Stability of Arched Roof Made of Profiled Steel Sheeting

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ABSTRACT

An investigation of the structural behaviour and design criteria of an arched roof structural system is described. Double shell arch system constructed by using corrugated steel sheets bent to a form of two-layered curved roof vault is specially investigated by applying specific structural model developed for the system. In this model consisting of a plane bar system, transverse hat profiles connecting two curved shell layers together are stated as radial connection bars and stiffness characteristics for these bars simulating structural behaviour of transverse hat profiles are experimentally determined by shear tests. Analyses and calculations made by applying the double shell arch model are based both on geometrically linear and non-linear behaviour of the arch. Stability of the arched roof is also studied in the analyses by determining critical loads both for global buckling of the roof vault and for local buckling of the curved shell layer between transverse hat profiles.

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1. INTRODUCTION

Free span of a steel roof deck can be considerably extended by using an arched roof instead of flat decking with profiled sheetings. These self-supported arched roof-vaults are made of trapezoidally corrugated steel sheetings bent during cold-forming process in form of an arch usually having geometrical proportion of L (span) = R (radius). An arched roof structure is especially effective in form of a double-arch system i.e. a two-layered arch of profiled steel sheetings connected with transverse members e.g. with hat profiles. This new type of arched roof system was first developed and patented some two years ago by Austrian company ZEMAN & Co (GmbH) in Vienna. In applications of this arched roof structure, a steel tie-bar connecting arch-bases in span is usually added to the structural system. In Finland this arched roof system was first adopted and applied by company PAAVO RANNILA Oy.

Structural behaviour of arched roof system described above is studied in this reserch for finding out and introducing design criteria for arch in respect to design specifications and recommendations. This study is made by using specific structural model for two-layered arch with profiled sheetings connected by transverse hat profiles. For this model consisting of a plane bar system, stiffness characteristics of the bars simulating hat profiles between two arched sheetings are experimentally determined.

2. STRUCTURAL MODEL

2.1 Plane frame model of the arch

In the structural model used for analysis and calculations, the original two-layered arch (Fig. 1a) is replaced by a plane frame as illustrated in Fig. 1b. In this frame, the curved parts of profiled steel sheetings between transverse connecting members i.e. hat profiles (spacing 1.2 - 1.5 m) are replaced by straight beam elements. This means that in dimensioning the arch, original curvature of the arch between hat profiles is to be taken into account as an eccentricity causing extra bending moment to the beam element.

Hat profiles connecting the two profiled steel sheetings are replaced in the model by short bar elements and these bars are assumed to be clamped to the lower and pinned to the upper profiled steel sheeting. Values of the bending and shear stiffness characteristics for these bar elements are experimentally determined.

The so called system lines of the structural model (Fig. 1c) are thus defined by gravity center axes (G.C.A.) of hat profiles and by neutral axes (N.A.) of profiled steel sheetings determined by applying effective cross-sectional area for the compression side of the profiled cross-section.

As cross-sectional forces and moments of an arch usually are also dependent upon deflections caused by external loading then for analysing this geometrically non-linear behaviour, calculation methods used are to be based on the second order theory. In this study, both linear and non-linear behaviour of the arch is analysed by using the plane frame model described above.

2.2 Cross-sectional and stiffness properties for the model

For calculating bending moment, normal force, and shear force (M, N,V) in an arch cross-section, bending stiffness (EI) and axial stiffness (EA) is to be known in each cross-section of the arch. Because of the local buckling phenomenon on compression side of the cross-section, stiffnesses EI and EA are dependent upon loading state. Thus for determining these stiffnesses, effective cross-sectional areas are to be applied for compressed parts of the profiled sections.

Effective cross-sectional areas can be determined by reducing certain parts (widths) of the profiled section on compression side. Furthermore, in case of edge stiffeners and intermediate flange and web stiffeners reduction for effective area is also to be applied to the sheet thickness (t) on these parts of the compressed cross-section. In this study, effective cross-sectional areas are calculated according to the Finnish code /1/. This corresponds mainly to the Swedish code /2/ and to the German code DIN 18807 /3/.

