Aug 20th, 12:00 AM

Cold-formed Folded Plate Structures

Wilbur C. Schoeller

Richard H. J. Pian

Harry R. Lundgren

COLD-FORMED FOLDED PLATE STRUCTURES

by

Wilbur C. Schoeller,[1] M. ASCE
Richard H. J. Pian,[1] F. ASCE
and Harry R. Lundgren,[2] M. ASCE

INTRODUCTION

The object of the research work on steel folded plates conducted at Arizona State University has been to investigate the behavior of cold-formed steel folded plate structures including both parallel and tapered chords of rectangular, square, and circular plans (Figures 1 and 2). The studies have included (1) the formulation of theoretical methods to determine stresses and displacements based on simplified analysis, and (2) the development of stress, displacement, and stability analyses employing finite element techniques. Fabrication and material characteristics peculiar to cold-formed steel necessitate analysis procedures that are different from those employed for other materials.

Folded plate systems of cold-formed steel in long span roof may be considered as deep girders tilted from the usual vertical position so that the flanges of adjacent girders are in contact either at the ridge or valley line (Figure 3). Effective use of material is provided since the longitudinal sloping plates (or panels) not only carry the web shear of the tilted girders but also serve as the transverse structural roof deck system. The various components can be used in many different combinations to provide attractive variation in appearance. In addition to performing a structural function, the roof deck may serve as the finished interior ceiling.

SIMPLIFIED ANALYSIS

The following general assumptions are used in the simplified analysis of cold-formed steel folded plates:

(a) Material is homogeneous and elastic.

(b) Longitudinal edges (at ridge and valley) are simply supported (not continuous).

(c) Principle of superposition is applicable.

(d) Individual plates possess negligible torsional resistance.

(e) The supporting members (diaphragms, frames, walls, columns, etc.) do not provide restraint against rotation of the ends of the plates.

(f) The effects of deformations are neglected for stress analysis of the system.

The cold-formed steel folded plate system with parallel chords and rectangular plans may be considered as divided into two parts: (1) a simple transverse span supported at folded lines, and (2) a series of simple girders spanning longitudinally between end supports. The load normal to the surface is carried by the flexural strength of the individual cold-formed steel panels and the reaction of the transverse panel strip is applied as a line loading along the fold lines. The action of the folded plate units in resisting this load is similar to that of inclined deep girders braced by adjacent plates with top flange (bent plate) in compression, bottom flange in tension, and web elements in shear (Figure 4). If the web contribution for flexural strength is neglected, the longitudinal flange force in fold line members is obtained by dividing the bending moment of the deep girder by its depth. The shear force is resisted by the shear diaphragm action of the steel panels.

For tapered folded plates (or pleated dome) with circular or square plans, a similar approach can be followed. Here, radial tapered plate elements span between a high-level compression ring near the center and a lower tension ring near the perimeter (Figure 2). Adjacent plates are attached to a common fold line member (bent plate) at the ridge or valley by screws, bolts, or welds. If the secondary effects due to the deformations of the fold line members, the tapered plates (or panels), and the inner and outer rings are assumed to be negligible, the following procedure may be used for analysis:

(1) First consider each inclined tapered plate to be a simple transverse (tangential) span supported at fold lines. The load normal to this surface is carried by the flexural strength of the individual light gage steel panels. The panel span length varies from a maximum at the perimeter to a minimum near the center.

(2) The reaction of the tangential panel strip is applied as a line loading along the fold lines. The resulting loads are resolved into plate loads that are carried to the support at the perimeter by the inclined plate (or panel) acting as a girder with its top and bottom flanges each being half of a fold line member and the web being the steel panels. The center ring supplies a couple and a horizontal force to the girder, the outer ring provides a horizontal force, and a web stiffener at the outer ring transmits the shear reaction from the girder to the ring.

(3) Using a simplified procedure it is possible to determine the forces (or stresses) of various parts of the structure neglecting the deformations of the inner and outer rings, the axial deformations of the flanges, and the shear deformation of the web.

In addition to the consideration of strength, the deflection of the folded plate structures should be investigated, particularly for long-span structures. The plate elements may be assumed as temporarily separated at the fold lines, and the in-plane deflections of the individual plates can be computed. The true deflection of the fold line can be found if the plates are brought back into coincidence. This can be done analytically or graphically (Figure 5).

RECTANGULAR PLANFORM FOLDED PLATES USING FINITE ELEMENTS

General Method of Analysis. A more precise method of analysis than that previously described has been developed in which the method of finite elements is utilized to obtain stresses and displacements for cold-formed steel folded plate structures. It applies to structures that are rectangular in plan in which the plates can be orthotropic and the properties can vary from plate to plate.

Support restraints can be varied, although restraints must be rigid; that is, elastic supports cannot be accommodated. Loads are defined as vertical and horizontal projection uniformly distributed loads and vertical and horizontal line loads on fold lines.

The solution is formulated using the basic stiffness method, wherein the plate element stiffnesses are obtained by employing the finite element techniques based upon an assumed displacement function. Fold line member strains are compatible with the extensional strains of the plate elements. Displacements are obtained for a specific loading and then back substituted to yield the stresses.

