# Warping Restraint, Member Stability and Standards

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# WARPING RESTRAINT, MEMBER STABILITY AND STANDARDS

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## SUMMARY

A preliminary experimental investigation into the influence of warping restraint on axially loaded unlipped channels is presented together with a description of a rig for testing thin-walled beams subject to different boundary conditions and loading. The results from the column tests are compared with various Standards and the current differences in the degree of warping restraint assumed by various design standards examined. Finally, suggestions are made to change the philosophy used in the treatment of warping restraint in design.

## 1 INTRODUCTION

The instability of thin-walled members has been under investigation for many years and extensive research work[1,7,8,9,10,13,14] and Standards, Codes and Recommendations[2,3,5,6] are available. The majority of this work, however, is based on ideal support conditions which rarely occur in practice. Without additional information on practical connections designers must resort to choosing those ideal conditions which most closely fit the practical situation. The choice of these conditions is of the utmost importance as they can have a considerable effect on both the member's buckling mode and failure load. The degree of warping restraint applied to a member is one such important boundary condition.

Warping occurs when the twisting of a member results in the cross-sections distorting out-of-plane along the direction of the member's longitudinal axis. Most cold-formed members (i.e. all except closed hollow circular sections) have cross-sections which tend to warp when subject to torsion. If the out-of-plane distortion is restrained or prevented at any particular cross-section, longitudinal shear stresses and strains are developed in the member. These shear stresses act in conjunction with those due to St. Venant torsion to resist the applied torque. Hence, if warping restraint is applied to a member, the torsional stiffness may be considerably greater than it would be if the section were allowed to warp freely. Warping restraint, therefore, can have a considerable influence on the behaviour of cold-formed steel members, particularly columns.

Different degrees of warping restraint are assumed by various Standards. This leads to significant differences in the importance placed on flexural-torsional buckling in the determination of the mode of failure by different design standards and hence on the magnitude of the design load. The two boundary conditions most commonly adopted for warping are free and fully-fixed. These ideal conditions are rarely attainable in practice, with the degree of restraint provided by actual connections in service lying somewhere between these two ideal extremes.

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It would therefore be beneficial to be able to assess the degree of warping restraint provided by different connections and the influence this has on member stability, and input this information into design standards.

This paper describes the preliminary experimental investigations conducted at the Building Research Establishment into the influence of warping restraint on member stability and examines the current differences in the degree of warping restraint assumed by various design standards. Recommendations are made to change the philosophy used in the treatment of warping restraint in design

## 2 EXPERIMENTAL INVESTIGATION

### 2.1 Column tests

To establish the limits of warping restraint on column instability a series of tests on various lengths of an unlipped channel section with fixed and free warping was undertaken.

The end supports were designed to enable the following boundary conditions to be applied:-

(1) Simple supports about both principal axes and free warping.
(2) Simple supports about both principal axes and fixed warping.

Pinned supports were achieved by loading the ends of each column through rigid steel plates which were free to rotate about the major and minor axes of the cross-section. Furthermore, these steel plates were designed so that the axial load was applied to the centre of gravity of the unlipped channel's gross cross-section.

The free warping boundary condition was attained by designing a loading plate which loaded the channel section at it's three positions of zero sectorial co-ordinate (for the definition of sectorial co-ordinates see ref. [15]). This loading plate consisted of a stiff steel plate on which two knife egdes were mounted at right angles. These knife edges located into three knife edge supports which were bolted to the channel section such that the supports were at the points of zero sectorial co-ordinate. The whole assembly was then mounted on a spherical bearing and was free to rotate about the major and minor axes of the cross-section. This detail is shown in figure 1.

Figure 2 shows the arrangement for fixed warping which consisted of a rigid steel plate into which the channel section was clamped. Again this was mounted on a spherical bearing.

The dimensions of the unlipped channel are given in figure 3. Each section was formed from mild-steel coil using a brake press. In the case of fixed warping twenty tests were undertaken on six different lengths(1000mm, 1500mm, 2000mm 3000mm, 3500mm and 3700mm) of the unlipped channel. For free warping a total of thirteen tests were carried out on six different lengths(250mm, 500mm, 2000mm, 2500mm, 3000mm and 3500mm) of unlipped channel. A minimum of two tests were carried out for each different length.

Coupons taken from the undamaged parts of the columns were used to determine the yield stress in accordance with BS 18 part 2[4] and the average results for both flanges and the web are shown in Table 1.

Table 1 Material properties

| | |
|---|---|
| Average yield for flange 1. | 258 $N/mm^2$ |
| Average yield for flange 2. | 230 $N/mm^2$ |
| Average yield for web | 235 $N/mm^2$ |

A 5000kN(500Ton) Avery Universal testing machine was used to apply the load to each column and the axial displacements were measured with a dial gauge. A general arrangement of the test set-up is shown in figure 4.

The load-deflection characteristic shown in figure 5 is for a simply supported and free to warp channel section column subject to an axial load and is typical of those obtained for all the tests identified above.

For each of the two series of tests (i.e.tests on columns with and without warping restraint) three different modes of failure were identified. These are shown in figure 6. At low slenderness ratios the columns failed by local buckling of the flanges, while at high slenderness ratios the columns failed by buckling about their minor axis. Between these two ranges failure was by a combination of twisting and bending about the major axis, i.e. flexural-torsional buckling.

Figure 7 compares the results of columns with free and fixed warping, from which it is observed that columns with fixed warping buckle at a significantly higher axial load than those with free warping over the middle range of slenderness ratio. This is because warping restaint has more effect on flexural-torsional buckling than on local or purely flexural buckling. These results also indicated that the difference in buckling load can be as much as 64% between the fixed and free warping conditions.

## 2.2 Beam tests

As part of investigations on the influence of practical boundary conditions and loading on beams, the test rig shown in figure 8 has be designed to enable different degrees of warping restraint and different combinations of bending and torsion to be applied to the cold-formed steel beam under test.