In case of the two-layered arch made of profiled steel sheetings, for calculating exactly the effective cross-sectional area A_e and the effective moment of inertia I of section an iterative procedure is to be applied. In practice, however, values for $A_{\bullet}$ and $I_{\bullet}$ can be taken in calculations with an adequate accuracy as minimum values of A and I determined from all possible cases i.e. $A_{\bullet} = \min A$ and $I_{\bullet} = \min I$. These values are then applied in each cross-section of the arch. Table 1 compares the effective values of A and I as minA and minI with the corresponding gross-values maxA and maxI for profiled sheetings of the arch used in this study.

3. EXPERIMENTAL INVESTIGATION

3.1 Test set-up

Experimental investigation for determining stiffness values for hat profiles (RA-120/1.0 and RA-170/1.2) connecting steel sheetings of the two-layered arch was carried out in the Laboratory of Structural Engineering at Helsinki University of Technology. Fig. 2 shows the types of steel sheetings and hat profiles used in assembling test specimens in a form corresponding to a unit part of the structural model of the arch i.e. a part of two-layered sheeting over two adjacent hat profiles. Altogether 13 test specimens according to Fig. 3 were assembled by using self-tapping screws (Ø 4.8 mm) and these screws were applied to fasten flanges of two hat profiles to each bottom flange of the profiled sheeting.

Test specimen was loaded horizontally (Fig. 3) by compressive force (P) applied with servo-controlled hydraulic jack through a wooden strip to cross-section of upper sheeting while lower sheeting was fixed in longitudinal direction. Force P was controlled to act exactly in horizontal plane perpendicular to cross-section of upper sheeting. Values of P were measured by load cell connected with hydraulic jack. For measuring deflections and shear strains in specimens, deflection transducers were used in points shown in Fig. 3.

3.2 Test results

Measured force (P) and deflection (f) values were used to determine P-f plots for transverse deflection behaviour of hat profiles. Fig. 4 shows a typical P-f plot having first a linear part about up till half of ultimate force value ($P_M/2$) and then gradually curving to almost a straight line of lower inclination ending up at ultimate point (force P_M) just before final collapse of test specimen.

Initial or tangent stiffness EI_t of hat profile corresponding to bar element in structural model can be determined from linear part of P-f plot by using following simple relationship between P and f:

$$f = (P/2)H^3/3EI \quad \Rightarrow \quad EI = (P/2)H^3/3f \qquad (1)$$

In formula (1), total force P is equally divided between the two adjacent hat profiles (P/2) having profile height of H.

Secant stiffess EI_s of bar element corresponding to the behaviour of hat profile is also determined from P-f plot and as shown in Fig. 4 I_s-values are based on loading level of $P_M/1.5$ (= $2P_M/3$). These secant stiffness values (divided by E) determined for test specimens are listed in Table 2.

From test results recorded on P-f plots, it was clearly noticed for various specimen types that both in case of hat profile 120/1.0 and 170/1.2 stiffness was dependent upon lower sheeting type: for lower sheeting 45/0.9 (7 fastener-pairs/m) stiffness was greater than for lower sheeting 120/0.9 (4 fastener-pairs/m). Type of upper sheeting was indicating no significant influence on stiffness values.

Four basic specimen types (I - IV) were obtained in testing stiffness properties of hat profiles between two profiled steel sheetings:

Type	Test No.	Upper sheeting	Hat profile	Lower sheeting
I	1,2;7,8	120/0.9;45/0.9	120/1.0	45/0.9
II	3,4;5,6	120/0.9;45/0.9	120/1.0	120/0.9
III	9A,9B	45/0.9	170/1.2	45/0.9
IV	10A,10B;11	45/0.9;120/0.9	170/1.2	120/0.9

Table 3 lists average tangent (EI_t) and secant (EI_s) stiffness values of hat profiles for four different types I - IV of test specimens. As can be seen from Table 3, differences between I_t- and I_s-values in all four cases are almost insignificant. This means that in calculations as basic stiffness value EI_0 of hat profile EI_s-value can be used ($I_0=I_s$) provided that shear forces (Q) in hat profiles are not exceeding value $P_M/1.5$ used in defining secant stiffness.