Plate Element Stiffness. A stiffness matrix is developed for each panel. The panel is first divided into a specified number of elements each of which has five degrees of freedom at each corner, consisting of the two rotations

[1]Prof. of Engrg., Arizona State Univ., Tempe, Az.

[2]Assoc. Prof. of Engrg., Arizona State Univ., Tempe, Az.

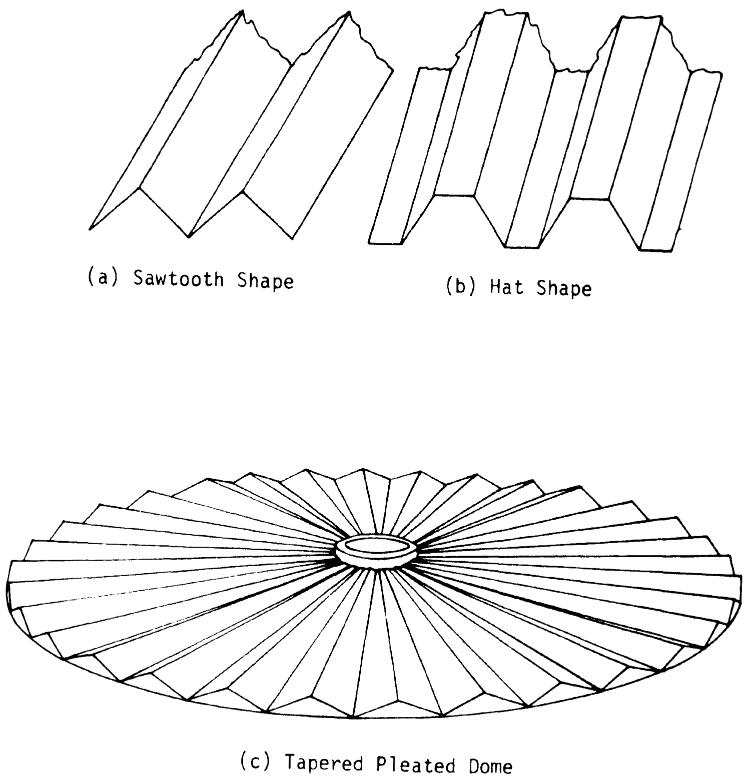

FIGURE 1 FOLDED PLATE SHAPES

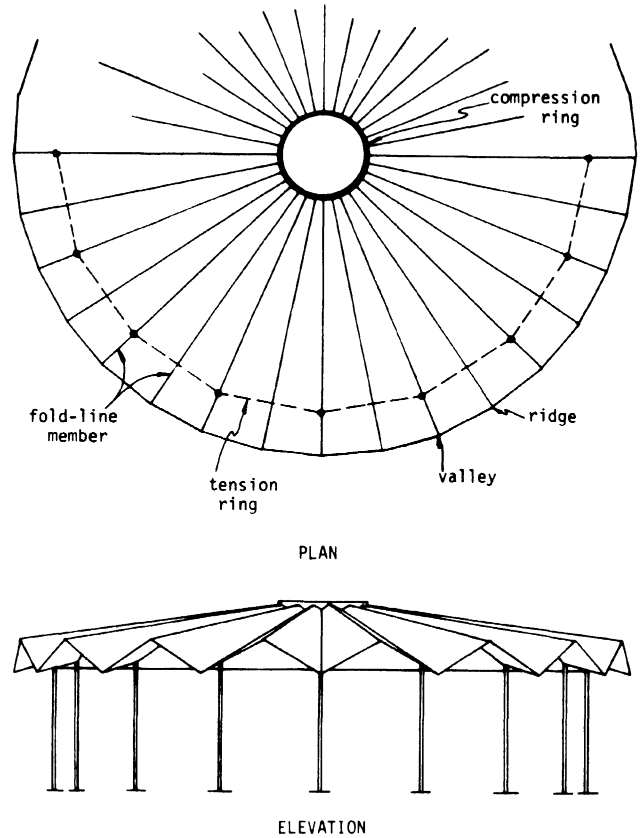

FIGURE 2 TAPERED FOLDED PLATE

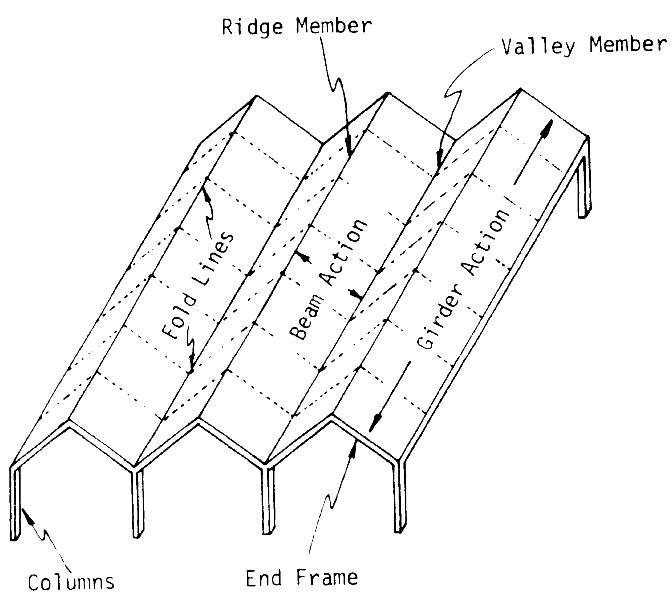

FIGURE 3 FOLDED PLATE BUILDING

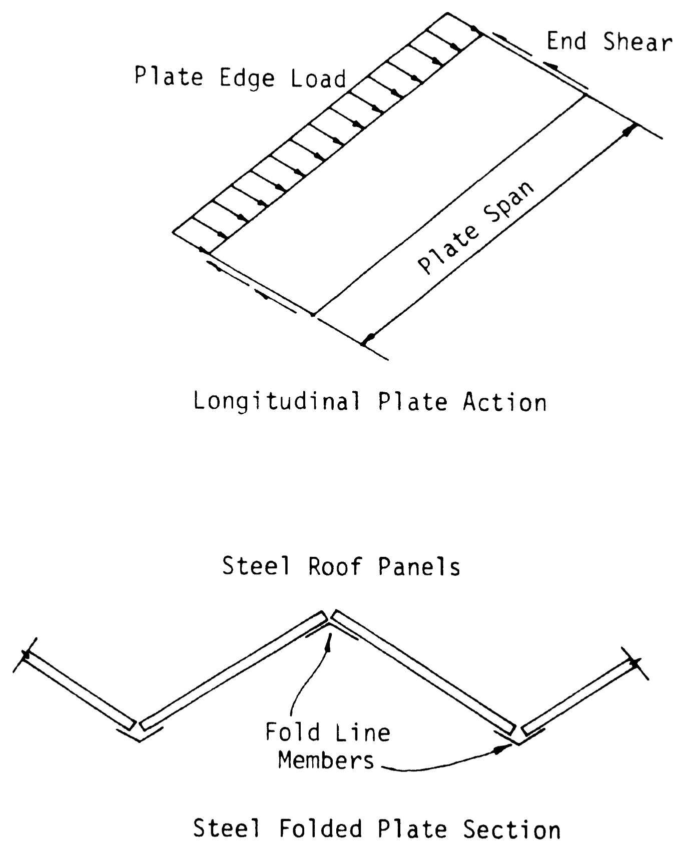

FIGURE 4

about the axes in the plane of the plate and the three linear displacements coincidental with the orthogonal axes. A vector representation of the degrees of freedom is shown in Figure 6. w, θ_x, and θ_y represent the bending displacements and are related to the "bending element." u and v represent the in-plane displacements and are related to the "plane stress element." Stiffnesses of the "bending element" and of the "plane stress element" are generated independently and superimposed.

The bending element stiffness is formulated for the basic rectangular element (Figure 7) using the displacement function presented by Zienkiewicz and Cheung (4).*

Plane stress triangular elements based on linear strain (3) are used to provide the in-plane behavioral characteristics of the plate. Linear strain is selected in preference to constant strain in order to describe the plate strains with as few elements as necessary. In addition, the elements are overlapped to minimize the directional effects inherently caused by the geometric proportions of the individual elements.

The linear strain triangle requires six nodal points; therefore the three corner nodes are supplemented by nodes at the mid-length of each side, thereby providing six per element.

Since the elemental pattern for rectangular plates most naturally leads to a rectilinear nodal pattern, the triangular elements and nodes are arranged

half the actual element thickness, are superimposed to provide the total thickness (Figure 9). For example, one element has nodes a, b, d, and e at its corners. Another has nodes b, c, e, and f.

The combination of these two elements provides the five degrees of freedom per node shown in Figure 6. The normal finite element procedure is deviated from at this point to accommodate a special problem caused by the lack of moment continuity across the fold lines. This problem has been resolved by the formulation of special bending elements to be used at the fold lines that incorporate a hinged edge condition into the formulation.