At the supports, the web of the beam is fixed in aluminium diaphragms cut to accommodate the beam profile; these are mounted directly on ball bearing units to provide a simple support. These supports allow the beam to rotate in both the horizontal and the vertical planes but, the captive roller bearings at the ends of the support frames prevent the beam from twisting at the support in the vertical plane. One supporting frame also allows movement in the longitudinal direction of the beam.

The degree of warping restraint can be varied by the number of thin, profile cut aluminium restraint diaphragms located on the beam behind the support diaphragm. These restraint diaphragms are designed to prevent movement of the flanges due to warping. A beam without any restraint diaphragms will tend towards free warping, whereas one with several, closely-spaced diaphragms will tend towards fixed warping. The weight of this restraint arrangement behind the support is counterbalanced by a weight on a threaded rod attached to the front of the end support frames.

Loads are applied to the beam through a profile cut aluminium loading diaphragm which carries the loading weights on a rolling carriage. This system was developed to load all the cross-section rather than one locality. It also allows the applied load to move with the deflecting and rotating beam so that the initial point of application of the load to the section remains unchanged. The profile cut in the loading diaphragm is so positioned such that the initial point of load application on the section coincides with the centroid of the diaphragm. The rolling carriage allows the section and loading diaphragm to rotate under the load but the applied weights always remain vertical. Rotation is about the centre of the circular diaphragm and the line of action of the applied loads also passes through this point due to the movement of the rolling carriage.

All movements and rotations at the supports are measured by deflection transducers and systems of pointers and angular and linear scales. These systems are shown in figure 9. Vertical and horizontal deflections and rotation of the beam web are measured by deflection transducers attached by thin wires to three points on the web at several cross-sections along the length of the beam. This system of wires and transducers has to be employed because of the coincident horizontal and vertical deflections. The distance between the section and the transducer is such that the greatest inclination of the wire never exceeds 5 degrees so that errors in displacement due to this inclination are negligible.

At the time of writing this paper, the test rig for beams is still undergoing trials to prove its operation and no results are as yet available. BRE is also undertaking a survey to establish a set of commonly used cold-formed section connections and the degree of warping restraint provided by these connections. Once their degree of warping restraint has been determined the beam rig described above will be used to determine the influence of warping restraint on the stability of beams with practical end connections.

## 3. COMPARISON OF STANDARDS WITH EXPERIMENTAL RESULTS FOR COLUMNS

Comparing the BS 5950 part 5 curve in figure 10 with the experimental results shows BS 5950 to be very conservative. These members buckle locally below their ultimate capacity causing some parts of the section to become ineffective resulting in a movement in the position of the centroid. BS 5950 part 5 overestimates the effect of this centroidal shift. If, however, we ignore the effect of centroidal shift then the curve shown in figure 11 is produced. This curve underestimates the buckling load for columns with fixed warping and overestimates the buckling load for the columns with free warping. Such a result is to be expected since the warping condition adopted in BS 5950 part 5 is between the extreme conditions but closer towards fixed warping.

The European recommendations[6] for "Torsional or torsional-flexural buckling" give little guidance on this subject and refer the designer to a suitable text. As flexural-torsional buckling is a relatively unknown phenomenon for designers these recommendations should include something on the conditions to be taken into account and the degree of warping fixity[12]. Furthermore, inspection of those clauses dealing with "Members in Flexure and Compression" and in particular the sections on "Flexure in the plane of symmetry - Lateral torsional buckling not prevented" reveal some ambiguity regarding the orientation of the member's axes. Because of this confusion the buckling load for comparison with the experimental results has not been calculated.

At the start of the section relating to concentrically loaded compression members the AISI Specification[2] states:-

> "This section applies to members in which the resultant of all loads acting on the member is an axial load passing through the centroid of the effective section.."

The literal interpretation of this statement prevents its application to concentrically loaded unlipped channels because these members experience centroidal shift due to local buckling. However, it could be argued that such a section should be designed as a beam-column and the section on combined axial load and bending be used. But here again the interaction formula requires the calculation of the allowable axial load in accordance with the previous section and therefore no comparison of experimental results with this Specification has been made. However, the equations used for flexural-torsional buckling in this Specification are, in general, conservative and assume no warping restraint.

## 4. DISCUSSION

It can be seen from the foregoing discussion that each of the Standards considered are applicable for a different set of warping boundary conditions. The AISI Specification assumes no warping restraint whereas BS 5950 part 5 assumes a partial warping restraint. The ECCS Recommendations do not quantify the degree of warping restraint to be used. As practical connections apply a variety of different warping restraints and these are unlikely to agree with the values given in the relevant design documents a better approach would be to let the designer choose that boundary condition which most closely fits the practical situation. This philosophy cannot be sucessfully included in Standards unless it is founded on a comprehensive set of commonly used connections and the warping restraint which they apply to members. BRE, in collaboration with the Universities of Strathclyde and Salford and supported by the Cold Rolled Sections Association and the Science and Engineering Research Council, are currently undertaking a survey to identify commonly used, cold-formed connections and determine their warping restraint. The results of this work should then provide the information required to implement such a philosophy.

## 5. METHODS OF INCORPORATING VARIABLE WARPING RESTRAINT IN STANDARDS

It would be possible to incorporate the phiosophy described in section 4 into design standards with relatively minor amendments to the existing provisions. Possible methods of amending the various Standards are described below but in the present absence of experimental data on the warping restraint provided by practical connections one arbitrary value of partial warping restraint has been adopted.

For flexural-torsional buckling, BS 5950 Part 5 uses effective length multipliers (α factor) in the Perry-Robertson interaction curve. This approach is simple to use and the only amendment required would be the production of an alternative set of effective-length multipliers for the different warping restraints. Rhodes[11], suggested that three sets of effective length multipliers could be used. One corresponding to a fully effective connection (i.e. fixed warping), a partially effective connection and an inffective connection with regard to warping. The variation in the effective length factors can easily be determined for these three different connections by using the effective column length factors 0.5, $1/\sqrt{2}$ and 1 in the equation for torsional buckling. The results of these calculations for a unlipped channel section are illustrated in table 2.