Table 4 shows for different specimen types (I - IV) average ultimate loads (P_M) and corresponding failure mechanisms observed in tests. Maximum shear forces ($1/2\ P_M/1.5$) in hat profiles used in stiffness calculations (corresponding allowable shear forces caused by nominal loading) are also listed in Table 4. As can be seen from Table 4, these maximum shear force values are greater in cases of lower sheeting section 45/0.9 (failure mechanisms 2 and 3) than in case of lower sheeting section 120/0.9 (failure case 1). This means that shear forces corresponding to failure mechanism 1 are significant in dimensioning. This means also that stiffness values determined for hat profile RA-120/1.0 can be used up till shear force value of $Q_{allow.}$ = 7 kN/m and for hat profile RA-170/1.2 correspondigly up till value of 12 kN/m.

3.3 Stiffness values for structural model

Transverse bending stiffness values EI_0 determined by tests for hat profiles RA-120/1.0 and RA-170/1.2 are listed in Table 5 together with maximum allowable values $Q_{allow.}$ of shear forces caused by nominal loading in validity ranges of I_0-values. For bar elements in structural model, bending stiffness values EI_H can then be determined with I_0-values by using following formula

$$I_H = (L_H/H)^3 I_0 \qquad (2)$$

where L_H represents theoretical length of bar elements in structural model (distance between neutral axes of sheetings) and H height of hat profiles (120 mm or 170 mm). In cases of variable sheeting thicknesses ($t \neq 0.9$ mm) I_0- and $Q_{allow.}$-values are to be reduced.

4. STRUCTURAL DESIGN OF ARCH

In basic design of two-layered arch with profiled sheetings connected with hat profiles, combined compression (N) and bending (M) is to be separetaly checked both for upper and lower sheeting section under maximum effect of combined loads. After cross-sectional forces (M,N,V) are determined by usual linear analysis, local and global buckling of arch can be taken into account by following interaction formula

$$(1+0.5\bar{\lambda}_{ki}(1-N_d/N_{Rci}))N_d/N_{Rci}+(1+0.5\bar{\lambda}_{kg}(1-N_d/N_{Rcg}))N_d/N_{Rcg}+(M_d+N_d e/2)/M_R \leq 1 \tag{3}$$

where N_d = design value of normal force

N_{Rcg} = global buckling force of arch baced on ECCS buckling curve "c"

N_{Rci} = local buckling force of arch-layer between hat profiles based on ECCS buckling curve "c"

M_d = design value of bending moment

M_R = $f_y W_e$ = bending capacity of arch section

e = eccentricity i.e. distance between original curved axis of sheeting and straight axis of sructural model

$\bar{\lambda}_{kg}$ = modified slenderness of arch based on combined cross-section of sheetings

$\bar{\lambda}_{ki}$ = modified slenderness of arch layer between hat profiles based on sheeting section

In case when cross-sectional forces (M,N,V) are determined by using non-linear (second-order) theory, formula (3) is reducing to form

$$(1+0.5\bar{\lambda}_{ki}(1-N_d/N_{Rci}))N_d/N_{Rci} + (M_d+N_d e/2)/M_R \leq 1 \tag{4}$$

In desing formulae (3) and (4), for effective cross-sectional area A_e and effective moment of inertia I_e minimum possible values of A and I (minA and minI) are used and also for effective elastic section modulus W_e corresponding minimum value of W in case of pure bending.

