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Illustrative examples based on the ASCE standard specification for the design of cold-formed stainless steel structural members

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Cold-Formed Steel Series

Final Report

ILLUSTRATIVE EXAMPLES BASED ON THE ASCE STANDARD SPECIFICATION FOR THE DESIGN OF COLD-FORMED STAINLESS STEEL STRUCTURAL MEMBERS

by

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PREFACE

During the past four years, two methods were developed for the design of stainless steel structural members at the University of Missouri-Rolla with consultation of Professor T. V. Galambos at the University of Minnesota. One of the methods is based on the load and resistance factor design (LRFD) and the other is based on the allowable stress design (ASD). Both design methods are now included in the new ASCE Standard 8-90, Specification for the Design of Cold-Formed Stainless Steel Structural Members.

At the September 21, 1990 meeting of the Control Group of the ASCE Stainless Steel Cold-Formed Section Standards Committee held in Washington, D.C., the urgent need for design examples using the new ASCE Standard was discussed at length. The University of Missouri-Rolla was asked to submit a proposal for preparation of such illustrative examples beginning October 1, 1990.

During the period from October 1990 through December 1991, a total of 27 illustrative problems have been prepared as included herein. Most of the given data used for these examples are similar to those used in the 1986 edition of the AISI Cold-Formed Steel Manual except that for each problem, two examples are illustrated by using LRFD and ASD methods.

The research work reported herein was conducted in the Department of Civil Engineering at the University of Missouri-Rolla with the consulting work provided by Dr. Shin-Hua Lin and Professor T. V. Galambos. The financial assistance provided by the Nickel Development Institute and the Chromium Centre is gratefully acknowledged. Appreciation is also expressed to Dr. W. K. Armitage, Mr. J. P. Schade, Professor P. Van der Merwe and Professor G. J. Van den Berg for their technical review and suggested revisions.

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I. INTRODUCTION

This publication cantains 54 examples for calculation of sectional properties, and the design of beams, compression members, beam-columns, and connections. They are prepared for the purpose of illustrating the application of various provisions of the new ASCE Standard 8-90, Specification for the Design of Cold-Formed Stainless Steel Structural Members.

II. COMPUTATION OF SECTIONAL PROPERTIES OF COLD-FORMED SECTION USING LINEAR METHOD

In the calculation of sectional properties of cold-formed stainless steel sections, the computation can be simplified by using a so-called linear method, in which the material of the section is considered to be concentrated along the centerline of the steel sheet and the area elements replaced by straight or curved "line elements." The thickness dimension, t, is introduced after the linear computations have been completed. This method has long been used for the design of cold-formed carbon steel sections.*

In the application of the linear method, the total area of the section is found from the following relation:

Area = L x t

where "L" is the total length of all line elements.

The moment of inertia of the section, I, is found from the following relation:

I = I' x t

---

* Cold-Formed Steel Design Manual (1986). American Iron and Steel Institute, Washington, D.C.

where "I'" is the moment of inertia of the centerline of the steel sheet.

The section modulus is computed as usual by dividing I or I' x t by the distance from the neutral axis to the extreme fiber, not to the centerline of the extreme element.

First power dimensions, such as x, y, and r (radius of gyration) are obtained directly by the linear method and do not involve the thickness dimension.

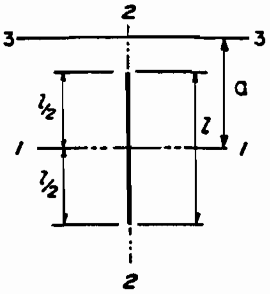
When the flat width of an element is reduced for design purpose, the effective design width, b, is used directly to compute the total effective length, L_{eff}, of the line elements, as shown in the examples.

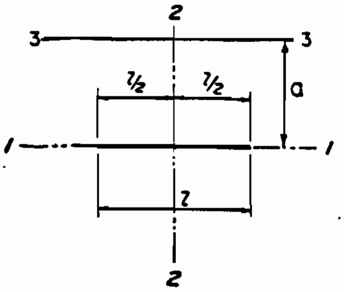
The element into which most sections may be divided for application of the linear method consist of straight lines and circular arcs. For convenient reference, the moments of inertia and location of centroid of such elements are identified in the sketches and formulas in Fig. 1, Properties of Line Elements.

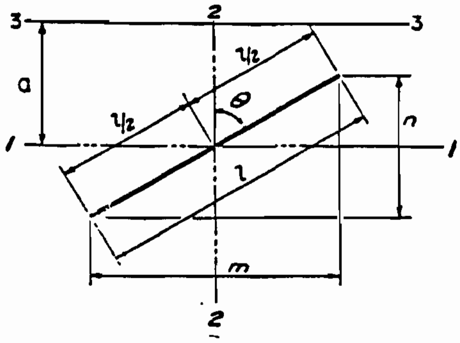
The formulas for line elements are exact, since the line as such has no thickness dimension; but in computing the properties of an actual element with a thickness dimension, the results will be approximate for the reasons given in the AISI Manual.

III. CORRELATION OF SPECIFICATION AND ILLUSTRATIVE EXAMPLES

The tables on pages 4 through 11 provide an easy cross reference between design provisions of the Specification and the illustrative examples. The first table is based on the type of design examples. The second table is based on various sections of the Specification.







$$I_1 = \left[\frac{\cos^2\theta}{12}\right] l^3 = \frac{ln^2}{12}$$

$$I_2 = \left[\frac{\sin^2\theta}{12}\right] l^3 = \frac{lm^2}{12}$$

$$= \frac{l^3}{12}, \qquad I_2 = 0$$

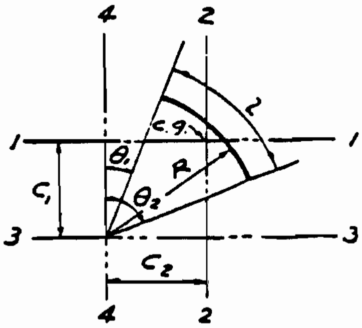
$$= la^2 + \frac{l^3}{12} = l\left(a^2 + \frac{l^2}{12}\right)$$

$$I_1 = 0, \qquad I_2 = \frac{l^3}{12}$$

$$I_3 = la^2$$

$$I_{12} = \left[\frac{\sin\theta\cos\theta}{12}\right] l^3 = \frac{lmn}{12}$$

$$I_3 = la^2 + \frac{ln^2}{12} = l\left(a^2 + \frac{n^2}{12}\right)$$



θ (expressed in radians) = 0.01745 θ (expressed in degrees and decimals thereof)

$$l = (\theta_2 - \theta_1)\,R$$

$$c_1 = \frac{\sin\theta_2 - \sin\theta_1}{\theta_2 - \theta_1} R, \qquad c_2 = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1} R$$

$$I_1 = \left[\frac{\theta_2 - \theta_1 + \sin\theta_2\cos\theta_2 - \sin\theta_1\cos\theta_1}{2} - \frac{(\sin\theta_2 - \sin\theta_1)^2}{\theta_2 - \theta_1}\right] R^3$$

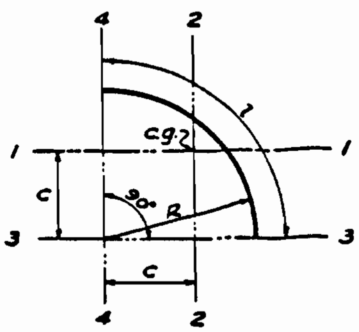
$$I_2 = \left[\frac{\theta_2 - \theta_1 - \sin\theta_2\cos\theta_2 + \sin\theta_1\cos\theta_1}{2} - \frac{(\cos\theta_1 - \cos\theta_2)^2}{\theta_2 - \theta_1}\right] R^3$$

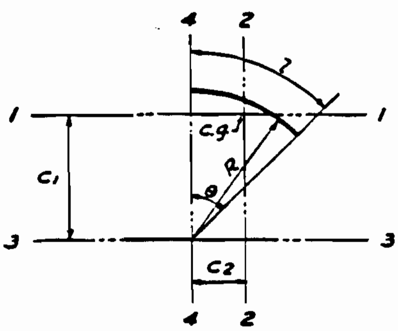
$$I_{12} = \left[\frac{\sin^2\theta_2 - \sin^2\theta_1}{2} + \frac{(\sin\theta_2 - \sin\theta_1)(\cos\theta_2 - \cos\theta_1)}{\theta_2 - \theta_1}\right] R^3$$

$$_3 = \left[\frac{\theta_2 - \theta_1 + \sin\theta_2\cos\theta_2 - \sin\theta_1\cos\theta_1}{2}\right] R^3, \quad I_4 = \left[\frac{\theta_2 - \theta_1 - \sin\theta_2\cos\theta_2 + \sin\theta_1\cos\theta_1}{2}\right] R^3, \quad I_{34} = \left[\frac{\sin^2\theta_2 - \sin^2\theta_1}{2}\right] R^3$$

ASE I: $\theta_1 = 0$, $\theta_2 = 90°$

CASE II: $\theta_1 = 0$, $\theta_2 = \theta$





$$l = \theta R$$

$$c_1 = \frac{R\sin\theta}{\theta}$$

$$c_2 = \frac{R(1 - \cos\theta)}{\theta}$$

$l = 1.57\,R$, $c = 0.637\,R$

$I_1 = I_2 = 0.149\,R^3$

$I_{12} = -0.137\,R^3$

$I_3 = I_4 = 0.785\,R^3$

$I_{34} = 0.5\,R^3$

$$I_1 = \left[\frac{\theta + \sin\theta\cos\theta}{2} - \frac{\sin^2\theta}{\theta}\right] R^3, \quad I_2 = \left[\frac{\theta - \sin\theta\cos\theta}{2} - \frac{(1 - \cos\theta)^2}{\theta}\right] R^3$$

$$I_{12} = \left[\frac{\sin^2\theta}{2} + \frac{\sin\theta(\cos\theta - 1)}{\theta}\right] R^3$$

$$I_3 = \left[\frac{\theta + \sin\theta\cos\theta}{2}\right] R^3, \qquad I_4 = \left[\frac{\theta - \sin\theta\cos\theta}{2}\right] R^3$$

$$I_{34} = \left[\frac{\sin^2\theta}{2}\right] R^3$$

Figure 1 Properties of Line Elements

CROSS REFERENCE BY EXAMPLE TO SPECIFICATION SECTION

PROBLEM NO. *	TITLE OF EXAMPLE	USE OF SECTION NO.
A. FLEXURAL MEMBERS		
1	Channel w/Unstiffened Flanges	1.5.1,1.5.2,1.5.5, 2.1.1,2.2.1,2.2.2,2.3.1, 3.3.1.1, App. E.
2	Channel w/Stiffened Flanges	1.5.1,1.5.2,1.5.5,2.1.1, 2.1.2,2.2.1,2.2.2, 2.4,2.4.2,3.3.1.1, App. E.
3	C-Section w/Bracing	1.5.1,1.5.2,1.5.5,2.1.1, 2.1.2,2.2.1,2.2.2,2.4, 2.4.2,3.3.1.1, App. E.
4	Z-Section w/Stiffened Flanges	1.5.1,1.5.2,1.5.5,2.1.1, 2.1.2,2.2.1,2.2.2,2.4, 2.4.2,3.3.1.1, App. E.

