. The compact form of Figure 8 and Figure 9 does not illustrate these factors, giving simply the final optimal relationship. The effect is apparent, however in Figures 6 and 7.

From a comparison of Figures 4 and 6 (or 5 and 7) it is clear that the aspect ratio q requires more precise determination than the sheet parameter s, to which the optimal load capacity is relatively insensitive. Hence, the difference in uncertainty intervals in Figures 8 and 9: 20 for s in a range of 60 to 320, and .05 for q in a range of about 0 to 1.0. This precision is likely to be satisfactory for most purposes. For smoother graphs (such as Figures 10 and 11) the intervals may be decreased to about 2 for S and .01 for q, at a cost of about double the computation time.

The optimal design charts (Figures 8 to 10) can easily be interpreted as follows: for a specified load, span length and material strength, the parameter $\bar{P}$ is calculated. Proceeding with this value horizontally to the left and right, the appropriate curves are intersected and, proceeding down to the horizontal axis q_{opt} and r_{opt} are obtained. The value of s_{opt} is obtained in a similar way, completely defining the optimal cross section for the prescribed load, span and material. It should be noted that the charts given are valid only for a steel with yield strength $F_y = 50$ ksi. Appendix 4 illustrates numerically how a producer might use the optimal design charts.

Fluctuation in the optimal values of q, s, and r result from shifts in dominance of local maxima with respect to $\bar{P}$ (Figure 5). This is accentuated by the dependence of the optimum upon a combination of two parameters, q and s, rather than a single parameter. The practical implications of setting a constant or average value for one or more of the parameters is discussed in the next section.

3.4 Practical Implications

Engineering does not necessarily demand rigorous optimality, particularly when the real problem may not be as precise as the mathematical model which can be solved. It may be a practical objective that the optimal s or q should not depend on the specified load. Figure 8 and Figure 9 show one example (dashed lines) of an alternative to the strictly optimal curve. For plain channels the value of s is fixed at 120, and for lipped channels 200. If web crippling is included in the optimization, s could be set at about 120 for both lipped and unlipped channels. The resulting deviations from the true optimum ($\bar{P}_{opt}$ curve) does not appear to be excessive.

Similarly, the aspect ratio, q, might be set at a constant value (about 0.4 for lipped channels and 0.25 for unlipped channels) which is independent of the specified load intensity and span length. This would seem to be of great practical significance, since the producer does not often have exact knowledge of the demanded structural configuration, and since it is efficient to produce several thicknesses of the same beam profile.

Of course, other manipulations are also possible. For example, it may be preferable to consider discrete thicknesses, since this is the one parameter over which the cold formed steel producer has not control. Another feature might be to consider groups of thicknesses with a common cross-sectional shape, since it is economical to run several thicknesses through the same operation of roll-forming.

The results seem to imply that despite its rigorous approach, "stereotype" optimization (in other words, point optimization) may be inadequate from a practical standpoint. Any optimization procedure applied to cold formed steel products should allow considerable interpretation of the results, and manipulation in order to reflect reality. These objectives are met by maximum load design. "Range Optimization", since it allows for parameter insensitivity and the possibility of local maxima (which may be close to the point optimum and much more stable) may offer a better approach.

Finally, it is noted from the applications that the optimal dimensionless load (for fixed r) increases significantly when a lipped channel is used. The additional strength thus provided is mostly lost, however, unless web crippling is removed from the design procedure. The following section discusses this very important aspect of the optimization.

3.5 Effect of Web Crippling

From Figure 6 and Figure 7, it is apparent that web crippling as a design incident has little effect on the optimal profile of plain channel beams, but a great effect on that of lipped channel beams. In fact, less than half the benefit occurs with the addition of a lip as would occur if web reinforcement were added also. The increase in load capacity from unlipped to lipped, and to lipped plus web-reinforced beams is in the order of 20% for the first change, and an additional 40% for the second.

Web crippling, then, has been identified as a very important active constraint uncovered in the optimization analysis. Especially for small bearing lengths (the maximum was assumed in this example) and small values of the sheet parameter $r(=L^2/at)$ the likelihood of dominance of the design by web crippling is high. Furthermore, as noted before, the benefits of flange stiffening can be almost completely lost in such cases, even if a rigorous optimization takes place. Finally, it is probably undesirable from the viewpoint of efficient design that such a local phenomenon governs the optimal design of a global system. Although web stiffening may involve some expense in material and construction costs, it is clear from the optimal design charts that a considerable benefit results.

3.6 Comparison with Available Products

For comparison, the products of a Canadian manufacturer were examined to see how much deviation from the optimal geometry occur. The producer practically ignored plain channels, providing standard profiles of lipped channels from 6 × 2-3/4 to 18 × 3 1/2 (depth × width in inches). Groups of sizes were arranged so that only the thickness was a variable in many cases. This indicated that the aspect ratio (q is constant whenever thickness is the only changing parameter) should be of prime concern for optimization. However, the value of q ranged from 0.19 for the large sections to 0.46 for the small ones. This appeared to be too great a range, according to this study, although the average (0.31) was not blatantly non-optimal.

Similarly, the parameter r showed much too great a variability and the value of s, although not as variable, was often far from optimal. To achieve conclusive results, however, a much more detailed comparison would have to be made.

3.7 Continuous Beams

Two and three equal-span continuous beams were optimized as for single span beams. The extra design incidents included were web crippling at an interior support, and combined bending and shear stresses in the web. The results are largely consistent with those for the simple case. Figures 10 and 11 show the effect of the number of spans on optional load intensity and geometry for plain and lipped channels respectively. The optimal load intensities are somewhat larger or smaller for the above cases respectively, whereas the optimal section parameters are not greatly changed. The same trends were observed in general. It may be that optimal section geometry is not significantly a function of the number of spans.

4. CONCLUSIONS

The rationale of maximum load design has been presented as an alternative to that of conventional minimum weight design. A simple and efficient optimization procedure is possible in this approach, which features (i) a simple concept of structural optimization in terms of design, (ii) isolation of structural analysis and code provisions from the optimization algorithm, (iii) ease of manipulation and interpretation of results in a very broad sense and (iv) dimensionless formulation.

The procedure has been applied to simple cold formed steel channel-shaped beams, with and without lip stiffeners. Although obtained by virtue of maximum load design, the results are the equivalent of weight-optimal design.

An optimal shape is a function of both load intensity and span length from the view of strict point optimization. However, when some allowance is made with regard to the maximum load, the optimal shape may be determined without respect for load intensity or span length. This seems of great practical importance. The optimal lipped channel has a stiffener depth prescribed by the code minimum.

The most important possible active constraint has been found to be web crippling. Significant gains in strength by the stiffening of the flange are seen to be lost unless web reinforcement is added.

Common manufactured products do not appear to be far from optimal, on the average, but exhibit too great a variation in geometric parameters, according to the results of this particular study.

A study of continuous equal-span beams indicates that the number of spans is not likely to be a significant factor in the optimization procedure. It may be sufficient to optimize a set of structural beam shapes for a single simply-supported span, insofar as relative section geometry is concerned. If support costs are neglected, in some cases several simple spans are preferable to one continuous multiple span.

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APPENDIX 1

EQUIVALENCE OF MINIMUM COST (WEIGHT) AND MAXIMUM LOAD DESIGNS

For the two basic approaches to optimal design to be equivalent, the following conditions must be met concerning load capacity, P, and cost, C:

(i) $\frac{\partial P}{\partial C} > 0$. That is, an extra expenditure of money can always be used to increase load capacity, and

(ii) C is a single-valued and piecewise differentiable continuous function in P.

Figure 12 illustrates (in three dimensions for simplicity only) a section of the structural configuration space:

$$P = P(C, X_j), \text{ or alternatively,} \quad (a)$$

$$C = C(P, X_j) \quad (b)$$

wherein X_j represents the geometrical parameter(s) for which the optimizer seeks to identify the optimal values. Part (a) of the figure indicates the maximization of load capacity and part (b) minimization of cost. The equilibrium point A is the same for both cases unde the above conditions.

