01 Sep 1985

Load and resistance factor design of cold formed steel comparative study of design methods for cold formed steel

Lan-Cheng Pan

Wei-wen Yu

Missouri University of Science and Technology, wwy4@mst.edu

Brian K. Snyder

CIVIL ENGINEERING STUDY 85-3
STRUCTURAL SERIES

Eighth Progress Report

LOAD AND RESISTANCE FACTOR DESIGN OF COLD-FORMED STEEL

COMPARATIVE STUDY OF DESIGN METHODS FOR COLD-FORMED STEEL

by

Brian K. Snyder
Research Assistant

Lan-Cheng Pan
Research Assistant

Wei-Wen Yu
Project Director

A Research Project Sponsored by American Iron and Steel Institute

September 1985

DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF MISSOURI-ROLLA
ROLLA, MISSOURI

PREFACE

This report is based on Brian Snyder's thesis presented to the Faculty of the Graduate School of the University of Missouri-Rolla (UMR) in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering. Revisions have been made to reflect some recent changes of resistance factors in the revised tentative recommendations (UMR Civil Engineering Study 85-2) dated September 1985.

This investigation was sponsored by American Iron and Steel Institute. The technical guidance provided by the AISI Task Group on Load and Resistance Factor Design (K.H. Klippstein, Chairman, D.H. Hall, R.L. Cary, members), the advisors for the AISI Task Group (R. Bjorhovde, C.W. Pinkham, R.M. Schuster, and Late Professor G. Winter), former members of the AISI Task Group (N.C. Lind, R.B. Matlock, W. Mueller, F.J. Phillips, and D.S. Wolford), the AISI Staff (A.L. Johnson and D.P. Cassidy), and our consultant, M.K. Ravindra, is gratefully acknowledged.

Special thanks are extended to Mrs. De Anne Larson for the typing that she did in this publication.

ABSTRACT

Allowable Stress Design is the current method used to design cold-formed steel structural members and connections. In this design approach, factors of safety are used to compute the allowable design stresses which are compared to the actual maximum stresses that will occur in the member during the life of the structure.

In recent years, the Load and Resistance Factor Design (LRFD) method has been developed for the design of hot-rolled steel shapes and the design of cold-formed steel structural members. This method is based on probabilistic and statistical techniques to account for the many uncertainties involved with the actual design. The LRFD criteria use load factors which are applied to the external load and resistance factors that are applied to the internal resistance capacities of the structure.

The allowable unfactored loads based on each design method for different types of structural members are compared and shown in graphical forms. For structural members with one type of loading, the dead-to-live load ratio contributes to the difference between the two allowable loads. For members with a combination of loads, cross-sectional geometry, loading conditions, material strength, member length, along with dead-to-live load ratio will affect the difference between the allowable loads computed from allowable stress design and LRFD.

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd)

TABLE OF CONTENTS (Cont'd)

TABLE OF CONTENTS (Cont'd)

TABLE OF CONTENTS (Cont'd)

TABLE OF CONTENTS (Cont'd)

TABLE OF CONTENTS (Cont'd)

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Cont'd)

LIST OF ILLUSTRATIONS (Cont'd)

LIST OF ILLUSTRATIONS (Cont'd)

LIST OF ILLUSTRATIONS (Cont'd)

LIST OF TABLES

I. INTRODUCTION

A. GENERAL

The 1980 Edition of the Specification for the Design of Cold-Formed Steel Structural Members published by the American Iron and Steel Institute (AISI) applies to steel members cold-formed to shape from carbon or low-alloy steel sheet, strip, plate or bar not more than one inch in thickness and used for load-carrying purposes in buildings[1]. The specification provides design formulas for determining allowable stresses or allowable loads for tension members, compression members, flexural members, and connections. In the design of such members and connections, the actual stresses are computed from service loads that include dead, live, snow, wind, and earthquake loads. The allowable stresses or allowable loads are based on appropriate factors of safety recommended by AISI for different types of structural members.

The Load and Resistance Factor Design (LRFD) criteria for steel members and connections have recently been developed by using probabilistic and statistical techniques to account for the uncertainties in design, fabrication, material properties, and applied loads. The proposed LRFD criteria for hot-rolled shapes, built-up members, and connections[2] are being considered for inclusion in the Specification for the Design, Fabrication and Erection of Structural Steel for Buildings published by the American Institute of Steel

Construction[3]. For cold-formed steel structural members, the Tentative Recommendations on the LRFD Criteria were developed from a joint research project entitled "Load and Resistance Factor Design of Cold-Formed Steel" conducted at the University of Missouri-Rolla and Washington University[4-10].

B. PURPOSE OF INVESTIGATION

The primary purpose of this investigation was to study and compare the Proposed Load and Resistance Factor Design (LRFD) Criteria for Cold-Formed Steel[10] with the existing Allowable Stress Design (ASD) Criteria included in the 1980 Specification for the Design of Cold-Formed Steel Structural Members[1]. This comparison involved studies of different variables used for the design of various types of structural members and discussions of different load carrying capacities determined by these two methods.

In addition, design examples were prepared to illustrate the application of the proposed Load and Resistance Factor Design Method for the purpose of comparison.

C. SCOPE OF INVESTIGATION

This study compares the existing Allowable Stress Design Method with the proposed Load and Resistance Factor Design Method for cold-formed steel structural members generally used in building construction. These shapes include channels with stiffened or unstiffened flanges, I-sections made from channels, and hat sections with unreinforced webs. The yield points of steel range from 33 to 50 ksi.

The AISI Specification and the proposed LRFD criteria can be used for the design of tension members, flexural members, compression members, members subjected to a combination of bending and axial loads, bolted connections, and weld connections. Even though the allowable stress design provisions and the proposed LRFD criteria were prepared for any combinations of different loads, only dead and live loads were used in this comparison for each type of structural members. Ratios of load carrying capacities were computed and evaluated for different shapes of structural members which are used in typical design situations.

II. REVIEW OF LITERATURE

A. GENERAL

Because of the growing need for a unified approach to structural design for all types of construction materials, many studies have been conducted in recent years. In early 1978, the LRFD criteria for hot-rolled steel shapes [2] were proposed by Galambos as alternative design methods. This proposal was a result of a research project conducted at Washington University under the sponsorship of the American Iron and Steel Institute. This subject was subsequently discussed by Galambos, Ravindra, Yura, Bjorhovde, Cooper, Hansell, Viest, Fisher, Kulak, and Cornell in References 11 through 18. In addition, numerous papers were published in the proceedings of the American Society of Civil Engineers (ASCE) Specialty Conference on Probabilistic Mechanics and Structural Reliability held in January 1979. In Reference 19, Grigoriu, Veneziano, and Cornell discuss the importance of decision making in probability distribution modeling. Chalk and Cortis studied a collection of live load data to develop a probabilistic format for the determination of design live loads for building floors[20].

During the period from 1979 to 1982, Ellingwood studied statistical information in reinforced concrete[21,22], wood[23], and masonry[24] structures for developing a probability-based limit states design criteria. In a recent study sponsored by the National Bureau of Standards, Galambos, Ellingwood, MacGregor, and Cornell developed a set of load factors, load combinations, and methodology for material specification groups [25-27]. More recently, the ASCE Committee on Fatigue and Fracture Reliability published a series of reports on fatigue reliability [28-30].

With regard to cold-formed steel design, a study on reliability based criteria for temporary cold-formed steel building was conducted by Knob and Lind [31] in 1975. A joint research project entitled "Load and Resistance Factor Design of Cold-Formed Steel" was conducted by Rang, Supornsilaphachai, Galambos, and Yu at the University of Missouri-Rolla and Washington University since 1976. This project was also under the sponsorship of AISI. References 4 through 8 summarize the studies of the LRFD criteria for cold-formed steel tension members, beams, columns, beam-columns, and connections. The research findings have been discussed at various engineering and specialty conferences and published in several conference proceedings [32-34]. In March 1980, the Tentative Recommendations on the LRFD Criteria for Cold-Formed Steel Structural Members and Commentary [9] were prepared according to the 1968 edition of the AISI Specification for allowable stress design. These tentative recommendations were updated in 1982 [10] on the basis of the 1980 edition of the AISI Specification [1] and the additional study conducted by Supornsilaphachai in 1980 [35].

In Canada, the Canadian Standards Association permits the use of either allowable stress design or limit states design in their standard for cold-formed steel [36].

B. LOAD AND RESISTANCE FACTOR DESIGN CRITERIA

The Tentative Recommendations on the Load and Resistance Factor Design Criteria for Cold-Formed Steel[10] are based on the first-order principles of probabilistic theory. The general format for the LRFD criteria is

$$\phi R_n \geq \sum_{k=1}^{j} \gamma_k Q_{kn} \tag{2.1}$$

In the above,

ϕ = resistance factor

R_n = nominal resistance

γ_k = load factor

Q_{kn} = nominal load effect

On the left side of Eq. (2.1), the resistance factor, ϕ, is a nondimensional factor less than or equal to one that accounts for the uncertainties in calculating the nominal resistance. The nominal resistance of the structure is the predicted ultimate resistance or load determined from design formulas using specified mechanical properties of material and section properties. It could be a bending moment, axial load, shear force, or an interaction formula when load combinations are present.

On the right side of the equation, factor γ is a nondimensional load factor used to reflect the possiblity of overloads and uncertainties in computing the load effect. Each load factor applies to a nominal load effect Q_n and the subscript k corresponds to different types of loads. Only dead and live load effects were used to develop the LRFD criteria for cold-formed steel.

Instead of a safety factor, a safety index is used to determine structural reliability. The safety index, β, indicates the probability of failure as shown in Figure 1. The distribution of the R/Q ratio was assumed to be lognormal. The safety index can be determined by using Eq. (2.2):[4,35]

$$\beta = \frac{\ln(R_m/Q_m)}{\sqrt{V_R^2 + V_Q^2}} \tag{2.2}$$

where

R_m = mean value of resistances

Q_m = mean value of load effects

V_R = coefficient of variation of resistances

V_Q = coefficient of variation of load effects

The target values of safety index used in the development of the LRFD criteria for cold-formed structural members and connections are 2.5 and 4.0, respectively. A probability of failure of 9.8×10^{-3} is obtained from the cumulative lognormal distribution for the value of safety index equal to 2.5[35].

Unlike the traditional design methods, the resistance of the structure is considered to be a random variable because of variations in mechanical properties and fabrication and uncertainties involved in calculations of the resistance. The mean value of the resistances was assumed to be a product of several values as given in Eq. (2.3).

$$R_m = R_n M_m F_m P_m \tag{2.3}$$

where M_m, F_m and P_m are the mean values of nondimensional variables reflecting the uncertainties in mechanical properties, sectional

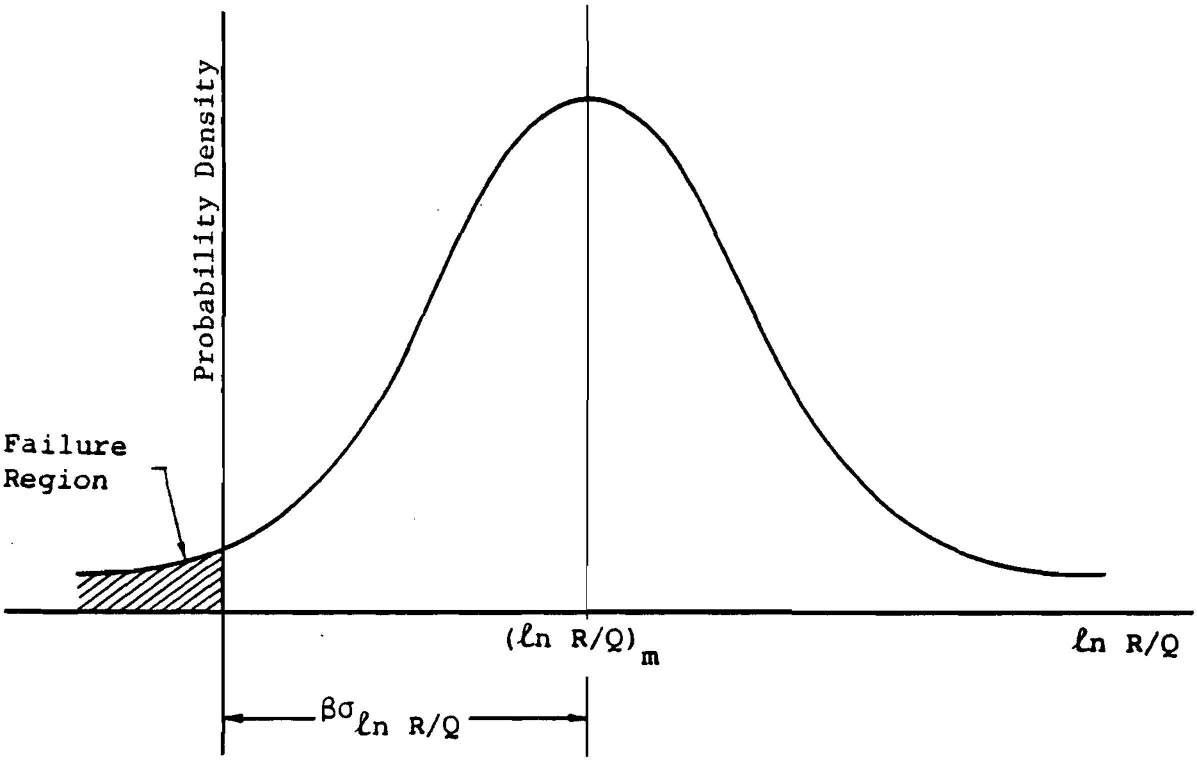


Figure 1. Probability Distribution of $\ln R/Q$

properties, and calculation of the resistance.

In Eq. (2.3), M is the material factor which is determined by the ratio of the tested mechanical properties to the specified values. Mechanical properties include yield point, modulus of elasticity, and tensile strength values. The fabrication factor, F, accounts for variations of geometric dimensions and uncertainties caused by initial imperfections and tolerances. The professional factor, P, accounts for uncertainties that results from the use of approximations and simplifications of complex design formulas based on ideal situations. It is obtained from the ratio of the tested failure loads to the predicted failure loads computed from design formulas.

From statistical studies of applied loads and reliability calculations[26,27], the following load combinations and load factors were used for cold-formed steel:[46]

1. $1.4\ D_n$
2. $1.4\ D_n + L_n$
3. $1.2\ D_n + 1.6\ L_n + 0.5(L_{rn} \text{ or } S_n \text{ or } R_n)$
4. $1.2\ D_n + 1.6(L_{rn} \text{ or } S_n \text{ or } R_n) + (0.5\ L_n \text{ or } 0.8\ W_n)$
5. $1.2\ D_n + 1.3\ W_n + 0.5\ L_n + 0.5(L_{rn} \text{ or } S_n \text{ or } R_n)$
6. $1.2\ D_n + 1.5\ E_n + (0.5\ L_n \text{ or } 0.2\ S_n)$
7. $0.9\ D_n - (1.3\ W_n \text{ or } 1.5\ E_n)$

where D_n = nominal dead load

E_n = nominal earthquake load

L_n = nominal live load

L_{rn} = nominal roof live load

R_n = nominal roof rain load

S_n = nominal snow load

W_n = nominal wind load (Exception: For wind load on individual purlins, girts, wall panels and roof decks, multiply W_n by 0.9)

Exception: The load factor on L_n in combination (4), (5), and (6) shall be equal to 1.0 for garages, areas occupied as places of public assembly, and all areas where the live load is greater than 100 psf.

For roof and floor construction, the load combination for dead load, weight of wet concrete, and construction load including equipment, workmen and formwork is suggested in Section 8.3.(2)(a) of the Commentary.[10]

When the structure effects of F,H,P, or T are significant, they shall be considered in design as the following factored loads: 1.3F, 1.6H, 1.2P, and 1.2T, where

F = loads due to fluids with well-defined pressures and maximum heights

H = loads due to the weight and lateral pressure of soil and water in soil

P = loads, forces, and effects due to ponding

T = self-straining forces and effects arising from contraction or expansion resulting from temperature changes, shrinkage, moisture changes, creep in component materials, movement due to differential settlement, or combinations thereof

The preceding load combinations are listed in Section 8.3.4 of the Tentative Recommendations(10) and should be used in the computation of the load effects. The combination of dead and live load with an assumed dead-to-live load ratio of 1/5 were used to develop the LRFD criteria for cold-formed steel.

The coefficient of variation of the resistances, V_R, is related to the coefficient of variation of M, F, and P as follows:

$$V_R = \sqrt{V_M^2 + V_F^2 + V_P^2} \quad (2.4)$$

The coefficient of variation of the load effects, V_Q, can be computed from the nominal dead-to-live load ratio and the coefficient of variation of the dead and live loads. For a dead-to-live load ratio equal to 1/5, V_Q is equal to 0.21.

The resistance factor can be obtained from the following equation developed in Reference 10.

$$\phi = \frac{1.481 M_m F_m P_m}{\exp(\beta\sqrt{V_R^2 + V_Q^2})} \quad (2.5)$$

All statistical data and calculations for material factors, fabrication factors, professional factors, coefficients of variation of resistances, and resistance factors can be found in References 4 through 10.

In the LRFD criteria, the factored nominal resistance for design is ϕR_n. For the purpose of comparison, the unfactored load combination $(D_n + L_n)$ or allowable load can be computed from the nominal resistance R_n, the resistance factor ϕ, and a given D_n/L_n ratio as follows:

$$\phi R_n \geq c(1.2\, D_n + 1.6\, L_n)$$

$$\phi R_n \geq c(1.2\, D_n/L_n + 1.6) L_n$$

$$\phi R_n \geq c(1.2\, D_n/L_n + 1.6)[(D_n + L_n)/(D_n/L_n + 1)]$$

Therefore,

$$c(D_n + L_n) \leq \frac{R_n}{(1.2\, D_n/L_n + 1.6)/[\phi(D_n/L_n + 1)]} \qquad (2.6)$$

where c is the deterministic influence coefficient to transform the load to load effect.

From Eq. (2.6), the factor of safety against the nominal resistance used in the LRFD criteria is:

$$(F.S.)_{LRFD} = (1.2\, D_n/L_n + 1.6)/[\phi(D_n/L_n + 1)] \qquad (2.7)$$

Equation (2.6) was used in this study to compare the AISI Specification for allowable stress design and the Tentative Recommendations on the LRFD criteria. The results are presented and discussed in Chapters III through VII.

III. TENSION MEMBERS

A. ALLOWABLE STRESS DESIGN (ASD)

According to Section 3.1 of the AISI Specification[1], cold-formed steel tension members should be designed to satisfy the following requirement:

"Stress on the net section of tension members, and tension and compression on the extreme of flexural members, shall not exceed the value F specified below, except as otherwise specifically provided herein.

$$F = 0.60\ F_y \tag{3.1}$$

where F_y is the specified minimum yield point."

B. LOAD AND RESISTANCE FACTOR DESIGN (LRFD)

Based on Section 9.2 of the Proposed Tentative Recommendations[10], the following provisions are used for the design of cold-formed steel tension members:

"For axially loaded tension members, the factored nominal tensile strength, ϕR_{nt}, shall be determined according to the following formulas:

$$\phi = 0.95$$

$$R_{nt} = A_n\ F_y \tag{3.2}$$

In the above,

ϕ = resistance factor for tension

R_{nt} = nominal strength of the member when loaded in tension, kips

A_n = net area of the cross section, in.^{2}"

C. COMPARISON

For a comparison between the allowable stress design and the LRFD approach, the unfactored load can be calculated by using the following equation for both design methods:

$$P_T = P_{DL} + P_{LL} \qquad (3.3)$$

where

P_T = total unfactored load applied to the member, kips

P_{DL} = axial tension due to the nominal dead load, kips

P_{LL} = axial tension due to the nominal live load, kips

This total unfactored load should be less than or equal to the allowable load. For allowable stress design, the allowable load is

$$(P_a)_{ASD} = A_n F = A_n (0.60 F_y) \qquad (3.4)$$

For LRFD, the allowable load can be calculated by using Eq. (2.6).

$$(P_a)_{LRFD} = \phi R_{nt}(D/L + 1)/(1.2 D/L + 1.6) \qquad (3.5)$$

Because $R_{nt} = A_n F_y$, Eq. (3.5) can be rewritten as

$$(P_a)_{LRFD} = \phi A_n F_y (D/L + 1)/(1.2 D/L + 1.6) \qquad (3.6)$$

where D/L is the ratio of the nominal dead load to the nominal live load. From Eq. (3.6) it is clear that the allowable load based on LRFD is a function of not only cross-sectional area and yield strength of the steel but also the dead-to-live load ratio. This will be true for all structural members designed by LRFD method.

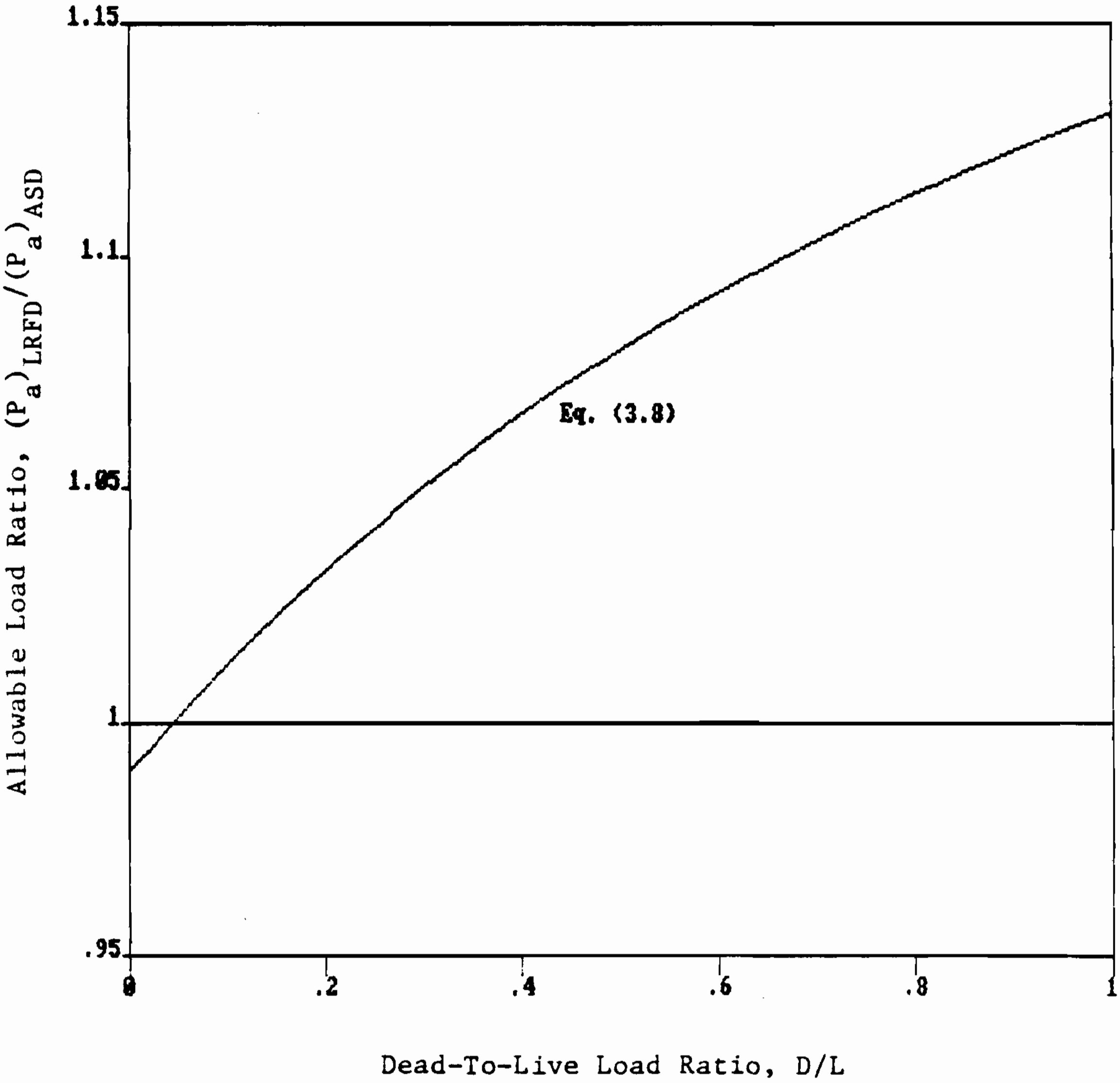


Figure 2. Allowable Load Ratio vs. D/L Ratio for Tension

Therefore, based on Eqs. (3.4) and (3.6), the allowable load ratio for tension members is

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \frac{\phi}{0.60} \; \frac{D/L + 1}{1.2D/L+1.6} \tag{3.7}$$

For the value of $\phi = 0.95$

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.58 \; \frac{D/L + 1}{1.2D/L+1.6} \tag{3.8}$$

Figure 2 shows the allowable load ratio versus the dead-to-live load ratio. When $D/L < 1/25$, the allowable load determined by the LRFD method is slightly less than that determined by the allowable stress design. For $D/L = 1/5$, ASD is about 3.2% conservative compared to LRFD.

D. DESIGN EXAMPLE

See Problem No. 1 in Appendix C for a design example of a tension member using Load and Resistance Factor Design.

IV. FLEXURAL MEMBERS

A. GENERAL

Cold-formed steel flexural members have several possible modes of failure. In the design of beams, consideration should first be given to the section strength or the moment-resisting capacity based on the type of compression elements present. For beams with inadequate lateral bracing, lateral buckling may limit the moment-resisting capacity. Beam webs have to be designed for shear, bending, and combined bending and shear. Because of highly localized concentrations of stress resulting from applied concentrated loads or reactions, web crippling and combined bending and web crippling have to be checked. Excessive deflection due to service live load could also be a problem.

B. BENDING STRENGTH

1. Allowable Stress Design. The section reaches its maximum allowable moment when the stress on the outer fibers of the flanges reaches an allowable stress. If the compression flange is a stiffened type, then the basic design stress, F, is the maximum allowable stress and an effective width of the compression flange is used. This effective width is calculated by using Section 2.3.1.1 of the AISI Specification[1]. If the compression flange is an unstiffened type, then a reduced allowable compressive stress, F_c, is used with the reduction depending upon the flat width-to-thickness ratio of the compression flange. The following equations are based on Section 3.1 and 3.2 of the AISI Specification[1]:

Basic design stress,

$$F = 0.60\ F_y \tag{4.1}$$

For $w/t \leq 63.3/\sqrt{F_y}$,

$$F_c = 0.60\ F_y \tag{4.2}$$

For $63.3/\sqrt{F_y} < w/t \leq 144/\sqrt{F_y}$,

$$F_c = F_y[0.767-(2.64\text{x}10^{-3})(w/t)\sqrt{F_y}] \tag{4.3}$$

For $144/\sqrt{F_y} < w/t \leq 25$,

$$F_c = 8000/(w/t)^2 \tag{4.4}$$

For $25 < w/t \leq 60$,

$$F_c = 8000/(w/t)^2\text{, for any struts and} \tag{4.5}$$

$$F_c = 19.8-0.28(w/t)\text{, for all other sections} \tag{4.6}$$

where

w/t = flat width-to-thickness ratio of the compression flange.

2. LRFD Criteria. The section reaches its ultimate moment when the stress on the extreme fibers of the beam having a stiffened compression flange reaches the yield point of the steel. For sections with unstiffened compression flanges, the ultimate moment may be limited by local buckling of the compression flange. Based on Section 9.3.1 of the Tentative Recommendations[10], the factored nominal section strength, ϕM_u, shall be determined by using $\phi = 0.95$ and the applicable value of M_u given as follows:

For members with stiffened compression flanges,

$$M_u = S_{eff}\ F_y \tag{4.7}$$

For members with unstiffened compression flanges,

$$M_u = S_c\ F_{cr} \leq S_t F_y \tag{4.8}$$

where

S_{eff} = elastic section modulus of effective section determined according to Section 8.4[10], in.^{3}

S_c = elastic section modulus of entire section about axis of bending; moment of inertia divided by distance to extreme compression fiber, in.^{3}

F_{cr} = critical stress determined according to Section 8.5[10], ksi

S_t = elastic section modulus of entire section about axis of bending; moment of inertia divided by distance to extreme tension fiber, in.^{3}

The critical stress, F_{cr}, on the basis of Section 8.5[10] is as follows:

For $w/t \leq 63.3/\sqrt{F_y}$,

$$F_{cr} = F_y \tag{4.9}$$

For $63.3/\sqrt{F_y} < w/t \leq 144/\sqrt{F_y}$,

$$F_{cr} = F_y[1.28-0.0044(w/t)\sqrt{F_y}\ ] \tag{4.10}$$

For $144/\sqrt{F_y} < w/t \leq 25$,

$$F_{cr} = 13{,}300/(w/t)^2 \tag{4.11}$$

For $25 < w/t \leq 60$,

$$F_{cr} = 13{,}300/(w/t)^2 \text{ for angle struts and} \tag{4.12}$$

$$F_{cr} = 33.0-0.467(w/t) \text{ for all other sections} \tag{4.13}$$

3. <u>Comparison</u>. The unfactored moment can be calculated by using Eq. (4.14) for both methods (ASD and LRFD) for comparison.

$$M_{TL} = M_{DL} + M_{LL} \tag{4.14}$$

where

M_{TL} = total unfactored moment, kip-in.

M_{DL} = moment due to the nominal dead load, kip-in.

M_{LL} = moment due to the nominal live load, kip-in.

For allowable stress design, the allowable stresses are determined from either the yield point of steel or the critical local buckling stress with a factor of safety of 1.67. Therefore, the allowable moment for beams with stiffened flanges is

$$(M_a)_{ASD} = F\, S_{eff} = 0.60\, F_y\, S_{eff} \tag{4.15}$$

and the allowable moment for beams with unstiffened flanges is

$$(M_a)_{ASD} = F_c S_c = 0.60\, F_{cr}\, S_c \tag{4.16}$$

For LRFD, the allowable moment can be computed by using the following equation developed from Eq. (2.6).

$$(M_a)_{LRFD} = \phi M_u (D/L+1)/(1.2D/L+1.6) \tag{4.17}$$

For beams with stiffened flanges,

$$(M_a)_{LRFD} = \phi F_y\, S_{eff} (D/L+1)/(1.2D/L+1.6) \tag{4.18}$$

For beams with unstiffened flanges,

$$(M_a)_{LRFD} = \phi F_{cr} S_c (D/L+1)/(1.2D/L+1.6) \tag{4.19}$$

The ratio of the allowable moments for beams with both stiffened and unstiffened compression elements is

$$\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = 1.67\phi \frac{D/L+1}{1.2D/L+1.6} \tag{4.20}$$

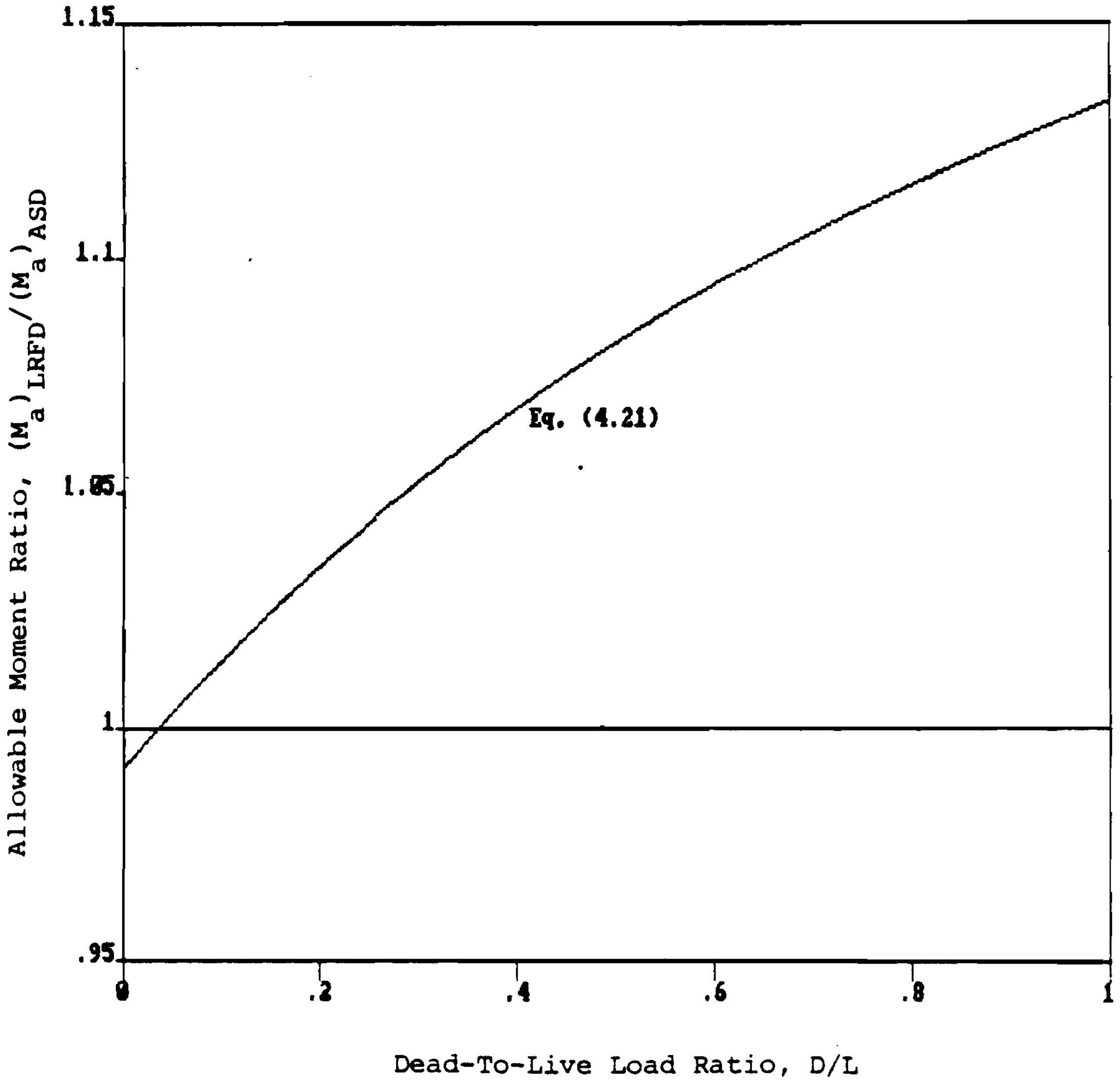


Figure 3. Allowable Moment Ratio vs. D/L Ratio for Bending Strength of Beams

By using $\phi = 0.95$,

$$\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = 1.58 \; \frac{D/L + 1}{1.2D/L + 1.6} \tag{4.21}$$

Figure 3 shows the allowable moment ratio versus dead-to-live load ratio for beams based on the section strength. For D/L = 1/25 both design methods will give the same value of allowable moment. However, LRFD will be conservative for D/L < 1/25 and unconservative for D/L > 1/25 as compared with the allowable stress design method.

C. LATERAL BUCKLING

1. Allowable Stress Design. To prevent lateral buckling, the maximum compression stress, in kips per square inch, on extreme fibers of laterally unsupported straight flexural members should not exceed the allowable stress, F_b, as specified in Sections 3.1 and 3.2 nor the following allowable stresses in accordance with Section 3.3 of the AISI Specification(1).

a. Singly-Symmetric and Doubly-Symmetric Shapes. When bending is about the centroidal axis perpendicular to the web for either I-shaped sections symmetrical about an axis in the plane for the web or symmetrical channel-shaped sections:

When $0.36\pi^2 EC_b/F_y < L^2 S_{xc}/dI_{yc} < 1.8\pi^2 EC_b/F_y$,

$$F_b = \frac{2}{3} F_y - \frac{F_y^2}{5.4\pi^2 EC_b} \left(\frac{L^2 S_{xc}}{dI_{yc}} \right) \tag{4.22}$$

When $L^2 S_{xc}/dI_{yc} \geq 1.8\pi^2 EC_b/F_y$,

$$F_b = 0.6\pi^2 EC_b \frac{dI_{yc}}{L^2 S_{xc}} \tag{4.23}$$

b. Point-Symmetric Shapes. For point-symmetrical Z-shaped sections bent about the centroidal axis perpendicular to the web:

When $0.18\pi^2 EC_b/F_y < L^2 S_{xc}/dI_{yc} < 0.9\pi^2 EC_b/F_y$,

$$F_b = \frac{2}{3} F_y - \frac{F_y^{\,2}}{2.7\pi^2 EC_b}\left(\frac{L^2 S_{xc}}{dI_{yc}}\right) \tag{4.24}$$

When $L^2 S_{xc}/dI_{yc} \geq 0.9\pi^2 EC_b/F_y$,

$$F_b = 0.3\pi^2 EC_b \frac{dI_{yc}}{L^2\ S_{xc}} \tag{4.25}$$

where

L = the unbraced length of the member, in.

I_{yc} = the moment of inertia of the compression portion of a section about the gravity axis of the entire section parallel to the web, in.^{4}

S_{xc} = compression section modulus of entire section about major axis, in.^{3}

C_b = bending coefficient which can be conservatively be taken as unity, or calculated from

$$C_b = 1.75+1.05(M_1/M_2)+ 0.3(M_1/M_2)^2 \tag{4.26}$$

but not more than 2.3 where M_1 is the smaller and M_2 the larger bending moment at the ends of the unbraced length, taken about the strong axis of the member, and where M_1/M_2, the ratio of end moments, is positive when M_1 and M_2 have the same sign (reverse curvature bending) and negative when they are of opposite sign (single curvature bending). When the bending moment at any point within an unbraced length is larger than that at both ends of this length the ratio C_b shall be taken as unity. For members subject to combined axial and bending stress (Section 3.7[1]), C_b shall be 1.0.

E = modulus of elasticity = 29,500 ksi

d = depth of section, in.

2. <u>LRFD Criteria</u>. According to Section 9.3.2 of Reference 10, the factored nominal strength of laterally unbraced I, channel, or Z-shaped members, ϕM_u, should be determined with $\phi = 0.90$ and

For $M_y/M_e \leq 0.36$,

$$M_u = M_y \tag{4.27}$$

For $0.36 \leq M_y/M_e \leq 1.8$,

$$M_u = M_y\ (10/9)[1- (5/18)(M_y/M_e)] \tag{4.28}$$

For $M_y/M_e \geq 1.8$,

$$M_u = M_e \tag{4.29}$$

where

$M_y = S_{xc}\ F_y$

M_e = critical moment, kip-in.

a. Singly-Symmetric and Doubly-Symmetric Shapes. For bending about the centroidal axis perpendicular to the web for either I-shaped sections symmetrical about an axis in the plane of the web, or symmetric channel-shaped sections,

$$M_e = \pi^2 E C_b d I_{yc}/L^2 \tag{4.30}$$

b. Point-Symmetric Shapes. For point-symmetrical Z-shaped sections bent about the centroidal axis perpendicular to the web,

$$M_e = \pi^2 E C_b d I_{yc}/2L^2 \tag{4.31}$$

3. Comparison. The unfactored moment can also be calculated by using Eq. (4.14) for the consideration of lateral buckling. This unfactored moment should be less than or equal to the allowable moment. For allowable stress design, the allowable moment for beams based on lateral buckling is

$$(M_a)_{ASD} = F_b\, S_{xc} \tag{4.32}$$

For LRFD, the allowable moment can be computed by using Eq. (2.6).

$$(M_a)_{LRFD} = \phi M_u (D/L+1)/(1.2D/L+1.6) \tag{4.33}$$

In view of the fact that the limits for the buckling modes are the same for both design methods and that the allowable compressive stress, F_b, is derived from the ultimate stress on the basis of the ultimate moment, M_u, with a factor of safety equal to 1.67, the ratio of the allowable moments is

$$\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = 1.67\phi \frac{D/L + 1}{1.2D/L+1.6} \tag{4.34}$$

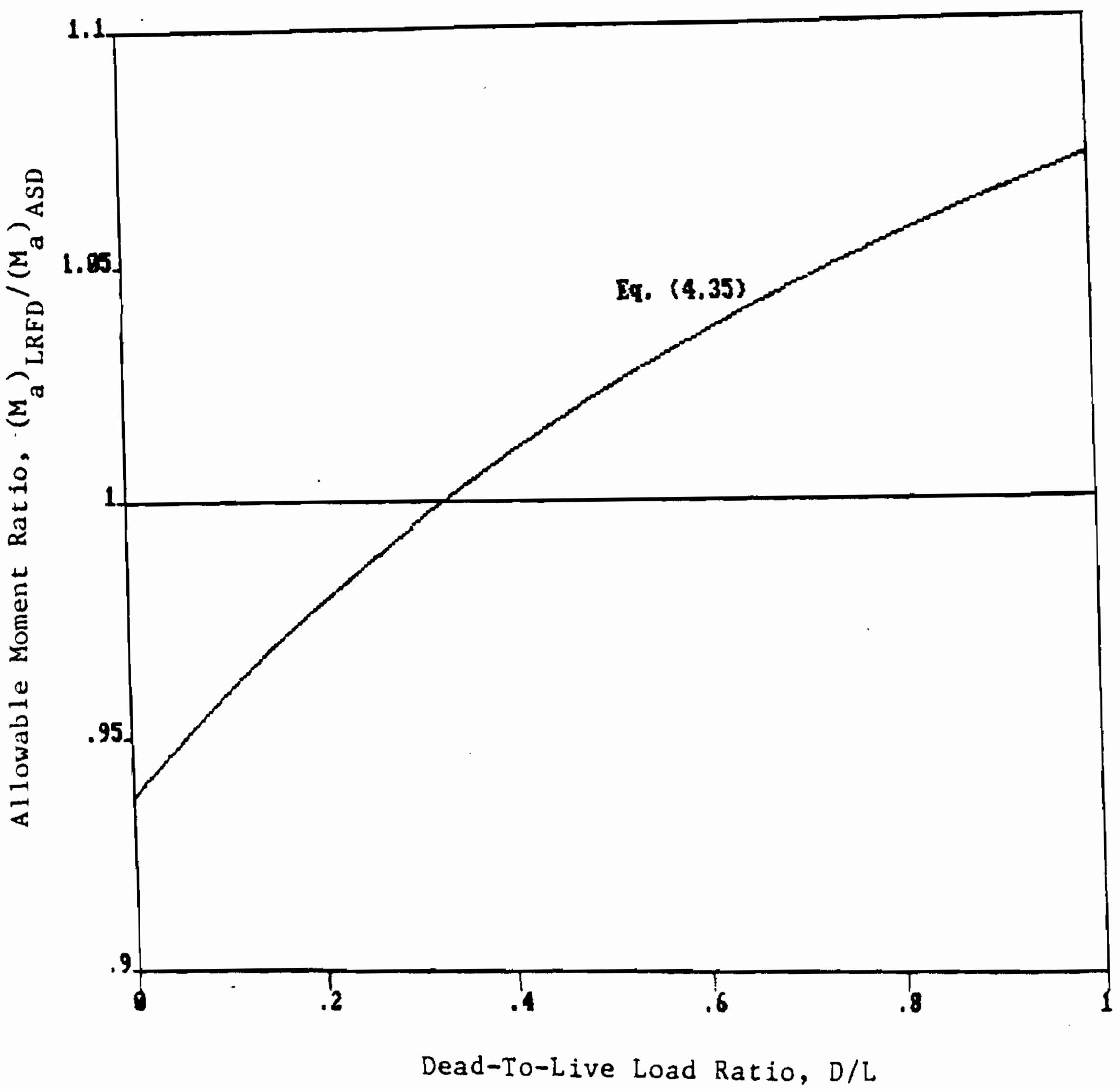


Figure 4. Allowable Moment Ratio vs. D/L Ratio for Lateral Buckling of Beams

Since $\phi = 0.90$

$$\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = 1.50 \; \frac{D/L+1}{1.2D/L+1.6} \qquad (4.35)$$

Figure 4 shows the allowable moment ratio versus the dead-to-live load ratio for this case. The two design methods give the same value for D/L = 1/3. For D/L = 0.5, the allowable moment based on LRFD is about 2.3% larger than the value obtained from allowable stress design. When the dead-to-live load ratio for cold-formed steel is less than 1/3, the LRFD criteria are found to be conservative for lateral buckling as compared with the allowable stress design method.

D. <u>WEB STRENGTH</u>

Beam webs should be designed for shear, bending, combined bending and shear, and web crippling. The AISI provisions on web design have recently been revised in the 1980 Edition of the Specification based on a research project conducted at the University of Missouri-Rolla[37-40]. Because some beam webs may require transverse stiffeners to improve the shear strength, new requirements for stiffeners are included in Reference 1.

1. <u>Shear Strength of Beam Webs</u>. There are three possible modes of shear failure in beam webs. For a relatively small h/t ratio, shear yielding will be the failure mode. For webs with large h/t ratios, the webs will fail in elastic shear buckling. For moderate values of h/t, the shear buckling will be in the inelastic range.

a. Allowable Stress Design. The maximum average shear stress in kips per square inch, on the gross area of a flat web should not exceed the allowable shear stress, F_v, specified in Section 3.4.1 of the Specification[1] as follows:

For $h/t \leq 237\sqrt{k_v/F_y}$,

$$F_v = 65.7\sqrt{k_v F_y}/(h/t) \leq 0.40\, F_y \qquad (4.36)$$

For $h/t > 237\sqrt{k_v/F_y}$,

$$F_v = 15{,}600\, k_v/(h/t)^2 \qquad (4.37)$$

where

F_y = yield point of the beam web, ksi

t = base steel thickness of the web element, in.

h = clear distance between flanges measured along the plane of web, in.

k_v = shear buckling coefficient determined as follows:

For unreinforced webs, $k_v = 5.34$

For beam webs with transverse stiffeners satisfying the requirements of Section 2.3.4.2,

$k_v=4.00+5.34/(a/h)^2$, when $a/h \leq 1.0$

$k_v=5.34+4.00/(a/h)^2$, when $a/h > 1.0$

In the above expressions, a is equal to the shear panel length of the unreinforced web element, in. For a reinforced web element, a is the distance between transverse stiffeners, in.

Where the web consists of two or more sheets, each sheet shall be considered as a separate member carrying its share of the shear.

b. LRFD Criteria. According to Section 9.3.3 of the Tentative Recommendations[10], the factored nominal shear strength of flat beam webs, $\phi_v V_u$, shall be determined as follows:

For $h/t \leq 171\sqrt{k_v/F_y}$,

$$\phi_v = 1.0$$

$$V_u = A_w F_y/\sqrt{3} \tag{4.38}$$

For $171\sqrt{k_v/F_y} < h/t \leq 243\sqrt{k_v/F_y}$,

$$\phi_v = 0.90$$

$$V_u = 110 A_w \sqrt{k_v F_y}/(h/t) \tag{4.39}$$

For $h/t > 243\sqrt{k_v/F_y}$,

$$\phi_v = 0.90$$

$$V_u = 26{,}700\, k_v A_w/(h/t)^2 \tag{4.40}$$

where

ϕ_v = resistance factor for shear

A_w = area of beam web (ht), in.^{2}

c. Comparison. The unfactored shear force can be calculated for both ASD and LRFD methods by using the following equation.

$$V_T = V_{DL} + V_{LL} \tag{4.41}$$

where

V_T = total unfactored shear force, kips

V_{DL} = shear force due to the nominal dead load, kips

V_{LL} = shear force due to the nominal live load, kips

This total unfactored shear force should be less than or equal to the allowable shear capacity. For allowable stress design, the allowable shear load for beam webs is

$$(V_a)_{ASD} = F_v\ ht \tag{4.42}$$

For LRFD, the allowable shear load equation was developed from Eq. (2.6) and is

$$(V_a)_{LRFD} = \phi_v V_u (D/L+1)/(1.2D/L+1.6) \tag{4.43}$$

The allowable shear stress, F_v, is determined from shear yielding with a factor of safety of 1.44, from the critical stress for elastic shear buckling with a factor of safety of 1.71, and from the critical stress for inelastic shear buckling with a factor of safety of 1.67. The limits of the h/t ratios were obtained by equating the formulas for the three shear failure modes for both allowable stress and LRFD criteria. Because each failure mode has a different factor of safety, the h/t limits are slightly different for both design criteria. For example, for h/t greater than $237\sqrt{k_v/F_y}$ and less than $243\sqrt{k_v/F_y}$, inelastic shear buckling will govern for LRFD.

The allowable shear ratios are:

For $h/t \leq 171\sqrt{k_v/F_y}$ and $\phi_v = 1.0$,

$$\frac{(V_a)_{LRFD}}{(V_a)_{ASD}} = 1.443\phi_v \frac{D/L+1}{1.2D/L+1.6} = 1.443\ \frac{D/L+1}{1.2D/L+1.6} \tag{4.44}$$

For $171\sqrt{k_v/F_y} < h/t \leq 237\sqrt{k_v/F_y}$ and $\phi_v = 0.90$

$$\frac{(V_a)_{LRFD}}{(V_a)_{ASD}} = 1.674\phi_v \frac{D/L+1}{1.2D/L+1.6} = 1.507\ \frac{D/L+1}{1.2D/L+1.6} \tag{4.45}$$

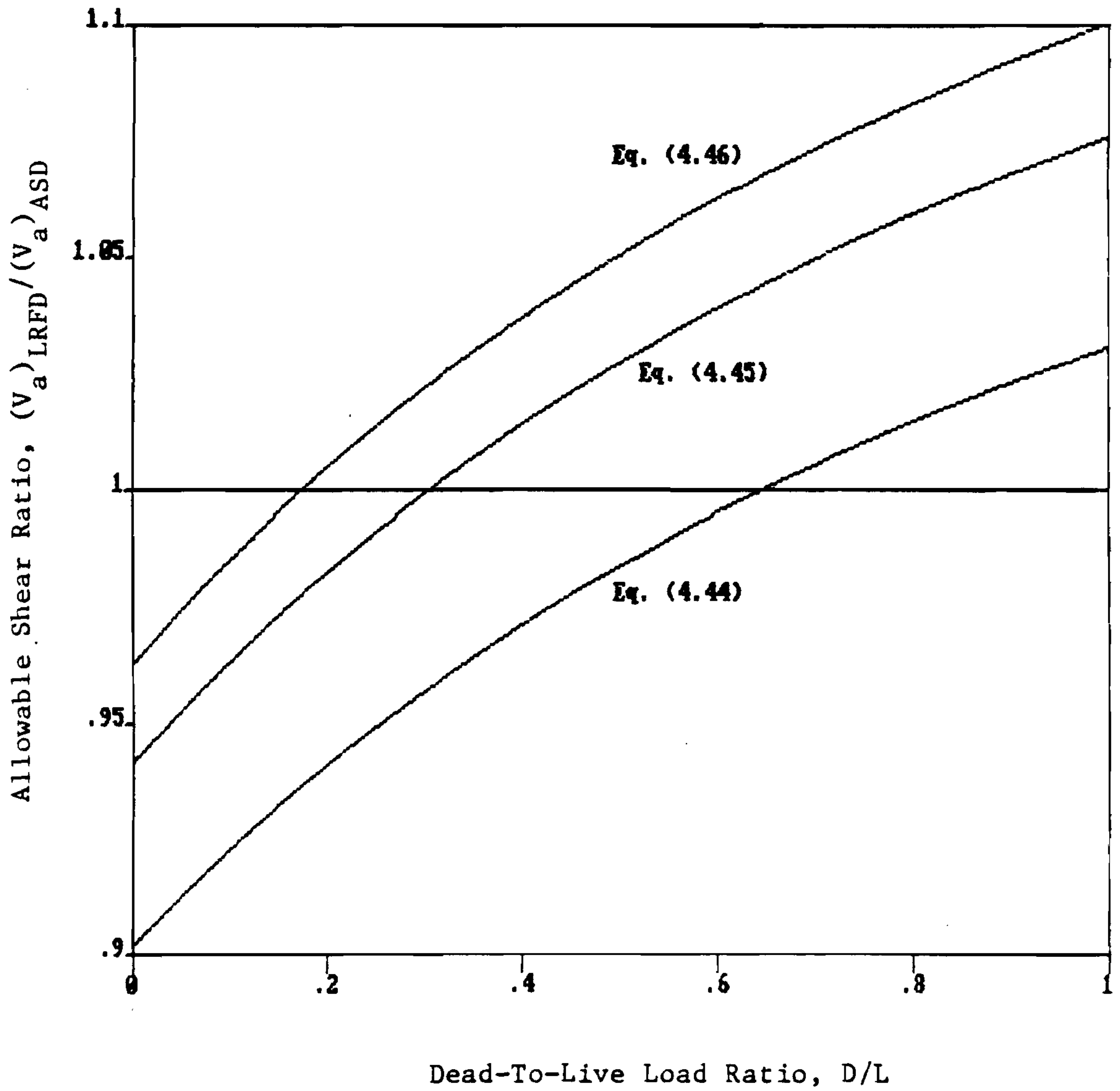


Figure 5. Allowable Shear Ratio vs. D/L Ratio for Shear Strength of Beam Webs

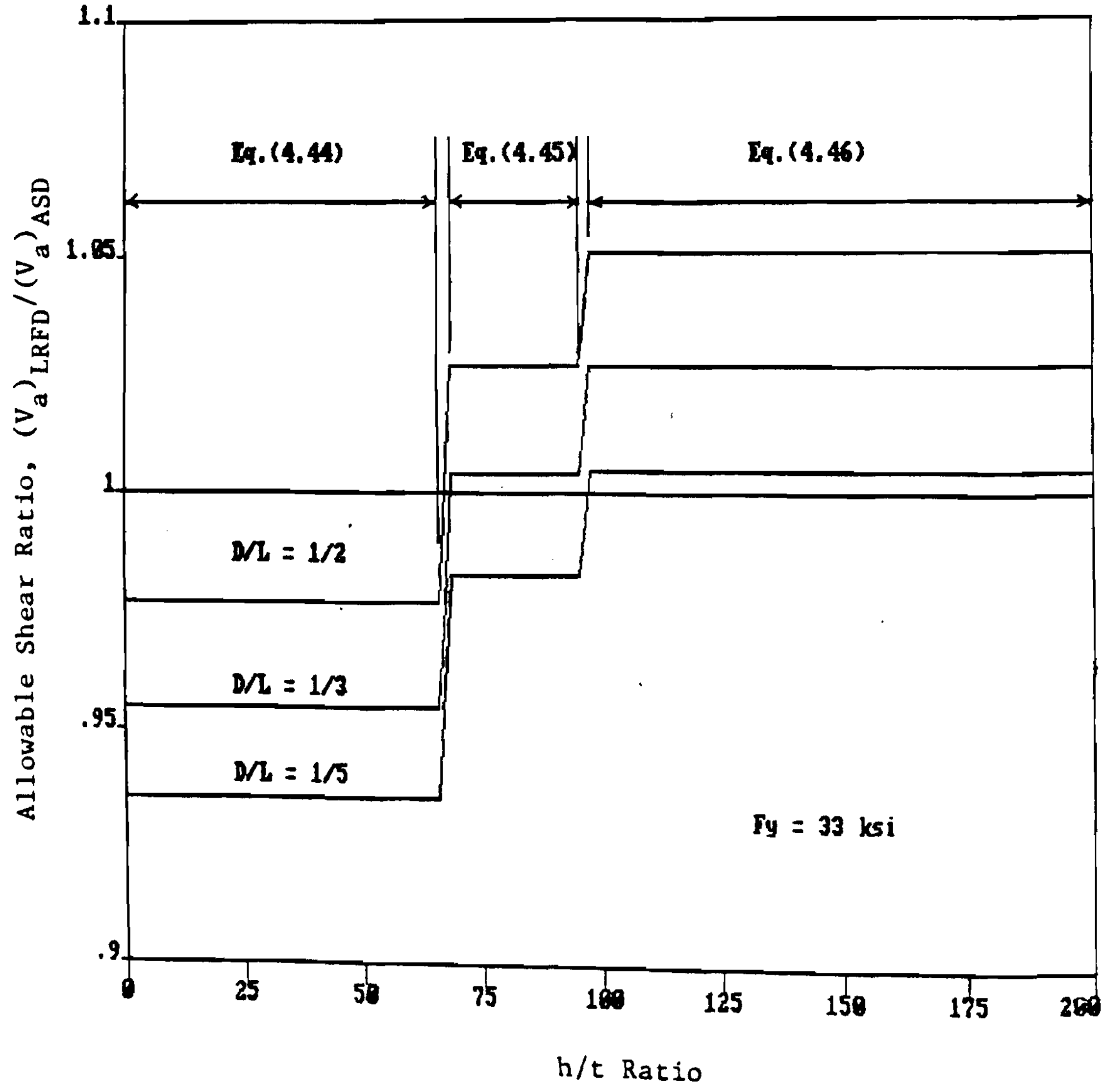


Figure 6. Allowable Shear Ratio vs. h/t Ratio for Shear Strength of Beam Webs

For $h/t > 243\sqrt{k_v/F_y}$ and $\phi_v = 0.90$

$$\frac{(V_a)_{LRFD}}{(V_a)_{ASD}} = 1.712\phi_v \frac{D/L+1}{1.2D/L+1.6} = 1.541\ \frac{D/L+1}{1.2D/L+1.6} \qquad (4.46)$$

Figure 5 shows the allowable shear ratio versus dead-to-live load ratio for the three failure modes. For D/L = 0.5, the allowable shear determined according to LRFD may be up to 5% higher than the value obtained from allowable stress design. For D/L < 0.17, LRFD is generally conservative. When D/L > 0.65, LRFD gives larger values of the allowable shear capacity.

In Figure 6, the relationships of the allowable shear ratio and the h/t ratio are shown graphically for dead-to-live load ratios equal to 1/5, 1/3, and 1/2. The transition zones between h/t limits can be seen clearly in this figure.

2. Flexural Strength of Beams Governed by Webs. For cold-formed steel beams, the bending stress may be reduced due to local buckling in the beam webs. For this reason, due consideration is given in the AISI Specification[1] and the Tentative Recommendations[10].

a. Allowable Stress Design. Based on Section 3.4.2 of Reference 1, the compressive stress in a flat web that results from bending in its plane, computed on the basis of the effective compression flange area for stiffened flanges and the reduced compression flange area for unstiffened flanges and full web area, should not exceed the following allowable stress:

For beams having stiffened compression flanges,

$$F_{bw} = [1.21-0.00034(h/t)\sqrt{F_y}](0.60F_y) \leq 0.60\ F_y \tag{4.47a}$$

For beams having unstiffened compression flanges,

$$F_{bw} = [1.26-0.00051(h/t)\sqrt{F_y}](0.60F_y) \leq 0.60\ F_y \tag{4.47b}$$

b. LRFD Criteria. In Section 9.3.3.2 of the Tentative Recommendations[10], the flexural strength of beams is also limited by the factored strength governed by webs, $\phi_{bw}M_{ubw}$, determined from $\phi_{bw} = 0.90$ and the value of M_{ubw} computed by using Eq. (4.48):

$$M_{ubw} = S_{eff}\ (\lambda F_y) \tag{4.48}$$

where

ϕ_{bw} = resistance factor for bending

S_{eff} = elastic section modulus of the effective section determined by using full areas of the web and the tension flange and the effective compression flange area, in.^{3} For beams having stiffened compression flanges, the effective compression area shall be determined according to Section 8.4.1[10]. For beams having unstiffened compression flanges, the effective compression flange area is equal to the gross flange area times the stress ratio F_{cr}/F_y, where F_{cr} is the critical stress computed according to Section 8.5[10].

λ = $1.21-0.00034(h/t)\sqrt{F_y} \leq 1.0$ for beams having stiffened compression flanges

$\lambda = 1.26-0.0005(h/t)\sqrt{F_y} \leq 1.0$ for beams having unstiffened compression flanges

c. Comparison. The unfactored moment resulting from the applied loads can be calculated for both methods using Eq. (4.14). This moment should be less than or equal to the allowable moment. For allowable stress design, the allowable moment for beam webs is based on an allowable compressive stress in the web. The section modulus is computed using the distance from the neutral axis to the extreme compression fibers. Because the thickness of the flange is usually very small as compared to this distance, the allowable moment is

$$(M_a)_{ASD} = S_{eff}\ F_{bw} = S_{eff}\ \lambda(0.60F_y) \tag{4.49}$$

For LRFD, the moment capacity for beams is based on a maximum stress in the extreme compression fibers. The allowable moment for LRFD was computed from Eq. (2.6) and is

$$(M_a)_{LRFD} = \phi_{bw} M_{ubw}(D/L)/(1.2D+1.6) \tag{4.50}$$

The ratio of allowable moment capacities from Eqs. (4.49) and (4.50) is

$$\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = 1.67\phi_{bw}\frac{D/L+1}{1.2D/L+1.6} = 1.50\ \frac{D/L+1}{1.2D/L+1.6} \tag{4.51}$$

in which $\phi_{bw} = 0.90$. This expression is identical to the allowable moment ratio obtained from the lateral buckling criteria because of identical safety factors and resistance factors used. Figure 7 shows the graph of the moment capacity ratio versus dead-to-live load ratio. For the case of D/L = 0.5, the nominal capacity permitted by LRFD is about 2.3% larger than the value on the basis of the allowable stress design method. The LRFD criteria are found to be conservative

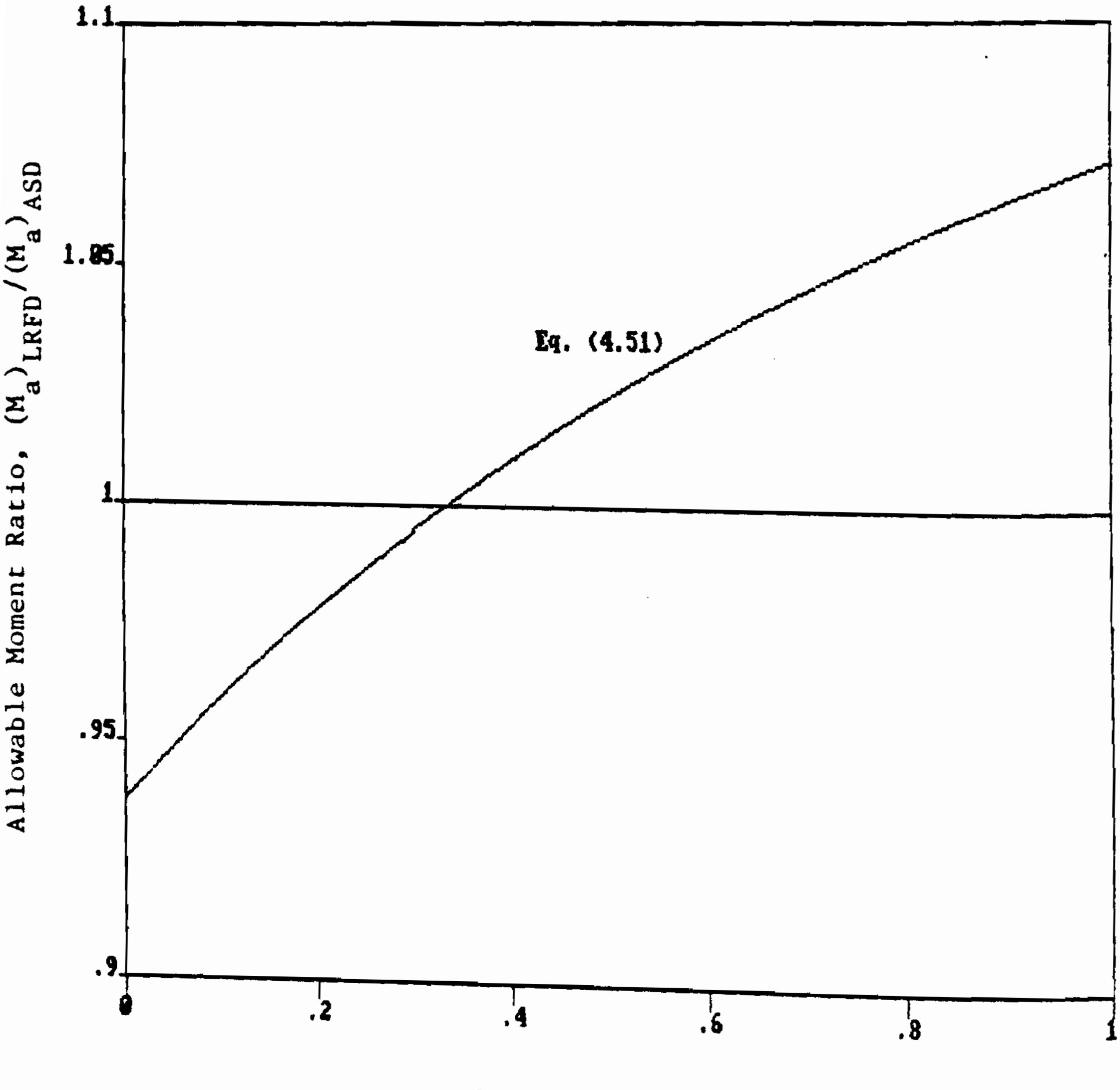


Figure 7. Allowable Moment Ratio vs. D/L Ratio for Web Strength of Beams

for webs strength in bending when D/L ratio is smaller than 1/3.

3. Combined Bending and Shear in Webs. For continuous beams and cantilevers, maximum bending stress and shear stress act simultaneously at supports. The webs will fail at a lower stress than if only one stress were present. The interaction between bending and shear must also be checked in beam webs.

a. Allowable Stress Design. For unreinforced beam webs subjected to both bending and shear stresses, the member should be so proportioned that such stresses do not exceed the allowable values specified in Sections 3.4.1 and 3.4.2 of the AISI Specification[1] and that the following equation should be satisfied in accordance with Section 3.4.3 of the Specification[1]:

$$\left(\frac{f_{bw}}{F_{bw}}\right)^2 + \left(\frac{f_v}{F_v}\right)^2 \leq 1.0 \tag{4.52}$$

For beam webs with transverse stiffeners satisfying the requirements of Section 2.3.4.2 of the Specification[1], the member may be proportioned so that the shear and bending stresses do not exceed the allowable values specified in Sections 3.4.1 and 3.4.2 of the Specification and that

$$0.6\left(\frac{f_{bw}}{F_{bw}}\right)^2 + \left(\frac{f_v}{F_v}\right)^2 \leq 1.3 \tag{4.53}$$

when $f_{bw}/F_{bw} > 0.5$ and $f_v/F_v > 0.7$

In the above expressions,

F_{bw} = allowable compression stress as specified in Section 3.4.2[1], except that for substitution in Eqs. (4.47) and (4.48), the limit of $0.60F_y$ shall not apply, ksi

F_v = allowable shear stress as specified in Section 3.4.1[1] except that for substitution in Eq. (4.36), the limit of $0.40F_y$ shall not apply, ksi

f_{bw} = actual compression stress at junction of flange and web, ksi

f_v = actual average shear stress, i.e., shear force per web divided by web area, ksi

b. LRFD Criteria. Section 9.3.3.3 of the Tentative Recommendations[10] specifies that for unreinforced beam webs subject to a combination of bending and shear, the members should be so proportioned that the factored shear force and the factored bending moment computed on the basis of the factored loads do not exceed the values specified in Sections 9.3.3.1 and 9.3.3.2 of Reference 10 and the following requirement be satisfied:

$$\left(\frac{V_D}{\phi_v V_u}\right)^2 + \left(\frac{M_D}{\phi_{bw} M_{ubw}}\right)^2 \leq 1.0 \qquad (4.54)$$

For beam webs with transverse stiffeners satisfying the requirements of Section 8.4.4.2 of Reference 10, the member may be proportioned so that the factored shear force and the factored bending moment do not exceed the values specified in Sections 9.3.3.1 and 9.3.3.2 of Reference 10 and that

$$\left(\frac{V_D}{\phi_v V_u}\right)^2 + 0.6\left(\frac{M_D}{\phi_{bw} M_{ubw}}\right)^2 \leq 1.3 \qquad (4.55)$$

when $M_D/(\phi_{bw} M_{ubw}) > 0.5$ and $V_D/(\phi_v V_u) > 0.7$

In the above expressions,

V_D = factored shear force computed on the basis of the factored loads, kips

M_D = factored bending moment computed on the basis of the factored loads, kip-in.

