Oct 17th, 12:00 AM

Cold-formed Steel Design by Spreadsheet Program

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Thirteenth International Specialty Conference on Cold-Formed Steel Structures
St. Louis, Missouri U.S.A., October 17-18, 1996

COLD-FORMED STEEL DESIGN BY SPREADSHEET PROGRAM

by Scott A. Burns *

Summary
This paper demonstrates how to use advanced features of a spreadsheet program to design cold-formed steel members efficiently. The example presented in the paper concerns a hat section in flexure which is to be designed for maximum bending strength with a restriction on the total amount of steel that can be used. The nature of the formulas and data entered into the spreadsheet program are presented.

1. Introduction

Today's spreadsheet programs have features that go beyond standard "what-if" type analysis. Microsoft Excel[3] for example has a "Solver" module that will automatically adjust the values of specified cells in order to achieve a desired condition, such as producing a minimum or maximum value in another cell. This module can easily be applied to the design of cold-formed steel members to achieve efficient designs.

Cold-formed steel structural members are used in a wide variety of ways, such as in building wall systems and automobile frames. Relatively simple forming operations (brake pressing, stamping, or roll forming) can produce a wide variety of structural shapes and sizes. Cold-formed sections can be very economical, particularly if production costs can be spread over a large number of identical units manufactured.

The design of light-gage cold-formed members involves considerations such as local buckling and post-buckling behavior that can make the design process somewhat complicated and iterative in nature. The automated goal-seeking features of the spreadsheet program can assist in finding section dimensions that satisfy all design requirements, relieving the designer of the more tedious aspects of light gage steel design.

In this paper, we focus on the selection of the cross-sectional dimensions of a hat section loaded in flexure. Previously, Seaburg and Salmon have investigated the minimum weight design of cold-formed flexural members.[4] Here, we approach the somewhat different problem of sizing the cross section to make the bending strength as large as possible while maintaining a fixed upper limit on volume of steel and depth of section. The techniques presented here are extendable to other section types, member types, or even to entire structural systems.

2. Problem Statement

The *Illustrative Examples* section of the *AISI Cold-Formed Steel Design Manual*[1] presents the steps that one would take to analyze a hat section in flexure (see Figure 1 in this paper or Example 5 in the *Design Manual*). This example provides us with a good starting point for redesign by spreadsheet. Thus, our goal will be to optimize this hat section to maximize the allowable bending moment that it can safely support while maintaining the existing cross-sectional area (1.43 in^2 or 921. mm^2) and overall section depth (4 in or 102. mm).

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3. Procedure

The problem will be posed in the form of an optimization statement known as a "mathematical program." It will treat key dimensions of the hat section (e.g., flange width, web height, sheet thickness, etc.) explicitly as *design variables* that will appear within *constraint expressions* that reflect the rules of the AISI specification.[2] An *objective function* that expresses the bending strength in terms of the design variables completes the formulation.

Since optimization is typically an iterative procedure that requires the selection of a starting point, the cross-sectional dimensions provided in the AISI example will be used to initiate the solution process. The mathematical program will be solved using Microsoft Excel, and the solution will be compared to the original section to assess the increase in moment capacity achieved by the optimization process.

4. Formulation

One of the most difficult aspects of designing cold-formed steel beams is how to properly treat the effects of local buckling. Light-gage steel members are very likely to have compression elements with large width-to-thickness ratios that are susceptible to local buckling. In many cases, the local buckling of a compression element does not cause global failure. If the element is stiffened, then the section can sometimes carry additional load beyond that causing first buckling. The local buckling causes a redistribution of stress toward the stiffeners, and overall failure does not occur. This phenomenon is known as "post-buckling strength."

The AISI specification treats local buckling by eliminating a portion from the center of each stiffened compression element in the modeled cross section. Each resulting compression element has an "effective width" that is used instead of the actual width for calculating the section properties. Figure 2 shows the modeled cross section of the hat section, where both the compression flange and the compression portion of the webs have been modified to account for local buckling. The magnitude of the effective width of each element depends on several factors, including the actual stress level in the element. This can make the design process tricky, since calculating the actual stress in the element requires knowing the section properties, but the section properties are dependent upon the effective widths of the compression elements, which in turn depend on the actual stress in the elements. Thus, a simple analysis of a given cross section can sometimes require an iterative process.

4.1 Design Variables

Figure 2 shows the five independent design variables:

- w, the flat width of the compression flange;
- h, the flat height of the web;
- t, the nominal thickness;
- w_t, the flat width of each tension flange;
- h_t, the flat height of the lip.

The inside bend radius, R, is not an independent variable since it usually depends on the sheet thickness. It will be specified to be twice the sheet thickness through an equality constraint in the formulation. Likewise, the effective width of the compression flange, b, the overall depth, d, and the distance from the neutral axis to the extreme compression fiber, y_c, are also dependent variables that will be specified as functions of the five independent variables in the formulation. Other dependent variables will be introduced later to simplify the constraint expressions in the formulation.