In determining values for N_{Rcg} in formula (3) or (4), buckling shape of arch is to be known. In case of a circular arch critical buckling mode is asymmetric and thus effective buckling length can be determined with multiplying half arch length by a trigonometric factor dependent upon central angle of arch. In case of arch geometry of R = L, this factor is equal to 1.02.

For determining global buckling force N_{Rcg} of two-layered arch, ideal slenderness λ_{id} of double-arch is to be determined for a plane frame system when interaction between two layers connected by bar elements is to be taken into account. This ideal slenderness can be determined by following formula

$$\lambda_{id} = (\lambda^2 + n\, a\, \lambda_i^2)^{1/2} \tag{5}$$

where λ = slenderness of arch with fully compact two-layered cross-section

λ_i = slenderness of separate arch sheeting layer between hat profiles having minor section stiffness value

a = constant, a = 1.3 for screwed fastenings

n = 2 in case of major axis bending

Modified slendernesses in design formulae (3) and (4) can be written as follows:

$$\bar{\lambda}_{kg} = \lambda_{id}\,(f_y/E)^{1/2}/\pi \tag{5}$$

$$\bar{\lambda}_{ki} = L_i\,(f_y/E)^{1/2}/\pi\, i_e \tag{6}$$

L_i in formula (6) is the length between transverse bar elements in structural model and i_e is effective radius of gyration of cross-section

$$i_e = (I_e/A_e)^{1/2} \tag{7}$$

In ECCS buckling curves used for N_{Rci} - and N_{Rcg} -values choice of type of curve (a,b,c,d) is dependent upon initial imperfections in arch. Usually curve "c" can be used for arch and then maximum initial deflection is assumed as L/400.

In formulae (3) and (4), design bending moment is added by term $N_d e/2$ where distance e is caused by difference between axis of curved sheeting part between hat profiles and corresponding straight line axis in structural model.

5. DESIGN CALCULATIONS AND COMPARISONS

In design calculation by using a plane bar system as structural model for two-layered arch of profiled steel sheetings connected with transverse hat profiles, validity of the model was tested by comparing results of calculations based on linear (first-order) and non-linear (second-order) theories. Also influence of horizontal restraint stiffnes at base supports on arch behaviour was studied by comparing design calculations in case of horizontally fully restrained support and of tied arch with different axial stiffnesses in the tie between base supports of arch.

Calculations were performed for a fixed arch geometry i.e. L (span) = R (radius) with three L-values of 10, 15 and 22 m. In all cases both linear and non-linear calculation methods were applied. In non-linear case, initial imperfections (max. L/400) according to ECCS buckling curve "c" were assumed in arch geometry as downward initial deflection on half of heavier loading and upward deflection on half of lighter loading. Loading cases were dead load ($g = 0.3 - 0.4\ kN/m^2$) and asymmetric live load (snow) as shown in Fig. 5.

Table 6 shows as an example results of design calculations on arch having span L (=R) = 22 m, both upper and lower sheeting of 120/1.0 (f_y = 320 MPa) and hat profiles of 170/1.2 (f_y = 320 MPa). As can be seen from Table 6, horizontal restraint stiffness at base supports seems to have a minor influence on load-bearing capacity of arch, at the utmost some percents between two extreme restraint stiffness cases of arch supports.

6. CONCLUSIONS

In analysis made for evaluating structural behaviour and load-bearing capacity of arched roof with profiled sheetings connected with transverse hat profiles, both linear and non-linear calculation methods were applied. It was found out by these calculations that as effective cross-sectional areas and section stiffness values could be chosen minimum values determined on the basis of all possible loading situations of the arch. The effective cross-sectional areas for arched roof sheeting sections can thus be determined correspondingly to values of flat sheeting sections.

Structural model for two-layered arch roof consisting of a plane bar system with double polygonal frame of beam elements connected with transverse bar elements was succesfully used in analyses and design calculations of arch after stiffness values for the bar elements simulating behaviour of transverse hat profiles between two arched sheetings were experimentally determined in connection of this research project.

