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ANALYSIS OF CYLINDRICAL GRAIN STORAGES

MADE OF COLD FORMED STEEL

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Windsor, Ontario, Canada

INTRODUCTION

Farm grain storages (bins) are often constructed in the shape of cylindrical cantilever shells using cold formed steel sheets. Their diameters are normally in the range of 10 to 40 feet and their heights correspond to one time to twice the diameter. They are subjected to axially symmetric pressures exerted by stored materials as well as to non-symmetric loading due to wind or eccentric discharge of the stored grain.

The present design of these bins follows a very simplified approach which is only adequate in dealing with the cases of symmetric loading since it accounts for the membrane forces only. However, bending moments are developed in the shells because of the axially non-symmetric loadings.

The bending state of cantilever cylinders was examined for isotropic shells (4, 5, 7, 10). Similar studies are needed for cantilever shells made of cold formed steel sheets in order to improve the design of the grain bins. It follows that the invest-igation presented here is focused on cantilever orthotropic cylindrical shells made of cold formed steel sheets and subjected to axially non-symmetric loadings. This analysis is approximate, it is based on Vlasov's approach in which the longitudinal bending and twisting moments are neglected (11).

LOADING

The present investigation deals mainly with cantilever ortho-tropic shells subjected to wind loading and to change in pressure

as a result of eccentric discharge of the stored solids (6). This loading is axially non-symmetric (Figs. la, b, c) and acts in the z-direction only $(p_\phi = p_x = 0)$. It can be presented analytically in the following form of Fourier's series:

$$p_z = p \sum_{m=1,3,5}^{\infty} \sum_{n=0,1,2}^{\infty} q_m q_n \sin \frac{m\pi}{2H} \overline{x} \cos n\phi \tag{1}$$

in which p = the maximum positive intensity of load; q_m, q_n = constants governing the distribution of loading in the vertical and horizontal directions, respectively; they are given as follows:

(a) Wind Loading (10):

$$q_{n=0,1,2,\ldots} = -0.387, 0.338, 0.533, 0.471, 0.166,$$
$$-0.066, -0.055 \tag{2a}$$

and

$$q_{m=1,3,\ldots} = \frac{4}{m\pi} \tag{2b}$$

(b) Eccentric Discharge:

$$q_{n=1,3,\ldots} = \frac{4}{1.115 \pi n} \sin \frac{n\pi}{8} \tag{3a}$$

and

$$q_{m=1,3} = 0.667 \tag{3b}$$

GOVERNING DIFFERENTIAL EQUATIONS

Grain bins built of cold formed steel sheets can be treated as cylindrical shells made of orthotropic material in which the

mechanical properties are equal to the average properties of the sheets (1). The behaviour of these shells is adequately governed by the following differential equations (8):

$$D_x \frac{\partial^2 u}{\partial x^2} + D_{x\phi} \left(\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 v}{\partial x \partial \phi}\right) = -R^2 P_x \qquad (4a)$$

$$D_\phi \left(\frac{\partial^2 v}{\partial \phi^2} - \frac{\partial w}{\partial \phi}\right) + D_{x\phi} \left(\frac{\partial^2 u}{\partial x \partial \phi} + \frac{\partial^2 v}{\partial x^2}\right) = -R^2 P_\phi \qquad (4b)$$

$$B_\phi \left(\frac{\partial^4 w}{\partial \phi^4} + 2\frac{\partial^2 w}{\partial \phi^2} + W\right) + B_x \frac{\partial^4 w}{\partial x^4} + 2 B_{x\phi} \frac{\partial^4 w}{\partial x^2 \partial \phi^2}$$

$$- R^2 D_\phi \left(\frac{\partial v}{\partial \phi} - W\right) = R^4 P_z \qquad (4c)$$

in which x = non-dimensional coordinate = actual length, $\bar{x}$, divided by the radius, R; D_x, D_ϕ = axial rigidity in the x- and ϕ- directions, respectively; $D_{x\phi}$ = the shear rigidity in the $x\,\phi$- plane; B_x and B_ϕ = bending rigidity in the xz- and ϕz- planes, respectively; $B_{x\phi}$ = torsional rigidity; and P_x, P_ϕ, and P_z = external loading per unit area of the middle surface acting in the x-, ϕ- and z- directions, respectively; u, v and w = displacement in the x-, ϕ- and z-direction respectively (Fig. 1d).

The internal force components can be calculated as follows:

$$N_\phi = \frac{D_\phi}{R} \left(\frac{\partial v}{\partial \phi} - W\right) - \frac{B_\phi}{R^3} \left(W + \frac{\partial^2 w}{\partial \phi^2}\right) \qquad (5a)$$

$$N_x = \frac{D_x}{R} \frac{\partial u}{\partial x} \qquad (5b)$$

$$N_{x\phi} = N_{\phi x} = \frac{D_{x\phi}}{R} \left(\frac{\partial u}{\partial \phi} + \frac{\partial v}{\partial x}\right) \qquad (5c)$$

$$M_\phi = - \frac{B_\phi}{R^2} (w + \frac{\partial^2 w}{\partial \phi^2}) \tag{5d}$$

$$M_x = - \frac{B_x}{R^2} \frac{\partial^2 w}{\partial x^2} \tag{5e}$$

$$M_{x\phi} = M_{\phi x} = - \frac{B_{x\phi}}{R^2} \frac{\partial^2 w}{\partial x \partial \phi} \tag{5f}$$