Table 2. α factors for a unlipped channel with free,partial and fixed warping.

| D/B | 1.0 | | | 1.2 | | | 1.6 | | |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{L}{r} \times \frac{t}{B}$ | FREE | PART′ | FIX′ | FREE | PART′ | FIX′ | FREE | PART′ | FIX′ |
| 1 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 2 | 2.07 | 1.62 | ---- | 1.86 | 1.43 | ---- | ---- | ---- | ---- |
| 3 | 1.84 | 1.52 | 1.24 | 1.70 | 1.36 | 1.08 | 1.41 | 1.09 | ---- |
| 4 | 1.63 | 1.41 | 1.19 | 1.53 | 1.28 | 1.05 | 1.31 | 1.04 | ---- |
| 5 | 1.46 | 1.31 | 1.15 | 1.38 | 1.20 | 1.02 | 1.21 | 1.00 | ---- |
| 6 | 1.33 | 1.23 | 1.10 | 1.26 | 1.13 | ---- | 1.11 | ---- | ---- |
| 7 | 1.22 | 1.15 | 1.06 | 1.16 | 1.06 | ---- | 1.04 | ---- | ---- |
| 8 | 1.14 | 1.09 | 1.02 | 1.07 | 1.00 | ---- | ---- | ---- | ---- |
| 9 | 1.08 | 1.04 | ---- | 1.01 | ---- | ---- | ---- | ---- | ---- |
| 10 | 1.03 | 1.00 | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 11 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |
| 12 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |

For the unlipped channel illustrated in figure 3, (D/B=1.6 and t/B=0.04) we can use Table 2 to determined the Alpha factors for free, partial and fixed warping and use these values inplace of the ones given in BS 5950 part 5 to predict the buckling load. Figure 12 compares these three curves with the experimental results. As in figure 11 the effect of centroidal shift has been omitted. The curve corresponding to free warping shows good agreement with the experimental results but the fixed warping curve is conservative. This is because the imperfection factor used in the Perry-Robertson formula is also conservative.

The AISI code could be amended by replacing the equation for torsional buckling with the following equation which has an additional factor for warping restraint:-

$$\sigma_t = \frac{1}{Ar_o^2}\left[GJ + \frac{W_r \pi^2 EC_w}{(K_t L_t)^2}\right]$$

where

$W_r$ is the warping restraint factor and can have the following values:-

| | | |
|---|---|---|
| $W_r=1$ | - | Free warping |
| $W_r=2$ | - | Partial warping |
| $W_r=4$ | - | Fixed warping |

The ECCS Recommendations could be amended if the relevant clause was expanded to include the following:-

Flexural-tosional buckling of a column is given by:-

$$P_{TF} = \frac{1}{2\kappa}\left[(P_{Ex}+P_T) - \{(P_{Ex}+P_T)^2 - 4\kappa P_{Ex} P_T\}^{\frac{1}{2}}\right]$$

where

$P_{Ex}$ is the elastic flexural buckling load for a column about the x axis and is given by:-

$$P_{Ex} = \frac{\pi^2 EI_x}{L_E^2}$$

$P_T$ is the torsional buckling load of a column and is given by:-

$$P_T = \frac{1}{r_o}\left(GJ + \frac{W_r \pi^2 EC_w}{L_E}\right)$$

$\kappa$ is a constant and is given by:-

$$\kappa = 1 - \left(\frac{x_o}{r_o}\right)^2$$

$W_r$ is the warping restraint factor and has the following values:-

| | | |
|---|---|---|
| $W_r=1$ | - | Free warping |
| $W_r=2$ | - | Partial warping |
| $W_r=4$ | - | Fixed warping |

All other symbols are defined in the appendix - notation

## 6. CONCLUSIONS AND RECOMMENDATIONS

1. The limited experimental work presented indicates that the degree of warping restraint applied to axially loaded, cold-formed unlipped channel section columns can have a significant influence on the load at which these columns become unstable. This result prompted a study into the main rules pertaining to the treatment of flexural-torsional buckling in Standards for Cold-formed steel. The results of this study indicated:-

a. Each code adopts a different approach for the treatment of flexural-torsional buckling and a different philosphy for the treatment of warping restraint.

b. BS 5950 Part 5 adopts a single value of partial warping restraint in its equations for buckling and this approach may produce non-conservative results when used to design columns with less warping restraint.

c. The literal interpretation of the AISI code prevents it application to the design of concentrically loaded unlipped channels. The equations for flexural-torsional buckling assume no warping restraint.

d. The ECCS Recommendations do not give specific guidance on the degree of warping restraint to be used in design.

2. As practical connections apply a variety of different warping restraints and these are unlikely to agree with the values given in the relevant design documents, a better approach would be to let the designer choose that boundary condition which most closely fits the practical situation. Incorportating such a philosophy into current design Standards would be possible with relatively minor amendents and the following recommendations are made:-

a. The degree of warping restraint provided by practical connections should be determined.

b. The treatment of warping restraint in design Standards should be amended to allow the designer to choose the warping restraint which most closely fits the practical situation.

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APPENDIX:--REFERENCES

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APPENDIX.--Notation

$B$ Breadth of section
$C_W$ Warping constant.
$D$ Depth of section.
$E$ Youngs modulus of elasticity.
$G$ Shear modulus.
$I_x$ Second moment of area about the x axis.
$J$ Torsion constant.
$K_t$ Effective length factor for twisting.
$L$ Length of member between supports.
$L_E$ Effective length.
$P_{EX}$ Elastic flexural buckling load for a column about the x axis.
$P_T$ Torsional buckling load of a column.
$P_{TF}$ Flexural-torsional buckling load for a column.
$r$ Radius of gyration.
$r_o$ Polar radius of gyration about the shear centre.
$t$ Material thickness.
$W_r$ Warping restraint factor.
$x_o$ Distance from the shear centre to the centroid measured along the x axis.
$\alpha$ Effective length multiplier for warping restraint.