ϕ_v = resistance factor for shear = 0.90

ϕ_{bw} = resistance factor for bending = 0.90

V_u = nominal maximum shear strength determined according to Section 9.3.3.1 of Reference 10 except that the equation $V_u = 110A_w\sqrt{k_v F_y}/(h/t)$ shall be used for $h/t \leq 171\sqrt{k_v/F_y}$, kips

M_{ubw} = nominal maximum bending moment determined according to Section 9.3.3.2 of Reference 10 except that for the computation of λ, the limit of 1.0 shall not apply, kip-in.

c. <u>Comparison</u>. A typical design example was selected for comparison purposes. The example deals with a three-equal-span continuous beam subjected to a uniformly distributed dead and live load. The combination of the following maximum moment and shear would occur at the interior supports.

$$M_{TL} = M_{DL} + M_{LL} = c_m w_T L^2 \tag{4.56}$$

$$V_T = V_{DL} + V_{LL} = c_v w_T L \tag{4.57}$$

where c_m and c_v are the deterministic influence coefficients for applied moment and shear based on support conditions and number of spans and w_T is the unfactored applied uniform load.

The allowable uniform loads were calculated for both design methods. Since each design procedure utilizes separate design variables, the allowable uniform loads were expressed using nominal resistances instead of allowable stresses. The allowable load based on allowable stress design was calculated as follows:

$$\frac{f_{bw}}{F_{bw}} = \frac{M_{TL}}{0.6\ M_{ubw}} = \frac{1.667\ c_m w_T L^2}{M_{ubw}} \tag{4.58}$$

For $h/t \leq 237\sqrt{k_v/F_y}$,

$$\frac{f_v}{F_v} = \frac{V_T}{V_u/1.674} = \frac{1.674 c_v w_T L}{V_u} \tag{4.59}$$

By substituting Eqs. (4.58) and (4.59) into Eq. (4.52),

$$\left(\frac{f_{bw}}{F_{bw}}\right)^2 + \left(\frac{f_v}{F_v}\right)^2 = w_T^{\,2}\left[\left(\frac{1.667 c_m L^2}{M_{ubw}}\right)^2 + \left(\frac{1.674 c_v L}{V_u}\right)^2\right] = 1$$

Therefore,

$$(w_T)_{ASD} = \frac{1}{L\sqrt{\left(\frac{1.667 c_m L}{M_{ubw}}\right)^2 + \left(\frac{1.674 c_v}{V_u}\right)^2}} \tag{4.60}$$

For $h/t > 243\sqrt{k_v/F_y}$,

$$\frac{f_v}{F_v} = \frac{V_T}{V_u/1.712} = \frac{1.712 c_v w_T L}{V_u} \tag{4.61}$$

By substituting Eqs. (4.58) and (4.61) into Eq. (4.52),

$$\left(\frac{f_{bw}}{F_{bw}}\right)^2 + \left(\frac{f_v}{F_v}\right)^2 = w_T^{\,2}\left[\left(\frac{1.667 c_m L^2}{M_{ubw}}\right)^2 + \left(\frac{1.712 c_v L}{V_u}\right)^2\right] = 1$$

Therefore,

$$(w_T)_{ASD} = \frac{1}{L\sqrt{\left(\frac{1.667c_mL}{M_{ubw}}\right)^2 + \left(\frac{1.712c_v}{V_u}\right)^2}} \tag{4.62}$$

The allowable uniform load based on LRFD was calculated as follows:

$$\frac{M_D}{\phi_{bw}M_{ubw}} = \frac{1.2D/L+1.6}{D/L+1}\,\frac{M_{TL}}{\phi_{bw}M_{ubw}} = \frac{1.2D/L+1.6}{D/L+1}\,\frac{c_m w_T L^2}{\phi_{bw}M_{ubw}} \tag{4.63}$$

$$\frac{V_D}{\phi_v V_u} = \frac{1.2D/L+1.6}{D/L+1}\,\frac{V_T}{\phi_v V_u} = \frac{1.2D/L+1.6}{D/L+1}\,\frac{c_v w_T L}{\phi_v V_u} \tag{4.64}$$

By substituting Eqs. (4.63) and (4.64) into Eq. (4.54),

$$\left(\frac{M_D}{\phi_{bw}M_{ubw}}\right)^2 + \left(\frac{V_D}{\phi_v V_u}\right)^2 = w_T^{\,2}\left(\frac{1.2D/L+1.6}{D/L+1}\right)^2\left[\left(\frac{c_m L^2}{\phi_b M_{ubw}}\right)^2 + \left(\frac{c_v L}{\phi_v V_u}\right)^2\right] = 1$$

Therefore,

$$(w_T)_{LRFD} = \frac{D/L+1}{1.2D/L+1.6}\,\frac{1}{L\sqrt{\left(\frac{c_m L}{\phi_{bw}M_{ubw}}\right)^2 + \left(\frac{c_v}{\phi_v V_u}\right)^2}} \tag{4.65}$$

For the design example used in this comparison, the coefficients, c_m and c_v, are equal to 0.10 and 0.60, respectively. Therefore, the allowable uniform load ratios for $\phi_{bw} = 0.90$ and $\phi_v = 0.90$ are as follows:

For $h/t \leq 237\sqrt{k_v/F_y}$,

$$\frac{(w_T)_{LRFD}}{(w_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} \sqrt{\frac{2.803+0.07716\,(V_u L/M_{ubw})^2}{1.235+0.03429\,(V_u L/M_{ubw})^2}} \qquad (4.66)$$

For $h/t > 243\sqrt{k_v/F_y}$,

$$\frac{(w_T)_{LRFD}}{(w_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} \sqrt{\frac{2.929+0.07716\,(V_u L/M_{ubw})^2}{1.235+0.03429\,(V_u L/M_{ubw})^2}} \qquad (4.67)$$

Equations (4.66) and (4.67) can be expressed in the following form:

$$\frac{(w_T)_{LRFD}}{(w_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6}\,(K_w) \qquad (4.68)$$

where K_w is a variable determined from section properties, material strength, and span length for a particular design example.

For combined bending and shear in beam webs, the allowable load ratio can be determined by using Eq. (4.68) as given above. It is not only a function of dead-to-live load ratio but is also a function of h/t, sectional geometry, and material strength. Because of the complexity involved in the comparison, several individual beam sections of different depths and thicknesses were studied.

Figure 8 shows the allowable load ratio versus dead-to-live load ratio for 5 in. x 2 in. standard channel sections with stiffened flanges which are listed in Table 1 of Part V of the AISI Design Manual [41]. Different curves represent the relationships for different thicknesses by using the same span length and material. Table 4.1 shows the sectional properties and calculated values used to obtain the curves which indicate that thinner members result in slightly higher values for the allowable load ratio.

Table 4.1 Channels With Stiffened Flanges, 5 in. Depths-Case A

Section	h/t	S_{eff} (in.^{3})	A_w (in.^{2})	V_u (kips)	M_{ubw} (k-in.)	$V_u L/M_{ubw}$	K_w
5x2x0.135	35.04	1.87	0.6386	26.612	70.45	22.66	1.5005
0.105	45.62	1.50	0.5030	16.100	55.48	17.41	1.5007
0.075	64.67	1.12	0.3638	8.215	40.05	12.31	1.5013
0.060	81.33	0.891	0.2928	5.257	30.91	10.20	1.5017
0.048	102.17	0.722	0.2354	3.215	24.08	8.011	1.5147

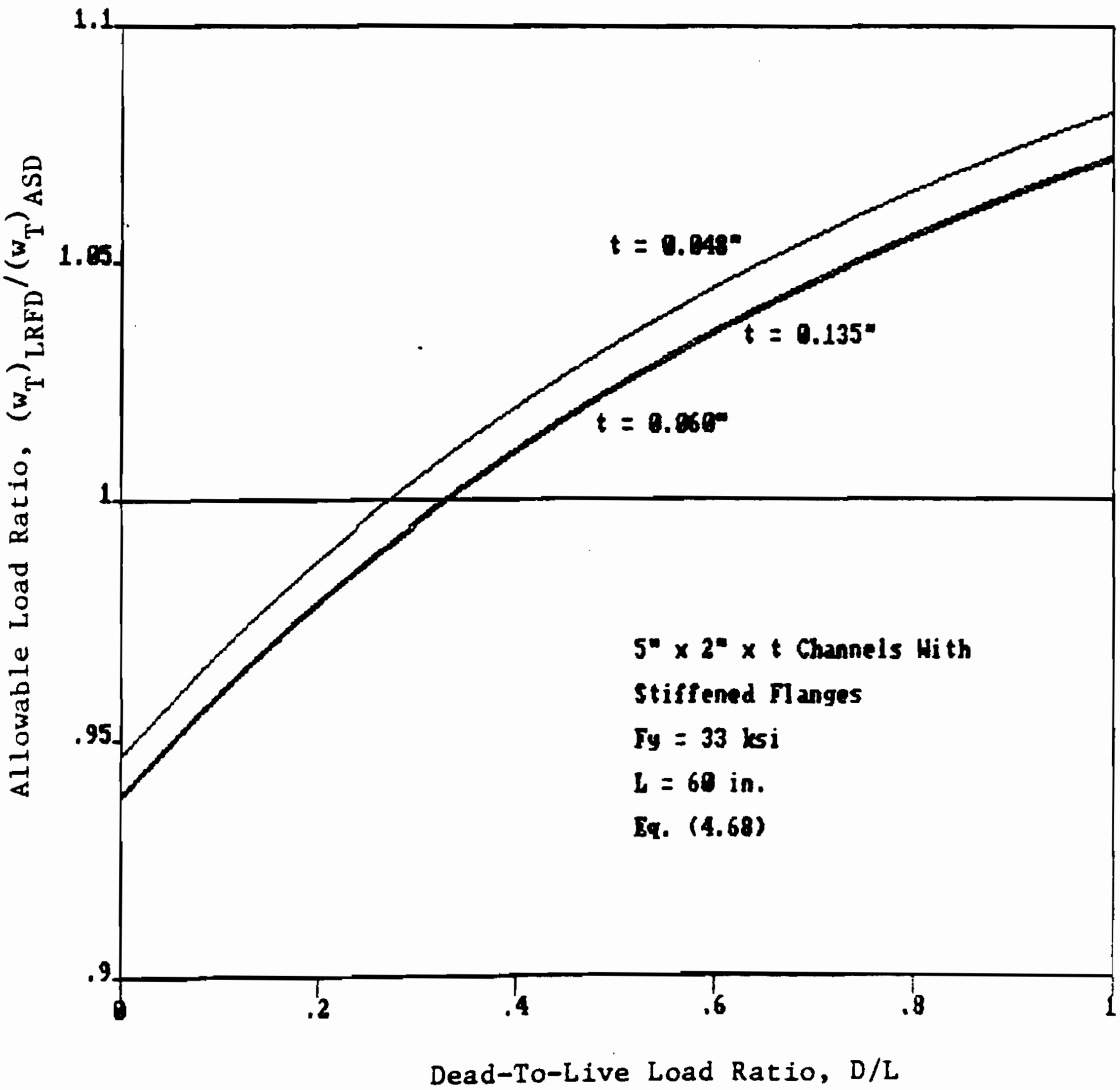


Figure 8. Allowable Load Ratio vs. D/L Ratio for Combined bending and Shear in Beam Webs-Case A

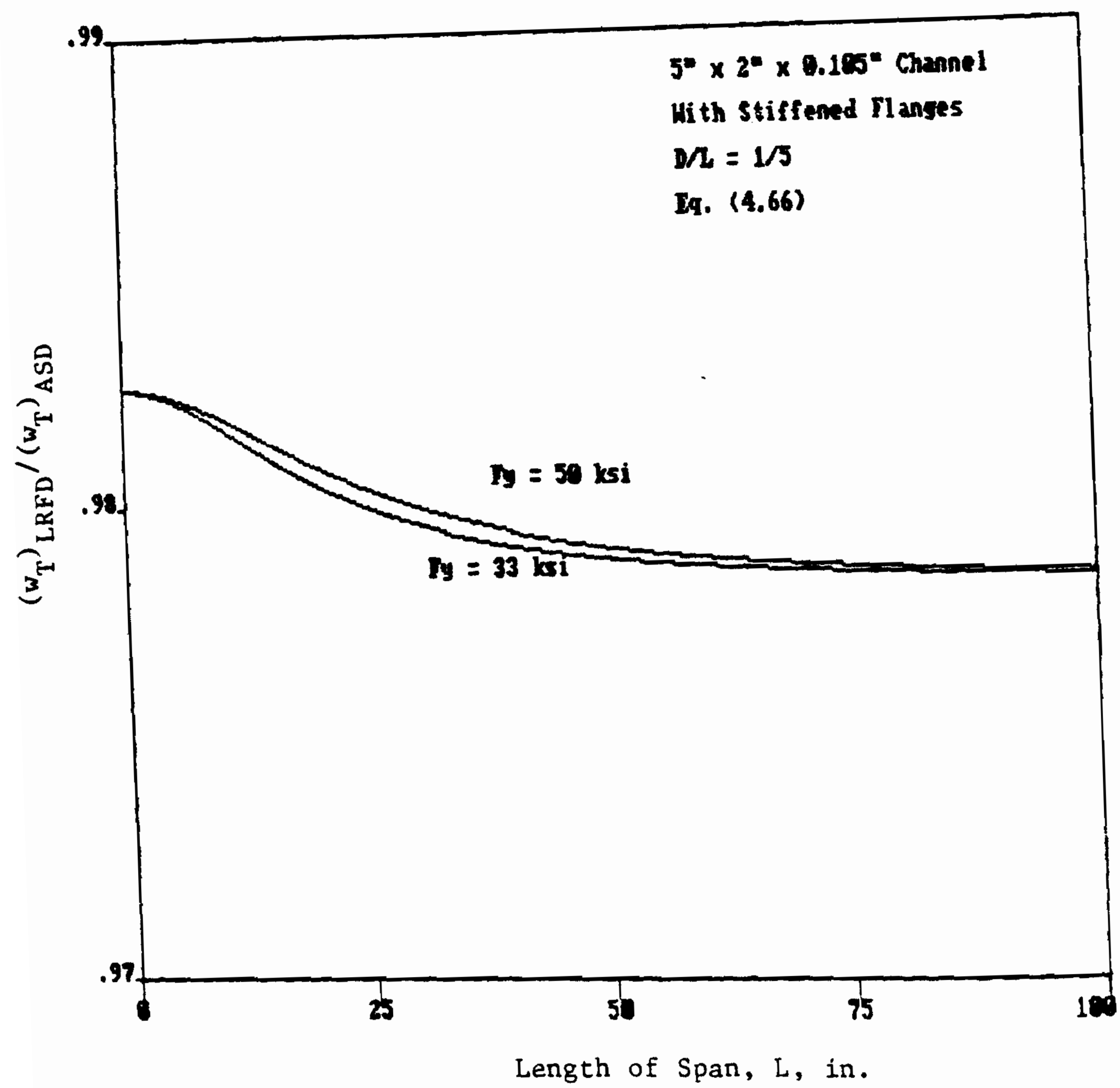


Figure 9. Allowable Load Ratio vs. Span Length-Case A

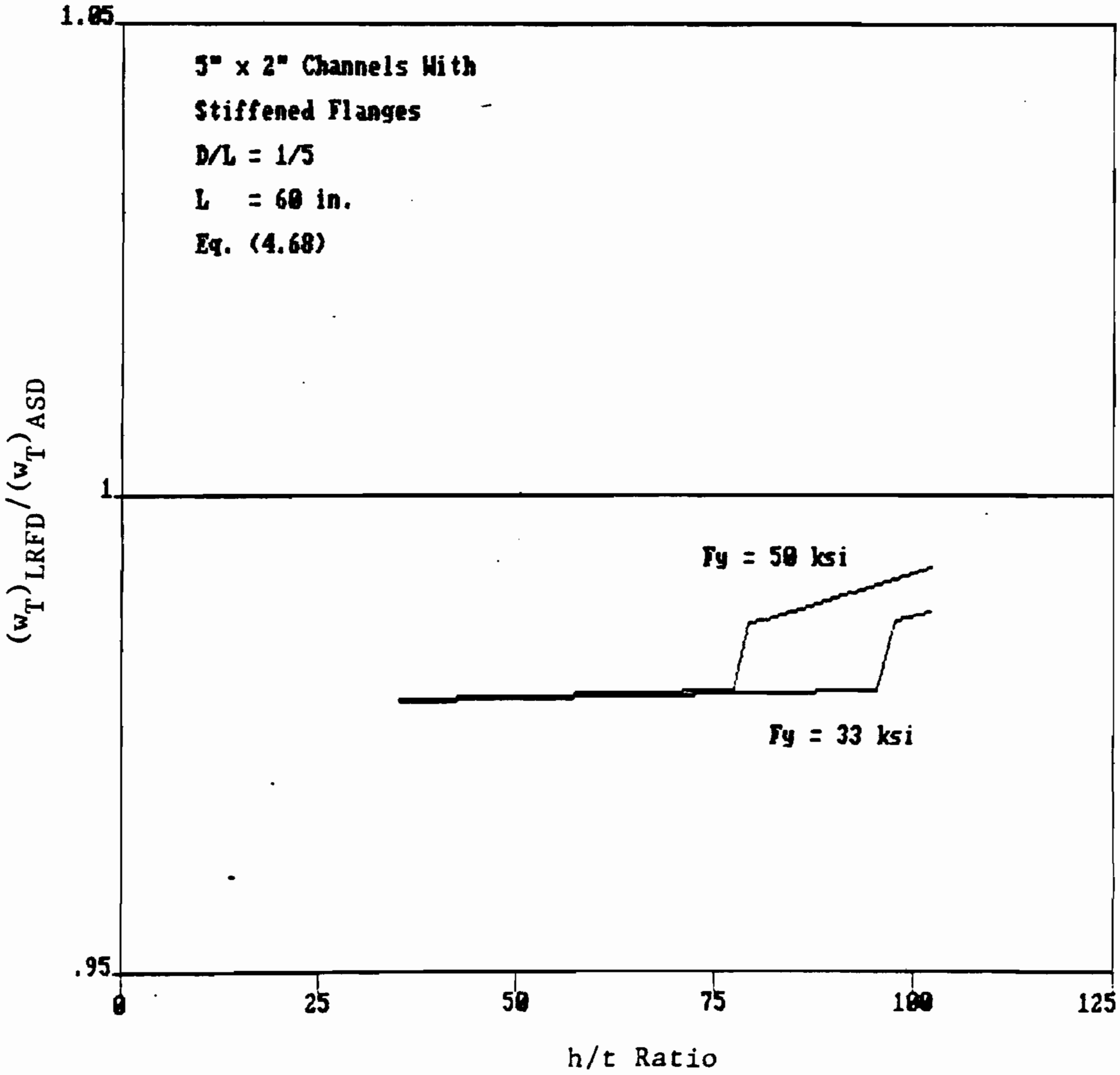


Figure 10. Allowable Load Ratio vs. h/t Ratio for Combined Bending and Shear in Beam Webs-Case A

In Figure 9, the span length was varied for a 5 in. x 2 in. x 0.105 in. channel with stiffened flanges for D/L = 1/5 and F_y = 33 to 50 ksi. It can be seen that the material strength has little effect on the allowable uniform load ratio. This figure also shows that when for the channel section used in this comparison, the allowable load permitted by LRFD is about 2% less than that determined by ASD for various span lengths.

Figure 10 shows the allowable uniform load ratio versus the h/t ratio for the 5 in. - deep channels used in Figure 8 and Table 4.1 for a dead-to-live load ratio of 1/5 and a span length of 5 ft. For F_y = 33 and 50 ksi, this figure shows that higher h/t ratios give slightly larger values of allowable load ratio.

Figure 11 shows the relationships of allowable load ratio and dead-to-live load ratio for channels with stiffened flanges. Sectional properties and other related data are included in Table 4.2. Deeper sections with larger h/t ratios give larger values of the allowable load ratio as indicated in Figure 10.

Channels with unstiffened flanges were also studied and similar results were found as shown in Figures 12, 13, and 14. Table 4.3 lists sectional properties and computed member strengths for channels with unstiffened flanges.

For hat sections, one web was assumed to carry one-half of the load and, therefore, only half-sectional properties were used. Dimensions and sectional properties of standard hat sections are given

Table 4.2 Channels With Stiffened Flanges–Case B

Section	h/t	S_{eff} (in.^{3})	A_w (in.^{2})	V_u (kips)	M_{ubw} (k-in.)	V_uL/M_{ubw}	K_w
9x3.25x0.105	83.71	4.66	0.9230	16.10	160.93	6.002	1.5033
7x2.75x0.105	64.67	2.98	0.7130	16.10	106.57	9.064	1.5020
5x2x0.105	45.62	1.50	0.5030	16.10	55.48	17.409	1.5008
3.5x2x0.105	31.33	0.926	0.3455	16.10	35.11	27.516	1.5004

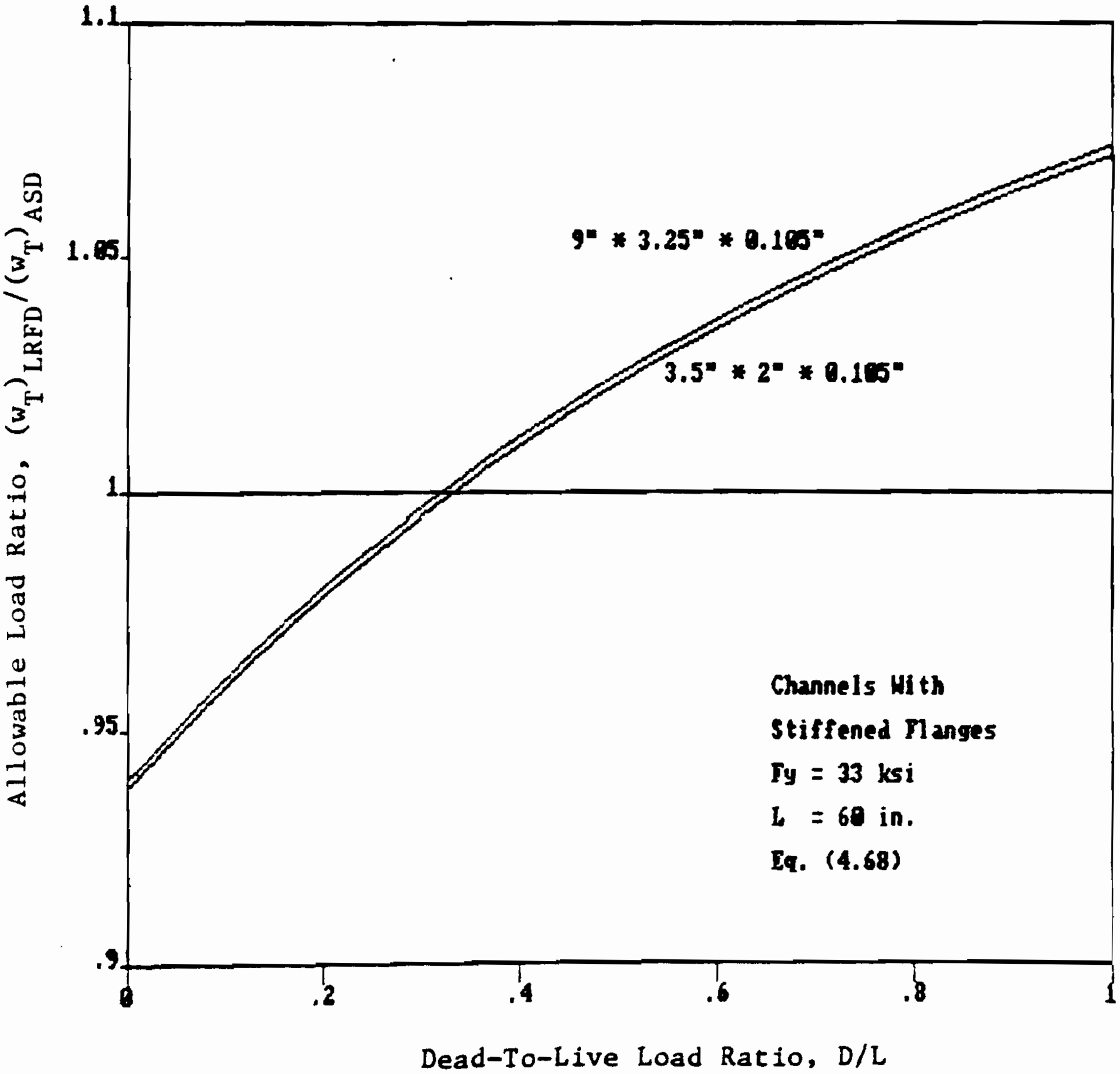


Figure 11. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Shear in Beam Webs–Case B

Table 4.3 Channels With Unstiffened Flanges, 6 in. Depths

Section	h/t	S_{eff} (in.^{3})	A_w (in.^{2})	V_u (kips)	M_{ubw} (k-in.)	V_uL/M_{ubw}	K_w
6x1.5x0.135	42.44	1.78	0.7736	26.61	66.85	23.89	1.5005
0.105	55.14	1.41	0.6080	16.10	51.26	18.85	1.5007
0.075	78.00	1.05	0.4388	8.125	35.90	13.73	1.5011
0.060	98.00	0.849	0.3528	5.238	27.42	11.46	1.5088
0.048	123.00	0.685	0.2834	2.671	20.50	7.819	1.5150

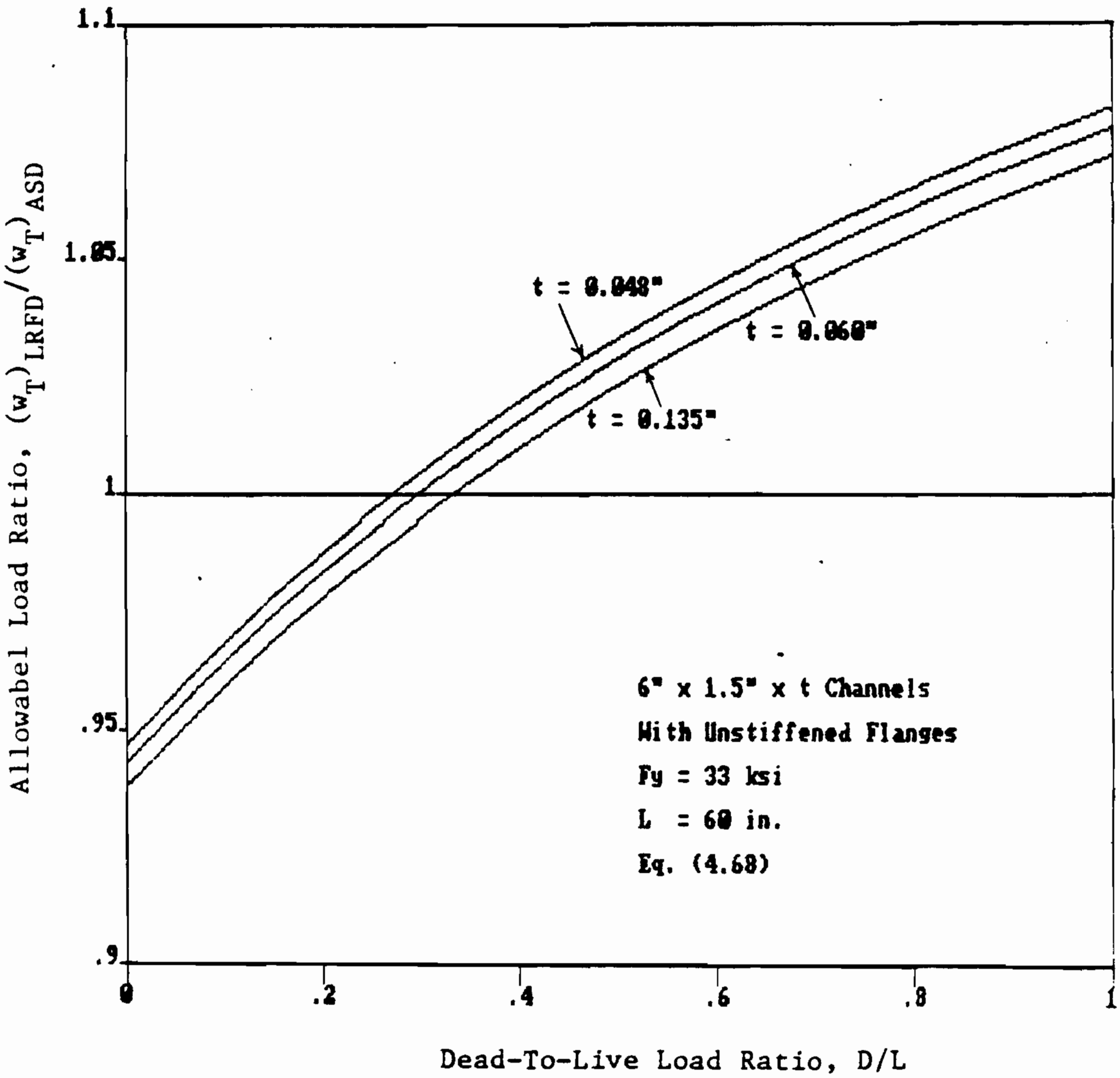


Figure 12. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Shear in Beam Webs-Case C

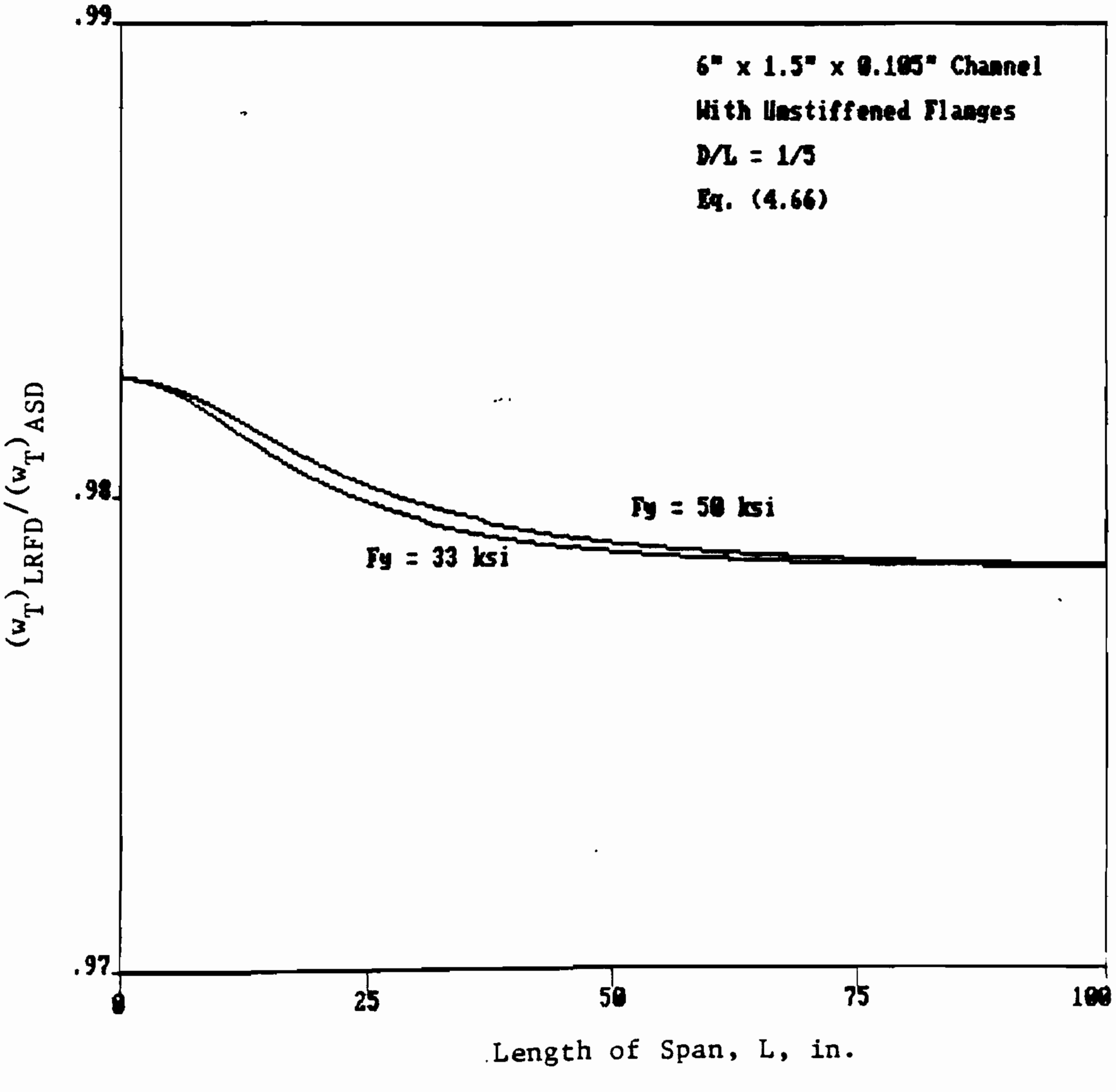


Figure 13. Allowable Load Ratio vs. Span Length-Case C

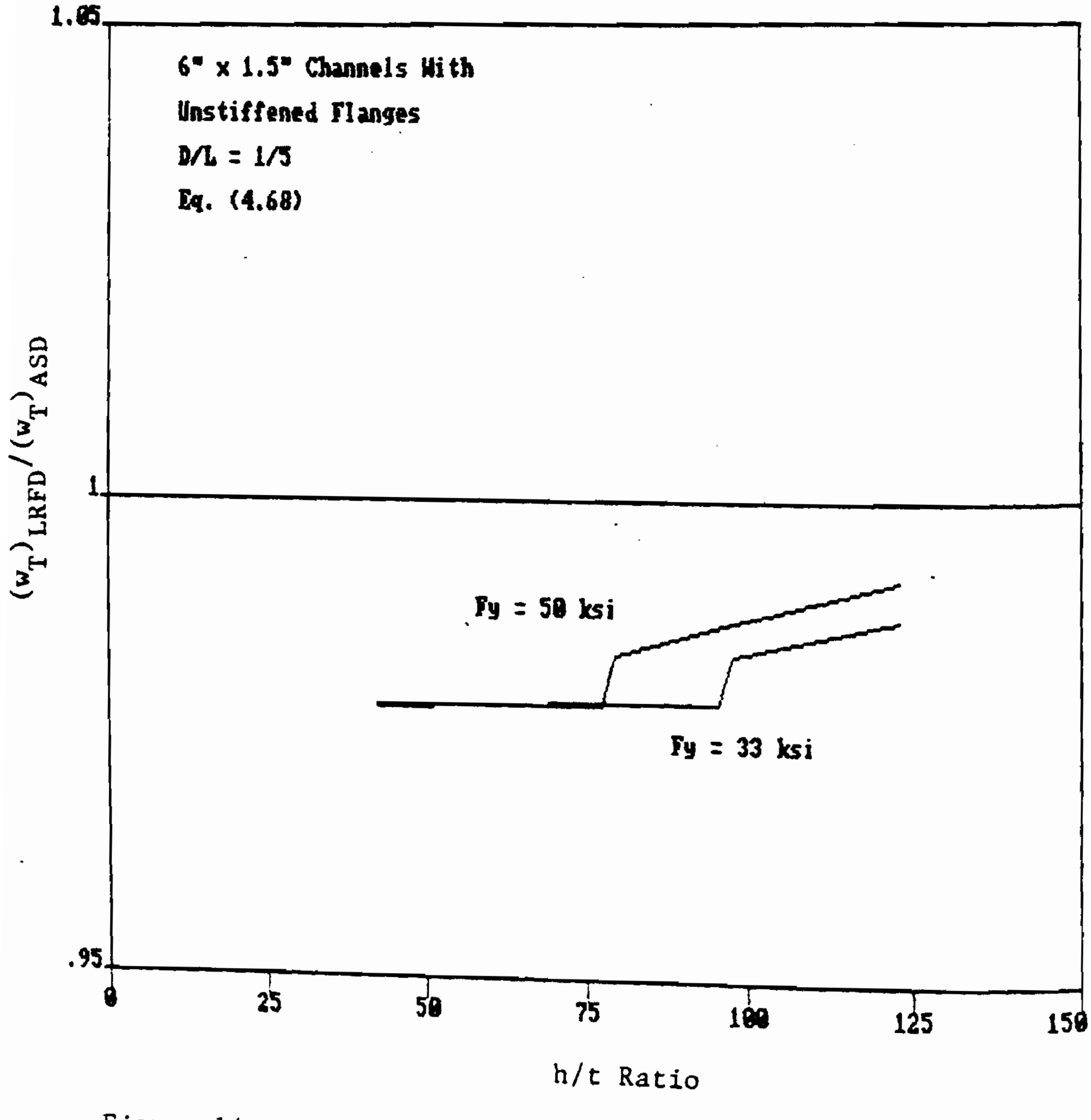


Figure 14. Allowable Load Ratio vs. h/t Ratio for Combined Bending and Shear in Beam Webs-Case C

in Table 9 of Part V of the AISI Design Manual[41] and Table 4.4 lists sectional properties and calculated member strengths used in this comparison. Figure 15 shows the relationships between allowable uniform load ratio and dead-to-live load ratio for three hat sections with a yield point of 33 ksi and a span length of 5 ft. All 4 in. deep hat sections resulted in the same curve regardless of h/t ratio. Hat sections with larger depths or larger h/t ratios resulted in larger values of allowable load ratio.

I-sections made of two channels back-to-back would result in the same comparison and conclusions as the single channel sections.

From Figure 8 through 15, it can be seen that for dead-to-live load ratios less than about 1/4, the LRFD criteria for combined bending and shear are usually conservative compared with the allowable stress design method. For D/L = 0.5, the differences range from 2.3% to 3.8%. For large D/L ratios, ASD method is always conservative than LRFD. Yield point of steel has little effect on the allowable load ratio. The lower the yield point, the larger the difference. Span length has little or no effect on the allowable uniform load ratio as shown in Fig. 13 on page 48. For channels and I-sections, smaller h/t ratios result in a slightly larger difference between allowable uniform loads obtained from the two design methods. For hat sections, smaller depths result in a larger difference between the allowable loads.

Table 4.4 Hat Sections (Positive Bending)

Section	h/t	S_{eff} (in.^{3})	A_w (in.^{2})	V_u (kips)	M_{ubw} (k-in.)	$V_u L/M_{ubw}$	K_w
4x2x0.075	51.33	0.863	0.2888	8.215	15.80	31.19	1.5003
4x4x0.105	36.10	1.55	0.3979	16.10	29.14	33.14	1.5003
4x4x0.075	51.33	0.954	0.2888	8.215	17.47	28.22	1.5004
4x6x0.135	27.63	2.34	0.5036	26.62	44.63	35.78	1.5002
4x6x0.105	36.10	1.63	0.3979	16.10	30.65	31.51	1.5003
6x9x0.105	55.14	3.01	0.6080	16.10	54.75	17.65	1.5007
10x5x0.075	131.33	4.04	0.7388	6.107	63.56	5.765	1.5210

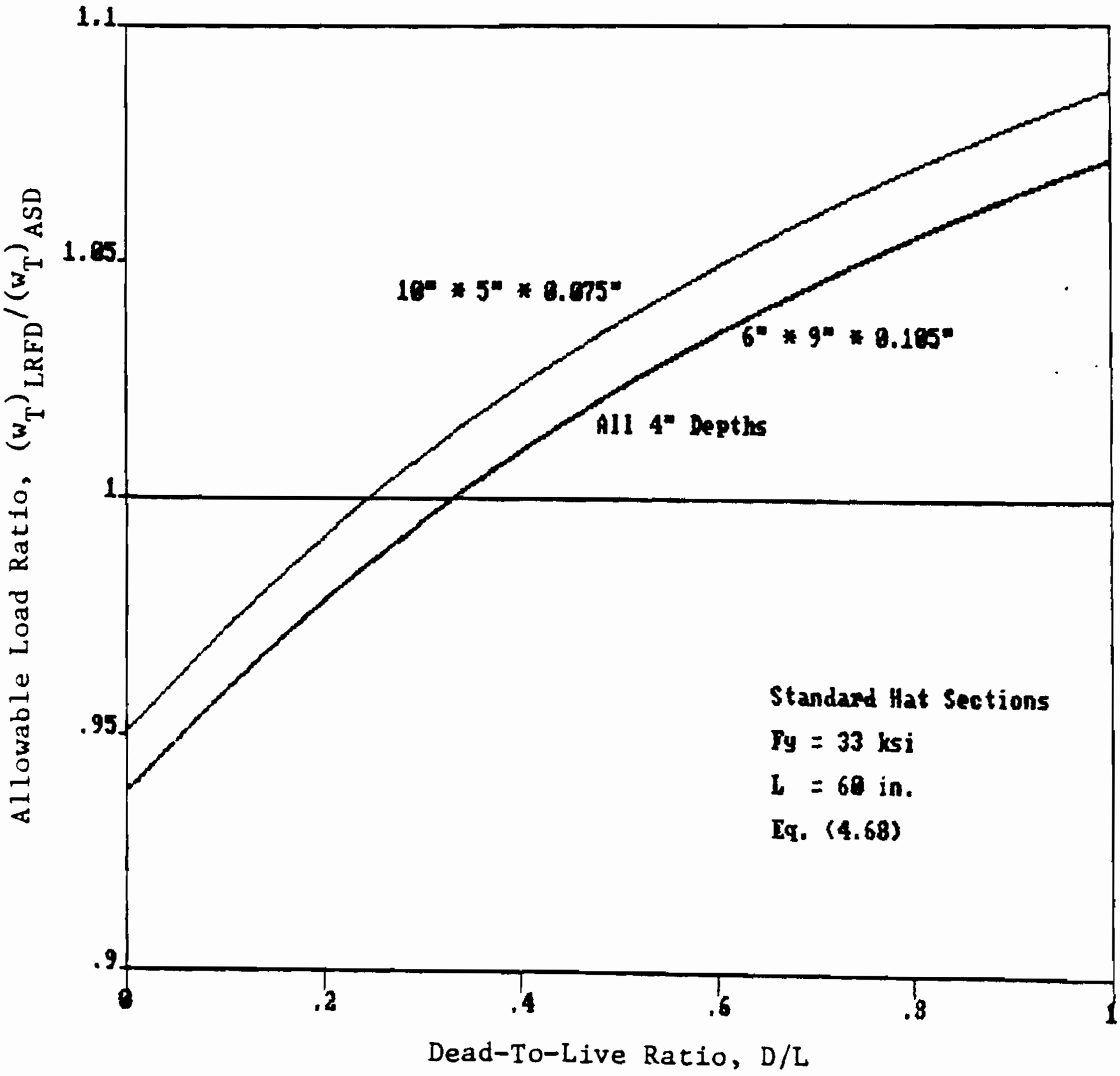


Figure 15. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Shear in Beam Webs-Case D

4. <u>Web Crippling</u>. Beam webs should also be checked for web crippling at locations of high intensity loads. This would occur under concentrated loads or support reactions.

a. <u>Allowable Stress Design</u>. To avoid crippling of unreinforced flat webs of flexural members having a flat width ratio, h/t equal to or less than 200, neither concentrated loads nor reactions should exceed the values of P_{allow} given below on the basis of Section 3.5.1 of the AISI Specification[1]. Webs of flexural members for which the ratio, h/t, is greater than 200 should be provided with adequate means of transmitting concentrated loads and/or reactions directly into the webs. The following formulas apply to beams when $R/t \leq 6$ and to decks when $R/t \leq 7$, $N/t \leq 210$ and $N/h \leq 3.5$.

(i) Shapes Having Single Webs: The allowable web crippling load is determined as follows:

<u>One Flange Loading:</u> At locations of one concentrated load or reaction acting either on the top or bottom flange,

For end reactions on beams with stiffened flanges,

$$P_{allow} = t^2 k C_3 C_4 C_\theta [179-0.33(h/t)][1+0.01(N/t)] \quad (4.69)$$

For end reactions on beams with unstiffened flanges,

$$P_{allow} = t^2 k C_3 C_4 C_\theta [117-0.15(h/t)][1+0.01(N/t)] \quad (4.70)$$

For interior loads on beams,

$$P_{allow} = t^2 k C_1 C_2 C_\theta [291-0.40(h/t)][1+0.007(N/t)] \quad (4.71)$$

Two Flange Loading: At locations of two opposite concentrated loads or of a concentrated load and an opposite reaction acting simultaneously on the top and bottom flanges,

For end reactions on beams,

$$P_{allow} = t^2 k C_3 C_4 C_\theta [132-0.31(h/t)][1+0.01(h/t)] \qquad (4.72)$$

For interior loads on beams,

$$P_{allow} = t^2 k C_1 C_2 C_\theta [417-1.22(h/t)][1+0.0013(N/t)] \qquad (4.73)$$

(ii) I-Sections: I-beams made of two channels connected back to back or for similar sections which provide a high degree of restraint against rotation of the web:

One Flange Loading: At locations of one concentrated load or reaction acting either on the top or bottom flange,

For end reactions on beams,

$$P_{allow} = t^2 F_y C_7 (5.0 + 0.63\sqrt{N/t}) \qquad (4.74)$$

For interior loads on beams,

$$P_{allow} = t^2 F_y C_5 C_6 (7.50 + 1.63\sqrt{N/t}) \qquad (4.75)$$

Two Flange Loading: At locations of two opposite concentrated loads or of a concentrated load and an opposite reaction acting simultaneously on the top and bottom flanges,

For end reactions on beams,

$$P_{allow} = t^2 F_y C_{10} C_{11} (5.0 + 0.63\sqrt{N/t}) \qquad (4.76)$$

For interior loads on beams,

$$P_{allow} = t^2 F_y C_8 C_9 (7.50+1.63\sqrt{N/t}) \qquad (4.77)$$

In all of the above, P_{allow} represents the load or reaction for one solid web connecting top and bottom flanges. For sheets consisting of two or more such adjacent webs, P_{allow} should be computed for each individual web and the results added to obtain the allowable load or reaction for the multiple web.

For built-up I-beams, or similar sections, the distance between the connector and beam flange should be kept as small as practical.

In the above formulas,

P_{allow} = allowable concentrated load or reaction, kips per web

$$C_1 = 1.22-0.22k \qquad (4.78)$$

$$C_2 = (1.06-0.06\ R/t) \le 1.0 \qquad (4.79)$$

$$C_3 = 1.33-0.33k \qquad (4.80)$$

$$C_4 = (1.15-0.15\ R/t) \le 1.0 \text{ but not less than } 0.50 \qquad (4.81)$$

$$C_5 = (1.49-0.53k) \ge 0.6 \qquad (4.82)$$

$$C_6 = 0.88+0.12m \qquad (4.83)$$

$$C_7 = 1+(h/t)/750 \text{ when } h/t \le 150 \qquad (4.84)$$

$$C_7 = 1.20 \text{ when } h/t > 150 \qquad (4.85)$$

$$C_8 = 1/k \text{ when } h/t \le 66.5 \qquad (4.86)$$

$$C_8 = [1.10-(h/t)/665]/k \text{ when } h/t > 66.5 \qquad (4.87)$$

$$C_9 = 0.82+0.15m \qquad (4.88)$$

$$C_{10} = [0.98-(h/t)/865]/k \qquad (4.89)$$

$$C_{11} = 0.64+0.31m \qquad (4.90)$$

$C_\theta = 0.7+0.3(\theta/90)^2$ (4.91)

F_y = yield point of the web, ksi

h = clear distance between flanges measured

$k = F_y/33$ (4.92)

$m = t/0.075$ (4.93)

t = web thickness, in.

N = actual length of bearing, in. For the case of two equal and opposite concentrated loads distributed over unequal bearing lengths, the smaller value of N shall be taken.

R = inside bend radius, in.

θ = angle between plane of web and plane of bearing surface $\geq 45°$ but no more than $90°$

b. LRFD Criteria. Section 9.3.3.4.1 of the Tentative Recommendation[10] specifies that to avoid crippling of unreinforced flat webs of flexural members having a flat width ratio, h/t, equal to or less than 200, neither concentrated loads nor reactions determined according to the factored design loads should exceed the values of $\phi_w P_u$ with $\phi_w = 0.85$ and P_u obtained from the equations below. Webs of flexural members for which the ratio, h/t, is greater than 200 should be provided with adequate means of transmitting concentrated loads and/or reactions directly into the webs. The following formulas apply to beams when $R/t \leq 6$ and to decks when $R/t \leq 7$, $N/t \leq 210$, and $N/h \leq 3.5$.

(i) Shapes Having Single Webs: The nominal ultimate web crippling load is determined as follows:

One Flange Loading: At locations of one concentrated load or reaction acting either on the top or bottom flange,

For end reactions on beams with stiffened flanges,

$$P_u = t^2 k C_3 C_4 C_\theta [331-0.61(h/t)][1+0.01(N/t)] \quad (4.94)$$

For end reactions on beams with unstiffened flanges,

$$P_u = t^2 k C_3 C_4 C_\theta [217-0.28(h/t)][1+0.01(N/t)] \quad (4.95)$$

For interior loads on beams,

$$P_u = t^2 k C_1 C_2 C_\theta [538-0.74(h/t)][1+0.007(N/t)] \quad (4.96)$$

Two Flange Loading: At locations of two opposite concentrated loads or of a concentrated load and an opposite reaction acting simultaneously on the top and bottom flange,

For end reactions on beams,

$$P_u = t^2 k C_3 C_4 C_\theta [244-0.57(h/t)][1+0.01(N/t)] \quad (4.97)$$

For interior loads on beams,

$$P_u = t^2 k C_1 C_2 C_\theta [771-2.26(h/t)][1+0.0013(N/t)] \quad (4.98)$$

(ii) I-Sections: I-beams made of two channels connected back to back or for similar sections which provide a high degree of restraint against rotation of the web:

One Flange Loading: At locations of one concentrated load or reaction acting either on the top or bottom flanges, For end reactions on beams,

$$P_u = t^2F_yC_7(10+1.25\sqrt{N/t}) \tag{4.99}$$

For interior loads on beams,

$$P_u = t^2F_yC_5C_6(15+3.25\sqrt{N/t}) \tag{4.100}$$

<u>Two Flange Loading</u>: At locations of two opposite concentrated loads or of a concentrated load and an opposite reaction acting simultaneously on the top and bottom flange,

For end reactions on beams,

$$P_u = t^2F_yC_{10}C_{11}(10+1.25\sqrt{N/t}) \tag{4.101}$$

For interior loads on beams,

$$P_u = t^2F_yC_8C_9(15+3.25\sqrt{N/t}) \tag{4.102}$$

c. <u>Comparison</u>. The unfactored concentrated load or reaction can be calculated for both methods by using Eq. (4.103):

$$P_T = P_{DL} + P_{LL} \tag{4.103}$$

where

P_T = total unfactored load, kips

P_{DL} = nominal dead load, kips

P_{LL} = nominal live load, kips

The total unfactored load should be less than or equal to the allowable load based on web crippling. For allowable stress design, the allowable load is P_{allow}. For LRFD, the allowable load is computed from Eq. (2.6) and is as follows:

$$(P_a)_{LRFD} = \phi_w P_u(D/L+1)/(1.2D/L+1.6) \tag{4.104}$$

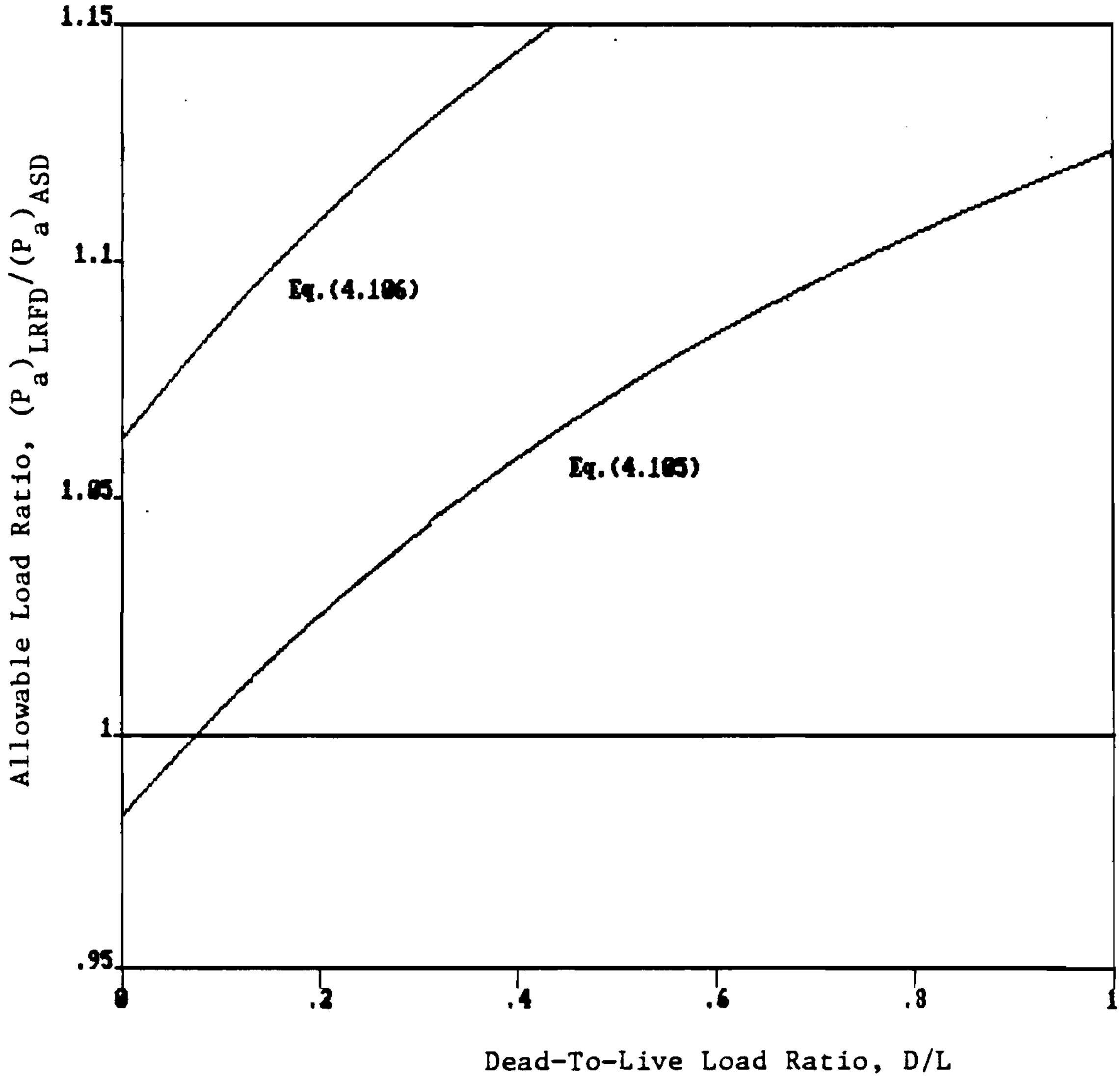


Figure 16. Allowable Load Ratio vs. D/L Ratio for Web Crippling

For shapes with single webs, the allowable load is derived from the ultimate value with a factor of safety of 1.85. For I-sections or similar shapes, the allowable load is derived from the ultimate web crippling load using a factor of safety of 2.00. Therefore, the allowable load ratios are as follows:

For shapes with single webs and $\phi_w = 0.85$,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.85\phi_w \frac{D/L+1}{1.2D/L+1.6} = 1.57 \frac{D/L+1}{1.2D/L+1.6} \tag{4.105}$$

For I-sections or similar shapes and $\phi_w = 0.85$

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.00\phi_w \frac{D/L+1}{1.2D/L+1.6} = 1.70 \frac{D/L+1}{1.2D/L+1.6} \tag{4.106}$$

Figure 16 shows the allowable load ratio versus dead-to-live load ratio for both types of beams based on the comparison of web crippling loads.

For single web beams, LRFD is conversative for $D/L < 0.08$ and for $D/L = 0.5$ the difference is 7.0%. For I-sections, the ASD approach is always conversative than LRFD. For $D/L = 0.5$, the allowable load permitted by the allowable stress design method for I-sections is about 17% lower than that permitted by the LRFD criteria.

5. Combined Bending and Web Crippling. The interaction between bending and web crippling is similar to that of combined bending and shear and exists when a large bending moment is applied close to concentrated loads or support reactions. The web crippling capacity may be reduced according to the following interaction equations provided in the specifications:

a. Allowable Stress Design. According to Section 3.5.2 of the AISI Specifications[1], unreinforced flat webs of shapes subjected to a combination of bending and reaction or concentrated load should be designed to meet the following requirements:

For shapes having single webs,

$$1.2\,\frac{P}{P_{allow}} + \frac{M}{M_{allow}} \leq 1.5 \tag{4.107}$$

At the interior supports in continuous spans the above formula is not applicable to deck or beams with two or more single webs provided the compression edges of adjacent webs are laterally supported in the negative moment region by continuous or intermittently connected flange elements, rigid cladding, or lateral bracing, and the spacing between adjacent webs does not exceed 10 in.

For I-beams made of two channels connected back to back or similar sections which provide a high degree of restraint against rotation of the web, such as I-beams made by welding two angles to a channel having unreinforced webs,

$$1.1\,\frac{P}{P_{allow}} + \frac{M}{M_{allow}} \leq 1.5 \tag{4.108}$$

When $h/t \leq 400/\sqrt{F_y}$ and $w/t \leq (w/t)_{lim}$, the allowable reaction or concentrated load may be determined for web crippling only. In the above formulas,

P = concentrated load or reaction in the presence of bending moment, kips

P_{allow} = allowable concentrated load or reaction in absence of bending moment determined in accordance with Section 3.5.1[1], kips

M = applied bending moment, at or immediately adjacent to the point of application of the concentrated load or reaction P, kip-in.

M_{allow} = allowable bending moment permitted if bending stress only exists, kip-in.

w = flat width of the beam flange which contacts the bearing plate, in.

t = thickness of web or flange, in.

$(w/t)_{lim}$ = limiting w/t ratio for the beam flange. Use Sections 2.3.1.1 and 3.2(a) of the AISI Specification[1] for stiffened flanges and unstiffened flanges, respectively.

b. LRFD Criteria. Section 9.3.3.4.2 of the Tentative Recommendations[10] specifies that unreinforced flat webs of shapes subjected to a combination of bending and reaction or concentrated load should be designed to meet the following requirements:

For shapes having single webs,[45]

$$1.07 \frac{P_D}{\phi_w P_u} + \frac{M_D}{\phi_b M_u} \leq 1.42 \tag{4.109}$$

At the interior supports in continuous spans the above formula is not applicable to deck or beams with two or more single webs provided the compression edges of adjacent webs are laterally supported in the negative moment region by continuous or intermittently connected flange elements, rigid cladding, or lateral bracing, and the spacing between adjacent webs does not exceed 10 in.

For I-beams made of two channels connected back to back or similar sections which provide a high degree of restraint against rotation of the webs, such as I-beams made by welding two angles to a channel having unreinforced webs,

$$0.82\,\frac{P_D}{\phi_w P_u} + \frac{M_D}{\phi_b M_u} \leq 1.32 \qquad (4.110)$$

when $h/t \leq 400/\sqrt{F_y}$ and $w/t \leq (w/t)_{lim}$, the reaction or concentrated load may be determined by Section 9.3.3.4.1 of the Tentative Recommendations[10] without considering the effect of bending moment on the reduction of the web crippling load.

In the above formulas,

ϕ_b = resistance factor for bending

ϕ_w = resistance factor for web crippling = 0.85

P_D = concentrated load or reaction in the presence of bending moment computed on the basis of factored loads, kips

P_u = nominal ultimate concentrated load or reaction in the absence of bending moment determined in accordance with Section 9.3.3.4.1 of the Tentative Recommendations[10], kips

M_D = applied bending moment, at or immediately adjacent to the point of application of the concentrated load or reaction, P_D, computed on the basis of factored loads, kip-in.

M_u = nominal ultimate bending moment permitted if bending stress only exists. The value of M_u should be M_u (Section 9.3.1 of Reference 10) or M_{ubw} (Section

9.3.3.2 of Reference 10) whichever is smaller, kip-in.

c. Comparison. A simple supported beam with a concentrated load at midspan was selected as a typical design example. This example has a maximum moment of PL/4 at midspan, under the concentrated load. The allowable loads, P_T, were calculated for both design methods. Since each design procedure utilizes separate design variables, the allowable loads were determined using nominal resistances.

The allowable load based on allowable stress design was calculated as follows:

$$\frac{M}{M_{allow}} = \frac{M_{TL}}{0.60M_u} = \frac{P_T L/4}{0.60M_u} = \frac{0.4167P_T L}{M_u} \tag{4.111}$$

For beams with single webs,

$$\frac{P}{P_{allow}} = \frac{P_T}{P_u/1.85} = \frac{1.85P_T}{P_u} \tag{4.112}$$

By substituting Eq. (4.111) and (4.112) into Eqs. (4.107),

$$1.2\,\frac{P}{P_{allow}} + \frac{M}{M_{allow}} = \frac{2.22P_T}{P_u} + \frac{0.4167P_T L}{M_u} = 1.5$$

Therefore,

$$(P_T)_{ASD} = \frac{3.6P_u}{5.328 + (P_u L/M_u)} \tag{4.113}$$

For I-sections,

$$\frac{P}{P_{allow}} = \frac{P_T}{P_u/2.00} = \frac{2.00P_T}{P_u} \tag{4.114}$$

By substituting Eqs. (4.111) and (4.114) into Eq. (4.108),

$$1.1\,\frac{P}{P_{allow}} + \frac{M}{M_{allow}} = \frac{2.20P_T}{P_u} + \frac{0.4167P_T L}{M_u} = 1.5$$

Therefore,

$$(P_T)_{ASD} = \frac{3.6P_u}{5.280+(P_u L/M_u)} \tag{4.115}$$

The allowable load based on LRFD criteria was calculated as follows:

$$\frac{M_D}{\phi_b M_u} = \frac{1.2D/L+1.6}{D/L+1}\,\frac{M_{TL}}{\phi_b M_u} = \frac{1.2D/L+1.6}{D/L+1}\,\frac{P_T L/4}{\phi_b M_u} \tag{4.116}$$

$$\frac{P_D}{\phi_w P_u} = \frac{1.2D/L+1.6}{D/L+1}\,\frac{P_T}{\phi_w P_u} \tag{4.117}$$

For beams with single webs, Eqs. (4.116) and (4.117) were substituted into Eq. (4.109) to obtain the following expression:

$$1.07\,\frac{P_D}{\phi_w P_u} + \frac{M_D}{\phi_b M_u} = \frac{1.2D/L+1.6}{D/L+1}(P_T)\left[\frac{1.07}{\phi_w P_u} + \frac{0.25L}{\phi_b M_u}\right] = 1.42$$

Therefore,

$$(P_T)_{LRFD} = \frac{D/L+1}{1.2D/L+1.6}\;\frac{5.680\ \phi_w P_u}{4.280+(\phi_w P_u L/\phi_b M_u)} \tag{4.118}$$

For I-sections, Eqs. (4.116) and (4.117) were substituted into Eq. (4.110) to obtain the following expression:

$$0.82\,\frac{P_D}{\phi_w P_u} + \frac{M_D}{\phi_b M_u} = \frac{1.2D/L+1.6}{D/L+1}(P_T)\left[\frac{0.82}{\phi_w P_u} + \frac{0.25L}{\phi_b M_u}\right] = 1.32$$

Therefore,

$$(P_T)_{LRFD} = \frac{D/L+1}{1.2D/L+1.6}\;\frac{5.280\phi_w P_u}{3.280+(\phi_w P_u L/\phi_b M_u)} \tag{4.119}$$

The allowable load ratios based on the design example for combined bending and web crippling are given in Eqs. (4.120) and (4.121) for $\phi_w = 0.85$ and $\phi_b = 0.90$ for preventing lateral buckling and 0.95 for sectional bending strength.

For beams with single webs,

$$\frac{(P_T)_{LRFD}}{(P_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} \quad \frac{7.145+1.341(P_uL/M_u)}{4.280+(0.85/\phi_b)(P_uL/M_u)} \tag{4.120}$$

For I-sections,

$$\frac{(P_T)_{LRFD}}{(P_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} \quad \frac{6.583+1.247(P_uL/M_u)}{3.280+(0.85/\phi_b)(P_uL/M_u)} \tag{4.121}$$

Eqs. (4.120) and (4.121) can be expressed in the following form:

$$\frac{(P_T)_{LRFD}}{(P_T)_{ASD}} = \frac{D/L+1}{1.2D/L+1.6} \, (K_w) \tag{4.122}$$

where K_w is a variable determined from section properties, material strength, span length, and the value of ϕ_b for a particular design example.

Because the interaction combines moment and web crippling, the allowable load ratio is rather complex. It is not only a function of dead-to-live ratio but is also a function of span length, sectional geometry, and material strength. Several individual beam sections with different conditions were studied due to the complexity involved in the comparison.

