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Abstract

Certain cold-formed sections like channels and zees, when subjected to bending in the plane of the web, often fail in laboratory tests at a load less than that predicted by standard specification equations. The mode of failure is an upward movement of the compression flange and lip relative to the web, which may remain fairly plane. When bends which were right angles change dramatically under load, the section changes shape, or distorts, resulting in a reduction of the section stiffness followed by buckling of the flange. Such a failure is referred to as "distortional buckling".

An analytical solution to the distortional buckling problem has been proposed by Hancock of the University of Sydney. This method was used to calculate distortional buckling capacities of around 200 shapes and a curve was fit to the results. Finally, a simplified expression was developed to check this limit state with minimal effort. Several full-scale tests were performed on channels and zees of various thicknesses and bracing conditions to verify the Hancock method.

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Introduction

Cold-formed channels and zee sections are the most common forms of pulins and girts used in metal building systems. It has long been known that such members in flexure not only deflect in the direction of the load, but move laterally and twist. When not attached to deck or sheathing, a certain amount of lateral bracing is required to prevent lateral-torsional buckling. Earlier versions of the AISI Specifications called for quarter-point bracing in such cases. Recently it has been observed that another type of failure---called distortional buckling---can occur even in well-braced beams.

An analytical procedure for distortional buckling has been proposed by Prof. Hancock of the University of Sydney (Hancock, 1995). It is an arduous procedure to do by hand and involves the calculation of section properties not generally given in any tables of standard shapes. The subject of this paper is an attempt to create a simplified model that will predict the distortional buckling moment using only the dimensions of the cross-section, the thickness and the yield stress. A series of full-scale tests was performed as well, as further verification of the Hancock's proposed mathematical solution.

Previous Research

In an attempt to prove that quarter-point bracing was not necessary for un-sheeted flexural members, as the AISI Specification (AISI, 1980) then required, a program of testing was carried out at the University of Florida (Ellifritt, 1992). Typical industry channel and zee sections were tested with (1) no bracing, (2) mid-point bracing, (3) third-point bracing, (4) quarter-point bracing and (5) full bracing. These tests did indeed show that quarter-point bracing was not necessary and that requirement was subsequently removed from the Specification (AISI, 1996)

In some of the tests, very little improvement in bending capacity was achieved by adding more braces beyond that at mid-point. This defied the existing AISI Specifications in which the only limit states treated in bending were yielding, local buckling or lateral-torsional buckling. Obviously, the lateral buckling strength is a function of unbraced length and one would expect bending capacity to improve as the unbraced length was reduced. But in fact it did not. The failure mode in most of the well-braced tests was distortional buckling, even though we barely had a name for it then. When unbraced lengths became large, lateral buckling controlled; for short lengths, distortional buckling buckling controlled. Subsequent research by the first author at the University of Western Australia (Kavanagh, 1993) produced similar results.

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Current Research

Additional tests of channels and zees was begun at the University of Florida in 1994, sponsored by the American Iron and Steel Institute and the Metal Building Manufacturers Association. The tests were divided into two groups: First, a series was conducted in which the load was applied directly to the top flange between braced points. A greased bearing assured that the loading device would not hinder the lateral deflection and twist. This loading produced a twisting in the member between braces and clouded the picture of distortional buckling a bit by introducing torsion stresses into the cross-section, which Prof. Hancock's method does not account for. In these tests a distortional type of failure tended to occur near the brace, where the member could not twist and thus relieve the compression on the lip. These tests produced results close to those tests mentioned earlier (Ellifritt, 92 and Kavanagh, 93).

In the second group of tests, the load was applied at the braced points. In fact, the load was actually applied through the brace itself, equally to both back-to-back sections. Twist was therefore completely prevented at the brace, but since the load did not touch the member, the compression flange and lip were free to rotate there. These two types of loading arrangements are shown in Figure 1.

Load cells were used at each loading point as a check on the distribution of loading between members. Strain gages were placed around the cross-section at a location on the member where distortional buckling was likely to occur. The stresses deduced from these strain measurements were used as input data to a finite strip buckling program called BFINST, also created by Prof. Hancock.

Test results are shown in Table 1 under the column headed, "M, Applied Moment". Tests 2 through 8 had loads applied directly to the member flanges between the braced points, as shown in Figure 1(a). Tests 9 through 17 had loads applied at the brace locations as shown in Figure 1(b). Examples of distortional buckling of both channels and zees are shown in Figure 3.