Five quantities will be specified as "design parameters." These quantities will be assigned fixed values during the optimization process, yet will be represented explicitly in the constraint expressions. They are:

m, the inside bend radius multiplier;
A_{max}, the maximum cross-sectional area;
d_{max}, the maximum overall depth of the section;
E, the modulus of elasticity;
F_y, the yield strength.

By representing these parameters explicitly in the formulation, we are able to extend the applicability of the formulation to a wider range of specific cases more easily. For example, by solving the optimization problem with a series of different maximum overall depth values, we may observe how the maximum depth requirement impacts the optimal design.

4.2 Section Properties

The AISI Design Manual recommends a tabular procedure to calculate the section properties, based on a line idealization of the cross section. The table contains six columns, as shown in Table 1. This table presents the section properties for the AISI example problem shown in Figure 1. Note that the length of element 5, the compression flange, has been given a reduced, effective length instead of its actual length in this table to account for local buckling. Section 4.3 will discuss how the effective length is calculated. Also note that the web is assumed to be fully effective (no portion is removed for local buckling effects). This assumption must, of course, be checked at the end of the design process to assure that it was justified. The distance from the neutral axis to the extreme compression fiber is calculated as the ratio of two column totals: $y_c = \Sigma(Ly)/\Sigma(L) = 43.54/17.70 = 2.46$ in (62.5 mm). The moment of inertia is also found from the column totals using the parallel axis theorem: $I_x = t\,[\Sigma(Ly^2) + \Sigma(I_1) - \Sigma(L)y_c^2] = 0.06\,[141.4 + 8.43 - 17.7\,(2.46)^2] = 2.56\ in^4$. $(107.\ cm^4)$.

To pose a mathematical program that reflects the effect that the design variables have on the behavior of the beam, the section properties must be expressed in terms of these design variables. Table 2 presents the modified section properties table. To simplify the expressions, five new dependent variables have been defined (d_1 through d_5). The following set of equations defines the location of the neutral axis (y_c) and the moment of inertia (I_x):

$$
\begin{aligned}
R &= mt \\
r &= R + t/2 \\
u &= 1.57r \\
c &= 0.637r \\
d_1 &= t/2 + r + h - h_t/2 \\
d_2 &= t/2 + r + h + c \\
d_3 &= t/2 + h + 2r \\
d_4 &= t/2 + r + h/2 \\
d_5 &= r + t/2 - c \\
\Sigma(L) &= 2h_t + 6u + 2w_t + 2h + b \\
\Sigma(Ly) &= 2h_t d_1 + 4ud_2 + 2w_t d_3 + 2hd_4 + bt/2 + 2ud_5 \\
\Sigma(Ly^2) &= 2h_t d_1^2 + 4ud_2^2 + 2w_t d_3^2 + 2hd_4^2 + bt^2/4 + 2ud_5^2 \\
y_c &= \Sigma(Ly)/\Sigma(L) \\
I_x &= t\,[\Sigma(Ly^2) + 2h_t^3/12 + 2h^3/12 - \Sigma(L)y_c^2]
\end{aligned}
$$

Note that by introducing the dependent variables (R, r, u, c, d_1, d_2, d_3, d_4, d_5, $\Sigma(L)$, $\Sigma(Ly)$, $\Sigma(Ly^2)$, b, y_c, and I_x), we increase the dimensionality of the problem, but minimize the algebraic manipulations that we must perform. Imagine how complex the moment of inertia equation would be if dependent variables were not used! We reduce the chance of making algebraic errors and make the problem easier to formulate at the expense of shifting more of the computational effort to the optimization computer program.

4.3 Effective Width

The effective width of the compression flange, b, is the only quantity in the preceding set of equations which has not yet been defined in terms of the independent variables, w, h, t, w_t, and h_t. If the w/t ratio of the compression flange is small enough, then the effective width equals the actual width because local buckling will not occur. In this case, the compression flange is termed "fully effective." The maximum value of w/t for which the flange is fully effective is expressed through a quantity called λ in the AISI specification:

$$\lambda = \frac{1.052}{\sqrt{k}}\left(\frac{w}{t}\right)\sqrt{\frac{f_c}{E}}\,.$$

If $\lambda \leq 0.673$, then $\rho = 1$.
If $\lambda > 0.673$, then $\rho = (1-0.22/\lambda)/\lambda$.
$b = \rho w$

The value of k in this case is 4. The quantity f_c is defined as the *actual* stress in the compression flange when the section first yields. Since the first yielding can occur in the tension flange, f_c might be less than F_y. Normally, f_c is initially assumed to equal F_y, and once the analysis of the section is complete, this assumption is checked. When this assumption is used and the tension flange yields first, then f_c must be adjusted, which leads to an iterative, trial-and-error procedure. We can avoid this iteration in our mathematical program if we define another dependent variable, M_n, the nominal bending moment which causes first yield of the section. Then f_c can be defined as $f_c = M_n y_c / I_x$ using an additional equality constraint in the mathematical program. This allows us to use f_c directly in the expression for λ without the need for iteration. The constraints establishing the dependent variable M_n will be developed in the following section.