Design calculations of the arch were showing that linear calculation methods can be applied with sufficient accuracy within the limits of arch spans (R = 10, 15, 22 m) and geometry (L = R) assumed in this study. It was also observed by analyses and calculations that the influence of horizontal restraint stiffness of arch base supports (i.e. axial stiffness of steel tie-bar between base supports) on stability and load-bearing capacity of arch is relatively small i.e. at maximum in extreme support stiffness cases only some percents on arch capacity values.

7. ACKNOWLEDGEMENTS

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8. APPENDIX I - REFERENCES

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9. APPENDIX II - NOTATION

A = cross-sectional area
A_e = effective cross-sectional area
EA = axial stiffness
EI = bending stiffness
EI_H = theoretical bending stiffness of bar element
EI_0 = basic bending stiffness of hat profiles
EI_s = secant stiffness of bar element
EI_t = tangent stiffness of bar element
H = height of hat profile
I_e = effective moment of inertia of cross-section
L = arch span
L_H = theoretical length of bar element
L_i = length between transverse bar elements
M = bending moment
M_d = design value of bending moment
M_R = bending moment capacity
N = normal force
N_d = design value of normal force
N_{Rcg} = global buckling force of arch
N_{Rci} = local buckling force of arch
P = force
P_M = failure load (ultimate load)
R = radius of arch
$Q_{allow.}$ = allowable shear force of hat profile
V = shear force in arch cross-section
W_e = effective elastic modulus of arch section

a = constant
e = eccentricity
f = deflection
f_y = yield stress of steel
g = dead load
g_d = design value of dead load
i_e = effective radius of gyration of cross-section
q = live load (snow)
q_d = design value of live load

λ = slenderness of arch
λ_i = slenderness of sheeting layer
λ_{id} = ideal slenderness of arch
$\bar{\lambda}_{kg}$ = modified slenderness of arch for combined cross-section
$\bar{\lambda}_{ki}$ = modified slenderness of arch layer

Table 1. Minimum effective cross-sectional values A_e and I_e for profiled sheeting sections compared with corresponding gross-sectional values.

Sheeting	t mm	$\frac{I_{min}}{I_{max}}$	$\frac{A_{min}}{A_{max}}$
45/0.7	0.63	0.82	0.82
45/0.9	0.82	0.89	0.88
45/1.1	1.01	0.94	0.92
120/0.8	0.67	0.91	0.75
120/1.0	0.85	0.94	0.82
120/1.2	1.03	0.97	0.88

Table 2. Tangent and secant stiffnesses (div. by E) of hat-profiles 120/1.0 and 170/1.2.

Specimen	P/2 kN	f mm	P/2f N/mm	$I_t \times 10^{-9}$ m^4/m	$I_s \times 10^{-9}$ m^4/m
120/1.0					
1	5.06	4.86	1042	3.01	
	8.05	8.28	972		2.81
2	5.10	5.65	902	2.60	
	8.11	9.50	854		2.47
3	4.06	6.41	633	1.83	
	6.11	9.97	613		1.77
4	3.41	5.25	649	1.87	
	6.03	10.11	596		1.72
5	4.47	7.12	628	1.81	
	5.98	9.56	625		1.81
6	4.11	6.11	763	1.94	
	6.16	9.69	636		1.84
7	5.03	5.05	996	2.88	
	7.04	7.41	950		2.74
8	3.99	3.62	1102	3.18	
	7.03	6.95	1012		2.92
170/1.2					
9A	8.06	5.51	1462	12.00	
	14.15	10.85	1304		10.71
9B	8.07	5.72	1412	11.59	
	15.13	11.91	1270		10.43
10A	6.00	5.91	1015	8.33	
	11.02	11.79	935		7.67
10B	5.95	5.37	1108	9.10	
	10.98	11.28	974		7.99
11	5.98	5.19	1153	9.47	
	9.99	9.08	1101		9.04

Table 3. Average I_t- and I_s-values of hat profiles 120/1.0 and 170/1.2 for specimen types I - IV.