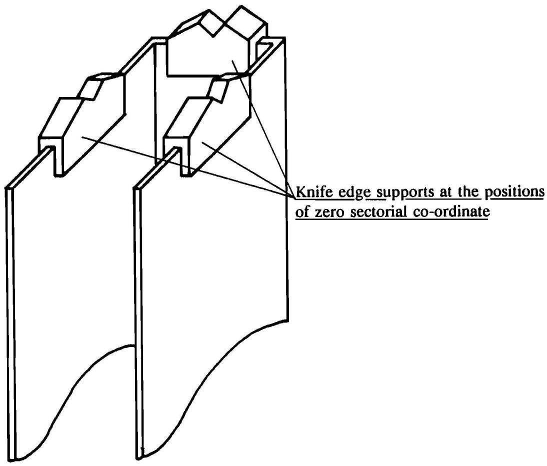



FIGURE 1 : Loading plate for free warping boundary condition for column tests

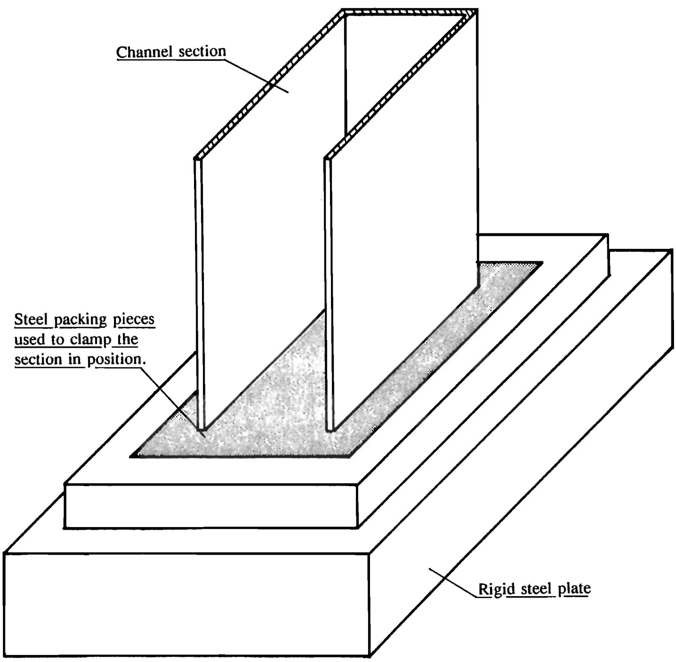



FIGURE 2 : Loading plate for fixed warping boundary condition for column tests

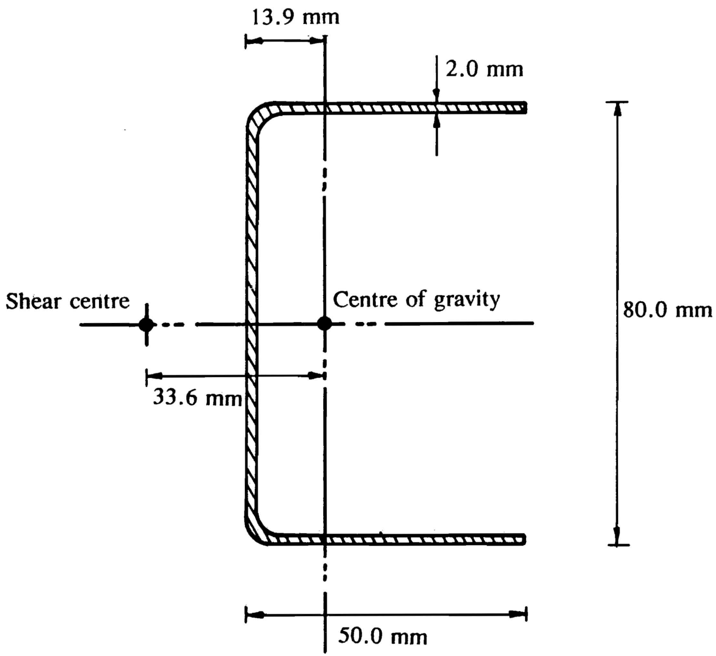



FIGURE 3 : Dimensions of the unlipped channel section used in the column tests

