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# Progress Report on Cold-formed Steel Purlin Design

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# PROGRESS REPORT ON
# COLD-FORMED STEEL PURLIN DESIGN

by Teoman Peköz[1]

## INTRODUCTION

Cold-formed steel panels and decks are frequently used for roofing or wall sheeting in industrial buildings. Such panels and decks are referred to as diaphragms when they are designed to transfer or support loads through in-plane shear resistance. In addition to their enclosure function diaphragms provide bracing to the individual purlins and columns thereby increasing their load carrying capacity.

A typical roof assembly with diaphragm-braced purlins is illustrated in Fig. 1. Diaphragms are usually thin-walled, corrugated or stiffened orthotropic steel panels. Cold-formed lipped channel and Z-section purlins are widely used since they are lightweight and are easy and economical to fabricate. However, these sections are weak in the lateral direction and in torsion. In order to use their full bending capacity in the strong direction they must be braced in the lateral direction and against twisting. Roof panels, by virtue of their shear rigidity and resistance to local bending at the connections may do just that.

In a great majority of metal building applications, the purlins are continuous over the building frames which serve as intermediate supports for the purlins. The continuity is accomplished by lapping the purlins over the supports. The Z-section purlins are lapped by nesting one inside the other. C-section purlins are lapped by placing them back-to-back over the supports. Typical details are illustrated

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in Fig. 7. In each case the purlins are bolted together and the screws connecting the roof panels to the purlins penetrate through both purlins.

Frequently, to reduce lateral deflection and in certain cases to facilitate the erection of the roof system, a brace is used at mid span. This brace, which will be referred to as the intermediate brace, reduces the twist and the lateral deflections and increases the load-carrying capacity of the purlins.

The research reported in this paper is aimed at obtaining an analytical formulation for the stresses and displacements of continuous lipped channel and Z-section purlins with or without an intermediate brace.

ANALYTICAL FORMULATION

The bracing action of the panels by virtue of their shear rigidity was first formulated by Larson (9). Subsequently, the general theory including both the bracing due to shear rigidity of the panels and the restraint offered by the panels to the twisting of beams and columns, was developed by Pincus (8, 14), Errera (2,3,8) and Apparao (2, 3) for doubly symmetric sections. Later Celebi (5) extended the treatment to lipped channel and Z-sections and derived the pertinent differential equations of equilibrium. These differential equations are in the following form:

$$\frac{E(I_x I_y - I_{xy}^2)}{I_x} \frac{d^4u}{dz^4} - Q\frac{d^2u}{dz^2} - Q\, e_D\frac{d^2\phi}{dz^2} + \frac{d^2}{dz^2}(M_x\phi) = -\frac{I_{xy}}{I_x} p_y \tag{1}$$

$$EC_w\frac{d^4\phi}{dz^4} - GK\frac{d^2\phi}{dz^2} - Q\, e_D\frac{d^2\phi}{dz^2} - p_y c\phi + F\phi + M_x\frac{d^2u}{dz^2} - Q\, e_D\frac{d^2u}{dz^2} = p_y\, a \tag{2}$$

where

| | |
|---|---|
| $u$, $v$ | are the deflections of the shear center in the direction of x and y axes, respectively. (Fig. 2) |
| $\phi$ | is the twist angle of the cross-section (Fig. 2) |
| $I_x$, $I_y$, $I_{xy}$ | are the moments of inertia and the product of inertia, respectively. |
| $E$, $G$ | are the elastic modulus and the shear modulus, respectively |
| $C_w$, $K$ | are the warping constant and the St. Venant torsion constant, respectively |
| $Q$, $F$ | are the shear rigidity and the rotational restraint of the diaphragm, respectively |
| $e_D$ | is the vertical distance (positive) of the diaphragm from the shear center (Fig. 3) |
| $a$, $e$ | are the horizontal and the vertical distances (positive) from the shear center to the point of application of the load $p_y$ (Fig. 3) |
| $p_y$ | is the distributed load in the plane of the web (positive for gravity, negative for uplift) |
| $M_x$ | is the bending moment due to $p_y$ |

Celebi obtained a solution for the differential equations of equilibrium using the Galerkin method. For this solution the displacement components are assumed in the form of series:

$$u = \sum_{n=1}^{\infty} u_n \; X_n \qquad (3)$$

$$\phi = \sum_{n=1}^{\infty} \phi_n \; Z_n \qquad (4)$$

$X_n$, $Z_n$ — are the eigenfunctions of nth mode that satisfy the kinematic boundary conditions