The longitudinal bending rigidity B_x and torsional rigidity $B_{x\phi}$ of grain bins are normally too small when compared to the bending rigidity B_ϕ. This observation enhances the confidence to utilize Vlasov's approximation in which the longitudinal bending moment, M_x, and the twist, $M_{x\phi}$, are neglected. Thus,

$$B_x \frac{\partial^4 w}{\partial x^4} = - R^2 \frac{\partial^2 M_x}{\partial x^2} = 0 \tag{6a}$$

$$2B_{x\phi} \frac{\partial^4 w}{\partial x^2 \partial \phi^2} = - R^2 \frac{\partial^2 M_{x\phi}}{\partial x \partial \phi} = 0 \tag{6b}$$

accordingly, Eq. 4C takes the following simplified form:

$$B_\phi (\frac{\partial^4 w}{\partial \phi^4} + 2 \frac{\partial^2 w}{\partial \phi^2} + w) - R^2 D_\phi (\frac{\partial v}{\partial \phi} - w) = R^4 P_z \tag{4d}$$

while Eqs. 4a and b remain unchanged

BOUNDARY CONDITIONS

The bin is attached to the foundation therefore:

at $x = 0$ $w = v = 0$ (7a)

and $u = 0$ (7b)

Along the upper edge, the axial force N_x (due to lateral loading)

should disappear;   i.e.

at $x = \dfrac{H}{R}$, $N_x = 0$ (7c)

The upper edge of the shell is connected to an edge member and to a conical roof. This roof system provides partial restrain to the upper edge of the shell. Therefore, two extreme cases are considered with regard to the remaining condition around the upper edge:

(a) Completely flexible edge:

at $x = \dfrac{H}{R}$, $N_{x\phi} = 0$ (7d)

or

(b) Completely restrained edge:

In this case the top section remain circular. It can undergo displacement only parallel to the $\phi = 0$ direction (without distortion) i.e.

at $x = \dfrac{H}{R}$, $M_\phi = 0$ (7e)

Note here that the differential equations, Eqs. 4a, b, d are formulated on the assumption that the moment M_x and the shear force Q_x are equal to zero throughout the shell as well as along the edges.

SOLUTION

The loading P_z can be written as follows:

$$P_z = P_{z0} + P_{z1} \cos \phi + \sum_{n=2,3,4}^{\infty} P_{zn} \cos n\phi \qquad (8)$$

in which P_{zo}, P_{z1} and P_{zn} = functions of x only. Separate solutions are to be obtained for each of the terms of loading given in Eq. 8

depending on the magnitude of n.

(a) Load Component With n = 0

The load here is independent of the angle ϕ, i.e. it is axially symmetric and the solution is:

$$w = \frac{R^2}{D_\phi} P_{z0} \tag{9}$$

(b) Load Component With n = 1

The loading:

$$P_{z1} = P q_{n=1} \cos \phi \sum_{1,3,5}^{\infty} q_m \sin \eta_m x \tag{10}$$

in which $\eta_m = \frac{m\pi}{2} \frac{R}{H}$

The corresponding deflection takes the form of a single cosine wave in the ϕ-direction, i.e. f(x) cos ϕ. Therefore:

$$B_\phi \left(\frac{\partial^4 w}{\partial \phi^4} + 2 \frac{\partial^2 w}{\partial \phi^2} + w\right) = 0 \tag{11a}$$

and also

$$M_\phi = - \frac{B_\phi}{R^2} \left(w + \frac{\partial^2 w}{\partial \phi^2}\right) = 0 \tag{11b}$$

Accordingly, the shell is subjected to a state of pure membrane forces, and the governing equations can be written as follows:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{x\phi}}{\partial \phi} = 0 \tag{12a}$$

$$\frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} = 0 \tag{12b}$$

and $N_\phi = -R P_{zl}$ (12c)

Eqs. 12a, b, c are integrated considering the loading P_{zl} as given in Eq. 10 and the boundary conditions, Eqs. 7. This leads to the following expressions for the internal force components.

$$N_\phi = -R \, p \, q_{n=1} \, \cos \phi \sum_{m=1,3,5}^{\infty} q_m \sin \eta_m \times \qquad (13a)$$

$$N_{\times\phi} = R \, p \, q_{n=1} \, \sin \phi \sum_{m=1,3,5}^{\infty} \frac{q_m}{\eta_m} \cos \eta_m \times \qquad (13b)$$

$$N_\times = -R \, p \, q_{n=1} \, \cos \phi \sum_{m=1,3,5}^{\infty} \frac{q_m}{\eta_m^2} \left(\sin \eta_m \times - (-1)^{\frac{m-1}{2}} \right) \qquad (13c)$$

and the displacement components:

$$u = \frac{R^2}{D_\times} p \, q_{n=1} \, \cos \phi \sum_{m=1,3,5}^{\infty} \frac{q_m}{\eta_m^2} \left((-1)^{\frac{m-1}{2}} \times \; + \right.$$

$$\left. + \frac{1}{\eta_m} (\cos \eta_m x - 1) \right) \qquad (14a)$$

$$v = R^2 \, p \, q_{n=1} \, \sin \phi \sum_{m=1,3,5}^{\infty} \frac{1}{D_{\times\phi}} \frac{q_m}{\eta_m^2} \sin \eta_m \times \; + \frac{1}{D_\times}$$

$$\frac{q_m}{\eta_m^2} \left[(-1)^{\frac{m-1}{2}} \frac{\times^2}{2} + \frac{1}{\eta_m} \left(\frac{1}{\eta_m} \sin \eta_m \times - \times \right) \right] \qquad (14b)$$

and

$$w = \frac{R^4 p}{B_\phi} \Omega \, q_{n=1} \, \cos \phi \sum_{m}^{\infty} \frac{q_m}{\eta_m^2} \left\{ \sin \eta_m \times \left(\eta_m^2 + \frac{D_\phi}{D_\times} \frac{1}{\eta_m^2} \; + \right. \right.$$

$$\left. \left. + \frac{D_\phi}{D_{\times\phi}} \right) + \frac{D_\phi}{D_\times} \left((-1)^{\frac{m-1}{2}} \frac{\times^2}{2} - \frac{\times}{\eta_m} \right) \right\} \qquad (14c)$$

in which $\Omega = \dfrac{B_\phi}{R^2 D_\phi}$

(c) Load Components With $n > 1$

Particular Solution:

To satisfy the differential equations, Eqs. 1a, b, d but not all boundary conditions.

$$w_p = \sum_n \sum_m A_{mn} \sin \eta_m x \cos n\phi \tag{15a}$$

$$v_p = \sum_n \sum_m B_{mn} \sin \eta_m x \sin n\phi \tag{15b}$$

$$u_p = \sum_n \sum_m C_{mn} \cos \eta_m x \cos n\phi \tag{15c}$$

A_{mn}, B_{mn} and C_{mn} are constants calculated by substituting Eqs. 15a, b and c in the governing equations, Eqs. 1a, b, d:

$$B_{mn} = \frac{\Omega\, q_m\, q_n\, (\frac{R^4}{B_\phi} p)}{K_{mn}(1+\Omega\,(n^2-1)^2) - n} \tag{16a}$$

$$C_{mn} = \frac{n\, \eta_m}{n^2 + \frac{D_x}{D_{x\phi}}\, \eta_m^2}\, B_{mn} \tag{16b}$$

$$A_{mn} = K_{mn}\, B_{mn} \tag{16c}$$

in which:

$$K_{mn} = \frac{1}{n}\, \{n^2 + \frac{D_{x\phi}}{D_\phi}\, \eta_m^2 + \frac{D_{x\phi}}{D_\phi}\, \frac{(\eta_m\, n)^2}{(n^2 + \frac{D_x}{D_{x\phi}}\, \eta_m^2)}\,\} \tag{16d}$$

The particular solution given by Eqs. 15 - a to c does not satisfy all

the boundary conditions. A homogenous solution is added so that the sum of the two solutions can satisfy all boundary conditions.

Homogenous Solution:

$$w = A e^{\gamma x} \cos n\phi \tag{17a}$$

$$v = B e^{\gamma x} \sin n\phi \tag{17b}$$

$$u = C e^{\gamma x} \cos n\phi \tag{17c}$$

A and B are real while C and γ could be complex numbers. Eqs. 17a, b, c are substituted in the governing equations, Eqs. 4a, b, d, after replacing P_z, P_x and P_ϕ by zero. A non-trivial solution of the resulting homogenous system of equations is governed by the following characteristic equation:

$$\gamma^4 - \gamma^2 \frac{D_\phi}{D_{x\phi}} \frac{\Omega n^2 (n^2 - 1)^2}{1 + \Omega(n^2 - 1)^2} + \frac{D_\phi}{D_x} \frac{\Omega n^4 (n^2 - 1)^2}{1 + \Omega(n^2 - 1)^2} = 0 \tag{18a}$$

The resulting roots are: $\gamma = \pm \alpha \pm i \beta$ $\tag{18b}$

α and β can be calculated using the formulas given in reference (2) and the homogenous solution can be written with four integration constants as follows:

$$w = \sum_{n=2,3,4}^{\infty} \cos n\phi \{e^{\alpha x} (K_1 \cos \beta x + K_2 \sin \beta x)$$

$$+ e^{-\alpha x} (K_3 \cos \beta x + K_4 \sin \beta x)\} \tag{19a}$$

$$v = \sum_{n=2,3,4}^{\infty} \sin n\phi \frac{1}{n} (1 + \Omega(n^2 - 1)^2) \{e^{\alpha x} (k_1 \cos \beta x + K_2 \sin \beta x)$$

$$+ e^{-\alpha x} (K_3 \cos \beta x + K_4 \sin \beta x) \} \qquad (19b)$$

$$u = \sum_{n=2,3,4}^{\infty} \cos n\phi \{K_1 e^{\alpha x} (a \cos \beta x - b \sin \beta x)$$

$$+ K_2 e^{\alpha x} (b \cos \beta x + a \sin \beta x) - K_3 e^{-\alpha x} (a \cos \beta x +$$

$$+ b \sin \beta x) + K_4 e^{-\alpha x} (b \cos \beta x - a \sin \beta x) \} \qquad (19c)$$

in which

$$a = \frac{-D_\phi n^2 \Omega (n^2 - 1)^2}{D_x \{ (\alpha^3 - 3\alpha\beta^2) + \frac{(\beta^3 - 3\alpha^2\beta)^2}{(\alpha^3 - 3\alpha\beta^2)} \}} \qquad (20a)$$

$$b = a \frac{\beta^3 - 3\alpha^2\beta}{\alpha^3 - 3\alpha\beta^2} \qquad (20b)$$

The four constants K_1, K_2, K_3 and K_4 are calculated by satisfying the boundary conditions, Eqs. 7a, b, c and d or e. Note here that the formulation of both particular and homogenous solution lead to one condition with regard to $v = w = 0$ at $x = 0$. Accordingly, the four integration constants can satisfy all boundary conditions.

OBSERVATIONS

1. A comparison is presented in Figs. 2a, b between the results obtained using the present approximate analysis for isotropic shell (as a special case of orthotropic shells) and the more rigorous analysis of isotropic shells reported in reference (4). It shows

that small differences exist between the two solutions. Improved

results are anticipated when applying the present solution to

orthotropic shells in which the rigidity in the x-direction is small

compared to that in the ϕ-direction.

2. A general program is written on IBM 360/50 computer of the

University of Windsor. The analysis is conducted for loadings due

to wind as well as due to eccentric discharge. Both cases of

flexible and rigid top edges are considered. The results of the

analysis are dependent mainly on the ratios D_ϕ/D_x, $\frac{H}{R}$ and on $\Omega = \dfrac{B_\phi}{R^2 D_\phi}$.

The ratio $D_\phi/D_{x\phi}$ is also encountered in the program, however, nu-

merical solutions show that it has negligible effects on the results.

This ratio can be replaced by a constant value (say 2.76 as in the

case of corrugated steel sheets).

3. An example is calculated for a shell made of standard corrugated

steel sheets with the following properties:

Height of the shell	H =	38.1 ft.
Diameter of shell	2R =	25.4 ft.
Depth of corrugation	2f =	0.625 in.
Thickness of sheet	t =	0.03 in.