Figures 17, 19 and 20 show the relationship between allowable load ratios and the ratio of dead-to-live load for various channel sections with L = 5 ft and F_y = 33 ksi. Tables 4.5, 4.6, and 4.7 present section properties and calculated member strengths for the standard channel sections selected from Tables 1 and 2 of Part V of the AISI Design Manual[41]. In these three figures for D/L = 0.5, the allowable web crippling loads determined by LRFD are from 2.5% to 6.5% larger than that permitted by allowable stress design. The

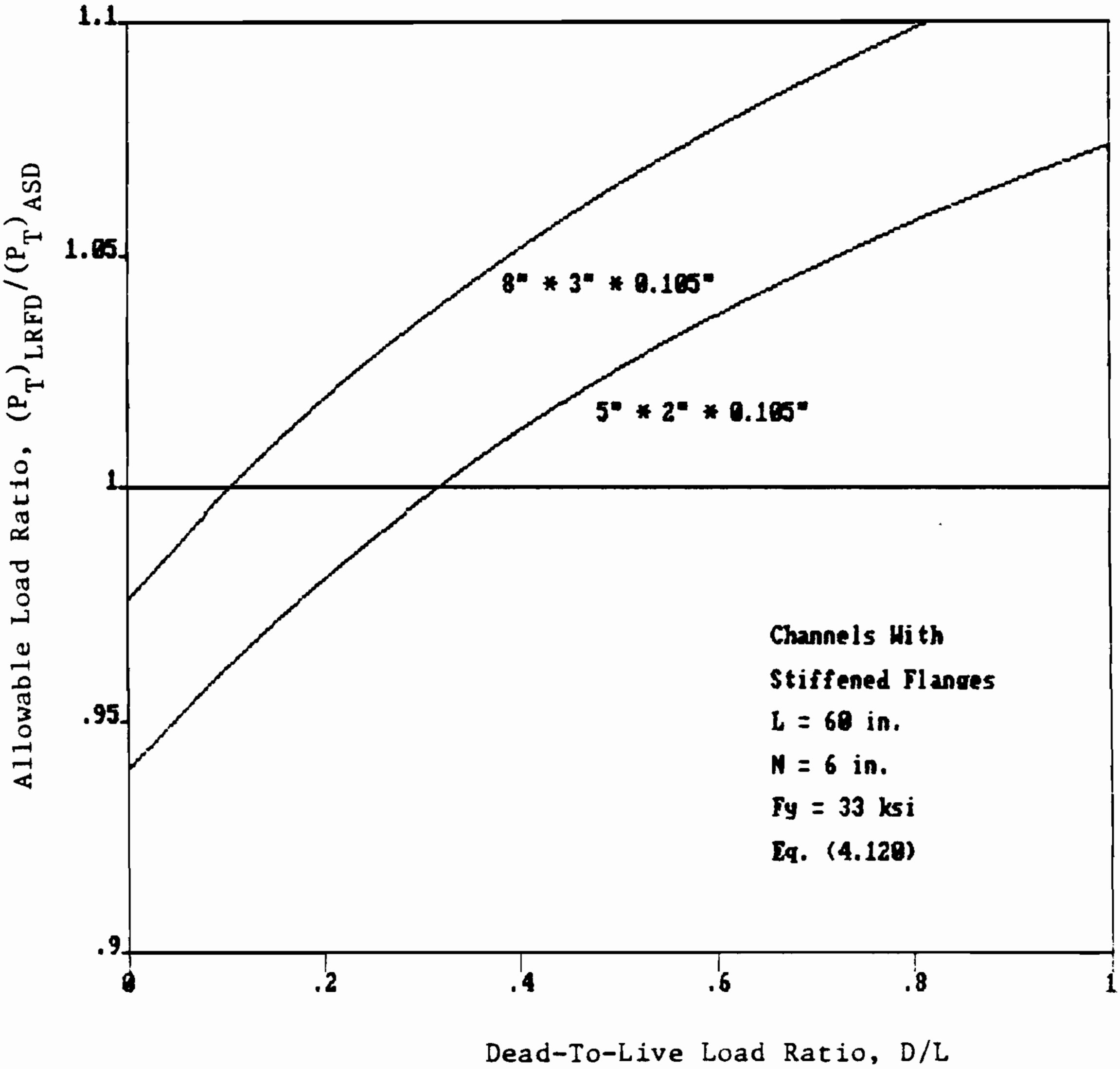


Figure 17. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Web Crippling-Case 1

Table 4.5 Channels With Stiffened Flanges

Section	S_{eff} (in.^{3})	h/t	M_u (k-in.)	M_{ubw} (k-in.)	P_u (kips)	$P_u L/M_u$	K_w
8x3x0.105	3.78	74.19	124.7	124.7	7.105	3.418	1.5621
5x2x0.105	1.50	45.62	49.50	49.50	7.416	8.989	1.5035

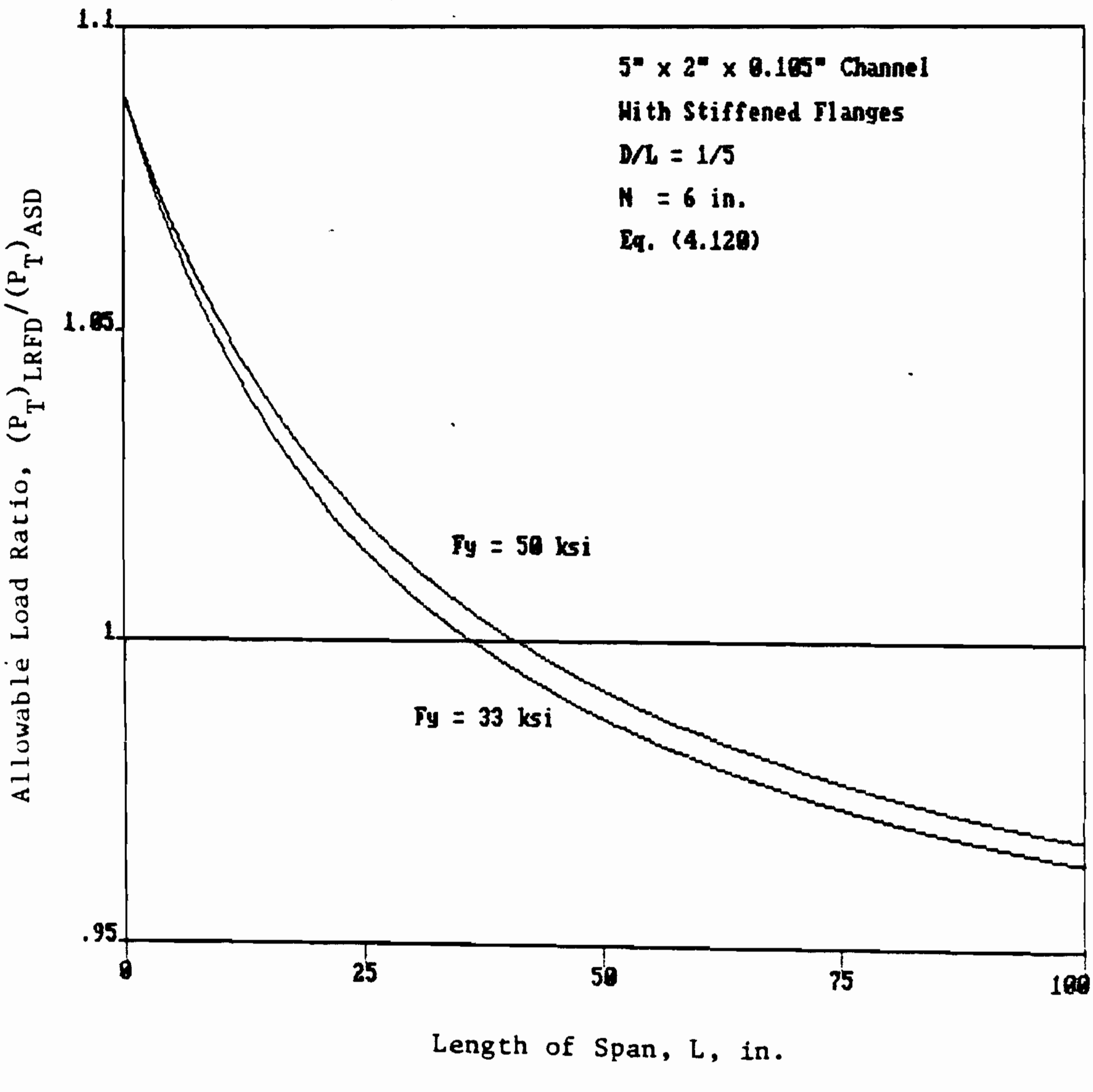


Figure 18. Allowable Load Ratio vs. Span Length for Combined Bending and Web Crippling-Case 2

Table 4.6 Channels With Stiffened Flanges, 5 in. Depths

Section	S_{eff} (in.^{3})	h/t	M_u (k-in.)	M_{ubw} (k-in.)	P_u (kips)	P_uL/M_u	K_w
5x2x0.075	1.12	64.67	36.96	36.96	4.237	6.878	1.5190
0.048	0.722	102.17	23.83	23.83	1.883	4.743	1.5418

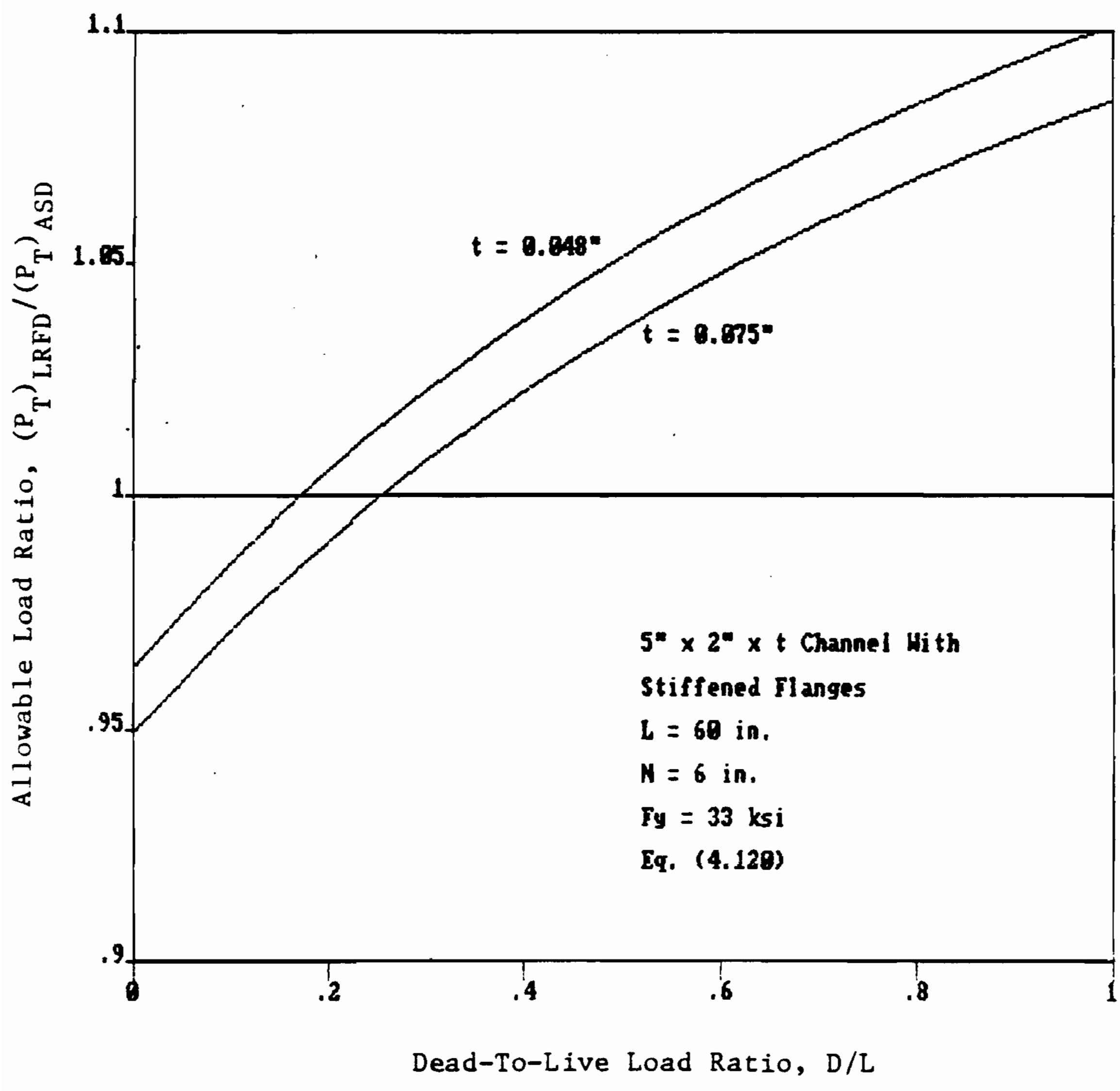


Figure 19. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Web Crippling-Case 2

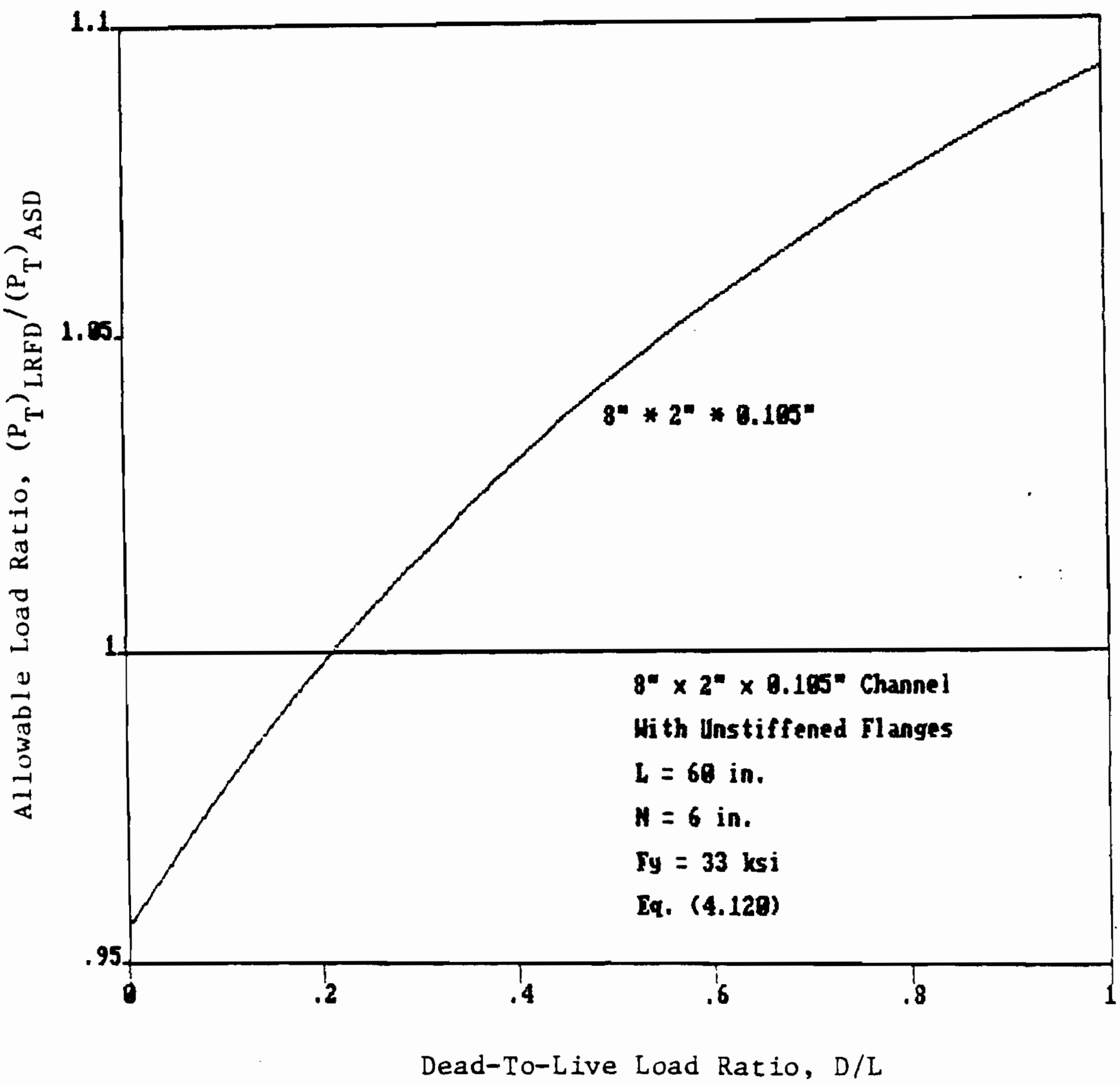


Figure 20. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Web Crippling-Case 3

Table 4.7 Channel With Unstiffened Flanges

Section	S_{xc} (in.^{3})	h/t	M_u (k-in.)	M_{ubw} (k-in.)	P_u (kips)	$P_u L/M_u$	K_w
8x2x0.105	2.58	74.19	73.98	73.98	7.105	5.762	1.5297

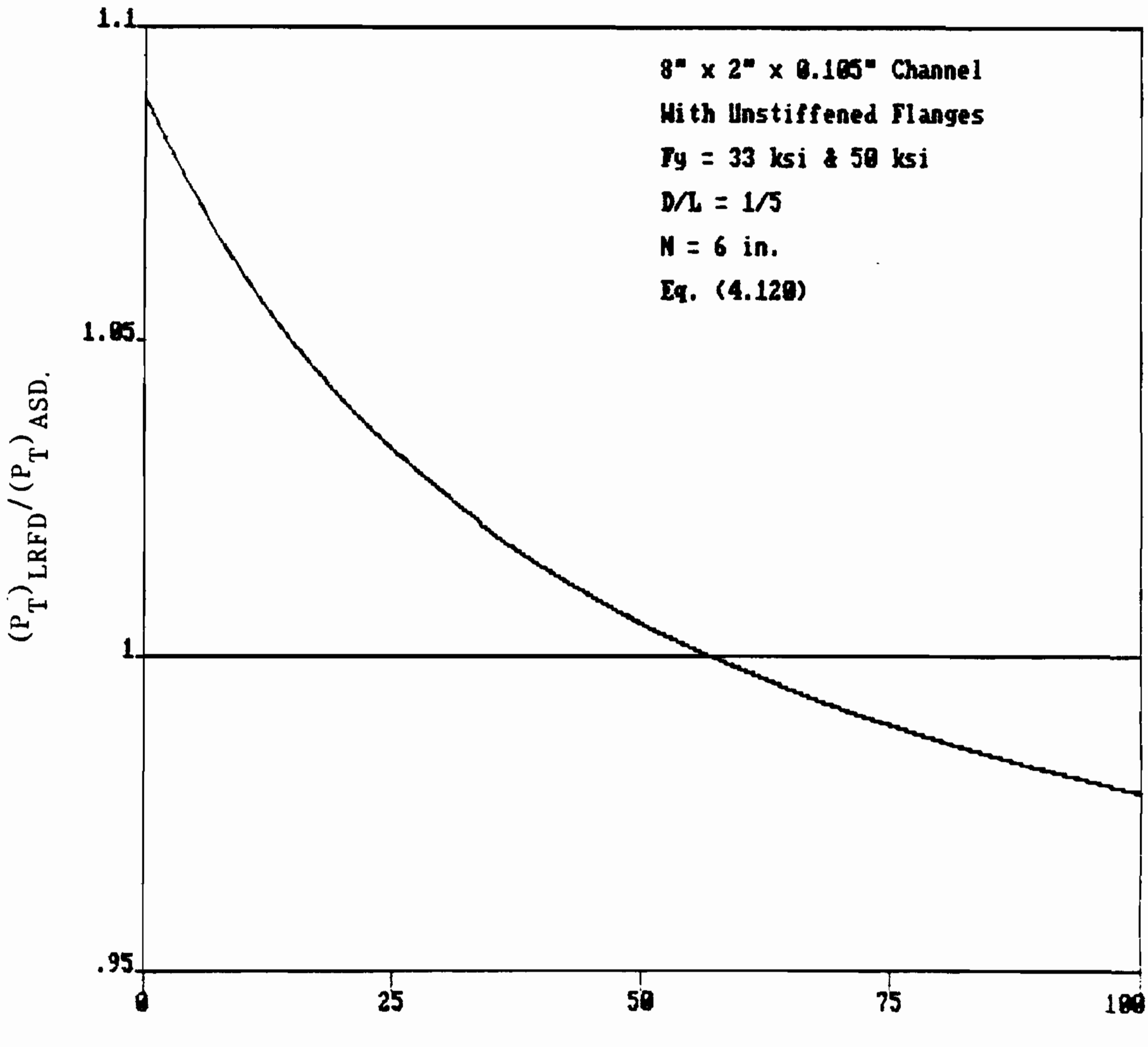


Figure 21. Allowable Load Ratio vs. Span Length for Combined Bending and Web Crippling-Case 3

channel sections with the larger h/t ratios resulted in larger values of allowable load ratio. Therefore, with increasing h/t ratio, the difference between the allowable loads obtained from the two design methods decreases.

Figures 18 and 21 show how the span length and yield point of steel affect the allowable load ratio. As shown in these two figures, larger span lengths will result in slightly lower values of the allowable load ratio. Also from Figures 18 and 21, it can be seen that the yield point of steel has a negligible effect on the allowable load ratio.

Figures 17 through 21 also show that channels with stiffened and unstiffened flanges give similar values of the allowable load ratio. In general, LRFD results in a somewhat conservative design for cold-formed steel channels as compared with allowable stress design for $D/L < 1/4$.

For I-section made from two channels back-to-back, Figure 22 shows the relationship between allowable load ratio and dead-to-live load ratio. Table 4.8 presents sectional properties and calculated values for the cold-formed I-section with $F_y = 33$ ksi and $L = 5$ ft. For the I-section with stiffened flange shown in Figure 22, LRFD would result in an allowable load about 5.6% higher than the load computed from allowable stress design for $D/L = 0.5$.

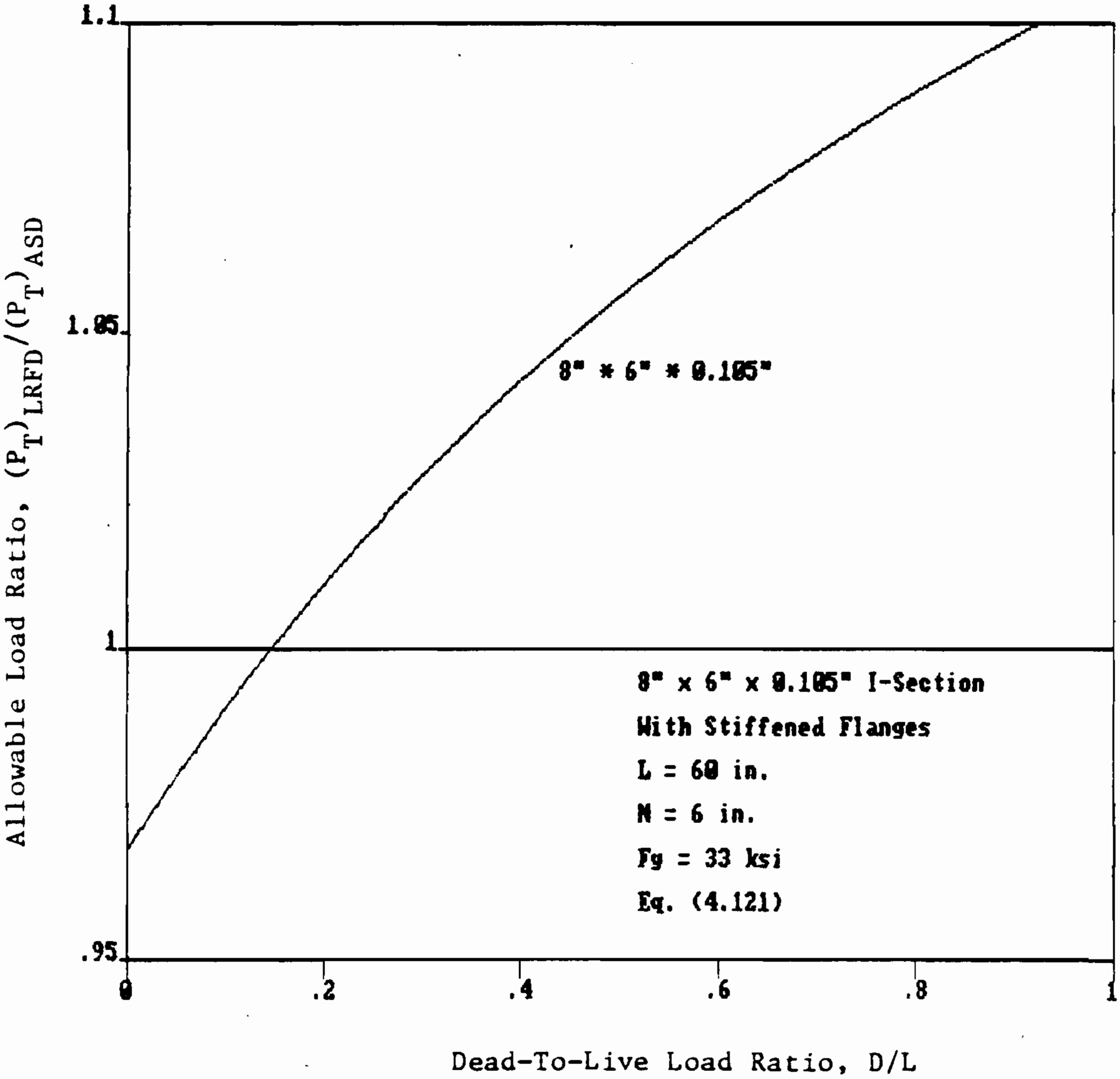


Figure 22. Allowable Load Ratio vs. D/L Ratio for Combined Bending and Web Crippling-Case 4

Table 4.8 I-Section With Stiffened Flanges

Section	S_{eff} (in.^{3})	h/t	M_u (k-in.)	M_{ubw} (k-in.)	P_u (kips)	$P_u L/M_u$	K_w
8x6x0.105	7.56	74.19	249.5	249.5	28.96	6.976	1.5486

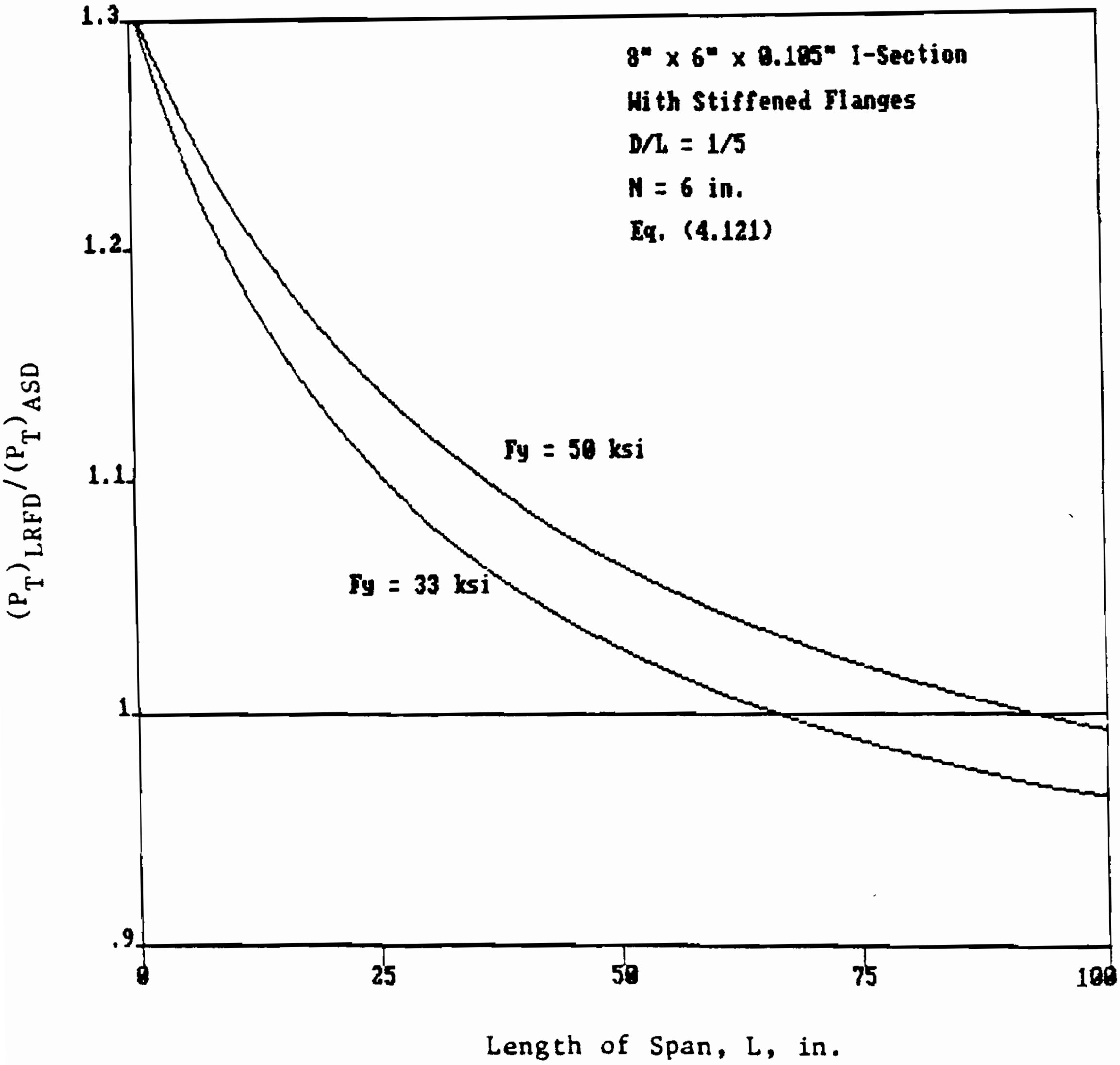


Figure 23. Allowable Load Ratio vs. Span Length for Combined Bending and Web Crippling-Case 4

Figure 23 shows how the span length and yield point of steel affect the allowable load ratio. A higher yield point of steel results in a larger value of the allowable load ratio. As shown in Figure 23, span length has a greater effect on the allowable load ratio for I-sections than it does on channel sections which are shown in Figures 18 and 21. In general, large span lengths result in lower values of the allowable load ratio.

E. INELASTIC RESERVE CAPACITY OF FLEXURAL MEMBERS

The inelastic reserve capacity of beams is a result of the partial plastification of the cross section. This pheonomenon is associated with web plastification which results from the continued plastic straining of one or both flanges[42]. Because buckling and other factors limit the strain capacity in the cross section, the inelastic flexural reserve capacity can be used only when the following conditions are met[1]:

(1) The member is not subjected to twisting, lateral, torsional, or torsional-flexural buckling

(2) The effect of cold-forming is not included in determining the yield point F_y

(3) The ratio of the depth of the compressed portion of the web to its thickness does not exceed $190/\sqrt{F_y}$

(4) The depth to thickness ratio of the entire web does not exceed $640/\sqrt{F_y}$

(5) The shear force does not exceed 0.58 F_y times the web area

(6) The angle between any web and the vertical does not exceed 20 degrees.

1. Allowable Stress Design. According to Section 3.9 of the AISI Specification[1], the design moment should not exceed $0.75M_y$ or $0.60M_u$

where

M_y = moment causing a maximum strain of e_y, kip-in.

e_y = yield strain = F_y/E

E = modulus of elasticity = 29,500 ksi

M_u = ultimate moment causing a maximum compression strain of $C_y e_y$ (no limit is placed on the maximum tensile strain), kip-in.

C_y = a factor determined as follows:

(1) Stiffened compression elements without intermediate stiffeners

$$C_y = 3 \text{ for } w/t \leq 190/\sqrt{F_y} \tag{4.123}$$

$$C_y = 3-[(w/t)\sqrt{F_y}-190]/15.5 \text{ for } 190/\sqrt{F_y} < w/t < 221/\sqrt{F_y} \tag{4.124}$$

$$C_y = 1 \text{ for } w/t \geq 221/\sqrt{F_y} \tag{4.125}$$

(2) Unstiffened compression elements

$$C_y = F_c/F \tag{4.126}$$

where F_c is defined in Section 3.2[1] and F is defined in Section 3.1[1]

(3) Multiple-stiffened compression elements and compression elements with edge stiffeners

$$C_y = 1 \tag{4.127}$$

When applicable effective design widths should be used in calculating section properties, M_u should be calculated considering equilibrium of stresses, assuming an ideally elastic-plastic stress-strain curve which is the same in tension as in compression, assuming small deformations and assuming that plane sections before bending remain plane during flexure.

2. LRFD Criteria. According to Section 9.7 of the Tentative Recommendations[10], the factored nominal bending strength, ϕM_{ul}, should be determined with $\phi = 0.95$ and M_{ul} is either 1.25 M_y or M_u, whichever is smaller. M_u and M_y are computed by the same formulas used in the AISI Specification[1] except that for unstiffened compression elements, C_y is calculated as follows:

$$C_y = F_{cr}/F_y \tag{4.128}$$

Where F_{cr} is defined in Section 8.5 of the Tentative Recommendations[10] and F_y is the minimum specified yield point.

3. Comparison. The unfactored applied moment can be calculated using Eq. (4.14) and should be less than or equal to the allowable moments. For allowable stress design, the allowable moment is computed from the ultimate inelastic reserve moment using a factor of safety of 1.67. The allowable moment for LRFD can be computed by using the following equation developed from Eq. (2.6) :

$$(M_a)_{LRFD} = \phi M_{ul}(D/L+1)/(1.2D/L+1.6) \tag{4.129}$$

Since the yield moment and the ultimate moment are calculated using the same formulas for allowable stress design and LRFD, the allowable moment ratio for $\phi = 0.95$ is as follows:

$$\frac{(M_a)_{LRFD}}{(M_a)_{ASD}} = 1.67\phi \frac{D/L+1}{1.2D/L+1.6} = 1.58 \frac{D/L+1}{1.2D/L+1.6} \qquad (4.130)$$

Equation (4.130) is identical with Eq. (4.21) used in the comparison of the allowable moments for bending strength. The relationship between allowable moment ratio and dead-to-live load ratio is illustrated in Figure 3, from which both design methods give the same allowable moment for $D/L = 1/25$. However, LRFD is conservative for $D/L < 1/25$ and unconservative for $D/L > 1/25$ as compared with the allowable stress design.

F. SERVICEABILITY

Similar to hot-rolled shapes, deflection of cold-formed steel beams with large span lengths has to be checked along with the load capacities. The deflection is a function of span length, bending stiffness EI, and type and magnitude of the applied load. The maximum live load deflection for beams and girders supporting plastered ceilings should not exceed 1/360 of the span length according to the AISC Specifications(3). The maximum deflection should be computed using unfactored live loads.

The moment of inertia, I, of the cross section is based on the type of compression flanges used in the beam section. For beams having unstiffened compression flanges, the moment of inertia is based on the full section. For beams with stiffened compression flanges, an effective width of the compression flange is used to compute the moment of inertia. The effective width is determined from the level of stress in the compression flange and the flat-width ratio, w/t.

Formulas used for calculating the effective width of a stiffened compression flange for deflection determination are identical for allowable stress design and LRFD. From Section 2.3.1.1 of the AISI Specifications(1) and Section 8.4.1.1 of the Tentative Recommendations(10), the procedure for calculating the effective width for deflection determination is as follows:

Flanges are fully effective up to

$$(w/t)_{lim} = 221/\sqrt{f}$$

For flanges with w/t larger than $(w/t)_{lim}$,

$$\frac{b}{t} = \frac{326}{\sqrt{f}}\left[1 - \frac{71.3}{(w/t)\sqrt{f}}\right] \tag{4.131}$$

Exception: Flanges of closed rectangular tubes are fully effective up to $(w/t)_{lim} = 237/\sqrt{f}$. For flanges with w/t larger than $(w/t)_{lim}$

$$\frac{b}{t} = \frac{326}{\sqrt{f}}\left[1 - \frac{64.9}{(w/t)\sqrt{f}}\right] \tag{4.132}$$

In the above,

w/t = flat-width ratio

b = effective design width, in.

f = actual stress in the compression element computed on the basis of the effective design width, ksi

When the flat-width ratio exceeds $(w/t)_{lim}$ the moment of inertia must frequently be determined by successive approximations or other appropriate methods, since the stress and the effective design width are interdependent. The actual stress is determined from unfactored service loads.

G. DESIGN EXAMPLE

See Problem No. 2 in Appendix C for a design example of a flexural member using Load and Resistance Factor Design.

V. COMPRESSION MEMBERS

A. GENERAL

Cold-Formed steel compression members have three possible modes of failure. Short and compact columns will fail by yielding. Local buckling of an individual element could occur if the flat-width to thickness ratio is large. Overall column buckling of intermediate and long columns could occur in one of three buckling modes: flexural buckling, torsional buckling, and torsional-flexural buckling.

B. FLEXURAL BUCKLING

Flexural buckling occurs when the member bends about a principal axis of the cross section. It can occur in the elastic or inelastic range depending upon the slenderness ratio.

1. Allowable Stress Design. For doubly-symmetric shapes, closed cross section shapes or cylindrical sections, and any other shapes which can be shown not to be subject to torsional or torsional-flexural buckling, and for members braced against twisting. Section 3.6.1.1 of the AISI Specification[1] specifies that the average axial stress, P/A, in compression members should not exceed the following values of F_{al}, except as otherwise permitted below.

For $KL/r < C_c/\sqrt{Q}$,

$$F_{al} = \frac{12}{23} QF_y - \frac{3(QF_y)^2}{23\pi^2 E}\left(\frac{KL}{r}\right)^2 \tag{5.1}$$

For $KL/r \geq C_c/\sqrt{Q}$,

$$F_{al} = \frac{12\pi^2 E}{23(KL/r)^2} \tag{5.2}$$

In the above,

$$C_c = \sqrt{2\pi^2 E/F_y}$$

P = total load, kips

A = full, unreduced cross-sectional area of the member, in.^{2}

F_{al} = allowable average compression stress under concentric loading, ksi

E = modulus of elasticity = 29,500 ksi

K = effective length factor

L = unbraced length of member, in.

r = radius of gyration of full, unreduced cross section, in.

F_y = yield point of steel, ksi

Q = a factor determined as follows:

(a) For members composed entirely of stiffened elements, Q, is the ratio between the effective design area, as determined from the effective design widths of such elements, and the full or gross area of the cross section. The effective design area used in determining Q is to be based upon the basic design stress F as defined in Section 3.1 of Reference 1.

(b) For members composed entirely of unstiffened elements, Q is the ratio between the allowable compression stress F_c for the element of the cross section having the largest flat-width ratio and the basic design stress, F, where F_c is as defined in Section 3.2 and F is as defined in

Section 3.1 of the AISI Specification[1].

(c) For members composed of both stiffened and unstiffened elements the factor Q is the product of a stress factor, Q_s, computed as outlined in paragraph (b) above and an area factor, Q_a, computed as outlined in paragraph (a) above, except that the stress upon which Q_a is to be based shall be that stress F_c which is used in computing Q_s; and the effective area to be used in computing Q_a shall include the full area of all unstiffened elements.

When the factor Q is equal to unity, the steel is 0.09 in. or more in thickness and KL/r is less than C_c:

$$F_{al} = \frac{\left[1 - \frac{(KL/r)^2}{2(C_c)^2}\right] F_y}{\frac{5}{3} + \frac{3(KL/r)}{8(C_c)} - \frac{(KL/r)^3}{8(C_c)^3}} \tag{5.3}$$

2. <u>LRFD Criteria</u>. For doubly symmetric shapes, closed cross section shapes or cylindrical sections, and any other shapes which can be shown not to be subject to torsional or torsional-flexural buckling, and for members braced against twisting, Section 9.4.1 of the Tentative Recommendations[10] specifies that the factored axial strength, $\phi_c P_u$, should be determined from $\phi_c = 0.85$ and the following formulas:

For $KL/r \le C_c/\sqrt{Q}$,

$$P_u = AQF_y \left[1 - \frac{QF_y}{4\pi^2 E} \left(\frac{KL}{r}\right)^2\right] \tag{5.4}$$

For $KL/r > C_c/\sqrt{Q}$,

$$P_u = \frac{\pi^2 EA}{(KL/r)^2} \tag{5.5}$$

(a) For members composed entirely of stiffened elements

$$Q = Q_a = A_{eff}/A$$

where A_{eff} is the effective area as determined for the effective design widths from Section 8.4 of Reference 10 for $f_{max} = F_y$.

(b) For members composed entirely of unstiffened elements

$$Q = Q_s = F_{cr}/F_y$$

where F_{cr} is the critical stress for the weakest element of the cross section as determined from the formulas given in Section 8.5 of Reference 10.

(c) For members composed of both stiffened and unstiffened elements

$$Q = Q_a Q_s$$

except that the stress upon which Q_a is to be based shall be that value of stress F_{cr} which is used in computing Q_s and the effective area to be used in computing Q_a shall include the full area of all unstiffened elements.

3. Comparison. The unfactored loads applied to the members can be computed for both design methods by using the following formula:

$$P_T = P_{DL} + P_{LL} \tag{5.6}$$

where

P_T = unfactored compressive load, kips

P_{DL} = compressive load due to the nominal axial dead load, kips

P_{LL} = compressive load due to the nominal axial live load, kips

The total unfactored load should be less than or equal to the allowable load computed from allowable stress design and LRFD. For allowable stress design, the allowable load is

$$(P_a)_{ASD} = AF_{al} \tag{5.7}$$

For LRFD, the allowable axial load can be computed by using the following equation developed from Eq. (2.6):

$$(P_a)_{LRFD} = \phi_c P_u (D/L+1)/(1.2D/L+1.6) \tag{5.8}$$

The allowable compressive stress, F_{al}, is derived from the buckling stress with a factor of safety of 23/12. When $Q = 1.0$, $t \geq 0.09$ in., and $KL/r < C_c$, the factor of safety is a function of the slenderness ratio and C_c.

$$F.S. = \frac{5}{3} + \frac{3(KL/r)}{8(C_c)} - \frac{(KL/r)^3}{8(C_c)^3} \tag{5.9}$$

Therefore, the allowable load ratios are:

For $Q = 1.0$, $t \geq 0.09$ in., and $KL/r < C_c$,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \phi_c \left[\frac{5}{3} + \frac{3(KL/r)}{8(C_c)} - \frac{(KL/r)^3}{8(C_c)^3} \right] \left[\frac{D/L+1}{1.2D/L+1.6} \right] \tag{5.10}$$

For all other cases,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \phi_c \frac{23}{12} \frac{D/L+1}{1.2D/L+1.6} = 1.629 \frac{D/L+1}{1.2D/L+1.6} \tag{5.11}$$

Figure 24 shows the allowable load ratio versus dead-to-live load ratio for the columns used to develop Eq. (5.11). For this case, the LRFD criteria always permit larger allowable loads than the allowable stress design. For $D/L = 0.5$, the LRFD criteria

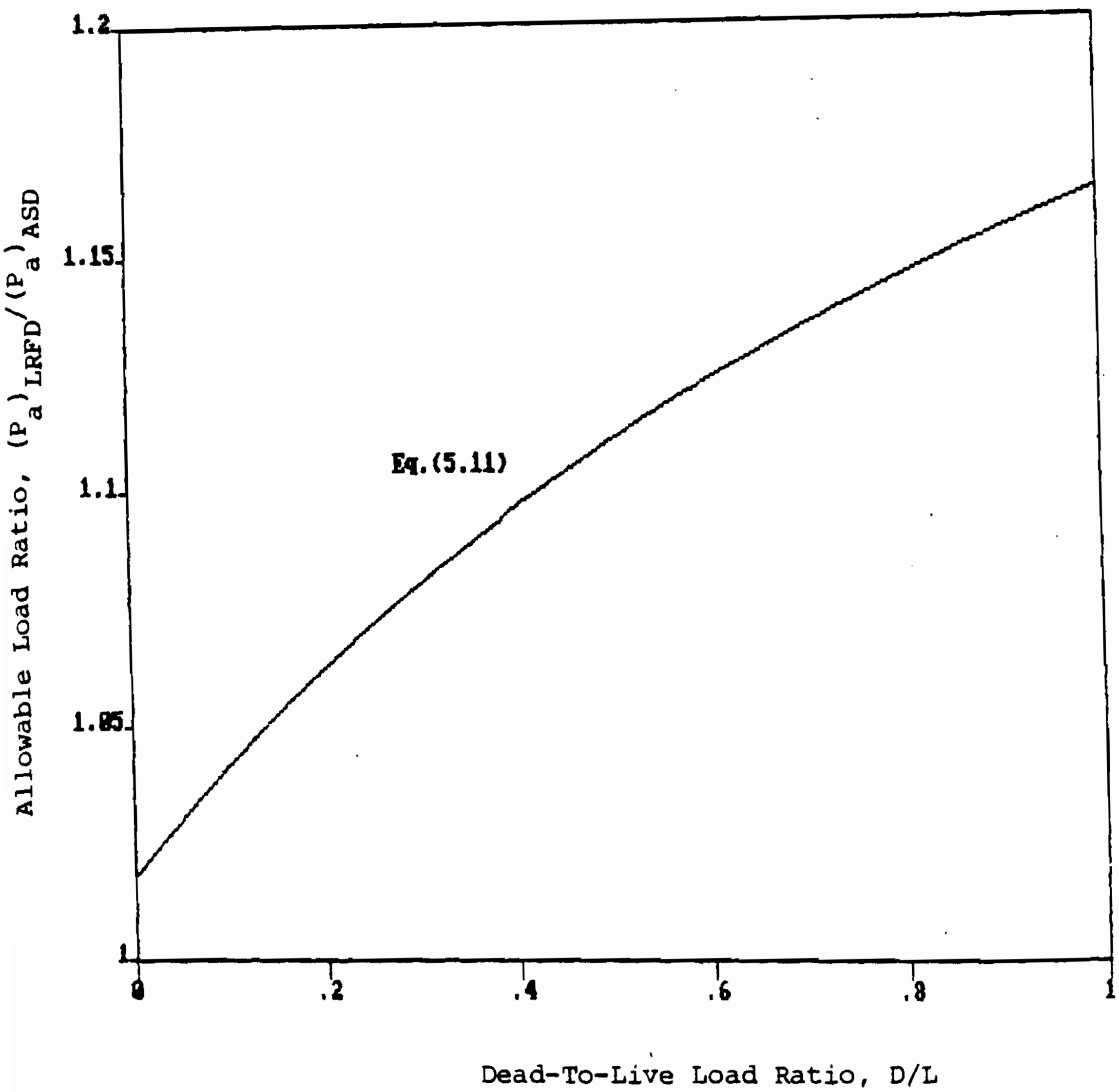


Figure 24. Allowable Load Ratio vs. D/L Ratio for Column Buckling

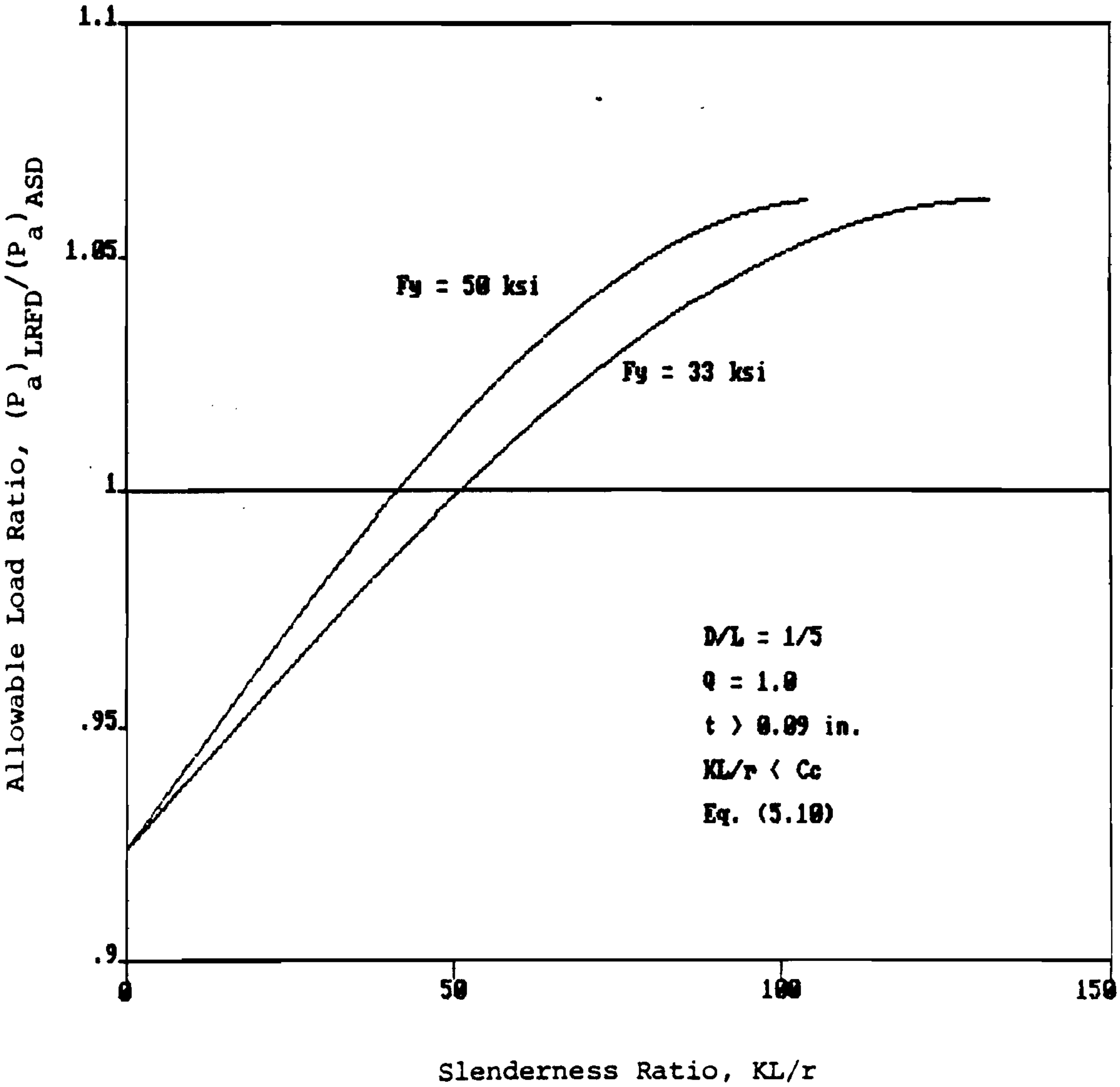


Figure 25. Allowable Load Ratio vs. Slenderness Ratio for Flexural Buckling of Columns

gives an allowable load about 11% greater than the load obtained by using allowable stress design.

The allowable load ratio versus slenderness ratio, KL/r, for columns with $Q = 1.0$, $t \geq 0.09$ in., and $KL/r < C_c$ is shown in Figure 25. For this case, the LRFD criteria were found to be conservative for short columns as compared with allowable stress design. As shown in Figure 25, higher yield point materials give slightly higher values of the allowable load ratio, $(P_a)_{LRFD}/(P_a)_{ASD}$.

C. <u>TORSIONAL-FLEXURAL BUCKLING</u>

Torsional-flexural buckling of singly-symmetric and nonsymmetric shapes can occur in open thin-walled columns. For these types of members, flexural buckling should also be checked.

1. <u>Allowable Stress Design</u>. Section 3.6.1.2 of the AISI Specifications[1] specifies that for singly-symmetric or nonsymmetric shapes of open cross-section or intermittently fastened singly-symmetric components of built-up shapes which may be subject to torsional-flexural buckling and which are not braced against twisting, the average axial stress, P/A, should not exceed F_{a1} specified in Section 3.6.1.1 of Reference 1 or F_{a2} given below:

For $\sigma_{TFO} > 0.5QF_y$,

$$F_{a2} = 0.522QF_y - (QF_y)^2/7.67\sigma_{TFO} \tag{5.12}$$

For $\sigma_{TFO} \leq 0.5QF_y$,

$$F_{a2} = 0.522\sigma_{TFO} \tag{5.13}$$

where

F_{a2} = allowable average compression stress under concentric loading, ksi

σ_{TFO} = elastic torsional-flexural buckling stress under concentric loading which shall be determined as follows:

(a) Singly-Symmetric Shapes. For members whose cross-sections have one axis of symmetry (x-axis), σ_{TFO} is less than both σ_{ex} and σ_t and is equal to:

$$\sigma_{TFO} = (1/2\beta)[(\sigma_{ex}+\sigma_t) - \sqrt{(\sigma_{ex}+\sigma_t)^2 - 4\beta\sigma_{ex}\sigma_t}] \tag{5.14}$$

where

$$\sigma_{ex} = \frac{\pi^2 E}{(KL/r)^2}\text{, ksi} \tag{5.15}$$

$$\sigma_t = \frac{1}{Ar_o^2}\left[GJ + \frac{\pi^2 EC_w}{(KL)^2}\right]\text{, ksi} \tag{5.16}$$

$$\beta = 1-(x_o/r_o)^2 \tag{5.17}$$

A = cross-sectional area

$r_o = \sqrt{r_x^2 + r_y^2 + x_o^2}$ = polar radius of gyration of cross-section about the shear center, in. (5.18)

r_x, r_y = radii of gyration of cross-section about centroidal principal axes, in.

E = modulus of elasticity = 29,500 ksi

G = shear modulus = 11,300 ksi

K = effective length factor

L = unbraced length of compression member, in.

x_o = distance from shear center to centroid along the principal x-axis, in.

J = St. Venant torsion constant of the cross section, in.^{4}

For thin-walled sections composed of n segments of uniform thickness,

$$J = (1/3)(\ell_1 t_1^3 + \ell_2 t_2^3 + \ldots + \ell_i t_i^3 + \ldots \ell_n t_n^3) \quad (5.19)$$

t_i = steel thickness of the member for segment i, in.

ℓ_i = length of middle line of segment i, in.

C_w = torsional warping constant of the cross-section, in.^{6}

(b) Nonsymmetric Shapes. For shapes whose cross-sections do not have any symmetry, either about an axis or about a point, σ_{TFO} shall be determined by rational analysis. Alternatively, compression members composed of such shapes may be tested in accordance with Section 6 of the AISI Specifications[1].

2. LRFD Criteria. For singly-symmetric or nonsymmetric shapes of open cross section or intermittenly fastened singly-symmetric components of build-up shapes which may be subject to torsional-flexural buckling and which are not braced against twisting, Section 9.4.2 of the Tentative Recommendations[10] specifies that the factored axial strength, $\phi_c P_u$, should be determined from ϕ_c = 0.85 and the load P_u which is the smaller of the values determined from Section 9.4.1 of Reference 10 and the following formulas:

For $\sigma_{TFO} > 0.5QF_y$,

$$P_u = AQF_y\ (1-QF_y/4\sigma_{TFO}) \quad (5.20)$$

For $\sigma_{TFO} \leq 0.5QF_y$,

$$P_u = A\sigma_{TFO} \quad (5.21)$$

3. Comparison. The applied unfactored load can be calculated using Eq. (5.6). This load should be less than or equal to the

allowable axial load determined from both design methods. The allowable load for torsional-flexural buckling based on allowable stress design is

$$(P_a)_{ASD} = AF_{a2} \tag{5.22}$$

The allowable load for LRFD was obtained by using the following equation developed from Eq. (2.6) :

$$(P_a)_{LRFD} = \phi_c P_u (D/L+1)/(1.2D/L+1.6) \tag{5.23}$$

In allowable stress design, the allowable compressive stress, F_{a2}, is derived from the torsional-flexural buckling stress with a factor of safety of 23/12. Therefore the allowable load ratio for this case with $\phi_c = 0.85$ is

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \phi_c \frac{23}{12} \frac{D/L+1}{1.2D/L+1.6} = 1.629 \frac{D/L+1}{1.2D/L+1.6} \tag{5.24}$$

This relation is similar to Eq. (5.11) illustrated graphically in Figure 24 which was discussed in the previous section on flexural buckling. The same conclusion applies to torsion-flexural buckling.

D. <u>TORSIONAL BUCKLING</u>

For point-symmetric shapes, torsional buckling along with flexural buckling should be considered in the design of columns.

1. <u>Allowable Stress Design</u>. In Section 3.6.1.3 of the AISI Specification[1], it is specified that for point-symmetric open shapes such as cruciform sections or such built-up shapes which may be subject to torsional buckling and which are not braced against twisting, the average axial stress, P/A, should not exceed F_{a1} specified in Section 3.6.1.1 of Reference 1 or F_{a2} given below:

For $\sigma_t > 0.5QF_y$,

$$F_{a2} = 0.522QF_y - (QF_y)^2/7.67\sigma_t \tag{5.25}$$

For $\sigma_t \leq 0.5QF_y$,

$$F_{a2} = 0.522\sigma_t \tag{5.26}$$

where σ_t is defined in Section 3.6.1.2.1 of Reference 1. If the section consists entirely of unstiffened elements Q should be taken as 1.0; otherwise Q should be determined in accordance with Section 3.6.1.1 of the AISI Specification.

2. LRFD Criteria. For point-symmetric open shapes such as cruciform sections or such built-up shapes which may be subject to torsional buckling and which are not braced against twisting, Section 9.4.3 of the Tentative Recommendations(10) specifies that the factored axial strength, $\phi_c P_u$, should be determined from $\phi_c = 0.85$ and the load P_u which is the smaller of the values determined from Section 9.4.1 of Reference 10 and the following formulas:

For $\sigma_t > 0.5QF_y$,

$$P_u = AQF_y(1-QF_y/4\sigma_t) \tag{5.27}$$

For $\sigma_t \leq 0.5QF_y$,

$$P_u = A\sigma_t \tag{5.28}$$

where σ_t is defined in Section 9.4.2 of Reference 10. If the section consists entirely of unstiffened elements Q should be taken as 1.0; otherwise Q should be determined in accordance with Section 9.4.1 of the Tentative Recommendations(10).

3. Comparison. The applied unfactored load can be calculated using Eq. (5.6). This applied load should be less than or equal to the allowable axial load determined from both design methods. The allowable load for torsional buckling according to allowable stress design is

$$(P_a)_{ASD} = AF_{a2} \tag{5.29}$$

For LRFD, the allowable axial load was obtained by using the following equation developed from Eq. (2.6):

$$(P_a)_{LRFD} = \phi_c P_u (D/L+1)/(1.2D/L+1.6) \tag{5.30}$$

In Eq. (5.29), the allowable design stress, F_{a2}, is derived from the torsional buckling stress with a factor of safety of 23/12. Therefore, the allowable load ratio for this case is similar to flexural and torsional-flexural buckling. For $\phi_c = 0.85$, the allowable load ratio is

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \phi_c \frac{23}{12} \frac{D/L+1}{1.2D/L+1.6} = 1.629 \frac{D/L+1}{1.2D/L+1.6} \tag{5.31}$$

Since Eq. (5.31) is identical to Eqs. (5.11) and (5.24), Figure 24 can also be used for the comparison of torsional buckling loads determined by using allowable stress design and LRFD.

E. DESIGN EXAMPLES

See Problems Nos. 3 and 4 in Appendix C for design examples of axially loaded compression members using Load and Resistance Factor Design.

VI. BEAM-COLUMNS

A. GENERAL

Beam-columns are structural members subjected to combined axial compression and bending stresses. The structural behavior of beam-columns depends on the shape and dimensions of the cross section, the location of the applied eccentric load, column length, and condition of bracing[43]. Interaction formulas are used to analyze beam-columns for flexural and torsional-flexural buckling.

B. DOUBLY-SYMMETRIC SHAPES

Doubly-symmetric shapes and shapes not subject to torsional or torsional-flexural buckling will fail by either flexural yielding or local buckling when subjected to axial compression and bending about its principal axis.

1. Allowable Stress Design. When the member is subject to both axial compression and bending, doubly-symmetric shapes or shapes which are not subject to torsional or torsional-flexural buckling should be proportioned to meet the following requirements in Section 3.7.1 of the AISI Specification[1]:

$$\frac{f_a}{F_{al}} + \frac{C_{mx} f_{bx}}{(1-f_a/F'_{ex})F_{bx}} + \frac{C_{my} f_{by}}{(1-f_a/F'_{ey})F_{by}} < 1.0 \qquad (6.1)$$

$$\frac{f_a}{F_{ao}} + \frac{f_{bx}}{F_{blx}} + \frac{f_{by}}{F_{bly}} \leq 1.0 \qquad (6.2)$$

When $f_a/F_{al} \leq 0.15$, the following formula may be used in lieu of the above two formulas:

$$\frac{f_a}{F_{al}} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \qquad (6.3)$$

The subscripts x and y in the above formulas indicate the axis of bending about which a particular stress or design property applies. In the above interaction equations,

C_m = a coefficient whose value shall be taken as follows:

(a) For compression members in frames subject to joint translation (sidesway),

$C_m = 0.85$

(b) For restrained compression members in frames braced against joint translation and not subject to transverse loading between their supports in the plane of bending,

$$C_m = 0.6 - 0.4(M_1/M_2) \geq 0.4 \qquad (6.4)$$

where M_1/M_2 is the ratio of the smaller to the larger moment at the ends of that portion of the member, unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature and negative when it is bent in single curvature.

(c) For compression members in frames braced against joint translation in the plane of loading and subject to transverse loading between their supports, the value of C_m may be determined by rational analysis. However, in lieu of such analysis, the following values may be used:

(1) for members whose ends are restrained,

$C_m = 0.85$,

(2) for members whose ends are unrestrained,

$C_m = 1.0$

F_{ao} = allowable compression stress under concentric loading determined by Section 3.6.1.1 of Reference 1 for $L = 0$, ksi

F_{a1} = allowable compression stress under concentric loading according to Section 3.6.1.1 of Reference 1 for buckling in the plane of symmetry, ksi

F_b = maximum bending stress in compression that is permitted by the AISI Specification where bending stress only exists (Section 3.1, 3.2, and 3.3 of References 1), ksi

F_{b1} = maximum bending stress in compression permitted by the AISI Specification where bending stress only exists and the possibility of lateral buckling is excluded (Sections 3.1 and 3.2 of Reference 1), ksi

$$F'_e = \frac{12\pi^2 E}{23(KL_b/r_b)^2}\text{, ksi} \qquad (6.5)$$

f_a = axial stress = axial load divided by full cross-sectional area of member, P/A, ksi

f_b = maximum bending stress = bending moment divided by appropriate section modulus of member, M/S, noting that for members having stiffened compression elements the section modulus shall be based upon the effective design widths of such elements, ksi

K = effective length factor in the plane of bending

L_b = actual unbraced length in the plane of bending, in.

r_b = radius of gyration about axis of bending, in.

2. LRFD Criteria. For shapes not subject to torsional or torsional-flexural buckling, the factored design forces P_D, M_{Dx}, and M_{Dy} should satisfy the following interaction equations obtained from Section 9.5.1 of the Tentative Recommendations[10]:

$$\frac{P_D}{\phi_c P_{uc}} + \frac{C_{mx} M_{Dx}}{\phi M_{ucx}[1-P_D/(\phi_c P_{Ex})]} + \frac{C_{my} M_{Dy}}{\phi M_{ucy}[1-P_D/(\phi_c P_{Ey})]} \le 1.0 \quad (6.6)$$

$$\frac{P_D}{\phi_s P_{us}} + \frac{M_{Dx}}{\phi_s M_{usx}} + \frac{M_{Dy}}{\phi_s M_{usy}} \le 1.0 \quad (6.7)$$

except that when $P_D/(\phi_c P_{uc}) \le 0.15$, the following formula may be used in lieu of the above two formulas:

$$\frac{P_D}{\phi_c P_{uc}} + \frac{M_{Dx}}{\phi M_{ucx}} + \frac{M_{Dy}}{\phi M_{ucy}} \le 1.0 \quad (6.8)$$

In the above interaction equations,

P_D = factored design axial load, kips

M_D = factored design moment, kip-in.

P_{uc} = axial strength determined by Section 9.4.1 of Reference 10, kips

$P_{us} = AQ_aQ_sF_y$, kips (6.9)

$P_{Ex} = \pi^2 EI_x/(KL)_x^2$, kips (6.10)

$P_{Ey} = \pi^2 EI_y/(KL)_y^2$, kips (6.11)

M_{uc} = factored nominal beam strength as determined from Sections 9.3.1 and 9.3.2 of Reference 10, whichever is smaller, kip-in.

M_{us} = beam strength as determined from Section 9.3.1 of Reference 10, kip-in.

E = modulus of elasticity = 29,500 ksi

I_x = moment of inertia of the section about the x-axis, in.^{4}

I_y = moment of inertia of the section about the y-axis, in.^{4}

Q_a, Q_s = factors determined according to Eqs. (9.4.1-3) and (9.4.1-4), respectively

A = cross-sectional area, in.^{2}

ϕ = 0.90 for using Section 9.3.2 to compute M_{uc}
= 0.95 for using Section 9.3.1 to compute M_{uc}

ϕ_c = 0.85

ϕ_s = 0.95

3. Comparison. For comparison, only bending about the x-axis was considered. A typical design example was selected and the allowable axial loads were calculated by using the three interaction equations for each design method. The example used a beam-column with equal moments applied to each end so that the member is bent in single curvature. Since the end moments are independent of the axial load, the ratio of the unfactored applied moment to the ultimate moment capacity based on section strength, M_T/M_{us}, was considered to be a parameter in the equations for determining allowable stresses to compute the allowable loads.

For allowable stress design the allowable axial loads were computed as follows:

$$\frac{f_a}{F_{al}} = \frac{P_T}{P_{uc}/(F.S.)} = \frac{(F.S.)P_T}{P_{uc}} \tag{6.12}$$

$$\frac{f_b}{F_b} = \frac{M_T}{0.6M_{uc}} = \frac{(M_T/M_{us})(M_{us}/M_{uc})}{0.6} \tag{6.13}$$

$$\frac{f_a}{F'_{ex}} = \frac{23P_T}{12P_{Ex}} \tag{6.14}$$

where

P_T = allowable axial load, kips

M_T = applied unfactored bending moment at each end of the member, kip-in.

F.S. = factor of safety of axially loaded compression members which is 23/12. If $Q = 1.0$, $t \geq 0.09$ in., and $KL/r < C_c$, then F. S. is determined from Eq. (5.9)

Substitution of Eqs. (6.12), (6.13), and (6.14) into Eq. (6.1) results in the following expression:

$$\frac{(F.S.)P_T}{P_{uc}} + \frac{C_m(M_T/M_{us})(M_{us}/M_{uc})}{0.6[1-(23/12)(P_T/P_{Ex})]} = 1.0 \tag{6.15}$$

By solving for P_T in the first term of Eq. (6.15), the following equation for allowable load is obtained :

$$(P_T)_{ASD1} = \left[1- \frac{C_m(M_T/M_{us})(M_{us}/M_{uc})}{0.6[1-(23/12)(P_T)(P_{Ex})]}\right]\frac{P_{uc}}{F.S.} \tag{6.16}$$

Equation (6.16) is based on Eq. (6.1) for failure at the midlength of the beam-column and requires a solution by iterations.

The following expressions were used to solve for the allowable load based on Eq. (6.2):

$$\frac{f_a}{F_{ao}} = \frac{P_T}{P_{us}/(F.S.)} = \frac{(F.S.)P_T}{P_{us}} \tag{6.17}$$

$$\frac{f_b}{F_{bl}} = \frac{M_T}{0.6M_{us}} = \frac{(M_T/M_{us})}{0.6} \tag{6.18}$$

Substitution of Eqs. (6.17) and (6.18) into Eq. (6.2) results in the following expression:

$$\frac{(F.S.)P_T}{P_{us}} + \frac{(M_T/M_{us})}{0.6} = 1.0 \tag{6.19}$$

By solving for P_T in Eq. (6.19), the following equation for allowable load is obtained:

$$(P_T)_{ASD2} = \left[1 - \frac{(M_T/M_{us})}{0.6}\right] \frac{P_{us}}{F.S.} \tag{6.20}$$

Equation (6.20) is based on Eq. (6.2) for failure at the braced points.