$u_n$, $\phi_n$ — are the amplitudes for the eigenfunctions $X_n$ and $Z_n$, respectively

The application of the Galerkin method leads to a solution in the following form:

$$\sum_{n=1}^{\infty} \int_0^L u_n \left[\frac{E(I_x I_y - I_{xy}^2)}{I_x L^2} X_n^{IV} - Q X_n''\right]$$

$$+ \phi_n \left[-QeZ_n'' + (M_x Z_n)''\right] X_m \, d\zeta = -\frac{I_{xy}}{I_x} L^2 \int_0^L p_y X_m d\zeta; \qquad (5)$$

$$m = 1, 2, \ldots, \infty$$

$$\sum_{n=1}^{\infty} \int_0^L u_n \left[M_x X_n'' - QeX_n''\right] + \phi_n \left[\frac{EC_w}{L^2} Z_n^{IV} - (GK + Qe^2) Z_n''\right.$$

$$\left. - p_y e L^2 Z_n + FL^2 Z_n\right] Z_m d\zeta = aL^2 \int_0^L p_y Z_m d\zeta; \qquad (6)$$

$$m = 1, 2, \ldots, \infty$$

The Galerkin method was used with the hope of obtaining a simple solution by considering only one term of the series in Eqs. 3 and 4. However, it was found (5) that three terms of each series need to be included.

A detailed discussion of the general behavior as depicted by this solution is given in Ref. 6.

The integrals in Eqs. 5 and 6 were evaluated by Celebi in the earlier Cornell University research (5) for the case of simply supported spans. This case is indicated as the C.U. solution in Fig. 4. Subsequently, the author in collaboration with Celebi, obtained solutions for the five additional cases shown in Fig. 4 (11). At an end shown as pinned the conditions $u'' = \phi' = 0$ is assumed, while at a fixed end $u' = \phi = 0$. In all but one case, closed form expressions for vibration eigenfunctions that satisfy the end conditions were used for displacement functions. For the case of one end fixed and the other end pinned, numerical solutions for the eigenfunctions were utilized. Functions up to third mode were used in all cases. The resulting solutions are too complex for manual computation. Therefore, a computer program (11) was prepared on the basis of these solutions.

THE COMPUTER PROGRAM (11)

The program was prepared as a design aid for continuous purlins. This program was made available to the profession both in the U.S. and abroad by the sponsors on a preliminary basis pending the experimental verification and their approval.

The program applies to all six cases shown in Fig. 4. The vertical deflection v is computed as in a continuous beam. The end moments are to be computed on the basis of a continuous beam and are input into the program. The continuous beam analysis may include the effect of doub le purlins in the area of overlap over the supports. The rotation and the lateral deflections are assumed to be prevented at an intermediate brace. The vertical deflection is assumed not to be affected by an intermediate brace.

The case of one end fixed and the other hinged is to be used, for end spans, whereas the case of both ends fixed is to be used for intermediate spans.

To facilitate nesting during transport and storage, most Z-section purlins have stiffening lips that are at an angle greater than ninety degrees with the flanges. The program also takes into account the effect of stiffening lips that are at an arbitrary angle with the flange for Z-sections.

BRACING ACTION PARAMETERS

The parameters that pertain to the bracing action of the diaphragm include shear rigidity Q and rotational restraint F. Though significant advances have been made in computing Q (4, 7, 10), currently the general practice is to determine it by a cantilever shear test. The procedure and the test set-up is described in Ref. 1. Fig. 5 gives a schematic of the test and a typical load deflection plot. A reliable design value for the effective shear modulus, $G'_{dr}$, for the diaphragm in question can be determined on the basis of the load-deflection plot as follows (2).

$$G'_{dr} = \frac{2}{3}\left[\frac{0.8\ P_{ult}\ a}{\Delta_d\ b}\right]$$

where

| | |
|---|---|
| $P_{ult}$ | is the ultimate load in the cantilever shear test. |
| $\Delta_d$ | is the deflection corresponding to a load of 0.8 $P_{ult}$. |
| a, b | are the dimensions as shown in Fig. 5. |
| $\frac{2}{3}$ | is a factor to obtain a reliable value for shear modulus for design. |

A reliable design value for Q is obtained using $G'_{dr}$ as:

$$Q_{dr} = G'_{dr} \cdot w$$

where w is the width of the diaphragm contributing to the bracing of one purlin.