Distortional Buckling Model

An analytical procedure for calculating distortional buckling was developed by Hancock (Hancock, 1994) and is reproduced in Appendix A. It involves several steps and the calculation of properties not generally known for any section. Hancock, Rogers and Schuster (1996)compared test results from various researchers with the proposed analytical method and found good agreement.

While computer solutions could easily be used on this model, it was felt that an easier "hand" solution could be developed which would make use of only the dimensions of the cross-section. The Hancock method was used to calculate the distortional buckling moment capacity for around 200 sections. The sections in Part 1 of the 1996 Edition of the AISI Manual were used for this study. Then a parameter study was done to see which variables had the most influence on the results as determined by Hancock's model. After this, a curve was fit to the Hancock model which was presented in the form of an equation involving thickness, yield strength, depth, flange width and lip length. These results are plotted in Figures 4 and 5. One can also use this simplified form to determine whether or not distortional buckling even needs to be checked. The dimensions of the full-scale tests are also shown on the Figures 4 and 5.

Design Example

In the Appendix, alongside the equations for distortional buckling, is a Mathcad solution for an 8 x 1 5/8 x .071 channel. Using the suggested simplified approach on the same section,

$$X = \left(\frac{B}{t}\right)^{1.1} \left(\frac{D}{d}\right)^{0.4} \frac{F_y}{50}$$

where B = flange width
t = thickness
D = overall depth
d = lip length

$$X = \left(\frac{1.625}{.071}\right)^{1.1} \left(\frac{8}{0.5}\right)^{0.4} \frac{70}{50} = 132.8 \text{ in.-k}$$

$$\frac{D_{Mn}}{M_y} = -8 \text{ x } 10^{-9} X^3 + 10^5 X^2 - 0.0048 X + 1.268$$

$$\frac{D_{Mn}}{M_y} = -8 \text{ x } 10^{-9} (132.8)^3 + 10^5 (132.8)^2 - 0.0048 (132.8) + 1.268 = 0.752$$

$M_y = S_x F_y = 1.751 \text{ x } 70 = 122.6$ in.-k

$D_{mn} = 0.752 \text{ x } 122.6 = \underline{\underline{92.2}}$ in.-k

Hancock's method gives $D_{mn} = \underline{\underline{95.1}}$ in.-k

Summary and Conclusions

From the far right-hand column of Table 1, it can be seen that the test results compared well with Hancock's analytical method. Keep in mind that the first eight tests included torsion as well, so those results are generally less than 1.0. The zee sections showed more scatter in the results than did the channel sections. The authors believe this can be attributed to the sensitivity of the lip angle to distortional buckling, both in calculation and in test.

The Hancock method was used to analyze some 200 shapes that are tabled in the AISI Manual. A formula was fit to the results and a simple equation developed whereby one could get a close estimate of the distortional buckling moment as a function of the yield moment. These curves are shown in Figures 4 and 5. This provides the designer with something that can be checked very quickly, using only the overall dimensions of the cross-section. If a more exact result is desired, one can always go back to the more exact method.

References

American Iron and Steel Institute, "Specification for the Design of Cold-Formed Steel Structural Members", 1980.

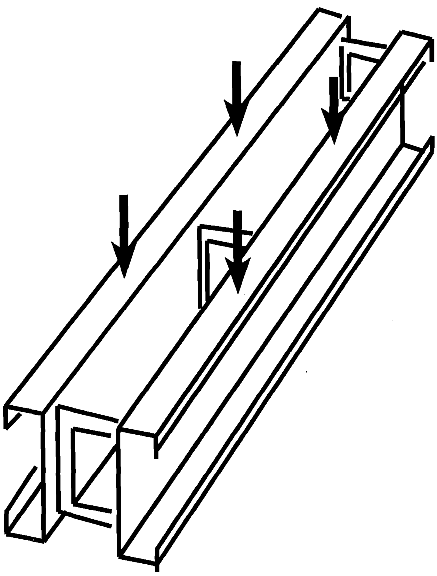
American Iron and Steel Institute, "Specification for the Design of Cold-Formed Steel Structural Members", 1996

Ellifritt, D. S., Sputo, T., and Haynes, J., "Flexural Capacity of Discretely Braced C's and Z's", *Proceedings, 11th International Specialty Conference on Cold-Formed Steel Design and Construction",* St. Louis, MO, October, 1992, pp. 109-129