* Two design examples are included for each problem. The first example uses the LRFD method and the second example uses the ASD method.

5	Deep Z-Section w/Stiffened Flanges	1.5.1,1.5.2,1.5.5, 2.2.1,2.1.1,2.2.2, 2.4,2.4.2,3.3.1.1, App. E.
6	Hat Section	1.5.1,1.5.2,1.5.5, 2.1.1,2.1.2,2.2.1, 2.2.2,3.3.1,3.3.1.1, 3.3.2,3.3.4,App. E.
7	Hat Section w/Intermediate Stiffener	1.5.1,1.5.2,1.5.5, 2.1.1,2.1.2,2.2.1,2.2.2, 2.4,2.4.1,3.3.1,3.3.1.1, App. E.
8	I-Section w/Unstiffened Flanges	1.5.5,2.1.1,2.2.1,2.2.2, 3.3.1.1,3.3.1.2, App. E.
9	Channel w/Lateral Buckling Consideration	1.5.5,2.1.1,2.2.1,2.2.2, 2.3.1,3.3.1.1,3.3.1.2, 3.3.2,3.3.3,3.3.4, 3.3.5,App. E.
10	Hat Section Using Inelastic Reserve Capacity	1.5.5,2.1.1,2.1.2,2.2.1, 2.2.2,3.3.1.1, App. E.
11	Deck Section	1.5.5,2.1.1,2.1.2, 2.2.1,2.2.2,2.4, 2.4.2,3.3.1.1,3.3.2, 3.3.3,3.3.4,App. E.

12	Cylindrical Tubular Section	1.5.5,3.6,3.6.1, App. E
13	Flange Curling	1.5.5,2.1.1,2.2.1,2.2.2, 2.3.1,3.3.1.1,App. E
14	Shear Lag	2.1.1,2.1.2,2.2.1,2.2.2, 3.3.1.1,App. E
B. COMPRESSION MEMBERS		
15	C-Section	1.5.5,2.1.1,2.2.1,2.4, 2.4.2,3.4,3.4.1,3.4.2, 3.4.3,App. B,App. E
16	C-Section w/Wide Flanges	1.5.5,2.1.1,2.2.1, 2.4,2.4.2,3.4,3.4.1, 3.4.2,3.4.3,App. E
17	I-Section	1.5.5,2.2.1,2.2.2,2.4, 2.4.2,3.4.1,3.4.2, 3.4.3,App. B,App. E
18	I-Section w/Lips	1.5.5,2.2.1,2.2.2,2.4, 2.4.2,3.4.1,3.4.2, 3.4.3,App. E
19	T-Section	1.5.5,2.2.1,2.2.2,2.4, 2.4.2,3.4.1,3.4.2, 3.4.3,App. E
20	Tubular Section - Square	1.5.5,2.1.1,2.2.1, 3.4,3.4.1,App. E
21	Tubular Section - Round	1.5.5,3.4.1,3.6, 3.6.1,3.6.2,App. E

C. BEAM-COLUMN MEMBERS

22	C-Section	1.5.5,2.1.2,2.2.1, 2.2.2,2.4,2.4.2,3.3.1.2, 3.4,3.4.1,3.4.2, 3.4.3,3.5,App. E
23	Tubular Section	1.5.5,2.1.1,2.1.2, 2.2.1,2.2.2,3.3.1.1, 3.4,3.4.1,3.5,App. E

D. CONNECTIONS

24	Flat Section w/Bolted Connection	3.2,5.3,5.3.1,5.3.2, 5.3.3,5.3.4,App. E
25	Flat Section w/Lap Fillet Welded Connection	5.2,5.2.2,App. E
26	Flat Section w/Groove Welded Connection in Butt Joint	5.2,5.2.1,App. E
27	Built-Up Section - Connecting Two Channels	4.1,4.1.1,5.2.3,App. E

CROSS REFERENCE BY SPECIFICATION SECTION TO DESIGN EXAMPLE

SECTION NO.	ILLUSTRATED IN PROBLEM NO. *
1. General Provisions	
1.1	
1.2	
1.3	
1.4	
1.5	
1.5.1	1,2,3,4,5,6,7
1.5.2	1,2,3,4,5,6,7
1.5.3	
1.5.4	
1.5.5	1,2,3,4,5,6,7,8,9,10,11,12,13, 15,16,17,18,19,20,21,22,23
1.6	
2. Elements	
2.1	
2.1.1	1,2,3,4,5,6,7,8,9,10,11,13,14, 15,16,17,20,21,22,23
2.1.2	2,3,4,5,6,7,10,11,14,19,20,21, 22,23

* Two design examples are included for each problem. The first example uses the LRFD method and the second example uses the ASD method.

2.2	
2.2.1	1,2,3,4,5,6,7,8,9,10,11,13,14, 15,16,17,18,19,20,22,23
2.2.2	1,2,3,4,5,6,7,8,9,10,11,13,14, 22,23
2.3	
2.3.1	1,9,13
2.3.2	1,9,13
2.4	2,3,4,5,7,11,15,16,17,18,19,22
2.4.1	7
2.4.2	2,3,4,5,11,15,16,17,18,19,22
2.5	
2.6	

3. Members	
3.1	
3.2	21
3.3	
3.3.1	6,7
3.3.1.1	1,2,3,4,5,6,7,8,9,10,11,13, 14,23
3.3.1.2	8,9,22
3.3.2	6,9,11
3.3.3	9,11
3.3.4	6,9,11
3.3.5	9
3.4	15,16,17,19,20,22,23

5.3.2		24
5.3.3		24
5.3.4		24
6. Tests		
6.1		
6.2		
6.3		
App. B	Modified Ramberg-Osgood Equation	15,17
App. E	Allowable Stress Design (ASD)	All examples using ASD method

IV. ILLUSTRATIVE EXAMPLE

EXAMPLE 1.1 CHANNEL W/UNSTIFFENED FLANGES (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi_b M_n$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use the following two types of stainless steels: (A) Type 301, 1/4-Hard and (B) Type 409. Assume dead load to live load ratio D/L = 1/5 and 1.2D+1.6L governs the design.

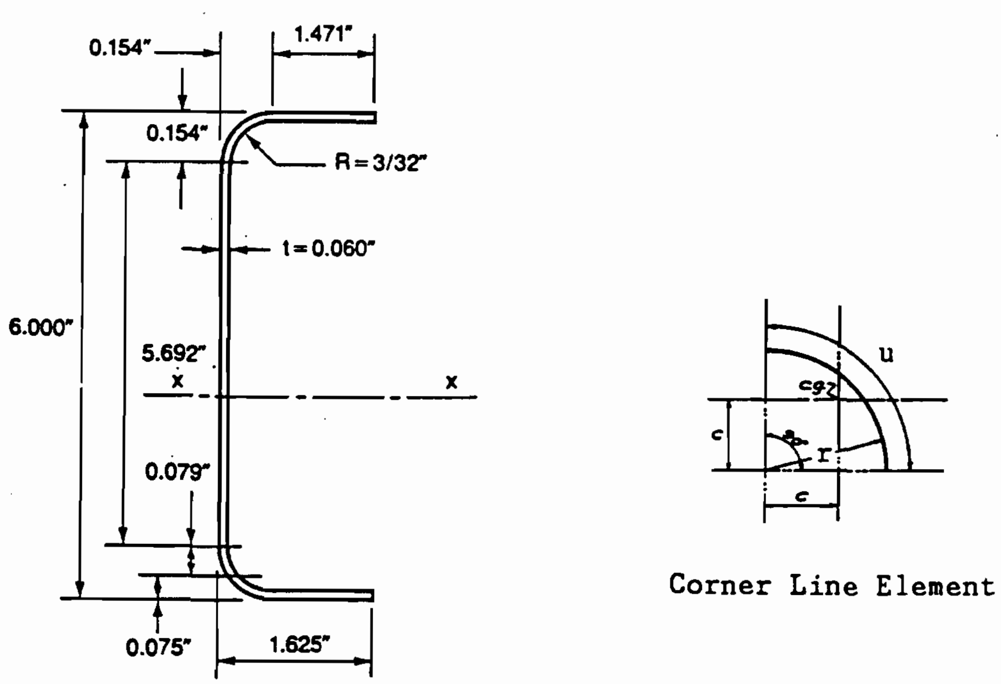


Figure 1.1 Section for Example 1.1

Given:

1. Section: 6" x 1.625" x 0.060" channel with unstiffened flanges.
2. Compression flange braced against lateral buckling.

Solution:

(A) Type 301 Stainless Steel, 1/4-Hard.

1. Calculation of the design flexural strength, $\phi_b M_n$:

a. Properties of 90^o corners:

$r = R + t/2 = 3/32 + 0.060/2 = 0.124$ in.

Length of arc, $u = 1.57r = 1.57 \times 0.124 = 0.195$ in.

Distance of c.g. from center of radius,

$c = 0.637r = 0.637 \times 0.124 = 0.079$ in.

b. Computation of I_x, S_e, and M_n:

For the first approximation, assume a compression stress of $f = F_y = 50$ ksi (yield strength in longitudinal compression, Table A1 of the Standard Specification) in the top fiber of the section and that the web is fully effective.

Compression flange: $k = 0.50$ (for unstiffened compression element, see Section 2.3.1)

$w/t = 1.471/0.060 = 24.52 < 50$ OK (Section 2.1.1-(1)-(iii))

$$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o} \qquad \text{(Eq. 2.2.1-4)}$$

The initial modulus of elasticity, E_o, for Type 301 stainless steel is obtained from Table A4 of the Standard, i.e., $E_o = 27000$ ksi.

$$\lambda = (1.052/\sqrt{0.50})(24.52)\sqrt{50/27000} = 1.570 > 0.673$$

$$\rho = [1-(0.22/\lambda)]/\lambda \qquad \text{(Eq. 2.2.1-3)}$$

$$= [1-(0.22/1.570)]/1.570 = 0.548$$

$$b = \rho w \qquad \text{(Eq. 2.2.1-2)}$$

$$= 0.548 \times 1.471$$

$$= 0.806 \text{ in.}$$

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Web	5.692	3.000	17.076	51.228	15.368
Upper Corner	0.195	0.075	0.015	0.001	--
Lower Corner	0.195	5.925	1.155	6.846	--
Compression Flange	0.806	0.030	0.024	0.001	--
Tension Flange	1.471	5.970	8.782	52.428	--
Sum	8.359		27.052	110.504	15.368

Distance from top fiber to x-axis is

y_{cg} = 27.052/8.359 = 3.236 in.