The rotational restraint is also to be determined experimentally. The rotational restraint is a function of both the cross bending rigidity of the diaphragm and the local rigidity of the diaphragm to resist the twisting of the purlin. According to Ref. 5, the cross bending rigidity is much higher than the local rigidity. Thus, the local rigidity determines the rotational restraint. For the case of uplift loading, the rotational restraint can be found using a test setup as shown in Fig. 6. A more complete description of the test setup and the procedure is given in Ref. 12. The load V simulates the effect of the uplift on the connection while the load P induces the rotation of the purlin. A test setup for determing F for the gravity loading case is currently being studied.

The rotational restraint is defined as

$$F = \frac{M}{\theta}$$

where

M is the torsional moment per unit length of the purlin.

$\theta$ is the rotational angle of the purlin.

It was found (12) that for simulated uplift case the value of F is sensitive to the uplift force V and the relationship between them is quite nonlinear. A recommendation for a reliable design value of F will be made after the completion of the experimental work described below.

SPECIAL CONSIDERATIONS

As mentioned above the formulation and the computer program gives the stresses and the displacements of the purlin for a given loading and a set of parameters of the problem. Engineering judgement and several special considerations are essential in the use of the program in design. The following is a brief discussion of some of these considerations.

Provided that the roof panels or the connection between the roof panels and the purlins do not fail prematurely, the failure of the purlins may be precipitated by yielding or by local buckling or by a combination of both. Thus the failure stress is to be determined on a basis that includes these possibilities. The design load, then, is to be determined by applying an appropriate factor of safety to the load that causes failure stress. This is because of the nonlinear relationship between the loading and the stresses.

The purlins may have sweep and twist before they are loaded. These could be due to the imperfections initially present in the purlins or due to those introduced during the erection process. The effect of the initial sweep and twist, though not included in the formulation, must be considered in the design process. This can perhaps be done in terms of an appropriate factor of safety.

The maximum allowable twist angle similar to that for vertical deflections also needs to be established.

Another important consideration relates to the connection between the roof panel and the purlin. In the case of uplift, the pull-over strength of the screw connection is influenced very significantly by the prying action caused by the twisting of the purlins. For this case, the test setup for the rotational restraint described in the preceding section may also be used to determine the pull-over strength. In the case of gravity loading local failure of the roof panels over the purlins also needs to be checked.

EXPERIMENTAL INVESTIGATION

The earlier research at Cornell University reported in Refs. 5 and 6 included several large scale single span purlin assembly tests. The agreement between the experimental and the analytical results was satisfactory.

The current phase of the research on continuous purlins also includes large scale testing. First, two Z-section and one lipped channel purlin assemblies were tested at Cornell University subjected to uplift loading. These tests will be discussed below. Second, one Z-section and one lipped channel purlin assembly was tested under gravity loading by Wiss, Janney, Elstner and Associates, Inc., of Northbrook, Illinois. These tests have just been completed (June 1975) and the results are being evaluated. A decision for the next phase of the research will be reached on the basis of the evaluated results.

The results of the three uplift tests are reported in Ref. 12. The test assembly was as shown in Figs. 7 and 8. The two-purlin assemblies had three continuous spans of 25 feet each. All the details were to the greatest extent possible as they are in construction practice. The loading was applied by two commercial vacuum cleaners. Polyethylene sheet and tape for sealing was placed between the purlin and the roof deck as shown in Fig. 8. To complete the set-up, polyethylene was taped securely to the floor to create an airtight space for pulling the vacuum needed to simulate wind uplift loading. Strains, deflections and twist angles of the purlins were measured at various points. Fig. 9 shows a general view of a failed Z-section purlin assembly. Fig. 10 shows a close up view of a typical local buckle that appeared in the same test. In addition to full scale tests, several tests were conducted to determine Q and F. In these tests the specimen configurations discussed above in the section "Bracing Action Parameters" were modified to reflect the effects of the special configuration of the large scale test assembly.

An evaluation of the results of uplift tests including their comparison with those predicted by the computer program is presented in Ref. 13. In general the test results are in good agreement with the computed results.

## SUMMARY AND CONCLUSIONS

An analytical formulation for the behavior of diaphragm braced cold-formed steel purlins was obtained. A program based on this formulation was prepared and made available to the profession on a preliminary basis. This program may be applied to simple span and continuous purlins with or without intermediate braces provided that proper end conditions are used to account for continuity and care is exercised with regard to the effects not included in the formulation. Such effects include local buckling, initial imperfections and special local conditions at lapped splices.