Assembly of Elements. The panel is formed by assembling the appropriate basic elements in such a way that the panel is constrained to conform to the physical support system.

In order to reduce the degrees of freedom (and thereby conserve core space) internal degrees of freedom are eliminated. That is, the stiffness of the panel is formulated in terms of the nodes on the panel boundary (Figure 10).

In the reduction process, all nodes designated "■" in Figure 10 will be eliminated from the formulation. This results in the stiffness being expressed in terms of the "o" nodes and hence eliminates 18 of the original 44 nodes--a considerable conservation of core space. It should be noted that this process yields the same accuracy of results as if the nodes had not been eliminated.

Assembly of Panels. After the stiffness of each panel has been obtained and stored (in auxiliary storage) the panel stiffnesses are combined to form the structure stiffness matrix.

Consider an example comprised of six panels in a "sawtooth"-type folded plate structure (see Figure 11 for the structure plan and elevation).

First panel I is transformed from its own axes system to the structure axes system (shown in Figure 11). Then panel II is also transformed to the structure axes. Now it can be seen that the two panels share common nodes along fold line B.

The same procedure is followed for the remaining fold lines resulting in the stiffness matrix for the entire structure being formed by superimposing the transformed stiffnesses of each element.

Program Utilization. The application of the program is limited to folded plate structures comprised of rectangular panels as well as having a rectangular plan. The number of panels that can be specified will vary with the storage capability of the computer on which the program is run. This is not considered a serious limitation, however, since solutions can be performed for typical interior and end panels that would probably provide the required design information. The present edition of the program has the supports located at the ends of the panels to simplify the input data. This could, however, be altered to allow for supports at interior positions. Figure 12 shows end elevations of some structures that can be analyzed by this program.