Since the distance from top compression fiber to the neutral axis is greater than one half the beam depth, a compression stress of 50 ksi will govern as assumed (i.e., initial yield is in compression).

To check if web is fully effective (Section 2.2.2):

f_1 = [(3.236-0.154)/3.236]x50 = 47.62 ksi(compression)

f_2 = - [(2.764-0.154)/3.236]x50 = -40.33 ksi(tension)

Ψ = f_2/f_1 = -40.33/47.62 = -0.847

k = $4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

= $4+2[1-(-0.847)]^3+2[1-(-0.847)]$

= 20.296

h = w = 5.692 in., h/t = w/t = 5.692/0.060 = 94.87

$h/t \quad = 94.87 < 200$ OK (Section 2.1.2-(1))

$\lambda \quad = (1.052/\sqrt{20.296})(94.87)\sqrt{47.62/29500} = 0.930 > 0.673$

$\rho \quad = [1-(0.22/0.930)]/0.930 = 0.821$

$b_e \quad = 0.821 \times 5.692 = 4.673$ in.

$b_2 \quad = b_e/2$ (Eq. 2.2.2-2)

$= 4.673/2 = 2.337$ in.

$b_1 \quad = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 4.673/[3-(-0.847)] = 1.215$ in.

The effective widths, b_1 and b_2, of web are defined in Figure 2 of the Standard.

$b_1+b_2 = 1.215 + 2.337 = 3.552$ in.

Compression portion of the web calculated on the basis of the effective section $= y_{cg} - 0.154 = 3.236 - 0.154 = 3.082$ in.

Since $b_1+b_2 = 3.552$ in. > 3.082 in., b_1+b_2 shall be taken as 3.082 in.. This verifies the assumption that the web is fully effective.

$I'_x \quad = Ly^2+I'_1-Ly^2_{cg}$

$= 110.504 + 15.368 - 8.359(3.236)^2$

$= 38.339$ in.^{3}

Actual $I_x = I'_x t$

$= 38.339 \times 0.060$

$= 2.300$ in.^{4}

$S_e \quad = I_x/y_{cg}$

$= 2.300/3.236$

$= 0.711$ in.^{3}

$M_n \quad = S_e F_y$ (Eq. 3.3.1.1-1)

$= 0.711x50$

$= 35.55$ kips-in.

c. The design flexural strength, $\phi_b M_n$, based on initiation of yielding is determined as follows: (Section 3.3.1.1(1))

ϕ_b = 0.85 (for section with unstiffened compression flanges)

$\phi_b M_n$ = 0.85x35.55 = 30.22 kips-in.(positive bending)

2. Calculation of the effective moment of inertia for deflection determination at the service moment M_s:

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

$$
\begin{aligned}
\phi_b M_n &= 1.2M_{DL} + 1.6M_{LL} \\
&= [1.2(M_{DL}/M_{LL})+1.6]M_{LL} \\
&= [1.2(1/5)+1.6]M_{LL} \\
&= 1.84M_{LL} \\
M_{LL} &= \phi_b M_n/1.84 = 30.22/1.84 = 16.42 \text{ kips-in.} \\
M_s &= M_{DL} + M_{LL} \\
&= (1/5+1)M_{LL} \\
&= 1.2(16.42) = 19.70 \text{ kips-in.}
\end{aligned}
$$

where

M_{DL} = Moment determined on the basis of nominal dead load

M_{LL} = Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive stress f under this service moment M_s. Knowing f, proceeds as usual to obtain S_e and checks to see if ($f \times S_e$) is equal to M_s as it should. If not, reiterate until one obtains the desired level of accuracy.

a. For the first iteration, assume a stress of $f = F_y/2 = 25$ ksi in the top and bottom fibers of the section and that the web is fully effective.

For deflection determination, the value of E_r, reduced modulus of elasticity determined by using Eq. (2.2.1-7), is substituted for E_o in Eq. (2.2.1-4). For a compression and tension stresses of f = 25 ksi, the corresponding E_{sc} and E_{st} values for Type 301 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:

$E_{sc} = 25650$ ksi, $E_{st} = 27000$ ksi

$E_r = (E_{sc}+E_{st})/2$ (Eq. 2.2.1-7)

$= (25650+27000)/2 = 26325$ ksi

Thus, for compression flange:

$\lambda = (1.052/\sqrt{0.50})(24.52)\sqrt{25/26325} = 1.124 > 0.673$

$\rho = [1-(0.22/1.124)]/1.124 = 0.716$

$b_d = \rho w$ (Eq. 2.2.1-6)

$= 0.716 \times 1.471 = 1.053$ in.

Effective section properties about x-axis:

$L = 8.359 - 0.806 + 1.053 = 8.606$ in.

$Ly = 27.052 - 0.024 + 1.053 \times 0.030 = 27.060$ in.^{2}

$Ly^2 = 110.504 - 0.001 + 1.053(0.030)^2 = 110.504$ in.^{3}

$I'_1 = 15.368$ in.^{3}

$y_{cg} = 27.060/8.606 = 3.144$ in. which is greater than one half beam depth. Thus, the top compression fiber controls the determination of S_e.

To check if web is fully effective (Section 2.2.2-(2)):

$f_1 = [(3.144-0.154)/3.144]\text{x}25 = 23.78$ ksi

$f_2 = -[(2.856-0.154)/3.144]\text{x}25 = -21.49$ ksi

$\Psi = -21.49/23.78 = -0.904$

$k = 4+2[1-(-0.904)]^3+2[1-(-0.904)] = 21.613$

For a compression stress of f = 23.78 ksi and a tension stress of f = 21.49 ksi, the values of E_{sc} and E_{st} are found as follows: $E_{sc} = 26244$ ksi, $E_{st} = 27000$ ksi.

$E_r = (E_{sc}+E_{st})/2$

$= (26244+27000)/2 = 26622$ ksi

$\lambda = (1.052/\sqrt{21.613})(94.87)\sqrt{23.78/26622} = 0.642 < 0.673$

$b_e = w$ (Eq. 2.2.1-1)

$= 5.692$ in.

$b_2 = 5.692/2 = 2.846$ in.

$b_1 = 5.692/[3-(-0.904)] = 1.458$ in.

Compression portion of the web calculated on the basis of the effective section = 3.144 - 0.154 = 2.990 in..

Since $b_1+b_2 = 4.304$ in. > 2.990 in., b_1+b_2 shall be taken as 2.990 in.. This verifies the assumption that the web is fully effective.

$I'_x = 110.504 + 15.368 - 8.606(3.144)^2$

$= 40.804 \text{ in.}^3$

Actual $I_x = 40.804 \times 0.060$

$= 2.448 \text{ in.}^4$

$S_e = 2.448/3.144 = 0.779 \text{ in.}^3$

$M = f \times S_e = 25 \times 0.779$

$= 19.48 \text{ kips-in.} < M_s = 19.70 \text{ kips-in.}$

Need to do another iteration by increasing f.

b. For the second iteration, assume f = 25.50 ksi in the top and bottom fibers of the section and that the web is fully effective.

Compression flange:

For a stress of f = 25.50 ksi, E_{sc} = 25375 ksi and E_{st} = 27000 ksi, and E_r = (25375+27000)/2 = 26188 ksi. Thus,

$\lambda = (1.052/\sqrt{0.50})(24.52)\sqrt{25.50/26188} = 1.138 > 0.673$

$\rho = [1-(0.22/1.138)]/1.138 = 0.709$

$b_d = 0.709 \times 1.471 = 1.043 \text{ in.}$

Effective section properties about x-axis:

$L = 8.359 - 0.806 + 1.043 = 8.596 \text{ in.}$

$Ly = 27.052 - 0.024 + 1.043 \times 0.030 = 27.059 \text{ in.}^2$

$Ly^2 = 110.504 - 0.001 + 1.043(0.030)^2 = 110.504 \text{ in.}^3$

$I'_1 = 15.368 \text{ in.}^3$

y_{cg} = 27.059/8.596 = 3.148 in. which is greater than one half beam depth. Thus, the top compression fiber controls the determination of S_e.

To check if web is fully effective:

f_1 = $[(3.148-0.154)/3.148]$x25.50 = 24.25 ksi

f_2 = $-[(2.825-0.154)/3.148]$x25.50 = -21.85 ksi

Ψ = -21.85/24.25 = -0.901

k = $4+2[1-(-0.901)]^3+2[1-(-0.901)]$ = 21.542

For a compression stress of f = 24.25 ksi, E_{sc} = 26063 ksi, and for a tension stress of f = 21.85 ksi, E_{st} = 27000 ksi. Thus,

E_r = (26063+27000)/2 = 26532 ksi.

λ = $(1.052/\sqrt{21.542})(94.87)\sqrt{24.25/26532}$ = 0.650 < 0.673

b_e = 5.692 in.

b_2 = 5.692/2 = 2.846 in.

b_1 = 5.692/[3-(-0.901)] = 1.459 in.

Compression portion of the web calculated on the basis of the effective section = 3.148 - 0.154 = 2.994 in..

Since b_1+b_2 = 4.305 in. > 2.994 in., b_1+b_2 shall be taken as 2.994 in.. This verifies the assumption that the web is fully effective.

I'_x = 110.504 + 15.368 - 8.596$(3.148)^2$

= 40.686 $in.^3$

Actual I_x = 40.686 x 0.060

= 2.441 $in.^4$

S_e = 2.441/3.148 = 0.775 $in.^3$

$M \quad = f \times S_e = 25.50 \times 0.775$

$= 19.76$ kips-in. $= M_s$ OK

Thus, use $I_x = 2.441$ in.^{4} for deflection determination.

(B) Type 409 Stainless Steel.

1. Calculation of the design flexural strength, $\phi_b M_n$:

a. Properties of 90° corners:

From Case (A) above,

r = 0.124 in., u = 0.195 in., c = 0.079 in.

b. Computation of I_x, S_e, and M_n:

For the first approximation, assume a compression stress of $f = F_y = 30$ ksi (yield strength in longitudinal compression, Table A1 of the Standard Specification) in the top fiber of the section and that the web is fully effective.

Compression flange: k = 0.50 (for unstiffened compression element, see Section 2.3.1)

$w/t = 1.471/0.060 = 24.52 < 50$ OK (Section 2.1.1-(1)-(iii))

$\lambda \quad = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

The initial modulus of elasticity, E_o, for Type 409 stainless steel is obtained from Table A5 of the Standard, i.e., $E_o = 27000$ ksi.