4.4 Inequality Constraints

The beam is designed so that the nominal bending moment causes first yielding at one of the extreme fibers of the section. The allowable bending moment results from dividing the nominal bending moment by a factor of safety (=1.67 for the AISI specifications). We may determine the nominal bending moment in terms of two inequality constraints, one for the compression side and the other for the tension side:

$$M_n \le F_y I_x / y_c \text{ and } M_n \le F_y I_x / y_t.$$

Here, we define $y_t = d - y_c$, where $d = h + 2r + t$. Since our objective is to maximize the allowable bending moment, $M_n/1.67$, one of the two inequality constraints will be forced to become active (become a strict equality) during the optimization process. We need not be concerned with which flange yields first; this will automatically be established by the optimization process.

The original problem statement was to maximize allowable bending moment while maintaining the same cross-sectional area and section depth of the AISI illustrative example. This leads to two additional inequality constraints:

$$t(2h_t + 6u + 2w_t + 2h + w) \le A_{max}$$
$$d \le d_{max}.$$

4.5 Complete Formulation

The design parameters for this example, which are fixed during the optimization process, are:

m	= 2,	the inside bend radius multiplier;
A_{max}	= 1.43 in^2 (921. mm^2),	the maximum cross-sectional area;
d_{max}	= 4.00 inches (102. mm),	the maximum overall depth of the section;
E	= 29,000 ksi (200,000 MPa),	the modulus of elasticity;
F_y	= 50 ksi (345. MPa),	the yield strength.

The design variables are:

w	flat width of the compression flange (independent);
h	flat height of the web (independent);
t	nominal thickness (independent);
w_t	flat width of each tension flange (independent);
h_t	flat height of the lip (independent);
R	inside bend radius;
b	effective width of the compression flange;
d	overall section depth;
y_c	distance from the neutral axis to the extreme fiber in compression flange;
r	distance from the center of radius of the bend to the centerline of the bend;
u	length of the bend centerline arc;
c	distance from the center of radius to the center of gravity of the bend;
d_1	distance from top fiber to c.g. of lip;
d_2	distance from top fiber to c.g. of lower bends;
d_3	distance from top fiber to c.g. of tension flanges;
d_4	distance from top fiber to c.g. of webs;
d_5	distance from top fiber to c.g. of upper bends;
$\Sigma(L)$	sum of column 2 of Table 2;
$\Sigma(Ly)$	sum of column 4 of Table 2;
$\Sigma(Ly^2)$	sum of column 5 of Table 2;
I_x	moment of inertia of the section;
λ	effective width cutoff parameter;
ρ	effective width multiplier;
f_c	actual stress in compression flange when first yielding occurs on either flange;
M_n	nominal bending moment causing first yielding;
y_t	distance from the neutral axis to the extreme fiber in tension flange.

The mathematical program contains an objective function and a set of constraints:

maximize

$M_n/1.67$

subject to

$R = mt$

$r = R + t/2$

$u = 1.57r$

$c = 0.637r$

$d_1 = t/2 + r + h - h_t/2$

$d_2 = t/2 + r + h + c$

$d_3 = t/2 + h + 2r$

$d_4 = t/2 + r + h/2$

$d_5 = r + t/2 - c$

$\Sigma(L) = 2h_t + 6u + 2w_t + 2h + b$

$\Sigma(Ly) = 2h_t d_1 + 4ud_2 + 2w_t d_3 + 2hd_4 + bt/2 + 2ud_5$

$\Sigma(Ly^2) = 2h_t d_1^2 + 4ud_2^2 + 2w_t d_3^2 + 2hd_4^2 + bt^2/4 + 2ud_5^2$

$y_c = \Sigma(Ly)/\Sigma(L)$

$I_x = t\,[\Sigma(Ly^2) + 2h_t^3/12 + 2h^3/12 - \Sigma(L)y_c^2]$

$\lambda = \frac{1.052}{\sqrt{k}}\left(\frac{w}{t}\right)\sqrt{\frac{f_c}{E}}$

if $\lambda \leq 0.673$, then $\rho = 1$, otherwise $\rho = (1-0.22/\lambda)/\lambda$

$b = \rho w$

$f_c = M_n y_c/I_x$

$M_n \leq F_y I_x/y_c$

$M_n \leq F_y I_x/y_t$

$y_t = d - y_c$

$d = h + 2r + t$

$t(2h_t + 6u + 2w_t + 2h + w) \leq A_{max}$

$d \leq d_{max}$.