The proof is by contradiction. We show that if the cost were not minimum the load-carrying capacity could not be maximum. Assume, for the sake of argument, that the cost has not the smallest possible value for a given maximum load capacity. We reduce this cost, then, keeping the load-carrying capacity constant.

By taking the money just obtained, and expending it appropriately we can increase the load-carrying capacity (recall $\partial P/\partial C > 0$) to a new value. The system is returned to the original cost, but has a higher load-carrying capacity.

The original assumption of a maximum load-carrying capacity is violated. Therefore, it is concluded that the state of minimum cost must exist for the state of maximum load capacity.

More should be said regarding conditions (i) and (ii) which are necessary for equivalence. They are not in themselves sufficient to define a manageable optimization problem. To obtain the optimal point A, particularly in minimum weight design, the Kuhn-Tucker conditions are used [1] as follows:
For a Lagrangian function L(Q,V) formed from a maximization problem with n variables and m constraints, the Kuhn-Tucker conditions are,

(I) $\partial L(Q,V)/\partial Q_j \leq 0$ $(j=1,2,\ldots,n)$

(II) $Q_j \partial L(Q,V)/\partial Q_j = 0$ $(j=1,2,\ldots,n)$

(III) $\partial L(Q,V)/\partial V_i \geq 0$ $(i=1,2,\ldots,m)$

(IV) $V_i \partial L(Q,V)/\partial V_i = 0$ $(i=1,2,\ldots,m)$

$Q_j \geq 0, \quad V_i \geq 0$

In words, the Kuhn-Tucker conditions may be interpreted as: "The negative gradient of the objective function must lie within the cone spanned by the gradients of the active constraints".

In order for the Kuhn-Tucker conditions to be both necessary and sufficient to define the optimal point, the following further conditions must be met:

(iii) The set of constraints must define a feasible region which is everywhere convex. That is, the function $P(C,X_j)$ or $C(P,X_j)$ must be convex.

(iv) The original objective function must be concave (convex) in the neighbourhood of the maximum (minimum) point.

If these conditions are met, the extreme point will be a global maximum (minimum). Maximum load design, as presented in this paper, takes on a simple form, such as to make these conditions trivial.

APPENDIX 2

OPTIMALITY OF THE LIP STIFFENER

With respect to every design incident, certain section properties can be identified for which the maximization thereof accomplishes maximum load design. Thus, for the case of flange yielding, the optimal geometry is that which maximizes section modulus, and so on. This appendix is intended to show that the lip stiffener will never be increased beyond the minimum depth required by the appropriate standard. In this way, the lip dimension is removed from the optimization analysis.

An optimal shape, for a given sheet of width a, thickness t and length L, will have an optimal web height h, flange width w and lip depth d. Let us suppose, since the optimal depth can be achieved either at the expense of the flange or of the web, that we achieve the maximum in two steps: (i) the sheet is bent at the optimal depth h*, leaving all remaining material in the flanges and (ii) a portion of each flange is bent to form the optimal lip depth d* and resulting flange width w* (Figure 13).

It is now shown that for every incident it is either meaningless or detrimental (in terms of maximizing load capacity) to perform step (ii):

j = 1: Flange yielding and web buckling in bending.

The first criterion involves the section modulus, S_x. It is apparent that this property can only be reduced by performing step (ii).

The second criterion involves the section modulus divided by the square of the height, S_x/h^2 (t is a constant for a given sheet). Step (ii) reduces S_x while maintaining h* as a constant. Hence, no benefit from such a move would result.

j = 2: Lateral-torsional buckling

This design incident is related to the parameter hI_{yc}, that is the height times the moment of inertia of the compression portion of the cross section. Where no flange width reduction is made, I_{yc} is equal to one-half the moment of inertia of the full section about the minor axis ($I_y/2$). Again, it is apparent that performing step (ii) can only reduce I_y, while maintaining h* constant. Even if the flange width were greater than the effective width, the material would be removed from the centre of the flange, so that the assumption $I_{yc} = I_y/2$ would remain nearly exact. It seems that there is again no reason to perform step (ii).

j = 3: Shear yielding and web buckling in shear.

The only geometric property pertinent to this incident is the web depth, h. Since h* is set by step (i) it is pointless to carry out step (ii).

j = 4: Elastic deflection.

The relevant property for this incident is the moment of inertia about the major axis, I_x. It is clear that step (ii) reduces I_x. In fact it is clear that even step (i) does so as well.

j = 5: Web crippling.

The pertinent variables for this incident are entirely independent of lip depth. The important one, h*, can be set in step (i), making step (ii) immaterial.

Conclusion: Because it is never clearly beneficial to provide any stiffener depth at all, the optimal depth will be exactly the minimum called for in the appropriate standard. The benefit of the lip is only in the stiffening of the compression flange and is not otherwise due to the presence of the lip stiffener itself.

APPENDIX 3

ANALYSIS AND CODE FUNCTIONS FOR THE APPLICATIONS

The following analysis and code functions were found in terms of dimensionless ratios, based on a parent sheet of width a, thickness t and length L. The formed cross section is comprised of a flange width w, web height h and in the case of lipped channels, lip depth d.

(i) Analysis Functions

a) For plain channels,

Incidents j=1,2,3: a dimensionless bending stress for flange yielding, web bending and lateral buckling respectively,

$$\bar{A}_1 = \bar{A}_2 = \bar{A}_3 = \frac{3}{4}\frac{L}{a}\frac{a}{t}\left(\frac{L}{h}\right)^2 \frac{1}{1 + 6(^w/h)}$$

j=4: a dimensionless shear stress parameter,

$$\bar{A}_4 = \frac{1}{2}\frac{L}{a}\frac{a}{t}\frac{L}{h}$$

j=5: for elastic deflection,

$$\bar{A}_5 = \frac{5}{35}\frac{L}{a}\frac{a}{t}\left(\frac{L}{h}\right)^3 \frac{1}{1 + 6(^w/h)}\frac{1}{\varepsilon_y{}^2}$$

j=6: for web crippling,

$$\bar{A}_6 = \frac{1}{2}\frac{L^2}{at}\frac{a}{t}$$

To modify the functions for continuous beams, the following multipliers were determined for two and three equal spans respectively:

$\bar{A}_1$: 1. , 0.936

$\bar{A}_2$: 1. , 0.936

$\bar{A}_3$: 0.76 , 0.808

$\bar{A}_4$: 1.25 , 1.234

$\bar{A}_5$: 0.74 , 0.82

$\bar{A}_6$: 0.876, 0.9

Two additional incidents considered for continuous beams are,

j=7: for web crippling at an interior support,

$$\bar{A}_7 = 2.5\ \bar{A}_6 \text{ (2-span)},\quad 2.4\ \bar{A}_6 \text{ (3-span)}$$

j=8: for combined web bending and shear,

$$\bar{A}_8 = 1. \text{ (2 and 3-span)}$$

b) For lipped channels the corresponding expressions are,

$$\bar{A}_1 = \bar{A}_2 = \frac{1}{8}\frac{h}{t}\ \alpha \qquad \text{for flange yielding and lateral buckling}$$

$$\bar{A}_3 = \frac{1}{2}\frac{L}{h}\frac{L}{a}\frac{a}{t} \qquad \text{for shear}$$

$$\bar{A}_4 = \frac{5}{384}\frac{L}{h}\frac{h}{t}\gamma / g_y^2 \qquad \text{for deflection}$$

and $\bar{A}_5 = A_6$ in a) for web crippling

and for continuous beams,

$$\bar{A}_6 = \bar{A}_1 \text{ in a)}$$

$$\bar{A}_7 = 1.$$

The multipliers for 2 and 3 span cases are as in a) except that $\bar{A}_1$ and $\bar{A}_2$ are condensed into one multiplier.