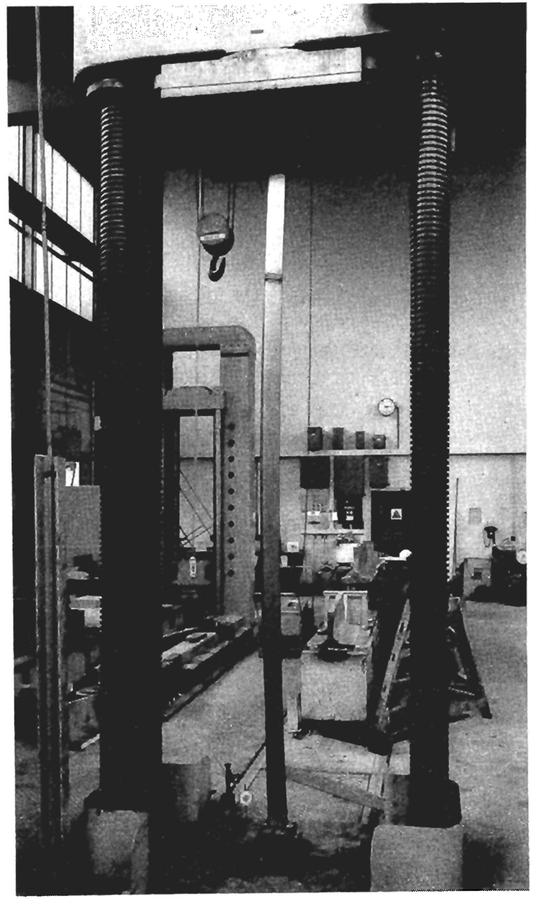


FIGURE 4 : General arrangement of the test set-up for columns

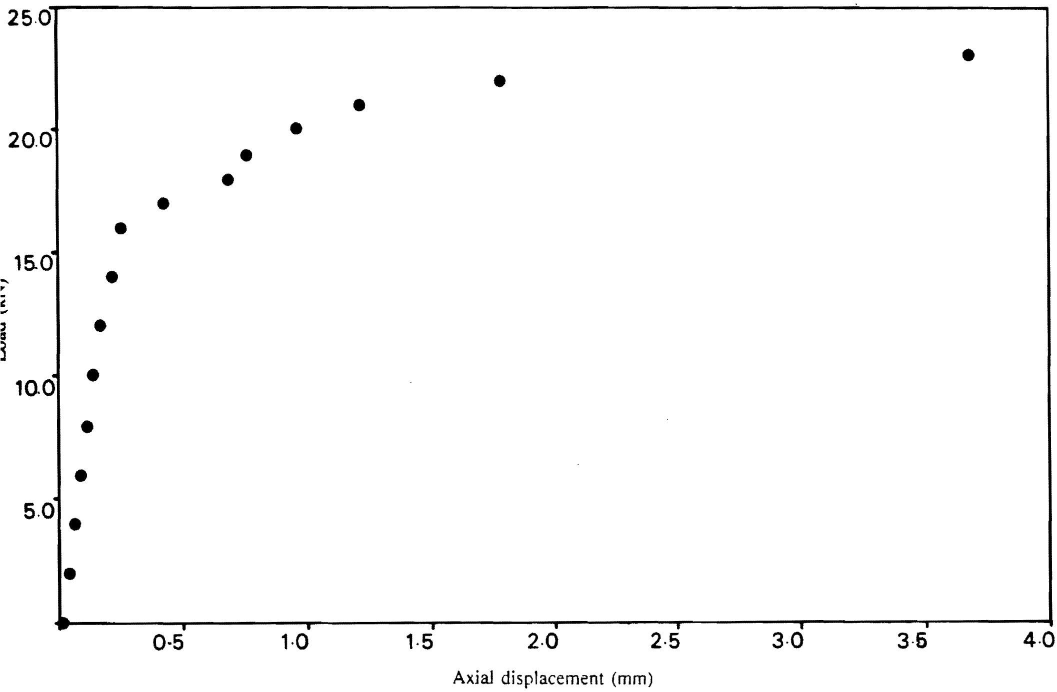



FIGURE 5 : Load vs deflection for simply supported, free to warp channel section column

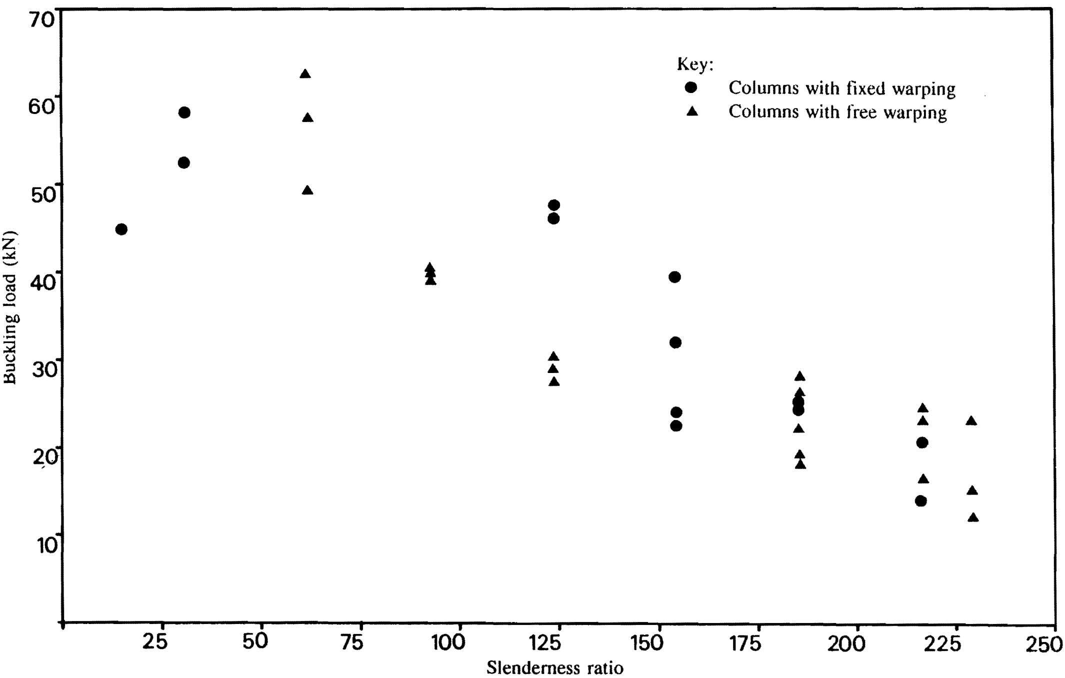
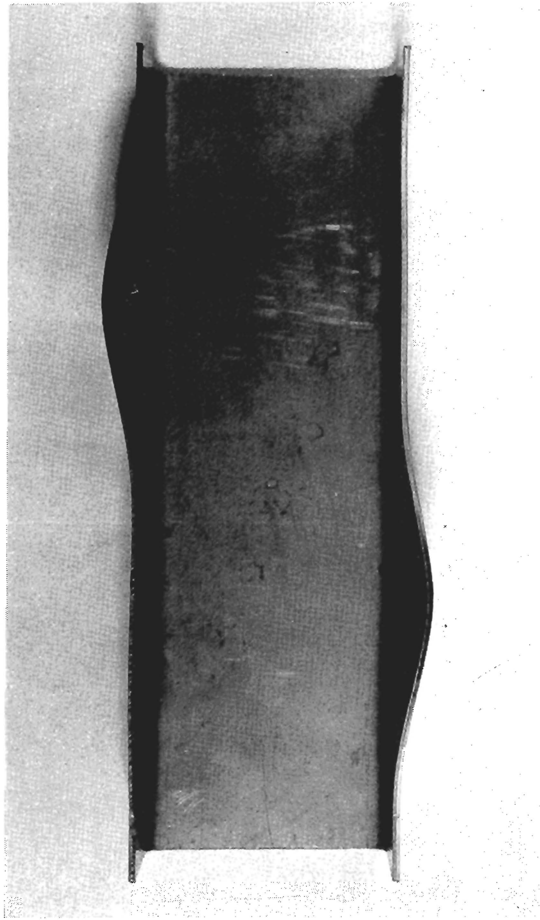
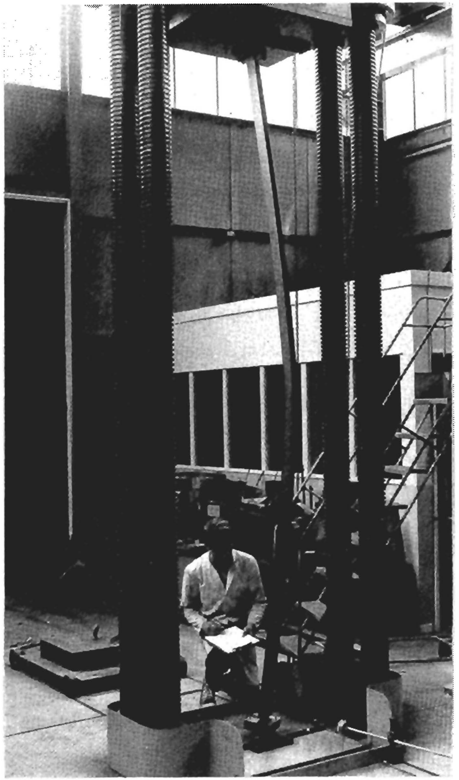