5. Spreadsheet Implementation and Solution

Figure 3 presents this problem as it might appear in spreadsheet form. The sheet has been divided into five regions, containing the independent variables, the parameters, the dependent variables, the objective function, and the constraints. The format is a matter of style; at a minimum, only the objective function value and independent variable values need to appear in the spreadsheet. The names, symbols, units shown in Figure 3 are provided to improve the readability of the spreadsheet.

Figure 4 shows the formulas that were entered into the cells. There are four sets of formulas defined. The first set comprises the dependent variable definitions in cells C20 through C39. Symbolic references to other cells have been defined to make formula entry easier. For example, typing "ht" into a formula would refer to the value in cell D7. The objective function value is also a formula, referring to the value of the nominal bending moment appearing in cell D9.

The graph of the section appearing in the sheet is produced by defining an X-Y plot using the values defined in cells F31 through G44. This diagram will change to reflect the new cross section as the optimization takes place.

Another set of formulas appear in the constraints portion of the sheet. Two formulas are defined for each constraint, and are associated with one another through either an equality or an inequality relationship. The equal sign or inequality sign appearing in column C is purely cosmetic. The relationship is established formally in Figure 5, when the "Solver" dialog is invoked in Excel (under the Tools menu). In this dialog, the objective function cell is identified, the independent variables are identified, and the constraints are defined. When the "Solve" button is pressed, Excel performs the optimization and returns the values listed below in the "Optimal Section" column:

Variable	Starting Value	Optimal Section	Final Solution
w	8.69 in (221. mm)	3.04 in (77.3 mm)	2.62 in (66.5 mm)
h	3.69 in (93.8 mm)	3.41 in (86.6 mm)	3.37 in (85.7 mm)
t	0.06 in (1.52 mm)	0.0987 in (2.51 mm)	0.1046 in (2.66 mm)
w_t	2.69 in (68.3 mm)	1.15 in (29.2 mm)	0.918 in (23.3 mm)
h_t	0.596 in (15.1 mm)	0.00 in (0.00 mm)	0.00 in (0.00 mm)
R	0.0938 in (2.38 mm)	0.197 in (5.00 mm)	0.209 in (5.31 mm)
b	2.57 in (65.3 mm)	3.04 in (77.2 mm)	2.62 in (66.5 mm)
d	4.00 in (102. mm)	4.00 in (102. mm)	4.00 in (102. mm)
y_c	2.46 in (62.5 mm)	2.00 in (50.8 mm)	2.00 in (50.8 mm)
r	0.124 in (3.15 mm)	0.247 in (6.27 mm)	0.262 in (6.65 mm)
u	0.195 in (4.95 mm)	0.387 in (9.83 mm)	0.411 in (10.4 mm)
c	0.079 in (2.01 mm)	0.157 in (3.99 mm)	0.167 in (4.24 mm)
d_1	3.55 in (90.2 mm)	3.70 in (94.0 mm)	3.69 in (93.7 mm)
d_2	3.93 in (99.8 mm)	3.86 in (98.0 mm)	3.85 in (97.8 mm)
d_3	3.97 in (101. mm)	3.95 in (100. mm)	3.95 in (100. mm)
d_4	2.00 in (50.8 mm)	2.00 in (50.8 mm)	2.00 in (50.8 mm)
d_5	0.075 in (1.91 mm)	0.139 in (3.53 mm)	0.147 in (3.73 mm)
$\Sigma(L)$	17.7 in (450. mm)	14.5 in (368. mm)	13.7 in (348. mm)
$\Sigma(Ly)$	43.5 in^2 (281. cm^2)	29.0 in^2 (187. cm^2)	27.3 in^2 (176. cm^2)
$\Sigma(Ly^2)$	141. in^3 (2311. cm^3)	86.3 in^3 (1414. cm^3)	80.0 in^3 (1311. cm^3)
I_x	2.56 in^4 (107. cm^4)	3.45 in^4 (144. cm^4)	3.32 in^4 (138. cm^4)
λ	3.14	0.673	0.547
ρ	0.296	1.000	1.00
f_c	50.0 ksi (345 MPa)	50.0 ksi (345 MPa)	50.0 ksi (345 MPa)
M_n	52.0 in·k (5.88 kN m)	86.3 in·k (9.75 kN m)	83.0 in·k (9.38 kN m)
y_t	1.54 in (39.1 mm)	2.00 in (50.8 mm)	2.00 in (50.8 mm)

The allowable bending moment was increased 66% from the initial starting design without an increase in volume of steel nor depth of section. Note that the optimized section has simultaneous yielding in both flanges at M_n. Also the compression flange is as slender as it can be without having to sacrifice material to a reduced effective width. The lip has disappeared in the optimal design.