$\lambda \quad = (1.052/\sqrt{0.50})(24.52)\sqrt{30/27000} = 1.216 > 0.673$

$\rho \quad = \left[1-(0.22/\lambda)\right]/\lambda$ (Eq. 2.2.1-3)

$= [1-(0.22/1.216)]/1.216 = 0.674$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.674 \times 1.471$

$= 0.991$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Web	5.692	3.000	17.076	51.228	15.368
Upper Corner	0.195	0.075	0.015	0.001	--
Lower Corner	0.195	5.925	1.155	6.846	--
Compression Flange	0.991	0.030	0.030	0.001	--
Tension Flange	1.471	5.970	8.782	52.428	--
Sum	8.544		27.058	110.504	15.368

Distance from top fiber to x-axis is

$y_{cg} = 27.058/8.544 = 3.167$ in.

Since the distance from top compression fiber to the neutral axis is greater than one half the beam depth, a compression stress of 30 ksi will govern as assumed (i.e., initial yield is in compression).

To check if web is fully effective (Section 2.2.2):

$f_1 = [(3.167-0.154)/3.167]\text{x}30 = 28.54$ ksi(compression)

$f_2 = -[(2.833-0.154)/3.167]\text{x}30 = -25.38$ ksi(tension)

$\psi = f_2/f_1 = -25.38/28.54 = -0.889$

$k = 4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

$= 4+2[1-(-0.889)]^3+2[1-(-0.889)]$

$= 21.259$

$h = w = 5.692$ in., $h/t = w/t = 5.692/0.060 = 94.87$

$h/t = 94.87 < 200$ OK (Section 2.1.2-(1))

$\lambda = (1.052/\sqrt{21.259})(94.87)\sqrt{28.54/27000} = 0.704 > 0.673$

$\rho = [1-(0.22/0.704)]/0.704 = 0.977$

$b_e = 0.977 \times 5.692 = 5.561$ in.

$b_2 = b_e/2$ (Eq. 2.2.2-2)

$= 5.561/2 = 2.781$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 5.561/[3-(-0.889)] = 1.430$ in.

The effective widths, b_1 and b_2, are defined in Figure 2 of the Standard.

$b_1+b_2 = 1.430 + 2.781 = 4.211$ in.

Compression portion of the web calculated on the basis of the effective section = $y_{cg} - 0.154 = 3.167 - 0.154 = 3.013$ in.

Since $b_1+b_2 = 4.211$ in. > 3.013 in., b_1+b_2 shall be taken as 3.013 in.. This verifies the assumption that the web is fully effective.

$I'_x = Ly^2+I'_1-Ly^2_{cg}$

$= 110.504 + 15.368 - 8.544(3.167)^2$

$= 40.177 \text{ in.}^3$

Actual $I_x = I'_x t$

$= 40.177 \times 0.060$

$= 2.411 \text{ in.}^4$

$S_e = I_x/y_{cg}$

$= 2.411/3.167$

$= 0.761$ in.^{3}

$M_n = S_e F_y$ (Eq. 3.3.1.1-1)

$= 0.761 \times 30$

$= 22.83$ kips-in.

c. The design flexural strength, $\phi_b M_n$, based on initiation of yielding is determined as follows: (Section 3.3.1.1(1))

$\phi_b = 0.85$ (for section with unstiffened compression flanges)

$\phi_b M_n = 0.85 \times 22.83 = 19.41$ kips-in. (positive bending)

2. Calculation of the effective moment of inertia for deflection determination at the service moment M_s:

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

$$\phi_b M_n = 1.2M_{DL} + 1.6M_{LL}$$

$$= [1.2(M_{DL}/M_{LL})+1.6]M_{LL}$$

$$= [1.2(1/5)+1.6]M_{LL}$$

$$= 1.84M_{LL}$$

$$M_{LL} = \phi_b M_n/1.84 = 19.41/1.84 = 10.55 \text{ kips-in.}$$

$$M_s = M_{DL} + M_{LL}$$

$$= (1/5+1)M_{LL}$$

$$= 1.2(10.55) = 12.66 \text{ kips-in.}$$

where

M_{DL} = Moment determined on the basis of nominal dead load

M_{LL} = Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive stress f under this service moment M_s. Knowing f, proceeds as usual to obtain S_e and checks to see if ($f \times S_e$) is equal to M_s as it should. If not, reiterate until one obtains the desired level of accuracy.

a. For the first iteration, assume a stress of $f = F_y/2 = 15$ ksi in the top and bottom fibers of the section and that the web is fully effective.

For deflection determination, the value of E_r, reduced modulus of elasticity determined by Eq. (2.2.1-7), is substituted for E_o in Eq. (2.2.1-4). For a compression and tension stress of f = 15 ksi, the corresponding E_{sc} and E_{st} values for Type 409 stainless steel are obtained from Table A3 or Figure A2 of the Standard as follows:

$E_{sc} = 26850$ ksi, $E_{st} = 26930$ ksi

$$E_r = (E_{sc}+E_{st})/2 \qquad \text{(Eq. 2.2.1-7)}$$

$$= (26850+26930)/2 = 26890 \text{ ksi}$$

Thus, for compression flange:

$$\lambda = (1.052/\sqrt{0.50})(24.52)\sqrt{15/26890} = 0.862 > 0.673$$

$$\rho = [1-(0.22/0.862)]/0.862 = 0.864$$

$$b_d = \rho w \qquad \text{(Eq. 2.2.1-6)}$$

$$= 0.864 \times 1.471 = 1.271 \text{ in.}$$

Effective section properties about x-axis:

$L = 8.544 - 0.991 + 1.271 = 8.824$ in.

$Ly = 27.058 - 0.030 + 1.271 \times 0.030 = 27.066$ in.^{2}

$Ly^2 = 110.504 - 0.001 + 1.271(0.030)^2 = 110.504$ in.^{3}

$I'_1 = 15.368$ in.^{3}

$y_{cg} = 27.066/8.824 = 3.067$ in. which is greater than one half beam depth. Thus, the top compression fiber controls the determination of S_e.

To check if web is fully effective (Section 2.2.2-(2)):

$f_1 = [(3.067-0.154)/3.067] \times 15 = 14.25$ ksi

$f_2 = -[(2.933-0.154)/3.067] \times 15 = -13.59$ ksi

$\Psi = -13.59/14.25 = -0.954$

$k = 4+2[1-(-0.954)]^3+2[1-(-0.954)] = 22.829$

For a compression stress of $f = 14.25$ ksi and a tension stress of $f = 13.59$ ksi, the values of E_{sc} and E_{st} are found as follows, respectively: $E_{sc} = 26890$ ksi, $E_{st} = 26940$ ksi.

$E_r = (E_{sc}+E_{st})/2$

$= (26890+26940)/2 = 26920$ ksi

$\lambda = (1.052/\sqrt{22.829})(94.87)\sqrt{14.25/26920} = 0.481 < 0.673$

$b_e = w$ (Eq. 2.2.1-1)

$= 5.692$ in.

$b_2 = 5.692/2 = 2.846$ in.

$b_1 = 5.692/[3-(-0.954)] = 1.440$ in.

Compression portion of the web calculated on the basis of the effective section = 3.067 - 0.154 = 2.913 in..

Since b_1+b_2 = 4.286 in. > 2.913 in., b_1+b_2 shall be taken as 2.913 in.. This verifies the assumption that the web is fully effective.

$$I'_x = 110.504 + 15.368 - 8.824(3.067)^2$$
$$= 42.869 \text{ in.}^3$$
$$\text{Actual } I_x = 42.869 \times 0.060$$
$$= 2.572 \text{ in.}^4$$
$$S_e = 2.572/3.067 = 0.839 \text{ in.}^3$$
$$M = f \times S_e = 15 \times 0.839$$
$$= 12.59 \text{ kips-in.} \cong M_s = 12.66 \text{ kips-in. (close enough)}$$

Therefore, need no further iteration. Use I_x = 2.572 in.^{4} for deflection determination.

EXAMPLE 1.2 CHANNEL W/UNSTIFFENED FLANGES (ASD)

Use the data given in Example 1.1 (Figure 1.1) to determine the allowable moment, M_a, by using the Allowable Stress Design (ASD) method on the basis of initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable moment. Use Type 301 stainless steel, 1/4-Hard: F_y = 50 ksi.

Solution:

1. Calculation of the allowable moment, M_a:

   The effective section properties calculated by the ASD method are the same as those determined in Example 1.1 by the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   Ω = 1.85 (Safety Factor stipulated in Table E of the Standard)

   M_n = 35.55 kips-in. (obtained from Example 1.1)

   $M_a = M_n/\Omega$ (Eq. E-1)

   $= 35.55/1.85$

   $= 19.22$ kips-in.

2. Calculation of the effective moment of inertia for deflection determination at the allowable moment M_a:

   For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 1.1 for the LRFD method, except that the computed moment M (= fxS_e) should be equal to M_a.

From the results of Example 1.1, it can be seen that by assuming a compression stress of $f=F_y/2=25$ ksi, the computed $f \times S_e$ = 25 x 0.779 = 19.48 kips-in., which is close to the allowable moment, M_a = 19.22 kips-in. Therefore, the computed I_x = 2.448 in^4 can be used for deflection determination.

EXAMPLE 2.1 CHANNEL W/STIFFENED FLANGES (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi_b M_n$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use Type 301 stainless steel, 1/4-Hard. Assume dead load to live load ratio D/L = 1/5 and 1.2D+1.6L governs the design.

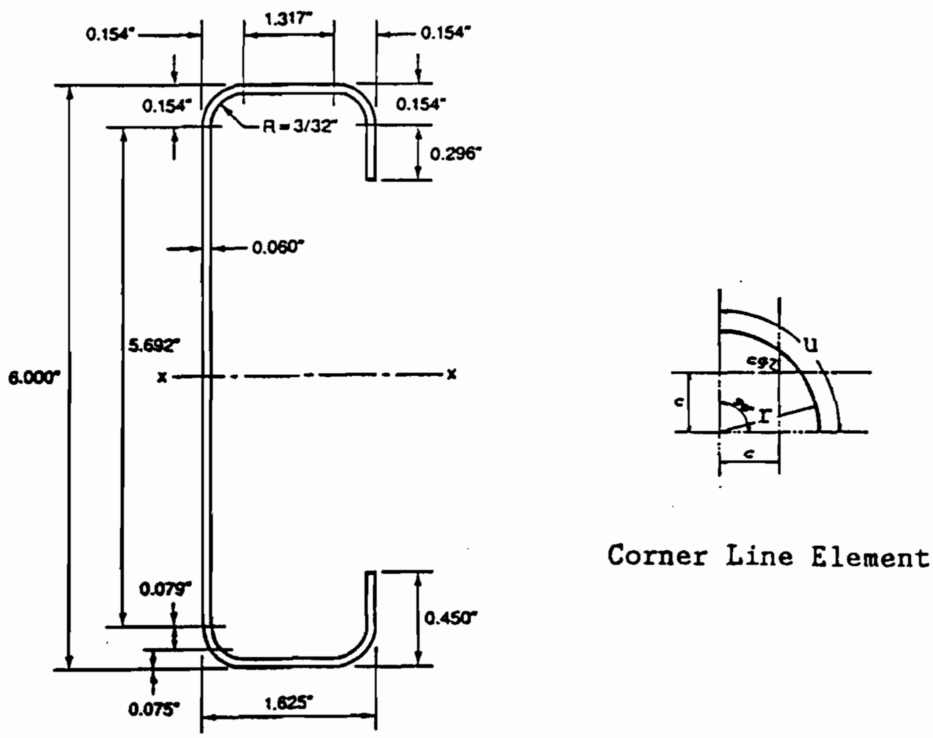


Figure 2.1 Section for Example 2.1

Given:

1. Section: 6" x 1.625" x 0.060" channel with stiffened flanges.
2. Compression flange braced against lateral buckling.

Solution:

1. Calculation of the design flexural strength, $\phi_b M_n$:

a. Properties of 90° corners:

$r = R + t/2 = 3/32 + 0.060/2 = 0.124$ in.