A common problem was solved using both the Simplified Method and the Finite Element program. A multiple sawtooth folded plate structure with a span of 40 feet was analyzed subjected to a uniformly distributed gravity load. Critical results were as follows:

	Simplified	Finite Element
Maximum Fold Line Force	22,500#	23,400#
Maximum Plate Shear	1,024#/ft	958#/ft

Since the simplified method yields only typical interior panel results, no comparison could be made for the end panels. The deflection pattern yielded in the computer analysis was consistent with physical restraints and symmetry and appeared reasonable.

CIRCULAR PLANFORM FOLDED PLATES USING FINITE ELEMENTS

Background. The initial studies that were made on tapered folded plate structures with circular plan form were based upon certain simplifying assumptions with regard to the action of the structure. The loads in the fold line members were determined on the assumption that each trapezoidal segment was rigid. These computed loads were then applied to the individual trapezoidal panels and the stress distribution in the fold line members and the webs were determined by method of finite elements.

The latest computer program to be developed considers the entire dome, rather than a segment of it, permits a more refined displacement analysis and thus allows a closer study of behavioral characteristics of pleated domes under various load conditions. This program assumes a uniform gravity load condition but can be readily adapted to partial loading as well as wind loading.

Method of Analysis. The stiffness method of analysis is used in which stiffness coefficients are obtained for nodal points that are assumed along the fold line members. Displacements are determined for load systems that are assumed to be applied only at these nodal points. Stresses in each panel are then determined from the displacements.

Plate Element Stiffness. Each trapezoidal panel is assumed to be subdivided into 28 trapezoidal segments (Figure 13) and each of these into four triangular segments (Figure 14) yielding in all 112 triangular segments. Using the finite element method, each triangle is assumed to be in a state of uniform plane stress throughout and stiffness coefficients are obtained in terms of the nodes at the vertices of the triangles. Bending of the elements is not considered. A force at each node can have two components in the plane of the panel and the node has only two degrees of freedom.