Specimen type	$I_t \times 10^{-9}$ m^4/m	$I_s \times 10^{-9}$ m^4/m
I	2.92	2.73
II	1.86	1.78
III	11.80	10.57
IV	8.97	8.23

Table 4. Ultimate loads and failure mechanisms for specimen types I - IV.

Specimen type	P_M / 0.95 m kN	$Q_{allow.}$ / 0.95 m kN	Failure type*
I	24.11	8.04	2
	(27.67	9.22	3)
II	19.78	7.01	1
III	44.66	15.61	3
IV	33.55	11.66	1

*Failure types: 1=Fasteners pulled out in lower sheeting
2=Fasteners pulled out in upper sheeting
3=Failure of hat profile

Table 5. I_0-values for hat profiles 120/1.0 and 170/1.2 with allowable shear force values.

Hat profile	Lower sheet	$I_0 \times 10^{-9}$ m^4/m	$Q_{allow.}$ kN/m
120/1.0	45/0.9	2.73	7
	120/0.9	1.78	7
170/1.2	45/0.9	10.57	12
	120/0.9	8.23	12

Table 6. Interaction of design stresses (M in kNm and N in kN) for arch R = L = 22 m calculated applying formulas (3) and (4).

Case		Lower sheeting		Upper sheeting	
		1. order	2. order	1. order	2. order
1	N	40.86	37.20	25.51	36.01
	M	7.59	10.13	5.97	8.24
	3;4	0.990	0.803	0.707	0.676
2	N	40.08	36.64	27.25	37.80
	M	7.72	10.28	6.06	8.32
	3;4	0.990	0.811	0.733	0.688
3	N	39.32	36.08	28.96	39.56
	M	7.85	10.44	6.15	8.40
	3;4	0.990	0.819	0.759	0.700

Case 1: Horizontally fully restrained base supports
Case 2: Steel tie-bar with cross-sectional area A = 400 mm^2/m
Case 3: Steel tie-bar with cross-sectional area A = 200 mm^2/m