The optimal design specifies a sheet thickness that does not match a standard gage of available sheets. To remedy this, the variable t was set equal to the nearest standard value (12 gage or 0.1046 in or 2.66 mm), and the optimization process was repeated. The final solution is shown above and is presented in Figure 6. Note that the variables change considerably to simultaneously accommodate the new sheet thickness and the maximum permitted cross-sectional area. The compression flange moves away from the slenderness cutoff value (λ = 0.673) in order to take full advantage of the maximum permissible section depth. The flanges still yield simultaneously at failure. The nominal bending moment decreases 4% as a result of the additional restriction placed on the problem (the forced sheet thickness).

There are a number of final checks that must be made. It was assumed that the web would be fully effective. Checking section B2.3 of the AISI specification confirms that the optimal design maintains a fully-effective web. Other checks that must be performed include maximum width-to-thickness ratios of each flange element (AISI section B1.1), maximum width-to-thickness ratios of each web (AISI section B1.2), maximum allowable shearing force (AISI section C3.2), web crippling (AISI section C3.4), and combined bending and web crippling (AISI section C3.5). The optimal solution passes all of these checks.

6. Summary and Discussion

A spreadsheet program has been programmed to design a cold-formed steel beam to maximize its bending strength while maintaining a fixed volume of steel and section depth. A mathematical program was formulated and solved using the Solver module in Microsoft Excel. The formulation was posed for a general hat section, so it can be re-used for a variety of different applications (i.e., different steel strength, allowable section depth, allowable cross-sectional area, etc.) With very little modification, the formulation could be changed to minimize volume of steel for a given applied bending moment, or to maximize or minimize any other quantity that can be expressed in terms of the variables. Other design considerations could be added to the formulation to customize it for specific applications, such as constraints that determine and limit the maximum displacement of the beam. By producing a series of optimal designs for a range of parameter values, sets of efficient standardized sections could be developed.

Appendix—References

1. American Iron and Steel Institute, "AISI Cold-Formed Steel Design Manual," Washington, D.C., 1989.
2. American Iron and Steel Institute, "Specification for the Design of Cold-Formed Steel Structural Members," August 19, 1986 Edition with December 11, 1989 Addendum, Washington, D.C., 1989.
3. Microsoft Corporation, *Microsoft EXCEL,* version 4.0, Redmond, WA.

4. Seaburg, P. A. and C. G. Salmon, "Minimum Weight Design of Light Gage Steel Members," *ASCE J. Struct. Div.*, Vol. 97, No. ST1, January, 1971, pp. 203-222.

Appendix—Notation

A_{max} maximum cross-sectional area;
b effective width;
c bend center of gravity location;
d overall depth;
d_i dependent variables;
d_{max} maximum overall depth of the section;
E modulus of elasticity;
f_c maximum compression at first yield;
F_y yield strength;
h flat height of the web;
h_t flat height of the lip;
I_x moment of inertia;
λ slenderness factor;
m inside bend radius multiplier;
M_n nominal bending moment;
r midsurface bend radius;
R inside bend radius;
ρ reduction factor
t nominal thickness;
u arc length of bend;
w flat width of the compression flange;
w_t flat width of each tension flange;
y_c distance from neutral axis to extreme compression fiber;
y_t distance from neutral axis to extreme tension fiber.

List of Figures

Figure 1. Example #5 of the Illustrative Examples section of the AISI Cold-Formed Steel Design Manual. (Bending moment capacity = 52.0 in-k; cross-sectional area = 1.43 in^2; overall depth = 4 in.)

Figure 2. Effective section with shaded areas removed to account for local buckling.

Figure 3. Spreadsheet structure and starting point for design.

Figure 4. Spreadsheet formulas corresponding to Figure 3.

Figure 5. Objective function and constraint definitions.

Figure 6. Final optimized section. (Bending moment capacity = 83.0 in-k; cross-sectional area = 1.43 in^2; overall depth = 4 in.)

Element	Effective Length, L (in*)	Distance from top fiber, y, (in)	Ly (in^2)	Ly^2 (in^3)	I_1' about its own axis (in^3)
1	1.192	3.548	4.229	15.005	0.035
2	0.780	3.925	3.062	12.016	negl.
3	5.384	3.970	21.375	84.857	negl.
4	7.384	2.000	14.768	29.536	8.388
5	2.573	0.030	0.077	0.002	negl.
6	0.390	0.075	0.029	0.002	negl.
sum	17.70		43.540	141.418	8.423

$y_c = 43.540 / 17.70 = 2.46$ in
$I_x = 0.06 [\ 141.418 + 8.423 - 17.70\ (2.46)^2\ ] = 2.56\ in^4$
*Note: 1 in = 25.4 mm

Table 1. Section properties table for the example in Figure 1.