When $f_a/F_{al} \leq 0.15$, Eq. (6.3) can be used in lieu of Eqs. (6.1) and (6.2). Equation (6.3) can be written in the following form by using Eqs. (6.12) and (6.13):

$$\frac{(F.S.)P_T}{P_{uc}} + \frac{(M_T/M_{us})(M_{us}/M_{uc})}{0.6} = 1.0 \tag{6.21}$$

By solving for P_T in Eq. (6.21), the following equation for allowable load is obtained:

$$(P_T)_{ASD3} = \left[1 - \frac{(M_T/M_{us})(M_{us}/M_{uc})}{0.6}\right] \frac{P_{uc}}{F.S.} \tag{6.22}$$

Equation (6.22) is based on Eq. (6.3) for flexural failure when the effect of the secondary moment is neglected.

For LRFD, the allowable axial loads were computed in accordance with Eq. (2.6) as follows:

$$\frac{P_D}{\phi_c P_{uc}} = \frac{1.2D/L+1.6}{D/L+1} \; \frac{P_T}{\phi_c P_{uc}} \tag{6.23}$$

$$\frac{M_D}{\phi \; M_{uc}} = \frac{1.2D/L+1.6}{D/L+1} \; \frac{(M_T/M_{us})(M_{us}/M_{uc})}{\phi} \tag{6.24}$$

$$\frac{P_D}{\phi_c P_{Ex}} = \frac{1.2D/L+1.6}{D/L+1} \; \frac{P_T}{\phi_c P_{Ex}} \tag{6.25}$$

Substitution of Eqs. (6.23),(6.24), and (6.25) into Eqs. (6.6) results in the following expression:

$$\frac{1.2D/L+1.6}{D/L+1} \left[\frac{P_T}{\phi_c P_{uc}} + \frac{C_m (M_T/M_{us})(M_{us}/M_{uc})}{\phi \left(1 - \frac{1.2D/L+1.6}{D/L+1} \; \frac{P_T}{\phi_c P_{Ex}} \right)} \right] = 1.0 \tag{6.26}$$

By solving for P_T in the first term of Eq. (6.26), the following equation for allowable load is obtained :

$$(P_T)_{LRFD1} = \left[\frac{D/L+1}{1.2D/L+1.6} - \frac{C_m (M_T/M_{us})(M_{us}/M_{uc})}{\phi \left(1 - \frac{1.2D/L+1.6}{D/L+1} \; \frac{P_T}{\phi_c P_{Ex}} \right)} \right] \phi_c P_{uc} \tag{6.27}$$

Equation (6.27) is based on Eq. (6.6) for flexural failure at the midlength of the beam-column and requires a solution by iterations.

The following expressions were used to solve for the allowable load based on Eq. (6.7):

$$\frac{P_D}{\phi_s P_{us}} = \frac{1.2D/L+1.6}{D/L+1} \; \frac{P_T}{\phi_s P_{us}} \tag{6.28}$$

$$\frac{M_D}{\phi_s M_{us}} = \frac{1.2D/L+1.6}{D/L+1} \frac{(M_T/M_{us})}{\phi_s} \tag{6.29}$$

Substitution of Eqs. (6.28) and (6.29) into Eq. (6.7) results in the following expression:

$$\frac{1.2D/L+1.6}{D/L+1} \left[\frac{P_T}{\phi_s P_{us}} + \frac{(M_T/M_{us})}{\phi_s} \right] = 1.0 \tag{6.30}$$

By solving for P_T in Eq. (6.30), the following equation for allowable load is obtained:

$$(P_T)_{LRFD2} = \left[\frac{D/L+1}{1.2D/L+1.6} - \frac{(M_T/M_{us})}{\phi_s} \right] \phi_s P_{us} \tag{6.31}$$

Equation (6.31) is based on Eq. (6.7) for failure at the braced points.

When $P_D/(\phi_c P_{uc}) \leq 0.15$, Eq. (6.8) can be used in lieu of Eq. (6.6) and (6.7). Equation (6.8) can be written in the following form by using Eqs. (6.23) and (6.24):

$$\frac{1.2D/L+1.6}{D/L+1} \left[\frac{P_T}{\phi_c P_{uc}} + \frac{(M_T/M_{us})(M_{us}/M_{uc})}{\phi} \right] = 1.0 \tag{6.32}$$

By solving for P_T in Eq. (6.32), the following equation for allowable load is obtained:

$$(P_T)_{LRFD3} = \left[\frac{D/L+1}{1.2D/L+1.6} - \frac{(M_T/M_{us})(M_{us}/M_{uc})}{\phi} \right] \phi_c P_{uc} \tag{6.33}$$

Equation (6.33) is based on (6.8) for flexural failure when the effect of the secondary moment is neglected.

Equations (6.16), (6.20), and (6.22) for determining the allowable axial load based on allowable stress design and Eqs. (6.27), (6.31), and (6.33) for determining the allowable axial load based on LRFD are very complex and utilize iterations with multiple variables. A computer program was used to calculate allowable axial loads for doubly-symmetric beam-columns based on allowable stress design and LRFD criteria. The program, listed in Appendix A, computes allowable loads and allowable load ratios, $(P_T)_{LRFD}/(P_T)_{ASD}$, for various lengths combined with different applied end moment ratios, M_T/M_{us}, with respect to the beam strength of the member. Standard I-sections and their section properties used in this study were obtained from Tables 5 and 6 of Part V of the AISI Cold-Formed Steel Design Manual(41).

An I-section (3.5 in. x 4 in. x 0.105 in.) with stiffened flanges was studied with a yield point of 33 ksi. Figure 26 shows the allowable load ratio versus dead-to-live load ratio for a 4 ft length with various end moment ratios, M_T/M_{us}. This figure is based on Eqs. (6.16) and (6.27) for flexural failure at the midlength of the beam-column. For a D/L ratio around 0.3, the LRFD criteria gives an allowable load about 1.3% more than the value computed from allowable stress design for all end moment ratios indicated in the figure. For other values of the D/L ratio, the difference between the allowable loads computed by using these two design methods depends on the end moment ratio as shown in Figure 26. For D/L > 0.3, the larger the end moment ratio, the higher the allowable load ratio. For example, for D/L = 0.5, the $(P_T)_{LRFD}/(P_T)_{ASD}$ ratios are 1.066 and 1.044 for M_T/M_{us} = 0.3 and 0.1, respectively.

Figure 27 shows the allowable load ratio based on Eqs. (6.20) and (6.31) versus dead-to-live load ratio for the same I-section used in Figure 26. Figure 27 is based on failure at the braced points which corresponds to Eqs. (6.20) and (6.31). For D/L = 0.05,

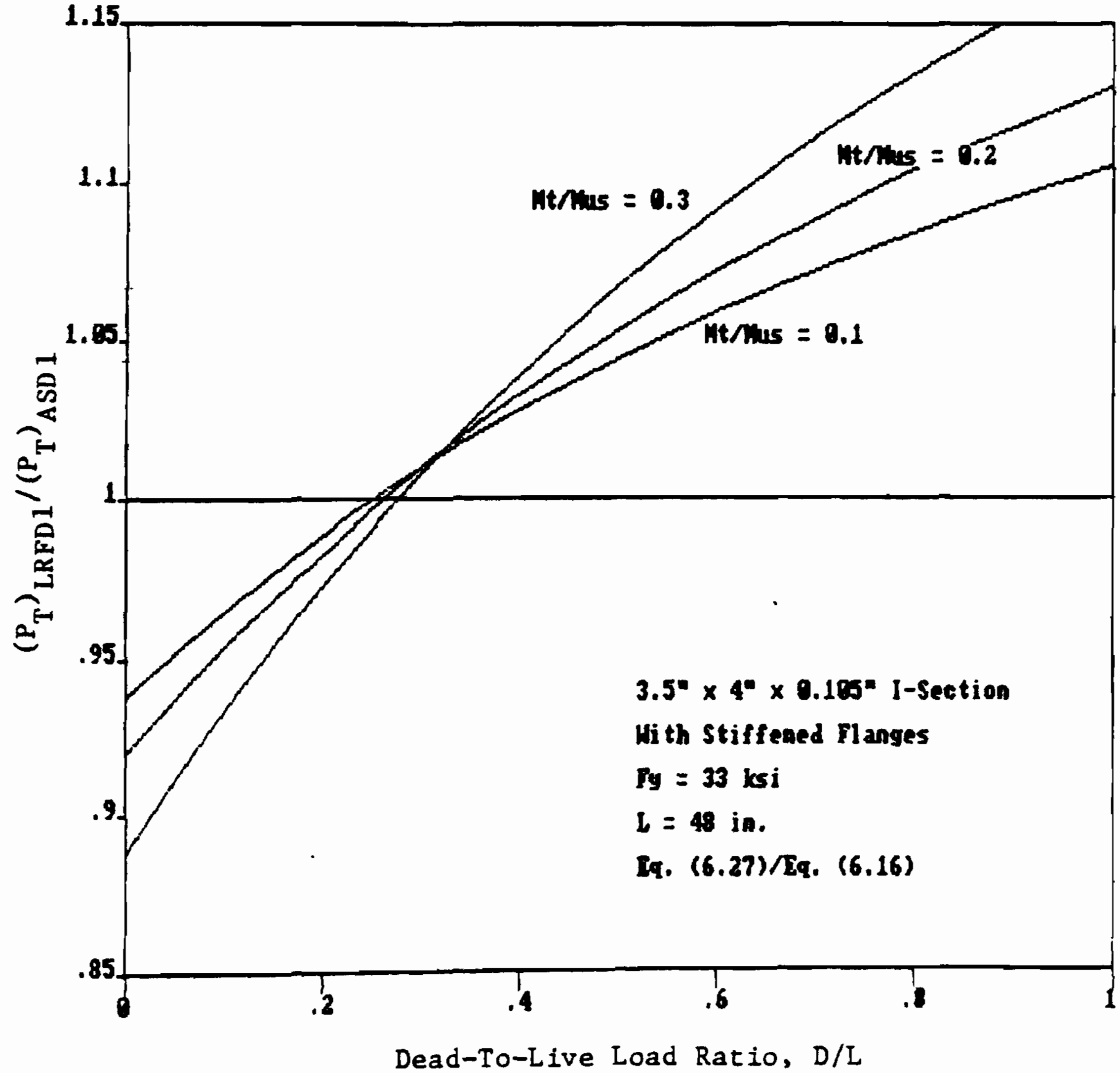


Figure 26. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case A

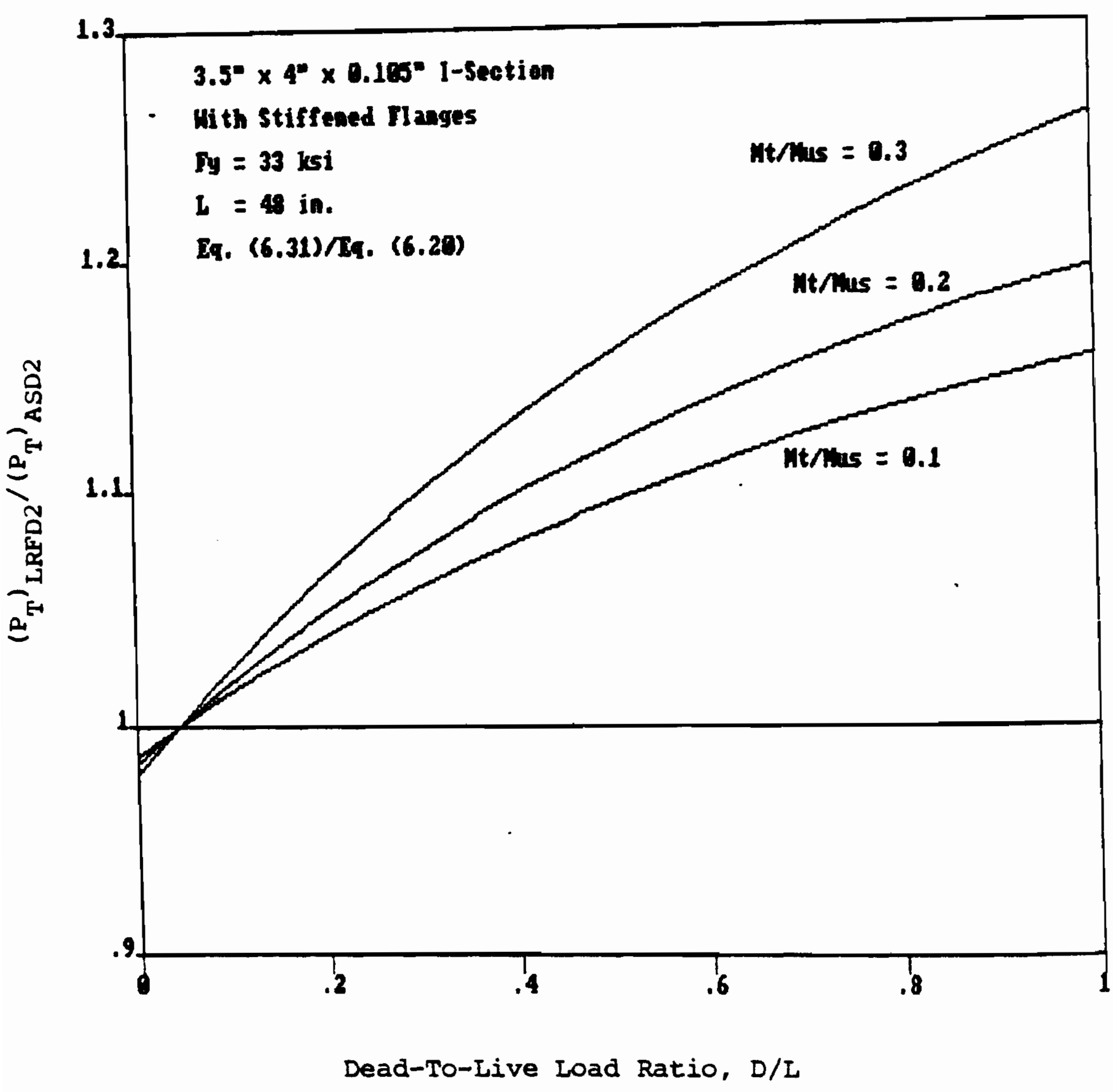


Figure 27. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case B

both design methods would result in the same allowable axial load for the end moment ratios shown in the figure. For other values of the D/L ratio, the end moment ratios would affect the allowable load ratio as shown in Figure 27.

Figures 28 and 29 show the relationships between the allowable load ratios and dead-to-live load ratios for end moment ratios of 0.2 and 0.3, respectively. The different curves in each figure represent different lengths of the 3.5 in. x 4 in. x 0.105 in. D/L = 0.5, ASD would provide conservative values up to 12% for column lengths from 4 ft increased to 9 ft as compared with the LRFD method. For the same column lengths and an end moment ratio of 0.3, ASD would be conservative (6.6% to 14%) as compared with the LRFD method for D/L = 0.5.

The relationship between the allowable load ratio and column length is shown in Figures 28 and 29 for various D/L ratios. Figures 30 and 31 show the allowable load ratio versus the slenderness ratio, KL/r_y, for end moment ratios of 0.2 and 0.3, respectively. Each curve in the figure represents a different D/L ratio for the same I-section used in Figures 26 through 29. As shown in these two figures, the allowable load ratio increases with increasing slenderness ratios for all D/L ratios. These two figures also show that for the D/L ratios between 0.2 and 0.5, the LRFD method would permit a slightly larger load than the ASD method when KL/r_y exceeds 68.

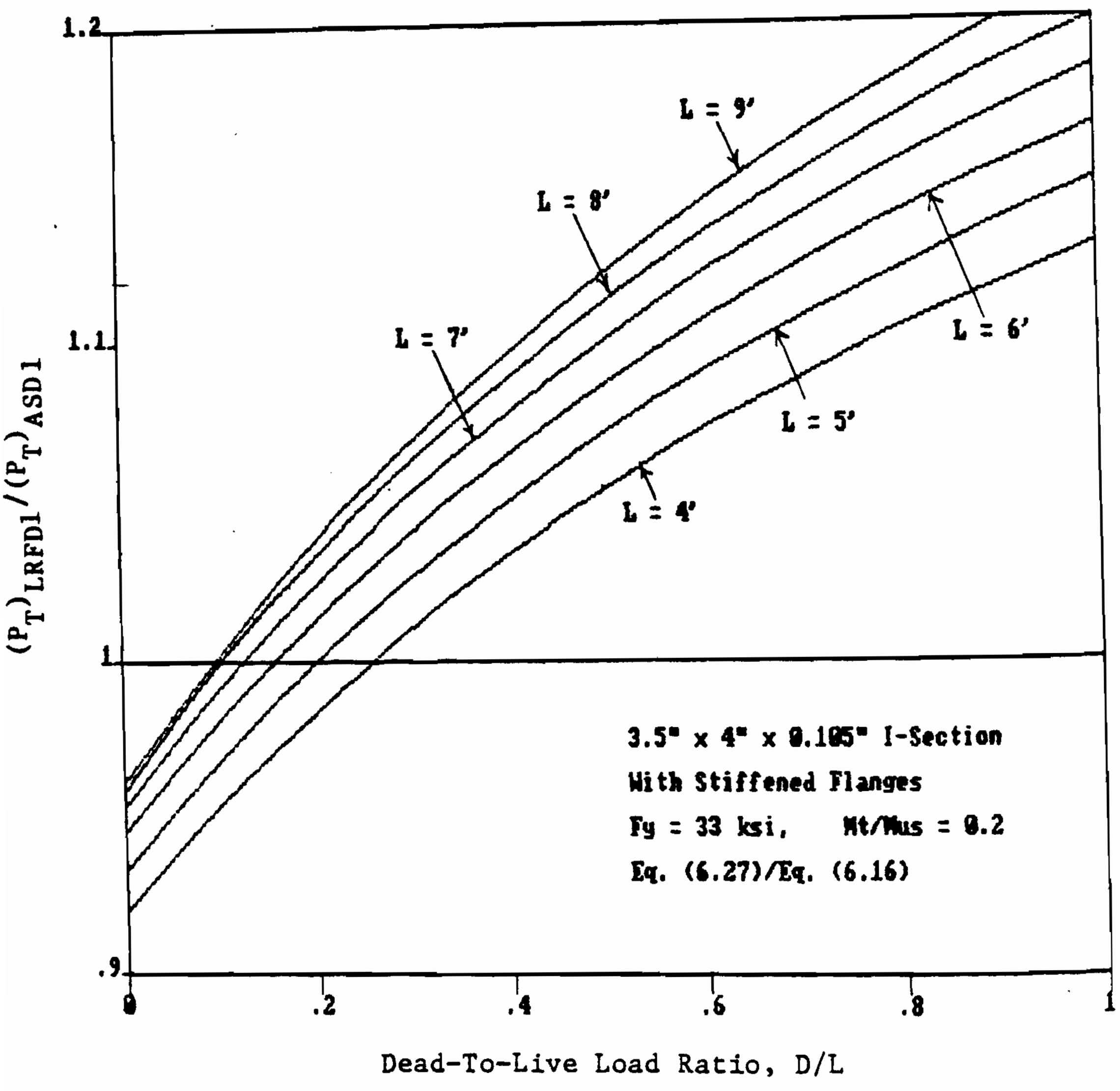


Figure 28. Allowable Load Ratio vs D/L Ratio for Beam-Columns-Case C

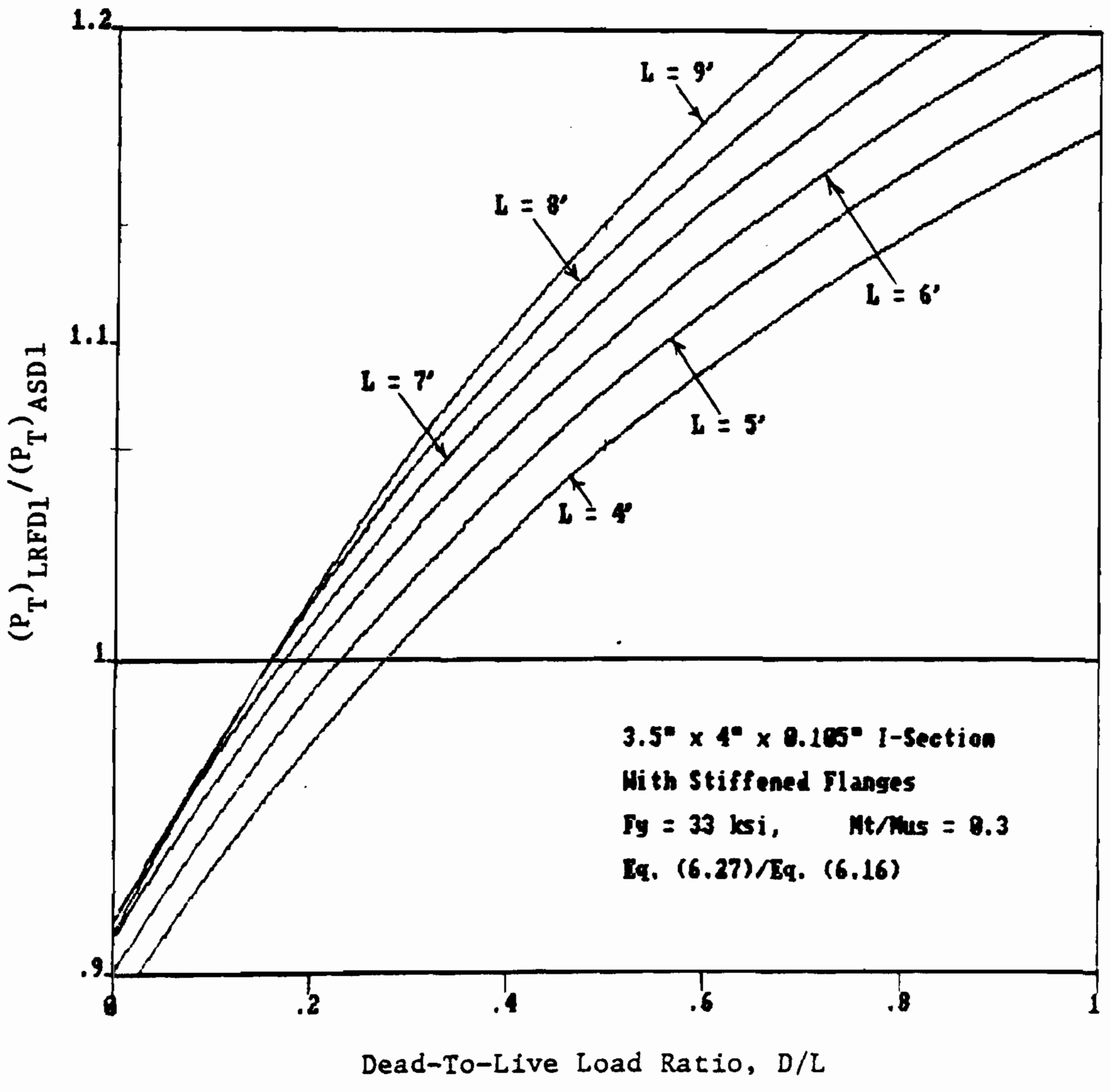


Figure 29. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case D

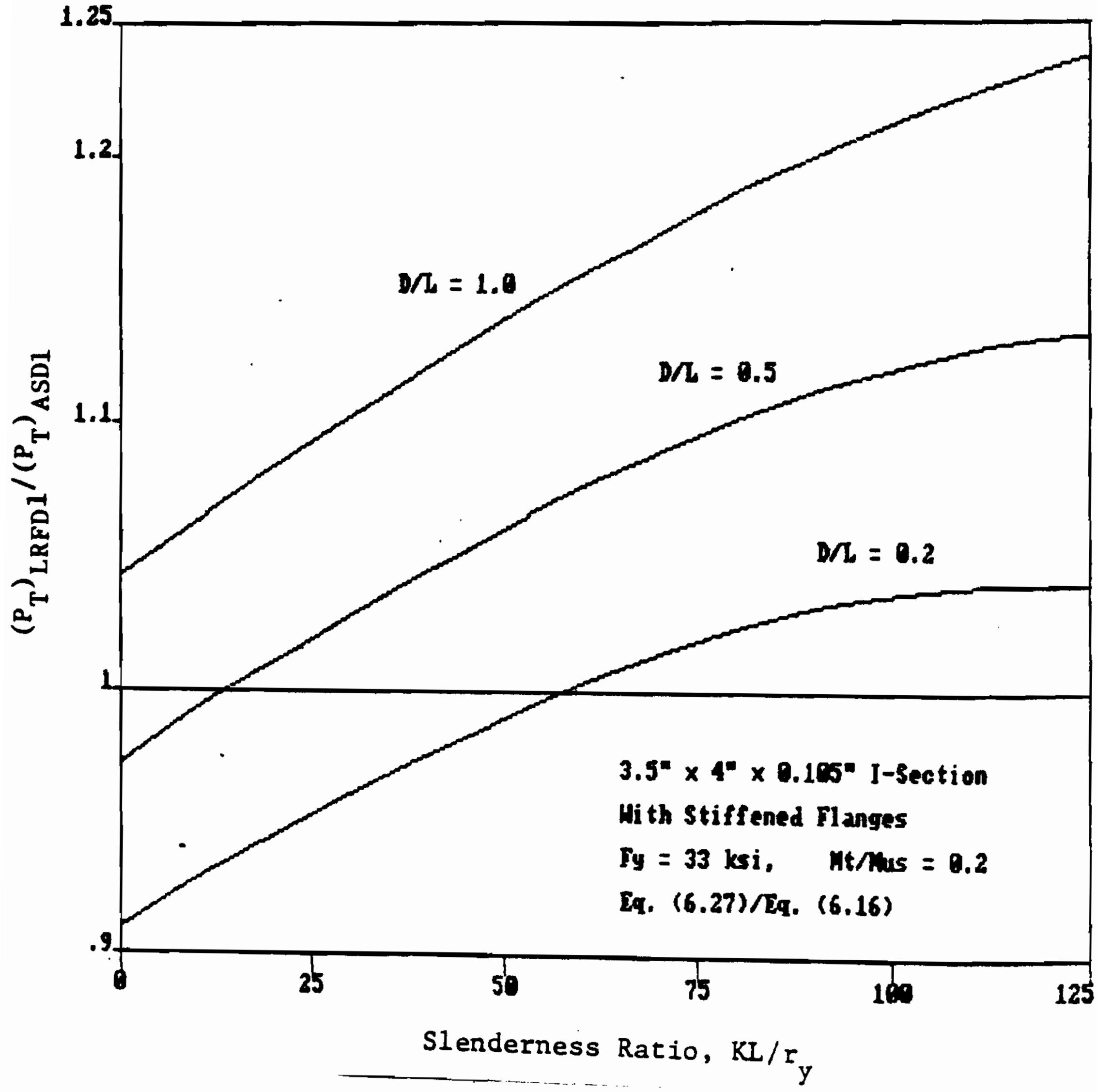


Figure 30. Allowable Load Ratio vs. Slenderness Ratio for Beam-Columns-Case C

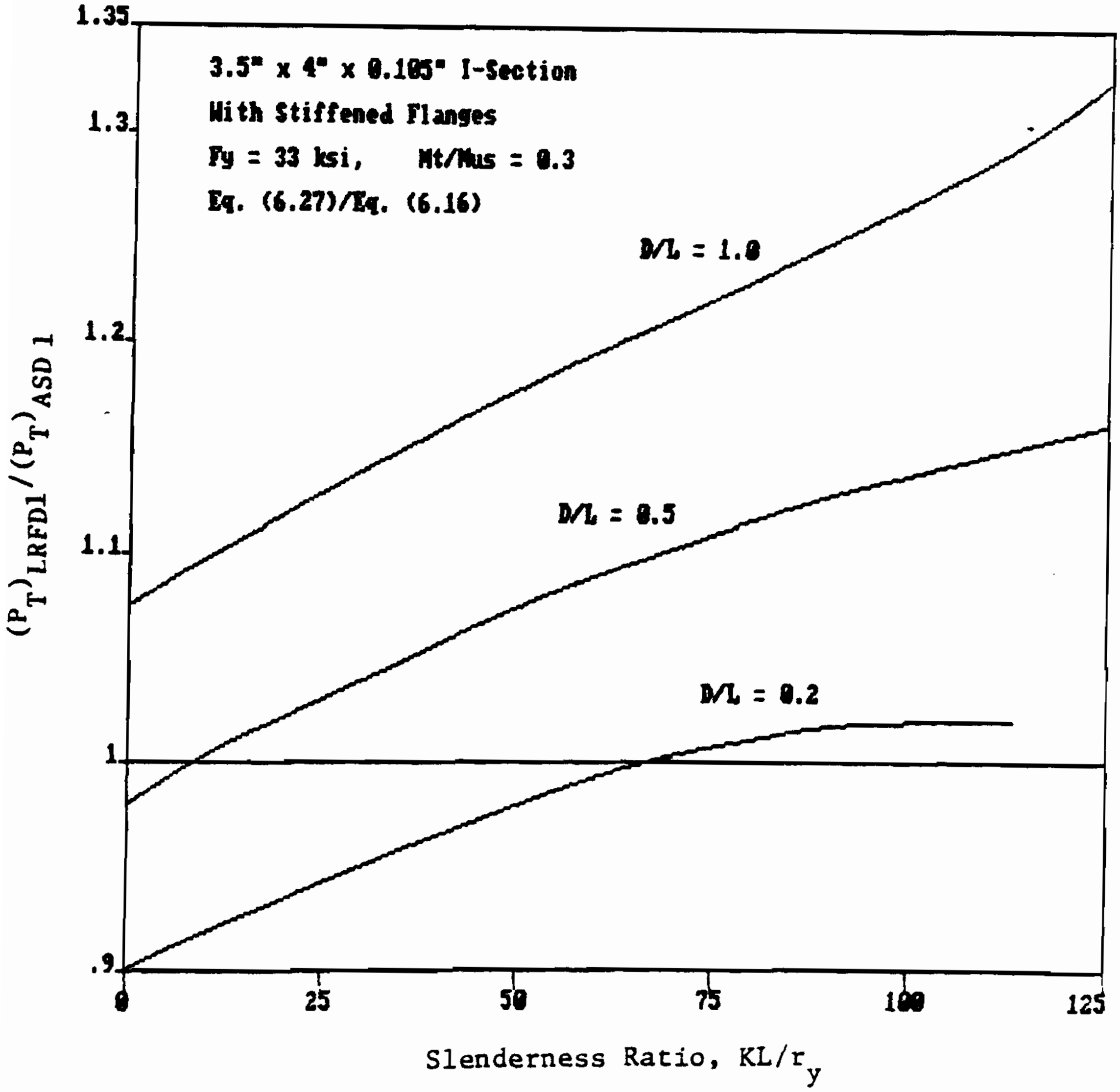


Figure 3I. Allowable Load Ratio vs. Slenderness Ratio for Beam-Columns-Case D

A deeper I-section (6 in. x 5 in. x 0.105 in.) with stiffened flanges was also studied for a length of 5 ft. Figure 32 shows the allowable load ratio based on Eqs. (6.16) and (6.27) versus dead-to live load ratio for various end moment ratios. This figure is also based on flexural failure at the midlength of the beam-column which governs the design for this case. The curves without triangular symbols are for C_m = 1.0. They are similar to those shown in Figure 26 for the 4 in. deep I-section execpt that the values of the allowable load ratio are about 7.5% more than the values shown in Figure 26. For this case, the yield point of steel would not affect the allowable load ratio. For D/L = 0.5 and M_T/M_{us} = 0.1, the allowable load computed from LRFD is 11.6% greater than the value determined from allowable stress design. However, for D/L = 0.5 and M_T/M_{us} = 0.3, the allowable load computed from LRFD is 13.4% higher than the value computed from allowable stress design.

The curves with triangular symbols in Figure 32 are for the same I-section execpt that the coefficient, C_m, is 0.85. The value of 0.85 is used for unbraced beam-columns and beam-columns with restrained ends subject to transverse loading between its supports. For small end moment ratios, the C_m value has a negligible effect on the allowable load ratio. The effect of C_m on the allowable load ratio increases as the end moment ratio increases as shown in Figure 32. It can be seen that for D/L < 1/3, the allowable load ratio computed for C_m = 0.85 is larger than that for C_m = 1.0.

Figure 33 shows the relationship between allowable load ratio and dead-to-live load ratio for the 6 in. deep I-section used in

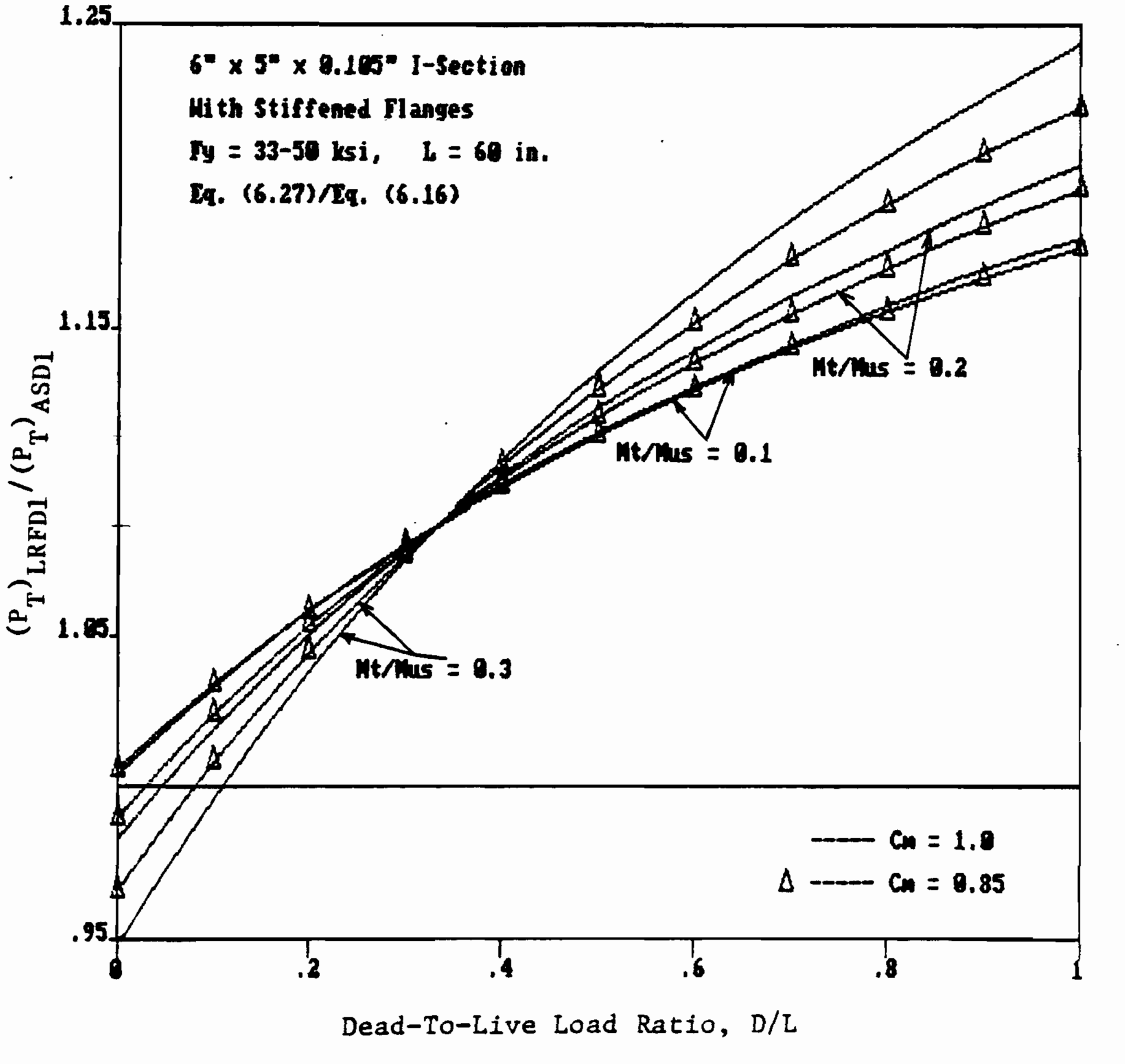


Figure 32. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case E

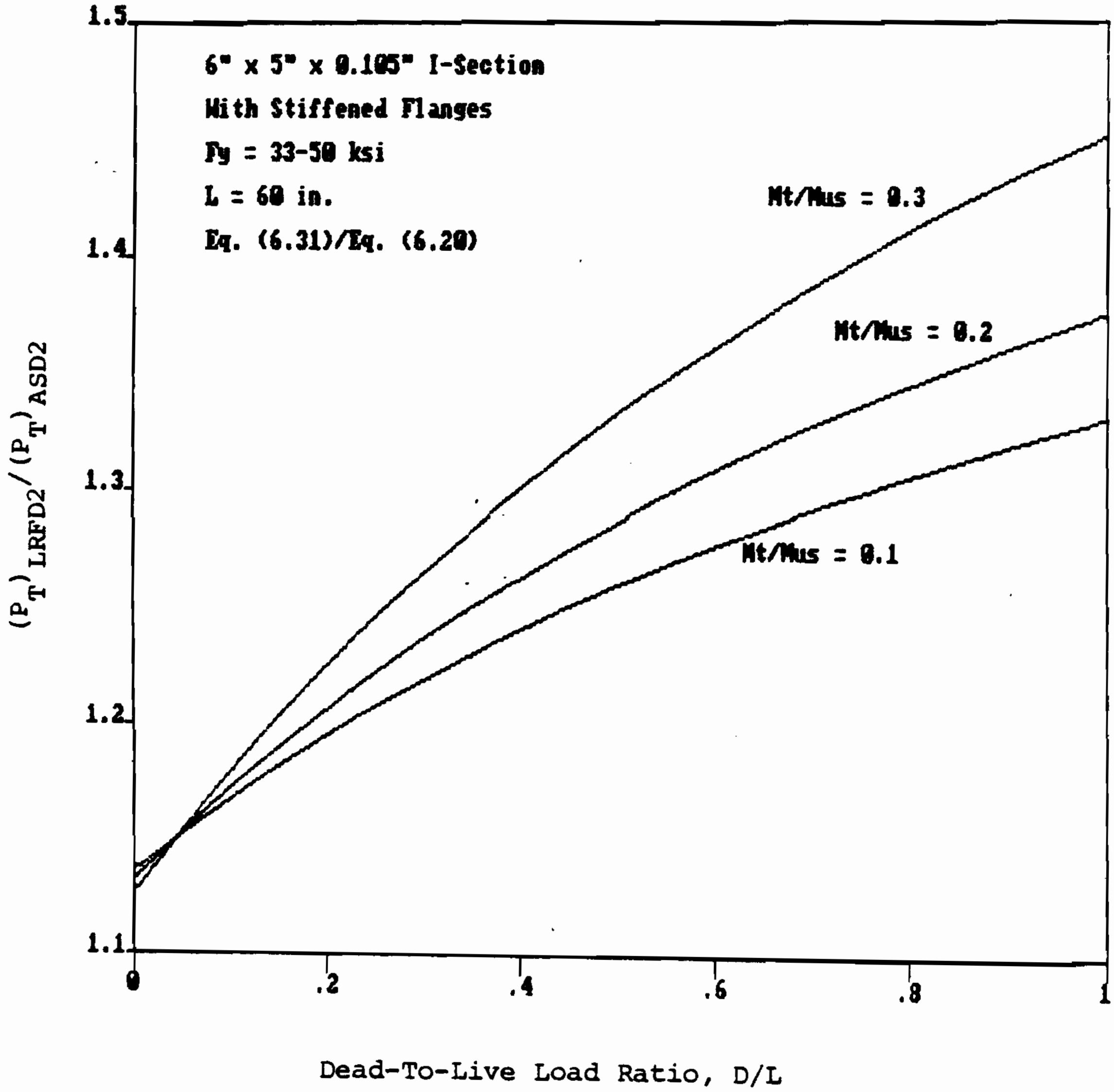


Figure 33. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case F

Figure 32 with a consideration of flexural failure at the braced points. This figure is similar to Figure 27 for the 4 in. deep I-section except that the values of the allowable load ratio are about 15% larger than the values computed for the smaller I-section. The curves shown in Figure 33 are applicable for yield points ranging from 33 to 50 ksi and all values of C_m.

I-sections with unstiffened flanges were studied in a similar manner. Figure 34 shows the allowable load ratio versus dead-to-live load ratio for an I-section (4 in. x 2.25 in. x 0.105 in.) having unstiffened flanges with F_y = 33 ksi and an effective column length of 4 ft. This figure is based on flexural failure at the midlength of the beam-column which would govern the design in this case. The allowable load ratio was determined from Eqs. (6.16) and (6.27). Figure 34 is similar to Figure 26 prepared for an I-section with stiffened flanges. For D/L = 0.5 and M_T/M_{us} = 0.1, the allowable load obtained from LRFD is 12% larger than the value obtained from allowable stress design. For D/L = 0.5 and M_T/M_{us} = 0.3, LRFD would result in an allowable load 15% higher than the value determined from allowable stress design.

Figure 35 shows the relationship between the allowable load ratio and dead-to-live load ratio for the same I-section used in Figure 34 by considering flexural failure at the braced points. Equations (6.20) and (6.31) are used for this type of failure. This figure is similar to Figure 27 which was prepared for an I-section of same depth with stiffened flanges. Both design methods result in the same allowable load for D/L = 0.05. For D/L = 0.5, the allowable

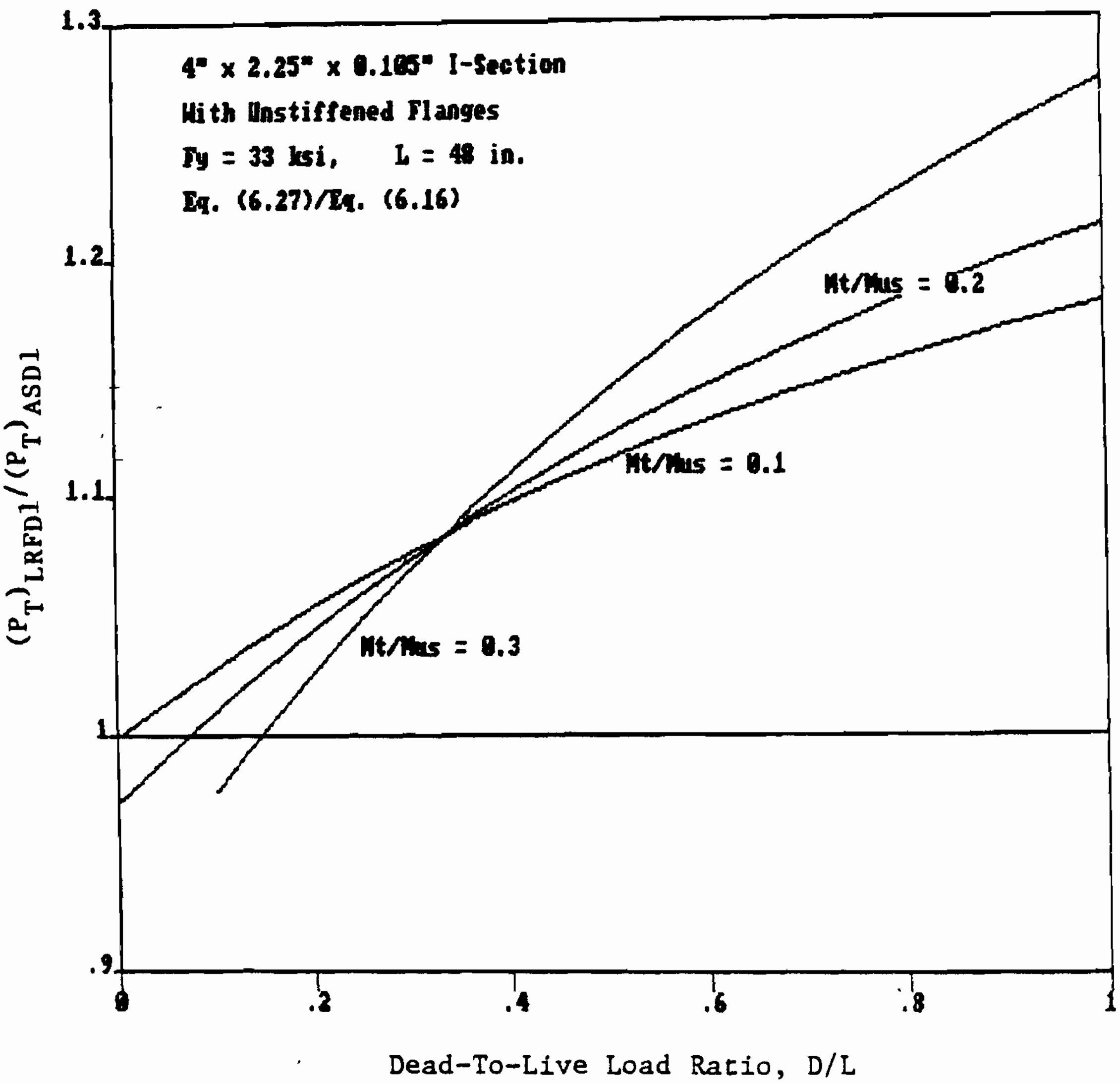


Figure 34. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case G

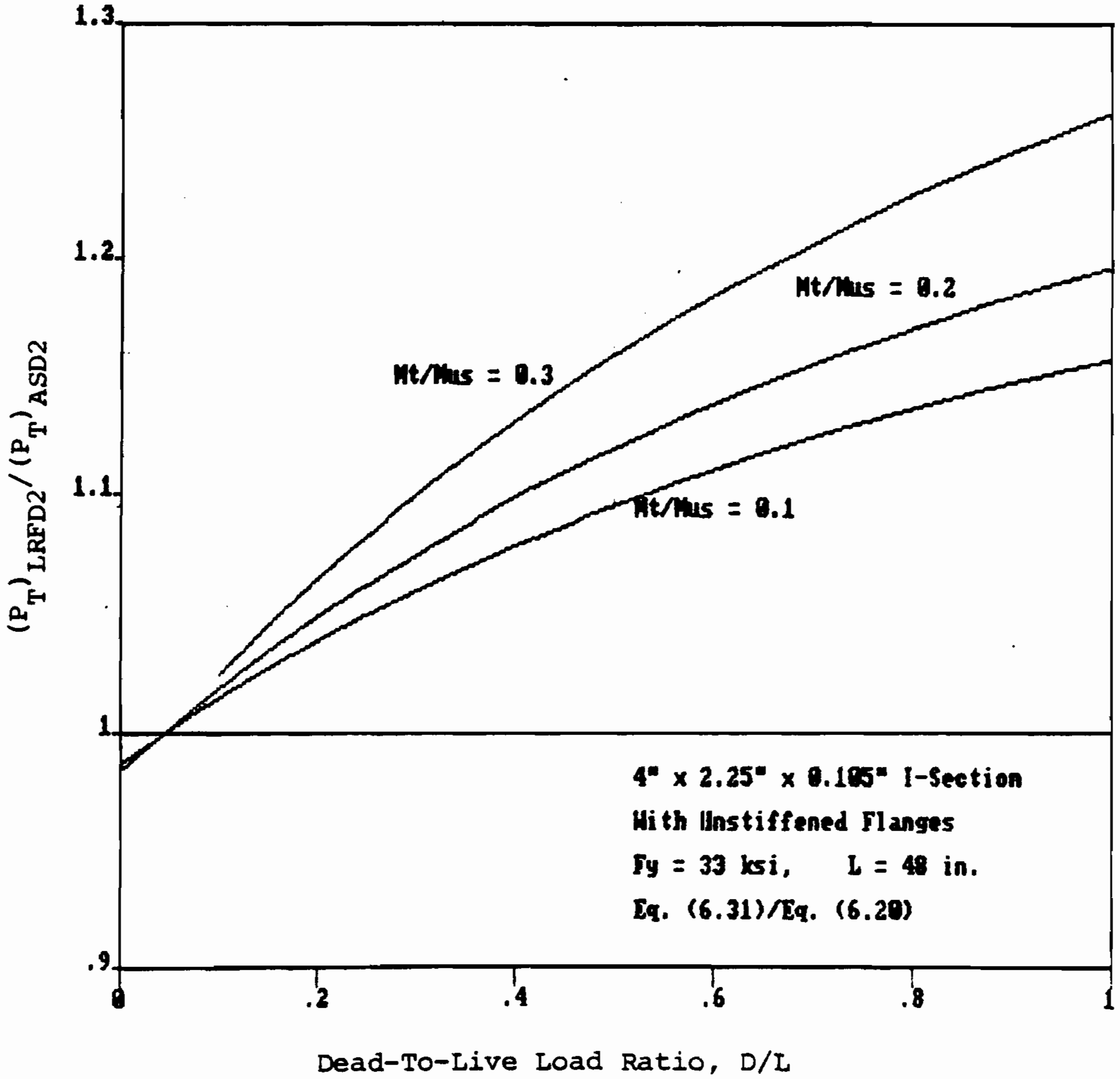


Figure 35. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case H

load obtained from LRFD is from 9.4% to 16% greater than the allowable load determined from allowable stress design for end moment ratios from 0.1 to 0.3.

Figures 36 and 37 show the allowable load ratio versus dead-to-live load ratio for end moment ratios of 0.1 and 0.2, respectively. Different curves represent different lengths of the I-section (4 in. x 2.25 in. x 0.105 in.) with F_y =33 ksi. It is noted that there is no clear pattern for the curves shown in Figures 36 and 37. For the values of M_T/M_{us} between 0.1 and 0.2 and D/L = 0.5, the allowable load values obtained from LRFD vary from 11.7% to 12.5% larger than the values obtained from the allowable stress design method.

Figure 38 shows the relationship between the allowable load ratio and the slenderness ratio, KL/r_y, for the same I-section used in previous figures and for an end moment ratio of 0.1. Each curve in the figure represents a different D/L ratio. The relationship in Figure 38 is similar to the relationship indicated in Figures 30 and 31 which are used in the study of I-sections with stiffened flanges. For D/L = 0.5 and 1.0, the allowable load ratio increases with increasing slenderness ratios. When the D/L ratio is between 0.2 and 0.5, the LRFD method would permit a slightly larger load than the ASD method for $KL/r_y > 50$.

A deeper I-section (6 in. x 3 in. x 1.05 in.) with unstiffened flanges was also included in this study for a length of 5 ft. The relationship between the allowable load ratio and dead-to-live ratio

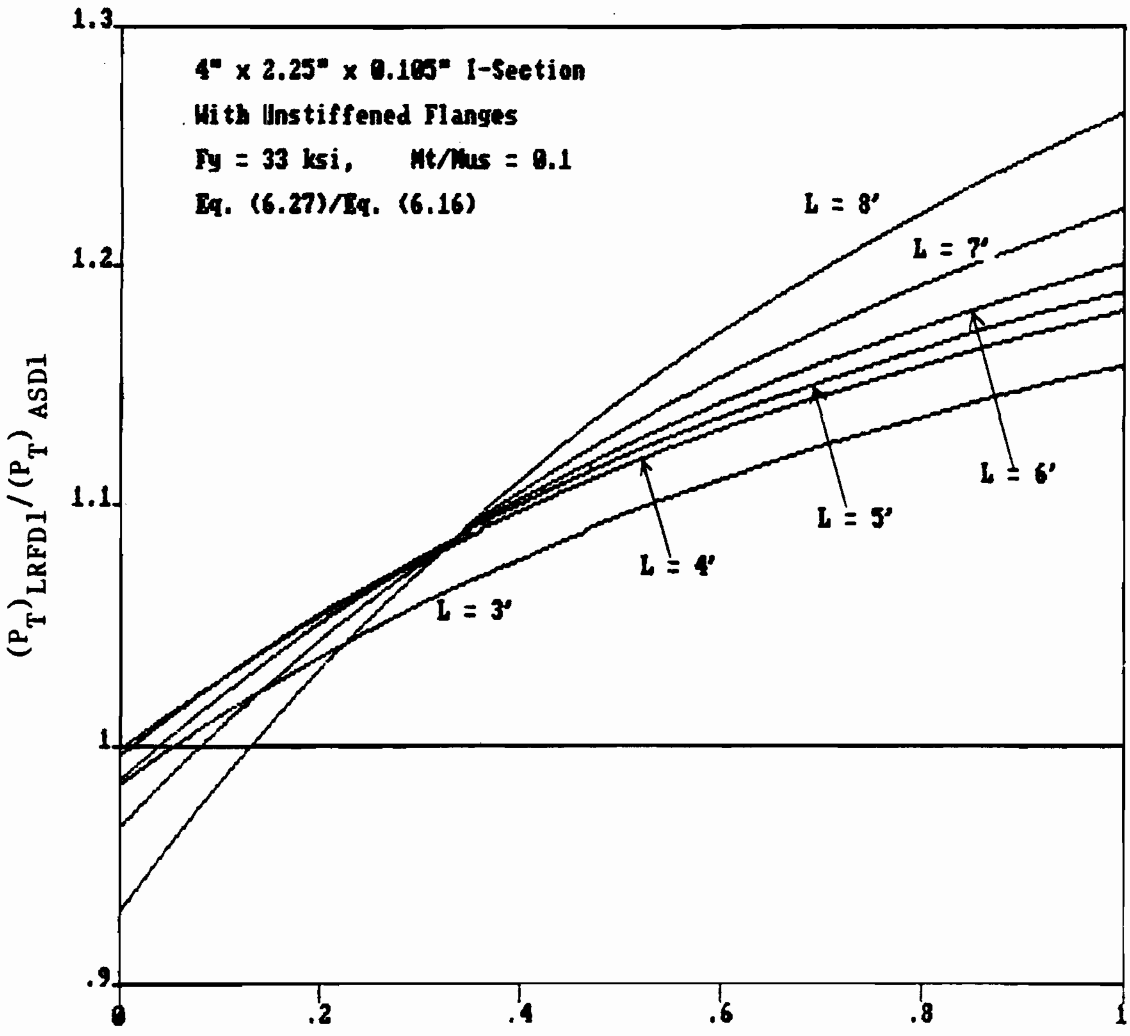


Figure 36. Allowable Load Ratio, D/L Ratio for Beam-Columns-Case I

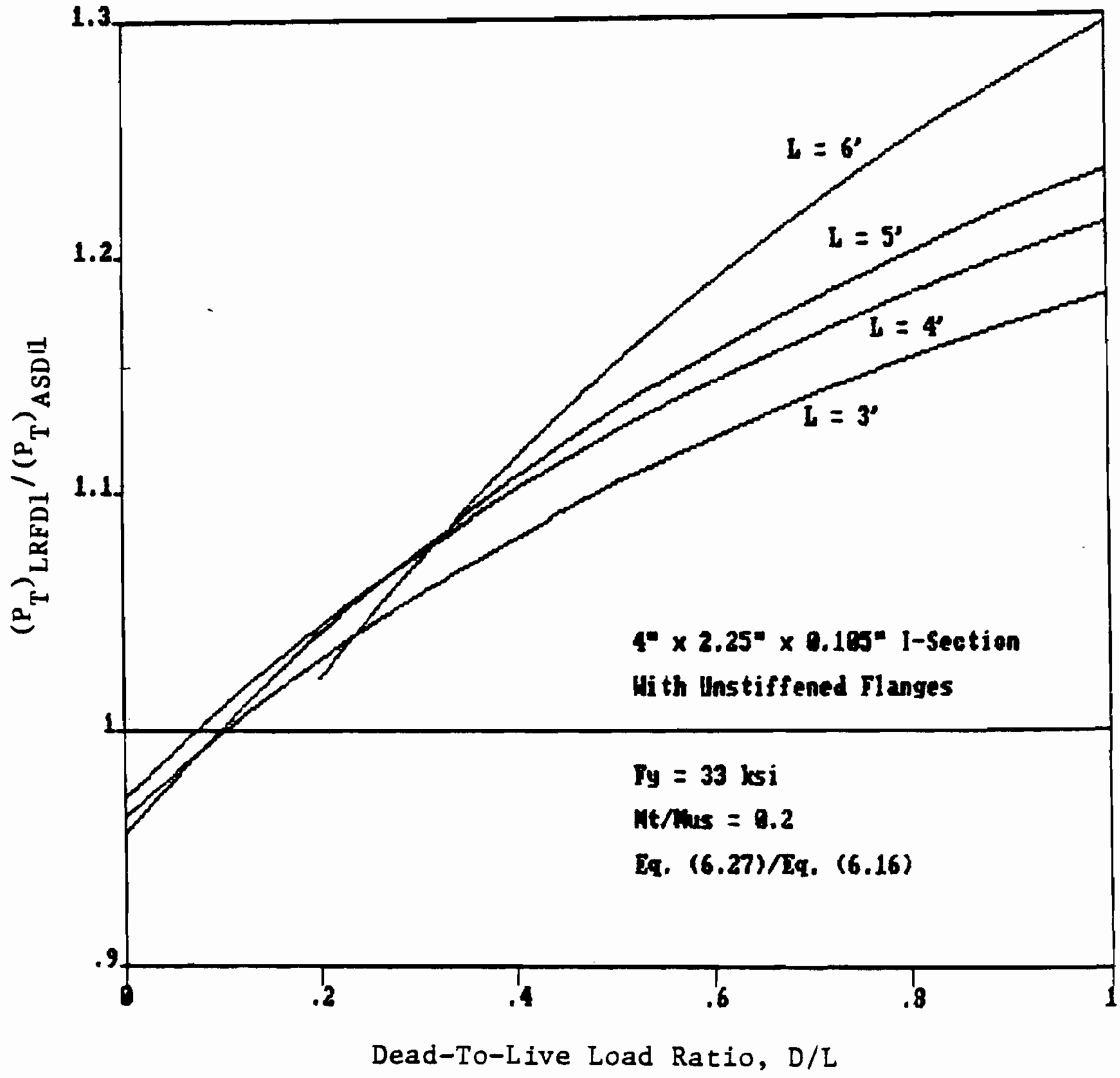


Figure 37. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case J

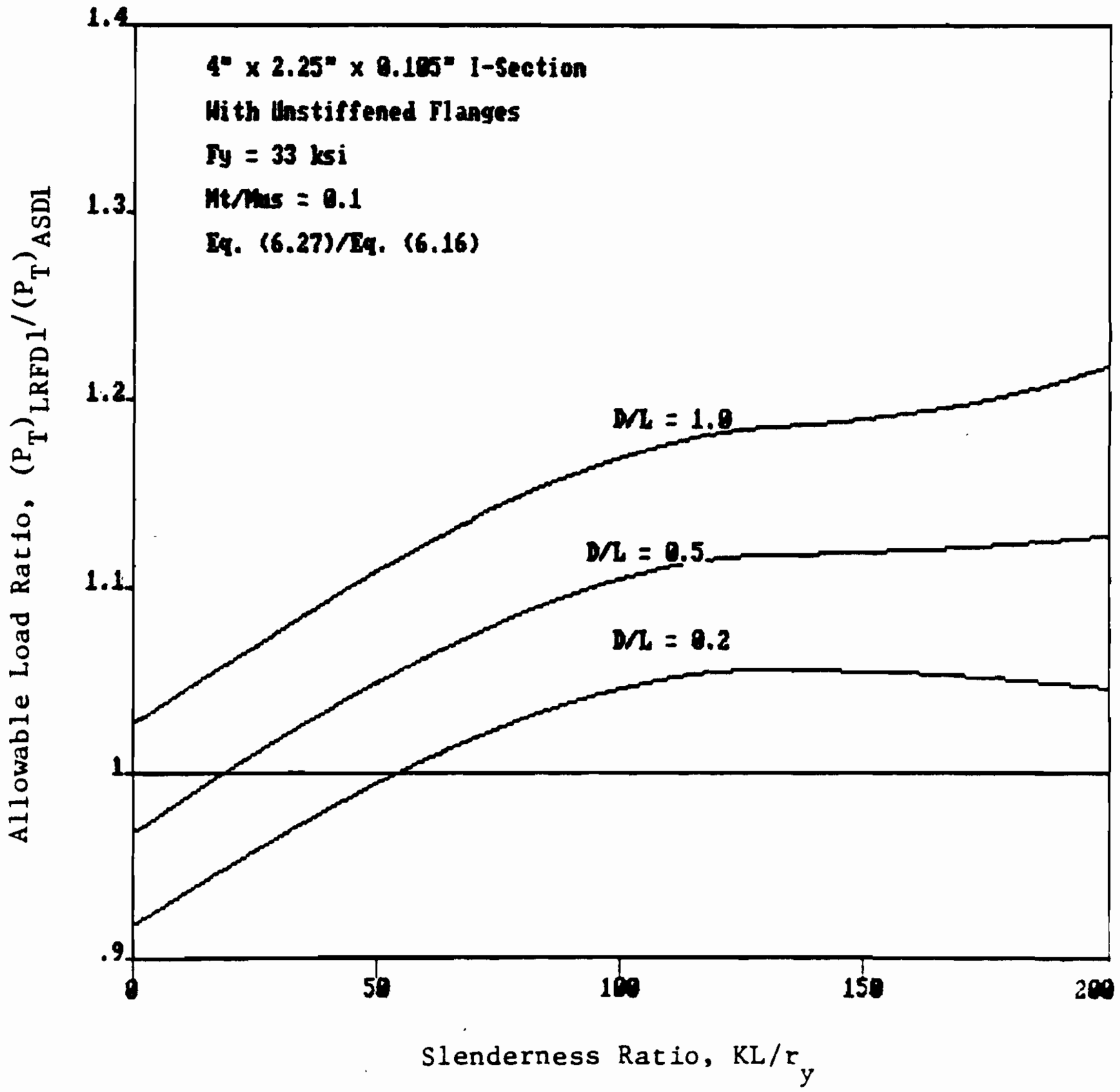


Figure 38. Allowable Load Ratio vs. Slenderness Ratio for Beam-Columns-Case I

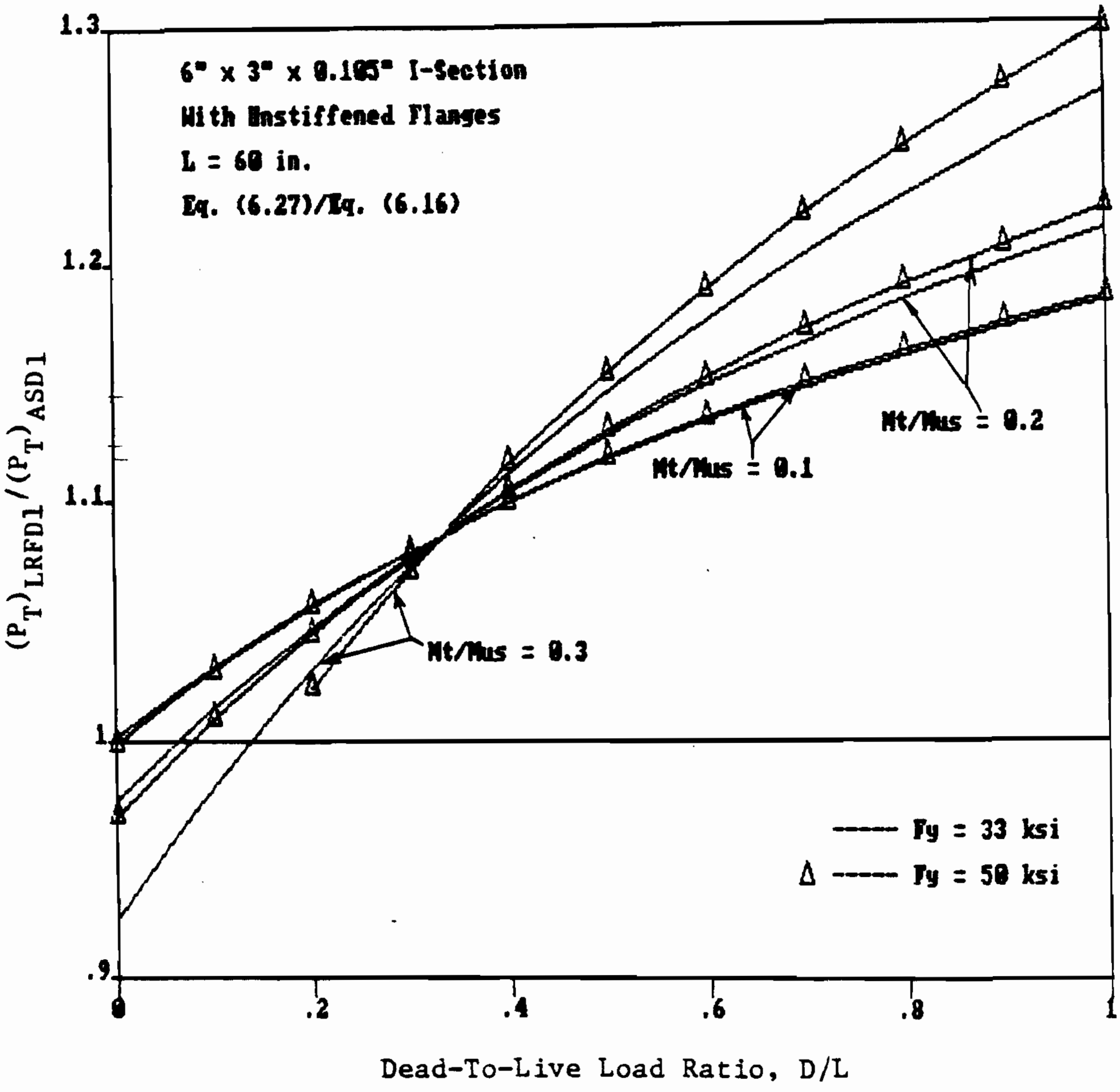


Figure 39. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case K

for the I-section is shown in Figure 39 for various end moment ratios. This figure is based on flexural failure at the midlength of the member. The curves computed for F_y = 33 ksi are similar to the curves shown in Figure 32 obtained for an I-section with stiffened flanges. For D/L = 0.5, the allowable load ratio varies from 1.12 to 1.14 for M_T/M_{us} ratios ranging from 0.1 to 0.3.

The lines with triangular symbols in Figure 39 represent the allowable load ratios determined for the same I-section by using F_y = 50 ksi. It can be seen that the allowable load ratios computed for F_y = 50 ksi are lower than that computed for F_y = 33 ksi when D/L < 1/3. This effect would be negligible for beam-columns with small end moment ratios as shown in Figure 39. This comparison does not agree with the results of a study of I-sections with stiffened flanges, for which the yield point had no significant effect on the allowable load ratio for the I-section with stiffened flanges illustrated in Figure 32.