Panel Stiffness Matrix. In all, there are 68 nodes of this type in a

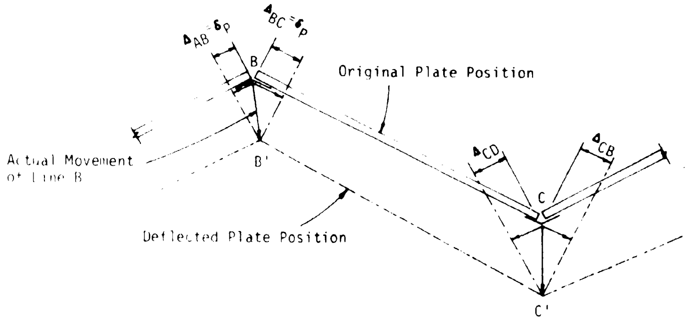

FIGURE 5 PLATE DEFLECTION DIAGRAM

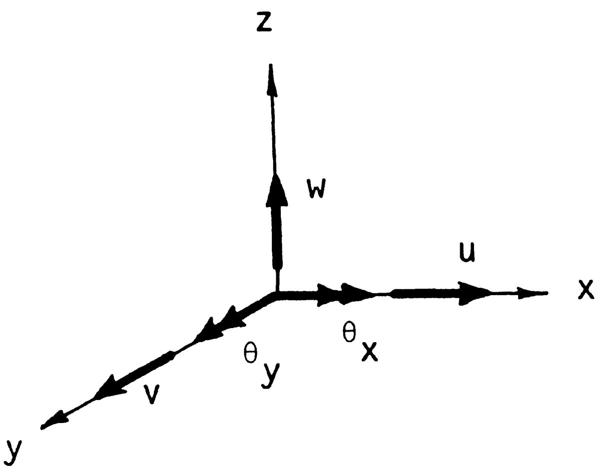

FIGURE 6

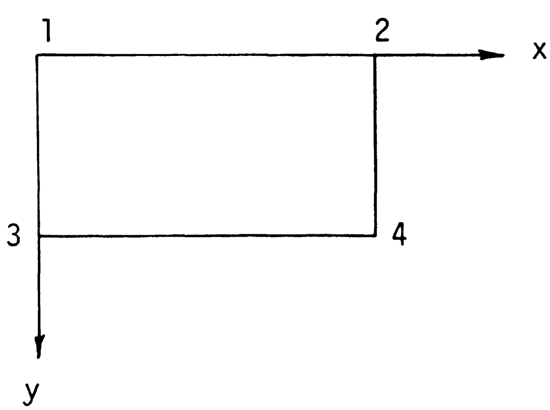

FIGURE 7

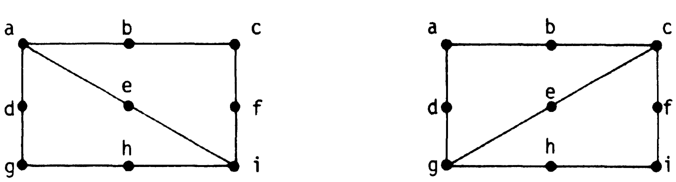

FIGURE 8

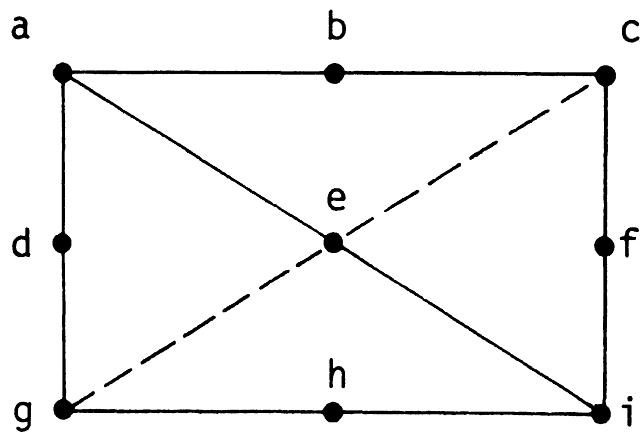

FIGURE 9

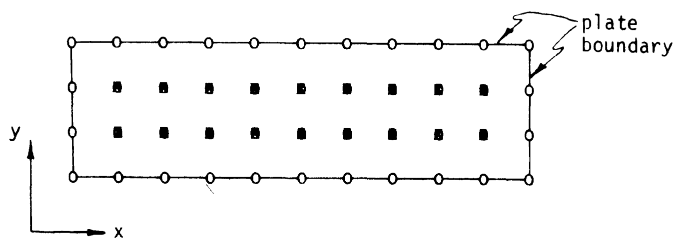

FIGURE 10

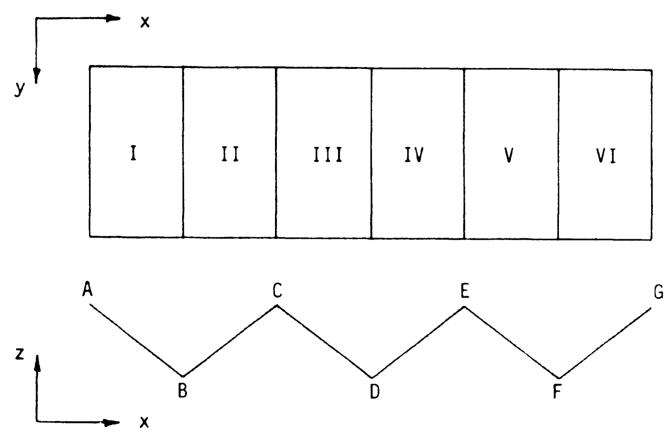

FIGURE 11

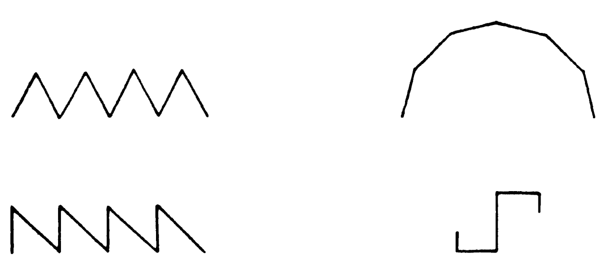

FIGURE 12

panel. Assuming two degrees of freedom for each node results in a total of 136 degrees of freedom for the panel. Providing for the necessary constraint conditions for equilibrium results in a total of 133 unknown displacements, requiring a like number of equilibrium equations. Considering the extremely large number of equations that there would be to solve for a complete roof structure, it was decided to reduce the stiffness matrix for each panel to include only nodal points along the fold lines. The interior nodal points are "condensed out" by matrix algebra.

Substructure Analysis of Complete Roof. The general stiffness matrix for the nodal points along the fold lines is built using three degrees of freedom per node. The additional degree of freedom is possible because of the angle made between the adjacent plates, so that although each plate may be considered to be loaded only in its plane, two plates together can carry components of load in any direction along their common boundary.

As this analysis was developed with 8 nodal points along each fold line, each with 3 degrees of freedom, the total number of equations that it would be necessary to solve for an actual roof would number several hundred. For this reason, it was decided to employ the method of substructure analysis in the formulation of the solution.

In the use of substructure analysis as applied herein, the structure is considered to be initially restrained completely. The constraints are released and the loads applied progressively until the structure is free of artificial constraint. This is accomplished by releasing one fold line first, arbitrarily starting with a ridge (Figure 15). Next, the two adjacent valleys are released. Then, two ridges are released, and so on. When the final fold line is released the deflections of the nodal points along it are calculated, next the displacements of nodal points along the two fold line members adjacent to it, and so on, progressively determining fold line deflections until the ridge that was initially released is reached.

This method of analysis permits the analysis of very large structures on computers that are limited in capacity. In its present form, it is capable of working with up to 24 fold lines (ridges and valleys combined) on the CDC 3400. It would be a simple matter to extend it by altering a dimension statement, but there might be some difficulty with roundoff error if larger numbers of fold lines were used.

STABILITY ANALYSIS BY FINITE ELEMENTS

An additional aspect of this study is the determination of critical stresses in elements of cold-formed folded plate structures.

It has long been understood that the load carrying capability of structures fabricated of thin plates, stiffened and unstiffened, may be a function of their buckling characteristics. The large flat surfaces inherent in folded plate structures, therefore, must be examined for this consideration.