Currently a test program is underway to study the applicability of the computer program. The analytical and experimental results so far are in general agreement.

## ACKNOWLEDGEMENT

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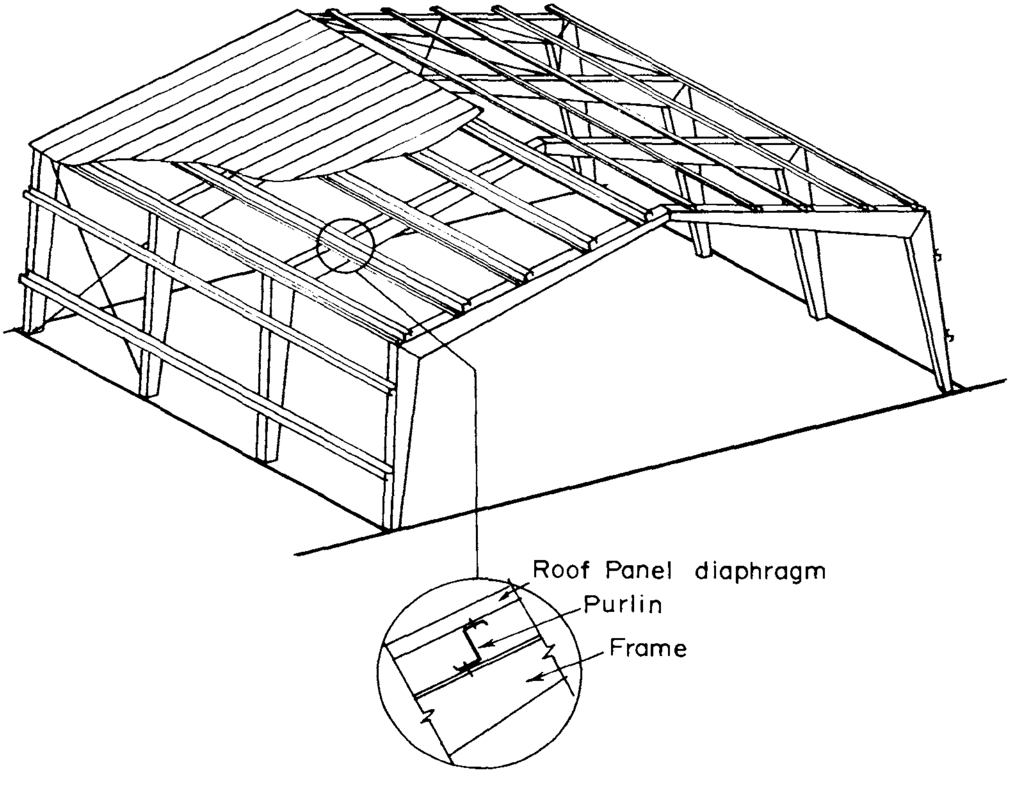