Figure 40 shows how the C_m coefficient affects the allowable load ratio for the I-section having unstiffened flanges. The curves without triangular symbols are plotted for C_m = 1.0. The lines with triangular symbols represent the allowable load ratios calculated by using C_m = 0.85. It should be noted that the relationship shown in Figure 40 is very similar to the relationship illustrated in Figure 32 obtained for an I-section with stiffened flanges. For D/L < 1/3, the allowable load ratios are larger for C_m = 0.85 as compared to the allowable load ratios computed with C_m = 1.0. In general, the effect of the C_m value on the allowable ratio is more important for beam-columns with large end moment ratios.

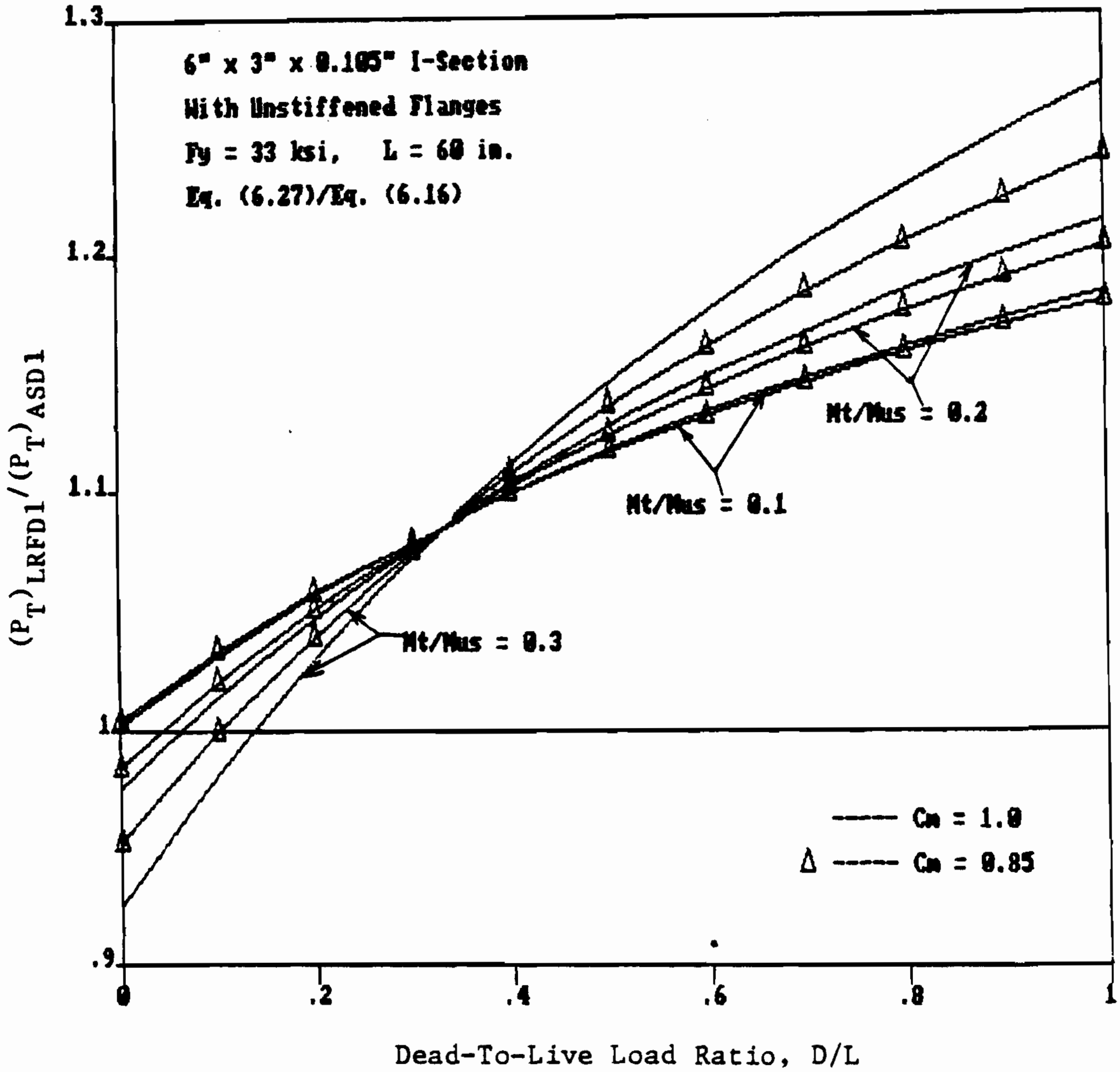


Figure 40. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case L

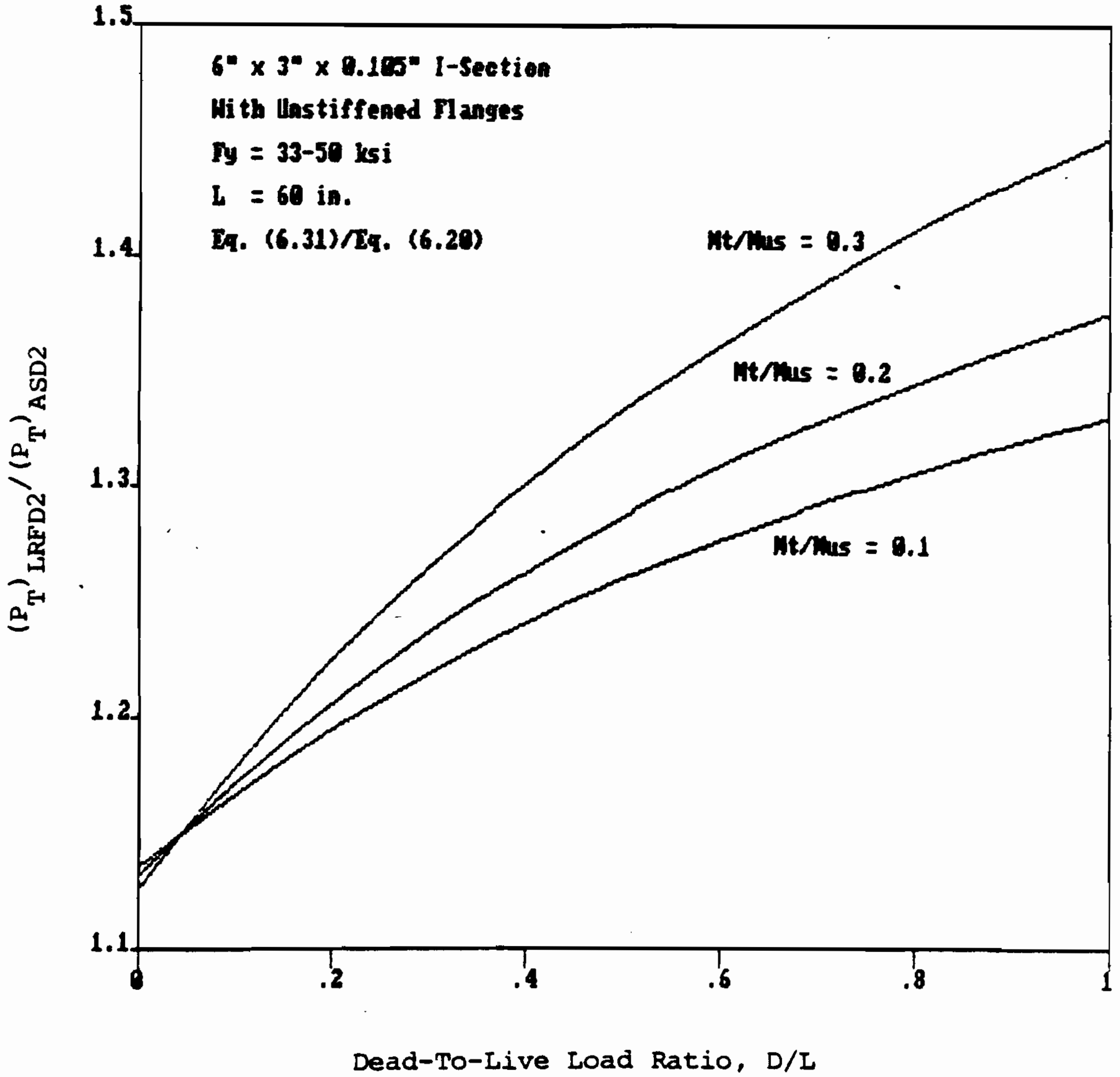


Figure 41. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case M

Figure 41 shows the allowable load ratio versus the dead-to-live load ratio for the same I-section used in Figures 39 and 40 but for flexural failure at the braced points. The relationship shown in this figure for an I-section with unstiffened flanges is similar to the relationship shown in Figure 33 for an I-section with stiffened flanges. For D/L = 0.5, the LRFD criteria result in a considerably larger allowable load than the value obtained from allowable stress design. For M_T/M_{us} ratios ranging from 0.1 to 0.3, the differences vary from 25.8% to 33.1%.

C. SINGLY-SYMMETRIC SHAPES

Singly-symmetric shapes will fail flexurally by yielding or local buckling or by torsional-flexural buckling when subjected to an eccentric compressive load or a combination of axial compression and bending.

1. Allowable Stress Design. According to Section 3.7.2 of the AISI Specifications,[1] singly-symmetric shapes subjected to both axial compression and bending applied in the plane of symmetry should be proportioned to meet the following four requirements as applicable:

(a)
$$\frac{f_a}{F_{a1}} + \frac{f_b C_m}{F_{b1}(1-f_a/F'_e)} \leq 1.0 \tag{6.34}$$

$$\frac{f_a}{F_{ao}} + \frac{f_b}{F_{b1}} \leq 1.0 \tag{6.35}$$

When $f_a/F_{a1} \leq 0.15$, the following formula may be used in lieu of the above two formulas:

$$\frac{f_a}{F_{a1}} + \frac{f_b}{F_{b1}} \leq 1.0 \tag{6.36}$$

(b) If the point of application of the eccentric load is located on the side of the centroid opposite from that of the shear center, i.e., if e is positive, then the average compression stress, f_a, also shall not exceed F_a given below:

For $\sigma_{TF} > 0.5QF_y$,

$$F_a = 0.522QF_y - (QF_y)^2/(7.67\sigma_{TF}) \qquad (6.37)$$

For $\sigma_{TF} \leq 0.5QF_y$,

$$F_a = 0.522\sigma_{TF} \qquad (6.38)$$

where σ_{TF} shall be determined according to the following formula:

$$\frac{\sigma_{TF}}{\sigma_{TFO}} + \frac{C_{TF}\sigma_{b1}}{\sigma_{bT}(1-\sigma_{TF}/\sigma_e)} = 1.0 \qquad (6.39)$$

(c) Except for T- or unsymmetric I-sections, if the point of application of the eccentric load is between the shear center and the centroid, i.e., if e is negative, and if F_{a1} is larger than F_{a2}, then the average compression stress, f_a, also shall not exceed F_a given below:

$$F_a = F_{a2} + (e/x_o)(F_{aE} - F_{a2}) \qquad (6.40)$$

(d) For T- and unsymmetric I-sections with negative eccentricities, (i) If the point of application of the eccentric load is between the shear center and the centroid, and if F_{a1} is larger than F_{a2}, then the average compression stress, f_a, also shall not exceed F_a given below:

$$F_a = F_{a2} + (e/x_o)(F_{ac} - F_{a2}) \qquad (6.41)$$

(ii) If the point of application of the eccentric load is located on the side of the shear center opposite from that of the centroid, then the average compression stress, f_a, also shall not exceed F_a given below:

For $\sigma_{TF} > 0.5QF_y$,

$$F_a = 0.522QF_y - (QF_y)^2/(7.67\sigma_{TF}) \tag{6.42}$$

For $\sigma_{TF} \leq 0.5QF_y$,

$$F_a = 0.522\sigma_{TF} \tag{6.43}$$

where σ_{TF} shall be determined according to the following formula:

$$\frac{\sigma_{TF}}{\sigma_{ex}} + \frac{C_{TF}}{\sigma_{bC}}\left[\frac{\sigma_{b1}}{1-\sigma_{TF}/\sigma_e} - \sigma_{b2}\right] = 1.0 \tag{6.44}$$

In this section, x and y are centroidal axes and the x-axis is the axis of symmetry whose positive direction is pointed away from the shear center. In the equations above,

C_{TF} = a coefficient whose value shall be taken as follows:

(a) For compression members in frames subject to joint translation (sidesway),

$C_{TF} = 0.85$

(b) For restrained compression members in frames braced against joint translation and not subject to transverse loading between their supports in the plane of bending,

$$C_{TF} = 0.6 - 0.4(M_1/M_2) \tag{6.45}$$

where M_1/M_2 is the ratio of the smaller to the larger moment at the ends of that portion of the member, unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature and negative when it is bent in single curvature.

F_a = maximum average allowable compression stress, ksi

F_{ac} = average allowable compression stress determined by both requirements (a) and (dii) if the point of application of the eccentric load is at the shear center, i.e., the calculated values of f_a and F_a, for $e = x_o$, ksi

F_{aE} = average allowable compression stress determined by requirement (a) if the point of application of the eccentric load is at the shear center, i.e., the calculated value of f_a for $e = x_o$, ksi

F_{a2} = allowable compression stress under concentric loading from Section 3.6.1.2 of Reference 1, ksi

σ_{TF} = average elastic torsional-flexural buckling stress, i.e., axial load at which torsional-flexural buckling occurs divided by the full cross-sectional area of member, ksi

$\sigma_{bC} = M_c c/I_y$ = maximum compression bending stress (6.46) caused by M_c, ksi. For I-sections with unequal flanges σ_{bC} may be approximated by $\pi^2 EdI_{xc}/(L^2 S_{yc})$

$\sigma_{bT} = M_t c/I_y$ = maximum compression bending stress (6.47) caused by M_t, ksi. For I-sections with unequal flanges σ_{bT} may be approximated by $\pi^2 EdI_{xc}/(L^2 S_{yc})$

$\sigma_{b1} = \sigma_{TF}\, ec/r_y^2$ = maximum compression bending stress in the section caused by σ_{TF}, ksi (6.48)

$\sigma_{b2} = \sigma_{TF} x_o c/r_y^2$, ksi (6.49)

$\sigma_e = \pi^2 E/(KL_b/r_b)^2$, ksi (6.50)

c = distance from the centroidal axis to the fiber with maximum compression stress, negative when the fiber is on the shear center side of the centroid, in.

d = depth of section, in.

e = eccentricity of the axial load with respect to the centroidal axis, negative when on the shear center side of the centroid, in.

$M_c = A\sigma_{ex}[j+\sqrt{j^2 + r_o^2(\sigma_t/\sigma_{ex})}\,]$ = elastic critical moment causing compression on the shear center side of the centroid, kip-in. (6.51)

$M_t = A\sigma_{ex}[j+\sqrt{j^2 + r_o^2(\sigma_t/\sigma_{ex})}\,]$ = elastic critical moment causing tension on the shear center side of the centroid, kip-in. (6.52)

$j = [\int_A x^3 dA + \int_A xy^2 dA]/(2I_y) - x_o$, in., where x is the axis of symmetry and y is orthogonal to x, in. (6.53)

I_{xc} = moment of inertia of the compression portion of a section about its axis of symmetry, in.^{4}

I_y = moment of inertia of the section about the y-axis, in.^{4}

2. LRFD Criteria. According to Section 9.5.2 of the Tentative Recommendations[10], singly-symmetric shapes subject to both axial compression and bending applied in the plane of symmetry should be proportioned to meet the following four requirements as applicable:

(a)
$$\frac{P_D}{\phi_c P_{uc}} + \frac{C_m M_D}{\phi_s M_{us}[1-P_D/(\phi_c P_{Ey})]} \leq 1.0 \tag{6.54}$$

$$\frac{P_D}{\phi_s P_{us}} + \frac{M_D}{\phi_s M_{us}} \leq 1.0 \tag{6.55}$$

when $P_D/(\phi_c P_{uc}) \leq 0.15$, the following formula may be used in lieu of the above two formulas:

$$\frac{P_D}{\phi_c P_{uc}} + \frac{M_D}{\phi_s M_{us}} \leq 1.0 \tag{6.56}$$

(b) If the point of application of the eccentric load is located on the side of the centroid opposite from that of the shear center, i.e., if e is positive, then

$$P_D \leq \phi_c P_u \tag{6.57}$$

In Eqs. (6.57), P_u is computed as follows:

For $\sigma_{TF} > 0.5QF_y$,

$$P_u = AQF_y[1-QF_y/(4\sigma_{TF})] \tag{6.58}$$

For $\sigma_{TF} \leq 0.50QF_y$,

$$P_u = A\sigma_{TF} \tag{6.59}$$

where σ_{TF} shall be determined according to the Eq. (6.39).

(c) Except for T- or unsymmetrical I-sections, if the point of application of the eccentric load is between the shear center and the centroid, i.e., if e is negative, and if P_{uc1} is larger than P_{uc2}, where P_{uc1} is determined from Section 9.4.1 of Reference 10 and P_{uc2} is determined from Section 9.4.2 of Reference 10, then the factored compressive load, P_D, also shall not exceed the following value:

$$P_D \leq \phi_c P_{uc2} + (e/x_o)(P_{DE} - \phi_c P_{uc2}) \qquad (6.60)$$

(d) For T- and I-sections with negative eccentricities

(i) If the point of application of the eccentric load is between the shear center and the centroid, and if P_{uc1} is larger than P_{uc2}, then the factored compressive load, P_D, also shall not exceed the following value:

$$P_D \leq \phi_c P_{uc2} + (e/x_o)(P_{DC} - \phi_c P_{uc2}) \qquad (6.61)$$

(ii) If the point of application of the eccentric load is located on the side of the shear center opposite from that of the centroid, then the factored compressive load, P_D, also shall not exceed $\phi_c P_u$ given below:

For $\sigma_{TF} > 0.5QF_y$,

$$P_u = AQF_y[1-QF_y/(4\sigma_{TF})] \qquad (6.62)$$

For $\sigma_{TF} \leq 0.5QF_y$,

$$P_u = A\sigma_{TF} \qquad (6.63)$$

where σ_{TF} shall be determined according to Eq. (6.44).

In this section, x and y are centroidal axes and the x-axis is the axis of symmetry whose positive direction is pointed away from the shear center. In the equations above,

P_{DC} = ultimate load determined by both requirements (a) and (dii) if the point of application of the eccentric load is at the shear center, i.e., the calculated values of P_D in requirement (a) and $\phi_c P_u$ in requirement (dii) for $e = x_o$, kips

P_{DE} = ultimate load determined by requirement (a) if the point of application of the eccentric load is at the shear center, i.e., the calculated value of P_D for $e = x_o$, kips

All other variables are defined in previous sections.

3. Comparison. The allowable eccentric axial loads were calculated for allowable stress design and LRFD. The applied end moments are a result of the eccentric axial loads and can be calculated using the following equation:

$$M_T = eP_T \tag{6.64}$$

Substitutions similar to the ones made to solve for the allowable loads of beam-columns with doubly-symmetric shapes in Section B of this chapter were used to solve for the allowable loads for members with singly-symmetric shapes.

Equation (6.34) for allowable stress design is based on flexural failure at the midlength of the beam-column. Equations (6.12), (6.14), (6.18), and (6.64) were substituted into Eq. (6.34) to obtain the following expression:

$$\frac{(F.S.)P_T}{P_{uc}} + \frac{eP_TC_m}{0.6M_{us}[1-(23/12)(P_T/P_{Ex})]} = 1.0 \qquad (6.65)$$

By solving for P_T in Eq. (6.65), the following equation for allowable load is obtained.

$$(P_T)_{ASD1} = \frac{1.0}{\frac{(F.S.)}{P_{uc}} + \frac{eC_m}{0.6M_{us}[1-(23/12)(P_T/P_{Ex})]}} \qquad (6.66)$$

Equation (6.66) requires a solution using iterations, since the allowable axial load is a function of the actual axial load, P_T.

Equation (6.35) for allowable stress design is based on flexural failure at the braced points. Equations (6.17),(6.18), and (6.64) were substituted into equation (6.35) to obtain the following expression:

$$\frac{(F.S.)P_T}{P_{us}} + \frac{eP_T}{0.6M_{us}} = 1.0 \qquad (6.67)$$

By solving for P_T in Eq. (6.67), the following equation for allowable load is obtained:

$$(P_T)_{ASD2} = \frac{1.0}{\frac{(F.S.)}{P_{us}} + \frac{e}{0.6M_{us}}} \qquad (6.68)$$

For allowable stress design, Eq. (6.36) is based on flexural failure when the effect of secondary moment is neglected. Equations (6.12),(6.18), and (6.64) were substituted into Eq. (6.36) to obtain the following expression:

$$\frac{(F.S.)P_T}{P_{uc}} + \frac{eP_T}{0.6M_{us}} = 1.0 \tag{6.69}$$

The following equation for allowable load is obtained by solving for P_T in Eq. (6.69):

$$(P_T)_{ASD3} = \frac{1.0}{\frac{(F.S.)}{P_{uc}} + \frac{e}{0.6M_{us}}} \tag{6.70}$$

For torsion-flexural failure, the allowable eccentric axial load based on allowable stress design can be computed using the following equation:

$$(P_T)_{ASD} = AF_a \tag{6.71}$$

Where the average allowable stress, F_a, can be computed from Eqs. (6.37) through (6.44), whichever is applicable.

For LRFD, Eq. (6.54) is based on flexural failure at the mid-length of the beam-column. Equations (6.23), (6.25), (6.29), and (6.64) were substituted into Eq. (6.54) to obtain the following expression:

$$\frac{1.2D/L+1.6}{D/L+1}\left[\frac{P_T}{\phi_c P_{uc}} + \frac{eP_T C_m}{\phi_s M_{us}\left(1- \frac{1.2D/L+1.6}{D/L+1}\,\frac{P_T}{\phi_c P_{Ey}}\right)}\right] = 1.0 \tag{6.72}$$

By solving for P_T in Eq. (6.72), the following equation for allowable load is obtained:

$$(P_T)_{LRFD1} = \frac{(D/L+1)/(1.2D/L+1.6)}{\frac{1}{\phi_c P_{uc}} + \frac{eC_m}{\phi_s M_{us}\left(1- \frac{1.2D/L+1.6}{D/L+1}\,\frac{P_T}{\phi_c P_{Ey}}\right)}} \tag{6.73}$$

Equation (6.73) requires a solution by using iterations, since the allowable axial load is also a function of the actual axial load.

Equation (6.55) for LRFD is based on flexural failure at the braced points. The following expression was obtained by substituting Eqs. (6.28), (6.29), and (6.64) into Eq. (6.55):

$$\frac{1.2D/L+1.6}{D/L+1}\left[\frac{P_T}{\phi_s P_{us}} + \frac{eP_T}{\phi_s M_{us}}\right] = 1.0 \qquad (6.74)$$

By solving for P_T in Eq. (6.74), the following equation for allowable load is obtained.

$$(P_T)_{LRFD2} = \frac{(D/L+1)/(1.2D/L+1.6)}{\dfrac{1}{\phi_s P_{us}} + \dfrac{e}{\phi_s M_{us}}} \qquad (6.75)$$

Equation (6.56) for LRFD is based on flexural failure when the effect of secondary moment is neglected. Equations (6.23), (6.29), and (6.64) were substituted into Eq. (6.56) to obtain the following expression:

$$\frac{1.2D/L+1.6}{D/L+1}\left[\frac{P_T}{\phi_c P_{uc}} + \frac{eP_T}{\phi_s M_{us}}\right] = 1.0 \qquad (6.76)$$

The following equation for allowable load was obtained by solving for P_T in Eq. (6.76):

$$(P_T)_{LRFD3} = \frac{(D/L+1)/(1.2D/L+1.6)}{\dfrac{1}{\phi_c P_{uc}} + \dfrac{e}{\phi_s M_{us}}} \qquad (6.77)$$

For torsional-flexural failure based on LRFD, the allowable eccentric axial load can be computed by using the following equation:

$$(P_T)_{LRFD} = \phi_c P_u \frac{D/L+1}{1.2D/L+1.6} \tag{6.78}$$

where $\phi_c P_u$ can be computed from Eqs. (6.57) through (6.63), whichever is applicable.

The equations to be used for the allowable eccentric axial load for allowable stress design and LRFD are very complex and utilize iterations with multiple variables and two failure modes. A computer program was used to calculate allowable axial loads for singly-symmetric shapes based on allowable stress design and LRFD criteria. The program, listed in Appendix B, computes allowable loads and allowable load ratios, $(P_T)_{LRFD}/(P_T)_{ASD}$, for various lengths and an array of eccentricities. Standard channel sections and their section properties used in this study, were obtained from Tables 1 and 2 of Part V of the AISI Cold-Formed Steel Design Manual[41].

A channel (4 in. x 2 in. x 0.105 in.) with stiffened flanges was studied as a beam-column subjected to an eccentric load applied at each end. Figure 42 show the allowable load ratio versus the eccentricity for the channel with an effective length of 5 ft, D/L = 0.5, and C_m = 1.0. From this figure, it can be seen that when the load is applied along the axis of symmetry between the centroid

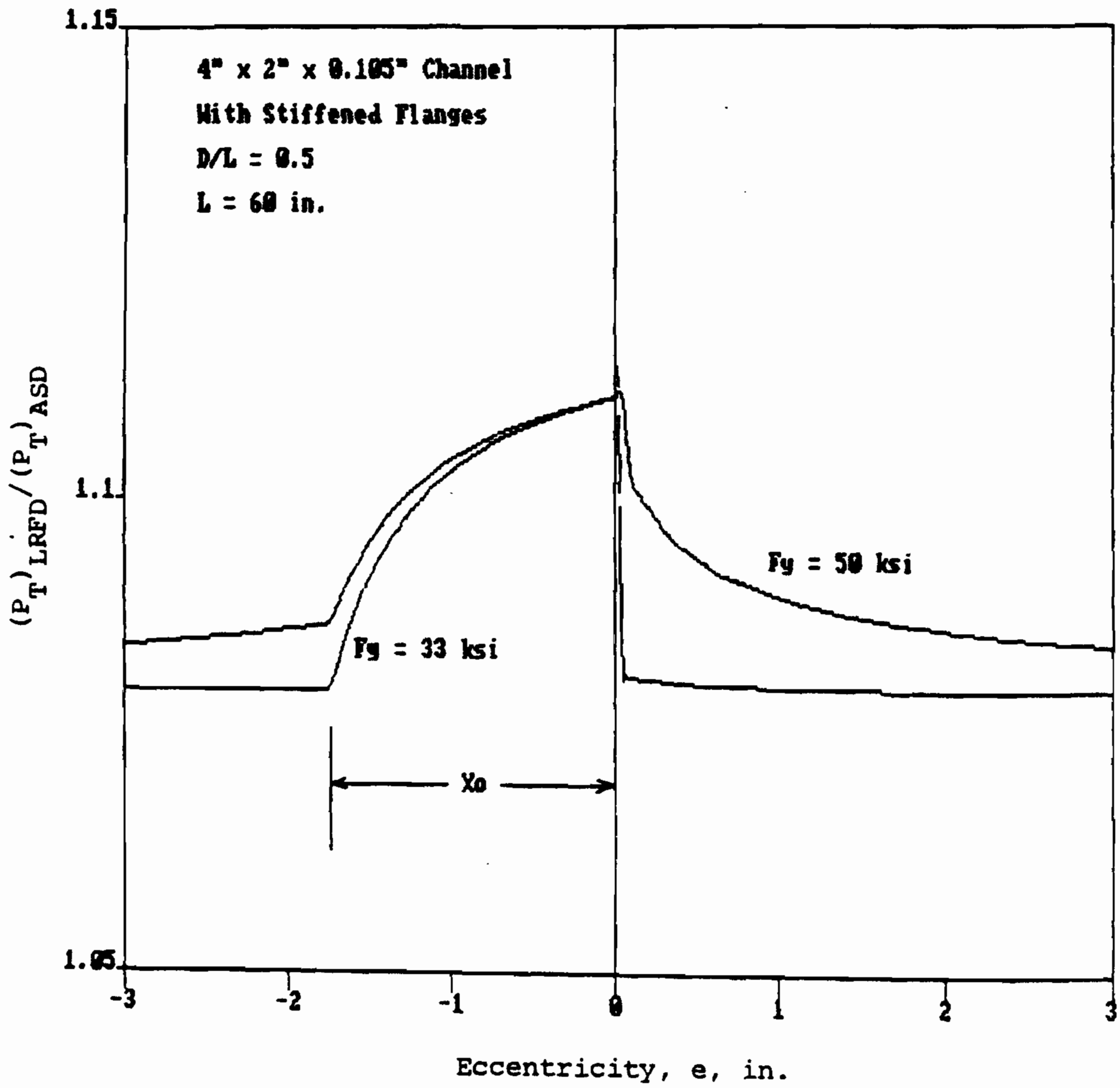


Figure 42. Allowable Load Ratio vs. Eccentricity for Beam-Columns-Case 1

and the shear center, the allowable load ratio is higher than the value computed for other eccentricities. The abrupt change in the curve at e = 0.04 in. is a result of the change of failure modes from torsional flexural to flexural buckling. For other eccentricities, the allowable load ratio is relatively a constant value and the allowable load determined from LRFD is 8.0% greater than the value obtained from allowable stress design for D/L = 0.5.

The top line in Figure 42 represents the same channel section with a yield point of 50 ksi. The allowable ratios in this case are slightly greater than that computed with F_y = 33 ksi for eccentricities greater than zero and less than x_o.

Figure 43 shows the relationship between the allowable load ratio and dead-to-live load ratio for the 4 in. deep channel with e = + 1.29 in. The two curves represent yield points of 33 and 50 ksi for the 5 ft long beam-column. The higher yield point steels result in slightly higher values of the allowable load ratio as seen in Figures 42 and 43. From the computer output, the value of F_y has a negligible effect on the allowable load ratio for the same channel with $x_o < e < 0$ and effective lengths greater than 6 ft.

Figure 44 shows the allowable load ratio versus slenderness ratio, KL/r_y, for the channel (4 in. x 2 in. x 0.105 in.) with stiffened flanges and D/L = 1/5. The curves represent yield points of 33 and 50 ksi for the channel with e = + 1.29 in. For F_y = 33 ksi, the allowable load ratio increases slightly as the slenderness

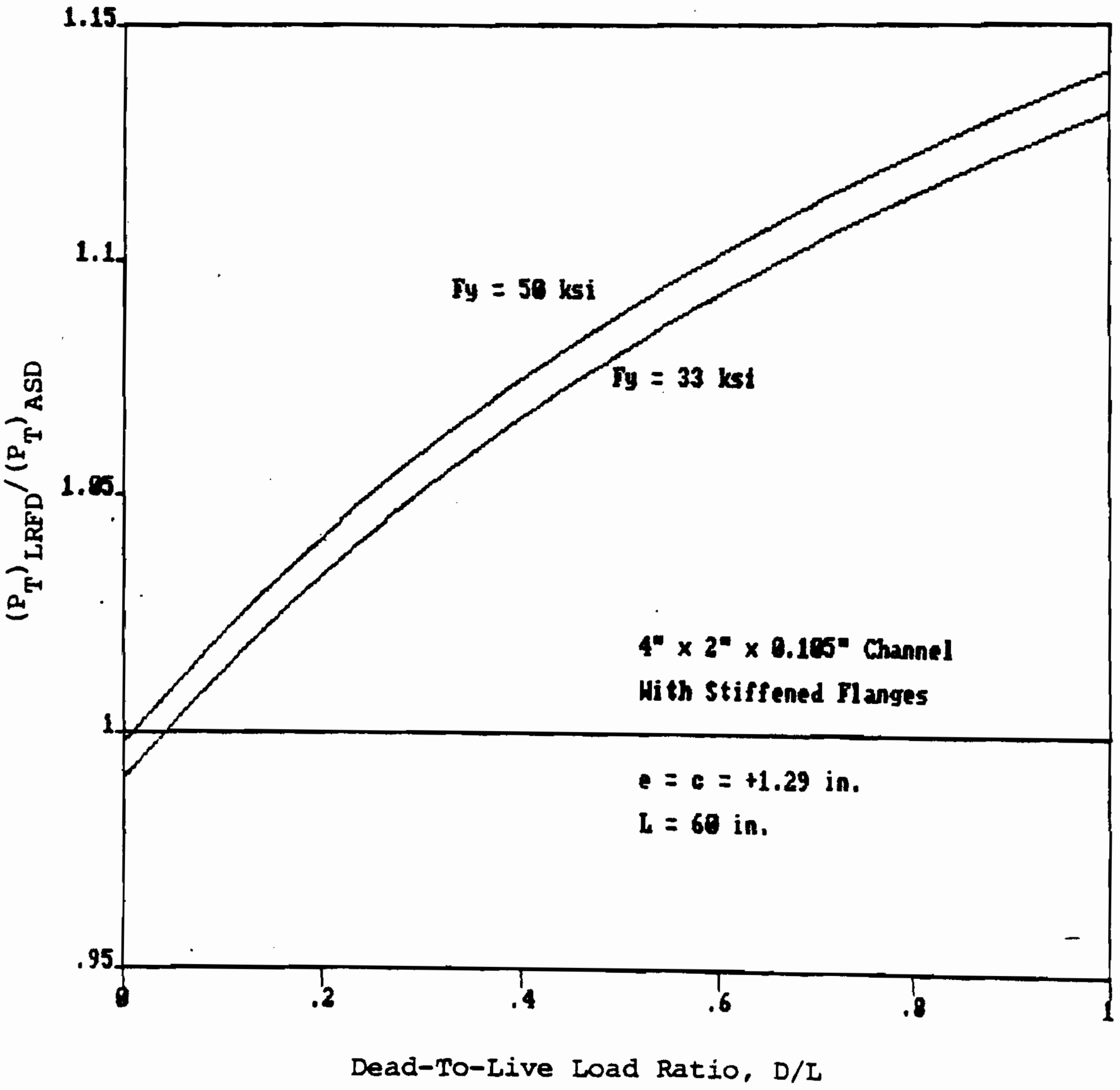


Figure 43. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case 1

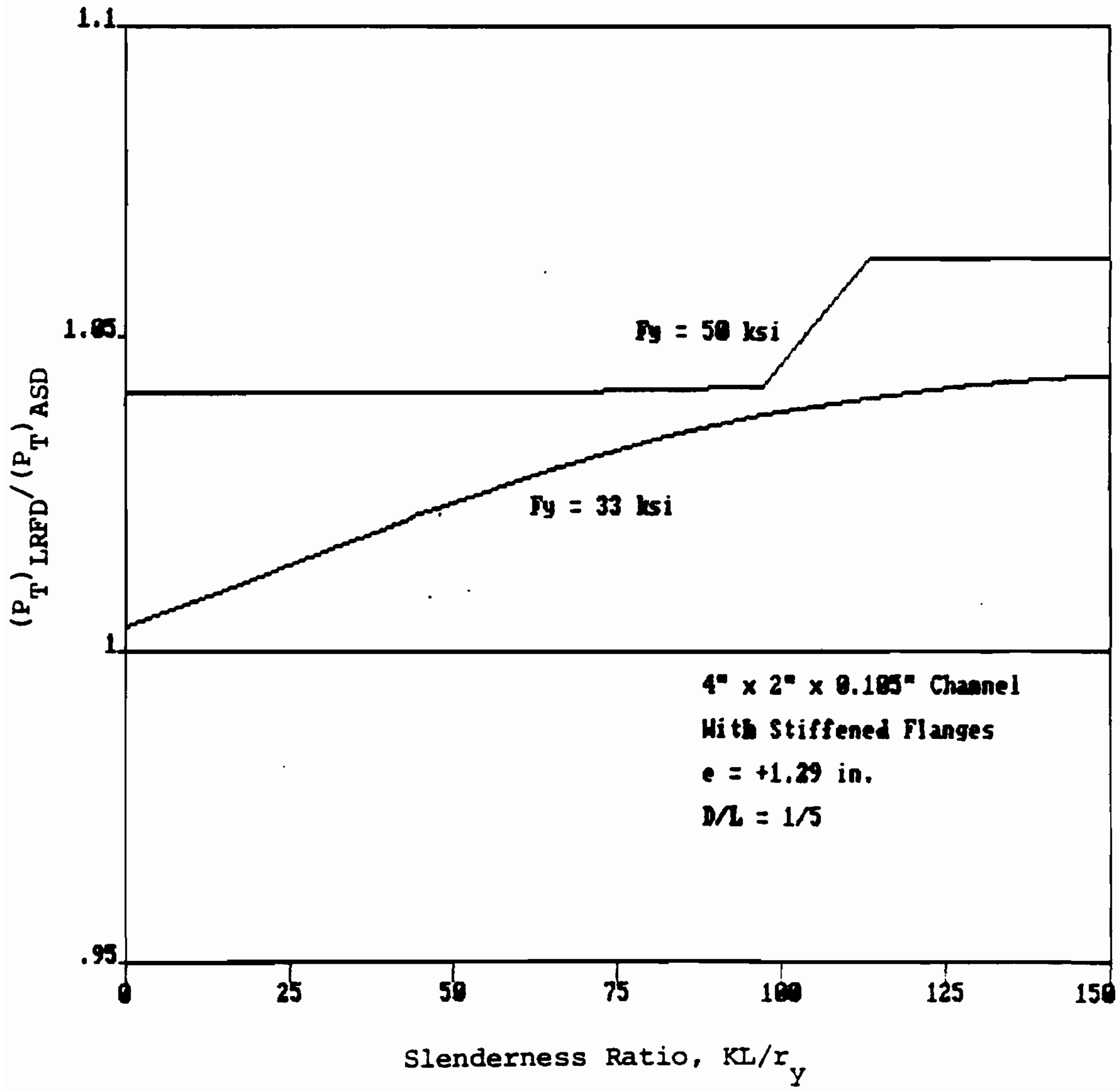


Figure 44. Allowable Load Ratio vs. Slenderness Ratio for Beam-Columns-Case 1

ratio increases. The slenderness ratio has a lesser effect on the allowable load ratio for the channel with F_y = 50 ksi as compared with F_y = 33 ksi. It can be seen from Figure 44 that the effect of yield point for short beam-columns is slightly greater than that for long members.

A channel (6 in. x 2.5 in. x 0.105 in.) with stiffened flanges was also studied. The relationship between the allowable load ratio and eccentricity for the channel with a length of 5 ft and D/L = 0.5 is shown in Figure 45. The bottom line represents the curve for C_m = 1.0 which would be used for braced frames. For this case, the curve is similar to that shown in Figure 42 for the 4 in. deep channel. The allowable load ratios are slightly higher in the region between the shear center and the centroid than they are outside this region.

The top line in Figure 45 represents the same channel with C_m = 0.85. This value of C_m is used for unbraced frames and beam-columns with restrained ends subject to transverse loading between its supports. The curve for C_m = 0.85 is similar to the curve for C_m = 1.0 except for $e > +1.8$ in. and $e < -2.0$ in. where the effect of the C_m value on the allowable load ratio is relatively large. The effect of the value of C_m on the allowable load ratio is negligible for -2.0 in $< e < +1.8$ in. as shown in Figure 45.

Figure 46 shows the allowable load ratio versus dead-to-live load ratio for the channel used in Figure 45. The curves represent the allowable load ratios for various eccentricities by using

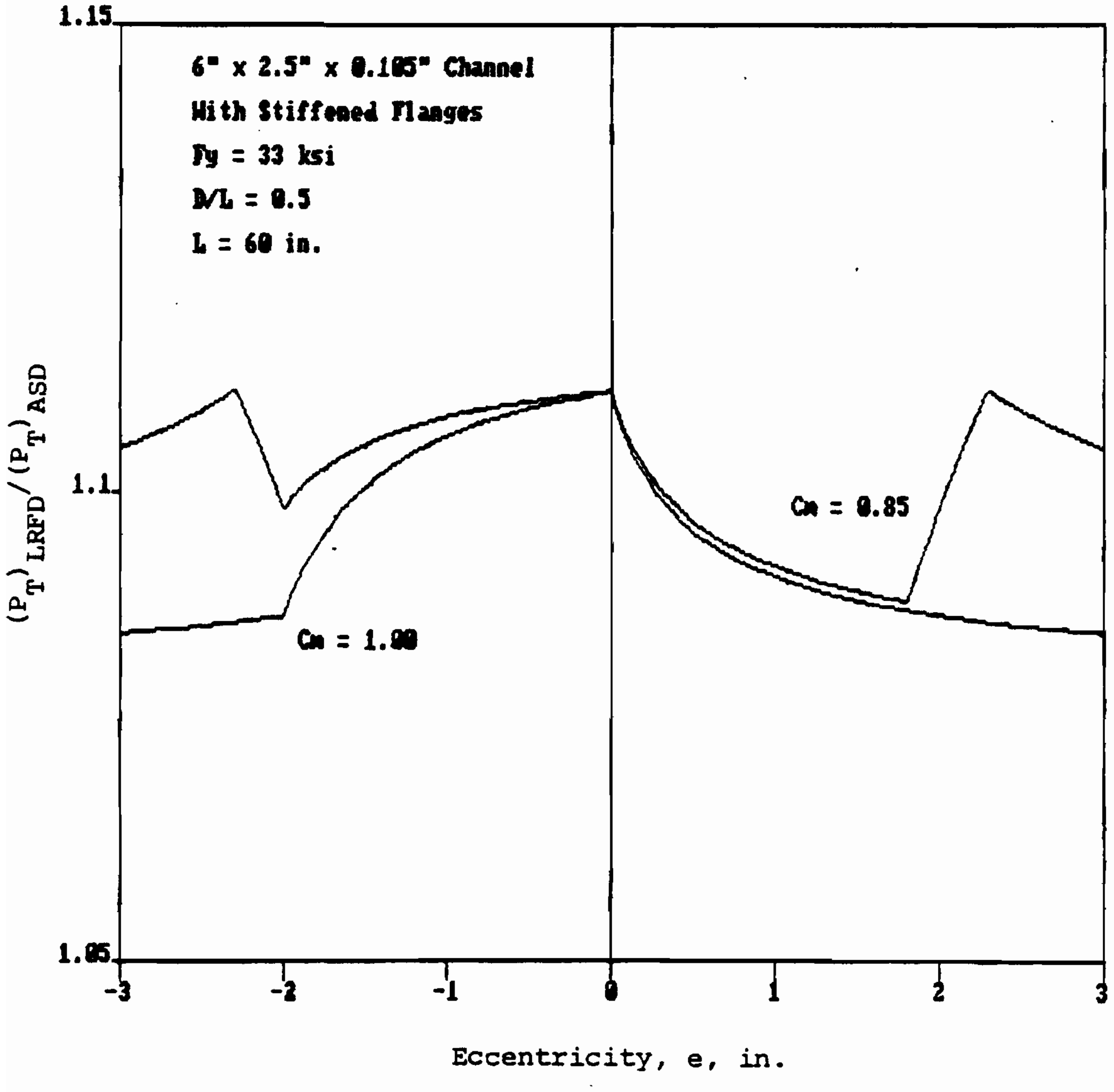


Figure 45. Allowable Load Ratio vs. Eccentricity for Beam-Columns-Case 2

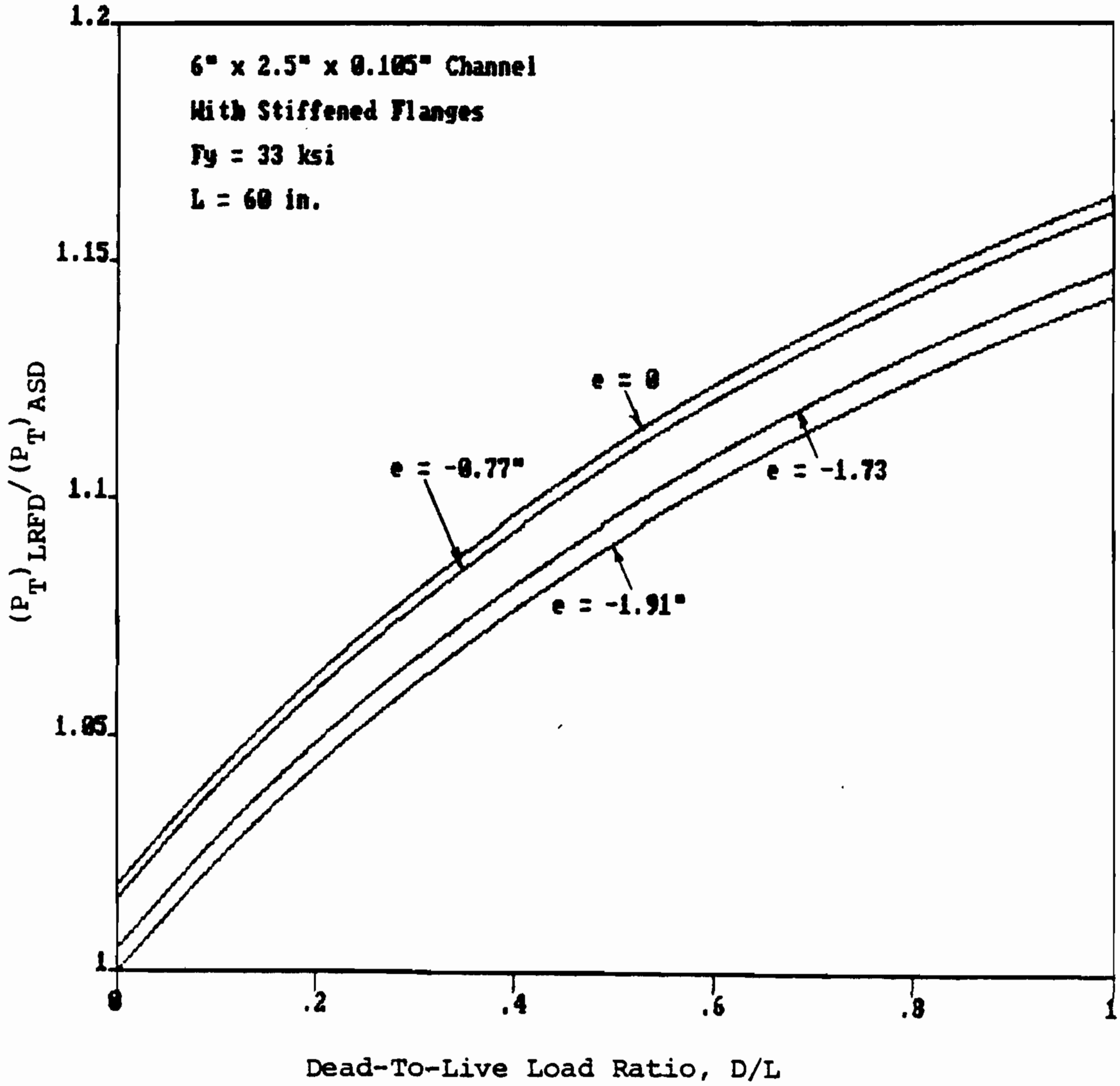


Figure 46. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case 2

F_y = 33 ksi and C_m = 1.0. It can be seen from this figure that the eccentricity does not affect the shape of the curve but slightly affects the value of the allowable load ratio.

The relationship between the allowable load ratio and dead-to-live load ratio for the 6 in. deep channel (6 in. x 2.5 in. x 0.105 in.) is shown in Figure 47 for various lengths. The curves represent the values of allowable load ratios for $e = \pm 1.73$ in. and effective lengths between 3 and 11 ft. It should be noted that the effective length has a small effect on the allowable load ratio.

Channels with unstiffened flanges were studied in a similar manner. Figure 48 shows the allowable load ratio versus eccentricity for a channel (4 in. x 1.125 in. x 0.105 in.) with unstiffened flanges and an effective length of 5 ft. The curves in the figure are allowable load ratios computed for yield points of 33 and 50 ksi, respectively. These curves indicate different relationships as compared with the curves in Figure 42 obtained from a 4 in. deep channel with stiffened flanges. The reason for these differences in the shape of the curves is that torsional-flexural buckling governs the design of the channel with stiffened flanges in Figure 42 for $x_o < e < 0$. For the channel with unstiffened flanges shown in Figure 48, flexural buckling governs the design for all values of eccentricities used in this study. However, the range of allowable load ratios are similar in both figures.

As shown in Figure 48, the value of yield point of steel has

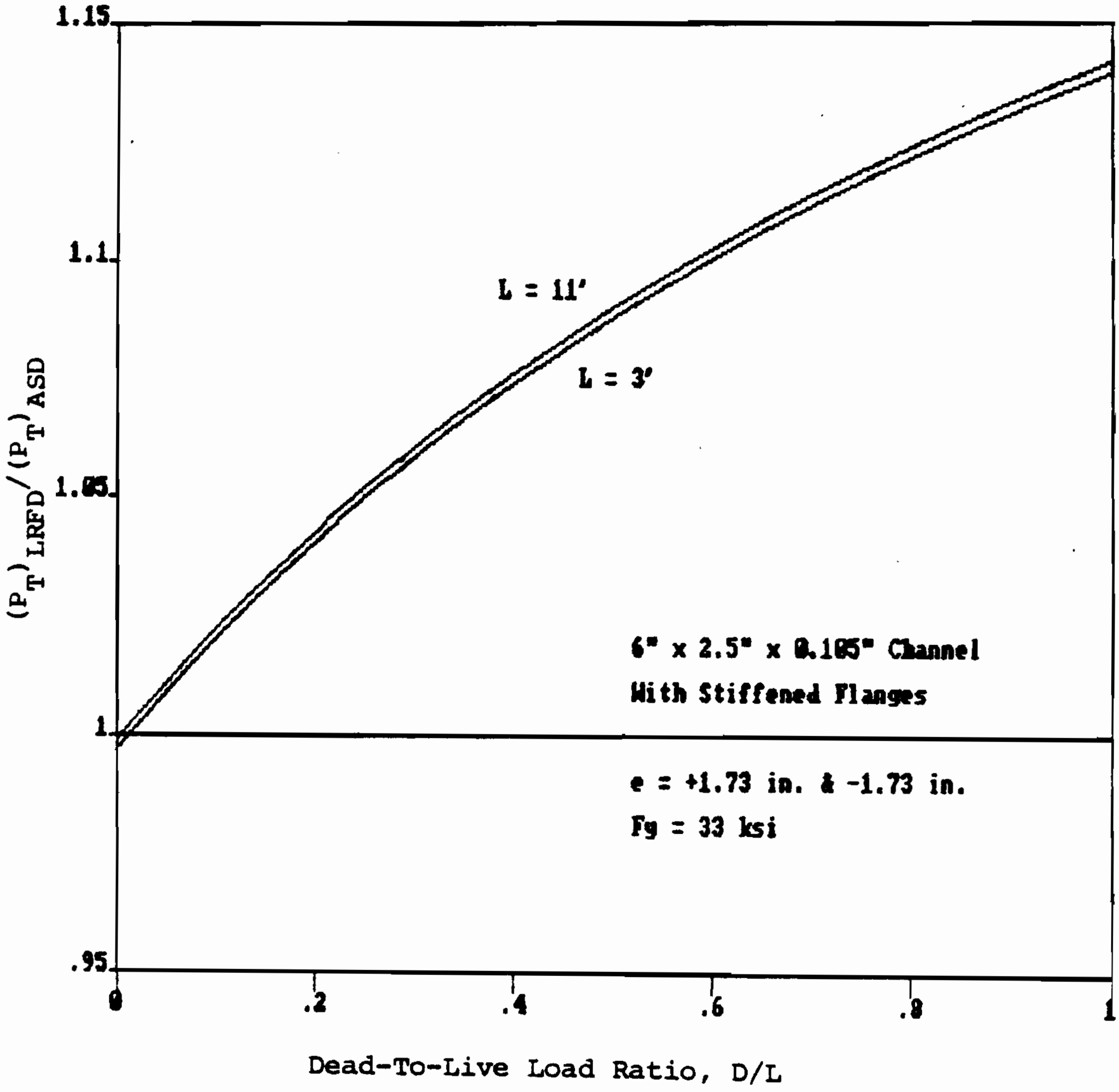


Figure 47. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case 3

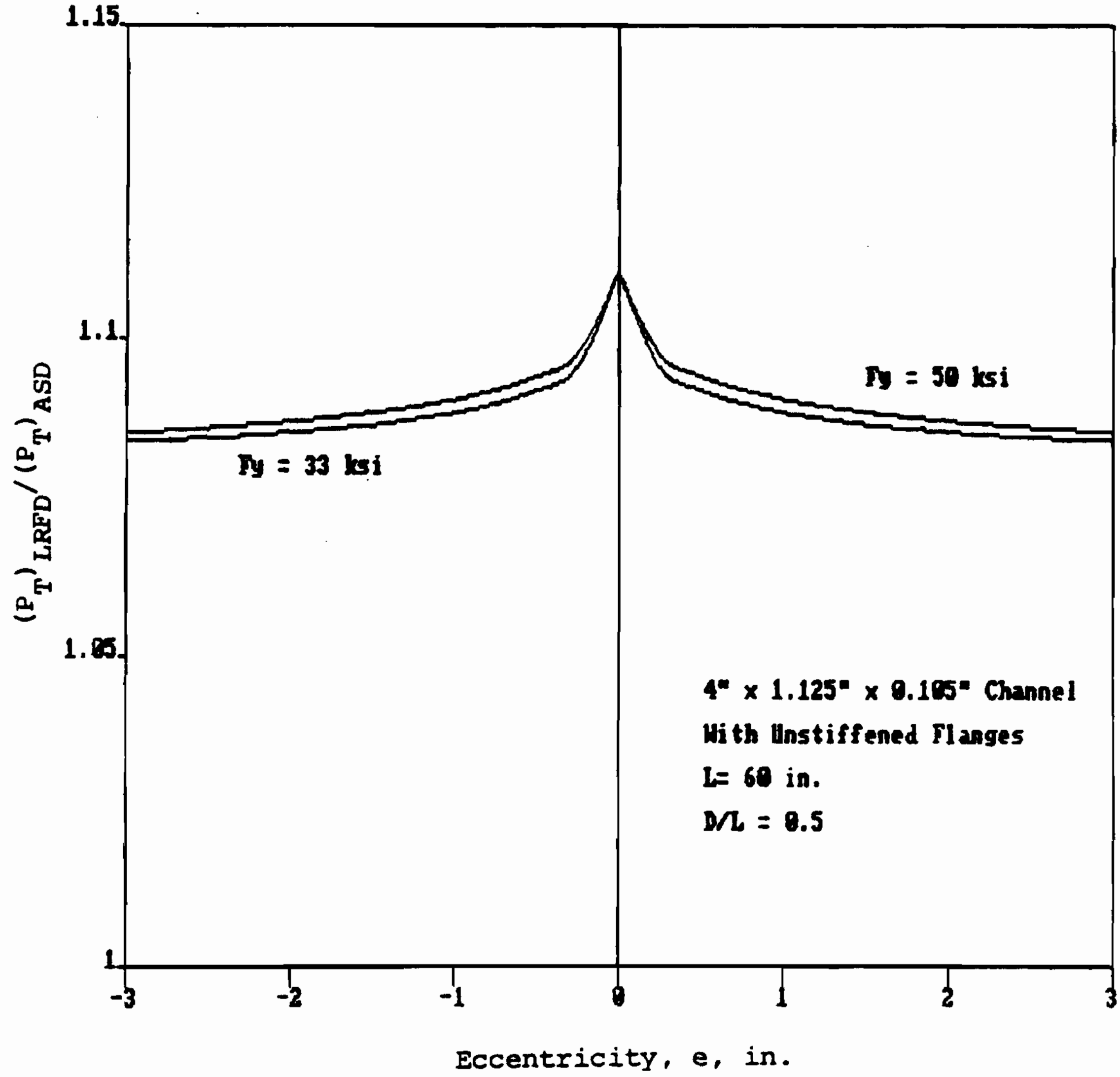


Figure 48. Allowable Load Ratio vs. Eccentricity for Beam-Columns-Case 4

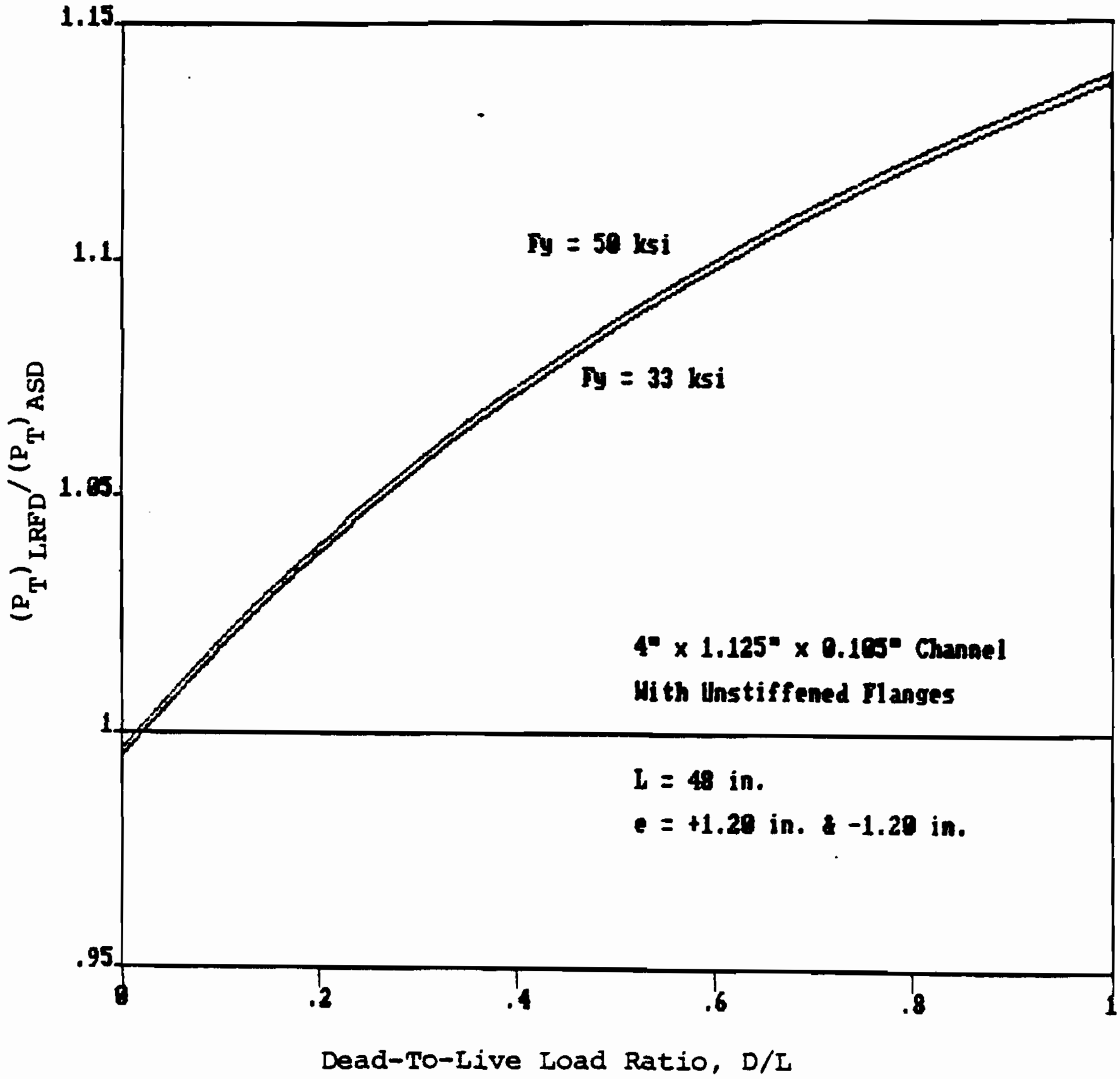


Figure 49. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case 4

a negligible effect on the allowable load ratio. Figure 49 also shows that the effect of the yield point on the allowable load ratio for various D/L ratios is negligible. The curves in this figure are for the same channel used in Figure 48 with an effective length of 4 ft. The yield points of steel vary from 33 to 50 ksi.

Figure 50 shows the allowable load ratio versus slenderness ratio, KL/r_y, for the same channel used in Figures 48 and 49 for D/L = 1/5 and e = ± 1.20 in. The curves computed for yield points of 33 and 50 ksi indicate that the allowable load ratio increases slightly with increasing slenderness ratios. The value of F_y has a negligible effect on the allowable load ratio particularly for long beam-columns.

A deeper channel (6 in. x 1.5 in. x 0.105 in.) with unstiffened flanges was also studied. Figure 51 shows the allowable load ratio versus eccentricity for the 5 ft long channel with D/L = 0.5. The curve shown in the figure is applicable for C_m values of 1.0 and 0.85. It is similar in shape and magnitude to the allowable load ratio curves shown in Figure 48 for a 4 in. deep channel with unstiffened flanges. As shown in Figures 48 and 51, small eccentricities will result in relatively high allowable load ratios.

The relationship between the allowable load ratio and dead-to-live load ratio for the channel used in Figure 51 is shown in Figure 52 for various lengths. The curves represent the values of allowable load ratio for e = ± 1.00 in. and effective lengths between 3 and 11 ft. This figure is similar to Figure 47 which was obtained from a channel of equal depth but with stiffened flanges. As shown

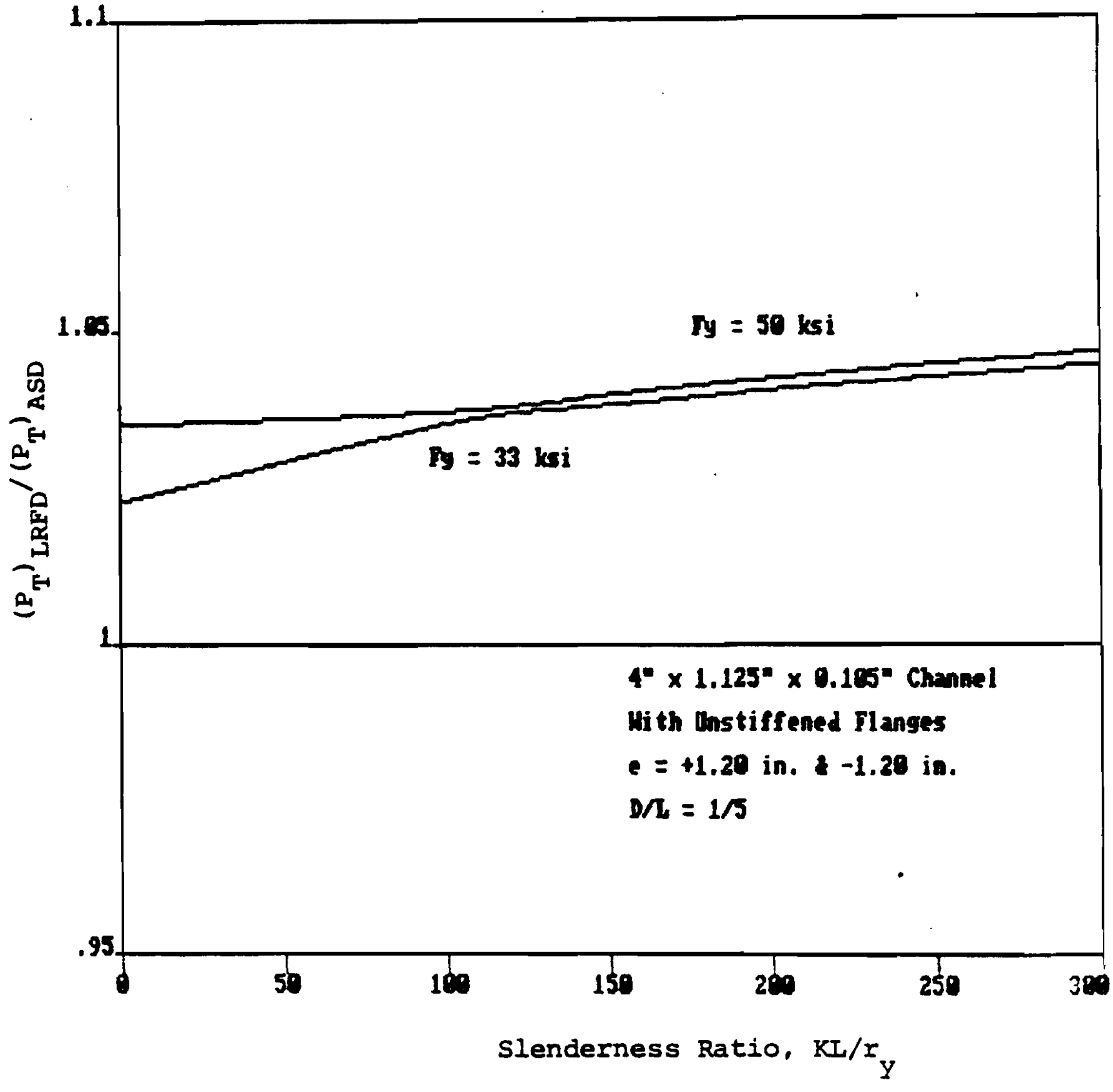


Figure 50. Allowable Load Ratio vs. Slenderness Ratio for Beam-Columns-Case 4

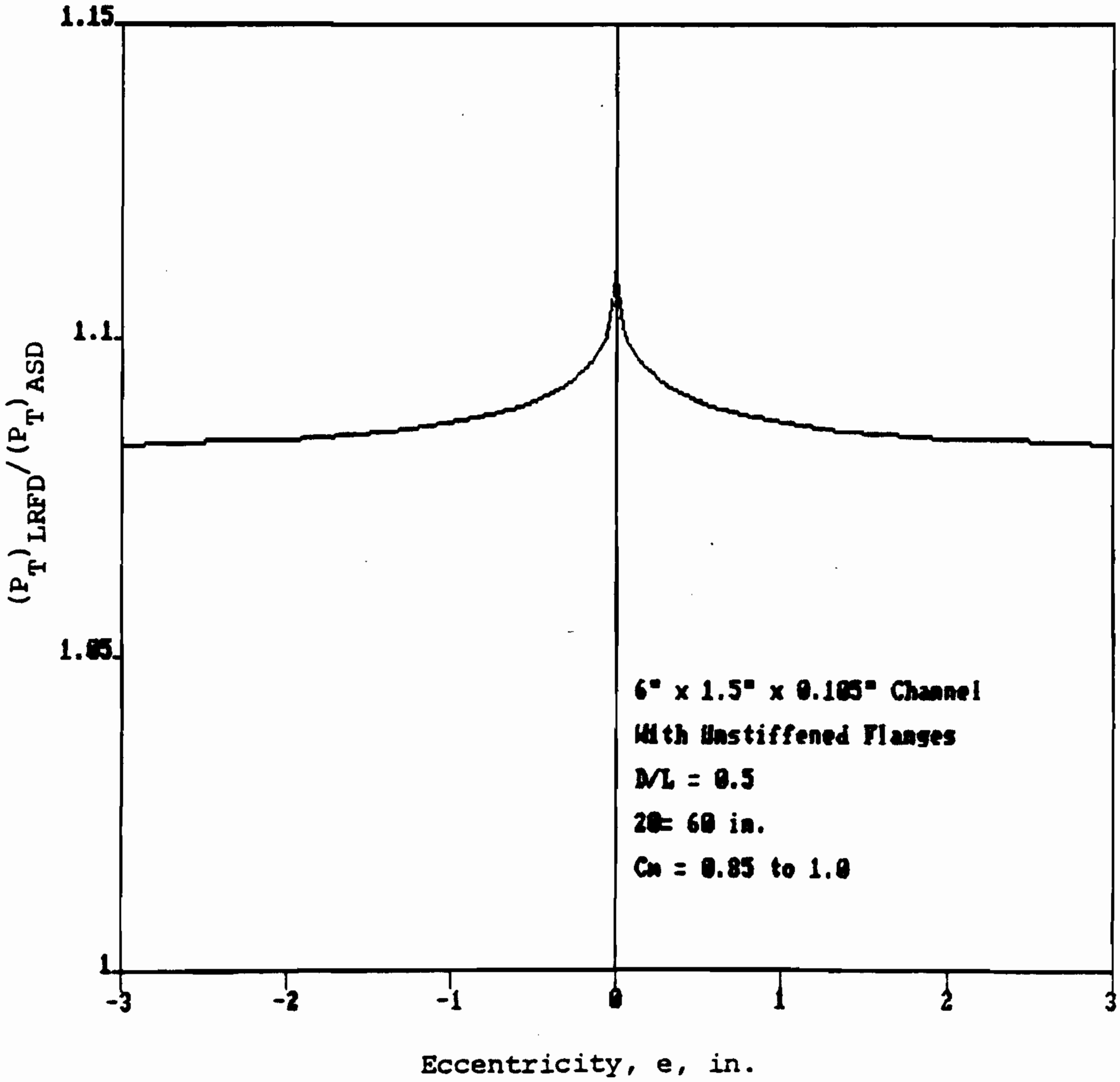


Figure 51. Allowable Load Ratio vs. Eccentricity for Beam-Columns-Case 5

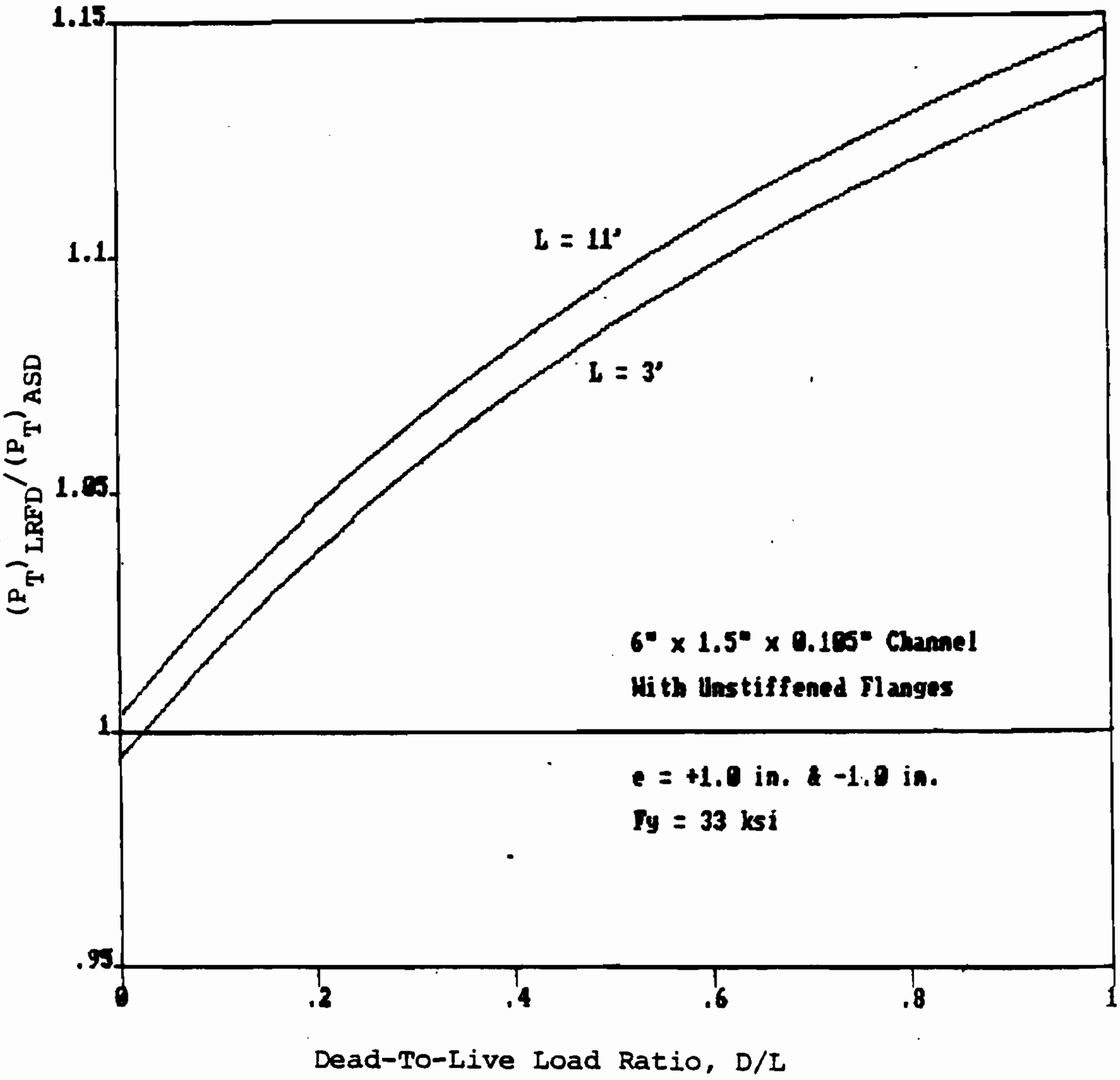


Figure 52. Allowable Load Ratio vs. D/L Ratio for Beam-Columns-Case 5

in the figure, the effective length has a small effect on the allowable load ratio.