In the above equations, g_y is the dimensionless material characteristic,

$$g_y \equiv \sqrt{E/F_y} \text{ in which E is Young's modulus,}$$

and

$$\frac{1}{\alpha} = \left[\frac{2}{3}\left(\frac{d}{L}\right)^3 + \frac{1}{3}\left(\frac{h}{L}\right)^3 + \left(\frac{w}{L}\right)\left(\frac{h}{L}\right)^2 + \frac{d}{L}\frac{h}{L}\left(\frac{h}{L} - \frac{d}{L}\right)\right] \frac{1 + \frac{b}{w}\frac{w}{h} + \frac{w}{h} + 2\frac{d}{h}}{\frac{1}{2} + \frac{w}{h} + \frac{d}{h}} - \left(\frac{h}{L}\right)^2\left(\frac{1}{2}\frac{h}{L} + \frac{w}{L} + \frac{d}{L}\right)$$

$$\frac{1}{\gamma} = \frac{\left[\frac{2}{3}\left(\frac{d}{L}\right)^3 + \frac{1}{3}\left(\frac{h}{L}\right)^3 + \left(\frac{w}{L}\right)\left(\frac{h}{L}\right)^2 + \frac{d}{L}\frac{h}{L}\left(\frac{h}{L} - \frac{d}{L}\right)\right]\left(1 + \frac{b}{w}\frac{w}{h} + \frac{w}{h} + 2\frac{d}{h}\right) - \left(\frac{h}{L}\right)^3\left(\frac{1}{2} + \frac{w}{h} + \frac{d}{h}\right)^2}{1 + \frac{b}{w}\frac{w}{h} + \frac{w}{h} + 2\frac{d}{h}}$$

in which b = effective width of compression flange.

(ii) Code Functions

a) The respective code functions for plain channels are given by,

$$\bar{C}_1 = q\,\frac{F}{F_y}$$

$$\bar{C}_2 = f_w$$

$$\bar{C}_3 = k$$

$$\bar{C}_4 = f_v$$

$$\bar{C}_5 = \delta/L$$

$$\bar{C}_6 = (P_{max})_{CSA}/t^2F_y = f(\nu,\ H,\ n,\ K)$$

and for continuous beams,

$$\bar{C}_7 = (P_{max})_{CSA}/t^2F_y = f(\nu,\ H,\ n,\ K)$$

$$\bar{C}_8 = [(\bar{A}_2/\bar{C}_2)^2 + (\bar{A}_4/\bar{C}_4)^2]^{-\frac{1}{2}}$$

b) and correspondingly for lipped channels,

$$\bar{C}_1 = f_w$$

$$\bar{C}_2 = k$$

$$\bar{C}_3 = f_v$$

$$\bar{C}_4 = \delta/L$$

$$\bar{C}_5 = \bar{C}_6 \text{ in a)}$$

and for continuous beams,

$$\bar{C}_6 = (P_{max})_{CSA}/t^2F_y$$

$$\bar{C}_7 = [(\bar{A}_1/\bar{C}_1)^2 + (\bar{A}_3/\bar{C}_3)^2]^{-1/2}$$

in which

$q = 1.0$	for	$W \le 0.37\ g_y$
$1.37(1 - 0.725\ W/g_y)$		$0.37\ g_y < W \le 0.84\ g_y$
$0.378\ g_y^2/W^2$		$0.84\ g_y < W \le 25$
$g_y^2(1 - 0.015\ W)/1000$		$25 < W \le 60$

$$f_w = \mathrm{Min}(F/F_Y\ ,\ \phi_a 21.7\ g_y{}^2/H^2)$$

$$f_v = \mathrm{Min}(F_s/F_Y\ ,\ \phi_a 1.5\ g_y/H\ ,\ \phi_a 4.81\ g_y{}^2/H^2)$$

$$k = \begin{cases} F/F_Y - 0.25\ F_Y/F_{be} \cdot (F/F_Y\ )^2 & F_Y/F \cdot F_{be}/F_Y \geq 0.5 \\ F_{be}/F_Y & F_Y/F \cdot F_{be}/F_Y < 0.5 \end{cases}$$

In the above $(P_{max})_{CSA}$ is the limit reaction prescribed by the CSA code, ν is the ratio of length of bearing to web depth, H is the width to thickness ratio of the web, $K = F_y(\mathrm{ksi})/33$, and n is the ratio of inside bend radius to web thickness. The explicit form of the function f is given in the CSA standard. ϕ_a is the performance factor (0.9) for tension, compression and shear, F is the basic normal design stress ($= \phi_a F_Y$), F_s is the basic shear design stress ($= 0.577\ \phi_a F_Y$), W is the flat width ratio of the flange ($= {}^w/t$), and

$$F_{be}/F_Y = \phi_c\ \pi^2\ g_y{}^2 \left(\frac{w}{L}\right)^2 \left(\frac{w}{h}\right) \frac{2 + {}^w/h}{(1 + 2\ {}^w/h)(1 + 6\ {}^w/h)}$$

for plain channels, and

$$F_{be}/F_Y = \phi_c\ \pi^2\ g_y{}^2 \left(\frac{w}{L}\right)^2 \left(\frac{h}{L}\right)^3\ \alpha \cdot \beta$$

for lipped channels, in which ϕ_c is the performance factor (= 0.75) for lateral buckling, and

$$\beta = \frac{\left(\frac{1}{3}\frac{w}{h} + \frac{d}{h}\right)\ \left(\frac{1}{2} + \frac{w}{h} + \frac{d}{h}\right)\ -\ \left(\frac{1}{2}\frac{w}{h} + \frac{d}{h}\right)^2}{\frac{1}{2} + \frac{w}{h} + \frac{d}{h}}$$

APPENDIX 4

OPTIMAL DESIGN EXAMPLE

To illustrate the use of the optimal design charts for specific designs, consider the following example in conjunction with Figure 14:

A producer has a large order for a single span simply supported stiffened channel beam. At a span length of 100 in., the beam must support a uniform load of 100 lb/in, and the producer intends to use cold formed steel having F_y = 50,000 psi. As a guideline for production, the optimal cross-section is found from Figure 14.

$\overline{P}$ is evaluated:

$$\overline{P} = P/F_y L = 100/50,000(100) = 20 \times 10^{-6} \quad . \tag{1}$$

From the chart the optimal values of sheet slenderness, section slenderness and aspect ratios are read:

$$r = L^2/at = (100)^2/at = 6500 \ , \tag{2}$$

$$s = a/t = 153 \ , \tag{3}$$

$$q = w/h = 0.37 \ . \tag{4}$$

Solving (2) and (3) simultaneously:

a = 15.3 in. sheet width required

t = 0.100 in. sheet thickness.

Equation (4) must be solved in conjunction with the minimum lip stiffener requirement (from CSA S136):

$$d/t = (24\ w/t - 156)^{1/2} \nless 4.8 \tag{5}$$

and with the additional compatibility requirement:

$$2d + 2w + h = a = 15.3 \text{ in.} \tag{6}$$

Since t has already been determined, equations (4), (5) and (6) may be solved simultaneously. This is somewhat laborious by hand, so a design aid is provided in the q - s quadrant of Figure 14. It is important to note that these contours of d/t do not change from chart to chart. d/t is the same function of q and s for all cases. The contours are drawn in Figure 14 only for compactness.

Using this aid, for $q = 0.37$ and $s = 153$:

$$d/t = 8.2 \ . \qquad (7)$$

Equations (4), (6) and (7) are now easily solved, knowing $t = 0.100$ in.:

$d = (d/t)t = 8.2(0.100) = 0.82$ in.

$w = qh = 0.37h$

$2d + 2w + h = 2(0.82) + 2(0.37h) + h = 15.3$ in.

$h = 7.86$ in.

$w = 0.37(7.86) = 2.90$ in.

Therefore, the optimal beam has flange width 2.90 in., web height 7.86 in., lip depth 0.82 in. and thickness 0.100 in.

APPENDIX 5

REFERENCES

1. BAUMOL, W. J., Economic Theory and Operations Analysis, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1972.

2. DOUTY, R. T. and CROOKER, J. O., "Optimization of Long-Span Cold-Formed Truss Purlins", Journal of Structural Division, ASCE, Vol. 100, No. ST11, November, 1974.

3. GALLAGHER, R. H. and ZIENKIEWICZ, O. C. (Eds.), Optimum Structural Design, Theory and Applications, John Wiley & Sons, Inc. London, 1973.

4. GOTTFRIED, B. S. and WEISMAN, J., Introduction to Optimization Theory, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1973.