The rigities are calculated using the formulas outlined in reference

(1) leading to the following input data to the computer program:

$$\frac{D_\phi}{D_{x\phi}} = 2.76, \quad \frac{D_\phi}{D_x} = 246, \quad \frac{H}{R} = 3.0, \quad \frac{B_\phi}{R^2 D_\phi} = 8.78 \times 10^{-6}$$

Figs. 3a, b show the maximum value of M_ϕ along the generator of the

shell. They also show that the conditions at the top edge (flexible or rigid) have considerable effect on the bending moment M_ϕ in the case of wind loading. However, these conditions become insignificant in the case of eccentric discharge since the maximum pressure is acting at one third of the height and reduces to zero at the top (Fig. 1c).

4. The maximum value of the bending moment M_ϕ can be calculated using the following formula:

$$M_\phi = \Theta \; p \; R^2$$

in which p = the maximum positive wind pressure or the maximum change in the horizontal pressure due to eccentric discharge. The factors Θ are calculated for different ratios of $\dfrac{D_\phi}{D_x}$, $\dfrac{H}{R}$ and $\dfrac{B_\phi}{D_\phi R^2}$ and are presented for practical use in Figs. 4a to d.

CONCLUSION

   Analysis is presented for orthotropic cylindrical cantilever shells. The differential equations are based on Vlasov's approximation in which the longitudinal bending moment and twisting moment are neglected.

   Contrary to the semi-membrane theory, the membrane strains, ε_ϕ and γ_{xy} are not ignored. This approximation is adequate for cantilever cylindrical shells especially when the bending rigidity in the longitudinal direction is too small as in the case of shells made of cold formed steel sheets.

The encountered characteristic equation is of the 4th order. Expressions are obtained in closed form for the displacements from which the internal force components can be calculated. Also, curves are given for practical use for calculating the ring moment, M_ϕ.

ACKNOWLEDGEMENT

APPENDIX A - NOTATIONS

B_χ, B_ϕ = bending rigidity in $\times z$ and ϕz planes, respectively;

$B_{\times\phi}$ = torsional rigidity;

c = corrugation pitch;

$D_\times$, D_ϕ = axial rigidity in $\times$ and ϕ-directions, respectively;

$D_{\times\phi}$ = shear rigidity in $\times$ ϕ plane;

E = modulus of elasticity for isotropic material;

f = half depth of corrugation;

H = height of shell;

L = half length of corrugation;

$M_\times$, M_ϕ = bending moment per unit length acting in $\times z$ and ϕz planes, respectively;

$M_{\times\phi} = M_{\phi\times}$ = torsional moment per unit length acting about ϕ and $\times$-axis, respectively;

γ = root of characteristic equation;

$N_\times$, N_ϕ = axial force per unit length acting in $\times$ and ϕ-direction, respectively;

$N_{\times\phi}$ = shear force per unit length acting in $\times\phi$-plane;

$P_\times$, P_ϕ, P_z = external loading per unit area of middle surface acting in $\times$, ϕ, and z-directions, respectively;

$Q_\times$, Q_ϕ = lateral shear force per unit length acting perpendicular to $\times$ and ϕ-axis, respectively;

R = radius of shell;

t = average thickness of corrugated sheet;

u, v, w = displacement in $\times$, ϕ, z directions, respectively;

$\varepsilon_\times$, ε_ϕ = axial strain in $\times$ and ϕ directions, respectively;

$\varepsilon_{\times\phi}$ = shear strain in $\times\phi$ plane;

η_m $= \dfrac{m\pi}{2} \dfrac{R}{H}$

μ = Poisson's ratio of material;

σ_x, σ_ϕ = axial stress in $\times$ and ϕ-directions, respectively;

$\tau_{\times\phi}$ = shear stress in $\times\phi$-plane;

Ω $= \dfrac{B_\phi}{R^2 D_\phi}$

APPENDIX B - REFERENCES

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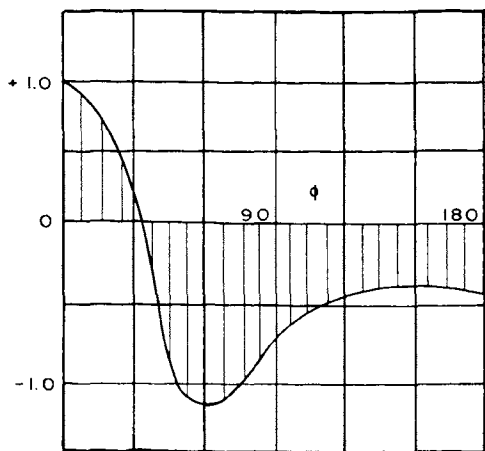
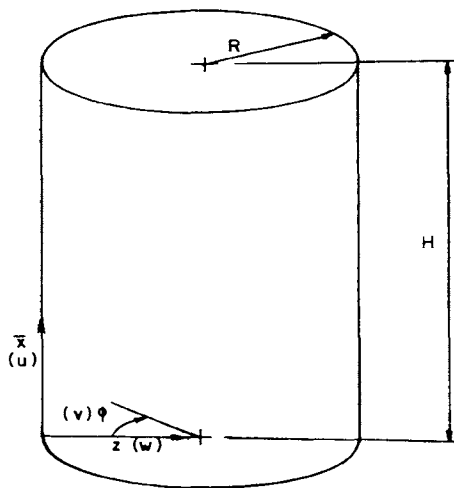a - Distribution of Wind Pressure on the outside of the Shell
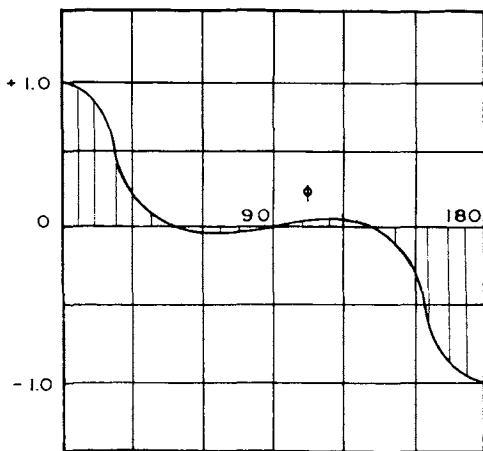
b - Distribution of Load due to eccentric discharge