The behavior of this type of structural element is complex. It is anticipated that very early in the loading history the plate will buckle elastically and change its load carrying system to one consisting of a tension field simulating plate girder behavior. The post-buckling strength may then extend to many times its pre-buckling strength. A mathematical analysis through the post-buckling range presents many problems such as determining the point of change of configuration and the buckled shape. These are particularly difficult to determine for plates fabricated of cold-formed material because of variations in material properties caused by manufacturing processes as well as the critical problem of fastening. After this introduction the question might well be posed--is an elastic critical stress analysis that does not consider the post-buckling configuration of any value? The worth can only be determined by comparing such an analysis to existing test results.

The problem of obtaining the critical buckling stresses of thin plates has been solved classically for only a relatively few cases. The large majority of these solutions are for rectangular plates with either simply supported or clamped edges. If stiffeners or orthotropy are considered the availability of solutions is drastically reduced. Recent innovations in cold-formed steel folded plates have created the need for calculating buckling loads for thin plates of not only the standard rectangular shape, but also such shapes as triangles, trapezoids, parallelograms, and even more exotic irregular shapes. In addition, in most of the plate buckling literature now available, a constant stress distribution is assumed, the notable exception being the solution for a rectangular plate under combined bending and compression. Obviously, more complicated stress distributions exist and should be taken into account along with a variety of boundary conditions.

In order to accommodate the variation in loading and to provide the necessary flexibility in support conditions, a finite element solution has been formulated and programmed.

General Method of Analysis. The finite element method is used to obtain the lowest elastic buckling load for a plate, either isotropic or orthotropic, with the in-plane stress distribution varying in a known manner throughout the plate.

Almost any shape of plate can be reasonably well approximated by a suitable combination of triangular and trapezoidal (of which a rectangle is a special case) elements. The elemental stiffness matrix for the trapezoidal-shaped element is based upon the derivation by Zienkiewicz (4). Tocher's (3) T-10 element is used to obtain the elemental stiffness of the triangular elements. The stiffness modifying matrices for both elements are derived following the method suggested by Kapur and Hartz (1), using the same displacement functions assumed in deriving the elemental stiffness matrices.

The stiffness modifying matrices are derived for only the case of a constant stress within the element (the stress may vary from element to element, however) although it is possible to extend the derivation to allow for various other stress distributions.

Since the finite element method, as do many other numerical techniques, depends somewhat upon the fineness of the element grid pattern for close convergence, an iterative technique which takes advantage of the symmetry and bandedness of the resulting system of equations is used.

The problem of calculating the buckling stresses for stiffened plates (including corrugated shear panels) is managed by consideration of an equivalent orthotropic flat plate.

The procedure used to obtain the critical stresses from the finite element formulations is a variation of the decomposition procedure. The value of the load that causes the modified stiffness matrix to become non-positive definite is determined, indicating an unstable structure.

The computer analysis assumes that fasteners retain their integrity throughout the loading. Also, the program is not capable of imposing non-rigid support conditions. Therefore, comparative analyses with test results are made with work performed by MacFarland (2) since his test reports indicated a failure mode primarily free of fastener effect.

In Figure 16, MacFarland's experimental results for 20 gage material are represented by solid lines for each of the two identical test specimens. Computer analysis results reflecting the allowable upper and lower material thicknesses are represented by dashed lines. Both fixed edges and simply supported edges were simulated in computer solutions.

The edge details of the tested panels are designed to restrict rotation resulting in a theoretical fixed edge, although local bending may have some effect. It can be seen that the computer solution for fixed edges does predict a failure load reasonably close to the test results.

Figure 17 shows similar results for 24 gage material. Again, the results agree most closely with the fixed edge solution although the lighter material might be expected to allow more rotation at the supports than is experienced by the 20 gage sample.

It does appear that a correlation may exist between the elastic buckling stress computations and the actual ultimate strength of a panel fabricated of cold-formed components and subjected to shear loading.