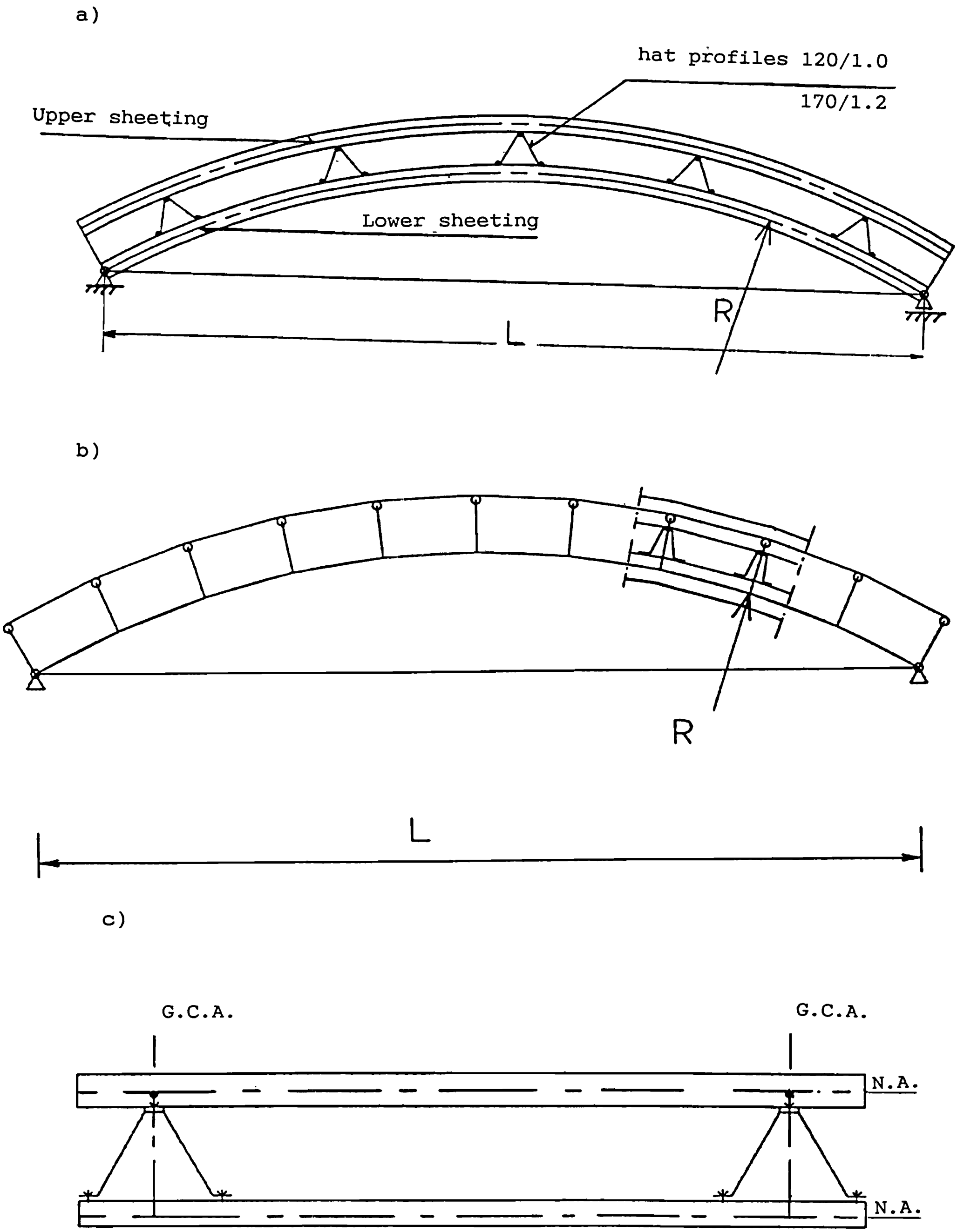
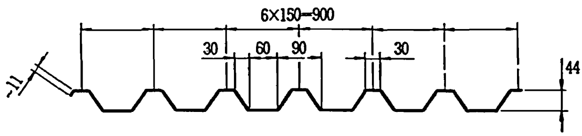
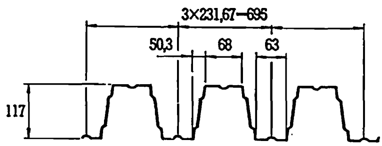


Fig. 1. Structural model of arch: a) Two-layered arch with profiled sheetings and transverse hat profiles, b) Plane frame model of arch, c) System lines of arch model.

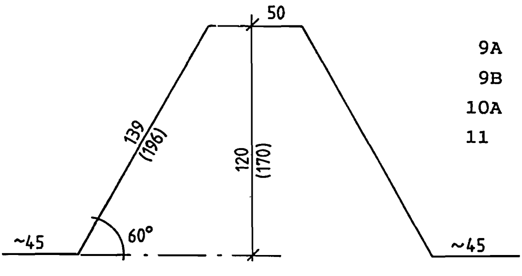
Sheeting 45/0.9



Sheeting 120/0.9



Hat profile 120/1.0 (170/1.2)



Spec.	Hat profile	Upper sheeting	Lower sheeting
1	120/1.0	45/0.9	45/0.9
2	120/1.0	45/0.9	45/0.9
3	120/1.0	45/0.9	120/0.9
4	120/1.0	45/0.9	120/0.9
5	120/1.0	120/0.9	120/0.9
6	120/1.0	120/0.9	120/0.9
7	120/1.0	120/0.9	45/0.9
8	120/1.0	120/0.9	45/0.9
9A	170/1.2	45/0.9	45/0.9
9B	170/1.2	45/0.9	45/0.9
10A	170/1.2	45/0.9	120/0.9
11	170/1.2	120/0.9	120/0.9

Fig. 2. Types of steel sheetings and hat profiles.