D. DESIGN EXAMPLES

See Problems Nos. 5 and 6 in Appendix C for design examples of members subjected to bending and compression using Load and Resistance Factor Design.

VII. CONNECTIONS

A. GENERAL

Connections are required for joining individual structural members together and are used to fabricate structural members from sheet steel or structural components. The AISI Specification[1] and the Tentative Recommendations for Load and Resistance Factor Design[10] include requirements for welded and bolted connections which are frequently used in cold-formed steel construction. All connections should be designed to transmit the maximum load with proper regard for eccentricity.

B. WELDED CONNECTIONS

Welds are classified as fusion welds and resistance welds. Weld shearing and plate tearing are the common failure modes for welded connections.

1. Arc-Welds. Arc-welds are fusion welds produced by burning the metal to a molten state at the surface to be joined without the application of mechanical pressure or blows[43]. Pekoz and McGuire[44] studied the welding of sheet steel and provided most of the statistical test data for the development of the AISI design provisions for allowable stress design and the LRFD criteria for arc-welds.

a. Arc Spot Welds. Arc spot welds are produced by burning a hole in the top sheet and filling it with weld metal which fuses it to the bottom sheet or structural member. They are sometimes referred to as puddle welds.

i. <u>Allowable Stress Design</u>. Arc spot welds permitted by the AISI Specification[1] are for welding sheet steel to thicker supporting members in the flat position. Arc spot welds should not be made on steel where the thinnest connected part is over 0.15 in. thick, nor through a combination of steel sheets having a total thickness over 0.15 in. Weld washers should be used when the thickness of the sheet is less than 0.028 in. Weld washers should have a thickness between 0.05 in. and 0.08 in. with a minimum prepunched hole of 3/8 in. diameter.

Arc spot welds should be specified by minimum effective diameter of fused area, d_e. The minimum allowable effective diameter is 3/8 in. According to Section 4.2.1.2.2 of the AISI Specifications[1], the shear loads on each spot weld between sheet or sheets and supporting member should not exceed the smaller value of the following allowable shear loads:

(a) $$P = d_e^2 F_{xx}/4 \tag{7.1}$$

(b) For $d_a/t \leq 140/\sqrt{F_u}$,

$$P = 0.88\, td_aF_u \tag{7.2}$$

For $140/\sqrt{F_u} < d_a/t < 240/\sqrt{F_u}$,

$$P = 0.112[1+960t/(d_a\sqrt{F_u})]td_aF_u \tag{7.3}$$

For $d_a/t \geq 240/\sqrt{F_u}$,

$$P = 0.56\, td_aF_u \tag{7.4}$$

where

d = visible diameter of outer surface of arc spot weld, in.

d_a = average diameter of the arc spot weld at mid-thickness of t, in. (where d_a = (d-t) for a single sheet, and (d-2t) for multiple sheets (not more than four lapped sheets over a supporting member)), in.

d_e = effective diameter of fused area, in.

$d_e = 0.7d - 1.5t \leq 0.55d$

t = total combined base steel thickness (exclusive of coatings) of sheets involved in shear transfer, in.

F_{xx} = strength level designation in AWS electrode classification, ksi

F_y = specified minimum yield point of steel, ksi

F_u = specified minimum tensile strength of steel, ksi

ii. <u>LRFD Criteria</u>. According to Section 10.2.1.3 of the Tentative Recommendations[(10)], the factored nominal strength of each arc spot weld between sheet or sheets and supporting member should be determined by using the smaller value of ϕR_n from the following:

(a) $\phi = 0.70$, $R_n = (\pi d_e^2/4)(0.6F_{xx})$ (7.5)

(b) For $d_a/t \leq 114/\sqrt{F_u}$,

$\phi = 0.60$

$$R_n = 2.2td_aF_u \tag{7.6}$$

For $114/\sqrt{F_u} < d_a/t < 240/\sqrt{F_u}$

$\phi = 0.50$

$$R_n = 0.28[1+960t/(d_a\sqrt{F_u})]td_aF_u \tag{7.7}$$

For $d_a/t \geq 240/\sqrt{F_u}$,

$$\phi = 0.50$$

$$R_n = 1.4td_aF_u \tag{7.8}$$

where

ϕ = resistance factor for welded connections

R_n = nominal ultimate strength of an arc spot weld, kips

iii. Comparison. Equations (7.1) and (7.5) are based on shearing of the weld. The allowable load per spot for allowable stress design is P computed from Eq. (7.1) for this type of failure. For the LRFD criteria, the allowable load per spot based on weld shearing and plate failure can be calculated from the following equation developed from Eq. (2.6):

$$(P_a)_{LRFD} = \phi R_n(D/L+1)/(1.2D/L+1.6) \tag{7.9}$$

Based on the assumption that the shear strength of welds is approximately equal to 0.6 times the strength level designation F_{xx} used in the AWS electrode classification, a factor of safety of 0.6π was used against weld shear for the allowable load used in allowable stress design. Therefore, the allowable load ratio based on shearing of arc spot welds and ϕ = 0.70 is as follows:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \phi 0.6\pi \frac{D/L+1}{1.2D/L+1.6} = 1.319 \frac{D/L+1}{1.2D/L+1.6} \tag{7.10}$$

Figure 53 shows the allowable load ratio versus dead-to-live load ratio determined from Eq. (7.10) for weld shear failure of arc spot welds. For D/L = 0.5, the allowable load per spot determined from the LRFD criteria is 10% less than the value obtained from allowable stress design. As shown in the figure,

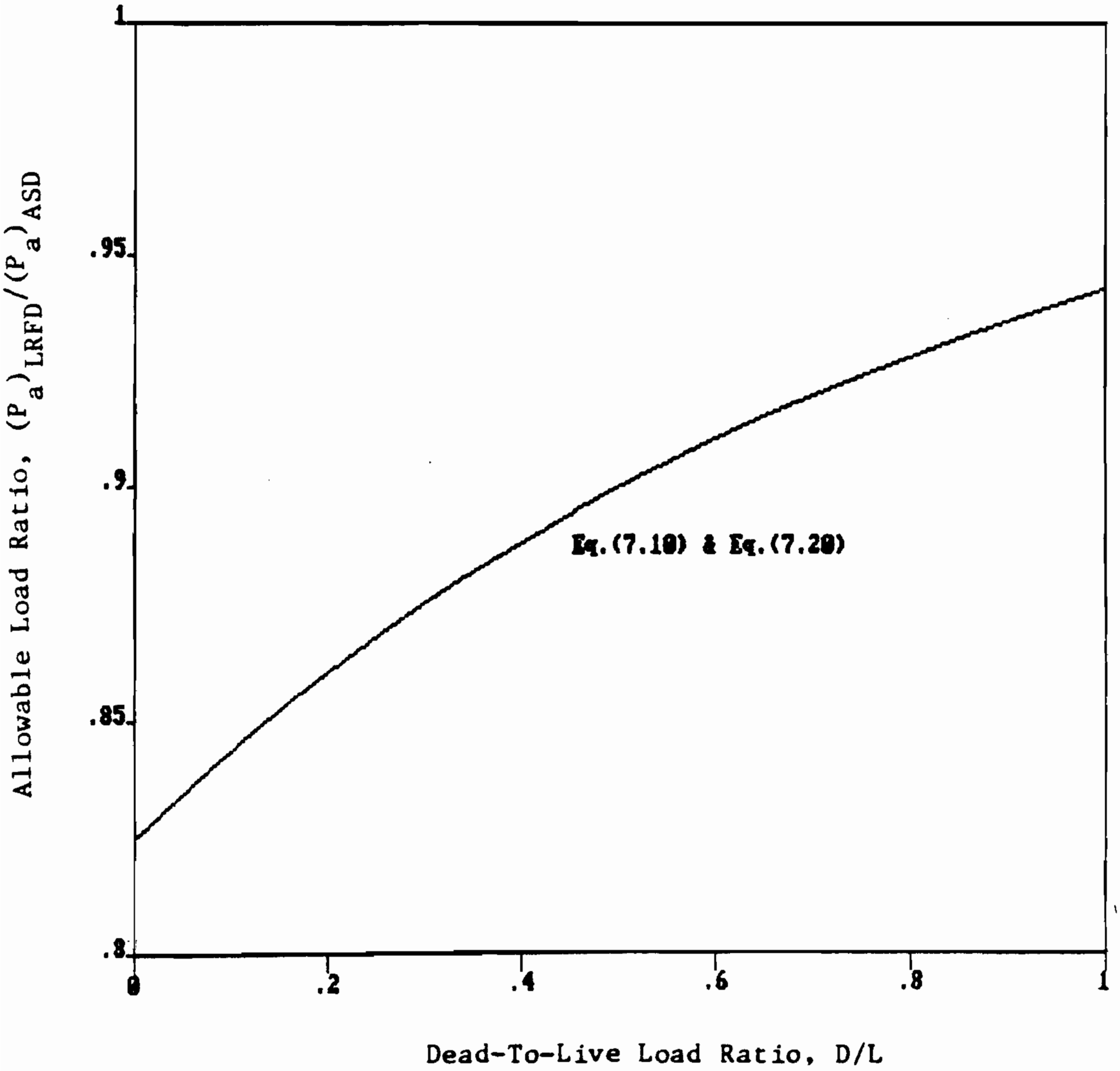


Figure 53. Allowable Load Ratio vs. D/L Ratio for Shear Failure of Arc Spot and Arc Seam Welds

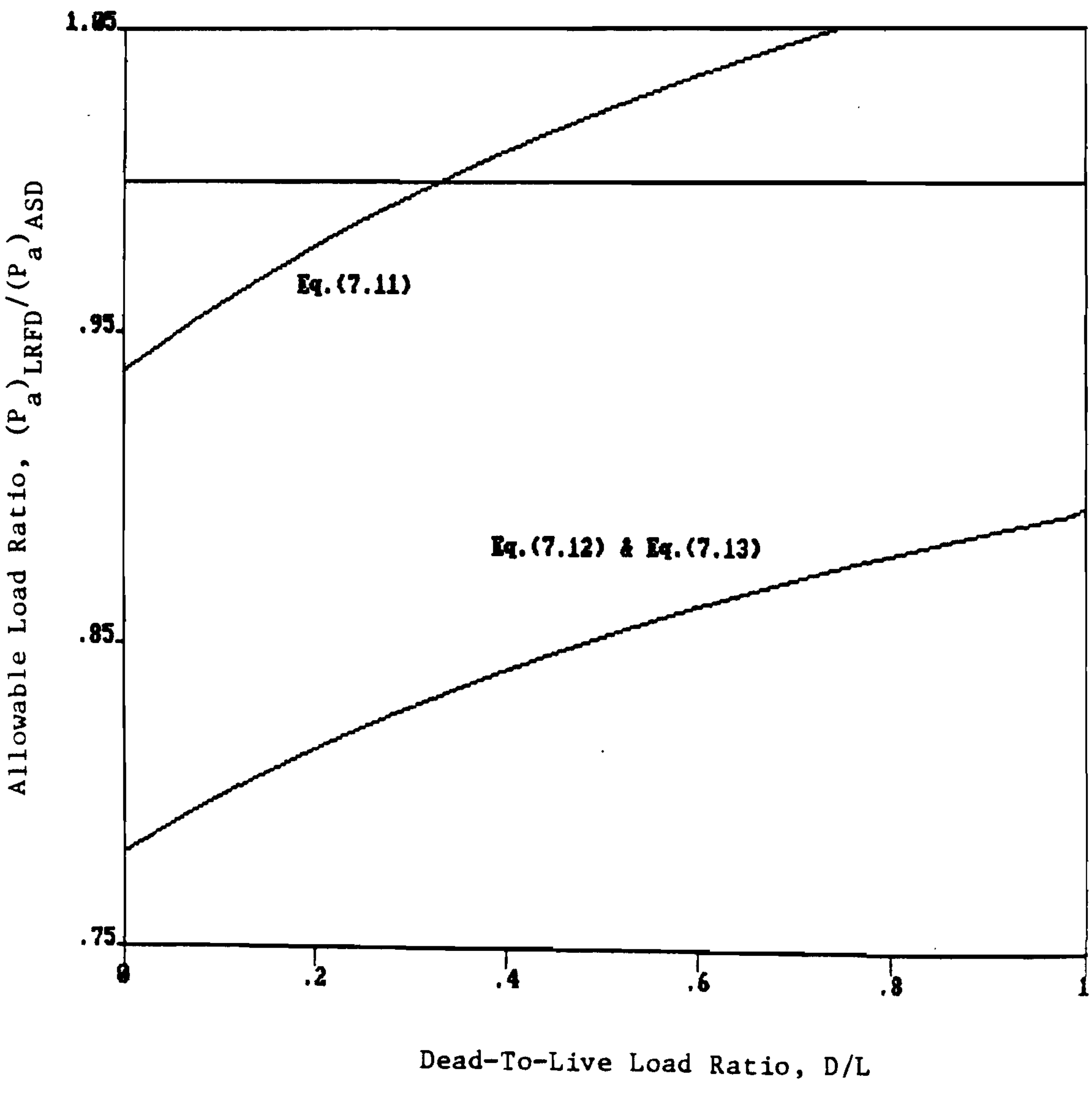


Figure 54. Allowable Load Ratio vs. D/L Ratio for Arc Spot Welds

LRFD is very conservative for shear failure in arc spot welds.

Equations (7.2), (7.3), and (7.4) from allowable stress design and Eqs. (7.6), (7.7), and (7.8) for LRFD are based on failure in the plate. The allowable load per spot for allowable stress design was derived from the nominal failure load of the welded plate using a factor of safety of 2.5. Therefore, the allowable load ratio for plate failure is as follows:

For $d_a/t \leq 114/\sqrt{F_u}$ and $\phi = 0.60$,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi\,\frac{D/L+1}{1.2D/L+1.6} = 1.50\,\frac{D/L+1}{1.2D/L+1.6} \tag{7.11}$$

For $114/\sqrt{F_u} < d_a/t < 240/\sqrt{F_u}$ and $\phi = 0.50$,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi\,\frac{D/L+1}{1.2D/L+1.6} = 1.25\,\frac{D/L+1}{1.2D/L+1.6} \tag{7.12}$$

For $d_a/t \geq 240/\sqrt{F_u}$ and $\phi = 0.50$

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi\,\frac{D/L+1}{1.2D/L+1.6} = 1.25\,\frac{D/L+1}{1.2D/L+1.6} \tag{7.13}$$

Equations (7.11), (7.12), and (7.13) are shown in Figure 54 and are based on plate failure of arc spot welds. As seen from the figure, for D/L = 0.5, the allowable load ratio computed from LRFD and ASD varies from about 0.85 to 1.02 depending upon the d/t ratio used in the connection. For the range of D/L ratios used in cold-formed steel, LRFD is conservative for the design of arc spot welds compared with allowable stress design.

b. <u>Arc Seam Welds</u>. Arc seam welds are produced in the same manner as arc spot welds except that a seam is formed.

i. Allowable Stress Design. Arc seam welds covered by the AISI Specification(1) apply only to the following joints:

(a) Sheet to thicker supporting member in the flat position

(b) Sheet to sheet in the horizontal or flat position

According to Section 4.2.1.2.3 of the AISI Specification(1), the load on each arc seam weld should not exceed the smaller value of the following allowable loads:

$$P = (d_e^2/4 + Ld_e/3)F_{xx} \tag{7.14}$$

$$P = tF_u(0.25L + 0.96d_a) \tag{7.15}$$

where

d = width of arc seam weld, in.

L = length of seam weld not including the circular ends, in. (For computation purposes, L shall not exceed 3d.)

d_a = average width of seam weld, in. (where d_a = (d-t) for a single sheet, and (d-2t) for a double sheet)

d_e = effective width of arc seam weld at fused surfaces.

$$d_e = 0.7d - 1.5t, \text{ in.} \tag{7.16}$$

ii. LRFD Criteria. According to Section 10.2.1.4 of the Tentative Recommendations(10), the factored nominal strength of arc seam welds should be determined by using the smaller value of ϕR_n from the following:

(a) $$\phi = 0.70,\ R_n = (\pi d_e^2/4 + Ld_e)(0.6F_{xx}) \tag{7.17}$$

(b) $$\phi = 0.60,\ R_n = (0.63L + 2.4d_a)tF_u \tag{7.18}$$

where

ϕ = resistance factor for welded connections

R_n = nominal ultimate strength of an arc seam weld, kips

iii. Comparison. Equations (7.14) and (7.17) are based on shearing of the weld. For allowable stress design the allowable load per weld is P computed from Eq. (7.14) for weld shearing. The allowable load per seam weld for weld shearing and plate tearing can be calculated from the following equation developed from Eq. (2.6):

$$(P_a)_{LRFD} = \phi R_n (D/L+1)/(1.2D/L+1.6) \tag{7.19}$$

Similar to arc spot welds a factor of safety of 0.6π was used against shearing of the weld for the allowable load value computed from allowable stress design. Therefore, the allowable load ratio based on shear failure of the arc seam weld and $\phi = 0.70$ is as follows:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = \phi 0.6\pi \frac{D/L+1}{1.2D/L+1.6} = 1.319 \frac{D/L+1}{1.2D/L+1.6} \tag{7.20}$$

Equation (7.20) is identical to Eq. (7.10) which is the allowable load ratio for arc spot welds based on weld shearing. Figure 53 shows the relationship between allowable load ratio and dead-to-live load ratio for this type of failure. As shown in the figure, LRFD is very conservative for shear failure of arc seam welds compared with allowable stress design.

Equations (7.15) and (7.18) are based on plate tearing. The allowable load, P, in Eq. (7.15) based on allowable stress design was derived from the nominal plate failure load using a factor of safety of 2.5. Therefore, the allowable load ratio for plate failure and $\phi = 0.60$ is as follows:

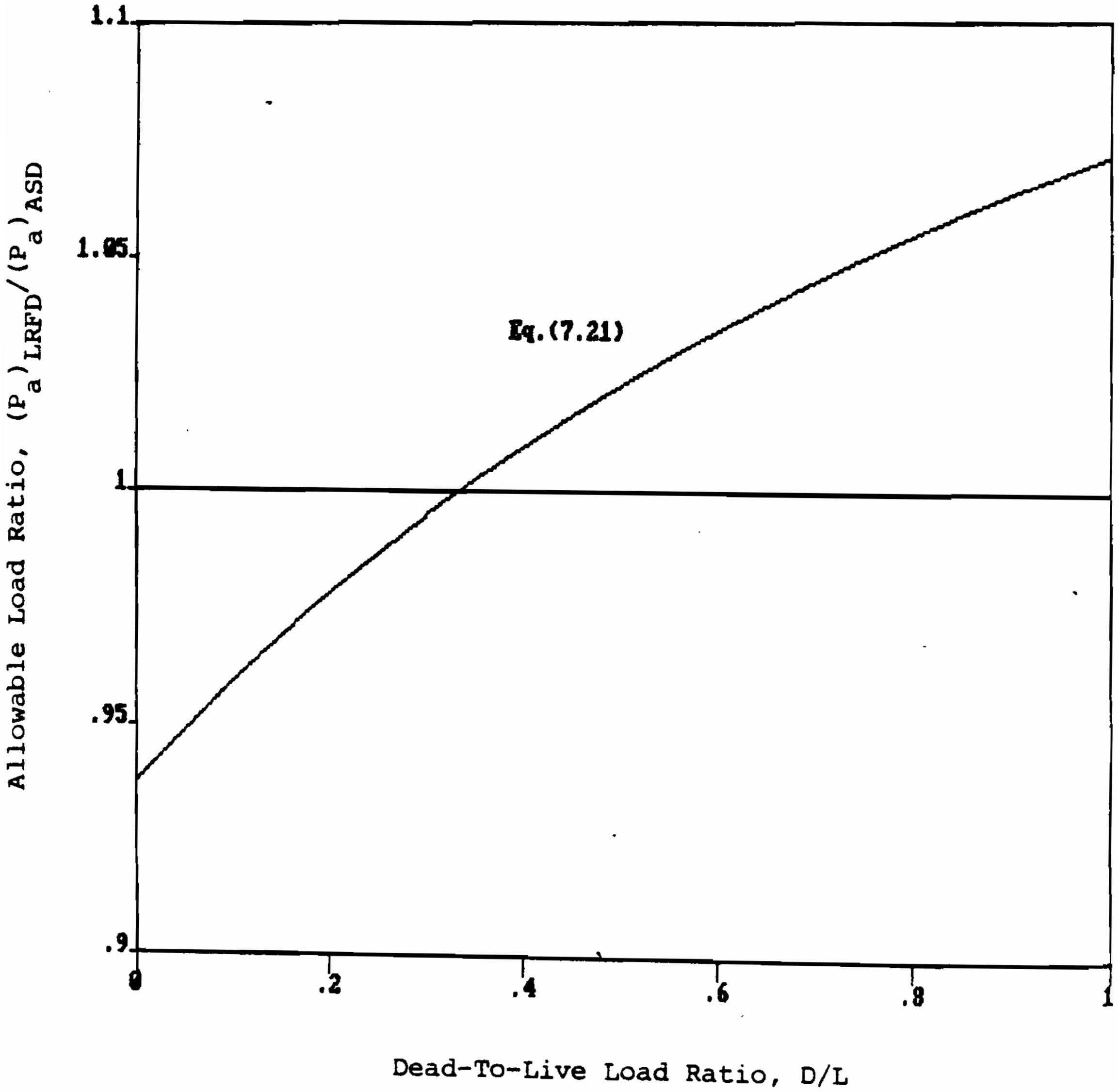


Figure 55. Allowable Load Ratio vs. D/L Ratio for Plate Tearing of Arc Seam Welds

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \frac{D/L+1}{1.2D/L+1.6} = 1.5 \frac{D/L+1}{1.2D/L+1.6} \quad (7.21)$$

Figure 55 shows the allowable load ratio versus dead-to-live load ratio determined from Eq. (7.21) for plate tearing failure. Both design methods result in the same value of allowable load for a D/L ratio of 1/3. The allowable load based on LRFD is 2.3% greater than the value based on allowable stress design for D/L = 0.5. However, LRFD is conservative for D/L < 1/3 compared with allowable stress design.

c. Fillet Welds. Fillet welds are used to connect lap joints and T-joints.

i. Allowable Stress Design. Fillet welds covered by the AISI Specification(1) apply to the welding of joints in any position, either

(a) Sheet to sheet, or

(b) Sheet to thicker steel member

According to Section 4.2.1.2.4 of the AISI Specification(1), the load on a fillet weld in lap and T-joints should not exceed the following allowable loads:

For longitudinal loading:

For $L/t < 25$,

$$P = 0.4[1-0.01(L/t)]tLF_u \quad (7.22)$$

For $L/t \geq 25$,

$$P = 0.3tLF_u \quad (7.23)$$

For transverse loading:

$$P = 0.4tLF_u \quad (7.24)$$

In addition, for $t > 0.150$ in., the load on a fillet weld in lap or T-joints should not exceed the following allowable load:

$$P = 0.3t_w LF_{xx} \tag{7.25}$$

where

L = length of fillet weld, in.

t_w = effective throat = $0.707w_1$ or $0.707w_2$, whichever is smaller. A larger effective throat may be taken if it can be shown by measurement that a given welding procedure will consistently give a larger value providing the particular welding procedure used for making the welds that are measured are followed.

w = leg on weld

ii. LRFD Criteria. According to Section 10.2.1.5 of the Tentative Recommendations[10], the factored nominal strength, ϕR_n, of a fillet weld should be determined as follows:

For longitudinal loading:

For $L/t < 25$,

$$\phi = 0.60$$

$$R_n = [1-0.01(L/t)]tLF_u \tag{7.26}$$

For $L/t \geq 25$,

$$\phi = 0.60$$

$$R_n = 0.75tLF_u \tag{7.27}$$

For transverse loading:

$$\phi = 0.60$$

$$R_n = tLF_u \tag{7.28}$$

In addition, for $t > 0.15$ in., the factored nominal strength determined above should not exceed the following value of ϕR_n:

$$\phi = 0.70$$

$$R_n = 0.6 t_w L F_{xx} \qquad (7.29)$$

where

ϕ = resistance factor for welded connections

R_n = nominal ultimate strength of a fillet weld, kips

iii. Comparison. For allowable stress design, the value of P is the allowable load per fillet weld. The allowable load based on the LRFD criteria can be calculated from the following formula developed from Eq. (2.6):

$$(P_a)_{LRFD} = \phi R_n (D/L+1)/(1.2D/L+1.6) \qquad (7.30)$$

Equations (7.22), (7.23), and (7.24) are based on plate tearing and a factor of safety of 2.5. Therefore, the allowable load ratio can be computed using the following formula:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \, \frac{D/L+1}{1.2D/L+1.6} \qquad (7.31)$$

For longitudinal loading with $L/t < 25$, the resistance factor is 0.60 . Therefore, the allowable load ratio can be computed using the following equation:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.50 \, \frac{D/L+1}{1.2D/L+1.6} \qquad (7.32)$$

For longitudinal loading with $L/t \geq 25$, the resistance factor is also 0.60 . Therefore, the following equation can be used to calculate the allowable load ratio:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.50 \; \frac{D/L+1}{1.2D/L+1.6} \qquad (7.33)$$

For transverse loading with $\phi = 0.6$, Eq. (7.34) can be used to calculate the allowable load ratio.

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.5 \; \frac{D/L+1}{1.2D/L+1.6} \qquad (7.34)$$

The relationship between the allowable load ratio and dead-to-live load ratio is shown on Figure 56 for plate tearing failure based on Eqs. (7.32), (7.33), and (7.34). For longitudinally loaded fillet welds and D/L = 0.5, the allowable load computed from LRFD is 2.3% higher than the value computed from allowable stress design.

For transverse loading of fillet welds, the allowable load based on the LRFD criteria is also 2.3% higher than the value based on allowable stress design for D/L = 0.5. From Figure 56 it can be seen that the LRFD criteria for plate tearing of fillet welds is similar to the allowable stress design criteria for D/L ratios around 1/3.

When the thickness of the plate is greater than 0.15 in., weld shearing has to be checked. Equations (7.25) and (7.29) are based on weld shearing of fillet welds. The allowable load, P, from Eq. (7.25) for allowable stress design was based on a factor of safety of 2.00 against weld failure. Therefore, the allowable load

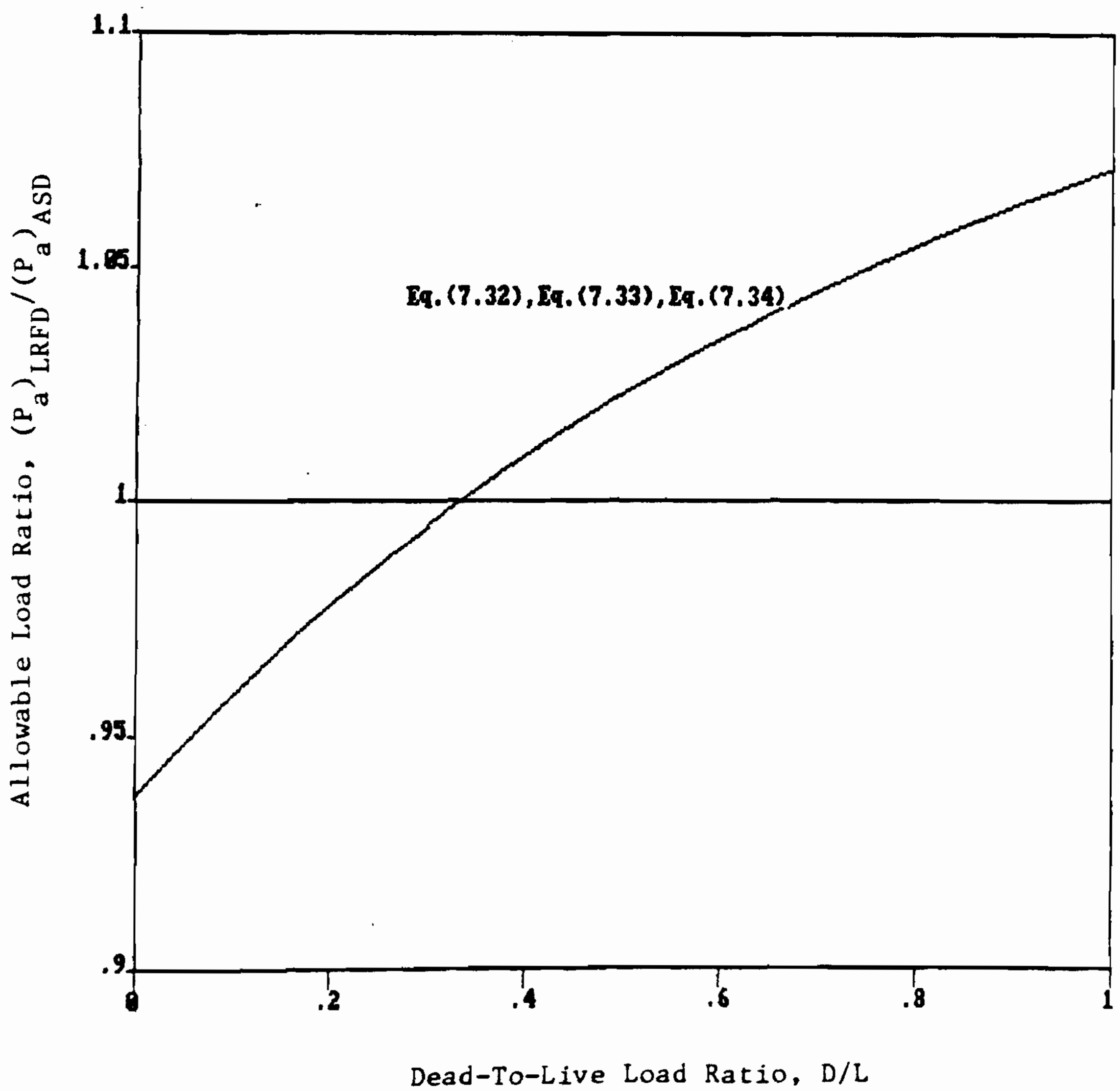


Figure 56. Allowable Load Ratio vs. D/L Ratio for Plate Tearing of Fillet Welds

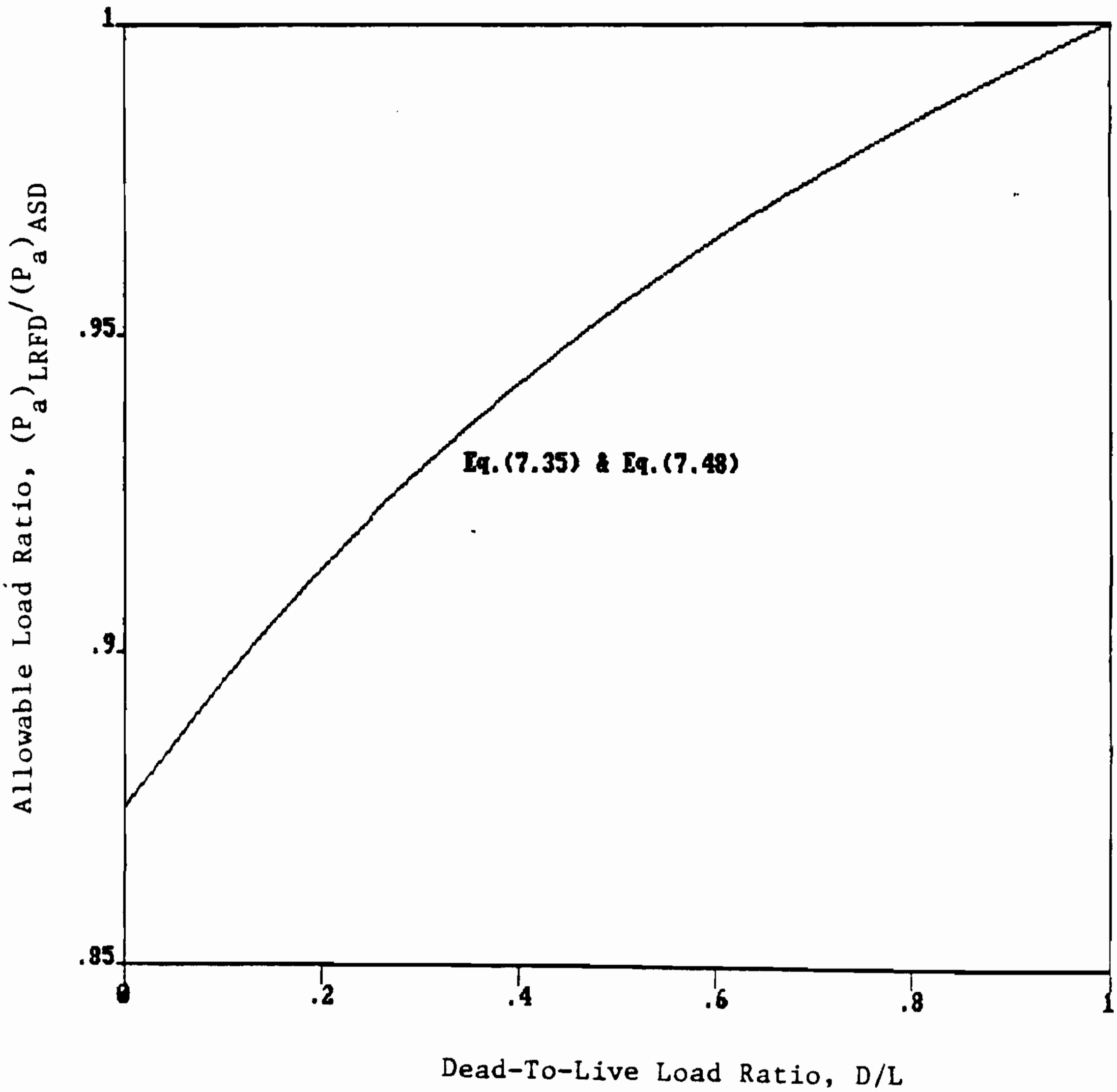


Figure 57. Allowable Load Ratio vs. D/L Ratio for Weld Failure of Fillet and Flare Groove Welds

ratio can be computed using the following formula with $\phi = 0.70$:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.0\phi \frac{D/L+1}{1.2D/L+1.6} = 1.40 \frac{D/L+1}{1.2D/L+1.6} \qquad (7.35)$$

The relationship between allowable load ratio and dead-to-live load ratio for weld failure of fillet welds is shown in Figure 57. For $D/L < 1.0$, LRFD is conservative compared with allowable stress design. Also from the figure, LRFD criteria result in an allowable load 4.5% smaller than the value computed from allowable stress design for $D/L = 0.5$.

d. Flare Groove Welds. Flare groove welds are used in cold-formed steel construction to join rolled corners to sheets and to join two rolled corners.

i. Allowable Stress Design. Flare groove welds covered by Section 4.2.1.2.5 of the AISI Specification(1) apply to welding of joints in any position, either:

(a) Sheet to sheet for flare-V groove welds, or

(b) Sheet to sheet for flare-bevel groove welds, or

(c) Sheet to thicker steel member for flare-bevel groove welds.

Allowable loads on welds should be governed by the thickness, t, of the sheet steel adjacent to the welds.

For transverse loading of flare-bevel groove welds, the allowable load should be computed by the following formula:

$$P = tLF_u/3 \qquad (7.36)$$

For longitudinal loading of flare groove welds, the allowable load should be computed as follows:

For $t \leq t_w < 2t$ or $L >$ lip height,

$$P = 0.3tLF_u \qquad (7.37)$$

For $t_w \geq 2t$ and $L \leq$ lip height,

$$P = 0.6tLF_u \qquad (7.38)$$

In addition, if $t > 0.15$ in., the allowable load computed above should not exceed the following allowable load:

$$P = 0.3t_wLF_{xx} \qquad (7.39)$$

ii. LRFD Criteria. According to Section 10.2.1.6 of the Tentative Recommendations(10), the factored nominal strength, ϕR_n, of a flare groove weld should be determined as follows:

(a) For flare-bevel groove welds, transverse loading:

$$\phi = 0.55$$

$$R_n = 0.8tLF_u \qquad (7.40)$$

(b) For flare groove welds, longitudinal loading:

For $t \leq t_w < 2t$ or $L >$ lip height,

$$\phi = 0.55$$

$$R_n = 0.75tLF_u \qquad (7.41)$$

For $t_w \geq 2t$ and $L \leq$ lip height,

$$\phi = 0.55$$

$$R_n = 1.5tLF_u \qquad (7.42)$$

In addition, if $t > 0.15$ in., the factored nominal strength determined above should not exceed the following value of ϕR_n:

$$\phi = 0.70$$

$$R_n = 0.6t_wLF_{xx} \qquad (7.43)$$

iii. Comparison. The allowable load based on allowable stress design can be calculated using Eqs. (7.36) through (7.39), whichever is applicable. For LRFD, the allowable load can be calculated from the following formula developed from Eq. (2.6):

$$(P_a)_{LRFD} = \phi R_n (D/L+1)/(1.2D/L+1.6) \qquad (7.44)$$

From allowable stress design, Eqs. (7.36), (7.37), and (7.38) were derived from the plate failure load using a factor of safety of 2.5. Therefore, the allowable load ratio can be computed using the following formula:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \frac{D/L+1}{1.2D/L+1.6} \qquad (7.45)$$

For flare-bevel groove welds loaded in the transverse direction and $\phi = 0.55$, the following equation can be used for allowable load ratio:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.375 \frac{D/L+1}{1.2D/L+1.6} \qquad (7.46)$$

For flare groove welds loaded in the longitudinal direction and $\phi = 0.55$, the allowable load ratio can be computed as follows:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.375 \frac{D/L+1}{1.2D/L+1.6} \qquad (7.47)$$

Figure 58 shows the relationship between allowable load ratio and dead-to-live load ratio computed from Eqs. (7.46) and (7.47). For transverse loading of flare-bevel groove welds and D/L = 0.5, the allowable load computed from LRFD is 6.3% lower than the value computed from allowable stress design. The same is true for flare groove welds loaded in the longitudinal direction.

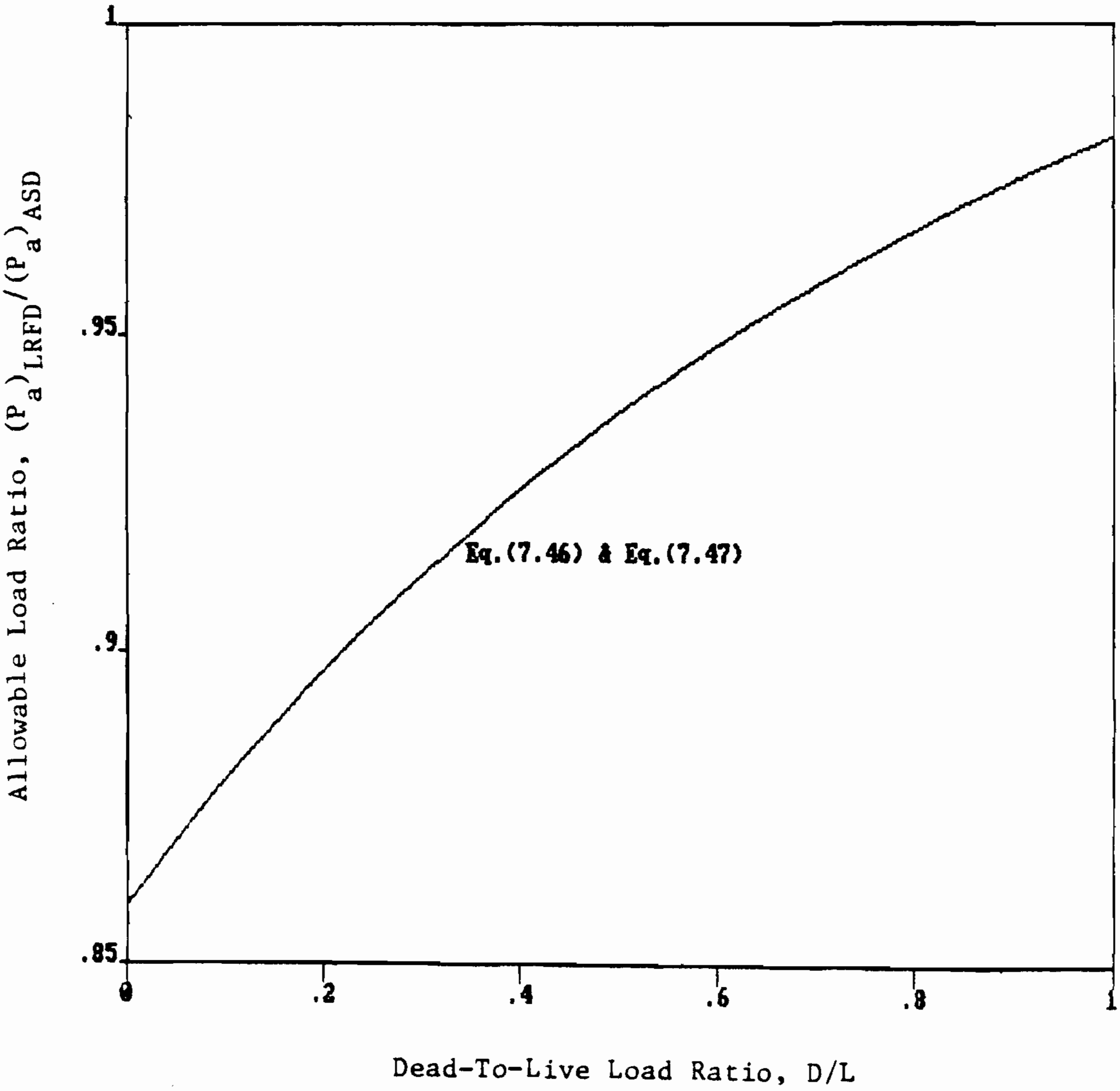


Figure 58. Allowable Load Ratio vs. D/L Ratio for Plate Tearing of Flare Groove Welds

As shown in the figure, the LRFD criteria for flare groove welds are slightly conservative for the values of D/L ratios generally used in cold-formed steel construction.

For flare groove welds on sheets thicker than 0.15 in., weld shearing may govern the design. Equation (7.39) from allowable stress design is based on shear failure of the weld with a factor of safety of 2.0. Equation (7.43) is the shear failure load of the weld used in LRFD with $\phi = 0.70$. Therefore, the allowable load ratio can be computed as follows:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.0\phi \frac{D/L+1}{1.2D/L+1.6} = 1.40 \frac{D/L+1}{1.2D/L+1.6} \qquad (7.48)$$

Equation (7.48) is identical to Eq. (7.35) which is the allowable load ratio for fillet welds based on the same type of failure. Figure 57 shows the allowable load ratio versus dead-to-live load ratio for weld failure of fillet and flare groove welds. The allowable load ratio based on LRFD is 4.5 % smaller than the value based on allowable stress design for D/L = 0.5.

2. Resistance Welds. Resistance welding is a group of welding processes wherein coalescence is produced by the heat obtained from resistance to electric current through the work parts held together under pressure by electrodes[43]. They are mostly used for shop welding in cold-formed steel fabrication.

a. Allowable Stress Design. According to Section 4.2.2 of the AISI Specification[1], the allowable shear per spot for sheets joined by spot welding should be determined from Table 7.1.

Table 7.1 Allowable Shear Per Spot for Resistance Welds

Thickness of Thinnest Outside Sheet, in.	Allowable Shear Strength per Spot, kips
0.010	0.050
0.020	0.125
0.030	0.225
0.040	0.350
0.050	0.525
0.060	0.725
0.080	1.075
0.094	1.375
0.109	1.650
0.125	2.000
0.188	4.000
0.250	6.000

Values for intermediate thicknesses may be obtained by straight-line interpolation.

b. LRFD Criteria. According to Section 10.2.2 of the Tentative Recommendations[10], the factored nominal shear strength, ϕR_n, of spot welding should be determined as follows:

$$\phi = 0.65$$

$$R_n = \text{tabulated value given in Table 7.2, kips}$$

Table 7.2 Nominal Shear Strength Per Spot for Resistance Welds

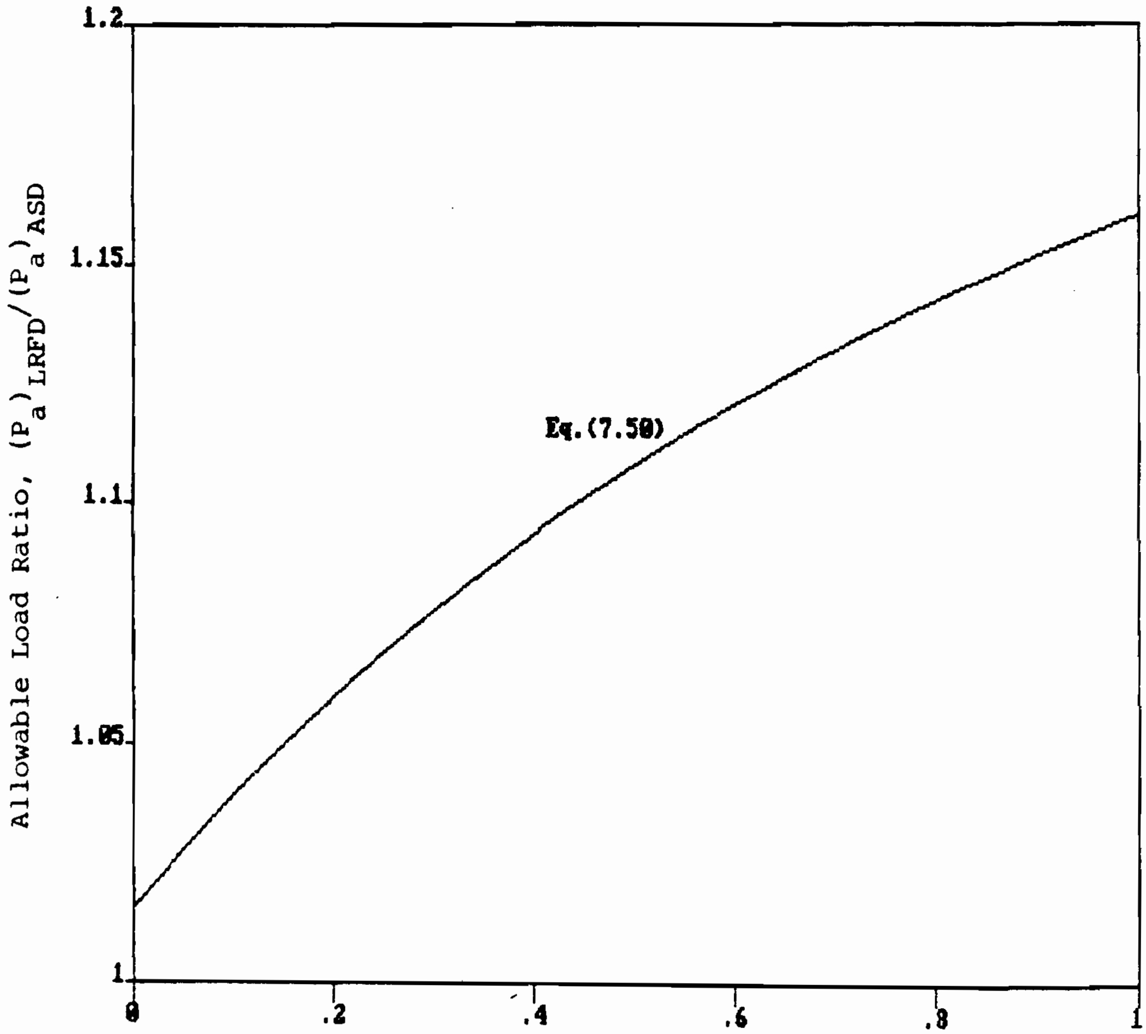
Thickness of Thinnest Outside Sheet, in.	Nominal Shear Strength per Spot, kips
0.010	0.125
0.020	0.313
0.030	0.563
0.040	0.875
0.050	1.310
0.060	1.810
0.080	2.690
0.094	3.440
0.109	4.130
0.125	5.000
0.188	10.000
0.250	15.000

c. Comparison. The allowable load based on LRFD can be calculated using the following equation derived from Eq. (2.6):

$$(P_a)_{LRFD} = \phi R_n (D/L+1)/(1.2D/L+1.6) \tag{7.49}$$

The allowable loads per spot weld for allowable stress design in Table 7.1 were derived from the values in Table 7.2 using a factor of safety of 2.5. Therefore, the following equation for allowable load ratio can be used for $\phi = 0.65$:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.5\phi \frac{D/L+1}{1.2D/L+1.6} = 1.625 \frac{D/L+1}{1.2D/L+1.6} \tag{7.50}$$



Dead-To-Live Load Ratio, D/L

Figure 59. Allowable Load Ratio vs. D/L Ratio for Resistance Welds

The relationship between the allowable load ratio and dead-to-live load ratio is shown in Figure 59 for resistance welds. As shown from the figure, LRFD criteria always result in higher values of allowable load than allowable stress design for all dead-to-live load ratios. For D/L = 0.5, the difference between the allowable loads is 10.8%.

3. Design Examples. See Problems Nos. 7 through 11 in Appendix C for design examples of welded connections using Load and Resistance Factor Design.

C. BOLTED CONNECTIONS

The AISI Specifications[1] and the Tentative Recommendations[10] for bolted connections of cold-formed steel structural members apply to members in which the thickness of the thinnest connected part is less than 3/16 in. The AISC Specifications[3] should be used for bolted connections when the thickness of the thinnest connected part is greater than or equal to 3/16 in.

1. Minimum Spacing and Edge Distance in Line of Stress. The minimum spacing and edge distance in the line of the stress has to be checked to prevent tearing of the steel sheet due to shear.

a. Allowable Stress Design. The distance e measured in the line of force from the center of a standard hole to the nearest edge of an adjacent hole or to the end of the connected part toward which the force is directed should not be less than the value of e_{min} determined from the following equations from Section 4.5.4 of the AISI Specifications[1]:

(i) When $F_u/F_y \geq 1.15$,

$$e_{min} = P/(0.5F_u t) \tag{7.51}$$

(ii) When $F_u/F_y < 1.15$,

$$e_{min} = P/(0.45F_u t) \tag{7.52}$$

where

P = force transmitted by bolt, kips

t = thickness of thinnest connected part, in.

F_u = specified minimum ultimate tensile strength of steel of the connected part, ksi

F_y = specified minimum tensile yield point of steel of the connected part, ksi

b. LRFD Criteria. According to Section 10.3.2 of the Tentative Recommendations[10], the factored nominal shear strength, ϕR_n, of the connected part along two parallel lines in the direction of applied force should be determined as follows:

(i) When $F_u/F_y \geq 1.15$,

$\phi = 0.70$

$$R_n = teF_u \tag{7.53}$$

(ii) When $F_u/F_y < 1.15$,

$\phi = 0.70$

$$R_n = 0.9teF_u \tag{7.54}$$

where

ϕ = resistance factor

R_n = nominal resistance per bolt, kips

e = the distance measured in the line of force from the center of a standard hole to the nearest edge of an adjacent hole or to the end of the connected part, ksi

c. Comparison. For allowable stress design, the allowable load can be computed for a given edge distance by solving for P in Eqs. (7.51) and (7.52).

For $F_u/F_y \geq 1.15$,

$$(P_a)_{ASD} = 0.5teF_u \tag{7.55}$$

For $F_u/F_y < 1.15$,

$$(P_a)_{ASD} = 0.45teF_u \tag{7.56}$$

The allowable load for LRFD can be computed using the following formula developed from Eq. (2.6):

$$(P_a)_{LRFD} = \phi R_n(D/L+1)/(1.2D/L+1.6) \tag{7.57}$$

The allowable loads from Eqs. (7.55) and (7.56) were derived from the ultimate loads in Eqs. (7.53) and (7.54) using a factor of safety of 2.00. Therefore, the allowable load ratio based on plate shearing around the bolt can be computed from the following formula and $\phi = 0.70$:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.0\phi \frac{D/L+1}{1.2D/L+1.6} = 1.4 \frac{D/L+1}{1.2D/L+1.6} \tag{7.58}$$

Figure 60 shows the relationship from Eq. (7.58) between allowable load ratio and dead-to-live load ratio. For D/L = 0.5, the allowable load based on the LRFD criteria is 4.5% lower than the value based on allowable stress design. It can also be seen in the

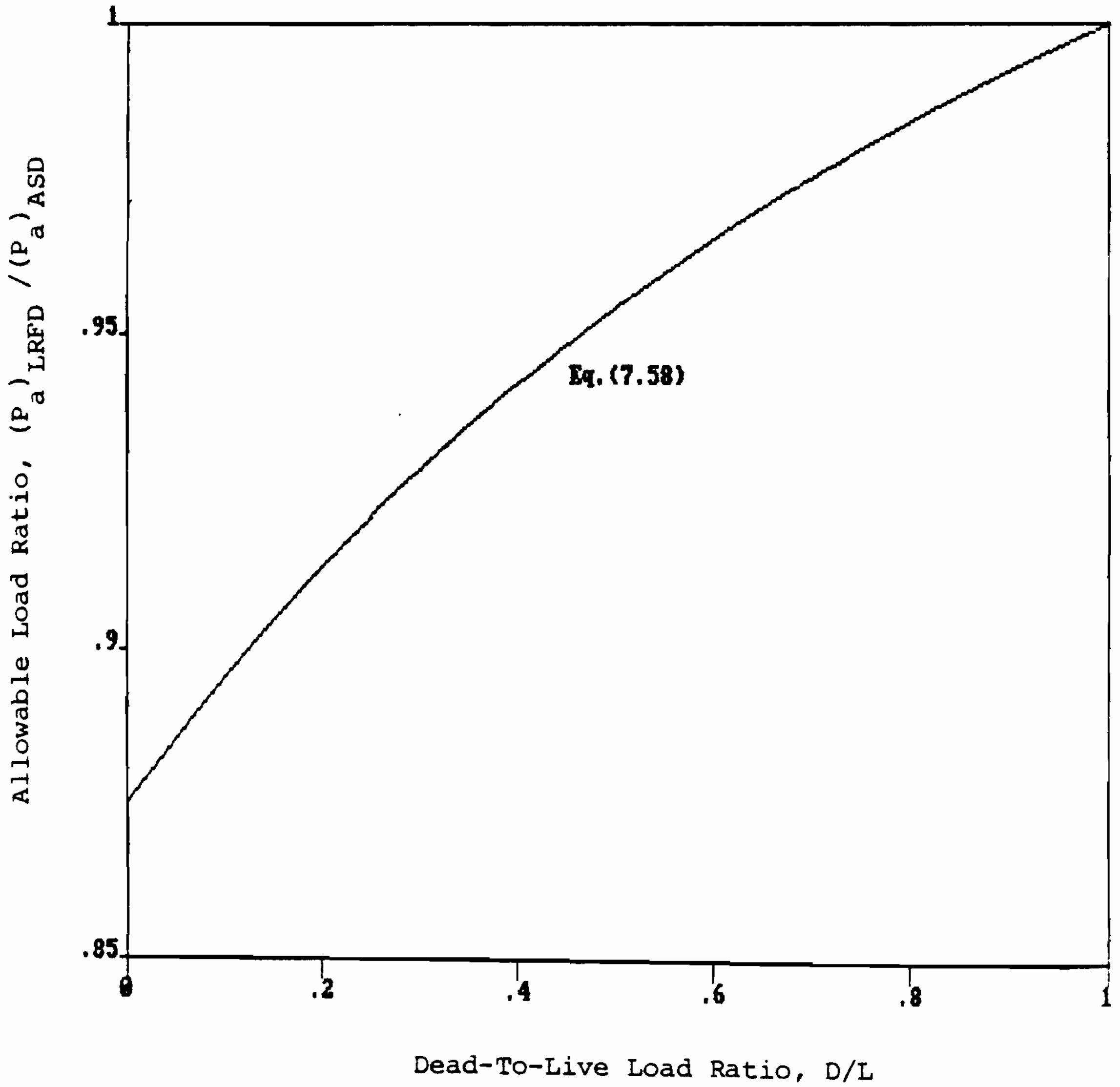


Figure 60. Allowable Load Ratio vs. D/L Ratio for Minimum Edge Distance of Bolts

figure that both design methods result in the same value of allowable load for D/L = 1.0.

2. Tensile Strength on Net Section. Tearing of the net section in tension is caused by stress concentrations resulting from the presence of holes and the concentrated force transmitted by the bolt to the sheets.

a. Allowable Stress Design. According to Section 4.5.5 of the AISI Specification(1), the tension stress on the net section of a bolted connection should not exceed $0.6F_y$ nor should it exceed the following allowable stress:

(i) With washers under both bolt head and nut:

For double shear connection,

$$F_t = (1.0-0.9r+3rd/s)0.50F_u \leq 0.50F_u \qquad (7.59)$$

For single shear connection,

$$F_t = (1.0-0.9r+3rd/s)0.45F_u \leq 0.45F_u \qquad (7.60)$$

(ii) Without washers under both bolt head and nut, or with only one washer:

$$F_t = (1.0-r+2.5rd/s)0.45F_u \leq 0.45F_u \qquad (7.61)$$

where

r = the force transmitted by the bolt or bolts at the section considered, divided by the tension force in the member at that section. If r is less than 0.2, it may be taken as zero.

s = spacing of bolts perpendicular to line of stress, in. In the case of a single bolt, s = width of sheet.

F_t = allowable tension stress on net section, ksi

b. LRFD Criteria. According to Section 10.3.3 of the Tentative Recommendations[10], the factored nominal tensile strength, ϕR_n, on the net section of the connected part should be determined as follows:

(i) With washers under both bolt head and nut,

$$R_n = (1.0-0.9r+3rd/s)F_uA_n \leq F_uA_n \tag{7.62}$$

ϕ = 0.65 for double shear connection

ϕ = 0.60 for single shear connection

(ii) Without washers under both bolt head and nut, or with only one washer,

$\phi = 0.65$

$$R_n = (1.0-r+2.5rd/s)F_uA_n \leq F_uA_n \tag{7.63}$$

In addition, the factored nominal tensile strength should not exceed the following value:

$\phi = 0.90$

$$R_n = F_yA_n \tag{7.64}$$

where

A_n = net area of the connected part, in.^{2}

c. Comparison. For allowable stress design, the allowable tension on the net section can be computed by Eq. (7.65).

$$(P_a)_{ASD} = A_nF_t \tag{7.65}$$

For LRFD, the allowable tension on the net section can be computed using the following equation developed from Eq. (2.6):

$$(P_a)_{LRFD} = \phi R_n(D/L+1)/(1.2D/L+1.6) \tag{7.66}$$

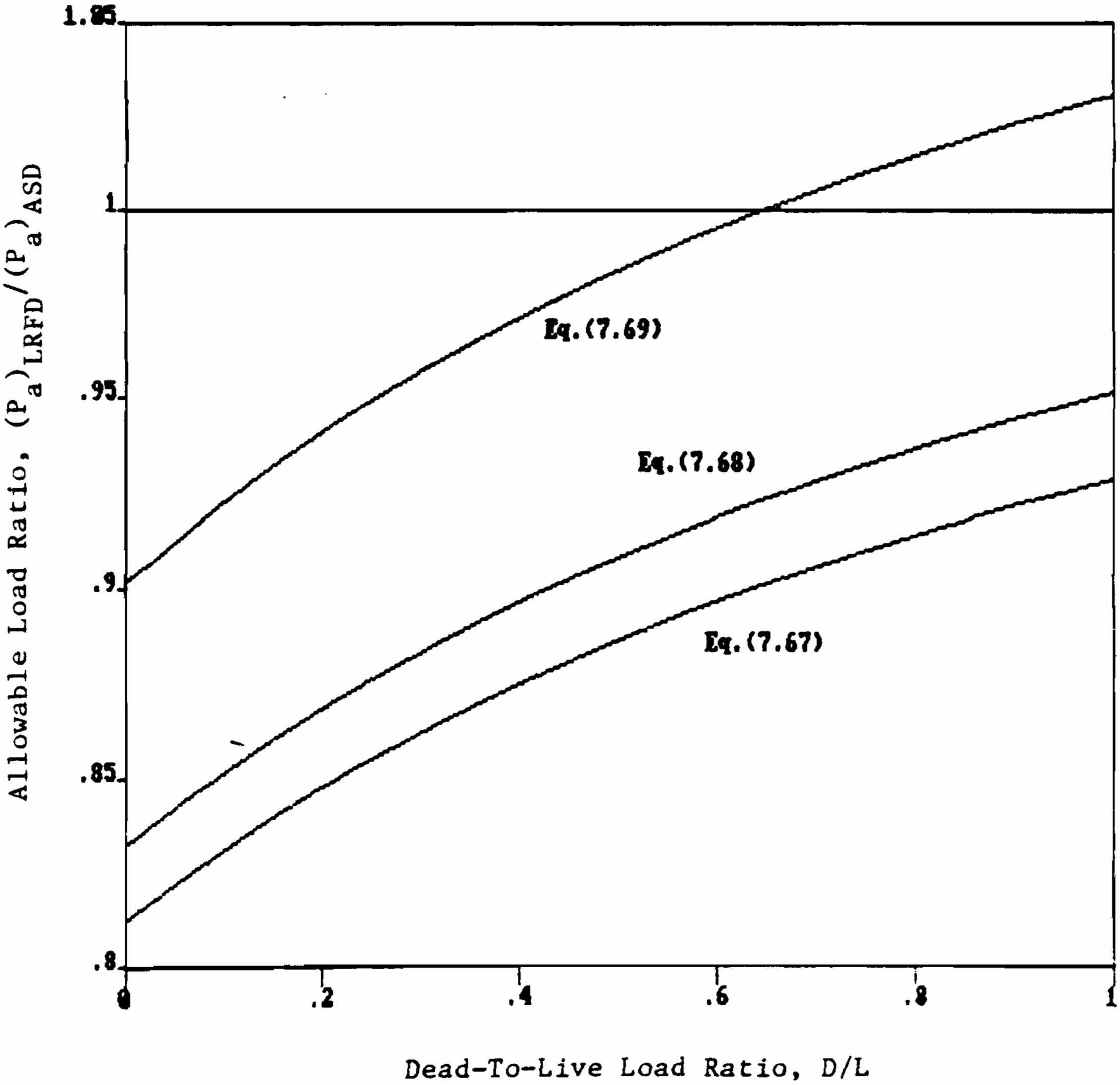


Figure 61. Allowable Load Ratio vs. D/L for Tension on Net Section

The allowable load for double shear connections with washers based on allowable stress design was derived from the nominal tearing load and a factor of safety of 2.0. For single shear connections and connections without washers, a factor of safety of 2.22 was used for allowable stress design. The yielding criteria for the net section was studied in Chapter III of this paper.