5. KELLISON, S. G., Fundamentals of Numerical Analysis, Richard D. Irwin, Inc., Homewood, Illinois, 1975.

6. MAJID, K. I., Optimum Design of Structures, Butterworth & Co., Ltd., London, 1974.

7. SCHUSTER, R. M. (Ed.), Cold Formed Steel Design Manual, Part I, CSA Standard S136-1974, University of Waterloo Press, Waterloo, Ontario, Canada, 1975.

8. SEABURG, P. A. and SALMON, C. G., "Minimum Weight Design of Light Gage Steel Members", Journal of Structural Division, ASCE, Vol. 97, No. ST1, January, 1971.

APPENDIX 6

NOTATION

The following symbols are used in this paper:

A_j	=	analysis function for jth state
a	=	width of steel sheet
b	=	effective width of compression flange
C_j	=	code function for jth state
D_j	=	design function for jth state.
d	=	lip depth
$\bar{\varepsilon}_y$	=	dimensionless property of steel
h	=	web depth
L_i	=	geometrical parameter
L	=	length of steel sheet = span of beam
P_j	=	incident capacity function for jth state
p	=	load
t	=	thickness of steel sheet
w	=	width of flange
s	=	section slenderness = a/t
r	=	member slenderness = L^2/at
q_k	=	shape parameter such as w/h
$(\bar{\ })$	=	symbol for dimensionless property

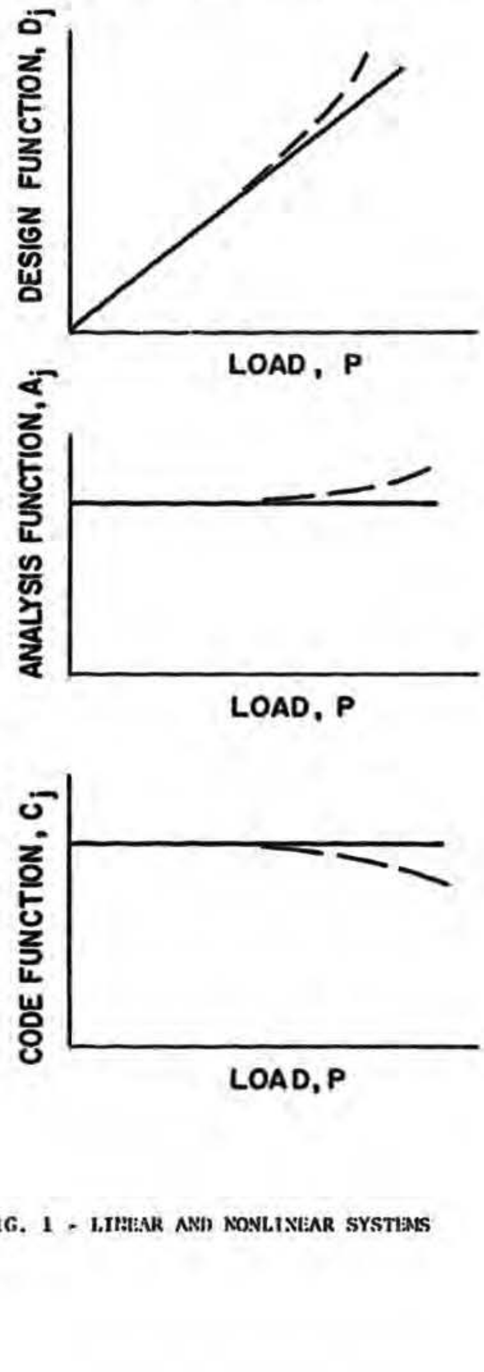


FIG. 1 - LINEAR AND NONLINEAR SYSTEMS

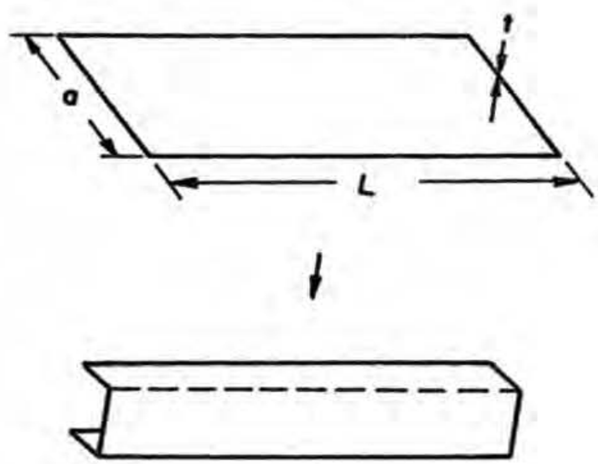
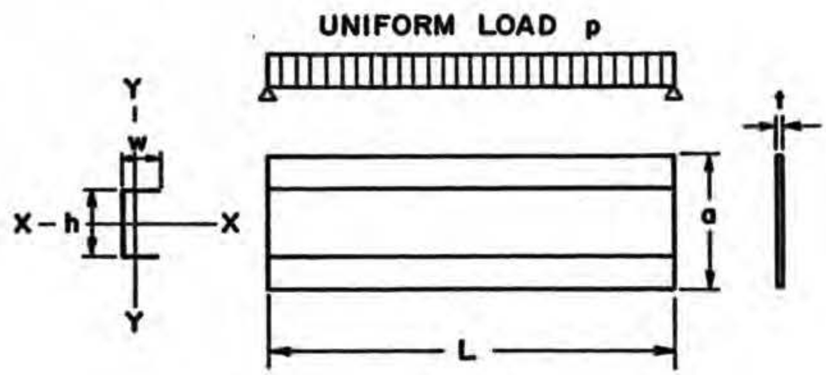
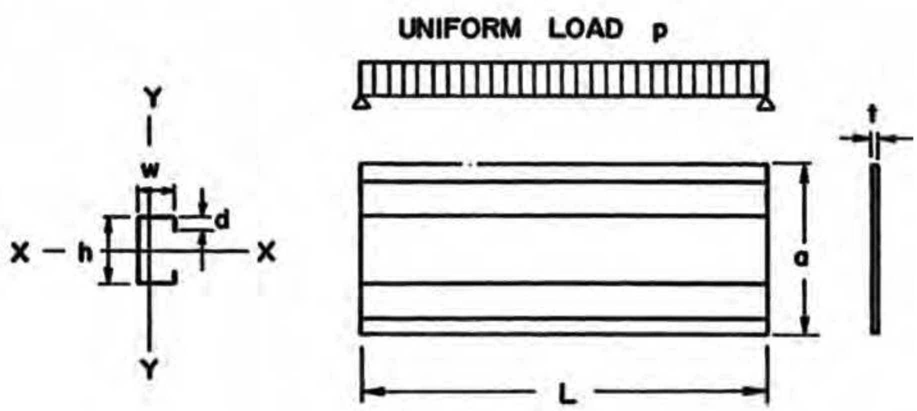