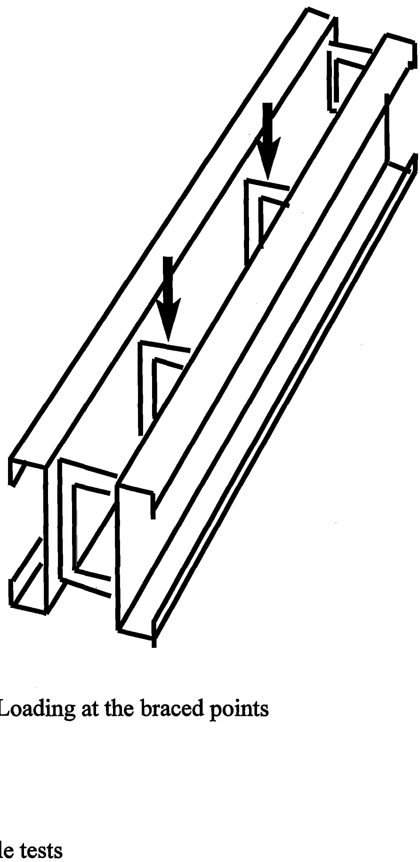
Hancock, G. J., "Design for Distortional Buckling of Flexural Members", *Proceedings, Third International Conference on Steel and Aluminum Structures,* Istanbul, Turkey, 1995, pp. 275-283

Hancock, G.J., Rogers, C. A., and Schuster, R.M., "Comparison of the Distortional Buckling Method for Flexural Members with Tests", *Proceedings, 13th International Specialty Conference on Cold-Formed Steel Design and Construction,* St. Louis, MO, October, 1996, pp. 125-139

Kavanagh, K. T., and Ellifritt, D. S., "Bracing of Cold-Formed Channels not Attached to Deck or Sheathing", *Proceedings, "Is Your Structure Suitably Braced?",* Structural Stability Research Council, Milwaukee, WI, 1993, pp. 3-18.



(a) Loading between braced points



(b) Loading at the braced points

Figure 1. Load arrangements for full-scale tests

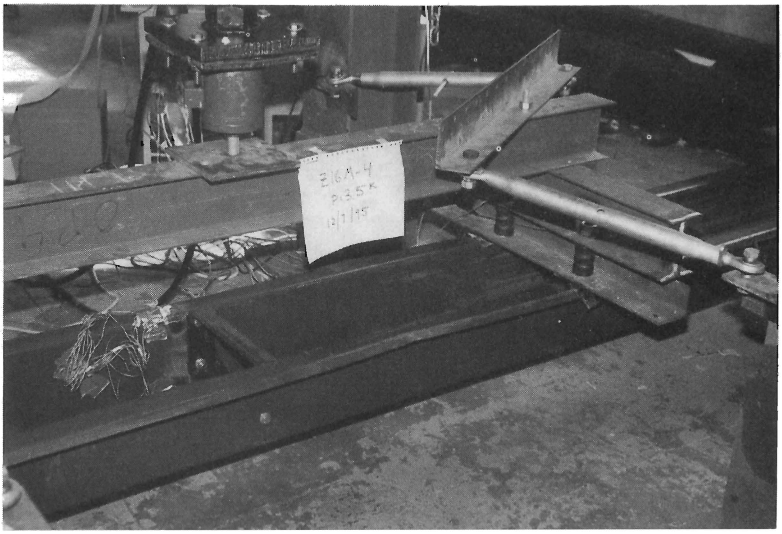


Figure 2(a) Example of load applied directly to the flange between braced points.