Element	Effective Length, L	Distance from top fiber, y	Ly	Ly^2	I_1' about own axis
1	$2\ h_t$	$t/2 + r + h - h_t/2$ (= d_1)	$2h_td_1$	$2h_td_1^2$	$2h_t^3/12$
2	$4\ u$	$t/2 + r + h + c$ (= d_2)	$4ud_2$	$4ud_2^2$	negl.
3	$2\ w_t$	$t/2 + h + 2r$ (= d_3)	$2w_td_3$	$2w_td_3^2$	negl.
4	$2\ h$	$t/2 + r + h/2$ (= d_4)	$2hd_4$	$2hd_4^2$	$2h^3/12$
5	b	$t/2$	$bt/2$	$bt^2/4$	negl.
6	$2\ u$	$r + t/2 - c$ (= d_5)	$2ud_5$	$2ud_5^2$	negl.
sum	$\Sigma(L)$		$\Sigma(Ly)$	$\Sigma(Ly^2)$	

$y_c = \Sigma(Ly) / \Sigma(L)$
$I_x = t\ [\Sigma(Ly^2) + 2h_t^3/12 + 2h^3/12 - \Sigma(L)\ y_c^2]$

Table 2. Section properties table for mathematical program.

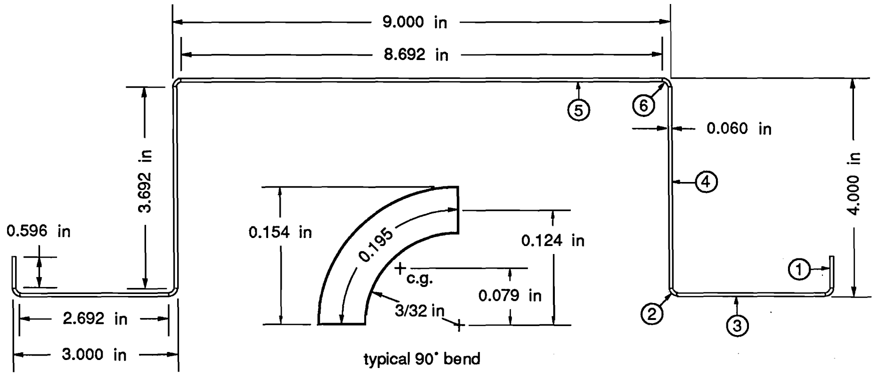

Figure 1. Example #5 of the Illustrative Examples section of the AISI Cold-Formed Steel Design Manual. (Bending moment capacity = 52.0 in-k; cross-sectional area = 1.43 in^2; overall depth = 4 in.)

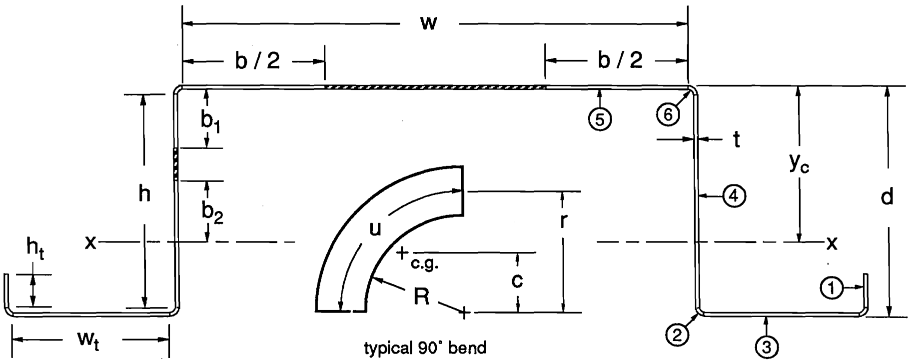

Figure 2. Effective section with shaded areas removed to account for local buckling.