FIG. 2 - PARENT SHEET



(a) Plain Channel Beam



(b) Lipped Channel Beam

FIG. 3 - APPLICATIONS

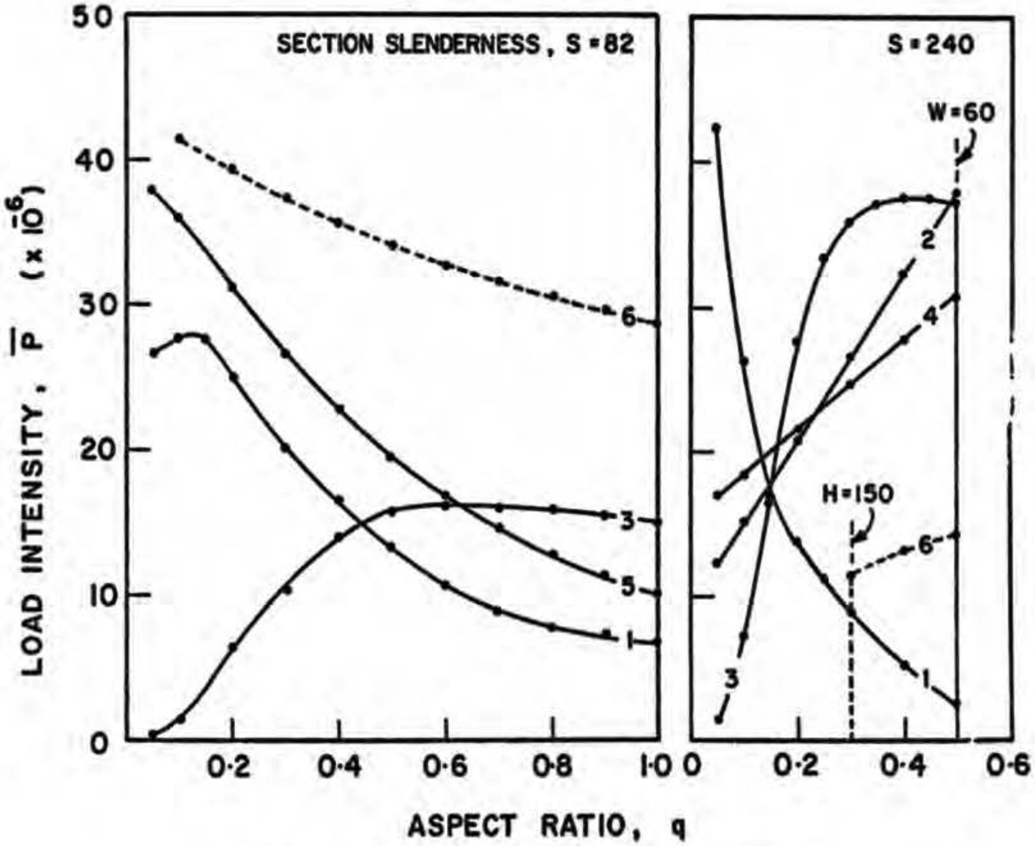
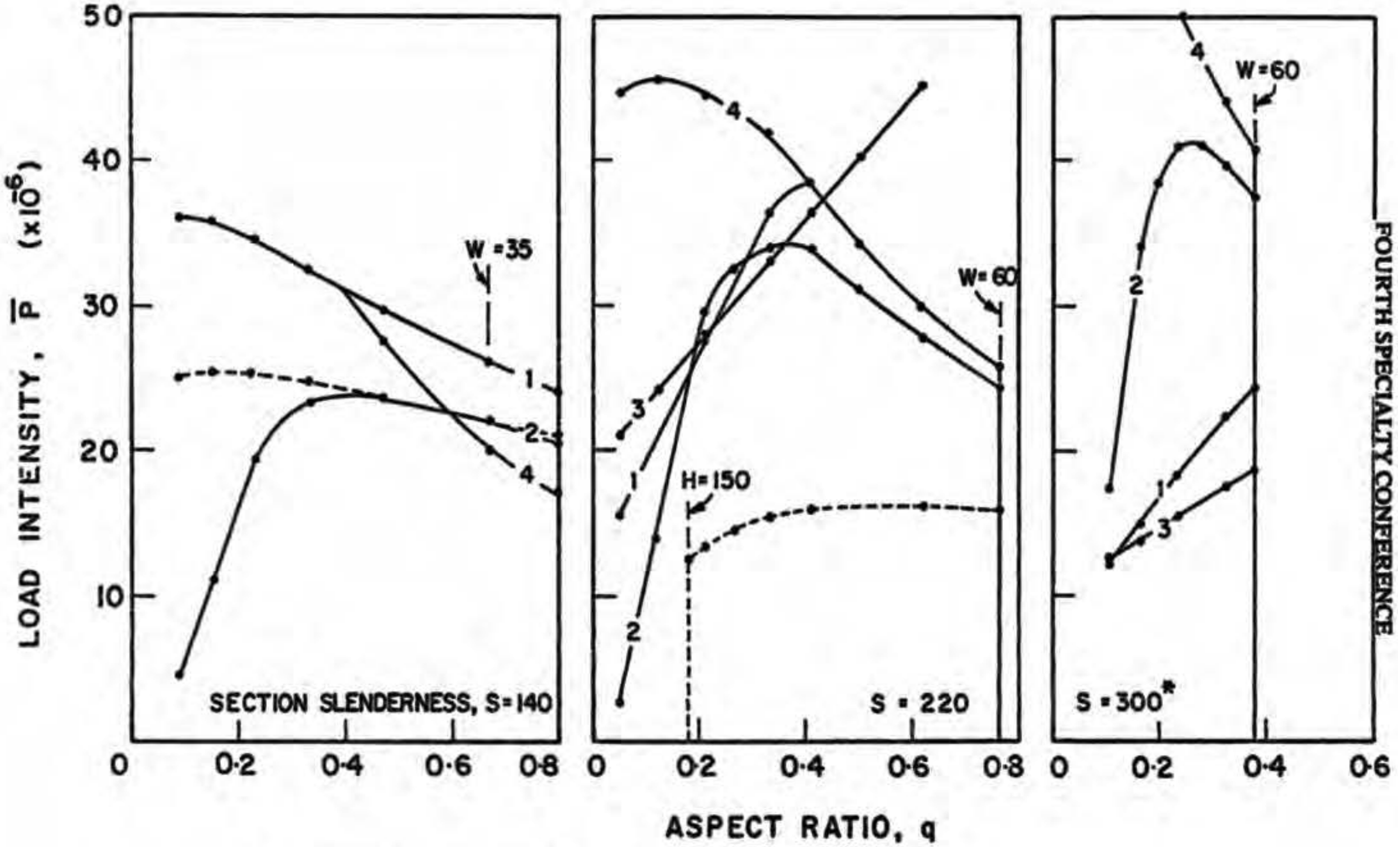


FIG. 4 - PLAIN CHANNELS. MEMBER SLENDERNESS, r = 5750.

(Dashed lines - Web Crippling considered, Maximum Bearing Length)



* No feasible domain exists when web crippling is included, for S >270.

FIG. 5 - LIPPED CHANNELS, MEMBER SLENDERNESS, r = 5750. (Dashed lines - Web Crippling Considered)

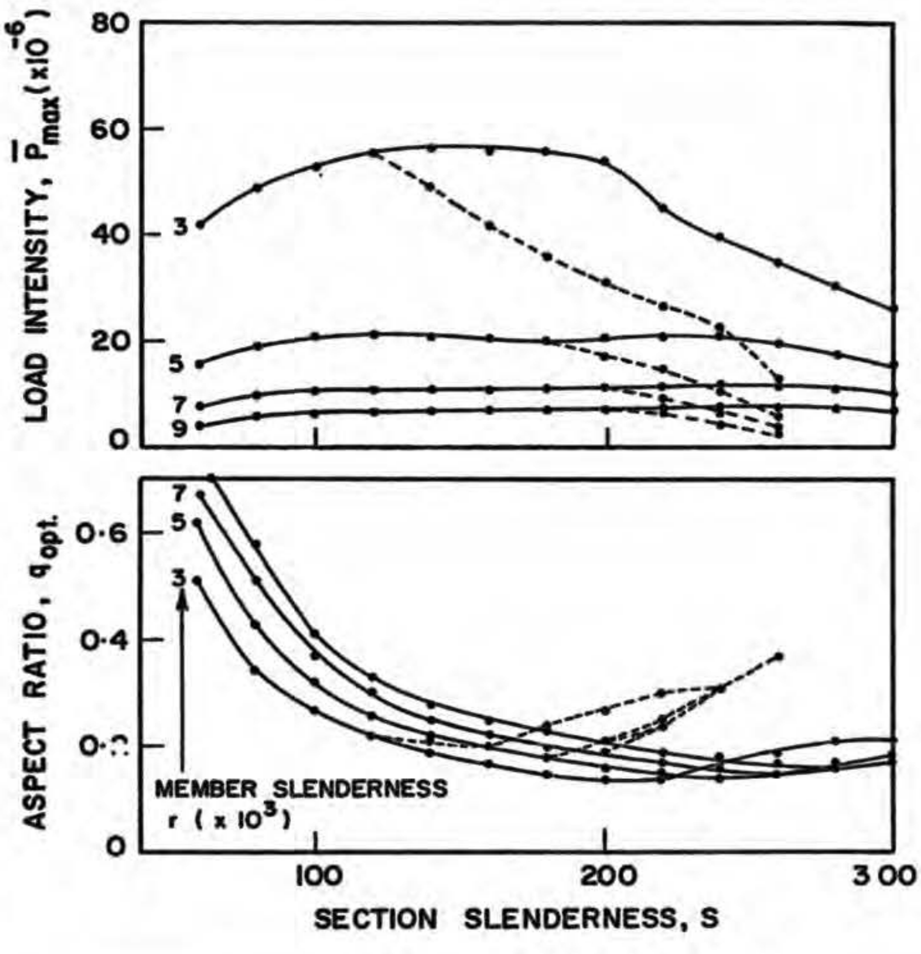


FIG. 6 - PLAIN CHANNELS (Dashed Lines - Web Crippling Considered)