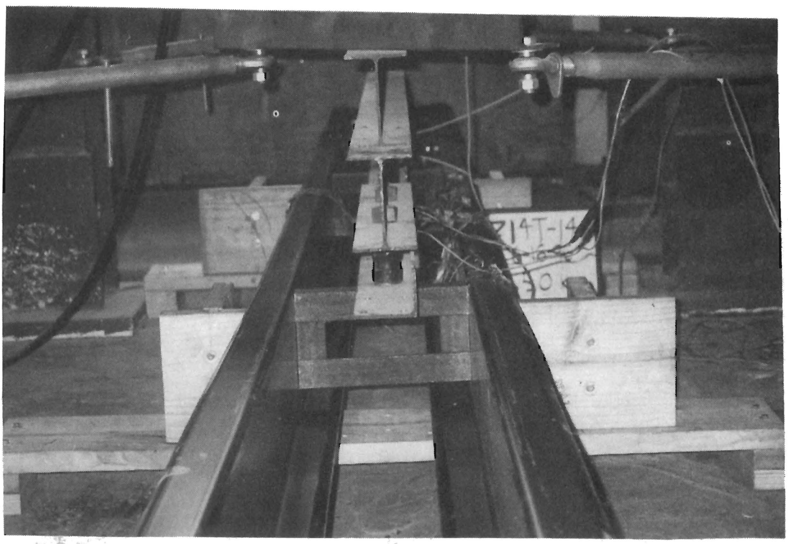


Figure 2(b) Example of load applied directly to the braces.

Figure 3(a) Distortional Buckling of Channel (Test C14M-3)

Figure 3(b) Distortional Buckling of Zee (Test Z16T-11)

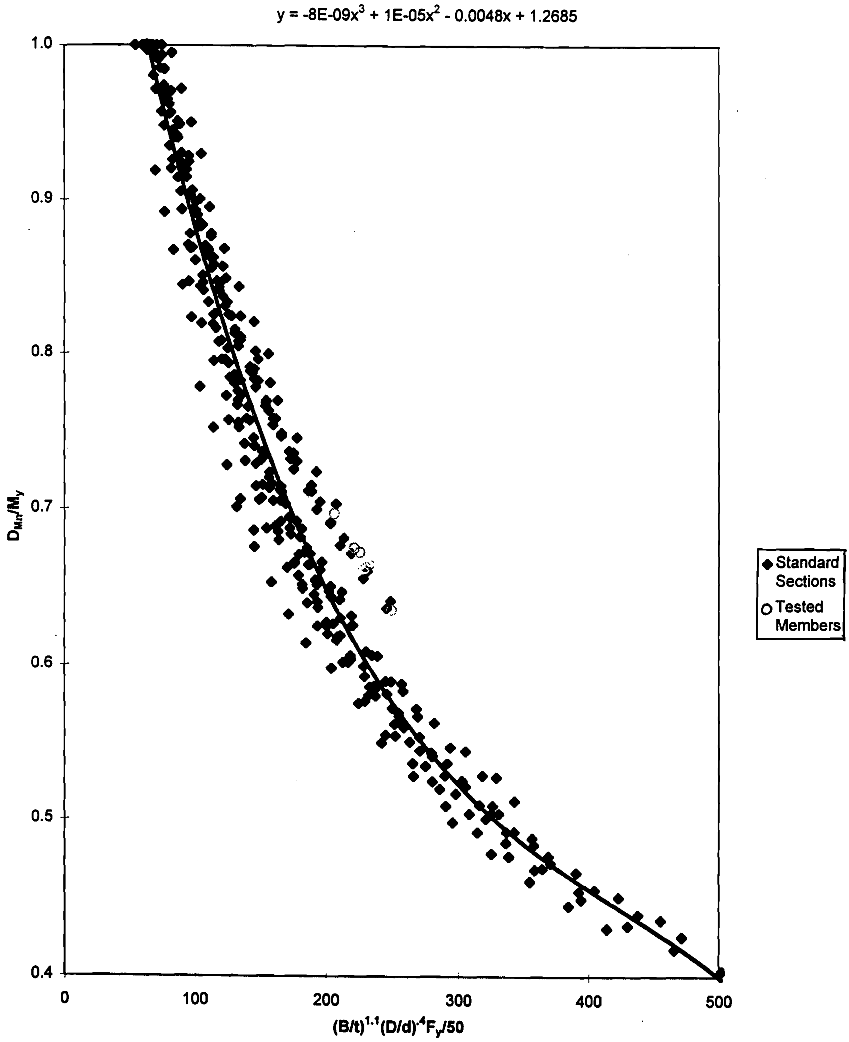


Figure 4 - Distortional Buckling Moment / Yield Moment for Channels vs. the Section Dimensional Parameter $(B/t)^{1.1}\ (D/d)^{0.4}\ F_y/50$