	A	B	C	D	E	F	G
1	*Hat Section*			*Scott Burns*		*4/5/96*	
2	**Independent Variables name**	**LB**	**symbol**	**value**	**UB**	**units**	
3	flat width of compression flange	0	w	8.692	20	in	
4	flat height of web	0	h	3.692	20	in	
5	nominal thickness	0	t	0.060	20	in	
6	flat width of each tension flange	0	wt	2.692	20	in	
7	flat width of lip	0	ht	0.596	20	in	
8	effective width	0	b	2.573	20	in	
9	nominal bending moment	0	Mn	53.000	500	in.k	
10							
11	**Parameters name**	**symbol**	**value**	**units**			
12	inside bend radius multiplier	m	2	none			
13	maximum cross-sectional area	Amax	1.428	in^2			
14	maximum overall section depth	dmax	4	in			
15	modulus of elasticity	E	29000	ksi			
16	yield strength	Fy	50	ksi			
17							
18							
19	**Dependent Variables name**	**symbol**	**eqn**	**units**			
20	inside bend radius	R	0.120	in			
21	height of bend	rb	0.150	in			
22	arc length of bend	ub	0.236	in			
23	centroid of bend	cb	0.096	in			
24	location of lip centroid	d_1	3.574	in			
25	location of lower bend centroid	d_2	3.968	in			
26	location of tension flange centroid	d_3	4.022	in			
27	location of web centroid	d_4	2.026	in			
28	location of upper bend centroid	d_5	0.084	in			
29	sum of effective lengths	sumL	17.946	in			
30	first moment of effective lengths	sumLy	44.729	in^2		x	y
31	second moment of effective lengths	sumLy2	147.466	in^3		0	0.746
32	neutral axis w.r.t. top fiber	yc	2.492	in		0	0.15
33	moment of inertia	Ix	2.664	in^4		0.15	0
34	total depth of section	d	4.052	in		2.842	0
35	neutral axis w.r.t. bottom fiber	yt	1.560	in		2.992	0.15
36	max compr stress @ first yield	fc	49.581	ksi		2.992	3.842
37	max tens stress @ first yield	ft	31.024	ksi		3.142	3.992
38	slenderness factor	lambda	3.151	none		11.834	3.992
39	effective width reduction factor	rho	0.295	none		11.984	3.842
40						11.984	0.15
41						12.134	0
42	**Objective Function name**	**symbol**	**eqn**	**units**		14.826	0
43	moment capacity	Mn	53.000	in.k		14.976	0.15
44						14.976	0.746
45	**Constraints name**	**value/eqn**	**</>/=**	**value/eqn**	**name**		
46	effective width, b	2.573	=	2.56608024	rho * w		
47	max compr stress @ first yield	49.5813404	<	50	yield stress		
48	max tens stress @ first yield	31.0243691	<	50	yield stress		
49	cross-sectional area	1.4439	<	1.428	allowable area		
50	overall depth	4.052	<	4	max depth		

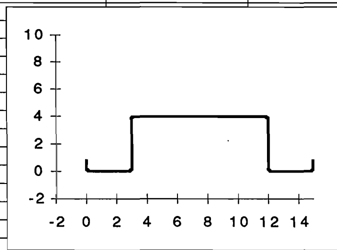

Figure 3. Spreadsheet structure and starting point for design.

	A	B	C	D	E	F	G
1	*Hat Section*			*Scott Burns*			
2	**Independent Variables name**	**LB**	**symbol**	**value**	**UB**	**units**	
3	flat width of compression flange	0	w	8.692	20	in	
4	flat height of web	0	h	3.692	20	in	
5	nominal thickness	0	t	0.06	20	in	
6	flat width of each tension flange	0	wt	2.692	20	in	
7	flat width of lip	0	ht	0.596	20	in	
8	effective width	0	b	2.573	20	in	
9	nominal bending moment	0	Mn	53	500	in.k	
10							
11	**Parameters name**	**symbol**	**value**	**units**			
12	inside bend radius multiplier	m	2	none			
13	maximum cross-sectional area	Amax	1.428	in^2			
14	maximum overall section depth	dmax	4	in			
15	modulus of elasticity	E	29000	ksi			
16	yield strength	Fy	50	ksi			
17							
18							
19	**Dependent Variables name**	**symbol**	**eqn**	**units**			
20	inside bend radius	R	=m*t	in			
21	height of bend	rb	=R_+t/2	in			
22	arc length of bend	ub	=1.57*rb	in			
23	centroid of bend	cb	=0.637*rb	in			
24	location of lip centroid	d_1	=t/2+rb+h-ht/2	in			
25	location of lower bend centroid	d_2	=t/2+rb+h+cb	in			
26	location of tension flange centroid	d_3	=t/2+h+2*rb	in			
27	location of web centroid	d_4	=t/2+rb+h/2	in			
28	location of upper bend centroid	d_5	=rb+t/2-cb	in			
29	sum of effective lengths	sumL	=2*ht+6*ub+2*wt+2*h+b	in			
30	first moment of effective lengths	sumLy	=2*ht*d_1+4*ub*d_2+2*wt*d_3+2*h*d_4+b*t/2+2*ub*d_5	in^2		x	y
31	second moment of effective lengths	sumLy2	=2*ht*d_1^2+4*ub*d_2^2+2*wt*d_3^2+2*h*d_4^2+b*t^2/2+2*ub*d_5^2	in^3		0	=ht+rb
32	neutral axis w.r.t. top fiber	yc	=sumLy/sumL	in		0	=rb
33	moment of inertia	Ix	=t*(sumLy2+2*ht^3/12+2*h^3/12-sumL*yc^2)	in^4		=rb	0
34	total depth of section	d	=h+2*rb+t	in		=wt+rb	0
35	neutral axis w.r.t. bottom fiber	yt	=d-yc	in		=wt+2*rb	=rb
36	max compr stress @ first yield	fc	=Mn*yc/Ix	ksi		=F35	=h+rb
37	max tens stress @ first yield	ft	=Mn*yt/Ix	ksi		=F36+rb	=G36+rb
38	slenderness factor	lambda	=1.052*(w/t)*SQRT(ABS(fc/E))/2	none		=F37+w	=G37
39	effective width reduction factor	rho	=IF(lambda<0.673205081, 1, (1-0.22/lambda)/lambda)	none		=F38+rb	=G36
40						=F39	=G35
41						=F39+rb	0
42	**Objective Function name**	**symbol**	**eqn**	**units**		=F41+wt	0
43	moment capacity	Mn	=Mn	in.k		=F42+rb	=rb
44						=F43	=G31
45	**Constraints name**	**value/eqn**	**</>/=**	**value/eqn**	**name**		
46	effective width, b	=b	=	=rho*w	rho * w		
47	max compr stress @ first yield	=fc	<	=Fy	yield stress		
48	max tens stress @ first yield	=ft	<	=Fy	yield stress		
49	cross-sectional area	=(sumL-b+w)*t	<	=Amax	allowable area		
50	overall depth	=d	<	=dmax	max depth		