The allowable load ratio can be computed as follows:

For double shear connections with washers and $\phi = 0.65$,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.0\phi \frac{D/L+1}{1.2D/L+1.6} = 1.30\frac{D/L+1}{1.2D/L+1.6} \quad (7.67)$$

For single shear connections with washers and $\phi = 0.60$,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.22\phi \frac{D/L+1}{1.2D/L+1.6} = 1.332 \frac{D/L+1}{1.2D/L+1.6} \quad (7.68)$$

For connections without washers and $\phi = 0.65$,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.22\phi \frac{D/L+1}{1.2D/L+1.6} = 1.443 \frac{D/L+1}{1.2D/L+1.6} \quad (7.69)$$

Figure 61 shows the allowable load ratio versus dead-to-live load ratio for the three cases represented by Eqs. (7.67), (7.68), and (7.69). As shown in the figure, the criteria for tension on the net section result in a wide range of allowable load ratios. For D/L = 0.5, the allowable load based on the LRFD criteria is from 1.8% to 12% lower than the value based on allowable stress design. The difference depends on the use of washers and the type of connections. Figure 61 also shows that LRFD is very conservative for connections with washers under the bolt head and nut compared with allowable stress design.

3. <u>Bearing Strength in Bolted Connections</u>. Bearing failure occurs when the steel sheet piles up in front of the bolts. This

occurs when the edge distance or longitudinal spacing of the bolts is relatively large.

a. Allowable Stress Design. The bearing stress on the area (dxt) should not exceed the allowable, F_p, computed from Section 4.5.6 of the AISI Specification[1] as follows:

(i) Bolted connections with washers under both bolt head and nut:

For inside sheets of double shear connections,

$$F_p = 1.50F_u, \text{ for } F_u/F_y \geq 1.15 \tag{7.70}$$

$$F_p = 1.35F_u, \text{ for } F_u/F_y < 1.15 \tag{7.71}$$

For single shear and outside sheets of double shear connections,

$$F_p = 1.35F_u \tag{7.72}$$

(ii) Bolted connections without washers or with only one:

For inside sheets of double shear connections,

$$F_p = 1.35F_u, \text{ for } F_u/F_y \geq 1.15 \tag{7.73}$$

For single shear and outside sheets of double shear connections,

$$F_p = 1.00F_u, \text{ for } F_u/F_y \geq 1.15 \tag{7.74}$$

For conditions not listed, stresses should be determined on the basis of test data using a factor of safety of 2.22.

b. LRFD Criteria. According to Section 10.3.4 of the Tentative Recommendations[10], the factored nominal bearing strength, ϕR_n, should be determined as follows:

(i) Bolted connections with washers under both bolt head and nut:

For inside sheets of double shear connections with $F_u/F_y \geq 1.15$,

$$\phi = 0.60$$

$$R_n = 3.5F_u dt \tag{7.75}$$

For inside sheets of double shear connections with $F_u/F_y < 1.15$,

$$\phi = 0.70$$

$$R_n = 3.0F_u dt \tag{7.76}$$

For single shear and outside sheets of double shear connections,

$$\phi = 0.65$$

$$R_n = 3.0F_u dt \tag{7.77}$$

(ii) Bolted connections without washers or with only one:

For inside sheets of double shear connections with $F_u/F_y \geq 1.15$,

$$\phi = 0.70$$

$$R_n = 3.0F_u dt \tag{7.78}$$

For single shear and outside sheets of double shear connections with $F_u/F_y \geq 1.15$,

$$\phi = 0.70$$

$$R_n = 2.2F_u dt \tag{7.79}$$

For conditions not listed, the factored nominal bearing strength of bolted connections should be determined by tests.

c. Comparison. The allowable load based on allowable stress design can be computed using the following equation:

$$(P_a)_{ASD} = F_p td \tag{7.80}$$

For LRFD, the following equation developed from Eq. (2.6) can be used to calculate the allowable load:

$$(P_a)_{LRFD} = \phi R_n (D/L+1)/(1.2D/L+1.6) \tag{7.81}$$

The factor of safety used in the development of the allowable stress design formulas was around 2.22. Therefore, the allowable load ratios can be computed as follows:

(i) Connections with washers:

For inside sheets of double shear connections with $F_u/F_y \geq 1.15$ and $\phi = 0.60$,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.40 \quad \frac{D/L+1}{1.2D/L+1.6} \tag{7.82}$$

For inside sheets of double shear connections with $F_u/F_y < 1.15$ and $\phi = 0.7$,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.556 \quad \frac{D/L+1}{1.2D/L+1.6} \tag{7.83}$$

For single shear and outside sheets of double shear connections with $\phi = 0.65$,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.444 \quad \frac{D/L+1}{1.2D/L+1.6} \tag{7.84}$$

(ii) Connections without washers or with only one washer:

For inside sheets of double shear connections with $F_u/F_y \geq 1.15$ and $\phi = 0.70$

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.556 \quad \frac{D/L+1}{1.2D/L+1.6} \tag{7.85}$$

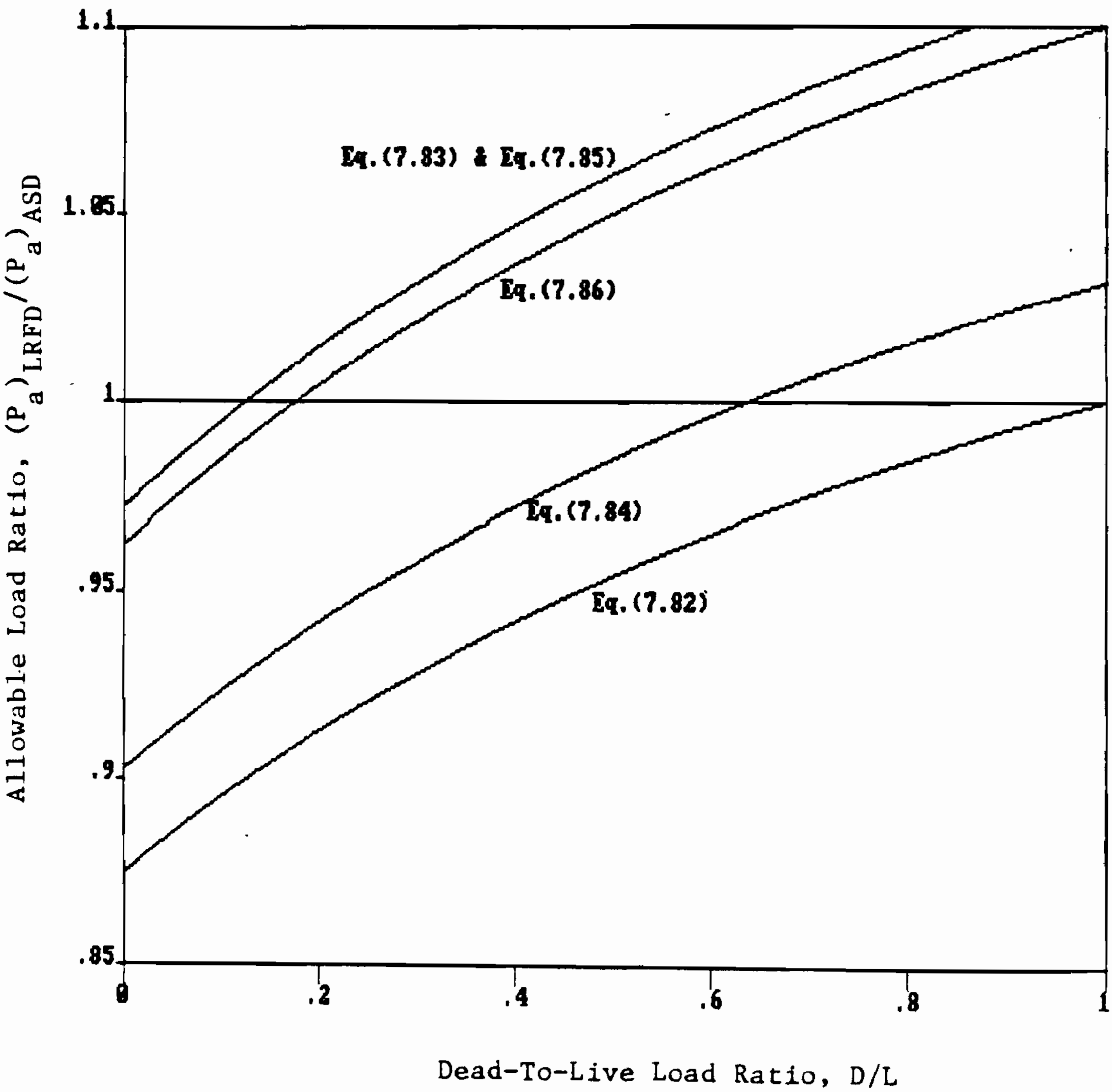


Figure 62. Allowable Load Ratio vs. D/L Ratio for Bearing Strength of Bolted Connections

For single shear and outside sheets of double shear connections with $F_u/F_y \geq 1.15$ and $\phi = 0.70$,

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 1.54 \frac{D/L+1}{1.2D/L+1.6} \tag{7.86}$$

The relationship between allowable load ratio and dead-to-live load ratio for Eqs. (7.82) through (7.86) are shown in Figure 62. As shown in the figure, the criteria for bearing strength of bolted connections result in a wide range of values for allowable load ratio. For D/L = 0.5, the allowable load based on LRFD is from 6.1% higher to 4.6% lower than the value obtained from allowable stress design. The difference between the allowable loads will depend upon the use of the washers, the shear conditions, and the F_u/F_y ratio. Inside sheets of double shear bolted connection with washers designed using LRFD will be very conservative compared with allowable stress design.

4. Shear Strength of Bolts. The strength of the bolts in shear have to be checked for bolted connections.

a. Allowable Stress Design. According to Section 4.5.7 of the AISI Specification[1], the shear stress on the gross cross-sectional area of bolts designed for dead and live loads should not exceed the following allowable shear stresses:

(i) ASTM A307-78 Bolts, Type A	10 ksi
(ii) ASTM A325-79 Bolts	
When threading is excluded from shear planes	30 ksi
When threading is not excluded from shear planes	21 ksi
(iii) ASTM A354-79 Grade BD Bolts (d < 1/2 in.)	
When threading is excluded from shear planes	40 ksi

When threading is not excluded from shear planes 24 ksi

(iv) ASTM A449-78a Bolts (d < 1/2 in.)

When threading is excluded from shear planes 30 ksi

When threading is not excluded from shear planes 18 ksi

(v) ASTM A490-79 Bolts

When threading is excluded from shear planes 40 ksi

When threading is not excluded from shear planes 28 ksi

b. LRFD Criteria. According to Section 10.3.5 of the Tentative Recommendations(10), the factored nominal shear strength, ϕR_n, of bolts should be determined as follows:

$$R_n = 0.6mA_{sA}F_u \tag{7.87}$$

$\phi = 0.65$, for A307

$\phi = 0.65$, for A325 and A449 bolts

$\phi = 0.65$, for A490 and A354 Grade BD bolts

where

m = the number of shear planes per bolt

A_{sA} = stress area when threading is included in shear planes; gross area when threading is excluded from shear planes, in.^{2}

F_u = ultimate tensile strength of bolt, ksi

c. Comparison. The allowable load based on allowable stress design can be computed as follows:

$$(P_a)_{ASD} = F_vA_g \tag{7.88}$$

where

F_v = allowable shear stress of bolt from Section 4.5.7 of the AISI Specification(1), ksi

A_g = gross cross-sectional area of bolt, in.^{2}

For LRFD, the ultimate load depends on the stress area of the bolt. When threading is excluded from the shear plane, the stress area is the gross cross-sectional area of the bolt. When threading is included in the shear plane, the stress area is the root area, A_r, of the bolt. Table 7.3 lists the cross-sectional areas and the A_r/A_g ratios used in this study. The ultimate tensile strengths of the different bolt types are listed in Table 7.4 along with allowable shear stresses. The allowable shear load based on LRFD can be calculated using the following formula developed from Eq. (2.6):

$$(P_a)_{LRFD} = \phi R_n (D/L+1)/(1.2D/L+1.6) \qquad (7.89)$$

For cases when threading is excluded in the shear plane, the allowable load based on LRFD can be obtained from the following equation:

$$(P_a)_{LRFD} = \phi(0.6A_gF_u)(D/L+1)/(1.2D/L+1.6) \qquad (7.90)$$

Therefore, the allowable load ratio for shear strength of bolts with threads excluded from the shear plane is:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 0.6\phi\left(\frac{F_u}{F_v}\right)\frac{D/L+1}{1.2D/L+1.6} \qquad (7.91)$$

For cases when threading is included in the shear plane, the allowable load based on LRFD can be obtained from the following equation:

$$(P_a)_{LRFD} = \phi(0.6A_rF_u)(D/L+1)/(1.2D/L+1.6) \qquad (7.92)$$

Therefore, the allowable load ratio for shear strength of bolts with threads included in the shear plane is:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 0.6\phi\left(\frac{F_u}{F_v}\right)\left(\frac{A_r}{A_g}\right)\frac{D/L+1}{1.2D/L+1.6} \qquad (7.93)$$

Table 7.3 Cross-Sectional Areas of Bolts

Diameter (in.)	Gross Area ($in.^2$)	Root Area ($in.^2$)	A_r/A_g
1/4	0.049	0.027	0.551
3/8	0.110	0.068	0.618
1/2	0.196	0.126	0.643
5/8	0.307	0.202	0.658
3/4	0.442	0.302	0.683
7/8	0.601	0.419	0.697
1	0.785	0.551	0.702

Table 7.4 Properties of Bolts

Bolt Type		F_v, (ksi) Threads Excluded	F_v, (ksi) Threads Included	F_u (ksi)
A307-78-A	1/4"-1"	10	10	60
A325-79	1/2"-1"	30	21	120
A354-79-BD	1/4"-3/8"	40	24	150
A449-78a	1/4"-3/8"	30	18	120
A490-79	1/2"-1"	40	28	150

Equations (7.91) and (7.93) can be expressed in the following form:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = (K_b) \quad \frac{D/L+1}{1.2D/L+1.6} \tag{7.94}$$

where

When threads are excluded,

$$K_b = 0.6\phi(F_u/F_v) \tag{7.95}$$

When threads are included,

$$K_b = 0.6\phi(F_u/F_v)(A_r/A_g) \tag{7.96}$$

Table 7.5 lists the values of K_b calculated from the bolt areas and properties provided in Tables 7.3 and 7.4. Figures 63 through 67 show the relationship between the allowable load ratio and dead-to-live load ratio for the bolts in Table 7.5 using Eq. (7.94).

Figure 63 shows the allowable load ratio versus dead-to-live load for A307-78 type A bolts based on shear strength. As seen from the figure, the allowable load ratio varies with the size of bolt and the D/L ratio. For D/L = 0.5 and when threads are included in the shear plane, allowable loads based on LRFD will be from 12% smaller to 12% greater than the values based on allowable stress design. The difference between the allowable loads increases as the bolt diameter increases.

For threads excluded from the shear plane of connections with A307-78 type A bolts, LRFD criteria result in allowable loads much greater than that obtained from allowable stress design. For D/L = 0.5, the difference would be 60%. This means the allowable

Table 7.5 K_b Values for Standard Bolts

Diameter (in.)	A307-78-A $\phi = 0.65$		A325-79 $\phi = 0.65$		A354-79-BD $\phi = 0.65$		A449-78a $\phi = 0.65$		A490-79 $\phi = 0.65$	
	EX	IN	EX	IN	EX	IN	EX	IN	EX	IN
1/4	2.340	1.289	--	--	1.463	1.343	1.560	1.432	--	--
3/8	2.340	1.446	--	--	1.463	1.506	1.560	1.607	--	--
1/2	2.340	1.505	1.560	1.433	--	--	--	--	1.463	1.343
5/8	2.340	1.539	1.560	1.467	--	--	--	--	1.463	1.375
3/4	2.340	1.598	1.560	1.522	--	--	--	--	1.463	1.427
7/8	2.340	1.630	1.560	1.554	--	--	--	--	1.463	1.456
1	2.340	1.642	1.560	1.564	--	--	--	--	1.463	1.466

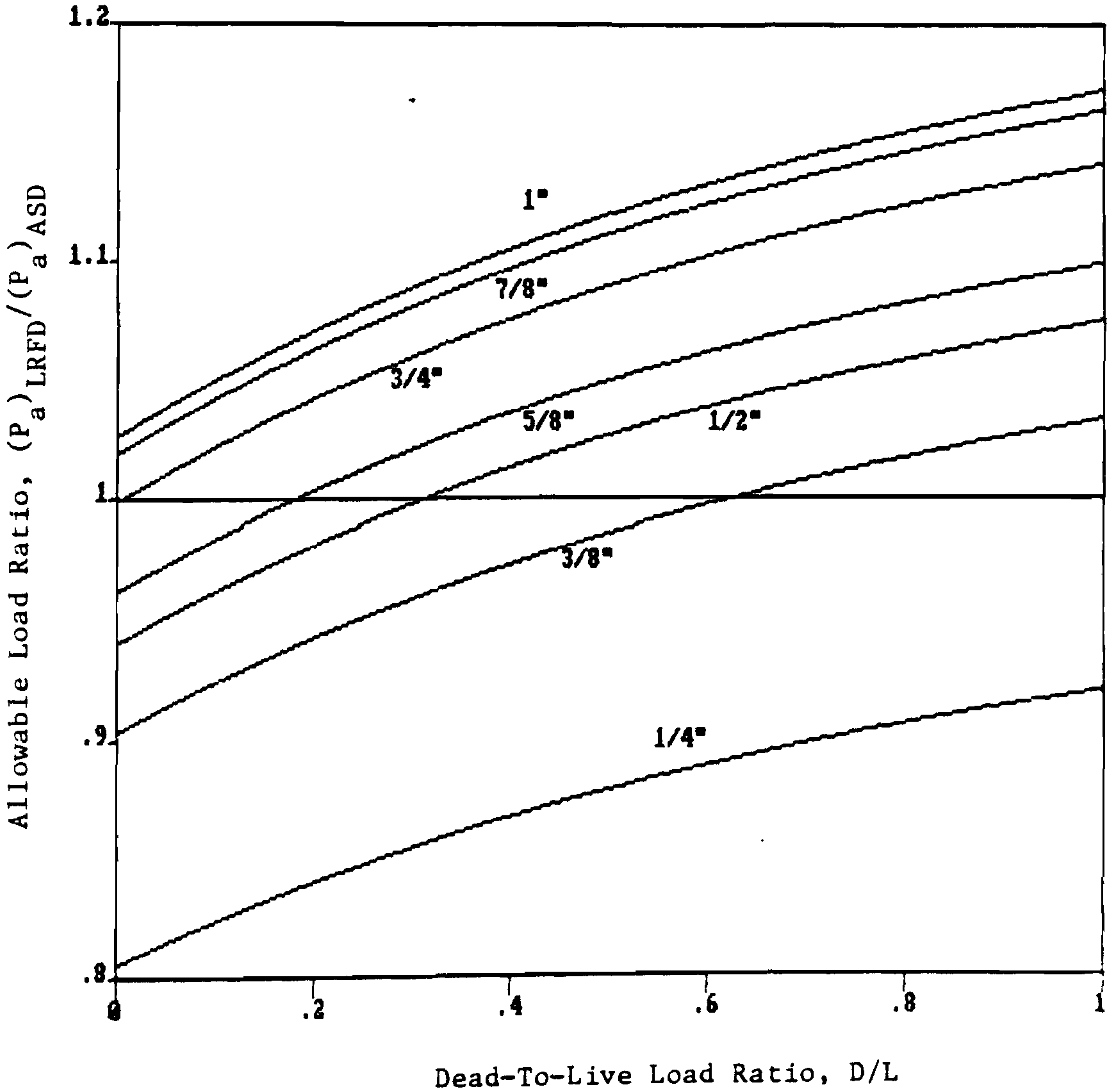


Figure 63. Allowable Load Ratio vs. D/L Ratio for Shear on A307-78 Type A Bolts

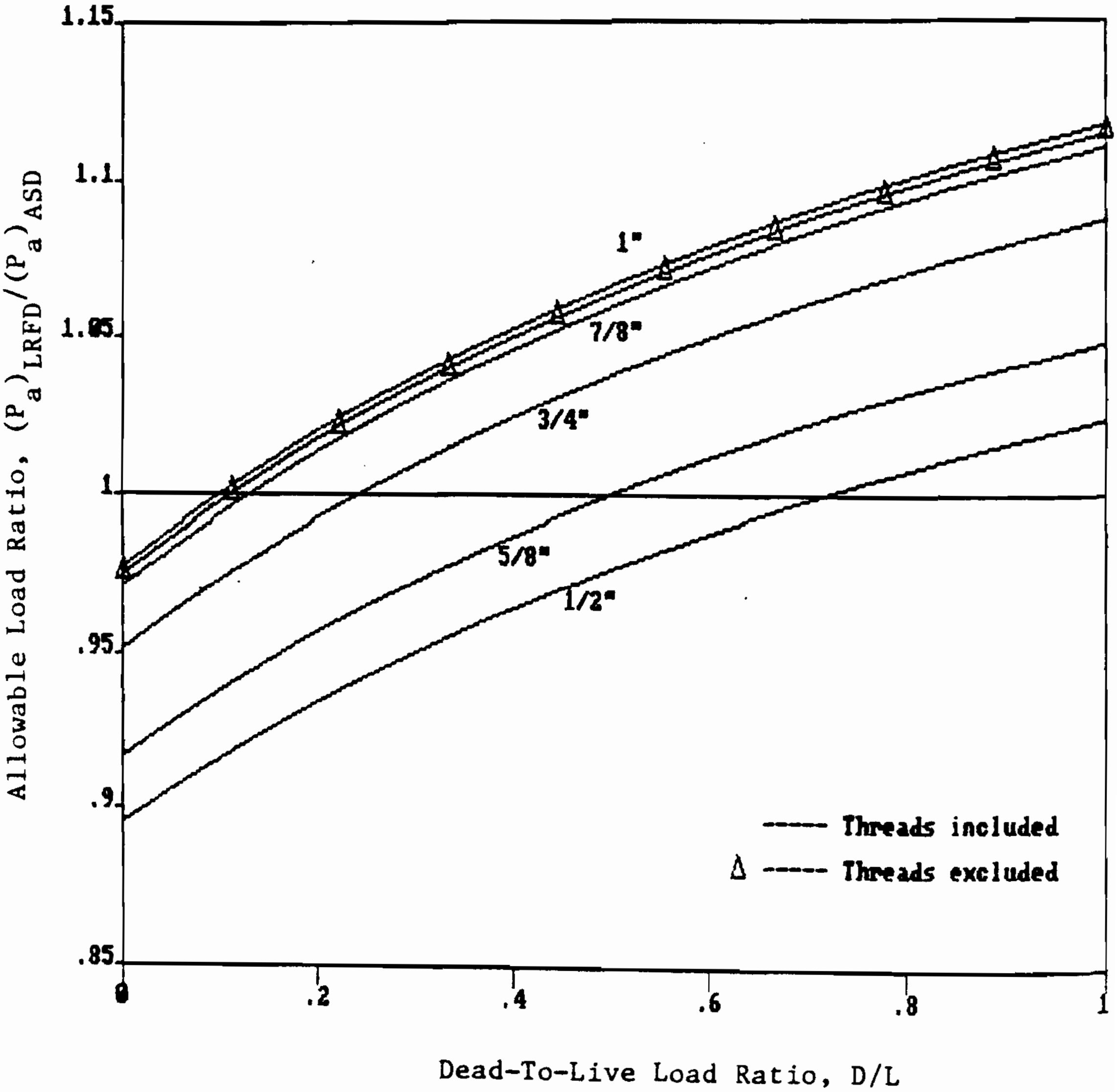


Figure 64. Allowable Load Ratio vs. D/L Ratio for Shear on A325-79 Bolts

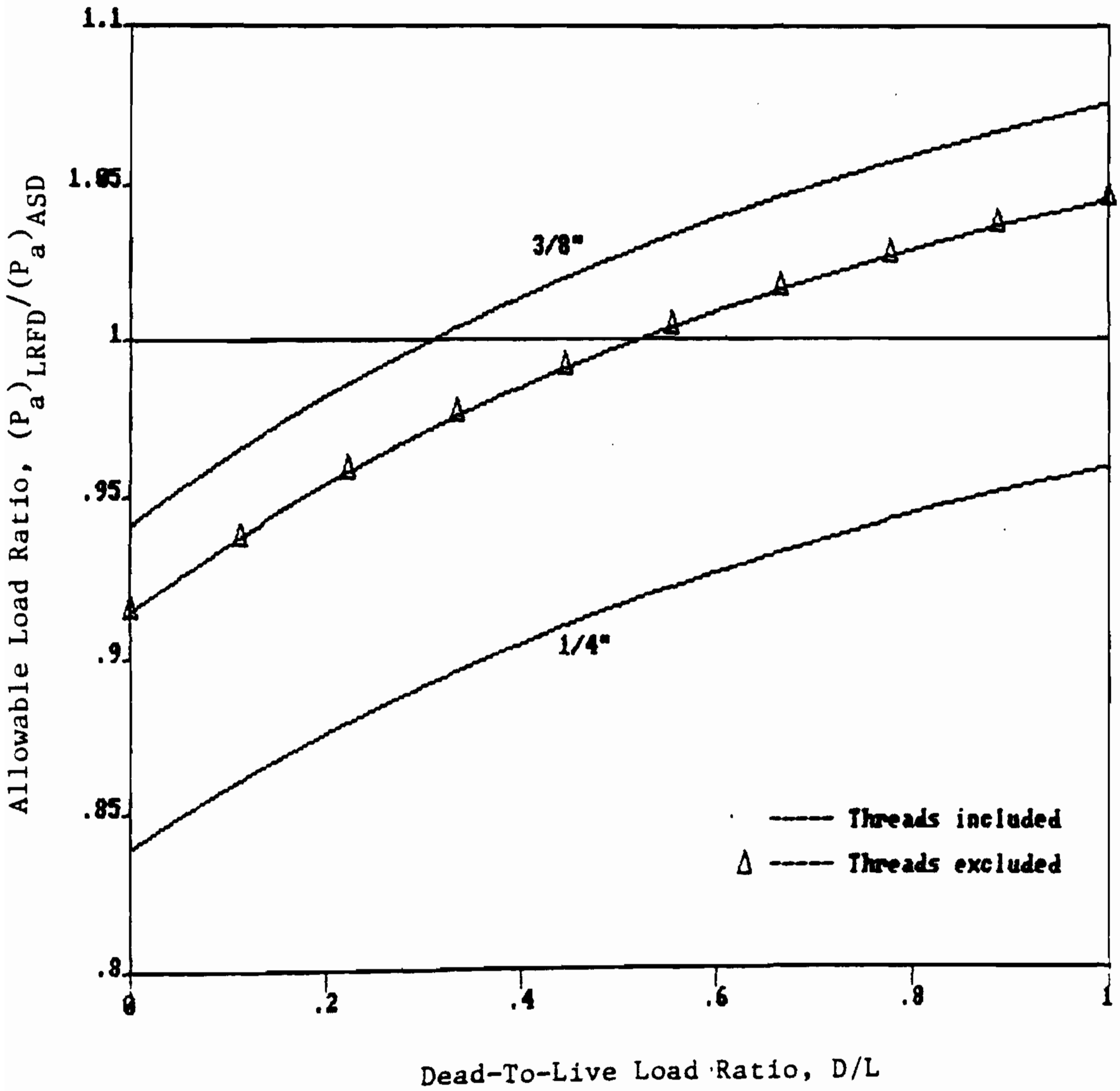


Figure 65. Allowable Load Ratio vs. D/L Ratio for Shear on A354-79 Type BD Bolts

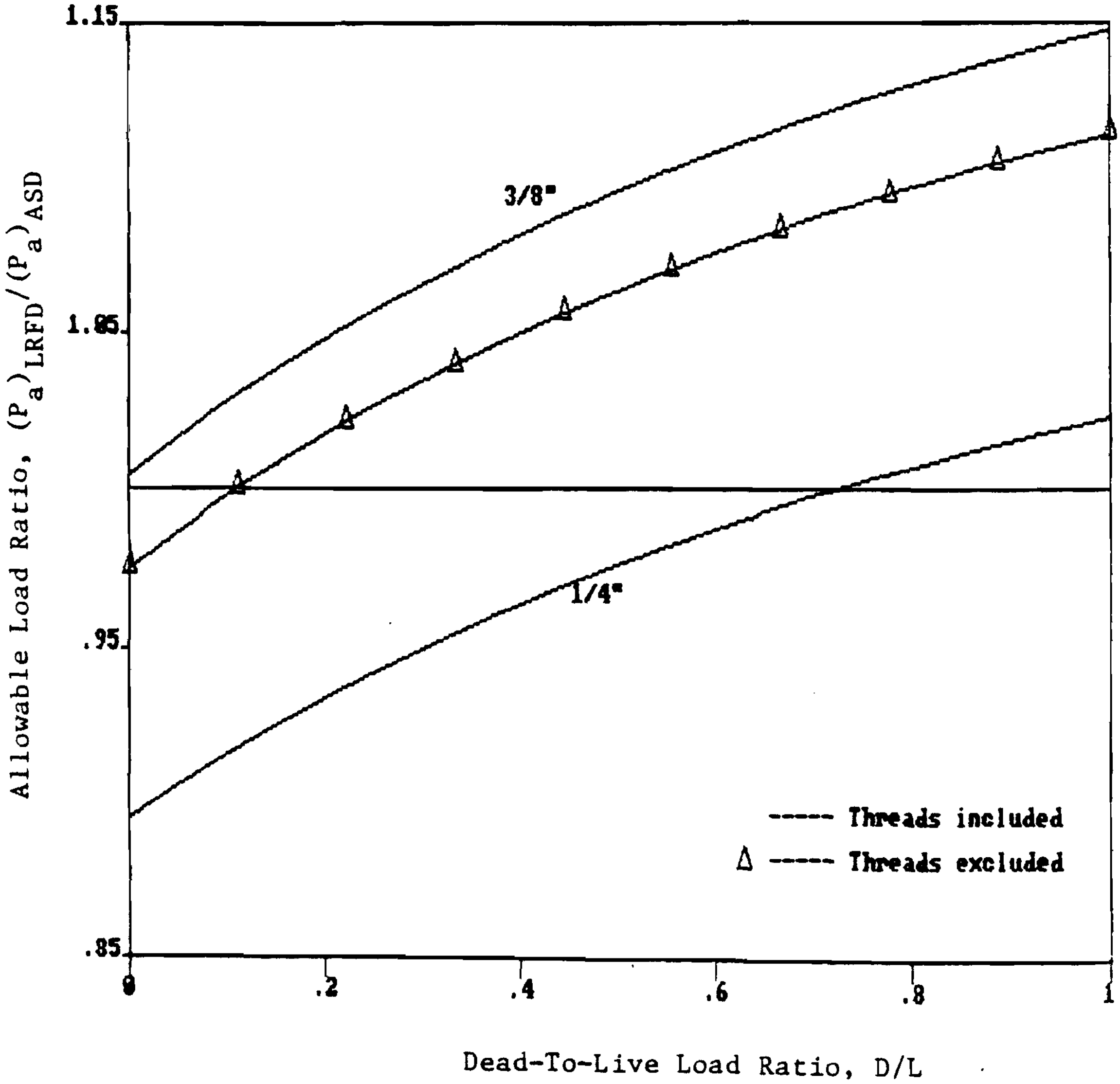


Figure 66. Allowable Load Ratio vs. D/L Ratio for Shear on A449-78a Bolts

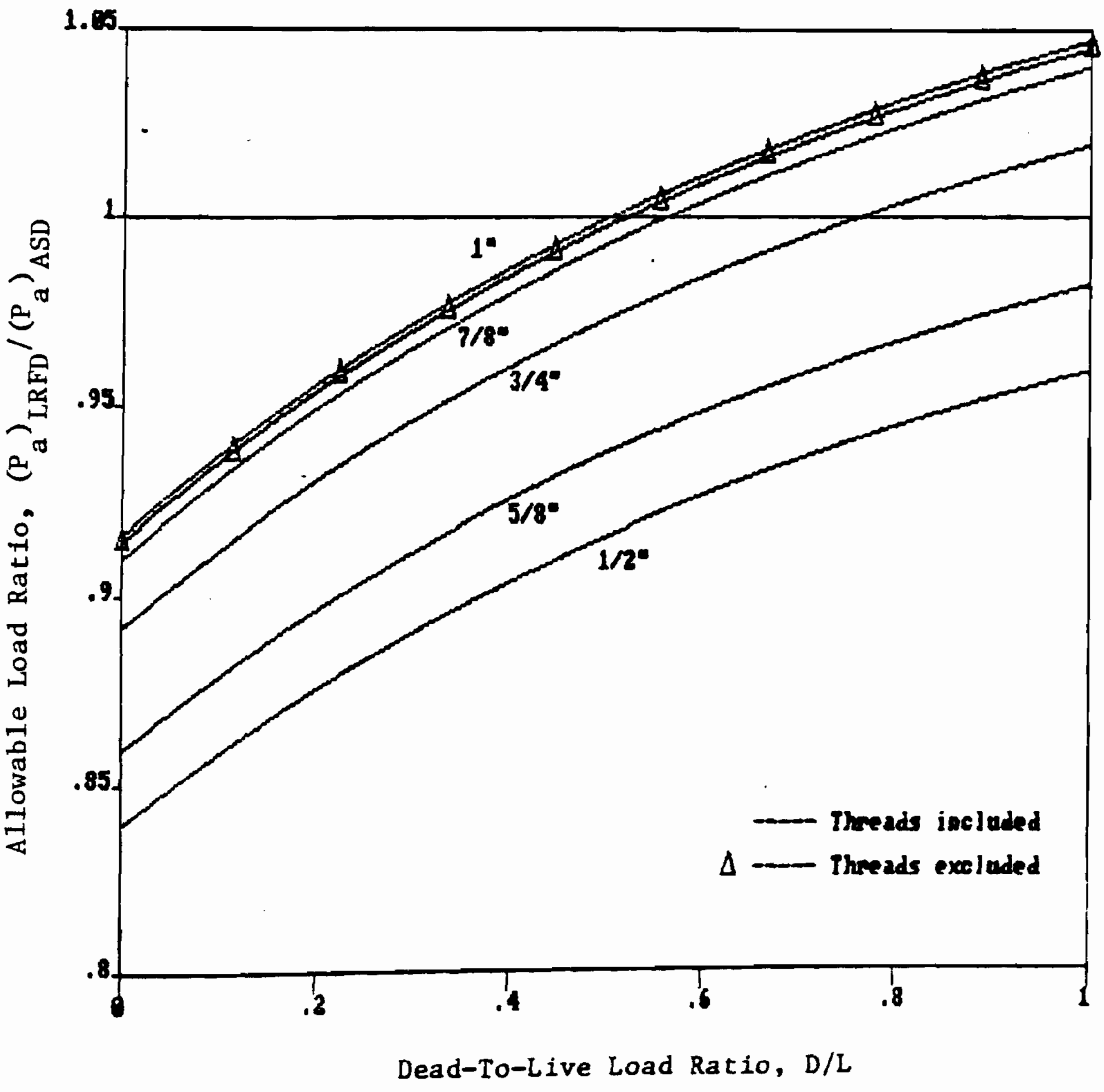


Figure 67. Allowable Load Ratio vs. D/L Ratio for Shear on A490-79 Bolts

load obtained from LRFD is almost 1.6 times the allowable load obtained from allowable stress design for this case.

The relationship between the allowable load ratio and dead-to-live load ratio for A325-79 bolts is shown in Figure 64. For D/L = 0.5, LRFD will result in an allowable load from 2.2% smaller to 6.7% higher than the value from allowable stress design. The curve represented by the line with triangular symbols is for all bolt diameters when threading is excluded from the shear plane.

For A354-79 type BD bolts, the relationship between the allowable load ratio and dead-to-live load ratio is shown in Figure 65. For D/L = 0.5, the allowable shear load based on LRFD will be from 8.5% smaller to 2.8% higher than that based on allowable stress design.

Figure 66 illustrates the same relationship for A449-78a bolts based on shear strength. For 3/8-in. diameter bolt, LRFD always results in allowable loads greater than that for allowable stress design. The load ratio ranges from 0.98 to 1.10, depending upon bolt diameter and position of threads for D/L = 0.5.

Figure 67 also illustrates the same relationship from Eq. (7.94) for A490-79 bolts. As shown in the figure, allowable load ratio increases as bolt diameter increases for cases when threading is included in the shear plane. For D/L = 0.5, the allowable load based on LRFD is 8.4% smaller than the value based on allowable stress design for 1/2-in. diameter bolt.

5. Design Example. See Problem No. 12 in Appendix C for a design example of a bolted connection using Load and Resistance Factor Design.

VIII. CONCLUSIONS

Currently, the 1980 Edition of the Specification for the Design of Cold-Formed Steel Structural Members published by the American Iron and Steel Institute applies to the design of cold-formed steel members and connections for load-carrying purposes in buildings[1]. This specification provides design formulas for determining allowable stresses or allowable loads for tension members, compression members, flexural members, and connections based on appropriate factors of safety recommended by AISI for different types of structural members.

The Load and Resistance Factor Design method for cold-formed steel members and connections has recently been studied by using probabilistic and statistical techniques to account for the uncertainties in design, fabrication, material properties, and applied loads. The Tentative Recommendations on the LRFD Criteria were developed from a joint research project conducted at the University of Missouri-Rolla and Washington University[10].

This report compares these two methods for the design of cold-formed steel structural members using the proposed load and resistance factor design criteria and the allowable stress design criteria being used in the AISI Specification. Following a review of literature and discussion of different design variables used in both criteria, allowable loads using each design method were calculated for tension members, flexural members, compression members, beam-columns, and connections. These allowable loads were then compared in Chapters III through VII for different types of structural members and connections. For some cases, specific examples were used in this study due to the complexity of the analysis.

For all types of structural members only the dead and live load combination was studied in this investigation. It was found that the D/L ratio has a significant effect on the allowable load ratio. In general, the allowable load ratio, $(P_a)_{LRFD}/(P_a)_{ASD}$, increases as the dead-to-live load ratio increases. Because cold-formed steel members are usually thin, the dead-to-live load ratios of such light weight members are expected to be lower than the ratios used for other building materials. In general practice, the dead-to-live load ratios used in building design of cold-formed steel members are less than 1/3. In view of the fact that the load factor used for live load is 1.6 which is larger than the load factor of 1.2 used for dead load and that the LRFD criteria were found to be conservative for unusually small D/L ratios.

In addition to the effect of the dead-to-live load ratio, the resistance factors used in the LRFD criteria and the factors of safety used in allowable stress design also contribute to the differences between the allowable loads computed from two different methods. As the safety factor or resistance factor increases, the ratio of $(P_a)_{LRFD}/(P_a)_{ASD}$ also increases. For a given set of statistical data and a selected safety index, the resistance factor can be determined by Eq. (2.5). This equation is a function of the mean value and coefficient of variation of the professional factor which is the ratio of the tested load to the predicted load. A low value of the resistance factor is resulted from a low value of P_m and a large value of V_p which represents a big scatter of test results. This was the case for welded connections and plate failure of bolted connections.

For each type of structural members and connections, design examples were prepared and presented in Appendix C. The answers for all problems were compared with the general curves discussed in the text.

The load and resistance factor design method is a rational approach for structural design. The research findings obtained from this comparative study of the current method based on allowable stress design and the proposed LRFD criteria can provide a useful reference for future revision of the current AISI Specification and the proposed tentative recommendations on LRFD criteria.

BIBLIOGRAPHY

1. "Specification for the Design of Cold-Formed Steel Structural Members," American Iron and Steel Institute, Washington, D. C. 1980.

2. Galambos, T.V., "Proposed Criteria for Load and Resistance Factor Design of Steel Building Structures," Bulletin No. 27, Washington University, St. Louis, January 1978.

3. "Specifications for the Design, Fabrication and Erection of Structural Steel for Buildings," American Institute of Steel Construction, November, 1978.

4. Rang, T.N., Galambos, T.V., and Yu, W. W., "Load and Resistance Factor Design of Cold-Formed Steel," First Progress Report, submitted to American Iron and Steel Institute, January 1979.

5. Rang, T.N., Galambos, T.V., and Yu, W. W., "Load and Resistance Factor Design of Cold-Formed Steel," Second Progress Report, submitted to American Iron and Steel Institute, January 1979.

6. Rang, T.N., Galambos, T.V., and Yu, W. W., "Load and Resistance Factor Design of Cold-Formed Steel," Third Progress Report, submitted to American Iron and Steel Institute, January 1979.

7. Rang, T.N., Galambos, T.V., and Yu, W. W., "Load and Resistance Factor Design of Cold-Formed Steel," Fourth Progress Report, submitted to American Iron and Steel Institute, January 1979.

8. Supornsilaphachia, B., Galambos, T.V., and Yu, W. W., "Load and Resistance Factor Design of Cold-Formed Steel," Fifth Progress Report, submitted to American Iron and Steel Institute, September 1979.

9. Galambos, T.V., and Yu, W. W., "Load and Resistance Factor Design of Cold-Formed Steel," Sixth Progress Report, submitted to American Iron and Steel Institute, March 1980.

10. Galambos, T.V., and Yu, W. W., "Load and Resistance Factor Design of Cold-Formed Steel," Seventh Progress Report, submitted to American Iron and Steel Institute, September 1985 (Draft).

11. Ravindra, M.K., and Galambos, T.V., "Load and Resistance Factor Design for Steel," Journal of the Structural Division, ASCE, Vol. 104, No. ST9, September 1978.

12. Yura, J.A., Galambos, T.V., and Ravindra, M.K., "The Bending Resistance of Steel Beams," Journal of the Structural Division, ASCE, Vol. 104, No. ST9, September 1978.

13. Bjorhovde, R., Galambos, T.V., and Ravindra, M.K., "LRFD Criteria for Steel Beam-Columns," Journal of the Structural Division, ASCE, Vol. 104, No. ST9, September 1978.

14. Cooper, P.B., Galambos, T.V., and Ravindra, M.K., "LRFD Criteria for Plate Girders," Journal of the Structural Division, ASCE, Vol. 104, No. ST9, September 1978.

15. Hansell, W.C., Galambos, T.V., Ravindra, M.K., and Viest, I.M., "Composite Beam Criteria in LRFD," Journal of the Structural Division, ASCE, Vol. 104, No. ST9, September 1978.

16. Fisher, J.W., Galambos, T.V., Kulak, G.L., and Ravindra, M.K., "Load and Resistance Factor Design Criteria for Connectors," Journal of the Structural Division, ASCE, Vol. 104, No. ST9, September 1978.

17. Ravindra, M.K., Cornell, C.A., and Galambos, T.V., "Wind and Snow Load Factors for Use in LRFD," Journal of the Structural Division, ASCE, Vol. 104, No. ST9, September 1978.

18. Galambos, T.V., and Ravindra, M.K., "Properties of Steel for Use in LRFD," Journal of the Structural Division, ASCE, Vol. 104, No. ST9, September 1978.

19. Grigoriu, M., Veneziano, D., and Cornell, C.A., "Probabilistic Modeling as Decision Making," Journal of the Engineering Mechanics Division, ASCE, Vol. 105, No. EM4, August 1979.

20. Chalk, P.L., and Corotis, R.B., "Probability Model for Design Live Loads," Journal of the Structural Division, ASCE, Vol. 106, No. ST10, October 1980.

21. Ellingwood, B., "Reliability Based Criteria for Reinforced Concrete Design," Journal of the Structural Division, ASCE, Vol. 105, No. ST4, April 1979.

22. Ellingwood, B., "Reliability of Current Reinforced Concrete Design," Journal of the Structural Division, ASCE, Vol. 105, No. ST4, April 1979.

23. Ellingwood, B., "Reliability of Wood Structural Elements," Journal of the Structural Division, ASCE, Vol. 107, No. ST1, January 1981.

24. Ellingwood, B., "Analysis of Reliability for Masonry Structures," Journal of the Structural Division, ASCE, Vol. 107, No. ST5, May 1981.

25. Galambos, T.V., Ellingwood, B., MacGregor, J.G., and Cornell, C.A., "Probability Based Load Criteria: Assessment of Current Design Practice," Journal of the Structural Division, ASCE, Vol. 108, No. ST5, May 1982.

26. Galambos, T.V., Ellingwood, B., MacGregor, J.G., and Cornell, C.A., "Probability Based Load Criteria: Load Factors and Load Combinations," Journal of the Structural Division, ASCE, Vol. 108, No. ST5, May 1982.

27. "Development of a Probability Based Load Criterion for American National Standard A58," National Bureau of Standards Special Publication No. 577, National Bureau of Standards, Washington, D.C., June 1980.

28. Committee on Fatique and Fracture Reliability, " Fatique Reliability: Introduction," Journal of the Structural Division, ASCE, Vol. 108, No. ST1, January 1982.

29. Committee on Fatique and Fracture Reliability, "Fatique Reliability: Quality Assurance and Maintainability," Journal of the Structural Division, ASCE, Vol. 108, ST1, January 1982.

30. Committee on Fatique and Fracture Reliability, "Fatique Reliability: Variable Amplitude Loading," Journal of the Structural Division, ASCE, Vol. 108, No. ST1, January 1982.

31. Knab, L.I., and Lind, N.C., "Reliability Based Design Criteria for Temporary Cold-Formed Steel Buildings," Proceedings of the Third International Specialty Conference on Cold-Formed Steel Structures, University of Missouri-Rolla, Missouri, 1975.

32. Rang, T.N., Galambos, T.V., Yu, W. W., and Ravindra, M. K., "Load and Resistance Factor Design of Cold-Formed Steel Structural Members," Proceedings of the Fourth International Specialty Conference on Cold-Formed Steel Structures, University of Missouri-Rolla, Missouri, 1978.

33. Galambos, T.V., Rang, T.N., Yu, W. W., and Ravindra, M.K., "Structural Reliability Analysis of Cold-Formed Steel Members," Proceedings of the ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability, Tucson, Arizona, January 1979.

34. Rang, T.N., Yu, W. W., Galambos, T.V., and Ravindra, M.K., "Load and Resistance Factor Design of Bolted Connections," Thin-Walled Structures (J. Rhodes and A.C. Walker, eds.), International Conference at the University of Strathclyde, Glasgow, Scotland, April 1979, Granada Publishing, 1980.

35. Supornsilaphachia, B., "Load and Resistance Factor Design of Cold-Formed Steel Structural Members," Thesis presented to the University of Missouri-Rolla, Missouri, in 1980, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

36. Canadian Standards Association, "Cold-Formed Steel Structural Members," CSA Standard S-136, 1974.

37. LaBoube, R.A., and Yu, W.W., "Study of Cold-Formed Steel Beam Webs Subjected to Bending Stresses," Proceedings of the Third International Specialty Conference on Cold-Formed Steel Structures, University of Missouri-Rolla, Missouri, November 1975.

38. LaBoube, R.A., and Yu, W. W., "Effective Web Depth of Cold-Formed Steel Beams," International Colloquium on Stability of Structures Under Static and Dynamic Loads, Washington, D.C., May 1977.

39. LaBoube, R.A., and Yu, W. W., "Cold-Formed Steel Web Elements under Combined Bending and Shear," Proceedings of the Fourth International Specialty Conference on Cold-Formed Steel Structures, University of Missouri-Rolla, Missouri, June 1978.

40. LaBoube, R.A., and Yu, W. W., "Bending Strength of Webs of Cold-Formed Steel Beams," Journal of the Structural Division, ASCE, Vol. 108, No. ST7, July 1982.

41. American Iron and Steel Institute, Cold-Formed Steel Design Manual, 1980 Edition.

42. Reck, H.P., Pekoz, T., and Winter, G., "Inelastic Strength of Cold-Formed Steel Beams, Journal of the Structural Division, ASCE, Vol. 101, No. ST11, November 1975.

43. Yu, W. W., Cold-Formed Steel Structures, McGraw-Hill Book Company Inc., New York, 1973.

44. Pekoz, T., and McGuire, W., Welding of Sheet Steel, Report SG 79-2, AISI, January 1979.

45. Yu, W. W., Cold-Formed Steel Design, John Wiley & Sons, Inc., New York, 1985.

46. "American National Standard Minimum Design Loads for Building and other Structures," ANSI, A58.1-1982.

APPENDIX A

COMPUTER PROGRAM FOR BEAM-COLUMNS WITH DOUBLY-SYMMETRIC SHAPES

```
      READ(5,1) NOPROB
  1   FORMAT(I5)
      DO 500 I=1,NOPROB
      READ(5,2)MN,NF,D,B,T,FY,A,Q
      READ(5,3)S,SEFF,RX,RY,RIY,CM,CB
  2   FORMAT(2I5,6F10.5)
  3   FORMAT(7F10.5)
      IF(T.GE.0.105)R=0.1875
      IF(T.LT.0.105.AND.T.GE.0.048)R=0.09375
      IF(NF.EQ.2)GO TO 102
      WRITE(6,101)I
101   FORMAT('1','PROBLEM NO. ',I3,' IS A I-SECTION WITH STIFFENED FLANG
     1ES')
      GO TO 104
102   WRITE(6,103)MN
103   FORMAT('1','PROBLEM NO. ',I3,' IS A I-SECTION WITH UNSTIFFENED FLA
     1NGES')
104   WRITE(6,105)D,B,T,FY
105   FORMAT(1X,F5.3,' X ',F5.3,' X ',F5.3,' WITH FY = ',F5.1,' KSI')
      WRITE(6,106)A,S,SEFF,RX,RY,RIY,Q,CM,CB
106   FORMAT(1X,'SECTION PROPERTIES'/1X,'A = ',F5.3,11X,'S = ',F5.3,11X,
     1'SEFF = ',F5.3/1X,'RX = ',F5.3,10X,'RY = ',F5.3,10X,'RIY = ',F5.3/
     11X,'Q = ',F5.3,11X,'CM = ',F5.3,10X,'CB = ',F5.3/)
      PHIS=0.95
      PHIC=0.85
      DO 200 N=5,6
      EFFL=12.0*N
      RB=AMIN1(RX,RY)
      EFFLR=EFFL/RB
      CC=SQRT(582307./FY)
      RLIM=CC/SQRT(Q)
      IF(EFFLR.LE.RLIM)GO TO 20
      PUC=291153.*A/(EFFLR**2)
      GO TO 21
 20   PUC=A*Q*FY*(1.0-Q*FY*(EFFLR**2)/1164613.)
 21   CONTINUE
      PUS=A*Q*FY
      IF(Q.EQ.1.0.AND.T.GE.0.09.AND.EFFLR.LT.CC)GO TO 23
      PUCA=PUC
      PUSA=PUS
      GO TO 24
 23   FS=5./3.+0.375*(EFFLR/CC)-0.125*(EFFLR/CC)**3
      PUCA=23./12.*PUC/FS
      PUSA=23./12.*FY*A*0.6
 24   CONTINUE
      PE=291153.*A/((EFFL/RX)**2)
      IF(NF.EQ.1)GO TO 40
      W=B/2.-(R+T)
      WTRAT=W/T
      ALIM=63.3/SQRT(FY)
      BLIM=144./SQRT(FY)
```

```
      CLIM=25.
      IF(WTRAT.LE.ALIM)GO TO 35
      IF(WTRAT.LE.BLIM)GO TO 36
      IF(WTRAT.LE.CLIM)GO TO 38
      IF(WTRAT.GT.CLIM)GO TO 37
 35   FCR=FY
      GO TO 39
 36   FCR=FY*(1.28-0.0044*WTRAT*SQRT(FY))
      GO TO 39
 37   FCR=33.0-0.467*WTRAT
      GO TO 39
 38   FCR=13300./(WTRAT**2)
 39   RMUS=S*FCR
      GO TO 41
 40   RMUS=SEFF*FY
 41   RMY=S*FY
      RME=145577.*CB*D*RIY/(EFFL**2)
      RMR=RMY/RME
      RMUC=(RMY/0.90)*(1.0-RMR/3.6)
      IF(RMR.LE.0.36)RMUC=RMY
      IF(RMR.GE.1.80)RMUC=RME
      PHI=0.90
      IF(RMUC.GT.RMUS)PHI=0.95
      RMUC=AMIN1(RMUC,RMUS)
      RMSCR=RMUS/RMUC
      WRITE(6,107)N
107   FORMAT(1X,'FOR KL = ',I3,' FT.')
      WRITE(6,108)PUC,PUCA,PUS,PUSA,RMUS,RMUC
108   FORMAT(1X,'PUC = ',F7.3,7X,'PUCA = ',F7.3,6X,'PUS = ',F7.3/1X,'PUS
     1A = ',F7.3,6X,'RMUS = ',F7.3,6X,'RMUC = ',F7.3/)
      IF(PHI.EQ.0.90)GO TO 110
      WRITE(6,109)PHI
109   FORMAT(1X,'LOCAL BUCKLING OR YIELDING GOVERNS WHERE PHI = ',F4.2/)
      GO TO 49
110   WRITE(6,111)PHI
111   FORMAT(1X,'LATERAL BUCKLING GOVERNS WHERE PHI = ',F4.2/)
 49   WRITE(6,112)
112   FORMAT(1X,100('*')/1X,'D/L',2X,'M-RATIO',3X,'KL',4X,'RATIO-A',2X,'
     1RATIO-B',2X,'RATIO-C',3X,'PLRFA',3X,'PASDA',3X,'PLRFB',3X,'PASDB',
     13X,'PLRFC',3X,'PASDC')
      DO 150 M=1,5
      RATIOM=0.1*M
      WRITE(6,113)
113   FORMAT(1X,100('*'))
      DO 100 K=1,11
      DLRAT=0.1*K-0.1
      DLFAC=(DLRAT+1.0)/(1.2*DLRAT+1.6)
      PRATA=0.0
      PRATB=0.0
      PRATC=0.0
      PLRFB=0.0
      PLRFC=0.0
```

```
      PASDB=0.0
      PASDC=0.0
      PLRFA=(DLFAC-RATIOM*RMSCR/PHI)*PHIC*PUC
      CKLRF=DLFAC*PLRFA/PUC/PHIC
      IF(CKLRF.GT.0.15)GO TO 50
      GO TO 59
   50 PLRFA=0.0
   51 PLRFB=(DLFAC-RATIOM/PHIS)*PHIS*PUS
      TRIAL=PLRFB
   55 PLRFC=(DLFAC-CM*RATIOM*RMSCR/PHI/(1.-TRIAL/PE/PHIC/DLFAC))*PHIC*PU
     1C
      DIFF=PLRFC-TRIAL
      DIFF=ABS(DIFF)
      IF(DIFF.LT.0.001)GO TO 59
      TRIAL=PLRFC
      GO TO 55
   59 PASDA=(1.0-RATIOM*RMSCR/0.6)*12.*PUCA/23.
      CKASD=PASDA*23./PUCA/12.
      IF(CKASD.GT.0.15)GO TO 60
      PRATA=PLRFA/PASDA
      GO TO 70
   60 PASDA=0.0
   61 PASDB=(1.0-RATIOM/0.6)*PUSA*12./23.
      TRIAL=PASDB
   65 PASDC=(1.0-CM*RATIOM*RMSCR/.6/(1.-23./12.*TRIAL/PE))*12./23.*PUCA
      DIFF=PASDC-TRIAL
      DIFF=ABS(DIFF)
      IF(DIFF.LT.0.001)GO TO 69
      TRIAL=PASDC
      GO TO 65
   69 PRATB=PLRFB/PASDB
      PRATC=PLRFC/PASDC
   70 WRITE(6,115)DLRAT,RATIOM,EFFL,PRATA,PRATB,PRATC,PLRFA,PASDA,PLRFB,
     1PASDB,PLRFC,PASDC
  115 FORMAT(1X,F3.1,4X,F3.1,4X,F5.1,2X,F6.4,3X,F6.4,3X,F6.4,3X,F6.2,2X,
     1F6.2,2X,F6.2,2X,F6.2,2X,F6.2,2X,F6.2)
  100 CONTINUE
  150 CONTINUE
  200 CONTINUE
  500 CONTINUE
      STOP
      END
```

APPENDIX B

COMPUTER PROGRAM FOR BEAM-COLUMNS WITH SINGLY-SYMMETRIC SHAPES

```
      DIMENSION EC(100)
      READ(5,1)NOPROB
    1 FORMAT(I5)
      DO 700 I=1,NOPROB
      READ(5,2)MN,NF,D,B,T,FY,A,Q
      READ(5,3)S,SY,RX,RY,RIY,CM
      READ(5,4)CE,SVJ,CW,SJ,XO,CTF,DE
    2 FORMAT(2I5,6F10.5)
    3 FORMAT(6F10.5)
    4 FORMAT(7F10.5)
      WRITE(6,101)MN,D,B,T,FY
  101 FORMAT('1','PROBLEM NO. ',I3,'***',F5.3,' X ',F5.3,' X ',F5.3,' WI
     1TH FY = ',F5.1,' KSI')
      WRITE(6,102)A,S,SY,RX,RY,RIY,Q,CM,CE,SVJ,CW,SJ,XO,CTF
  102 FORMAT(1X,'A = ',F5.3,10X,'S = ',F5.3,10X,' SY = ',F5.3,7X,'RX =
     1',F5.3,9X,'RY = ',F5.3,9X,'IY = ',F5.3,9X,'Q = ',F5.3/1X,'CM = ',F
     15.3,9X,'CE = ',F5.3,9X,'J = ',F8.6,7X,'CW = ',F6.4,8X,'SJ = ',F5.3
     1,9X,'XO = ',F5.2,9X,'CTF = ',F5.3)
      IF(T.GE.0.105)R=0.1875
      IF(T.LT.0.105.AND.T.GE.0.048)R=0.09375
      PHIC=0.85
      PHIS=0.95
      RO=SQRT(RX**2+RY**2+XO**2)
      BETA=1.-(XO/RO)**2
      IF(NF.EQ.1)GO TO 10
      W=DE-(R+T)
      WTRAT=W/T
      ALIM=63.3/SQRT(FY)
      BLIM=144./SQRT(FY)
      CLIM=25.
      IF(WTRAT.LE.ALIM)GO TO 5
      IF(WTRAT.LE.BLIM)GO TO 6
      IF(WTRAT.LE.CLIM)GO TO 8
      IF(WTRAT.GT.CLIM)GO TO 7
    5 FCR=FY
      GO TO 9
    6 FCR=FY*(1.28-0.0044*WTRAT*SQRT(FY))
      GO TO 9
    7 FCR=33.0-0.467*WTRAT
      GO TO 9
    8 FCR=13300./(WTRAT**2)
    9 RMUS=SY*FCR
      GO TO 11
   10 RMUS=SY*FY
   11 CONTINUE
      RB=AMIN1(RX,RY)
      CC=SQRT(582307./FY)
      RLIM=CC/SQRT(Q)
      PUS=A*Q*FY
      READ(5,12)NOE
      READ(5,13)(EC(NE),NE=1,NOE)
```

```
 12   FORMAT(I5)
 13   FORMAT(F10.5)
      DO 200 N=5,5
      EFFL=12.0*N
      EFFLR=EFFL/RB
      IF(EFFLR.LE.RLIM)GO TO 20
      PUC=291153.*A/(EFFLR**2)
      GO TO 21
 20   PUC=A*Q*FY*(1.-Q*FY*(EFFLR**2)/1164613.)
 21   CONTINUE
      IF(Q.EQ.1.0.AND.T.GE.0.09.AND.EFFLR.LT.CC)GO TO 23
      PUCA=PUC
      PUSA=PUS
      GO TO 24
 23   FS=5./3.+0.375*(EFFLR/CC)-0.125*(EFFLR/CC)**3
      PUCA=23./12.*PUC/FS
      PUSA=23./12.*FY*A*0.6
 24   PE=291153.*A/(EFFLR**2)
      SEX=291153./((EFFL/RX)**2)
      ST=(11300.*SVJ+291153.*CW/(EFFL**2))/A/(RO**2)
      RMT=-A*SEX*(SJ-SQRT(SJ**2+(RO**2)*(ST/SEX)))
      SBT=RMT*CE/RIY
      SE=291153./((EFFL/RB)**2)
      STFO=((SEX+ST)-SQRT((SEX+ST)**2-4.*BETA*SEX*ST))/2./BETA
      TFLIM=0.5*Q*FY
      PUCII=A*Q*FY*(1.-Q*FY/4./STFO)
      IF(STFO.LE.TFLIM)PUCII=A*STFO
      FAII=12./23./A*PUCII
      WRITE(6,103)N
103   FORMAT(1X,'FOR KL = ',I3,' FT.')
      WRITE(6,104)PUC,PUCA,PUS,PUSA,RMUS
104   FORMAT(1X,'PUC = ',F7.3,7X,'PUCA = ',F7.3,6X,'PUS = ',F7.3/1X,'PUS
     1A = ',F7.3,6X,'RMUS = ',F7.3/)
      DO 300 NOEC=1,NOE
      E=EC(NOEC)
      IF(E.GE.0.0)GO TO 39
      IF(E.LT.0.0.AND.E.GE.XO)GO TO 41
      WRITE(6,105)
105   FORMAT(1X,125('*')/1X,'D/L',7X,'E',9X,'KL',4X,'*',3X,'PLRFA',5X,'P
     1LRFB',5X,'PLRFC',3X,'*',3X,'PASDA',5X,'PASDB',5X,'PASDC',3X,'*',4X
     1,'PLFF',6X,'PASF',3X,'*',3X,'PRATIO'/1X,125('*'))
      GO TO 45
 39   PARMX=STFO*SE
      PARMY=STFO+SE+CTF*E*A*PARMX/RMT
      STF=(PARMY-SQRT(PARMY**2-4.*PARMX))/2.
 41   IF(E.LT.0.0)GO TO 42
      WRITE(6,106)RO,BETA,SEX,ST,RMT,SBT,SE,STFO,STF
106   FORMAT(1X,'RO = ',F8.3,10X,'BETA = ',F8.4,10X,'SEX = ',F9.3/1X,'ST
     1 = ',F9.3,9X,'RMT = ',F9.3,10X,'SBT = ',F9.3/1X,'SE = ',F9.3,9X,'S
     1TFO = ',F9.3,9X,'STF = ',F9.3)
      WRITE(6,107)
```

```
107  FORMAT(1X,125('*')/1X,'D/L',7X,'E',9X,'KL',4X,'*',4X,'PLFF',6X,'PA
    1SF',3X,'*',4X,'PLFT',6X,'PAST',3X,'*',5X,'PRF',8X,'PRT',4X,'*',5X,
    1'PR'/1X,125('*'))
     GO TO 45
 42  WRITE(6,108)
108  FORMAT(1X,125('*')/1X,'D/L',7X,'E',9X,'KL',4X,'*',4X,'PUE',7X,'FAE
    1',4X,'*',3X,'PUCII',6X,'FAII',3X,'*',4X,'PLFL',6X,'PASL',3X,'*',5X
    1,'PRL'/1X,125('*'))
 45  CONTINUE
     DO 400 K=1,1
     DLRAT=K/2.
     DLFAC=(DLRAT+1.0)/(1.2*DLRAT+1.6)
     IF(E.LT.0.0.AND.E.GE.XO)GO TO 117
 49  PLRFB=0.0
     PLRFC=0.0
     PASDB=0.0
     PASDC=0.0
     E=ABS(E)
     PLRFA=DLFAC/(1./PHIC/PUC+E/PHI/RMUS)
     CKLRF=PLRFA/PUC/PHIC/DLFAC
     IF(CKLRF.GT.0.15)GO TO 50
     PLFF=PLRFA
     GO TO 59
 50  PLRFA=0.0
 51  PLRFB=DLFAC/(1./PHIS/PUS+E/PHIS/RMUS)
     TRIAL=PLRFB
 55  DENOM=1.-TRIAL/PHIC/PE/DLFAC
     IF(DENOM.EQ.0.0)DENOM=0.0001
     PLRFC=DLFAC/(1./PHIC/PUC+E*CM/PHI/RMUS/DENOM)
     DIFF=PLRFC-TRIAL
     DIFF=ABS(DIFF)
     IF(DIFF.LT.0.001)GO TO 58
     TRIAL=PLRFC
     GO TO 55
 58  PLFF=AMIN1(PLRFB,PLRFC)
 59  PASDA=1./(23./12./PUCA+E/0.6/RMUS)
     CKASD=PASDA*23./PUCA/12.
     IF(CKASD.GT.0.15)GO TO 60
     PASF=PASDA
     GO TO 70
 60  PASDA=0.0
 61  PASDB=1./(23./12./PUSA+E/0.6/RMUS)
     TRIAL=PASDB
 65  DEMON=1.-23.*TRIAL/12./PE
     IF(DENOM.EQ.0.0)DENOM=0.0001
     PASDC=1./(23./12./PUCA+CM*E/0.6/RMUS/DENOM)
     DIFF=PASDC-TRIAL
     DIFF=ABS(DIFF)
     IF(DIFF.LT.0.001)GO TO 68
     TRIAL=PASDC
     GO TO 65
 68  PASF=AMIN1(PASDB,PASDC)
 70  PRF=PLFF/PASF
```

```
      E=EC(NOEC)
      IF(E.LT.0.0.AND.E.GE.XO.AND.PUC.LE.PUCII)GO TO 115
      IF(E.LT.0.0.AND.E.GE.XO)GO TO 75
      IF(E.LT.XO)GO TO 115
      TFLIM=0.5*Q*FY
      PUTF=A*Q*FY*(1.-Q*FY/4./STF)
      IF(STF.LE.TFLIM)PUTF=A*STF
      PLFT=DLFAC*PHIC*PUTF
      PAST=12./23.*PUTF
      PRT=DLFAC*23./12.*PHIC
      PLF=AMIN1(PLFF,PLFT)
      PAS=AMIN1(PASF,PAST)
      PR=PLF/PAS
      WRITE(6,114)DLRAT,E,EFFL,PLFF,PASF,PLFT,PAST,PRF,PRT,PR
114   FORMAT(1X,F3.1,5X,F5.2,5X,F5.1,3X,'*',3X,F5.2,5X,F5.2,3X,'*',3X,F5
     1.2,5X,F5.2,3X,'*',3X,F6.4,5X,F6.4,3X,'*',3X,F6.4)
      GO TO 400
115   WRITE(6,116)DLRAT,E,EFFL,PLRFA,PLRFB,PLRFC,PASDA,PASDB,PASDC,PLFF,
     1PASF,PRF
116   FORMAT(1X,F3.1,5X,F5.2,5X,F5.1,3X,'*',3X,F5.2,5X,F5.2,5X,F5.2,3X,'
     1*',3X,F5.2,5X,F5.2,5X,F5.2,3X,'*',3X,F5.2,5X,F5.2,3X,'*',3X,F6.4)
      GO TO 400
117   IF(PUC.GT.PUCII)GO TO 74
      GO TO 49
 74   E=XO
      GO TO 49
 75   PUE=PLFF/DLFAC
      FAE=PASF/A
      PUL=PHIC*PUCII+E/XO*(PUE-PHIC*PUCII)
      PLFL=DLFAC*PUL
      PASL=A*(FAII+E/XO*(FAE-FAII))
      PRL=PLFL/PASL
      WRITE(6,119)DLRAT,E,EFFL,PUE,FAE,PUCII,FAII,PLFL,PASL,PRL
119   FORMAT(1X,F3.1,5X,F5.2,5X,F5.1,3X,'*',3X,F5.2,5X,F5.2,3X,'*',3X,F5
     1.2,5X,F5.2,3X,'*',3X,F5.2,5X,F5.2,3X,'*',3X,F6.4)
400   CONTINUE
      WRITE(6,120)
120   FORMAT(1X,125('*'))
300   CONTINUE
200   CONTINUE
700   CONTINUE
      STOP
      END
```

APPENDIX C

DESIGN EXAMPLES

The following examples deal with the design of tension members, flexural members, axially loaded compression members, beam-columns, welded connections and bolted connections.