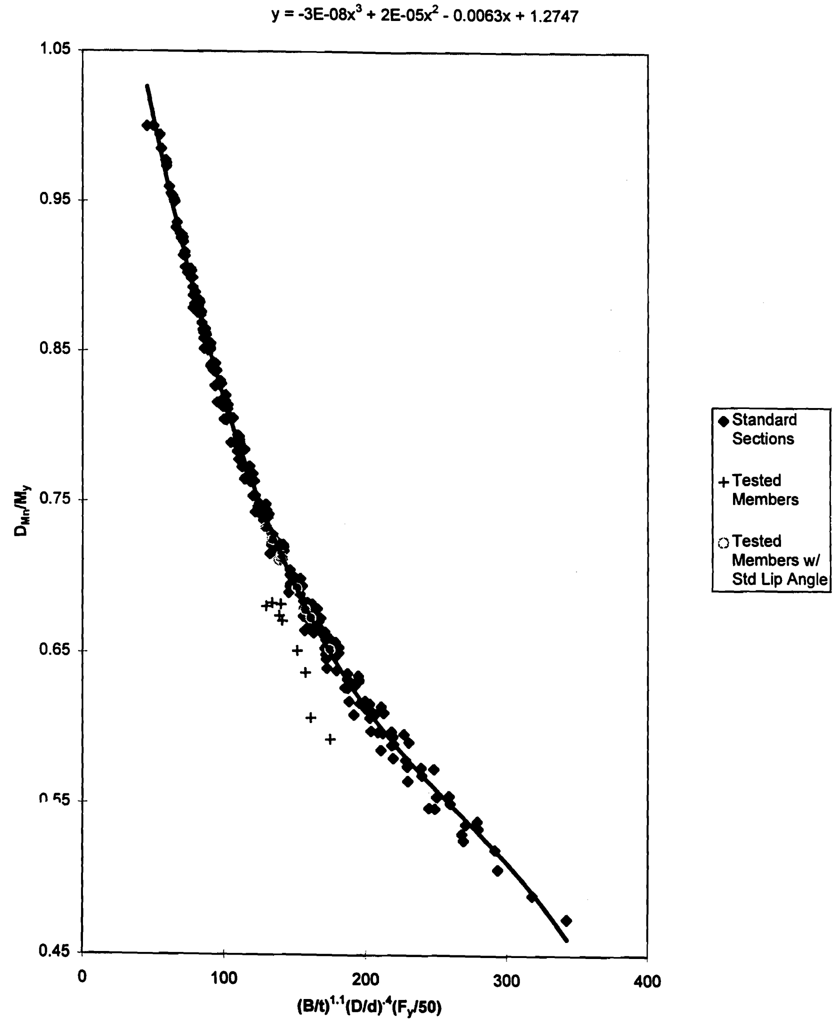


Figure 5 - Distortional Buckling Moment / Yield Moment for Zees vs. the Section Dimensional Parameter $(B/t)^{1.1}$ $(D/d)^{0.4}$ $F_y/50$

Table 1 - Test Results Compared to Distortional Buckling Calculation Method

Results for Channel Sections:

Test [4]	Depth of Section, D (in.)	Flange Width, B (in.)	Thick., t (in.)	Depth of Stiffener, L (in.)	Angle of Stiffener, θ (deg.)	Yield Strength F_y (ksi)	Calc. Half Wavelength λ_c (in.)	Meas. Half Wavelength λ_m (in.)	Buck. Load Factor BFINST	Dist. D_{Mn} [2] (k-in)	Nominal N_{Mn} [3] (k-in)	Applied Moment M (k-in)	M/D_{Mn}
C16M-2	8.209	3.394	0.066	0.841	90	60.2	25.78	30.0	1.14	105.9	129.2	82.8	0.782
C14M-3	8.179	3.348	0.072	0.888	90	66.8	25.20	24.0	1.32	129.2	156.5	92.9	0.719
C14M-7	8.231	3.466	0.072	0.940	90	66.8	26.92	22.0	1.51	133.7	161.5	129.9	0.972
C16T-9	8.326	3.170	0.066	0.909	90	60.2	25.93	40.0	1.54	109.5	132.1	102.7	0.938
C14T-10	8.166	3.455	0.072	0.878	90	66.8	25.58	30.0	1.29	129.1	157.0	120.2	0.932
C14T-13	8.247	3.424	0.071	0.811	90	69.5	24.36	5.0 [1]	0.97	126.9	157.9	128.7	1.014
C14F-16	8.322	3.368	0.071	0.930	90	69.5	26.45	5.0 [1]	1.02	134.8	164.9	138.9	1.030