Length of arc, $u = 1.57r = 1.57 \times 0.124 = 0.195$ in.

Distance of c.g. from center of radius,

$c = 0.637r = 0.637 \times 0.124 = 0.079$ in.

b. Computation of I_x, S_e, and M_n:

For the first approximation, assume a compression stress of $f = F_y = 50$ ksi (yield strength in longitudinal compression, Table A1 of the Standard) in the top fibers of the section and that the web is fully effective.

Compression flange: (Section 2.4.2)

$w = 1.317$ in.

$w/t = 1.317/0.060 = 21.95$

$$S = 1.28\sqrt{E_o/f} \quad \text{(Eq. 2.4-1)}$$

E_o value for Type 301 stainless steel is obtained from Table A4 of the Standard Specification, i.e., $E_o = 27000$ ksi.

$S = 1.28\sqrt{27000/50} = 29.74$

$S/3 = 9.91 < (w/t) = 21.95 < S = 29.74$

$$I_a = 399t^4\{[(w/t)/S]-0.33\}^3 \quad \text{(Eq. 2.4.2-6)}$$

$= 399(0.060)^4[(21.95/29.74)-0.33]^3$

$= 0.000351$ in.^{4}

$D = 0.450$ in.

$d = 0.296$ in., $d/t = 0.296/0.060 = 4.93$

$$I_s = d^3t/12 \quad \text{(Eq. 2.4-2)}$$

$= (0.296)^3(0.060)/12 = 0.000130$ in.^{4}

$D/w = 0.450/1.317 = 0.342$, $0.25 < D/w = 0.342 < 0.80$

$$k = [4.82-5(D/w)](I_s/I_a)^n+0.43 \le 5.25-5(D/w) \quad \text{(Eq. 2.4.2-9)}$$

$n = 1/2$

$[4.82-5(0.342)](0.000130/0.000351)^{1/2}+0.43 = 2.323$

$5.25-5(0.342) = 3.540 > 2.323$

Use $k = 2.323$

Since $I_s < I_a$, the stiffener is considered to be a simple lip.

$w/t = 21.95 < 50$ OK (Section 2.1.1-(1)-(i))

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{2.323})(21.95)\sqrt{50/27000} = 0.652 < 0.673$

$b = w$ (Eq. 2.2.1-1)

$= 1.317$ in. (i.e. compression flange fully effective)

Compression (upper) stiffener:

$k = 0.50$ (unstiffened compression element)

$d/t = 4.93$

Also conservatively assume f=50 ksi as used in top compression fiber.

$\lambda = (1.052/\sqrt{0.50})(4.93)\sqrt{50/27000} = 0.316 < 0.673$

therefore,

$d'_s = d = 0.296$ in.

$d_s = d'_s(I_s/I_a) \leq d'_s$ (Eq. 2.4.2-11)

$= 0.296(0.000130/0.000351)$

$= 0.110$ in. < 0.296 in.

$d_s = 0.110$ in. (i.e. compression stiffener is not fully effective)

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Web	5.692	3.000	17.076	51.228	15.368
Upper Corners	2x0.195 = 0.390	0.075	0.029	0.002	--
Lower Corners	2x0.195 = 0.390	5.925	2.311	13.691	--
Compression Flange	1.317	0.030	0.040	0.001	--
Upper Stiffener	0.110	0.209	0.023	0.005	--
Tension Flange	1.317	5.970	7.862	46.939	--
Lower Stiffener	0.296	5.698	1.687	9.610	0.002
Sum	9.512		29.028	121.476	15.370

Distance from top fiber to x-axis is

y_{cg} = 29.028/9.512 = 3.052 in.

Since the distance from top compression fiber to the neutral axis is greater than one half the beam depth, a compression stress of 50 ksi will govern as assumed (i.e., initial yield is in compression).

To check if web is fully effective (Section 2.2.2):

f_1 = [(3.052-0.154)/3.052]x50 = 47.48 ksi(compression)

f_2 = - [(2.948-0.154)/3.052]x50 = -45.77 ksi(tension)

Ψ = f_2/f_1 = -45.77/47.48 = -0.964

k = $4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

= $4+2[1-(-0.964)]^3+2[1-(-0.964)]$

= 23.079

$h = w = 5.692$ in., $h/t = w/t = 5.692/0.060 = 94.87$

$h/t = 94.87 < 200$ OK (Section 2.1.2-(1))

$\lambda = (1.052/\sqrt{23.079})(94.87)\sqrt{47.48/27000} = 0.871 > 0.673$

$\rho = [1-(0.22/\lambda)]/\lambda$ (Eq. 2.2.1-3)

$= [1-(0.22/0.871)]/0.871 = 0.858$

$b_e = \rho w$ (Eq. 2.2.1-2)

$= 0.858 \times 5.692 = 4.884$ in.

$b_2 = b_e/2$ (Eq. 2.2.2-2)

$= 4.884/2 = 2.442$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 5.037/[3-(-0.964)] = 1.232$ in.

The effective widths, b_1 and b_2, of web are defined in Figure 2 of the Standard Specification.

$b_1+b_2 = 1.232 + 2.442 = 3.674$ in.

Compression portion of the web calculated on the basis of the effective section = $y_{cg} - 0.154 = 3.052 - 0.154 = 2.898$ in.

Since $b_1+b_2 = 3.674$ in. > 2.898 in., b_1+b_2 shall be taken as 2.898 in.. This verifies the assumption that the web is fully effective.

$I'_x = Ly^2+I'_1-Ly^2_{cg}$

$= 121.476 + 15.370 - 9.512(3.052)^2$

$= 48.245$ in.^{3}

Actual $I_x = I'_x t$

$= 48.245 \times 0.060$

$= 2.895$ in.^{4}

$S_e = I_x/y_{cg}$

$= 2.895/3.052$

$= 0.949 \text{ in.}^3$

$M_n \quad = S_e F_y$ (Eq. 3.3.1.1-1)

$= 0.949 \times 50$

$= 47.45$ kips-in.

c. The design flexural strength, $\phi_b M_n$, based on initiation of yielding is determined as follows: (Section 3.3.1.1(1))

$\phi_b \quad = 0.90$ (for section with stiffened compression flanges)

$\phi_b M_n \quad = 0.90 \times 47.45 = 42.71$ kips-in.

2. Calculation of the effective moment of inertia for deflection determination at the service moment M_s:

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

$$\phi_b M_n = 1.2M_{DL} + 1.6M_{LL}$$

$$= [1.2(M_{DL}/M_{LL})+1.6]M_{LL}$$

$$= [1.2(1/5)+1.6]M_{LL}$$

$$= 1.84M_{LL}$$

$$M_{LL} = \phi_b M_n/1.84 = 42.71/1.84 = 23.21 \text{ kips-in.}$$

$$M_s = M_{DL} + M_{LL}$$

$$= (1/5+1)M_{LL}$$

$$= 1.2(23.21) = 27.85 \text{ kips-in.}$$

where

M_{DL} = Moment determined on the basis of nominal dead load

M_{LL} = Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive stress f under this service moment M_s. Knowing f, one proceeds as usual to obtain S_e and checks to see if ($f \times S_e$) is equal to M_s as it should. If not, reiterate until one obtains the desired level of accuracy. (Section 2.2.1-(2))

a. For the first iteration, assume a stress of $f = F_y/2 = 25$ ksi in the top and bottom fibers of the section andthat the web is fully effective.

Compression flange:

$S = 1.28\sqrt{27000/25} = 42.07$

$S/3 = 14.02 < (w/t) = 21.95 < S = 42.07$

$I_a = 399(0.060)^4[(21.95/42.07)-0.33]^3$

$= 0.000036$ in.^{4}

$I_s/I_a = 0.000130/0.000036 = 3.61; \quad 5.25-5(D/w) = 3.540$

$k = [4.82-5(0.342)](3.61)^{1/2}+0.43 = 6.339 > 3.540$

Use $k = 3.540$

For deflection determination, the reduced modulus of elasticity, E_r, shall be substituted for E_o in Eq. (2.2.1-4). For a compression and tension stresses of $f = 25$ ksi,

$E_r = 26325$ ksi as obtained from Example 1.1.

$\lambda = (1.052/\sqrt{3.540})(21.95)\sqrt{25/26325} = 0.378 < 0.673$

$b_d = 1.317$ in. (i.e. compression flange fully effective)

Compression (upper) stiffener:

Again assume conservatively f = 25 ksi as used in top compression fiber and the corresponding E_r = 26325 ksi.

$\lambda = (1.052/\sqrt{0.50})(4.93)\sqrt{25/26325} = 0.226 < 0.673$

Therefore, d'_s = 0.296 in.

Since $I_s/I_a = 3.25 > 1.0$, it follows that $d_s = d'_s$ = 0.296 in. (i.e. compression stiffener fully effective).

Thus, one concludes that the section is fully effective.

$y_{cg} = 6/2 = 3.000$ in. (from symmetry)

Full section properties about x-axis:

Element	L (in.)	y Distance from Centerline of Section (in.)	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Web	5.692	--	--	15.368
Stiffeners	2 x 0.296 = 0.592	2.698	4.309	0.004
Corners	4 x 0.195 = 0.780	2.925	6.673	--
Flanges	2 x 1.317 = 2.634	2.970	23.234	--
Sum	9.698		34.216	15.372

Since section is singly symmetric about x-axis and fully effective, top compression fiber may be used in computing S_e.