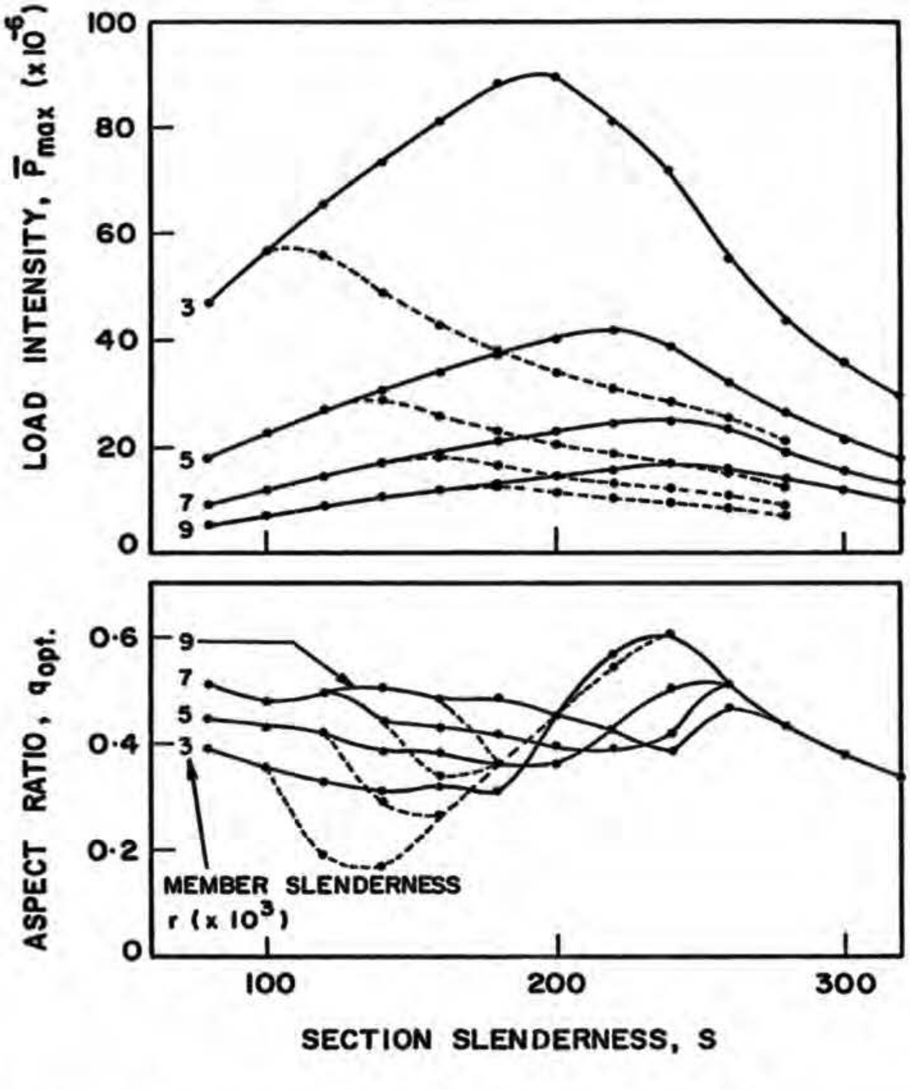


FIG. 7 - LIPPED CHANNELS (Dashed Lines - Web Crippling Considered)