Results for Zee Sections:

Test [4]	Depth of Section, D (in.)	Flange Width, B (in.)	Thick., t (in.)	Depth of Stiffener, L (in.)	Angle of Stiffener, θ (deg.)	Yield Strength F_y (ksi)	Calc. Half Wavelength λ_c (in.)	Meas. Half Wavelength λ_m (in.)	Buck. Load Factor BFINST	Dist. D_{Mn} [2] (k-in)	Nominal N_{Mn} [3] (k-in)	Applied Moment M (k-in)	M/D_{Mn}
Z16M-4	8.269	2.322	0.061	0.931	35	61.4	15.33	36.0	1.38	79.0	104.6	59.3	0.750
Z14M-5	8.318	2.330	0.072	0.921	40	60.6	14.04	38.0	1.79	103.2	130.9	86.4	0.837
Z14M-6	8.365	2.341	0.072	0.827	40	60.6	13.32	16.0	0.98	100.8	126.6	86.2	0.855
Z14M-8	8.300	2.416	0.072	0.911	42	60.6	14.37	30.0	1.34	104.5	129.3	103.2	0.987
Z16T-11	8.232	2.303	0.061	0.963	40	61.4	19.07	37.0	1.27	82.1	113.2	77.3	0.942
Z14T-12	8.196	2.314	0.072	0.963	38	60.6	17.23	6.0 [1]	2.11	101.4	140.8	109.8	1.083
Z14T-14	8.276	2.230	0.072	0.960	41	67.2	17.13	33.0	0.99	110.3	155.8	85.7	0.777
Z14F-15	8.333	2.353	0.072	0.904	40	67.2	17.18	36.0	1.70	109.8	150.3	85.8	0.782
Z16F-17	8.367	2.350	0.060	0.917	36	63.8	18.49	36.0	1.20	80.4	110.8	91.4	1.137

[1] Sections Buckled Locally

[2] Distortional Buckling Strength calculated as per Dr. Hancock's Ballot

[3] Nominal Section Strength from AISI C3.1.1

[4] 14, 16 = Gage thickness; M = Midpoint bracing, T = Third point bracing, F = Four braces in middle third of beam spaced 2 ft. apart

APPENDIX - Calculation of Distortional Buckling by Hancock's Method

(Mathcad Solution)

Standard Conditions:

$ksi := 1000 \cdot \frac{lb}{in^2}$ $\quad E := 29500 \cdot ksi$ $\quad kip := 1000 \cdot lb$

Input Data:

$D := 8 \cdot in$ $\quad B := 1.625 \cdot in$ $\quad L := .5 \cdot in$ $\quad t := .071 \cdot in$ $\quad \theta := 90 \cdot \left(\frac{\pi}{180}\right)$ $\quad F_y := 70 \cdot ksi$ $\quad S_x := 1.751 \cdot in^3$

CALCULATE SECTION PROPERTIES:

Centroidal Lengths:

$B_c := B - \frac{t}{2}$ $\qquad B_c = 1.589 \cdot in$

$L_c := L - \frac{t}{2}$ $\qquad L_c = 0.464 \cdot in$

Flange-Lip Area:

$A_f := (B_c + L_c) \cdot t$ $\qquad A_f = 0.146 \cdot in^2$

Flange-Lip Centroid Location:

$x_{bar} := \frac{t}{A_f} \cdot \left[\frac{B_c^{\,2}}{2} + L_c \cdot \left(B_c + L_c \cdot \frac{\cos(\theta)}{2}\right)\right]$ $\qquad x_{bar} = 0.974 \cdot in$

$y_{bar} := \frac{t}{2 \cdot A_f} \cdot L_c^{\,2} \cdot \sin(\theta)$ $\qquad y_{bar} = 0.053 \cdot in$

Flange-Lip Moment of Inertia:

$I_x := \left[\frac{L_c^{\,3} \cdot \sin(\theta)^2}{12} + L_c \cdot \left(\frac{\sin(\theta) \cdot L_c}{2} - y_{bar}\right)^2 + \frac{B_c \cdot t^2}{12} + B_c \cdot y_{bar}^{\,2}\right] \cdot t$ $\qquad I_x = 0.002 \cdot in^4$