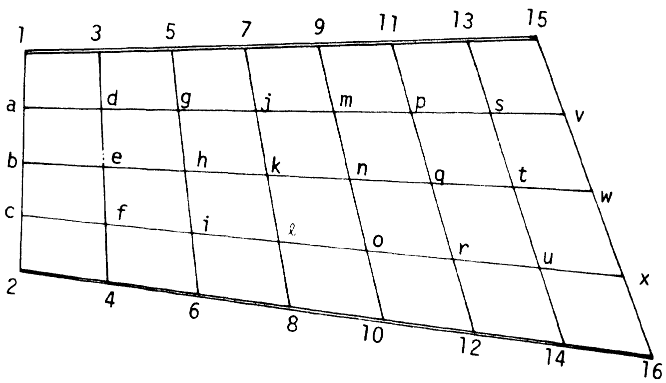

FIGURE 13

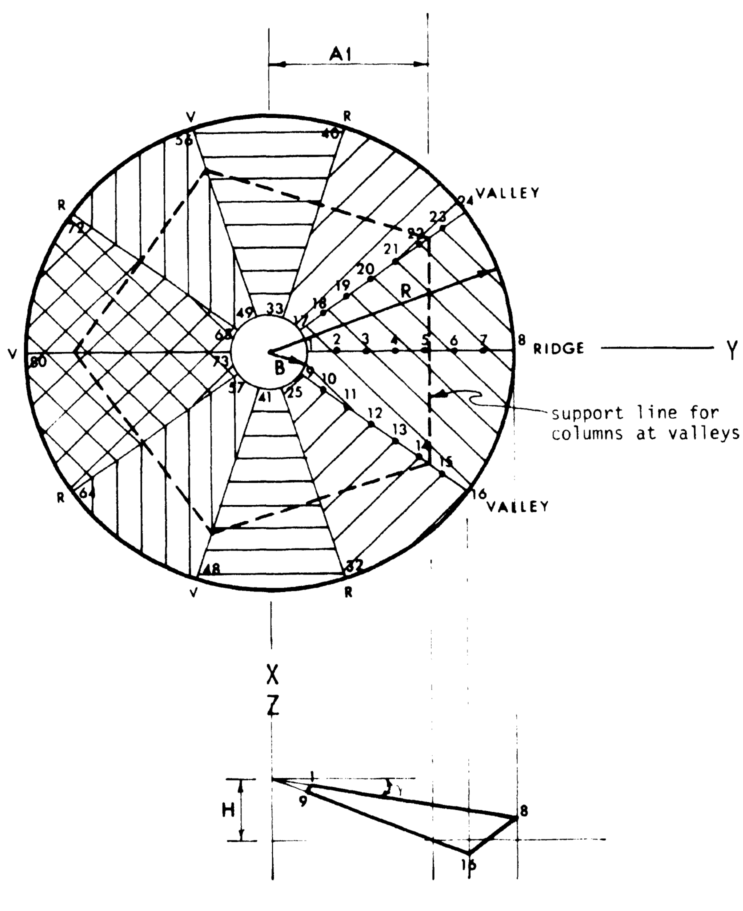

FIGURE 15 PLAN VIEW OF ROOF

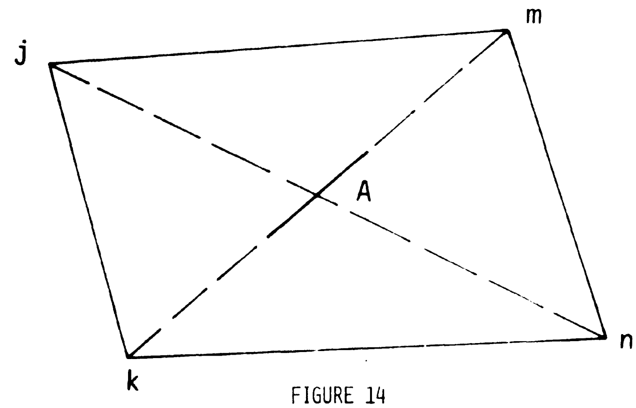

FIGURE 14

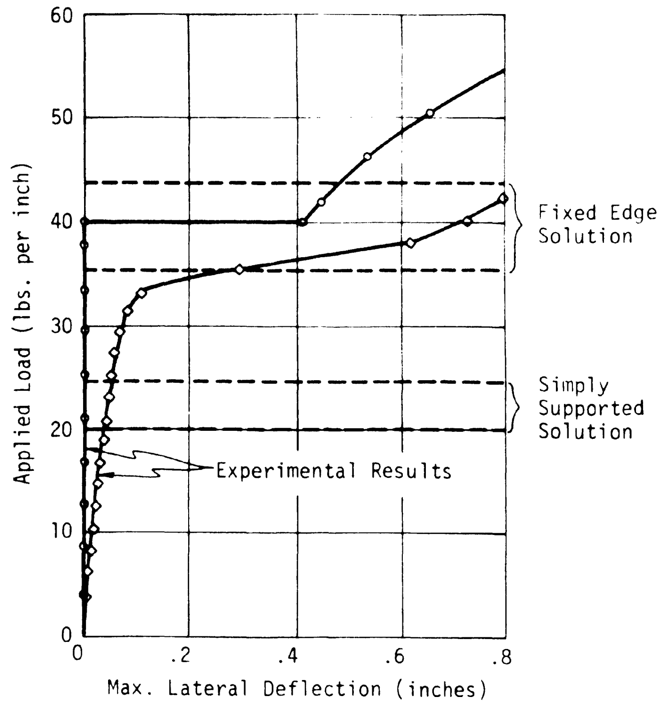

FIGURE 16 COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS (20 GAGE)