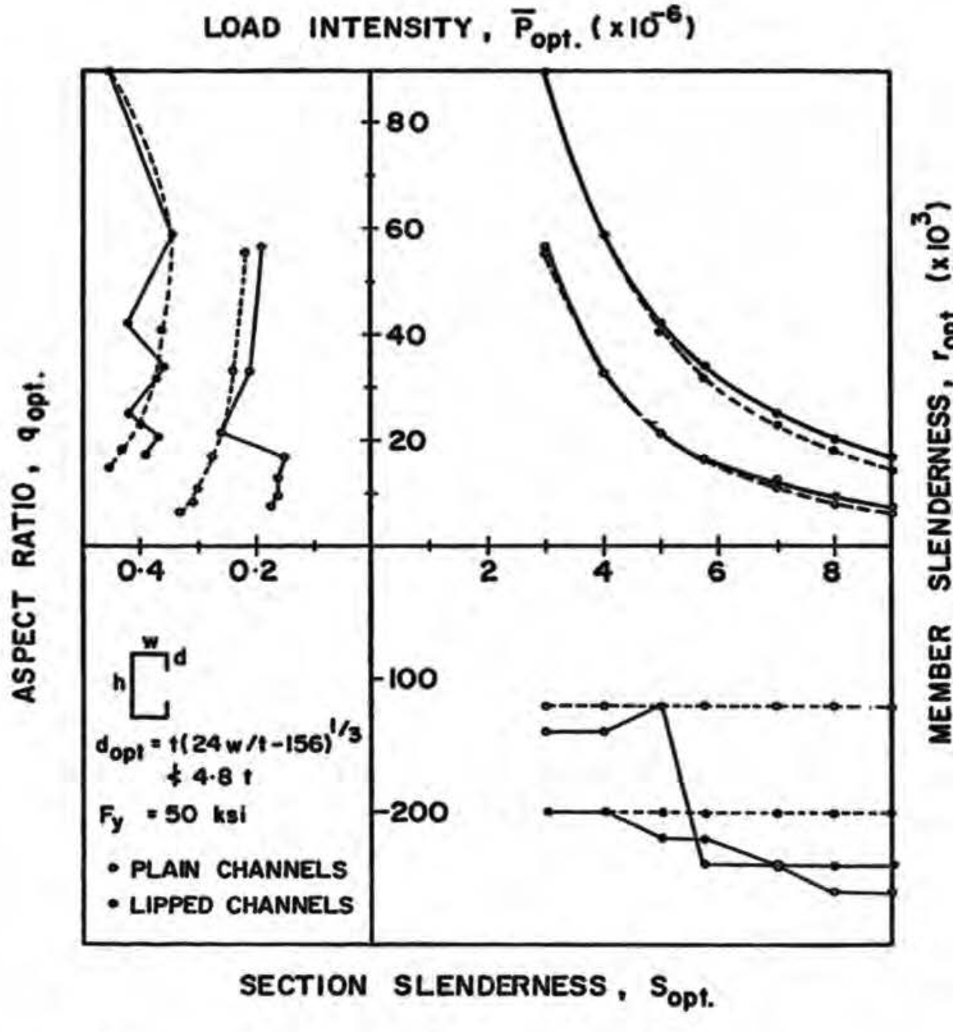


FIG. 8 - OPTIMAL DESIGN CHART - WEB CRIPPLING NOT CONSIDERED

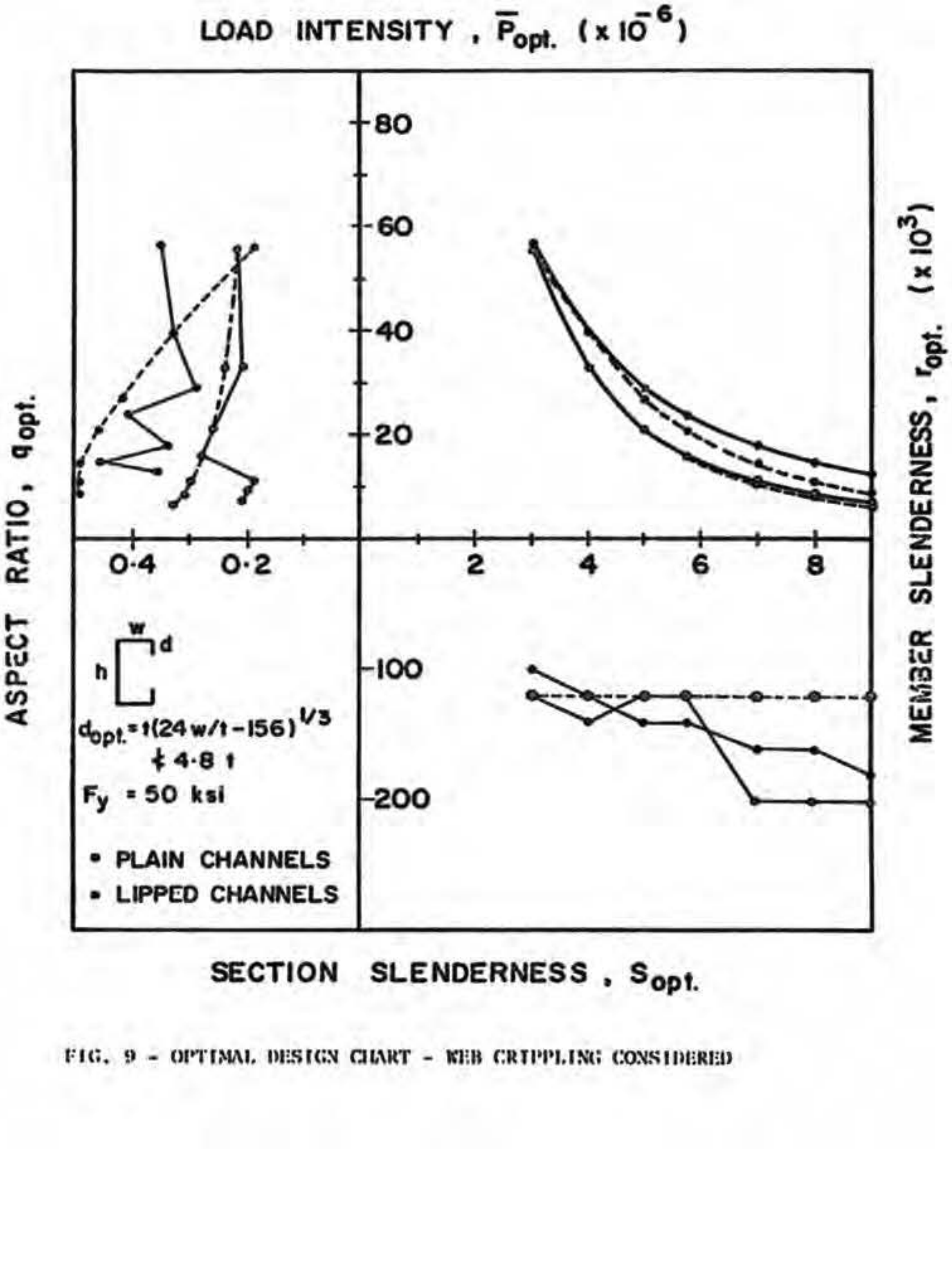


FIG. 9 - OPTIMAL DESIGN CHART - WEB CRIPPLING CONSIDERED

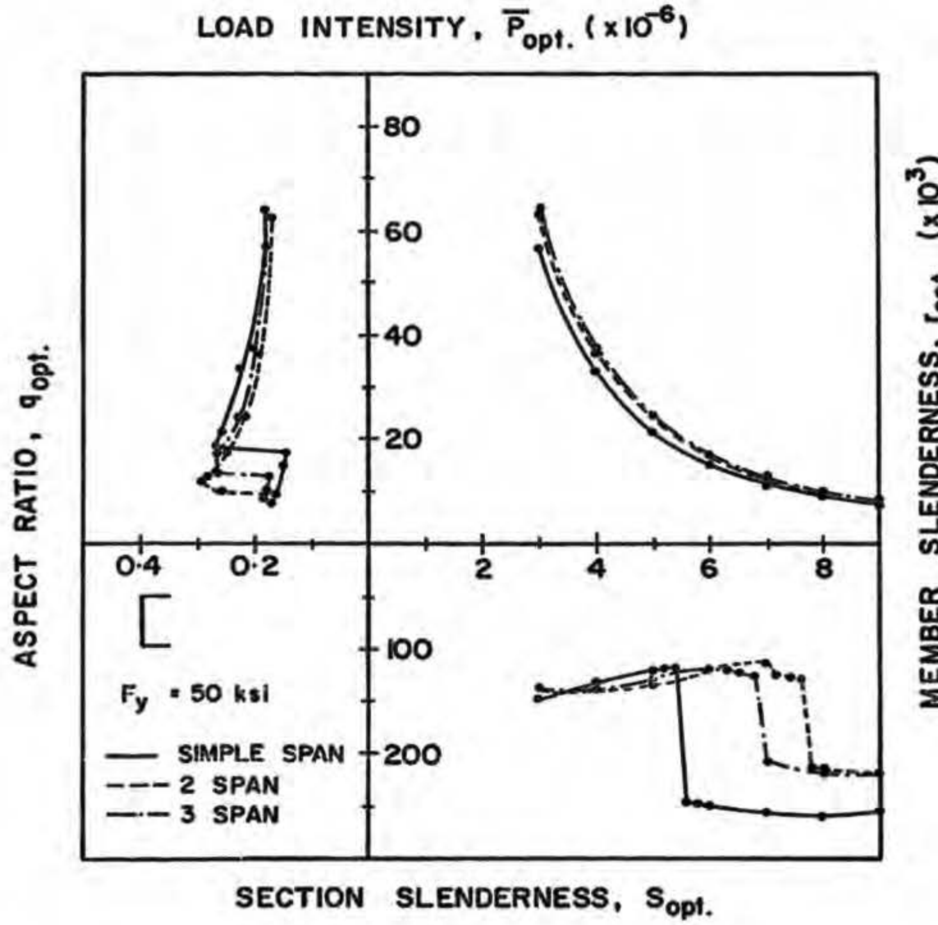


FIG. 10 EQUAL SPAN CONTINUOUS BEAMS
(FOR PLAIN CHANNELS. WEB CRIPPLING NOT CONSIDERED)