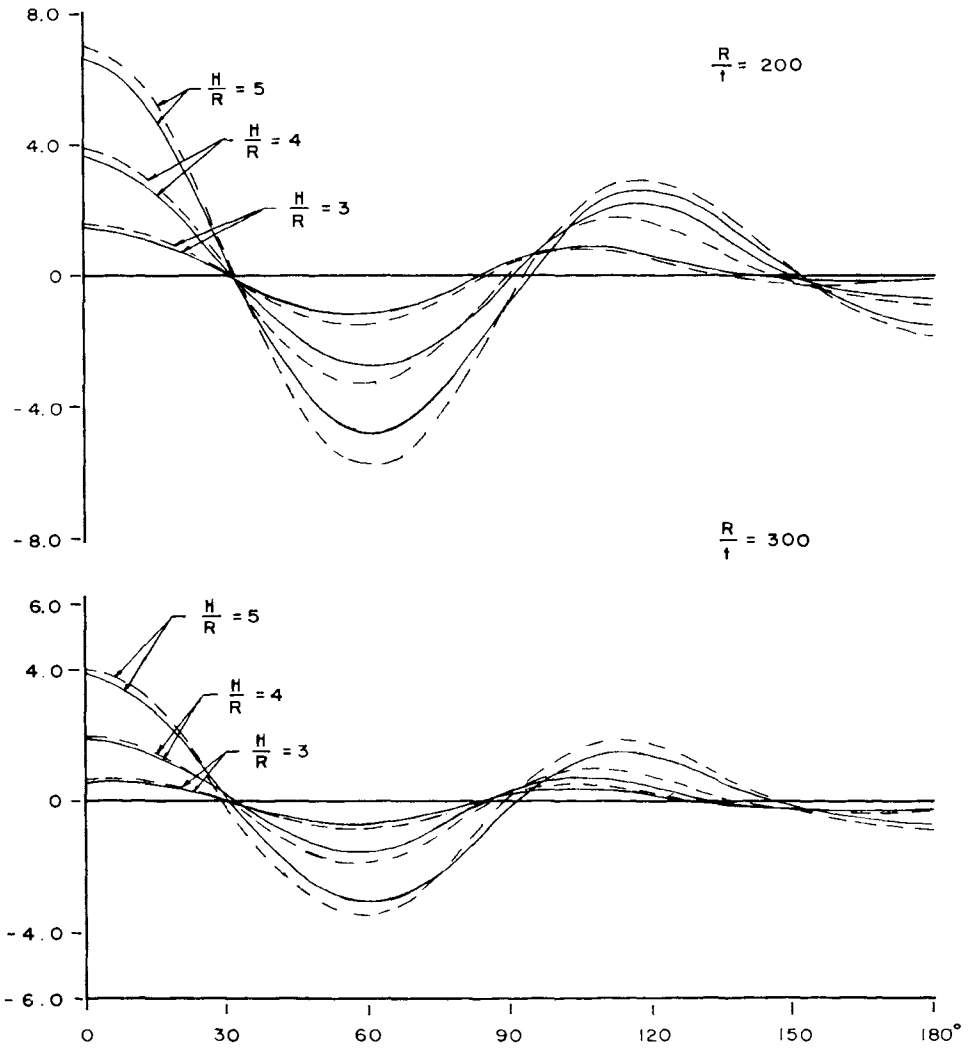c - Distribution of Loading in the Vertical Direction
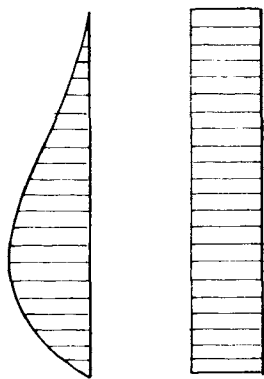
Eccentric Discharge   Wind Pressure