$I_y := \left[\frac{B_c^{\,3}}{12} + B_c \cdot \left(\frac{B_c}{2} - x_{bar}\right)^2 + L_c \cdot \left(B_c + \frac{L_c \cdot \cos(\theta)}{2} - x_{bar}\right)^2 + L_c^{\,3} \cdot \frac{\cos(\theta)^2}{12}\right] \cdot t$ $\qquad I_y = 0.04 \cdot in^4$

$I_{xy} := \left[B_c \cdot \left(x_{bar} - \frac{B_c}{2}\right) \cdot y_{bar} + L_c \cdot \left(B_c - x_{bar} + L_c \cdot \frac{\cos(\theta)}{2}\right) \cdot \left(L_c \cdot \frac{\sin(\theta)}{2} - y_{bar}\right)\right] \cdot t$ $\qquad I_{xy} = 0.005 \cdot in^4$

$J_f := \frac{A_f t^2}{3}$ $\qquad J_f = 2.45 \cdot 10^{-4} \cdot in^4$

CALCULATE DISTORTIONAL BUCKLING FORMULA VARIABLES:

$$\lambda := 4.8\cdot\left[\frac{I_x\cdot B_c^{\,2}\cdot D}{\left(2\cdot t^3\right)}\right]^{.25} \qquad \lambda = 13.186\cdot in$$

$$\eta := \left(\frac{\pi}{\lambda}\right)^2 \qquad \eta = 0.057\cdot in^{-2}$$

$$\beta := x_{bar}^{\,2} + \frac{I_x + I_y}{A_f} \qquad \beta = 1.237\cdot in^2$$

$$\alpha_1 := \frac{\eta}{\beta}\cdot\left(I_x\cdot B_c^{\,2} + .039\cdot J_f\lambda^2\right) \qquad \alpha_1 = 3.101\cdot 10^{-4}\cdot in^2$$

$$\alpha_2 := \eta\cdot\left(I_y + \frac{2}{\beta}\cdot y_{bar}\cdot B_c\cdot I_{xy}\right) \qquad \alpha_2 = 0.002\cdot in^2$$

$$\alpha_3 := \eta\cdot\left(\alpha_1\cdot I_y - \frac{\eta}{\beta}\cdot I_{xy}^{\,2}\cdot B_c^{\,2}\right) \qquad \alpha_3 = 5.56\cdot 10^{-7}\cdot in^4$$

CALCULATE DISTORTIONAL BUCKLING STRESS:

$$f_{ed\_minus} := \frac{E}{2\cdot A_f}\cdot\left[\left(\alpha_1 + \alpha_2\right) - \left[\left(\alpha_1 + \alpha_2\right)^2 - 4\cdot\alpha_3\right]^{.5}\right] \qquad f_{ed\_minus} = 6.816\cdot 10^6\cdot lb\cdot ft$$

$$f_{ed} := \mathrm{if}\left[f_{ed\_minus}>0, f_{ed\_minus}, \frac{E}{2\cdot A_f}\cdot\left[\left(\alpha_1 + \alpha_2\right) - \left[\left(\alpha_1 + \alpha_2\right)^2 - 4\cdot\alpha_3\right]^{.5}\right]\right] \qquad f_{ed} = 47.335\cdot ksi$$

$$k_\phi := \frac{2\cdot E\cdot t^3}{5.46\cdot\left(D + .06\cdot\lambda\right)}\cdot\left[1 - \frac{1.11\cdot f_{ed}\cdot D^4\cdot\lambda^2}{E\cdot t^2\cdot\left(12.56\cdot\lambda^4 + 2.192\cdot D^4 + 13.39\cdot\lambda^2\cdot D^2\right)}\right] \qquad k_\phi = 0.234\cdot kip\cdot\frac{in}{in}$$

If k > 0

$$\alpha_{1x} := \alpha_1 + \frac{k_\phi}{\beta\cdot\eta\cdot E} \qquad \alpha_{1x} = 4.231\cdot 10^{-4}\cdot in^2$$

$$\alpha_{3x} := \eta\cdot\left(\alpha_{1x}\cdot I_y - \frac{\eta}{\beta}\cdot I_{xy}^{\,2}\cdot B_c^{\,2}\right) \qquad \alpha_{3x} = 8.118\cdot 10^{-7}\cdot in^4$$

$$f_{edx} := \frac{E}{2\cdot A_f}\cdot\left[\left(\alpha_{1x} - \alpha_2\right) - \left[\left(\alpha_{1x} - \alpha_2\right)^2 - 4\cdot\alpha_{3x}\right]^{.5}\right] \qquad f_{edx} = 68.934\cdot ksi$$