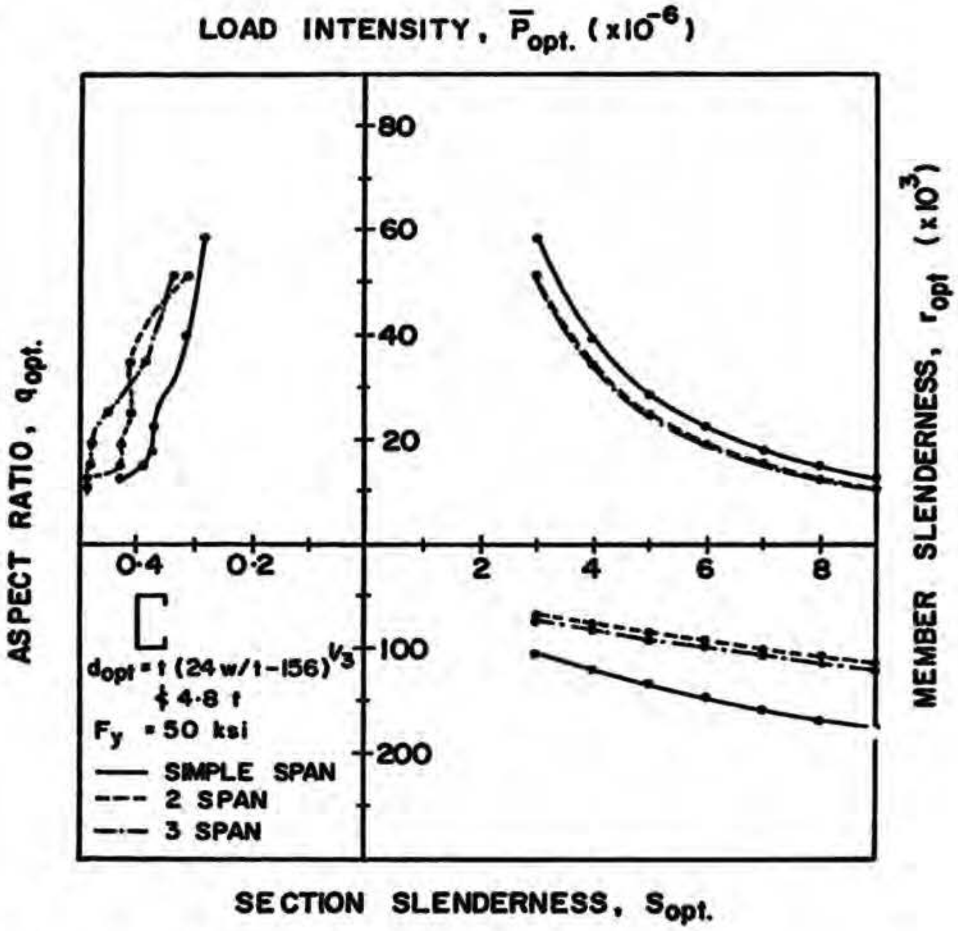
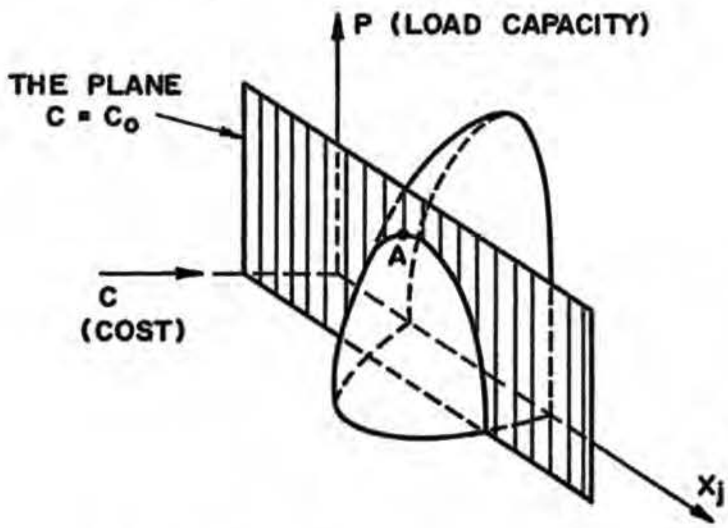
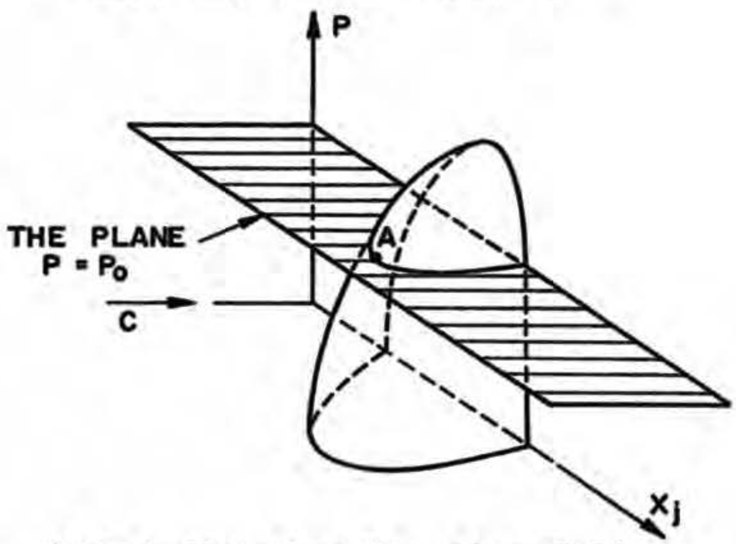


FIG. 11 - EQUAL SPAN CONTINUOUS BEAMS

(FOR LIPPED CHANNELS, WEB CRIPPING CONSIDERED)



(a) The Equilibrium State 'A' as a Point of Maximum Load Capacity 'P' For Constant Cost 'C'



(b) The Equilibrium State 'A' as a Point of Minimum Cost 'C' For Constant Load Capacity 'P'

FIG. 12 - EQUIVALENCE OF MAXIMUM LOAD AND MINIMUM COST DESIGNS

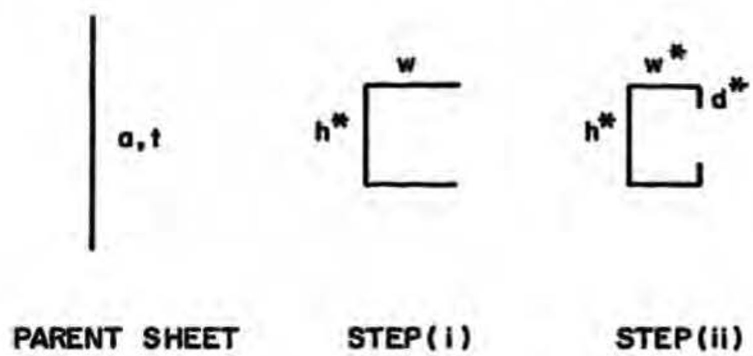


FIG. 13 - THE FORMATION OF AN OPTIMAL PROFILE

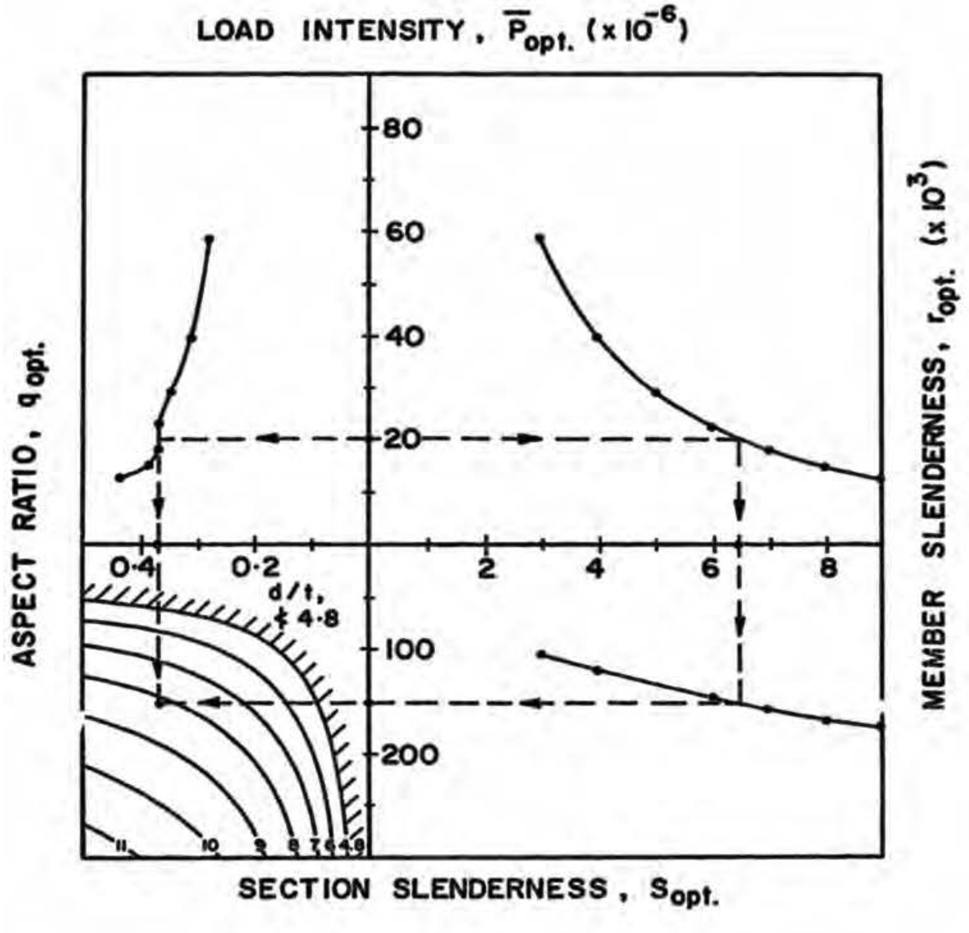


FIG. 14 - DESIGN EXAMPLE
(SINGLE SPAN LIPPED CHANNELS. WEB CRIPPLING CONSIDERED)