d - Systems of Coordinates and Displacements

FIGRE 1

Figures 2 a&b: Radial Deflection at the Free Top of Isotropic Shells
- - - - Analysis by Reference (4) $\mu = 0.0$
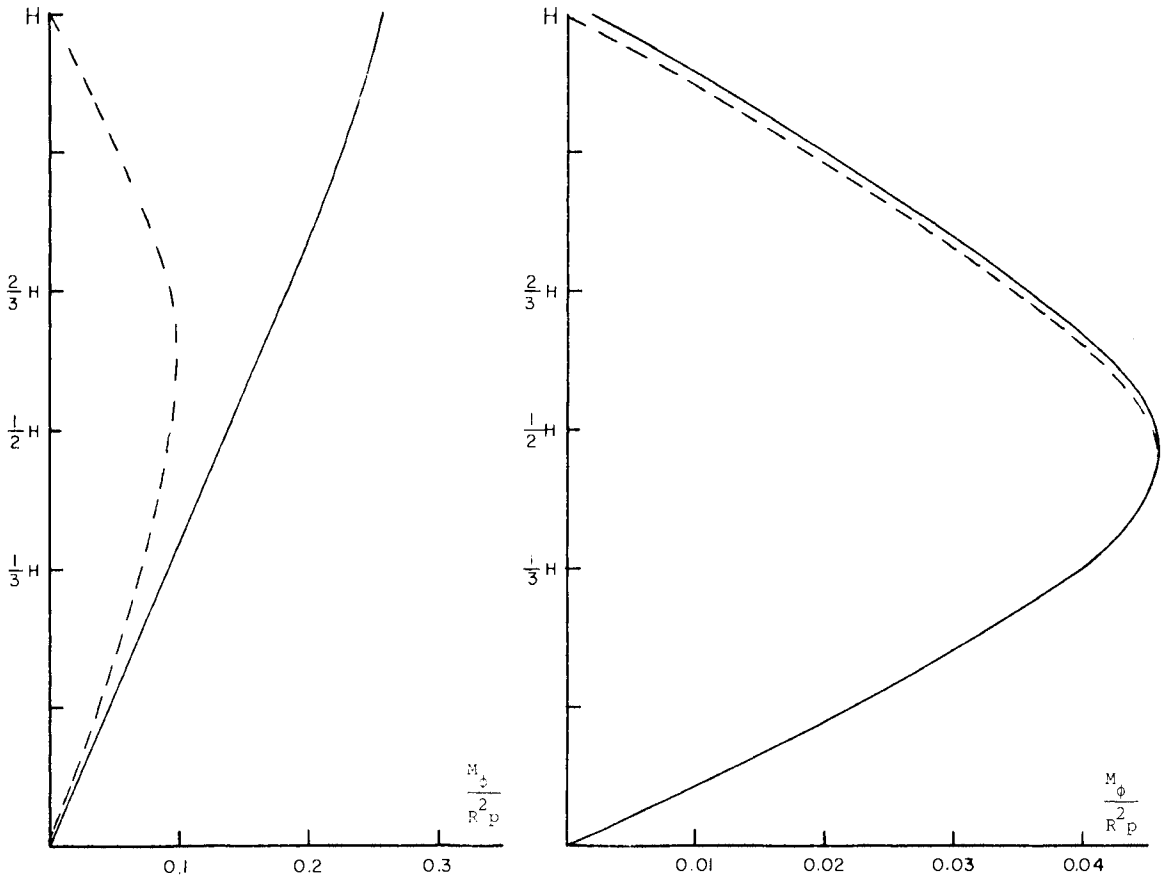———— Present Analysis $\mu = 0.0$

Figure 3 : Maximum Value of the Bending Moment, M_ϕ, along generator

———— Flexible Top Edge , ———— Restrained Top Edge

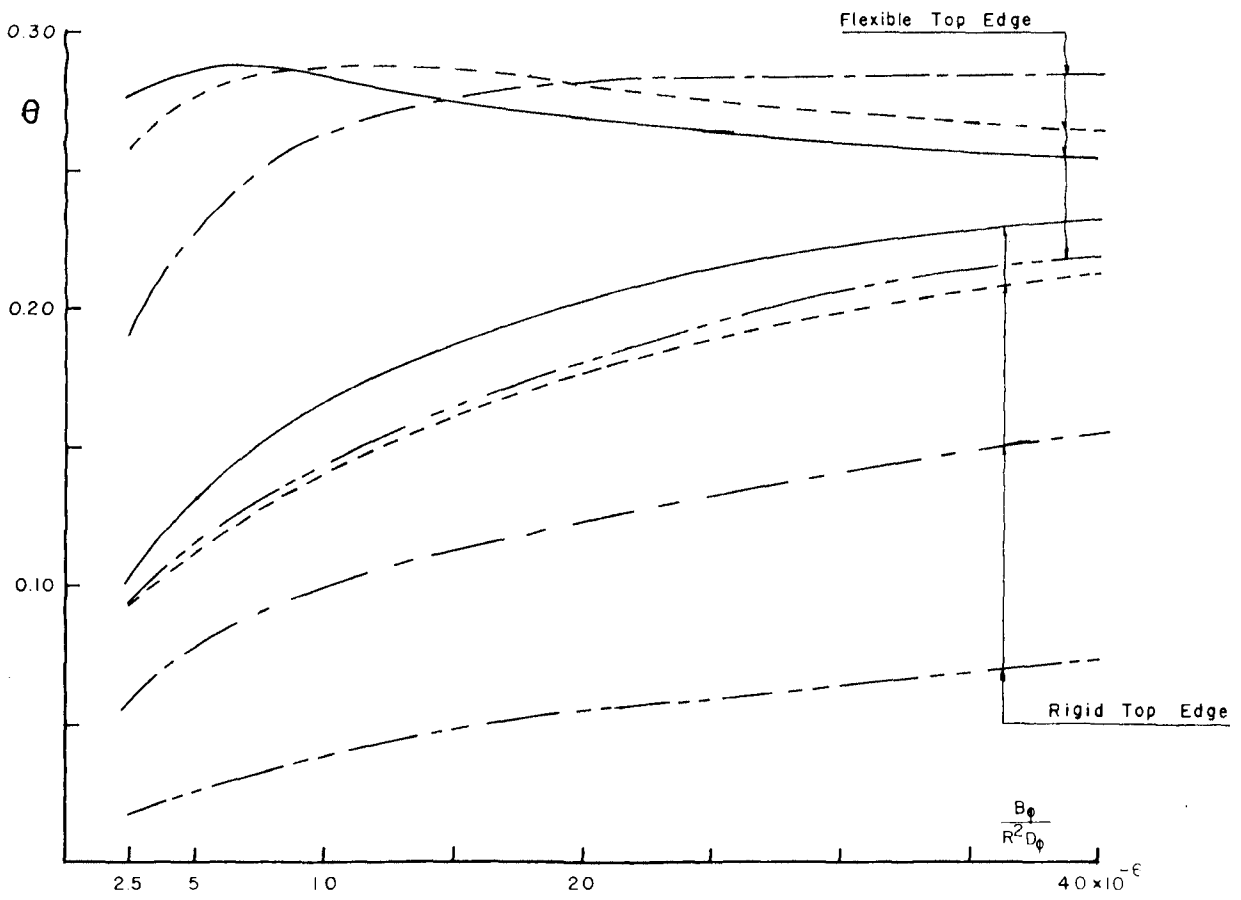Grain Bin, H=38.1 ft., 2R=25.4 ft., 2f=0.625 in., t=0.03 in.