FIGURE 7 : Comparison of test results for columns with free and fixed warping

(a) Local buckling



(b) Flexural buckling



(c) Flexural-torsional buckling

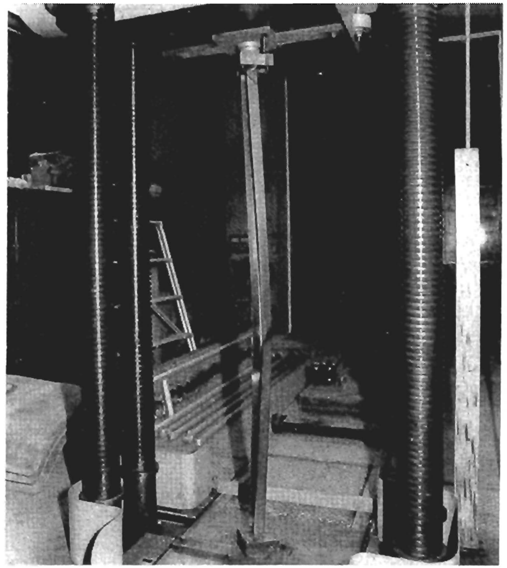


FIGURE 6 : Modes of failure identified in the column tests

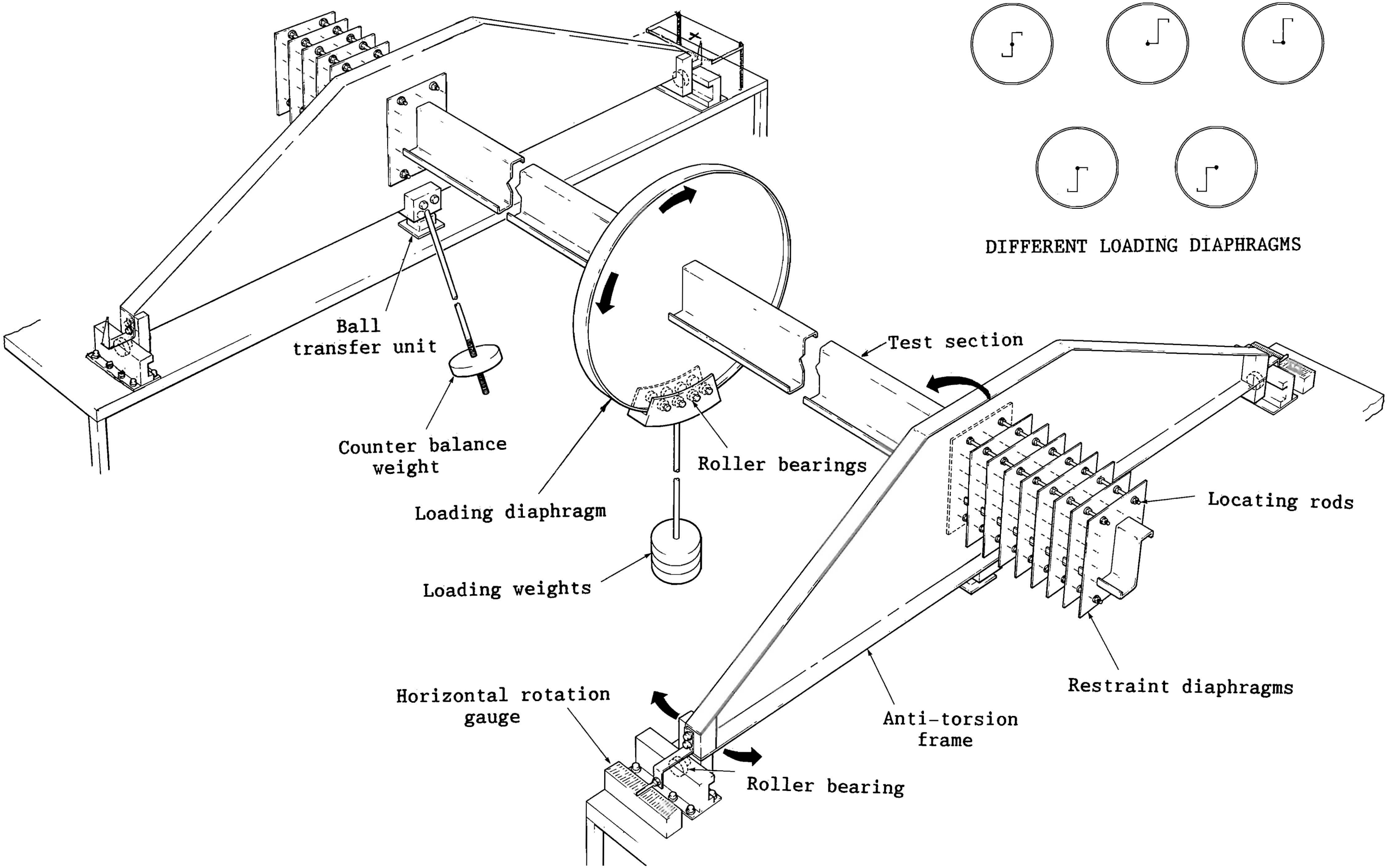