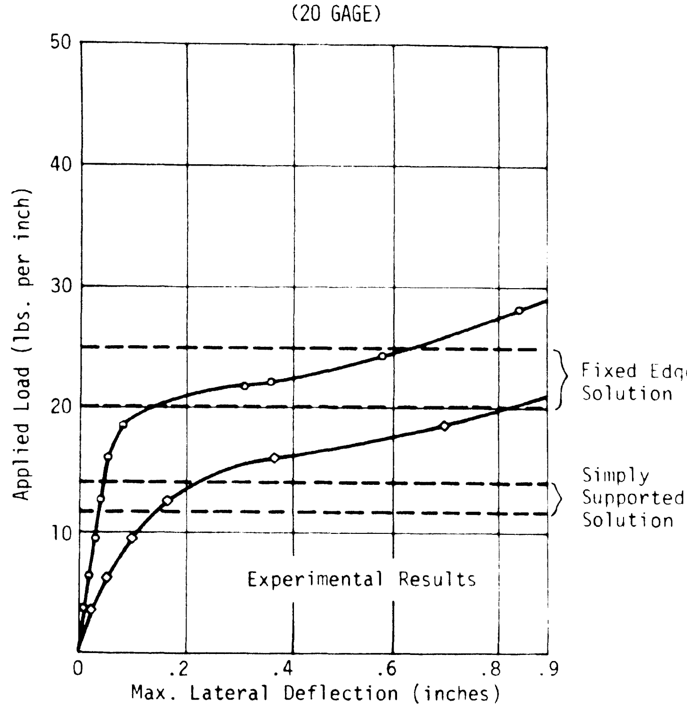

FIGURE 17 COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS (24 GAGE)

SUMMARY

The results of the theoretical portion of research on cold-formed steel folded plates are presented. The studies include stress, deflection and stability analyses of systems with rectangular as well as circular planforms.

CONCLUSIONS

This research provides design-oriented material that can be used to facilitate the design of folded plate structures fabricated of cold-formed elements.

The Simplified Method provides a direct technique that will suffice for use in final designs for many structures as well as for preliminary design of others.

The Finite Element stress analysis permits a more detailed analysis that allows additional freedom of loading, supports, and material variation.

The Buckling Analysis can utilize as input the results of one of the aforementioned stress analyses to compute the elastic buckling characteristics of the panels.

ACKNOWLEDGEMENTS

This paper is based on work carried out by the Engineering Research Center, Arizona State University, Tempe, Arizona, under a contract with the American Iron and Steel Institute of New York, New York, to perform the investigation of "Light Gage Steel Folded Plates." The writers wish to express their appreciation to William G. Kirkland and Dr. Albert L. Johnson of the A.I.S.I., and the A.I.S.I. Committee on Building Research and Technology (D. S. Wolford, Chairman of the Research and Specifications Subcommittee, and Dr. J. B. Scalzi, Chairman of the Task Group on Shell Roof Structures) for their helpful suggestions and continuing interest. They are also deeply indebted to John J. Glancy for his valuable assistance.

REFERENCES

(1) Kapur, K. K., and B. J. Hartz, "Stability of Plates Using the Finite Element Method," Proc. Amer. Soc. Civ. Engrs., Engr. Mech. Jour., (April 1966), pp. 177-195.

(2) MacFarland, D. E., "An Investigation of the Static Stability of Corrugated Rectangular Plates Loaded in Pure Shear" (unpublished Ph.D. dissertation, University of Kansas, January 1967).

(3) Tocher, J. L., "Analysis of Plate Bending Using Triangular Elements" (unpublished Ph.D. dissertation, University of California (Berkeley), 1962).

(4) Zienkiewicz and Cheung, "The Finite Element Method for Analysis of Elastic Isotropic and Orthotropic Slabs," Proc. Inst. Civ. Eng., 28 (1964), 471-88.

BIBLIOGRAPHY

Campbell, C. H. "Simplified Analysis of Tapered Light Gage Steel Folded Plates." Unpublished Master's thesis, Arizona State University, Tempe, Arizona, 1967.

"Design of Light Gage Steel Diaphragms." American Iron and Steel Institute, First Edition, 1967.

"Folded Plate Roof Design." Design procedure for light gage steel folded plate, Fenestra Company.

Kapur, K. K., and B. J. Hartz. "Stability of Plates Using the Finite Element Method." Proc. Amer. Soc. Civ. Engrs., Engr. Mech. Jour., (April 1966), pp. 177-195.

Luttrell, L. D. "Structural Performance of Light Gage Steel Diaphragms." Cornell University, Report No. 319, 1965.

MacFarland, D. E. "An Investigation of the Static Stability of Corrugated Rectangular Plates Loaded in Pure Shear." Unpublished Ph.D. dissertation, University of Kansas, January 1967.

Nilson, A. H. "Folded Plate Structures of Light Gage Steel." ASCE Trans., Vol. 128, 1963, Part II.

Nilson, A. H. "Diaphragm Action in Light Gage Steel Construction." AISI Regional Technical Paper, 1960.

Nilson, A. H. "Shear Diaphragms of Light Gage Steel." Proc. ASCE, J. Struc. Div., Vol. 86, No. ST 11, November 1960.

"Specifications for the Design of Cold-Formed Steel Structural Members." 1968 Edition. American Iron and Steel Institute.

Tocher, J. L. "Analysis of Plate Bending Using Triangular Elements." Unpublished Ph.D. dissertation, University of California (Berkeley), 1962.

Weaver, Wm. Jr. Computer Programs for Structural Analysis. Princeton, N.J.: D. Van Nostrand Co., Inc., 1967.

Winter, George. "Commentary on Light Gage Cold-Formed Steel Design Manual." For American Iron and Steel Institute, 1962.

Yu, Wei-Wen. "Design of Light Gage Cold-Formed Steel Structures." Engineering Experiment Station, West Virginia University, 1965.

Zienkiewicz and Cheung. "The Finite Element Method for Analysis of Elastic Isotropic and Orthotropic Slabs." Proc. Inst. Civ. Eng., 28 (1964), 471-88.