Figure 4a: θ For Bins Subjected to Wind Loading, $\frac{H}{R} = 4.0$
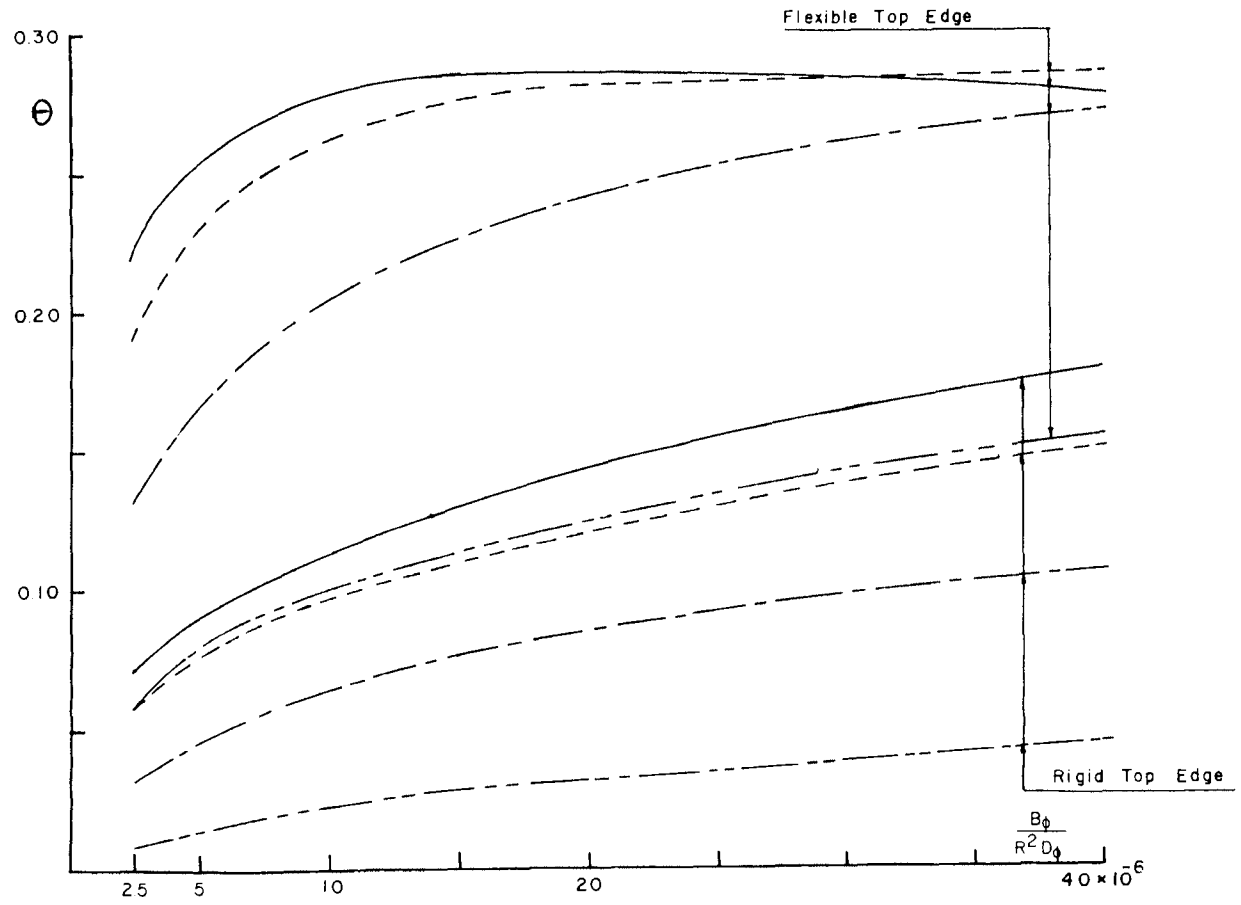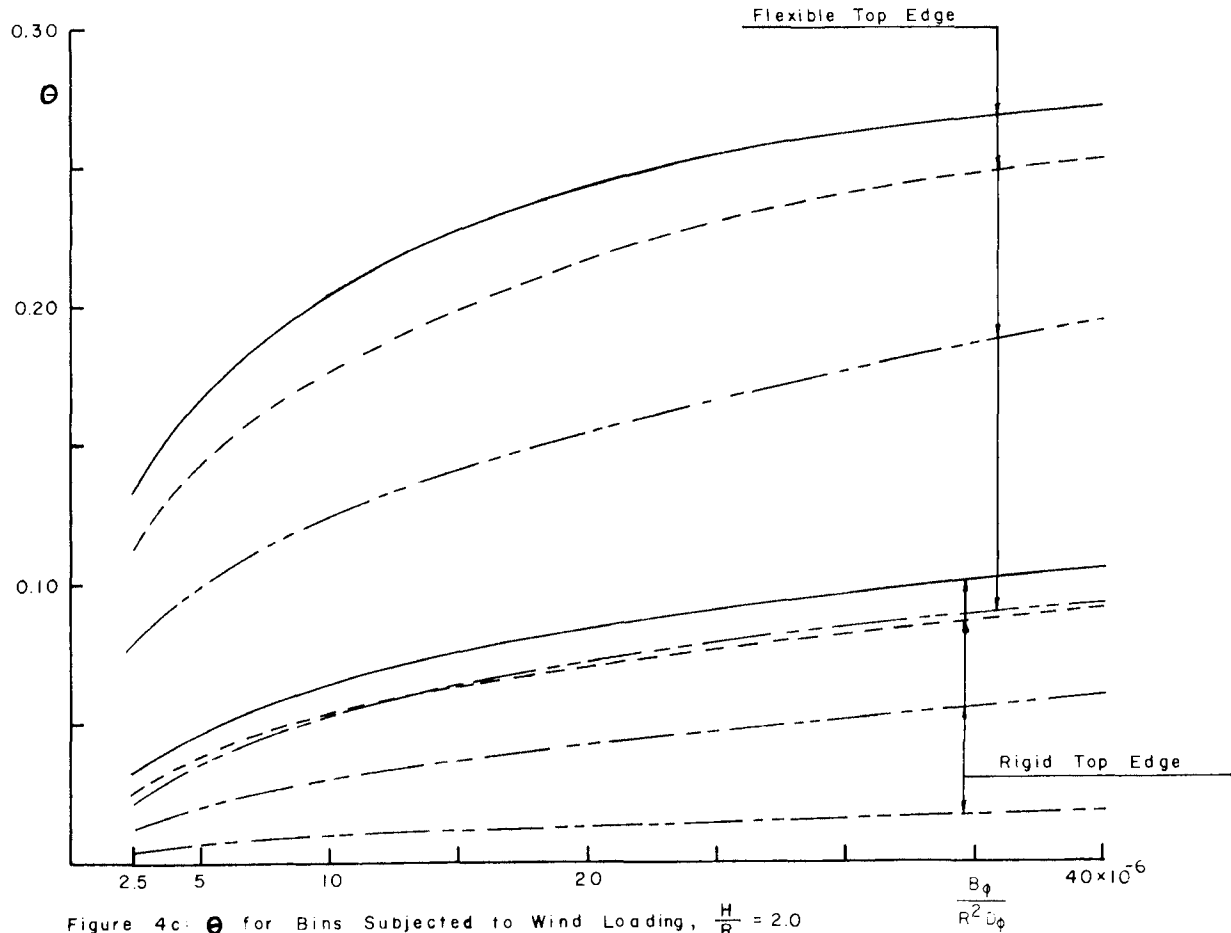
$\frac{D_\phi}{D_x} = 500$ ———— , 300 ----- , 100 —·—· , 10 — — —