Fig. 1 A typical roof assembly with diaphragm braced purlins

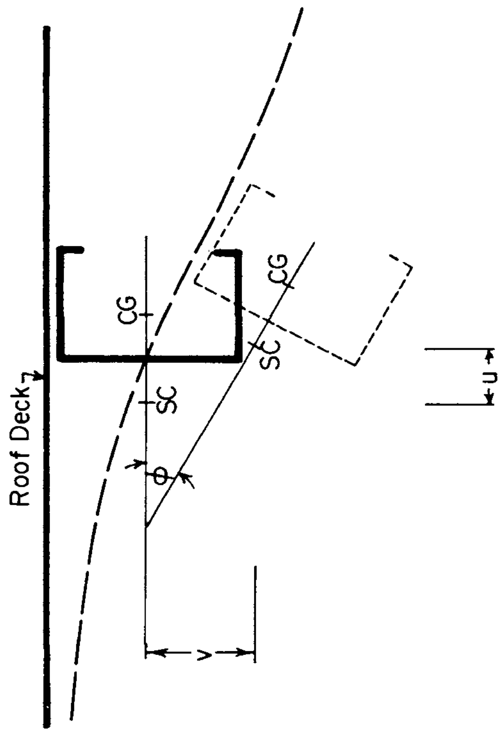


Fig. 2 Displacement components

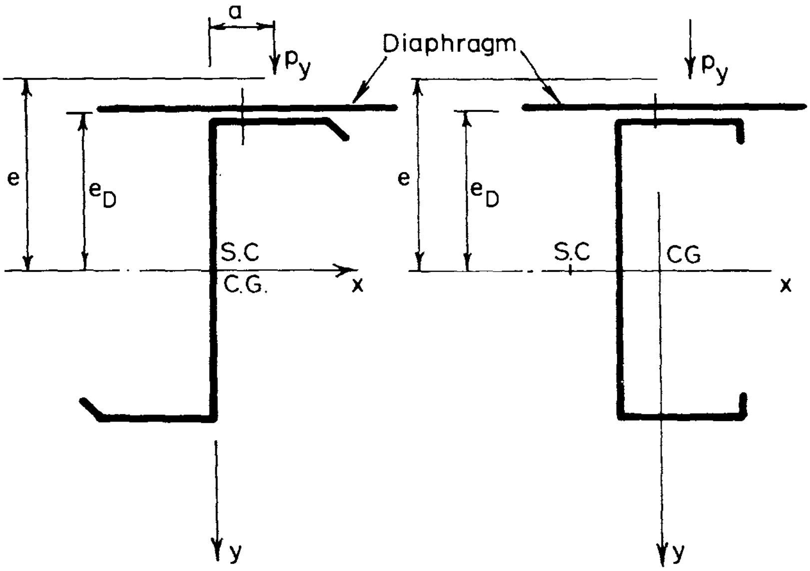


Fig. 3 Notation for the point of application of the load