To check if web is fully effective: (Section 2.2.2-(2))

$f_1 = [(3.000-0.154)/3.000]x25 = 23.72$ ksi(compression)

$f_2 = -23.72$ ksi(tension)

$\Psi \quad = -23.72/23.72 = -1.000$

$k \quad = 4+2[1-(-1)]^3+2[1-(-1)] = 24.000$

For a compression and tension stresses of f = 23.72 ksi, the corresponding E_{sc} and E_{st} values are as follows:

$E_{sc} \quad = 26256$ ksi, and $E_{st} = 27000$ ksi

$E_r \quad = (E_{sc}+E_{st})/2$ (Eq. 2.2.1-7)

$\quad = 26628$ ksi

$\lambda \quad = (1.052/\sqrt{24})(94.87)\sqrt{23.72/26628} = 0.608 < 0.673$

$b_e \quad = w$ (Eq. 2.2.1-1)

$\quad = 5.692$ in.

$b_2 \quad = 5.692/2 = 2.846$ in.

$b_1 \quad = 5.692/[3-(-1)] = 1.423$ in.

$b_1+b_2 = 4.269$ in.

Compression portion of the web = 3.000 - 0.154 = 2.846 in.

Since b_1+b_2 = 4.269 in. > 2.846 in., b_1+b_2 shall be taken as 2.846 in.. This verifies the assumption that the web is fully effective.

$I'_x \quad = 34.216 + 15.372 = 49.588$ in.^{3}

Actual I_x = 49.588 x 0.060 = 2.975 in.^{4}

$S_e \quad = 2.975/3.000 = 0.992$ in.^{3}

$M \quad = f \times S_e = 25 \times 0.992$

$\quad = 24.80$ kips-in. $< M_s = 27.85$ kips-in.

Need to do another iteration by increasing f.

b. After several trials, assume that a stress of f = 28.07 ksi in the top and bottom fibers of the section and that the web is fully effective.

Compression flange:

$S = 1.28\sqrt{27000/28.07} = 39.70$

$S/3 = 13.23 < (w/t) = 21.95 < S = 39.70$

$I_a = 399(0.060)^4[(21.95/39.70)-0.33]^3$

$= 0.000057$ in.^{4}

$I_s/I_a = 0.000130/0.000057 = 2.28$

$k = [4.82-5(0.342)](2.28)^{1/2}+0.43 = 5.126 > 3.540$

Use $k = 3.540$

For a compression and tension stresses of $f = 28.07$ ksi, it is found that E_{sc} and E_{st} are equal to 23950 ksi and 27000 ksi, respectively.

$E_r = (23950+27000)/2 = 25475$ ksi

$\lambda = (1.052/\sqrt{3.54})(21.95)\sqrt{28.07/25475} = 0.407 < 0.673$

$b_d = 1.317$ in. (i.e. compression flange fully effective)

Compression (upper) stiffener:

f conservatively taken as for top compression fiber.

$\lambda = (1.052/\sqrt{0.50})(4.93)\sqrt{28.07/25475} = 0.243 < 0.673$

$d'_s = 0.296$ in.

Since $I_s/I_a = 2.28 > 1.0$, it follows that $d_s = d'_s$ $= 0.296$ in. (i.e. compression stiffener fully effective).

Thus, the section is fully effective.

$y_{cg} = 6/2 = 3.000$ in. (from symmetry)

Full section properties are the same as were found in the first iteration. Thus, as before, top compression fiber may be used in computing S_e.

To check if web is fully effective:

$f_1 = [(3.000-0.154)/3.000]x28.07 = 26.63$ ksi(compression)

$f_2 = -26.63$ ksi(tension)

$\Psi = -26.63/26.63 = -1.000$

$k = 24.000$

For a compression and tension stresses of f = 26.63 ksi, it is found that E_{sc} and E_{st} are equal to 24754 ksi and 27000 ksi, respectively.

$E_r = (24754+27000)/2 = 25877$ ksi

$\lambda = (1.052/\sqrt{24})(94.87)\sqrt{26.63/25877} = 0.654 < 0.673$

$b_e = w = 5.692$ in.

Hence, as in first iteration, $b_1 + b_2 = 2.846$ in. and thus the web is fully effective as assumed.

$I_x = 2.975$ in.^{4}

$S_e = 0.992$ in.^{3}

$M = f \times S_e = 28.07 \times 0.992$

$= 27.85$ kips-in. $= M_s$ OK

Thus, use $I_x = 2.975$ in.^{4} for deflection determination.

EXAMPLE 2.2 CHANNEL W/STIFFENED FLANGES (ASD)

Use the data given in Example 2.1 (Figure 2.1) to determine the allowable moment, M_a, by using the Allowable Stress Design (ASD) method on the basis of initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable moment. Use Type 301 stainless steel, 1/4-Hard: F_y = 50 ksi.

Solution:

1. Calculation of the allowable moment, M_a:

   The effective section properties calculated by the ASD method are the same as those determined in Example 2.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   Ω = 1.85 (Safety Factor stipulated in Table E of the Standard)

   M_n = 47.45 kips-in. (obtained from Example 2.1)

   $M_a = M_n/\Omega$ (Eq. E-1)

   $= 47.45/1.85$

   $= 26.65$ kips-in.

2. Calculation of the effective moment of inertia for deflection determination at the allowable moment, M_a:

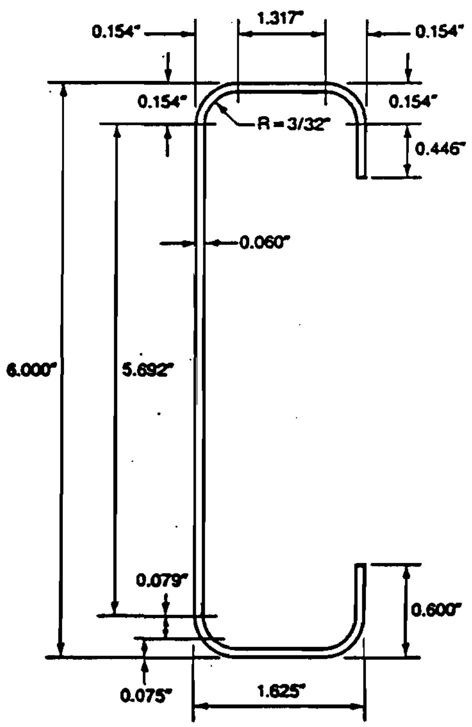
   For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 2.1 for the LRFD method, except that the computed moment M (= fxS_e) should be equal to M_a.

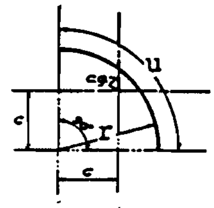
From the results of Example 2.1, it can be seen that by assuming a compression stress of f=28.07 ksi, the computed S_e = 0.992 in.^{3} which is based on the fully effective section. If the assumed stress is equal to f=26.86 ksi, the effective section modulus is also determined by the full section properties, i.e., S_e = 0.992 in.^{3}. This will give fxS_e = 26.65 kips-in., which is equal to M_a

Therefore, the computed I_x = 2.975 in^4 of the full section properties can be used for deflection determination.

EXAMPLE 3.1 C-SECTION W/BRACING (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi_b M_n$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use Type 304 stainless steel, 1/4-Hard. Assume dead load to live load ratio D/L = 1/5 and 1.2D+1.6L governs the design.





Corner Line Element

Figure 3.1 Section for Example 3.1

Given:

1. Section: 6" x 1.625" x 0.060" channel with stiffened flanges.
2. Compression flange braced against lateral buckling.

Solution:

1. Calculation of the design flexural strength, $\phi_b M_n$:

a. Properties of 90^o corners:

$r = R + t/2 = 3/32 + 0.060/2 = 0.124$ in.

Length of arc, $u = 1.57r = 1.57 \times 0.124 = 0.195$ in.

Distance of c.g. from center of radius,

$c = 0.637r = 0.637 \times 0.124 = 0.079$ in.

b. Computation of I_x, S_e, and M_n:

For the first approximation, assume a compression stress of $f = F_y = 50$ ksi (yield strength in longitudinal compression, Table A1 of the Standard) in the top fiber of the section and that the web is fully effective.

Compression flange:

$w = 1.317$ in.

$w/t = 1.317/0.060 = 21.95$

$$S = 1.28\sqrt{E_o/f} \qquad \text{(Eq. 2.4-1)}$$

The initial modulus of elasticity, E_o, for Type 304 stainless steel is obtained from Table A4 of the Standard, i.e., $E_o = 27000$ ksi.

$S = 1.28\ 27000/50 = 29.74$

$S/3 = 9.91 < (w/t) = 21.95 < S = 29.74$

$$I_a = 399t^4\{[(w/t)/S]-0.33\}^3 \qquad \text{(Eq. 2.4.2-6)}$$

$= 399(0.060)^4[(21.95/29.74)-0.33]^3$

$= 0.000351$ in.^{4}

$D = 0.600$ in.

$d = 0.446$ in., $d/t = 0.446/0.060 = 7.43$

$$I_s = d^3t/12 \qquad \text{(Eq. 2.4-2)}$$

$= (0.446)^3(0.060)/12 = 0.000444$ in.^{4}

$D/w = 0.600/1.317 = 0.456$, $0.25 < D/w = 0.456 < 0.80$

$$k = [4.82-5(D/w)](I_s/I_a)^n+0.43 \le 5.25-5(D/w) \qquad \text{(Eq. 2.4.2-9)}$$

$n = 1/2$

$[4.82-5(0.456)](0.000444/0.000351)^{1/2}+0.43 = 3.267$

$5.25-5(0.456) = 2.970 < 3.267$

Use $k = 2.970$

Since $I_s > I_a$ and $D/w < 0.8$, the stiffener is not considered as a simple lip.

$w/t = 21.95 < 90$ OK (Section 2.1.1-(1)-(i))

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{2.970})(21.95)\sqrt{50/27000} = 0.577 < 0.673$

$b = w$ (Eq. 2.2.1-1)

$= 1.317$ in. (i.e. compression flange fully effective)

Compression (upper) stiffener:

$k = 0.50$ (unstiffened compression element)

$d/t = 7.43$

f can be conservatively taken equal to 50 ksi as used in the top compression fiber.

$\lambda = (1.052/\sqrt{0.50})(7.43)\sqrt{50/27000} = 0.476 < 0.673$

Therefore,

$d'_s = d = 0.446$ in.

$d_s = d'_s(I_s/I_a) \leq d'_s$ (Eq. 2.4.2-11)

$= 0.446(0.000444/0.000351)$

$= 0.564$ in. > 0.446 in.

$d_s = 0.446$ in. (i.e. compression stiffener is fully effective)

Thus, one concludes that the section is fully effective.

$y_{cg} = 6/2 = 3.000$ in. (from symmetry)

Full section properties about x-axis:

Element	L (in.)	y Distance from Centerline of Section (in.)	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Web	5.692	--	--	15.368
Stiffeners	2 x 0.446 = 0.892	2.623	6.137	0.015
Corners	4 x 0.195 = 0.780	2.925	6.673	--
Flanges	2 x 1.317 = 2.634	2.970	23.234	--
Sum			36.044	15.383

Since section is singly symmetric about x-axis and fully effective, a compression stress of 50 ksi will govern as assumed. (At the bottom tension fiber a tensile stress of 50 ksi will develop simultaneously from symmetry).