If k < 0

$$k_{\phi y} := \frac{2\cdot E\cdot t^3}{5.46\cdot\left(D - .06\cdot\lambda\right)} \qquad k_{\phi y} = 0.44\cdot kip$$

$$\alpha_{1y} := \alpha_1 + \frac{k_{\phi y}}{\beta\cdot\eta\cdot E} \qquad \alpha_{1y} = 5.225\cdot 10^{-4}\cdot in^2$$

$$\alpha_{3y} := \eta\cdot\left(\alpha_{1y}\cdot I_y - \frac{\eta}{\beta}\cdot I_{xy}^{\,2}\cdot B_c^{\,2}\right) \qquad \alpha_{3y} = 1.037\cdot 10^{-6}\cdot in^4$$

$$f_{edy} := \frac{E}{2\cdot A_f}\cdot\left[\left(\alpha_{1y} - \alpha_2\right) - \left[\left(\alpha_{1y} - \alpha_2\right)^2 - 4\cdot\alpha_{3y}\right]^{.5}\right] \qquad f_{edy} = 87.819\cdot ksi$$

$$f_{ed} := \mathrm{if}\left(k_\phi>0, f_{edx}, f_{edy}\right) \qquad f_{ed} = 68.934\cdot ksi$$

$$f_c := \mathrm{if}\left[f_{ed}>2.2\cdot F_y, F_y, F_y\cdot\left(\frac{f_{ed}}{F_y}\right)^{.5}\cdot\left[1 - .22\cdot\left(\frac{f_{ed}}{F_y}\right)^{.5}\right]\right] \qquad f_c = 54.299\cdot ksi$$

$$D_{Mn} := S_x\cdot f_c \qquad D_{Mn} = 95.078\cdot kip\cdot in$$

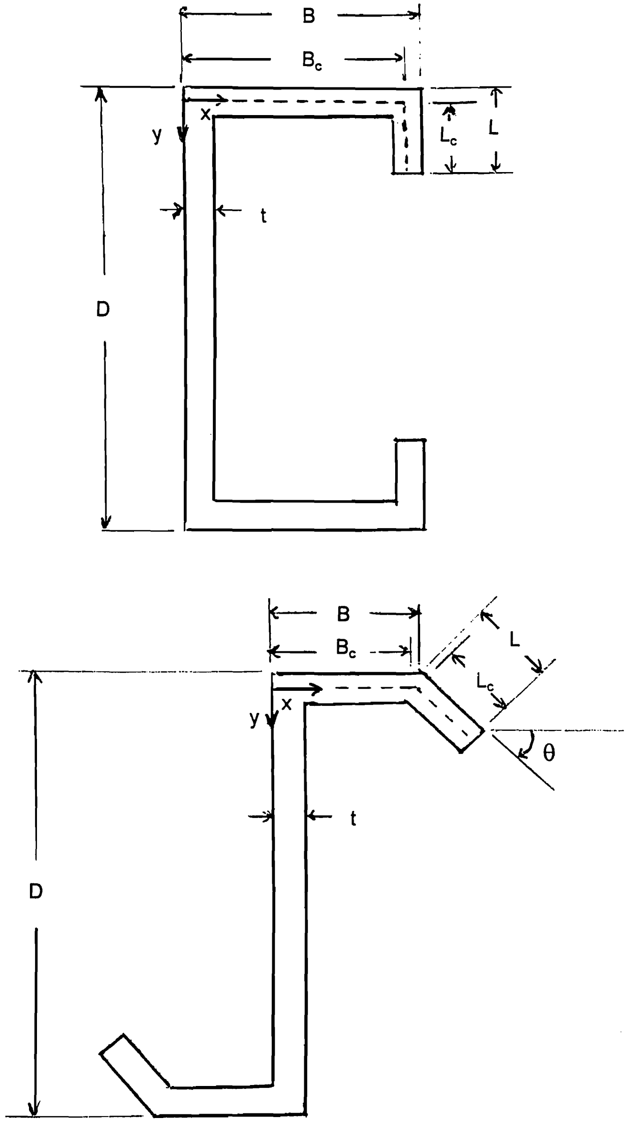


Figure A-1 - Dimension Variables Used for Distortional Buckling Equations