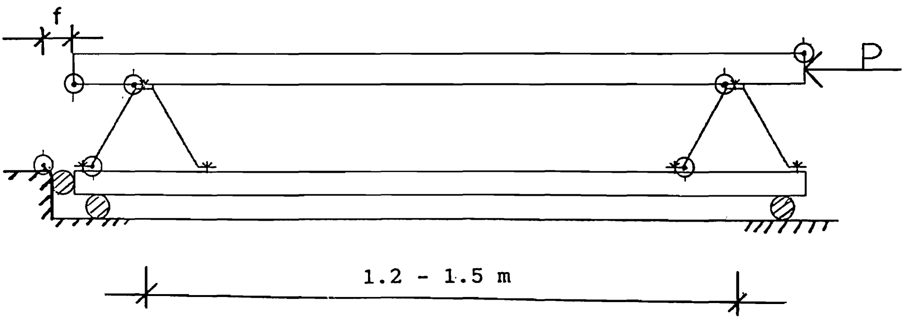


Fig. 3. Scheme of test set-up with deflection measurement points.

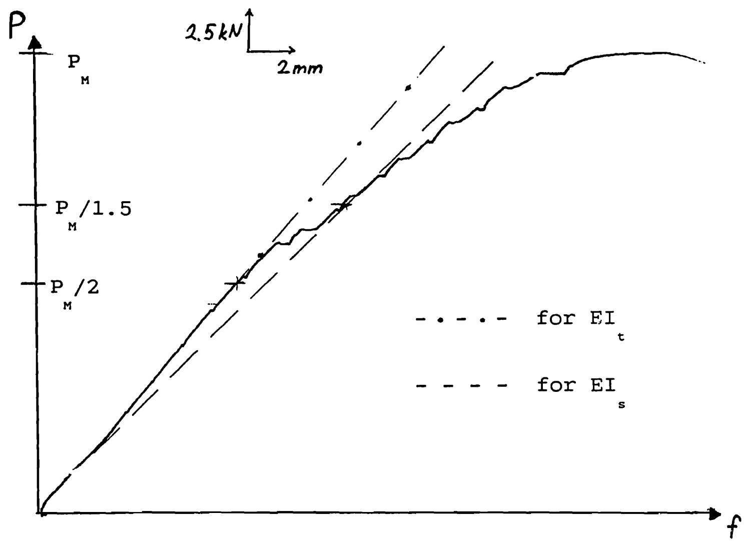


Fig. 4. Typical P-f plot with definitions of tangent and secant stiffnesses.

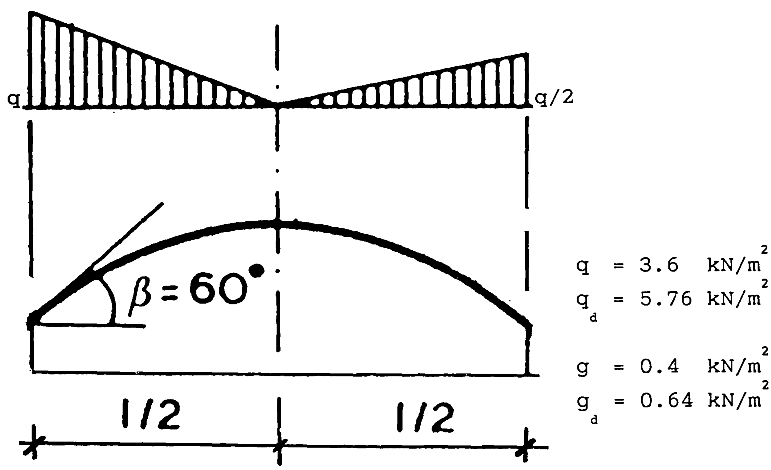


Fig. 5. Nominal and design loads for arch R = L = 22 m.