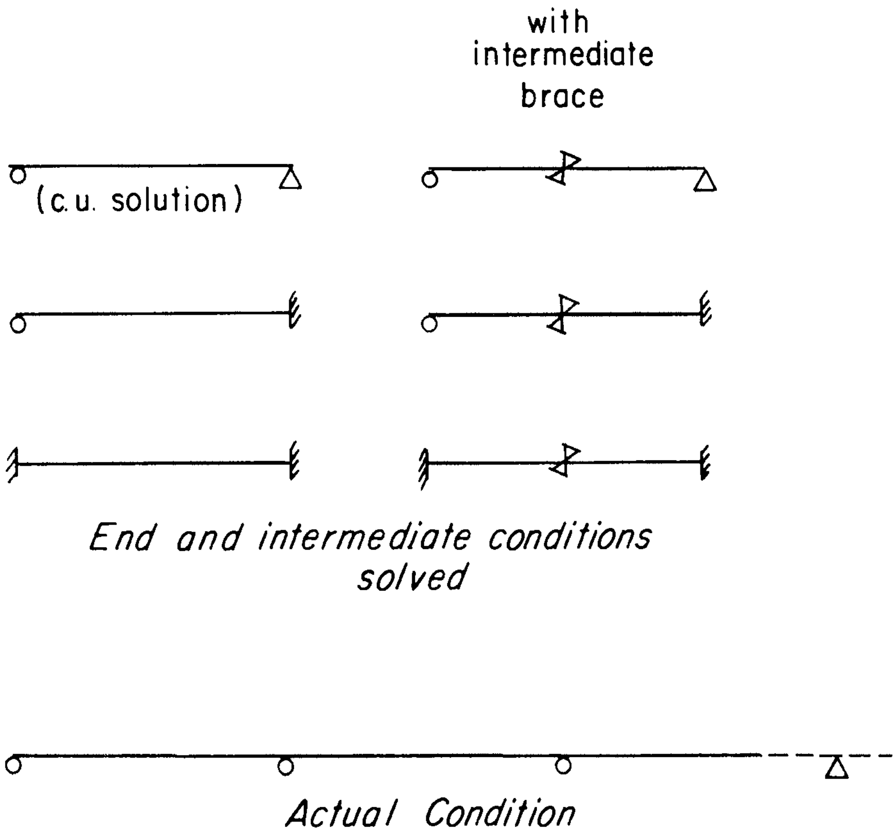


Fig. 4 Idealized end and intermediate conditions

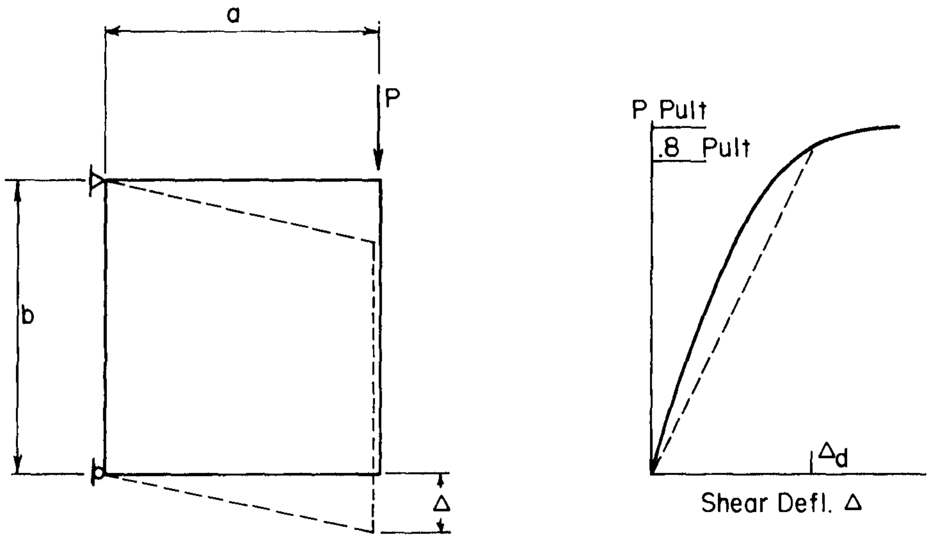


Fig. 5 Determination of shear rigidity Q