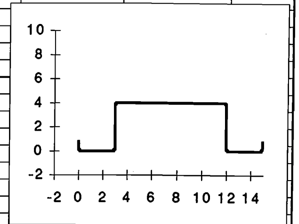

Figure 4. Spreadsheet formulas corresponding to Figure 3.

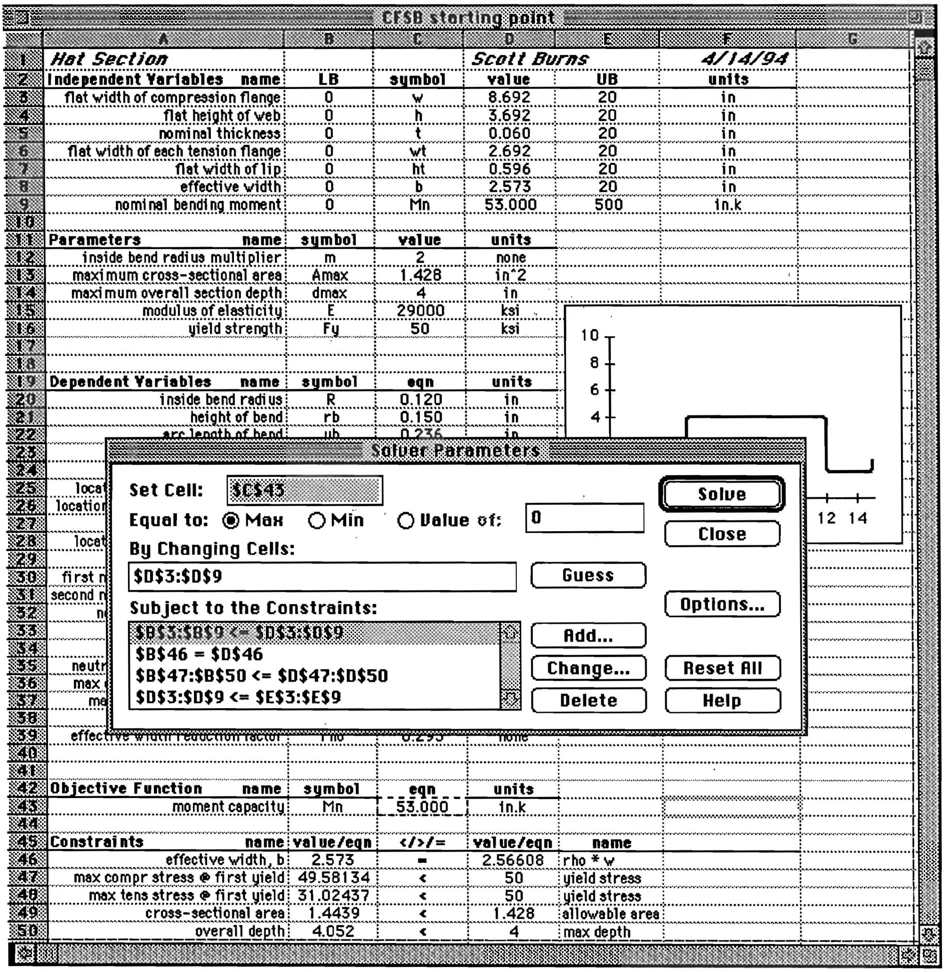

Figure 5. Objective function and constraint definitions.

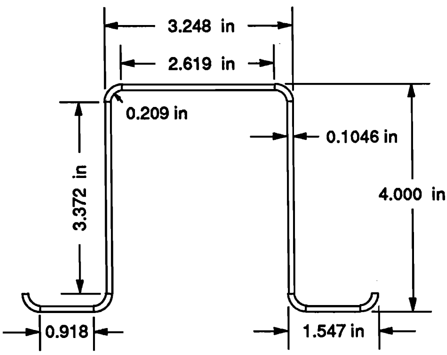

Figure 6. Final optimized section. (Bending moment capacity = 83.0 in-k; cross-sectional area = 1.43 in^2; overall depth = 4 in.)