FIGURE 8 : General arrangement of the test set-up for beams

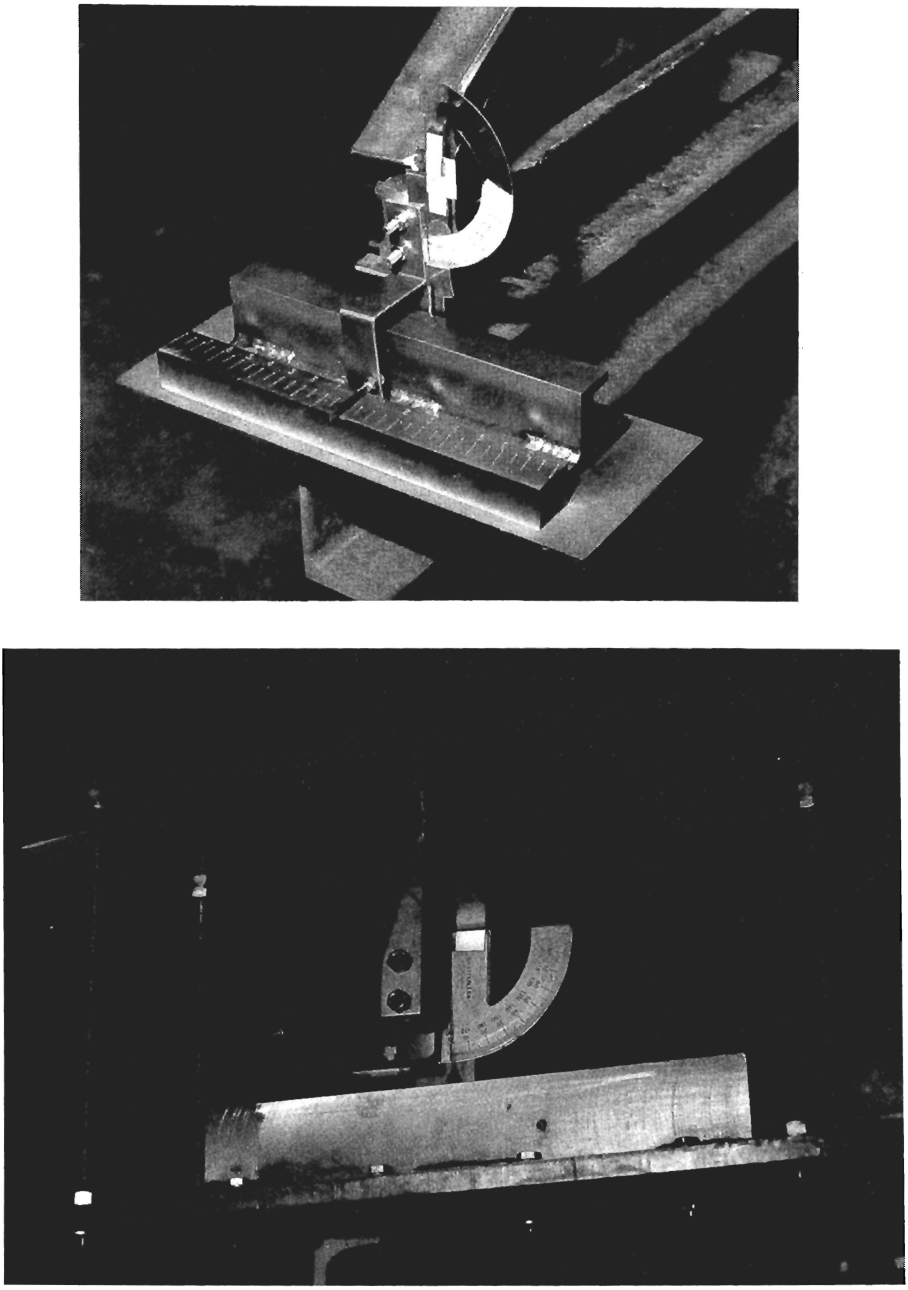


FIGURE 9 : Systems for measuring movements and rotations at the supports

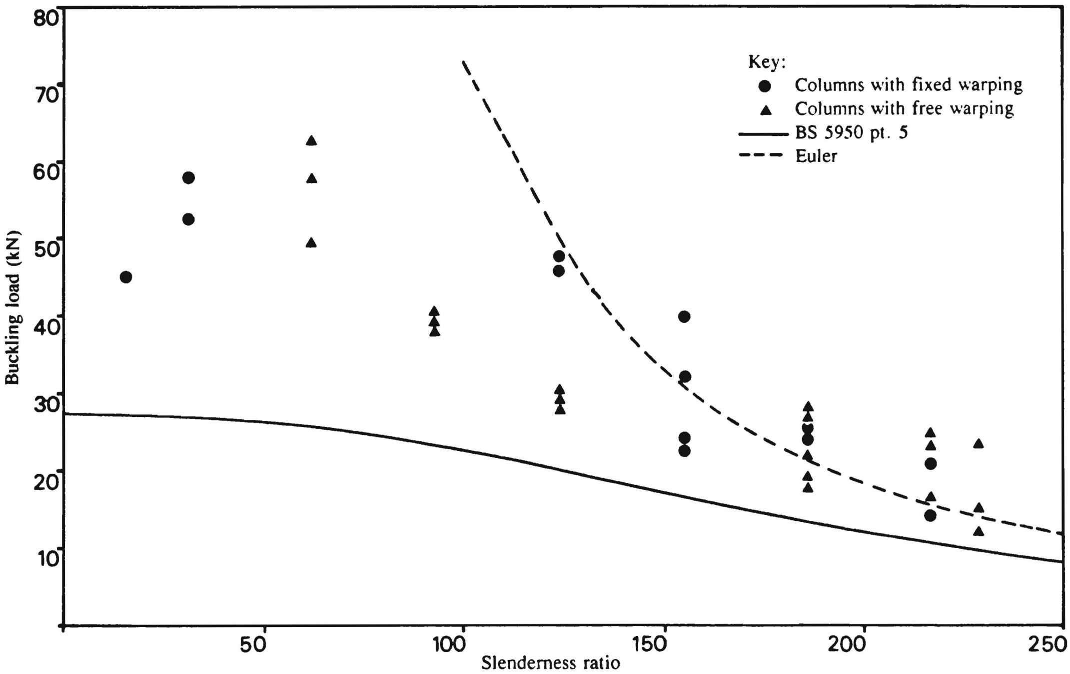



FIGURE 10 : Comparison of column test results with BS 5950 Part 5.