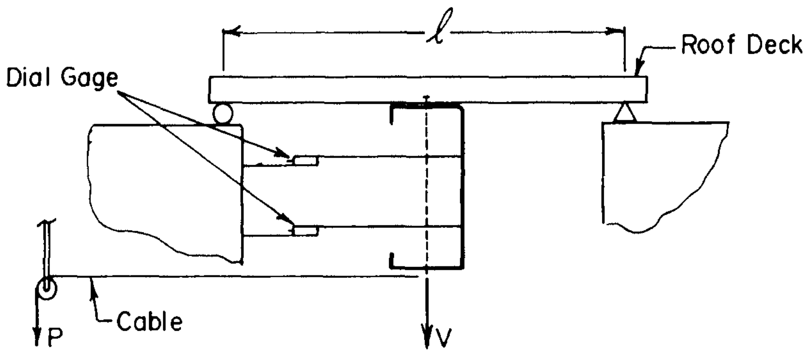


Fig. 6 Test setup to determine rotational restraint F

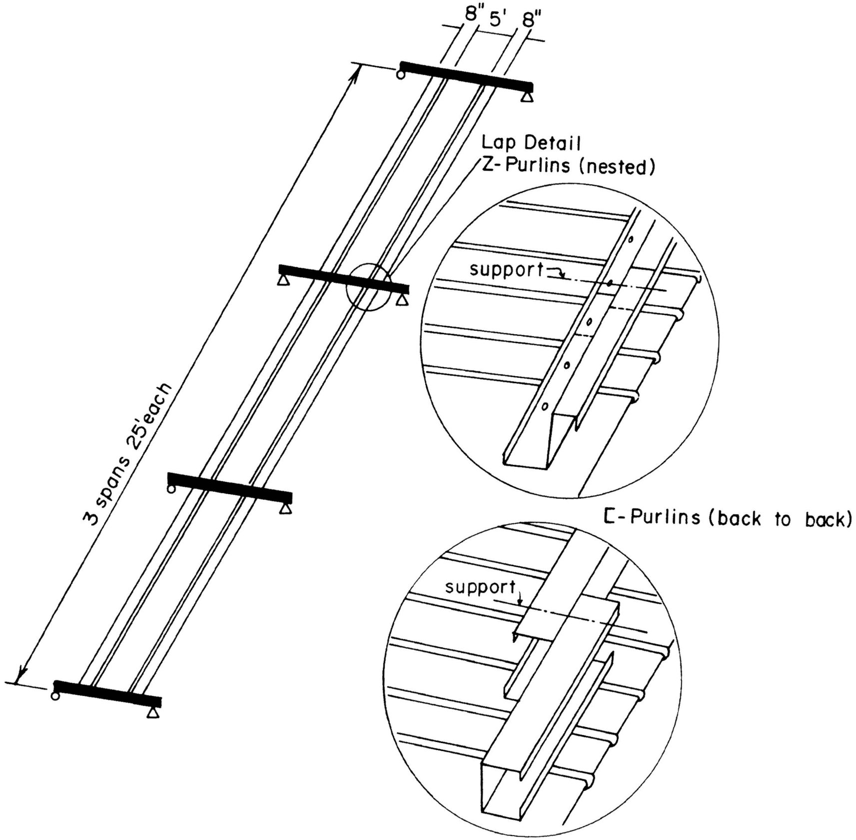


Fig. 7 Test assembly and overlapping details

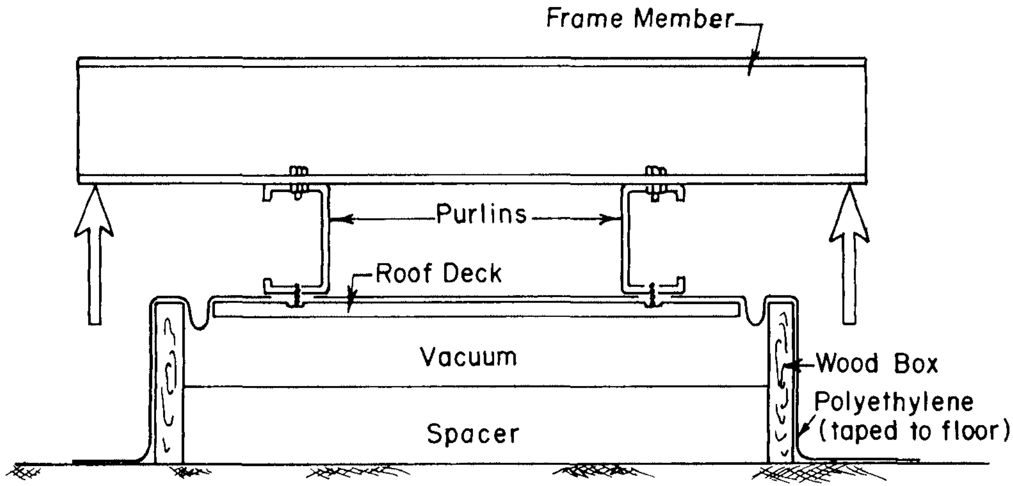