To check if web is fully effective: (Section 2.2.2)

$f_1 = [(3.000-0.154)/3.000]\text{x}50 = 47.43$ ksi(compression)

$f_2 = -47.43$ ksi(tension)

$\Psi = f_2/f_1 = -47.43/47.43 = -1.000$

$k = 4+2(1-\Psi)^3+2(1-\Psi)$

$= 4+2[1-(-1)]^3+2[1-(-1)]$

$= 24.000$

$h = w = 5.692$ in., $h/t = w/t = 5.692/0.060 = 94.87$

$h/t = 94.87 < 200$ OK (Section 2.1.2-(1))

$\lambda = (1.052/\sqrt{24})(94.87)\sqrt{47.43/27000} = 0.854 > 0.673$

$\rho = [1-(0.22/\lambda)]/\lambda$ (Eq. 2.2.1-3)

$= [1-(0.22/0.854)]/0.854 = 0.869$

$b_e = \rho w$ (Eq. 2.2.1-2)

$= 0.869 \times 5.692 = 4.946$ in.

$b_2 = b_e/2$ (Eq. 2.2.2-2)

$= 4.946/2 = 2.473$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 4.946/[3-(-1)] = 1.237$ in.

The effective widths, b_1 and b_2, of web are defined in Figure 2 of the Standard.

$b_1+b_2 = 1.237 + 2.473 = 3.710$ in.

Compression portion of the web = $y_{cg} - 0.154 = 3.000 - 0.154$ = 2.846 in.

Since $b_1+b_2 = 3.710$ in. > 2.846 in., b_1+b_2 shall be taken as 2.846 in.. This verifies the assumption that the web is fully effective.

$I'_x = Ly^2+I'_1$

$= 36.044 + 15.383$

$= 51.427$ in.^{3}

Actual $I_x = I'_x t$

$= 51.427 \times 0.060$

$= 3.086$ in.^{4}

$S_e = I_x/y_{cg}$

$= 3.086/3.000$

$= 1.029$ in.^{3}

$M_n = S_e F_y$ (Eq. 3.3.1.1-1)

$= 1.029x50$

$= 51.45$ kips-in.

c. The design flexural strength, $\phi_b M_n$, based on initiation of yielding is determined as follows: (Section 3.3.1.1(1))

$\phi_b = 0.90$ (for section with stiffened compression flanges)

$\phi_b M_n = 0.90x51.45 = 46.31$ kips-in.

2. Calculation of the effective moment of inertia for deflection determination at the service moment M_s:

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

$$\phi_b M_n = 1.2M_{DL} + 1.6M_{LL}$$
$$= [1.2(M_{DL}/M_{LL})+1.6]M_{LL}$$
$$= [1.2(1/5)+1.6]M_{LL}$$
$$= 1.84M_{LL}$$
$$M_{LL} = \phi_b M_n/1.84 = 46.31/1.84 = 25.17 \text{ kips-in.}$$
$$M_s = M_{DL} + M_{LL}$$
$$= (1/5+1)M_{LL}$$
$$= 1.2(25.17) = 30.20 \text{ kips-in.}$$

where

M_{DL} = Moment determined on the basis of nominal dead load

M_{LL} = Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive stress f under this service moment M_s. Knowing f, one proceeds as usual to obtain S_e and checks to see if (f x S_e) is equal to M_s as it should. If not, reiterate until one obtains the desired level of accuracy. (Section 2.2.1-(2))

After several iterations with beginning a stress of $f = F_y/2$, the following only gives the results of final iteration. Assume that a stress of f = 29.35 ksi in the top and bottom fibers of the section and that the web is fully effective.

Compression flange:

$S = 1.28\sqrt{27000/29.35} = 38.82$

$S/3 = 12.94 < w/t = 21.95 < S = 38.82$

$I_a = 399(0.060)^4[(21.95/38.82)-0.33]^3$

$= 0.000067 \text{ in.}^4$

$I_s/I_a = 0.000444/0.000067 = 6.627$

$k = [4.82-5(0.456)](6.627)^{1/2}+0.43 = 6.969 > 2.970$

$k = 2.970$

For deflection determination, the reduced modulus of elasticity, E_r, is substituted for E_o in Eq. (2.2.1-4). For a compression and tension stresses of f=29.35 ksi, the corresponding E_{sc} and E_{st} values for Type 304 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:

$E_{sc} = 23089$ ksi, and $E_{st} = 26933$ ksi.

$$E_r = (E_{sc}+E_{st})/2 \qquad \text{(Eq. 2.2.1-7)}$$

$= (23089+26933) = 25011$ ksi

$\lambda = (1.052/\sqrt{2.970})(21.95)\sqrt{29.35/25011} = 0.459 < 0.673$

b_d = 1.317 in. (i.e. compression flange fully effective)

Compression (upper) stiffener:

f can be conservatively taken equal to 29.35 ksi as used in the the compression fiber.

$\lambda = (1.052/\sqrt{0.50})(7.43)\sqrt{29.35/25011} = 0.379 < 0.673$

therefore, $d'_s = 0.446$ in.

Since $I_s/I_a = 6.627 > 1.0$, it follows that $d_s = d'_s$ = 0.446 in. (i.e. compression stiffener fully effective).

Thus the section is fully effective.

$y_{cg} = 6/2 = 3.000$ in. (from symmetry)

And since the section is singly symmetric about x-axis, top compression fiber (and also bottom tension fiber) may be used in computing S_e.

To check if web is fully effective:

$f_1 = [(3.000-0.154)/3.000] \text{x} 29.35 = 27.84$ ksi(compression)

$f_2 = -27.84$ ksi(tension)

$\Psi = f_2/f_1 = -27.84/27.84 = -1.000$

$k = 24.000$

For a compression and tension stresses of f=27.84 ksi, the values of E_{sc} and E_{st} are found as follows:

$E_{sc} = 24090$ ksi, $E_{st} = 27000$ ksi.

$E_r = (24090+27000)/2$ (Eq. 2.2.1-7)

$= 25550$ ksi

$\lambda = (1.052/\sqrt{24})(94.87)\sqrt{27.84/25550} = 0.672 < 0.673$

$b_e = w$ (Eq. 2.2.1-1)

$= 5.692$ in.

$b_2 = 5.692/2 = 2.846$ in.

$b_1 = 5.692/[3-(-1)] = 1.423$ in.

$b_1+b_2 = 4.269$ in. > compression portion of the web = 2.846 in.

thus b_1+b_2 shall be taken as 2.846 in.. This verifies the assumption that the web is fully effective.

Full section properties are the same as that used in determination of $\phi_b M_n$ since the section is fully effective.

$I_x = 3.086$ in.^{4}

$S_e = 1.029$ in.^{3}

$M = f \times S_e = 29.35 \times 1.029$

$= 30.20$ kips-in. $= M_s$ OK

Thus, use $I_x = 3.086$ in.^{4} for deflection determination.

EXAMPLE 3.2 C-SECTION W/BRACING (ASD)

Rework Example 3.1 to determine the allowable moment, M_a, by using the Allowable Stress Design (ASD) method on the basis of initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable moment.

Solution:

1. Calculation of the allowable moment, M_a:

   The effective section properties calculated by the ASD method are the same as those determined in Example 3.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   Ω = 1.85 (Safety Factor stipulated in Table E of the Standard)

   M_n = 51.45 kips-in. (obtained from Example 3.1)

   $M_a = M_n/\Omega$ (Eq. E-1)

   $= 51.45/1.85$

   = 27.81 kips-in.

2. Calculation of the effective moment of inertia for deflection determination at the allowable moment M_a:

   For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 3.1 for the LRFD method, except that the computed moment M (= fxS_e) should be equal to M_a.

From the results of Example 3.1, it is found that for a stress of $f=29.35$ ksi, the section is fully effective. Therefore, it can be seen that by assuming a stress of $f=27.03$ ksi (which isless than 29.35 ksi) the section will also be fully effective, i.e., $S_e = 1.029$ in.^{3} Thus,

$$M = S_e x 27.03$$
$$= 27.81 \text{ kips-in.} = M_a \quad \text{OK}$$

Therefore, the computed $I_x = 3.083$ in^4 can be used for deflection determination.

EXAMPLE 4.1 Z-SECTION W/STIFFENED FLANGES (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi_b M_n$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use Type 301 stainless steel, 1/4-Hard. Assume dead load to live load ratio D/L = 1/5 and 1.2D+1.6L governs the design.

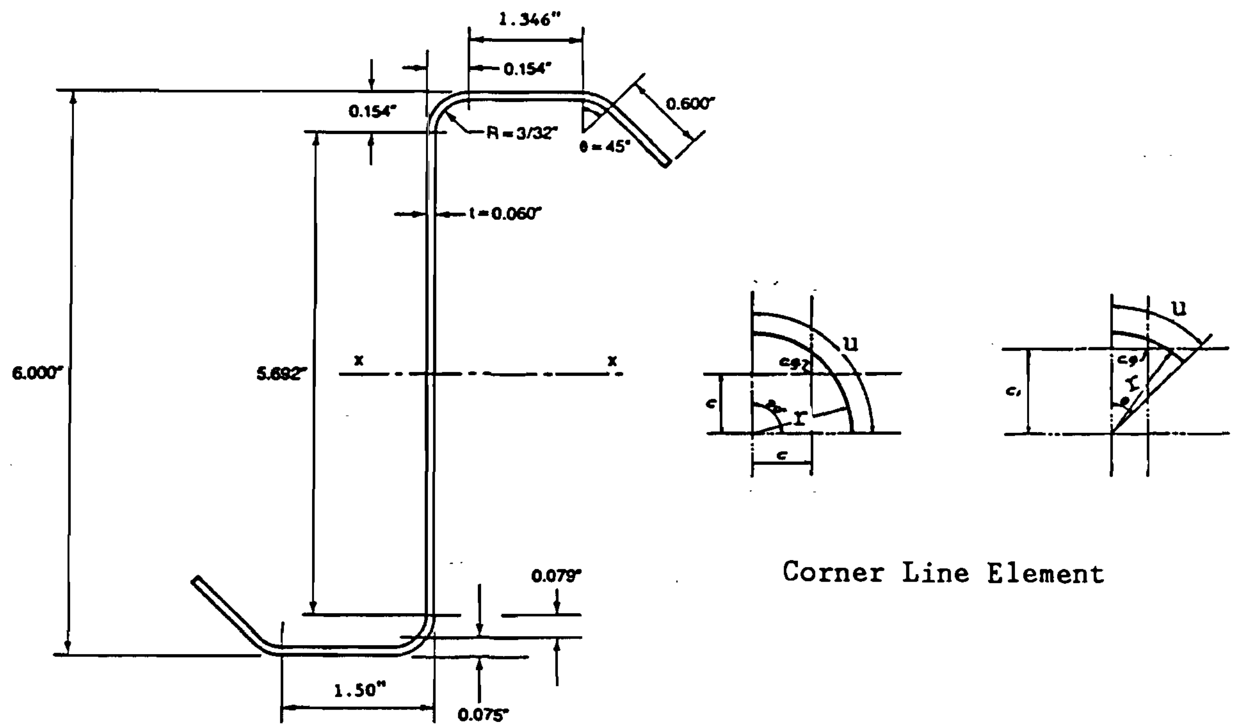


Figure 4.1 Section for Example 4.1

Given:

1. Section: 6" x 1.500" x 0.060" Z-section with stiffened flanges.
2. Compression flange braced against lateral buckling.