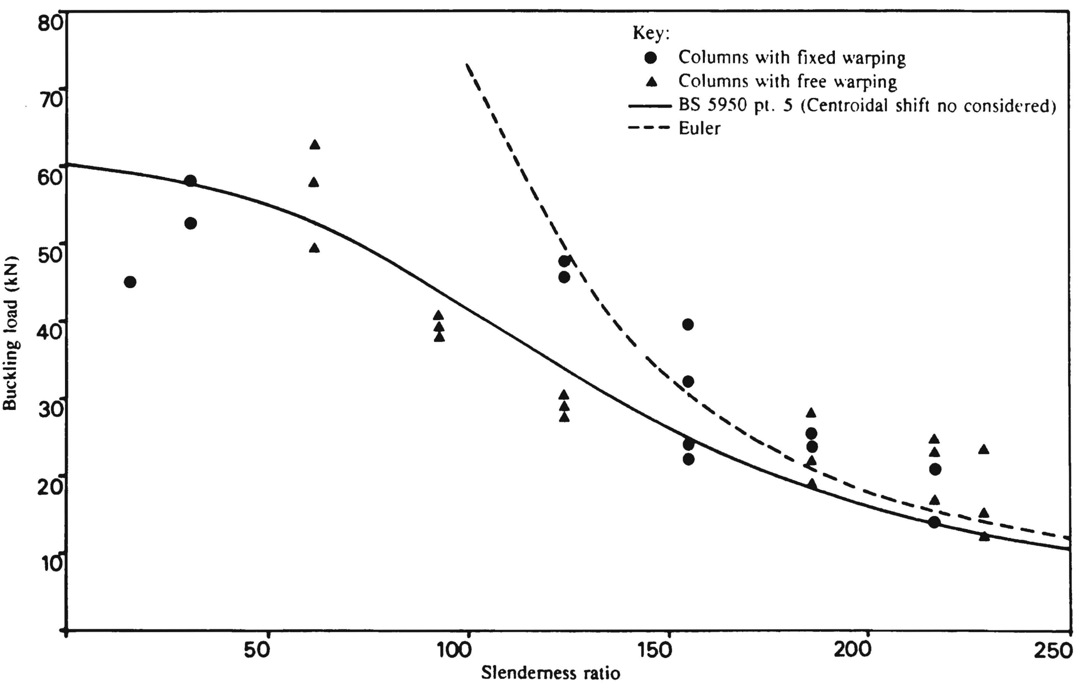



FIGURE 11 : Comparison of column test results with BS 5950
(centroidal shift not considered)

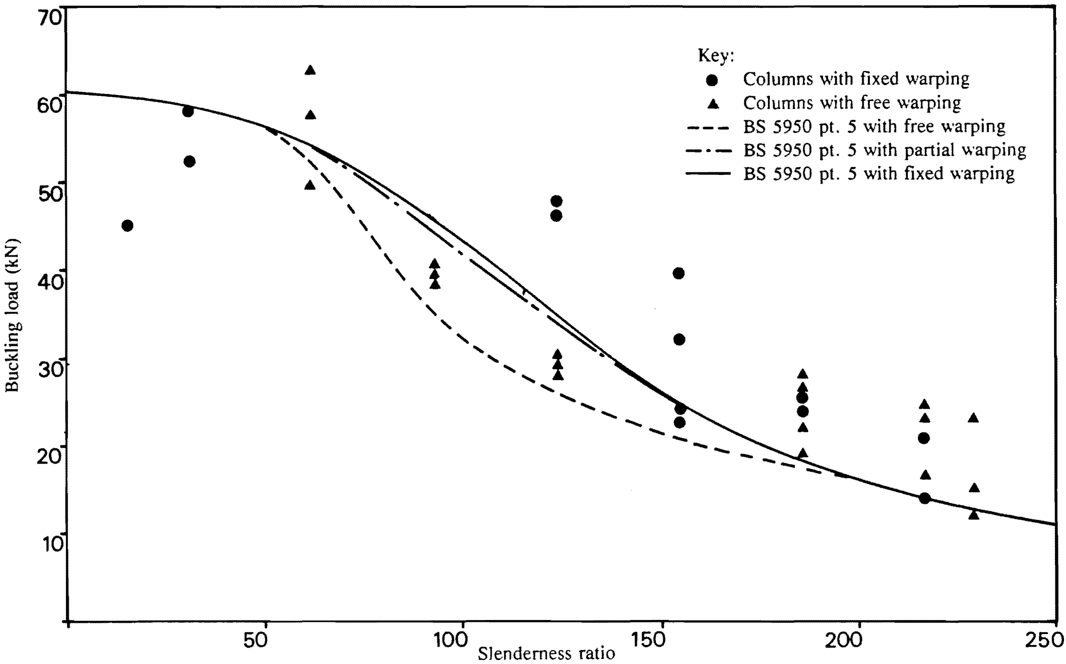



FIGURE 12 : Comparison of column test results with BS 5950 as amended to allow fixed, free or partial warping restraint