Figure 4b: Θ for Bins Subjected To Wind Loading, $\frac{H}{R} = 3.0$

$\frac{D_\phi}{D_x} = 500$ ———— , 300 ----- , 100 ——— , 10 —··—

Figure 4c: Θ for Bins Subjected to Wind Loading, $\dfrac{H}{R} = 2.0$

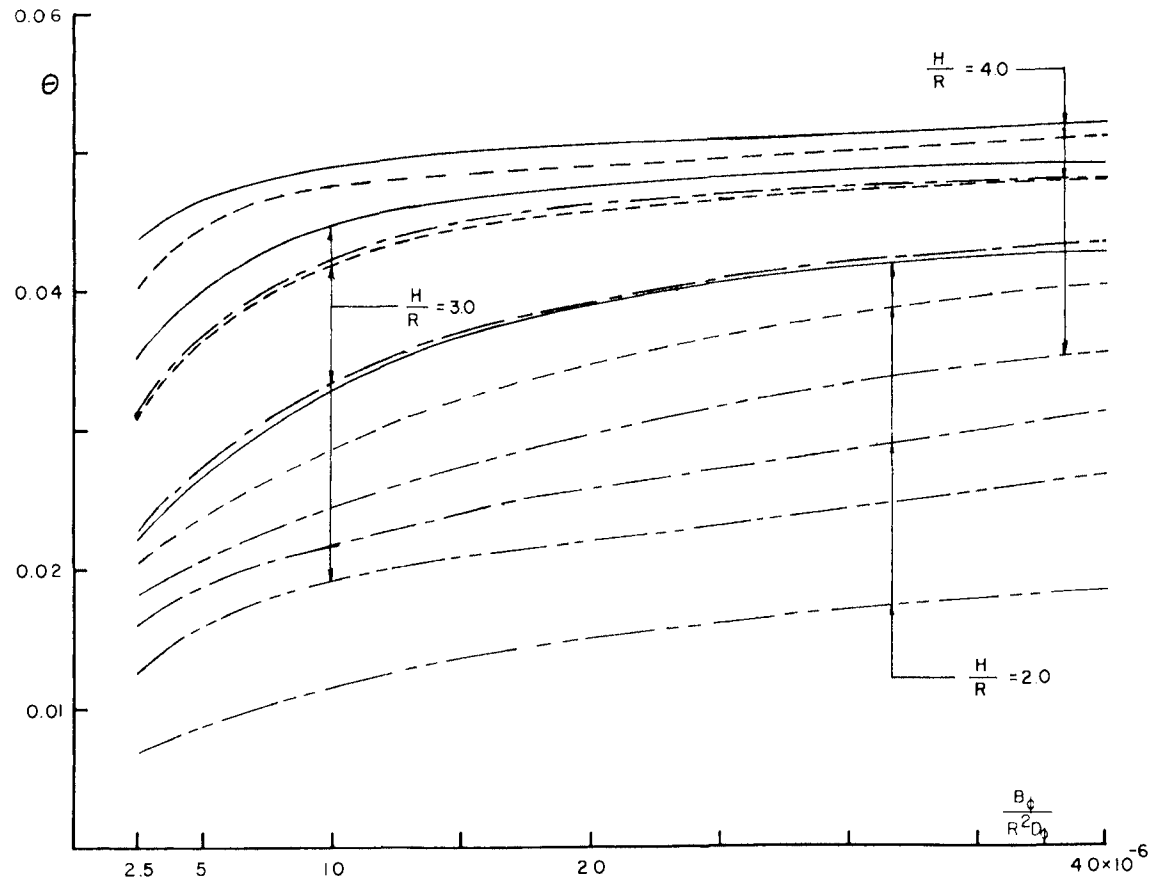$\dfrac{D_\phi}{D_x} = 500$ ———— , 300 - - - - , 100 —·—·— , 10 —··—··—

Figure 4d: θ for Bins Subjected to Eccentric Discharge, (Flexible Top Edge)

$\dfrac{D_\phi}{D_x} = 500$ ———— , 300 - - - - , 100 —·—·— , 10 ——————