PROBLEM NO. 1 - TENSION MEMBER

A. Problem Statement. The 3 in. x 3 in. x 0.105 in. cold-formed steel angle with equal unstiffened legs, shown in Figure C.1 is to be used as a tension member with weld connections. Determine the factored nominal tensile strength and the allowable load of the member based on the LRFD criteria. Use F_y = 33 ksi and D/L = 0.5.

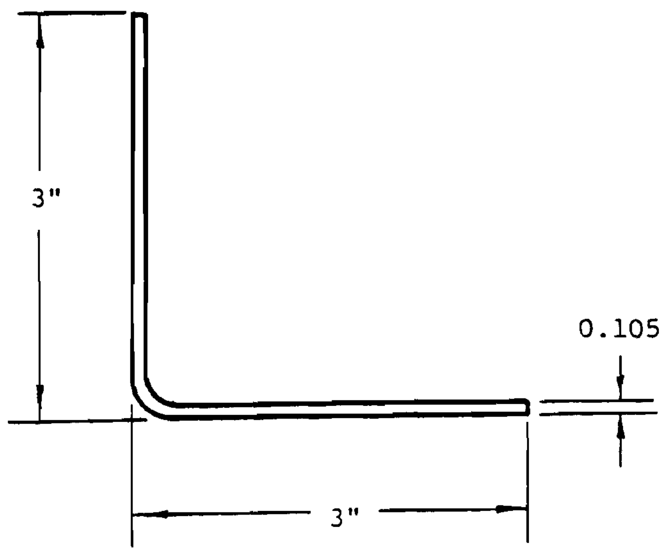


Figure C.1 Standard Angle With Equal Unstiffened Legs, 3 in. x 3 in. x 0.105 in., in Problem No. 1 (Selected from Table 8 of Part V in Reference 41)

B. Solution. The cross-sectional area for the cold-formed steel angle can be obtained from Table 8 of Part V of the Design Manual[41] and is equal to 0.608 in.^{2} The factored nominal tensile strength can be determined from Eq. (3.2) and $\phi = 0.95$, i. e.,

$$\phi R_{nt} = \phi A F_y = (0.95)(0.608)(33) = 19.06 \text{ kips}$$

The allowable unfactored load can be calculated from Eq. (3.5) with an assumption of D/L = 0.5.

$$(P_a)_{LRFD} = \phi R_{nt} \frac{D/L+1}{1.2D/L+1.6}$$

$$= 19.06 \frac{0.5+1}{1.2(0.5)+1.6} = 13.0 \text{ kips}$$

The allowable load based on allowable stress design, $(P_a)_{ASD}$, is $AF_t = (0.608)(0.6)(33) = 12.04$ kips. Therefore, the allowable load ratio for this case is 13.0 /12.04 = 1.079. This ratio agrees with the allowable load ratio computed from Eq. (3.8) shown in Figure 2.

PROBLEM NO. 2 - CONTINUOUS BEAM

A. Problem Statement. The 6 in. x 2.5 in. x 0.105 in. channel with stiffened flanges shown in Figure C.2 is to be used for supporting a uniform load over three equal spans. Assume that the span length is 10 ft, $F_y = 50$ ksi, and the dead-load to live-load ratio is 0.5. The following section properties were obtained from Table 1 of Part V of the Design Manual[41]:

$$R = 3/16 \text{ in.} \qquad S_{xc} = 2.28 \text{ in.}^3$$

$$I_y = 1.05 \text{ in.}^4 \qquad S_{eff} = 2.28 \text{ in.}^3$$

The beam is braced laterally at the supports and the web is

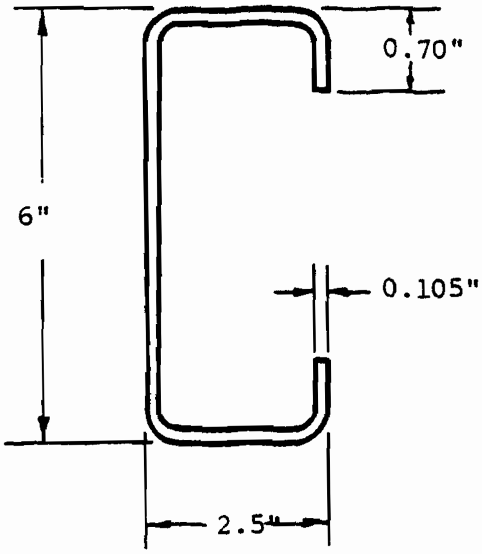


Figure C.2. Standard Channel With Stiffened Flanges, 6 in. x 2.5 in. x 0.105 in., in Problem Nos. 2, 4, & 6 (Selected from Table 1 of Part V in Reference 41)

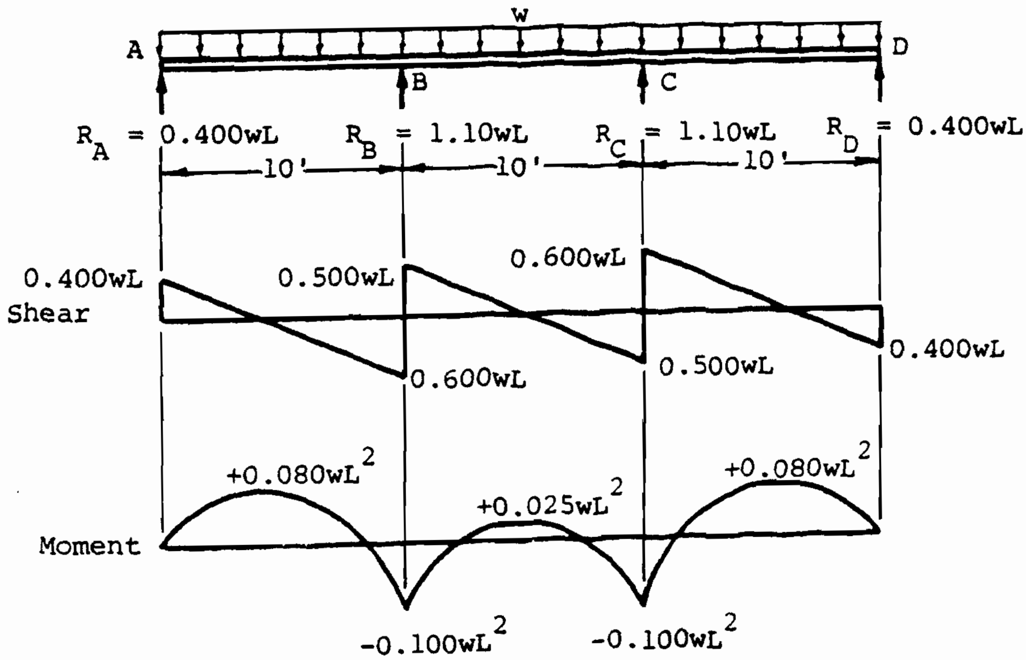


Figure C.3. Shear and Moment Diagram of Three Span Continuous Beam Subjected to Uniform Load unreinforced. Bearing plates are 6 in. long and are used at the end supports and interior supports.

Determine the factored nominal uniform load and the allowable uniform load for the beam based on the LRFD criteria.

B. Solution. The uniform load capacities were calculated based on bending strength, lateral buckling, shear strength of web, bending strength of web, combined bending and shear in web, web crippling, and combined bending and web crippling.

1. Bending Strength. The factored nominal moment, ϕM_u, based on section strength can be computed with $\phi = 0.95$ and Eq. (4.7) as follows:

$$\phi M_u = \phi S_{eff} F_y = (0.95)(2.28)(50) = 108.3 \text{ kip-in.}$$

The moment diagram of the beam is shown in Figure C.3. From the figure, the maximum factored moment occurs at the interior supports and is equal to

$$M_D = 0.100 w_D L^2 \tag{C-1}$$

where w_D is the applied factored uniform load and L is the span length. Let $M_D = \phi M_u$, therefore, the factored nominal uniform load capacity for this example is calculated as follows:

$$w_D = \frac{\phi M_u}{0.100 L^2} \tag{C-2}$$

$$w_D = \frac{108.3}{0.100(120)^2}(12) = 0.903 \text{ kips/ft}$$

Since the uniform load capacity is directly related to the bending moment capacity, the following equation developed from Eq. (4.17) is used to calculate the allowable uniform load based on the LRFD criteria:

$$(w_a)_{LRFD} = w_D \frac{D/L+1}{1.2D/L+1.6} \qquad \text{(C-3)}$$

$$(w_A)_{LRFD} = 0.903 \frac{0.5+1}{1.2(0.5)+1.6} = 0.615 \text{ kips/ft}$$

Because the allowable uniform load based on allowable stress design for bending strength is

$$(w_a)_{ASD} = \frac{(M_a)_{ASD}}{0.100L^2} = \frac{(0.6)(50)(2.28)}{0.100(120)^2}(12) = 0.570 \text{ kips/ft}$$

the allowable load ratio for the beam based on section strength is 0.615/0.570 = 1.08. This value agrees with the allowable load ratio determined from Eq. (4.21) and Figure 3.

2. <u>Lateral Buckling</u>. The factored nominal moment, ϕM_u, based on lateral buckling can be determined with ϕ = 0.90 and M_u computed from Eqs. (4.27), (4.28), or (4.29), whichever is applicable.

The bending coefficient, C_b, for the outer spans of the beam is determined from Eq. (4.26) with $M_1/M_2 = 0$.

$$C_b = 1.75 + 1.05(M_1/M_2) + 0.3(M_1/M_2)^2$$

$$C_b = 1.75 + 1.05(0) + 0.3\ (0)^2 = 1.75$$

For the center span, the C_b value is conservatively taken as 1.0. For this example, the center span will govern the design for lateral buckling.

From Eq. (4.30), the critical moment is determined as follows:

$$M_e = \frac{\pi^2 E C_b d I_{yc}}{L^2}$$

$$M_e = \frac{\pi^2(29500)(1.0)(6)(1.05/2)}{(120)^2} = 63.69 \text{ kip-in.}$$

$$M_y = S_{xc}F_y = (2.28)(50) = 114.0 \text{ kip-in.}$$

$$M_y/M_e = 114.0/63.69 = 1.79 < 1.8$$

Since $0.36 < M_y/M_e < 1.8$, Eq. (4.28) is used to calculated the factored nominal moment.

$$\phi M_u = \phi M_e = (0.90)(63.69) = 57.32 \text{ kip-in.}$$

The factored nominal uniform load for this example based on lateral buckling is calculated using Eq. (C-2).

$$w_D = \frac{57.32}{0.100(120)^2}(12) = 0.478 \text{ kips/ft}$$

The allowable uniform load capacity based on LRFD is calculated using Eq. (C-3).

$$(w_a)_{LRFD} = 0.478 \frac{0.5+1}{1.2(0.5)+1.6} = 0.326 \text{ kips/ft}$$

The allowable uniform load capacity based on allowable stress design for lateral buckling is determined as follows:

$$\frac{0.36\pi^2 EC_b}{F_y} = 2096 < \frac{L^2 S_{xc}}{dI_{yc}} = 10423 < \frac{1.8\pi^2 EC_b}{F_y} = 10482$$

$$F_b = \frac{2}{3} F_y - \frac{F_y^2}{5.4\pi^2 EC_b}\left(\frac{L^2 S_{xc}}{dI_{yc}}\right) = 16.76 \text{ ksi}$$

$$(M_a)_{ASD} = S_{xc}F_b = (2.28)(16.76) = 38.21 \text{ kip-in.}$$

$$(w_a)_{ASD} = \frac{38.21}{0.100(120)^2}(12) = 0.318 \text{ kips/ft}$$

The allowable load ratio is 0.326/0.318 = 1.023 which agrees with Eq. (4.35) shown in Figure 4.

3. <u>Shear Strength of Web</u>. The factored nominal shear strength of the web, $\phi_v V_u$, can be determined using the following h/t ratio:

$$\frac{h}{t} = \frac{6-2(0.105)}{0.105} = \frac{5.79}{0.105} = 55.14$$

Since the web is unreinforced, $k_v = 5.34$. Therefore,

$$171\sqrt{k_v/F_y} = 171\sqrt{5.34/50} = 55.88$$

Since $h/t < 171\sqrt{k_v/F_y}$, $\phi_v = 1.0$ and V_u can be calculated from Eq. (4.38).

$$V_u = A_w F_y/\sqrt{3}$$

$$V_u = (5.79 \times 0.105)(50)/\sqrt{3} = 17.55 \text{ kips}$$

$$\phi_v V_u = (1.0)(17.55) = 17.55 \text{ kips}$$

The shear diagram in Figure C.3 shows a maximum shear at the interior supports, i.e.,

$$V_D = 0.600 w_D L \tag{C-4}$$

For $\phi_v V_u = V_D$, the factored nominal uniform load can be calculated as follows:

$$w_D = \frac{\phi_v V_u}{0.600L} \tag{C-5}$$

$$= \frac{17.55}{0.600(120)}(12) = 2.925 \text{ kips/ft}$$

The allowable uniform load based on LRFD is calculated using Eq. (C-3).

$$(w_a)_{LRFD} = 2.925\,\frac{0.5+1}{1.2(0.5)+1.6} = 1.994 \text{ kips/ft}$$

For allowable stress design, the allowable uniform load based on the shear strength of the web is calculated as follows:

$$F_v = \frac{65.7\sqrt{k_v F_y}}{(h/t)} \leq 0.40F_y$$

$$F_v = \frac{65.7\sqrt{5.34 x 50}}{55.14} = 19.47 \text{ ksi} \leq 20 \text{ ksi}$$

$$(V_a)_{ASD} = A_w F_v = (5.79 \text{ x } 0.105)(19.47) = 11.84 \text{ kips}$$

$$(w_a)_{ASD} = \frac{11.84}{0.600(120)} \text{ x } 12 = 1.973 \text{ kips/ft}$$

The allowable load ratio is 1.994/1.973 = 1.011 which indicates that both methods permit about the same load.

4. <u>Flexural Strength Governed by Webs</u>. The factored nominal bending strength of the beam governed by the web, $\phi_{bw} M_{ubw}$, can be computed with ϕ_{bw}= 0.90 and M_{ubw} which is determined from Eq. (4.48).

$$M_{ubw} = S_{eff}(\lambda F_y)$$

For beams with stiffened flanges,

$$\lambda = 1.21-0.00034(h/t)\sqrt{F_y} \leq 1.0$$

$$\lambda = 1.21-0.00034(55.14)\sqrt{50} = 1.077, \lambda = 1.0$$

Therefore,

$$\phi_{bw} M_{ubw} = 0.90 \ (2.28)(1.0)(50) = 102.6 \text{ kip-in.}$$

The factored nominal uniform load can be calculated from Eq. (C-2) used previously for section strength and lateral buckling. Therefore,

$$w_D = \frac{102.6}{0.100(120)^2} (12) = 0.855 \text{ kips/ft}$$

The allowable uniform load based on LRFD is computed from Eq. (C-3) as follows:

$$(w_a)_{LRFD} = 0.855 \frac{0.5+1}{1.2(0.5)+1.6} = 0.583 \text{ kips/ft}$$

Same as the comparison for section strength, the allowable uniform load based on allowable stress design is 0.570 kips/ft. Therefore, the allowable load ratio is 0.583/0.57 = 1.023 which agrees with Eq. (4.51) shown in Figure 7.

5. Combined Bending and Shear in Web. The factored nominal uniform load capacity of the beam governed by combined bending and shear in the web can be determined from the interaction equation, Eq. (4.54).

$$\left(\frac{V_D}{\phi_v V_u}\right)^2 + \left(\frac{M_D}{\phi_{bw} M_{ubw}}\right)^2 \leq 1.0$$

From the shear and moment diagrams in Figure C.3, the maximum bending moment and shear combination occurs at the interior supports and are as follows:

$$V_D = 0.600\ w_D L^2\ ,\ M_D = 0.100\ w_D L^2$$

$$\phi_v V_u = (0.9)(110 A_w \sqrt{k_v F_y}/(h/t) = 17.83$$

$$\phi_{bw} M_{ubw} = (0.9)(2.28)(1.077)(50) = 110.5$$

From substitution into Eq. (4.54), the following expression is obtained:

$$\left(\frac{0.600\ w_D(10)}{17.83}\right)^2 + \left(\frac{0.100\ w_D(10)^2(12)}{110.5}\right)^2 \leq 1.0$$

By solving for w_D in the above expression, the factored uniform load capacity is 0.880 kip/ft. The allowable uniform load based on LRFD can be calculated from Eq. (C-3).

$$(w_a)_{LRFD} = 0.880 \; \frac{0.5+1}{1.2(0.5)+1.6} = 0.60 \text{ kips/ft}$$

The allowable load based on allowable stress design can be computed by Eq. (4.52) as follows:

$$f_{bw} = \frac{M}{S_{xc}} = \frac{0.100wL^2}{S_{xc}}$$

$$f_v = \frac{V}{A_w} = \frac{0.600wL}{ht}$$

$$\left(\frac{0.100w(10)^2(12)}{(32.31)(2.28)}\right)^2 + \left(\frac{0.600\ w(10)}{(19.47)(5.79)(0.105)}\right)^2 = 1.0$$

$$(w_a)_{ASD} = 0.586 \text{ kips/ft}$$

The allowable load ratio is 0.600/0.586 = 1.024. This value agrees with the allowable load ratio of 1.027 obtained from Eq. (4.66).

6. Web Crippling. The factored nominal reaction based on crippling of the channel with stiffened flanges at the interior supports can be calculated from Eq. (4.96).

$$P_u = t^2kC_1C_2C_\theta[538-0.74(h/t)][1+0.007(N/t)]$$

From Eqs. (4.79), (4.91), and (4.92),

$$k = F_y/33 = 50/33 = 1.515$$

$$C_1 = 1.22-0.22k = 1.22-0.22(1.515) = 0.8867$$

$$C_2 = 1.06-0.06R/t = 1.06-0.06(3/16)/0.105 = 0.9529$$

$$C_\theta = 1.0$$

For h/t = 55.14 and N = 6 in.,

$$P_u = (0.105)^2(1.515)(0.8867)(0.9529)[538-0.74(55.14)]\times$$
$$[1+0.007(6/0.105)] = 9.824 \text{ kips}$$

For $\phi_w = 0.85$,

$$\phi_w P_u = (0.85)(9.824) = 8.35 \text{ kips}$$

From Figure C.3, the reactions at the interior supports are

$$P_D = 1.10\ w_D L \qquad \text{(C-6)}$$

The factored nominal uniform load capacity based on web crippling of the beam web at the interior supports is calculated as follows:

$$w_D = \frac{\phi_w P_u}{1.10L} \qquad \text{(C-7)}$$

$$w_D = \frac{8.35}{(1.10)(10)} = 0.759 \text{ kips/ft}$$

The allowable uniform load based on LRFD is calculated from Eq. (C-3) as follows:

$$(w_a)_{LRFD} = 0.759\ \frac{0.5+1}{1.2(0.5)+1.6} = 0.518 \text{ kips/ft}$$

The allowable uniform load based on allowable stress design is 0.483 kips/ft. Therefore, the allowable load ratio is 0.518/0.483 = 1.072 which agrees with Eq. (4.105) shown in Figure 16.

The factored nominal reaction based on web crippling of the channel at the exterior supports was calculated from Eq. (4.94).

$$P_u = t^2 k C_3 C_4 C_\theta [331-0.61(h/t)][1+0.01(N/t)]$$

From Eqs. (4.80) and (4.81),

$$C_3 = 1.33-0.33k = 1.33-0.33(1.515) = 0.8300$$

$$C_4 = 1.15-0.15R/t = 1.15-0.15(3/16)/0.105 = 0.8821$$

For h/t = 55.14 and N = 6 in.,

$$P_u = (0.105)^2(1.515)(0.8300)(0.8821)[331-0.61(55.14)]\times$$
$$[1+0.01(6/0.105)] = 5.715 \text{ kips}$$

From Figure C.3, the reactions at the exterior supports are

$$P_D = 0.400\, w_D L \qquad \text{(C-8)}$$

The factored nominal uniform load capacity based on web crippling of the beam web at the exterior supports is calculated for $\phi_w = 0.85$ as follows:

$$w_D = \frac{\phi_w P_u}{0.400L} \qquad \text{(C-9)}$$

$$\phi_w w_u = \frac{(0.85)(5.715)}{(0.400)(10)} = 1.214 \text{ kips/ft}$$

The allowable uniform load based on LRFD is calculated from Eq. (C-3).

$$(w_a)_{LRFD} = 1.214 \, \frac{0.5+1}{1.2(0.5)+1.6} = 0.828 \text{ kips/ft}$$

The allowable uniform load based on allowable stress design is 0.773 kips/ft. Therefore, the allowable load ratio is 0.828/0.773 = 1.071 which agrees with Eq. (4.105) and the allowable load ratio based on web crippling of the beam at the interior support.

For the web crippling criteria, the reactions at the interior supports govern the design.

7. Combined Bending and Web Crippling. The factored nominal uniform load capacity of the beam governed by combined bending and

web crippling was determined from the interaction equation, Eq. (4.109).

$$1.07 \frac{P_D}{\phi_w P_u} + \frac{M_D}{\phi_b M_u} \leq 1.42$$

From Figure C.3, the maximum bending moment and support reaction combination occurs at the interior supports and are determined from Eqs. (C-1) and (C-6).

$$M_D = 0.100 \; w_D L^2$$

$$P_D = 1.10 \; w_D L$$

The values of $\phi_b M_u$ and $\phi_w P_u$ were calculated in parts 4 and 6 of this problem. From substitution into Eq. (4.109), the following expression is obtained:

$$1.07 \frac{1.10 w_D (10)}{8.35} + \frac{0.100 w_D (10)^2 (12)}{102.6} = 1.42$$

By solving for w_D in the expressive above, the factored uniform load capacity is 0.551 kips/ft. The allowable uniform load based on LRFD can be calculated from Eq. (C-3).

$$(w_a)_{LRFD} = 0.551 \frac{0.5+1}{1.2(0.5)+1.6} = 0.375 \text{ kips/ft}$$

The allowable load based on allowable stress design is calculated from Eq. (4.107) as follows:

$$1.2 \frac{1.10 w(10)}{5.314} + \frac{0.100 w(10)^2 (12)}{(30)(2.28)} = 1.5$$

$$(w_a)_{ASD} = 0.354 \text{ kips/ft}$$

The allowable load ratio is 0.375/0.354 = 1.059. This value does not correspond to the allowable load ratio of 1.019 obtained from Eq. (4.120). The reason for the difference is that Eq. (4.120) was developed for a concentrated load at the midspan of a simply supported beam.

8. Summary. Based on the above calculations, it can be seen that the factored nominal uniform load for the continuous beam in this example is 0.478 kips/ft based on lateral buckling. The allowable loads based on LRFD and allowable stress design are 0.326 and 0.318 kips/ft, respectively.

PROBLEM NO. 3 - AXIALLY LOADED COMPRESSION MEMBER (DOUBLY-SYMMETRIC SHAPE)

A. Problem Statement. The 6 in. x 3 in. x 0.105 in. cold-formed steel I-section with unstiffened flanges shown in Figure C.4 is to be used as an 8 ft long axially loaded column. The yield point of steel is 33 ksi and the D/L ratio is assumed to be 0.5. The column is assumed pinned at both ends. The following section properties are found from Table 6 of Part V of the Design Manual[41]:

$$A = 1.80 \text{ in.}^2 \qquad r_x = 2.17 \text{ in.}$$

$$Q = 0.864 \qquad r_y = 0.514 \text{ in.}$$

Determine the factored nominal axial strength and the allowable axial load based on the LRFD criteria.

B. Solution. The factored nominal axial strength, $\phi_c P_u$, can be computed with $\phi_c = 0.85$ and P_u computed from Eqs. (5.4) and (5.5).

$$C_c = \sqrt{2\pi^2 E/F_y}$$

$$= \sqrt{2\pi^2 (29500)/33} = 132.8$$

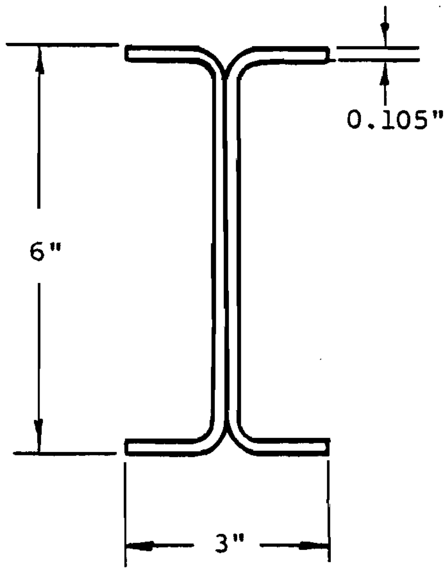


Figure C.4 Standard I-Section With Unstiffened Flanges, 6 in. x 3 in. x 0.105 in., in Problem Nos. 3 & 5 (Selected from Table 6 of Part V in Reference 41)

$$C_c/\sqrt{Q} = 132.8/\sqrt{0.864} = 142.9$$

$$KL/r_y = 8x12/0.514 = 186.8$$

Since $KL/r > C_c/\sqrt{Q}$, Eq. (5.5) was used to calculate P_u.

$$P_u = \pi^2 EA/(KL/r)^2$$

$$P_u = \pi^2(29500)(1.80)/(186.8)^2 = 15.02 \text{ kips}$$

$$\phi_c P_u = (0.85)(15.02) = 12.77 \text{ kips}$$

The allowable axial load based on LRFD is computed using Eq. (5.8) as follows:

$$(P_a)_{LRFD} = \phi_c P_u \frac{0.5+1}{1.2(0.5)+1.6}$$

$$(P_a)_{LRFD} = 12.77 \frac{0.5+1}{1.2(0.5)+1.6} = 8.705 \text{ kips}$$

The allowable axial load based on Eq. (5.2) from allowable stress design is 7.838 kips. Therefore, the allowable load ratio is 8.705/7.838 = 1.111 which agrees with Eq. (5.11) shown in Figure 24.

PROBLEM NO. 4 - AXIALLY LOADED COMPRESSION MEMBERS (SINGLY-SYMMETRIC SHAPE)

A. Problem Statement. The 6 in. x 2.5 in. x 0.105 in. cold-formed steel channel with stiffened flanges shown in Figure C.2 is to be used as an 8 ft long axially loaded column. The yield point of steel is 33 ksi and the D/L ratio is assumed to be 0.5. The column is assumed pinned at both ends. The following section properties were found from Table 1 of Part V of the Design Manual[41]:

$A = 1.24$ in.^{2}	$C_w = 8.44$ in.^{6}
$r_x = 2.35$ in.	$r_o = 3.22$ in.
$r_y = 0.921$ in.	$x_o = -2.00$ in.
$J = 0.00456$ in.^{4}	$Q = 0.908$

Determine the factored nominal axial strength and the allowable axial load based on the LRFD criteria.

B. Solution. Flexural or torsional-flexural buckling may govern the design of a column with a singly-symmetric cross section. For flexural buckling, $\phi_c P_u$ is computed as follows:

$$C_c = 2\pi^2(29500)/33 = 132.8$$

$$C_c/\sqrt{Q} = 132.8/\sqrt{0.908} = 139.4$$

$$KL/r = 8x12/0.921 = 104.2$$

Since $KL/r < C_c/\sqrt{Q}$, Eq. (5.4) was used to calculate P_u.

$$P_u = AQF_y \left[1- \frac{QF_y}{4\pi^2 E} \left(\frac{KL}{r}\right)^2 \right]$$

$$P_u = (1.24)(0.908)(33) \left[1- \frac{(0.908)(33)}{4\pi^2 (29500)} (104.2)^2 \right] = 26.78 \text{ kips}$$

Since $\phi_c = 0.85$,

$$\phi_c P_u = (0.85)(26.78) = 22.76 \text{ kips}$$

For torsion-flexural buckling, $\phi_c P_u$ was calculated from Section 9.4.1[10]. From Eqs. (5.14) through (5.17),

$$\beta = 1-(x_o/r_o)^2$$

$$= 1-(2.00/3.22)^2 = 0.6142$$

$$\sigma_t = \frac{1}{Ar_o^2} \left[GJ + \frac{\pi^2 EC_w}{(KL)^2} \right]$$

$$= \frac{1}{(1.24)(3.22)^2} \left[(11300)(0.00456) + \frac{\pi^2 (29500)(8.44)}{(96)^2} \right]$$

$$= 24.75 \text{ ksi}$$

$$\sigma_{ex} = \frac{\pi^2 E}{(KL/r_x)^2}$$

$$= \frac{\pi^2 (29500)}{(8x12/2.35)^2} = 174.5 \text{ ksi}$$

$$\sigma_{TFO} = [(\sigma_{ex}+\sigma_t) - \sqrt{(\sigma_{ex}+\sigma_t)^2 - 4\beta\sigma_{ex}\sigma_t}]/2\beta$$

$$= [199.2 - \sqrt{(199.2)^2 - 4(0.6142)(174.5)(24.75)}]/(2x0.6142)$$

$$= 23.36 \text{ ksi}$$

Since $\sigma_{TFO} > 0.5QF_y$, Eq. (5.21) is used to compute P_u.

$$P_u = AQF_y(1-QF_y/4\sigma_{TFO})$$

$$\phi_c P_u = (0.85)(1.24)(0.908)(33)[1-(0.908)(33)/(4x23.36)]$$

$$\phi_c P_u = 21.45 \text{ kips}$$

The above calculations indicate that torsional-flexural buckling governs the design because the value of $\phi_c P_u$ based on torsion-flexural buckling is less than that based on flexural buckling.

The allowable axial load based on LRFD is computed using Eq. (5.8) as follows:

$$(P_a)_{LRFD} = 21.45 \frac{0.5+1}{1.2(0.5)+1.6} = 14.62 \text{ kips}$$

The allowable axial load based on allowable stress design is 13.29 kips. Therefore, the allowable load ratio is 14.62/13.29 = 1.110 which agrees with Eq. (5.24) shown in Figure 24.

PROBLEM NO. 5 - BEAM-COLUMN (DOUBLY-SYMMETRIC SHAPE)

A. Problem Statement. The 6 in. x 3 in. x 0.105 in. I-section with unstiffened flanges shown in Figure C.4 is subjected to an axial load and bending moments applied to each end. The applied bending moments are equal and bend the member in a single curvature about the x-axis. The applied moment due to nominal dead load is 5.0 kip-in. and the applied moment due to nominal live load is 10.0 kip-in. The 8 ft long beam-column is braced at the end points only. The axial load is assumed to have a D/L ratio of 0.5.

Determine the factored nominal axial load capacity and the allowable axial load based on the LRFD criteria.

B. Solution. The factored axial load capacity of the beam-column can be determined from the interaction equations in Section 9.5.1[10]. For flexural failure at the midlength of the beam-column, Eq. (6.6) is used.

$$\frac{P_D}{\phi_c P_{uc}} + \frac{C_{mx} M_{Dx}}{\phi M_{ucx}[1-P_D/(\phi_c P_{Ex})]} \leq 1.0$$

$$M_{DX} = 1.2M_{DL} + 1.6M_{LL} = 1.2(5.0) + 1.6(10.0) = 22.0 \text{ kips}$$

$$\phi_c P_{uc} = 12.77 \text{ kips (see Problem No. 3)}$$

From Table 6 of Part V of the Design Manual[41], $I_x = 8.48$ in.^{4}, $S_{xc} = 2.83$ in.^{3}, and $I_y = 0.476$ in.^{4} From Eq. (6.10),

$$P_{Ex} = \pi^2 EI_x/(KL)_x^2$$

$$P_{Ex} = \pi^2 (29500)(8.48)/(8x12)^2 = 267.9 \text{ kips}$$

$$w/t = [1.5-2(3/16+0.105)]/0.105 = 8.714$$

$$(w/t)_{lim} = 63.3/\sqrt{F_y} = 63.3/\sqrt{33} = 11.02$$

Since $w/t < (w/t)_{lim}$, $F_{cr} = F_y$ according to Eq. (4.9). From Eq. (4.8), $M_u = S_{xc}F_{cr} = S_{xt}F_y$, i.e.

$$M_u = S_{xc}F_{cr} = (2.83)(33) = 93.39 \text{ kip-in.}$$

From Eq. (4.30),

$$M_e = \frac{\pi^2 (29500)(1.0)(6)(0.476/2)}{(8x12)^2} = 45.11 \text{ kip-in.}$$

$$M_y/M_e = 93.39/45.11 = 2.070$$

Since $M_y/M_e > 1.8$, $M_u = M_e$ according to Eq. (4.29) based on lateral buckling. Since lateral buckling governs the design of the moment capacity, $\phi = 0.90$ and

$$M_{uc} = 45.11 \text{ kip-in.}$$

$$M_{us} = 93.39 \text{ kip-in.}$$

From Eq. (6.4) and $M_1/M_2 = -1.0$

$$C_m = 0.6 - 0.4(M_1/M_2) = 0.6 - 0.4(-1.0) = 1.0$$

From Eq. (6.9),

$$P_{us} = A_{eff}F_y = QAF_y = (0.864)(1.80)(33) = 51.32 \text{ kips}$$

From substitution, Eq. (6.6) can be expressed in the following form:

$$\frac{P_D}{12.77} + \frac{(1.0)(22.0)}{(0.90)(45.11)[1-P_D/(0.85\text{x}267.9)]} = 1.0$$

From trial and error, $P_D = 5.672$ kips which is the factored axial load capacity for the beam-column to prevent flexural failure at the midlength.

For failure at the braced points, Eq. (6.7) is used.

$$\frac{P_D}{\phi_s P_{us}} + \frac{M_{Dx}}{\phi_s M_{usx}} \leq 1.0$$

$$\frac{P_D}{(0.95)(51.32)} + \frac{22.0}{(0.95)(93.39)} = 1.0$$

By solving for P_D, a factored axial load capacity of 36.67 kips is obtained for preventing failure at end points. This value is greater than that obtained from flexural failure at midspan. Since $P_D/\phi_c P_{uc} = 0.444 > 0.15$, Eq. (6.8) will not govern the design. Therefore, the factored axial load capacity for the beam-column based on LRFD is 5.672 kips.

The allowable unfactored load based on LRFD is calculated using an equation similar to Eq. (5.8).

$$(P_a)_{LRFD} = 5.672 \frac{0.5+1}{1.2(0.5)+1.6} = 3.867 \text{ kips}$$

The allowable axial load based on allowable stress design is 3.387 kips. Therefore, the allowable load ratio is 3.867/3.387 = 1.142. For this example M_T/M_{us} = 15/93.39 = 0.161. By interpolating Figure 39, a 5 ft I-section with the same dimensions will result in an allowable load ratio of 1.124. This comparison indicates that the increase in length of a beam-column will increase the allowable load ratio as shown in Figures 36 through 38.

PROBLEM NO. 6 - BEAM-COLUMN (SINGLY-SYMMETRIC SHAPE)

A. Problem Statement. The 6 in. x 2.5 in. x 0.105 in. cold-formed steel channel with stiffened flanges shown in Figure C.2 and used in Problem No. 4 is subjected to an eccentric load. The beam-column is 8 ft long and pinned at the end points. F_y = 33 ksi and D/L = 0.5. Section properties can be found in Problems 2 and 4.

Determine the factored eccentric load capacity and the allowable eccentric load based on LRFD and e = +1.73 in.

B. Solution. The failure of the singly-symmetric shape could be governed by flexural or torsional-flexural buckling according to Section 9.5.2[10]. For flexural failure at the midlength of the beam-column, Eq. (6.54) is used.

$$\frac{P_D}{\phi_c P_{uc}} + \frac{C_m M_D}{\phi M_s{}_{us}[1-P_D/(\phi_c P_{Ey})]} \leq 1.0$$

$$\phi_c P_{uc} = 22.76 \text{ kips (see Problem No. 4)}$$

From Eq. (6.66),

$$M_D = eP_D = 1.73P_D$$

$$S_y = 0.621 \text{ in.}^3 \text{ (Table 1 of Part V}^{(41)}\text{)}$$

From Eq. (4.7),

$$M_{us} = S_{eff}F_y = (0.621)(33) = 20.49 \text{ kip-in.}$$

$$\phi_s = 0.95$$

From Eqs. (6.4) and (6.11),

$$C_m = 0.6-0.4(-1.0) = 1.0$$

$$P_{Ey} = \pi^2(29500)(1.05)/(8x12)^2 = 33.17 \text{ kips}$$

From substitution, Eq. (6.54) can be expressed in the following form:

$$\frac{P_D}{22.76} + \frac{(1.0)(1.73P_D)}{(0.95)(20.49)[1-P_D/(0.85x\,33.17)]} = 1.0$$

By solving for P_D, a factored eccentric load capacity of 6.31 kips is obtained for flexural failure at the midlength. Equations (6.55) and (6.56) will not govern the design.

For torsional-flexural failure, $\phi_c P_u$ was computed using the value of σ_{TF} obtained from Eq. (6.60).

$$\frac{\sigma_{TF}}{\sigma_{TFO}} + \frac{C_{TF}\sigma_{b1}}{\sigma_{bT}(1-\sigma_{TF}/\sigma_e)} = 1.0$$

where

$$\sigma_{TFO} = 23.36 \text{ ksi (see Problem No. 4)}$$

$$j = 3.49 \text{ in. (Table 1 of Part V}^{(41)}\text{)}$$

$$\sigma_{ex} = 174.5 \text{ ksi (see Problem No. 4)}$$

$\sigma_t = 24.75$ ksi (see Problem No. 4)

$$M_t = -A\sigma_{ex}[j-\sqrt{j^2 + r_o^2(\sigma_t/\sigma_{ex})}]$$

$$= -(1.24)(174.5)[3.49-\sqrt{(3.49)^2+(3.22)^2(24.75/174.5)}]$$

$$= 44.29 \text{ kip-in.}$$

$C_{TF} = 1.0$

$$\sigma_{bT} = M_t c/I_y$$

$$= (44.29)(1.692)/1.05 = 71.37 \text{ ksi}$$

$$\sigma_e = \pi^2 E/(KL/r_y)^2$$

$$= \pi^2(29500)/(96/0.921)^2 = 26.80 \text{ ksi}$$

$$\sigma_{bl} = \sigma_{TF}\ ec/r_y^2$$

$$= \sigma_{TF}(1.73)(1.692)/(0.921)^2 = 3.451\sigma_{TF}$$

From substitution, Eq. (6.60) can be expressed in the following form:

$$\frac{\sigma_{TF}}{23.36} + \frac{3.451\sigma_{TF}}{71.37(1-\sigma_{TF}/26.80)} = 1.0$$

By solving for σ_{TF}, an average elastic torsional-flexural buckling stress of 8.734 ksi is obtained. Since $\sigma_{TF} < (0.5QF_y = 15.25 \text{ ksi})$, $\phi_c P_u$ can be computed according to Eq. (6.59).

$$\phi_c P_u = \phi_c A\sigma_{TF} = (0.85)(1.24)(8.734) = 9.21 \text{ kips}$$

Flexural buckling governs since 9.21 kips > 6.31 kips determined from flexural buckling. The allowable eccentric load based on LRFD is computed from Eq. (5.8) as follows:

$$(P_a)_{LRFD} = 6.31 \frac{0.5+1}{1.2(0.5)+1.6} = 4.30 \text{ kips}$$

From allowable stress design the allowable load is also governed by flexural buckling and is 3.94 kips. Therefore, the allowable load ratio is 4.30/3.94 = 1.091. This ratio agrees with the allowable load ratio from Figure 45.

PROBLEM NO. 7 - ARC SPOT WELD

A. Problem Statement. The arc spot welds shown in Figure C.5 connect two steel sheets (F_y = 50 ksi and F_u = 65 ksi). Calculate the factored nominal strength and the allowable load of the connection based on LRFD. Use E60 electrode (F_{xx} = 60 ksi) and D/L = 1/3.

B. Solution. According to Section 10.2.1.3[10], the factored nominal strength of each spot weld is computed as follows:

$d_a = d - t = 0.75 - 0.06 = 0.69$ in.

$d_e = 0.7d - 1.5t \leq 0.55d$

$= 0.7(0.75) - 1.5(0.06) = 0.435$ in. $> (0.55d = 0.4125$ in.)

$= 0.4125$ in.

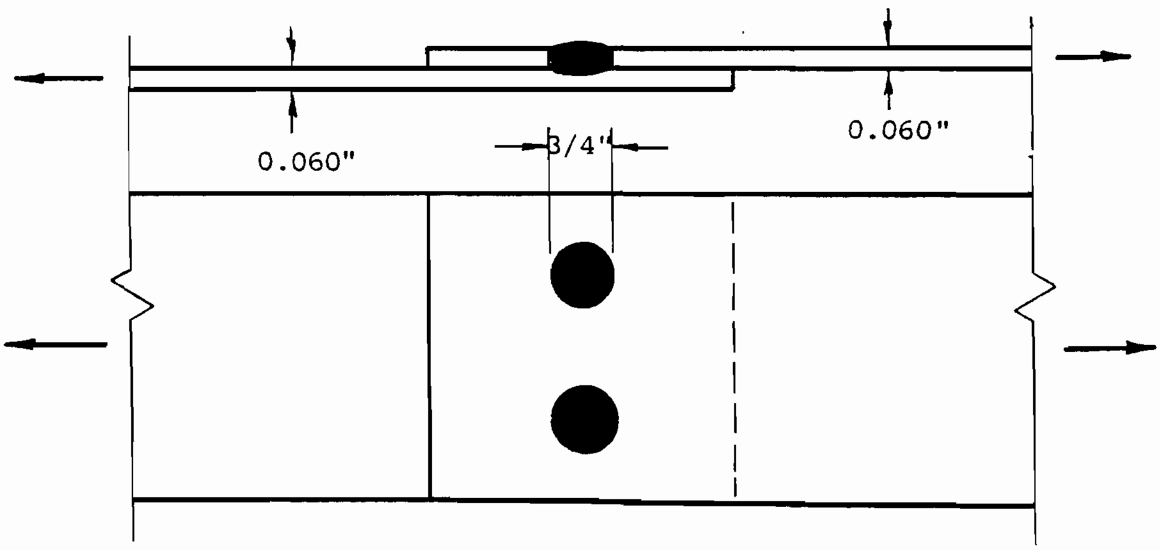


Figure C.5 Arc Spot Weld Connection in Problem No. 7

To prevent shear failure, Eq. (7.5) is used as follows:

$$R_n = (\pi d_e^2/4)(0.6F_{xx})$$

$$= [\pi(0.4125)^2/4](0.6x60) = 4.811 \text{ kips}$$

$$\phi R_n = (0.70)(4.811) = 3.368 \text{ kips}$$

To prevent plate failure, ϕR_n is computed as follows:

$$d_a/t = 0.69/0.060 = 11.5$$

$$114/\sqrt{F_u} = 114/\sqrt{65} = 14.14$$

Since $d_a/t < 114/\sqrt{F_u}$, Eq. (7.6) is used with $\phi = 0.60$.

$$R_n = 2.2td_aF_u$$

$$= 2.2(0.06)(0.69)(65) = 5.920 \text{ kips}$$

$$\phi R_n = (0.60)(5.920) = 3.552 \text{ kips}$$

Since 3.368 kips < 3.552 kips, shear failure groverns the design. Therefore, the factored nominal strength of the connection is 2x3.368 = 6.74 kips.

The allowable load based on LRFD can be calculated using Eq. (7.9) as follows:

$$(P_a)_{LRFD} = 6.74 \frac{1/3+1}{1.2(1/3)+1.6} = 4.49 \text{ kips}$$

The allowable load based on allowable stress design is 4.74 kips. Therefore, the allowable load ratio is 4.49/4.74 = 0.947. The disagreement between the above ratio and Fig. 53 is because $(P_a)_{LRFD}$ is based on shear failure and $(P_a)_{ASD}$ is based on plate failure.

<u>PROBLEM NO 8</u> - Arc SEAM WELD

A. <u>Problem Statement</u>. The arc seam weld shown in Figure C.6 connects two steel sheets (F_y = 50 ksi and F_u = 65 ksi). Calculate the factored nominal strength and the allowable load of the connection based on the LRFD criteria. Use E60 electrode (F_{xx} = 60 ksi) and D/L = 1/3.

B. Solution. According to Section 10.2.1.4[10], the factored nominal strength of the arc seam weld is computed as follows:

$$d_a = d-t = 0.75 - 0.06 = 0.69 \text{ in.}$$

$$d_e = 0.7d - 1.5t = 0.7(0.75) -1.5(0.06) = 0.435 \text{ in.}$$

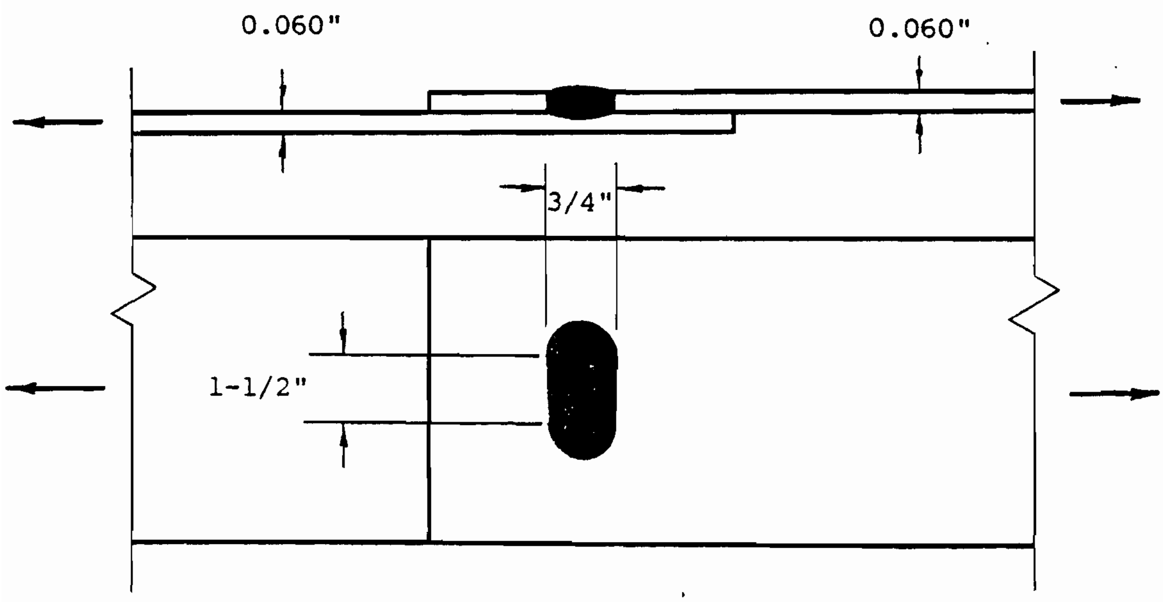


Figure C.6 Arc Seam Weld Connection in Problem No. 8

To prevent shear failure, Eq. (7.17) is used with $\phi = 0.70$ as follows:

$$R_n = (\pi d_e^2/4 + Ld_e)(0.6F_{xx})$$

$$= [\pi(0.435)^2/4+(1.5)(0.435)](0.6x60) = 28.84 \text{ kips}$$

$$\phi R_n = (0.70)(28.84) = 20.19 \text{ kips}$$

To prevent plate failure, Eq. (7.18) is used with $\phi = 0.60$ as follows:

$$R_u = (0.63L + 2.4d_a)tF_u$$

$$= [0.63(1.5) + 2.4(0.69)](0.06)(65) = 1.04 \text{ kips}$$

$$\phi R_u = (0.60)(10.14) = 6.09 \text{ kips}$$

Since 6.09 kips < 20.19 kips, plate failure governs the design. The factored nominal strength of the weld is 6.09 kips.

The allowable load based on LRFD can be calculated using Eq. (7.19) as follows:

$$(P_a)_{LRFD} = 6.09 \frac{1/3+1}{1.2(1/3)+1.6} = 4.06 \text{ kips}$$

The allowable load based on allowable stress design is 4.05 kips. Therefore, the allowable load ratio is 4.06/4.05 = 1.003 which agrees with Eq. (7.21) shown in Figure 55.

PROBLEM NO. 9 - FILLET WELD

A. Problem Statement. The fillet welds shown in Figure C.7 connects two steel sheets (F_y = 50 ksi and F_u = 65 ksi). Calculate the factored nominal strength and the allowable load of the connection based on LRFD. Use E60 electorde, F_{xx} = 60 ksi, and D/L = 1/3.

B. Solution. According to Section 10.2.1.5[10], the factored nominal strength of a fillet weld loaded in the longitudinal direction is computed as follows:

$$L/t = 2/0.06 = 33.3 > 25$$

Since $L/t > 25$, $\phi = 0.60$ and R_u is calculated from Eq. (7.27).

$$R_n = 0.75tLF_u$$

$$= 0.75(0.06)(2)(65) = 5.85 \text{ kips}$$

$$\phi R_n = (0.60)(5.85) = 3.51 \text{ kips}$$

Since the connection consists of two fillet welds, the factored nominal strength of the connection is 2 x 3.51 = 7.02 kips.

The allowable load based on LRFD can be calculated using Eq. (7.30) as follows:

$$(P_a)_{LRFD} = 7.02 \frac{1/3+1}{1.2(1/3)+1.6} = 4.68 \text{ kips}$$

The allowable load based on allowable stress design is 2(0.3)(0.06)(2)(65) = 4.68 kips. Therefore, the allowable load ratio is 4.68/4.68 = 1.00 which agrees with Eq. (7.33) shown in Figure 56.

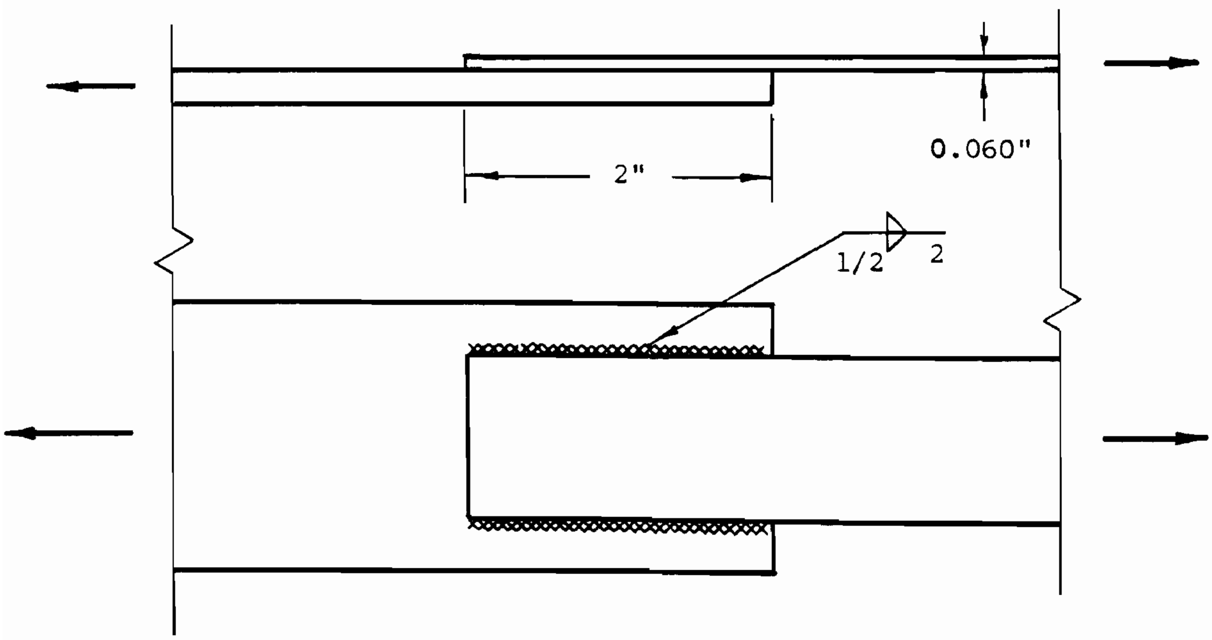


Figure C.7 Fillet Welded Connection in Problem No. 9

PROBLEM NO. 10 - FLARE-BEVEL GROOVE WELD

A. Problem Statement. The flare-bevel groove welded connection shown in Figure C.8 is loaded in the transverse direction. For the sheets, F_y = 50 ksi and F_u = 65 ksi. Calculate the factored nominal strength and the allowable load of the connection based on the LRFD criteria. Use E60 electrode (F_{xx} = 60 ksi) and D/L = 1/3. Assume $t \leq t_w < 2t$.

B. Solution. According to Section 10.2.1.6[10], the factored nominal strength of the flare-bevel groove weld is computed from Eq.(7.40) and ϕ = 0.55 as follows:

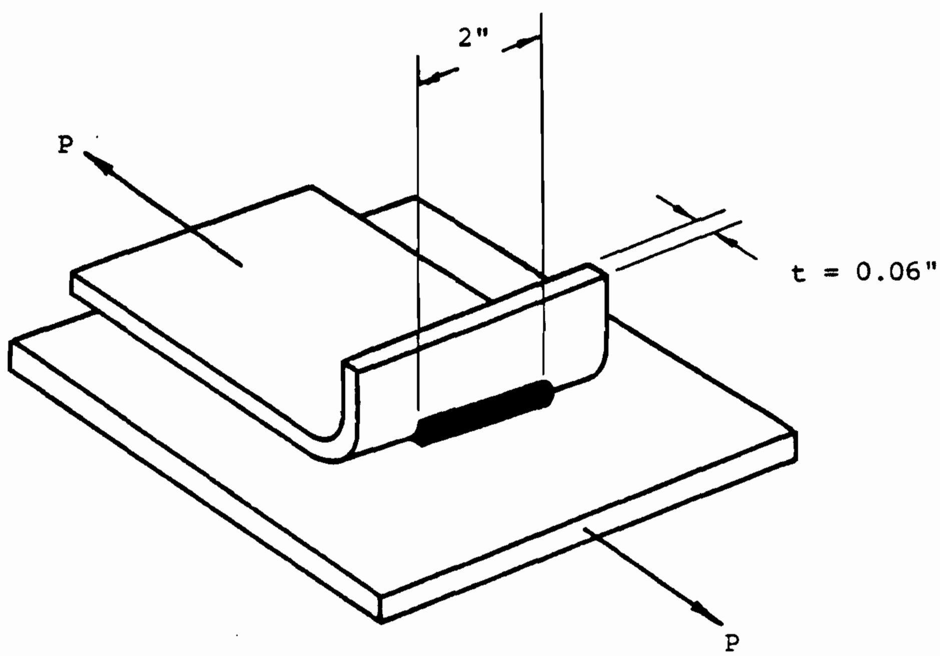


Figure C-8 Flare-Bevel Groove Welded Connection in Problem 10

$$R_n = 0.8tLF_u$$

$$= 0.80(0.06)(2)(65) = 6.24 \text{ kips}$$

$$\phi R_n = (0.55)(6.24) = 3.43 \text{ kips}$$

The allowable load based on LRFD can be calculated using Eq. (7.44) as follows:

$$(P_a)_{LRFD} = 3.43 \frac{1/3+1}{1.2(1/3)+1.6} = 2.29 \text{ kips}$$

The allowable load based on allowable stress design is (0.06)(2)(65)/3 = 2.60 kips. Therefore, the allowable load ratio is 2.29/2.60 = 0.881 which agrees with Eq. (7.46) shown in Figure 58.

PROBLEM NO. 11 - RESISTANCE WELD

A. Problem Statement. Two resistance spot welds connect two steel sheets ($t = 0.06$ in.) as shown in Figure C.5. Calculate the factored nominal strength of the connection based on weld strength and the LRFD criteria. Assume $D/L = 1/3$.

B. Solution. According to Section 10.2.2[10], the nominal shear strength per spot can be obtained from Table 7.2.

$$R_n = 1.810 \text{ kips/spot (for } t = 0.06 \text{ in.)}$$

$$\phi = 0.65$$

$$\phi R_n = (0.65)(1.810) = 1.177 \text{ kips/spot}$$

Since there are two spot welds in the connection, the factored nominal strength of the connection is 2 x 1.177 = 2.35 kips.

The allowable load based on the LRFD criteria can be computed from Eq. (7.49) as follows:

$$(P_a)_{LRFD} = 2.35 \frac{1/3+1}{1.2(1/3)+1.6} = 1.57 \text{ kips}$$

The allowable load based on allowable stress design is $2 \times 0.725 = 1.45$ kips (from Table 7.1). Therefore, the allowable load ratio is $1.57/1.45 = 1.082$ which agrees with Eq. (7.50) shown in Figure 58.

PROBLEM NO. 12 - BOLTED CONNECTION

A. Problem Statement. The bolted connection shown in Figure C.9 connects two steel sheets (F_y = 50 ksi and F_u = 65 ksi). 1/2 in. diameter A-307 bolts with washers under both bolt head and nut are used in the single shear connection.

Determine the factored nominal strength and the allowable load based on the LRFD criteria. Assume D/L = 1/3 and the threading is excluded from the shear plane.

B. Solution. For bolted connections, spacing and edge distances, tension on net section, bearing strength, and shear strength of the bolts have to be checked.

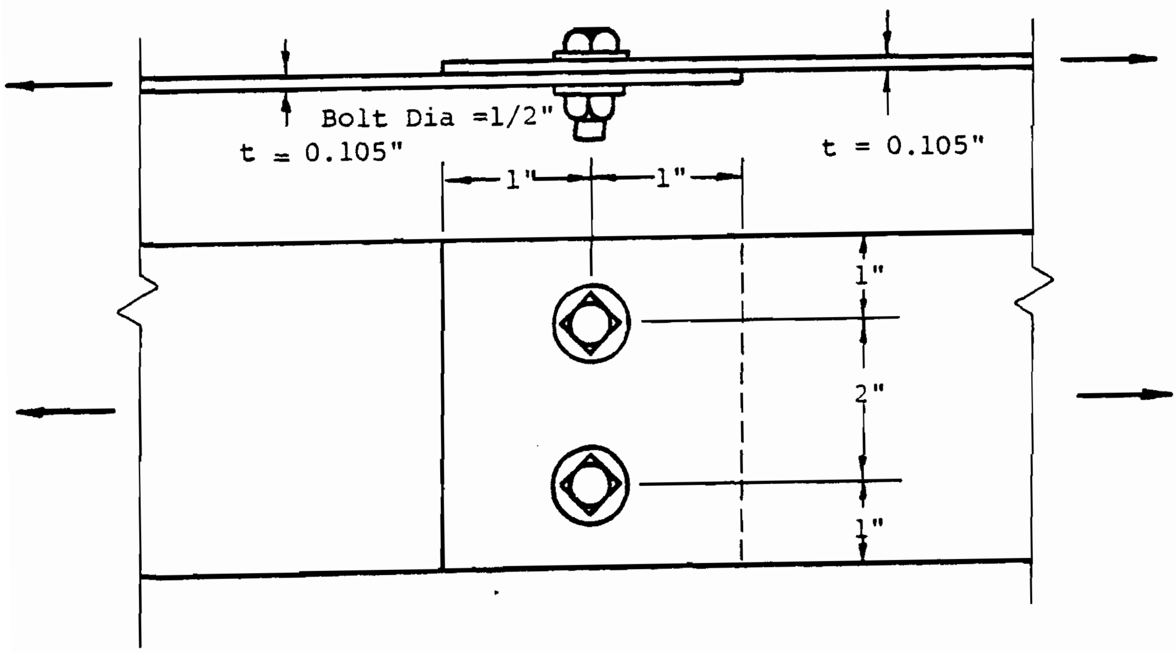


Figure C.9 Bolted Connection in Problem No. 12

1. Minimum Spacing and Edge Distance in Line of Stress. According to Section 10.3.2(10), the factored nominal shear strength of the connection can be computed with $\phi = 0.70$ as follows:

$$F_u/F_y = 65/50 = 1.3$$

Since $F_u/F_y > 1.15$, Eq. (7.53) is used.

$$R_n = 2(teF_u)$$
$$= (2)(0.105)(1)(65) = 13.65 \text{ kip}$$
$$\phi R_n = (0.70)(13.65) = 9.56 \text{ kips}$$

The allowable load based on the LRFD criteria can be calculated using Eq. (7.57) as follows:

$$(P_a)_{LRFD} = 9.56 \frac{1/3+1}{1.2(1/3)+1.6} = 6.37 \text{ kips}$$

The allowable load based on allowable stress design is (2)(0.5)(0.105)(1)(65) = 6.83 kips. Therefore, the allowable load ratio is 6.37/6.83 = 0.933 which agrees with Eq. (7.58) shown in Figure 60.

2. Tensile Strength on Net Section. According to Section 10.3.3(10), the factored nominal tensile strength can be computed using $\phi = 0.60$ and Eq. (7.62) as follows:

$$R_n = (1.0 - 0.9r + 3rd/s)F_uA_n \leq F_uA_n$$
$$r = P/P = 1.0$$
$$s = 2 \text{ in.}$$
$$A_n = [4 - 2(1/2 + 1/16)](0.105) = 0.3019 \text{ in.}^2$$
$$R_n = [1.0 - 0.9(1) + 3(1)(1/2)/2](65)(0.3019)$$
$$= 16.68 \text{ kips}$$
$$\phi R_n = (0.60)(16.68) = 10.01 \text{ kips}$$

In addition, the factored nominal tensile strength should not exceed the following value computed from Eq. (7.64):

$$\phi R_n = \phi F_y A_n = (0.90)(50)(0.3019) = 13.59 \text{ kips}$$

The factored nominal tensile strength of the connection based on tension on the net section is 10.01 kips.

The allowable load based on the LRFD criteria can be calculated from Eq. (7.66) as follows:

$$(P_a)_{LRFD} = 10.01\frac{1/3+1}{1.2(1/3)+1.6} = 6.67 \text{ kips}$$

The allowable load based on allowable stress design is (0.85)(0.45)(65)(0.3019) = 7.51 kips. Therefore, the allowable load ratio is 6.67/7.51 = 0.888 which agrees with Eq. (7.68) shown in Figure 61.

3. <u>Bearing Strength</u>. According to Section 10.3.4[10], the factored nominal bearing strength of the single shear connection with washers can be computed from Eq. (7.77) with $\phi = 0.65$ as follows:

$$F_u/F_y \geq 1.15 \text{ (see Part 2 of this problem)}$$

$$R_n = 2(3.0\ F_u\ dt)$$

$$= (2)(3.0)(65)(1/2)(0.105) = 20.48 \text{ kips}$$

$$\phi R_n = (0.65)(20.48) = 13.31 \text{ kips}$$

The allowable load based on the LRFD criteria can be computed from Eq. (7.81) as follows:

$$(P_a)_{LRFD} = 13.31\ \frac{1/3+1}{1.2(1/3+1.6} = 8.87 \text{ kips}$$

The allowable load based on allowable stress design is

(2)(1.35)(65)(1/2)(0.105) = 9.21 kips. Therefore, the allowable load ratio is 8.87/9.21 = 0.963 which agrees with Eq. (7.84) shown in Figure 62.

4. Shear Strength of Bolts. According to Section 10.3.5[10], the factored nominal shear strength of two 1/2 in. diameter bolts can be determined from Eq. (7.87) as follows:

$$\phi = 0.65 \text{ (for A307 bolts)}$$

$$R_n = 2(0.6m\ A_{sA}\ F_u)$$

$$A_{sA} = 0.196 \text{ in.}^2 \text{ (Table 7.3 for threading excluded)}$$

$$m = 1 \text{ (one shear plane)}$$

$$F_u = 60 \text{ ksi (Table 7.4 for A307-78-A)}$$

$$R_n = 2(0.6)(0.196)(60) = 14.11 \text{ kips}$$

$$\phi R_n = (0.65)(14.11) = 9.17 \text{ kips}$$

The allowable load based on the LRFD criteria can be computed using Eq. (7.89) as follows:

$$(P_a)_{LRFD} = 9.17\ \frac{1/3+1}{1.2(1/3)+1.6} = 6.12 \text{ kips}$$

The allowable load based on allowable stress design is (2)(10)(0.196) = 3.92 kips. Therefore, the allowable load ratio is 6.12/3.92 = 1.561. This ratio agrees with the allowable load ratio computed with K_b = 2.340 (Table 7.5) from Eq. (7.94) as follows:

$$\frac{(P_a)_{LRFD}}{(P_a)_{ASD}} = 2.340\ \frac{1/3+1}{1.2(1/3)+1.6} = 1.560$$

5. Summary. The factored nominal strength of the connection based on the LRFD criteria is 9.17 kips. This value is governed by shear strength of bolts. Consequently, the allowable load based on LRFD is 6.12 kips.