Fig. 8 Cross-section of the uplift load test assembly

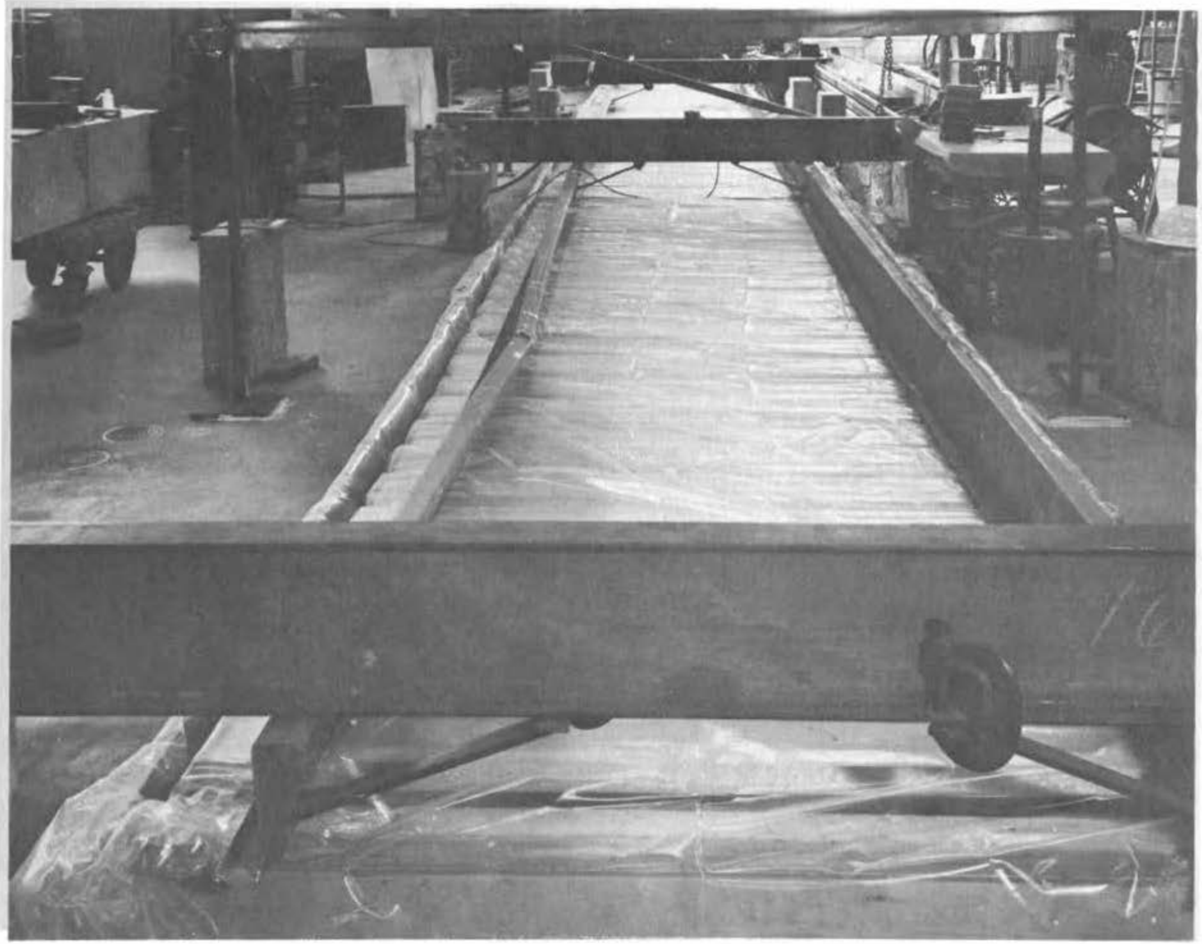

Fig. 9. View of tested Z-section purlin assembly

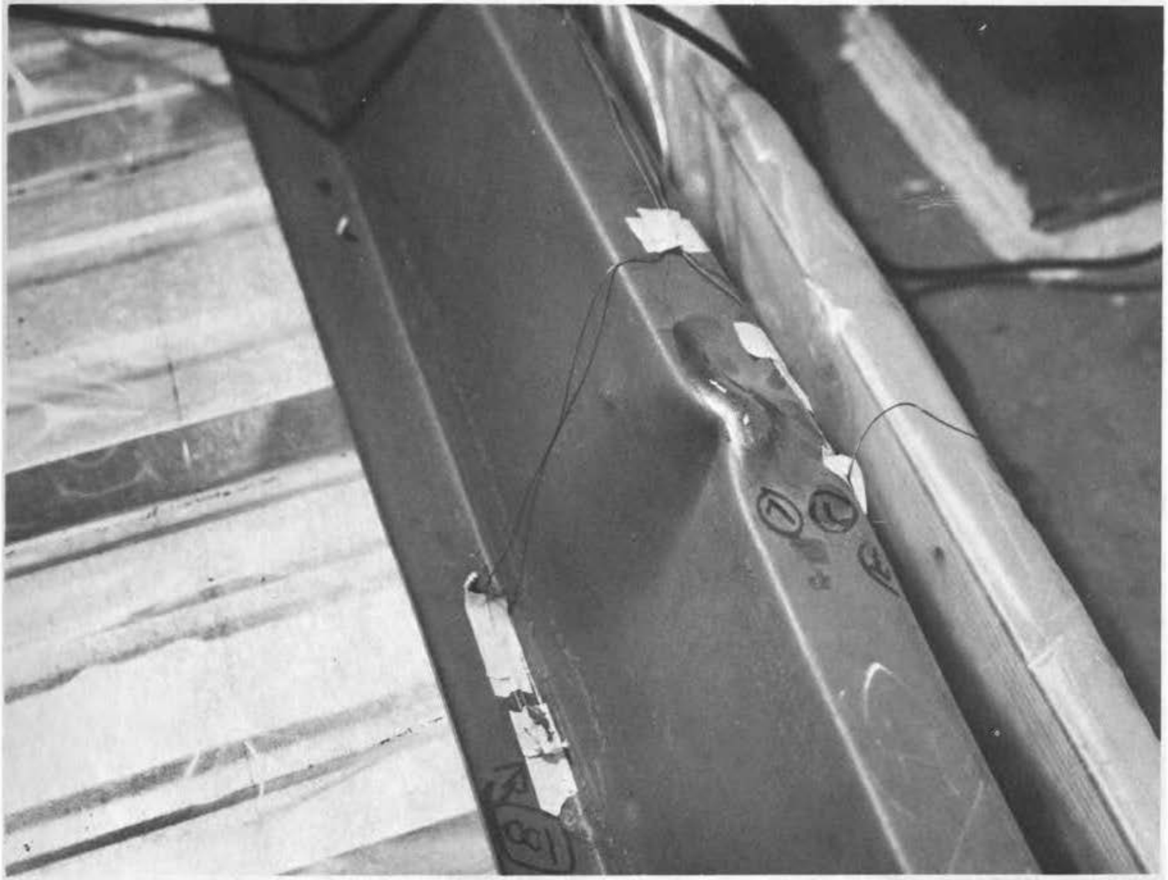

Fig. 10. View of typical local buckle