Solution:

1. Calculation of the design flexural strength, $\phi_b M_n$:

a. Properties of 90° corners:

$r = R + t/2 = 3/32 + 0.060/2 = 0.124$ in.

Length of arc, $u = 1.57r = 1.57 \times 0.124 = 0.195$ in.

Distance of c.g. from center of radius,

$c = 0.637r = 0.637 \times 0.124 = 0.079$ in.

b. Properties of 135^{o} corners:

$r = R + t/2 = 3/32 + 0.060/2 = 0.124$ in.

Length of arc, $u = (45^{o}/180^{o})(3.14)r = 0.785r = 0.785 \times 0.124$

$= 0.097$ in.

Distance of c.g. from center of radius,

$c_1 = r \sin\theta/\theta = (0.124 \times \sin 45^{o})/0.785 = 0.112$ in.

c. Computation of I_x, S_e, and M_n:

For the first approximation, assume a compression stress of $f = F_y = 50$ ksi (yield strength in longitudinal compression, Table A1 of the Standard) in the top fiber of the section and that the web is fully effective.

Compression flange:

$w = 1.346$ in.

$w/t = 1.346/0.060 = 22.43$

$S = 1.28\sqrt{E_o/f}$ (Eq. 2.4-1)

$E_o = 27000$ ksi (Table A4 of the Standard)

$S = 1.28\sqrt{27000/50} = 29.74$

$S/3 = 9.91 < w/t = 22.43 < S = 29.74$

$I_a = 399t^4\{[(w/t)/S]-0.33\}^3$ (Eq. 2.4.2-6)

$= 399(0.060)^4[(22.43/29.74)-0.33]^3$

$= 0.000395$ in.^{4}

$d = 0.600$ in., $d/t = 0.600/0.060 = 10$

$D = d+0.154\tan(\theta/2) = 0.600+0.154\tan(45^{\circ}/2) = 0.664$ in.

$I_s = d^3 t\sin^2\theta/12$ (Eq. 2.4-2)

$= (0.600)^3(0.060)\sin^2(45^{\circ})/12 = 0.000540$ in.^{4}

$I_s/I_a = 0.000540/0.000395 = 1.367$

$D/w = 0.664/1.346 = 0.493,\ 0.25 < D/w = 0.493 < 0.80$

$k = [4.82-5(D/w)](I_s/I_a)^n+0.43 \leq 5.25-5(D/w)$ (Eq. 2.4.2-9)

$n = 1/2$

$[4.82-5(0.493)](1.367)^{1/2}+0.43 = 3.183$

$5.25-5(0.493) = 2.785 < 3.183$

$k = 2.785$

Since $I_s > I_a$ and $D/w < 0.8$, the stiffener is not considered as a simple lip.

$w/t = 22.43 < 90$ OK (Section 2.1.1-(1)-(i))

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{2.785})(22.43)\sqrt{50/27000} = 0.608 < 0.673$

$b = w$ (Eq. 2.2.1-1)

$= 1.346$ in. (i.e. compression flange fully effective)

Compression (upper) stiffener:

$k = 0.50$ (unstiffened compression element)

$d/t = 10.00$

f conservatively taken equal to 50 ksi as in top compression fiber.

$\lambda = (1.052/\sqrt{0.50})(10.00)\sqrt{50/27000} = 0.640 < 0.673$

Therefore,

$d'_s = d = 0.600$ in.

$d_s = d'_s(I_s/I_a) \leq d'_s$ (Eq. 2.4.2-11)

$= 0.600(1.367)$

= 0.820 in. > 0.600 in.

d_s = 0.600 in. (i.e. compression stiffener is fully effective)

Thus, one concludes that the section is fully effective.

y_{cg} = 6/2 = 3.000 in. (from symmetry)

Full section properties about x axis:

Element	L (in.)	y Distance from Centerline of Section (in.)	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Web	5.692	--	--	15.368
Stiffeners	2 x 0.600 = 1.200	2.722	8.891	0.018
90^o Corners	2 x 0.195 = 0.390	2.925	3.337	--
135^o Corners	2 x 0.097 = 0.194	2.958	1.697	--
Flanges	2 x 1.346 = 2.692	2.970	23.746	--
Sum	10.168		37.671	15.386

Since section is singly symmetric about x-axis and fully effective, a compression stress of 50 ksi will govern as assumed. (At the bottom tension fiber a tensile stress of 50 ksi will develop simultaneously from geometry).

To check if web is fully effective: (Section 2.2.2)

f_1 = [(3.000-0.154)/3.000]x50 = 47.43 ksi(compression)

f_2 = -47.43 ksi(tension)

$\Psi = f_2/f_1 = -47.43/47.43 = -1.000$

$k = 4+2(1-\Psi)^3+2(1-\Psi)$

$= 4+2[1-(-1)]^3+2[1-(-1)]$

$= 24.000$

$h = w = 5.692$ in., $h/t = w/t = 5.692/0.060 = 94.87$

$h/t = 94.87 < 200$ OK (Section 2.1.2-(1))

$\lambda = (1.052/\sqrt{24})(94.87)\sqrt{47.43/27000} = 0.854 > 0.673$

$\rho = [1-(0.22/\lambda)]/\lambda$ (Eq.2.2.1-3)

$= [1-(0.22/0.854)]/0.854 = 0.869$

$b_e = \rho w$ (Eq. 2.2.1-2)

$= 0.869 \times 5.692 = 4.946$ in.

$b_2 = b_e/2$ (Eq. 2.2.2-2)

$= 4.946/2 = 2.473$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 4.946/[3-(-1)] = 1.237$ in.

The effective widths of web, b_1 and b_2, are defined in Figure 2 of the Standard.

$b_1+b_2 = 1.237 + 2.473 = 3.710$ in.

Compression portion of the web $= y_{cg} - 0.154$

$= 3.000 - 0.154$

$= 2.846$ in.

Since $b_1+b_2 = 3.710$ in. > 2.846 in., b_1+b_2 shall be taken as 2.846 in.. This verifies the assumption that the web is fully effective.

$I'_x = Ly^2+I'_1$

$= 37.671 + 15.386$

$= 53.057 \text{ in.}^3$

Actual $I_x = I'_x t$

$= 53.057 \times 0.060$

$= 3.183 \text{ in.}^4$

$S_e = I_x / y_{cg}$

$= 3.183/3.000$

$= 1.061 \text{ in.}^3$

$M_n = S_e F_y$ (Eq. 3.3.1.1-1)

$= 1.061 \times 50$

$= 53.05$ kips-in.

d. The design flexural strength, $\phi_b M_n$, based on initiation of yielding is determined as follows: (Section 3.3.1.1(1))

$\phi_b = 0.90$ (for section with stiffened compression flanges)

$\phi_b M_n = 0.90 \times 53.05 = 47.75$ kips-in.

2. Calculation of the effective moment of inertia for deflection determination at the service moment M_s:

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

$$\phi_b M_n = 1.2M_{DL} + 1.6M_{LL}$$

$$= [1.2(M_{DL}/M_{LL})+1.6]M_{LL}$$

$$= [1.2(1/5)+1.6]M_{LL}$$

$$= 1.84M_{LL}$$

$$M_{LL} = \phi_b M_n / 1.84 = 47.75/1.84 = 25.95 \text{ kips-in.}$$

$M_s = M_{DL} + M_{LL}$

$= (1/5+1)M_{LL}$

$= 1.2(25.95) = 31.14$ kips-in.

where

M_{DL} = Moment determined on the basis of nominal dead load

M_{LL} = Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive stress f under this service moment M_s. Knowing f, one proceeds as usual to obtain S_e and checks to see if ($f \times S_e$) is equal to M_s as it should. If not, reiterate until one obtains the desired level of accuracy. (Section 2.2.1-(2))

After several trials with first iteration using $f = F_y/2$, the following only gives the results of final iteration. Assume that a stress of f = 29.35 ksi in the top and bottom fibers of the section and that the web is fully effective.

Compression flange:

$S = 1.28 \sqrt{27000/29.35} = 38.82$

$S/3 = 12.94 < w/t = 22.43 < S = 38.82$

$I_a = 399(0.060)^4[(22.43/38.82)-0.33]^3$

$= 0.000079$ in.^{4}

$I_s/I_a = 0.000540/0.000079 = 6.835$

$k = [4.82-5(0.493)](6.835)^{1/2}+0.43 = 6.587 > 2.785$

Use k = 2.785

For a compression and tension stresses of f=29.35 ksi, the values of E_{sc} and E_{st} are found as follows:

E_{sc} = 23089 ksi, E_{st} = 26933 ksi.

E_r = (23089+26933)/2 (Eq. 2.2.1-7)

= 25011 ksi

$\lambda = (1.052/\sqrt{2.785})(22.43)\sqrt{29.35/25011} = 0.484 < 0.673$

b_d = 1.346 in. (i.e. compression flange fully effective)

Compression (upper) stiffener:

f can be conservatively taken equal to 29.35 ksi as used in the top compression fiber.

$\lambda = (1.052/\sqrt{0.50})(10.00)\sqrt{29.35/25011} = 0.510 < 0.673$

therefore, d'_s = 0.600 in.

Since $I_s/I_a = 6.835 > 1.0$, it follows that $d_s = d'_s$ = 0.600 in. (i.e. compression stiffener fully effective).

Thus, the section is fully effective.

y_{cg} = 6/2 = 3.000 in. (from symmetry)

And since the section is singly symmetric about x-axis, top compression fiber may be used in computing S_e.

To check if web is fully effective:

$f_1 = [(3.000-0.154)/3.000]x29.35 = 27.84$ ksi(compression)

$f_2 = -27.84$ ksi(tension)

$\psi = f_2/f_1 = -27.84/27.84 = -1.000$

$k = 24.000$

For a stress of f=27.84 ksi, the E_r = 25550 ksi, which is determined in Example 3.1.

$\lambda \quad = (1.052/\sqrt{24})(94.87)\sqrt{27.84/25550} = 0.672 < 0.673$

$b_e \quad = w$ (Eq. 2.2.1-1)

$\quad = 5.692$ in.

$b_2 \quad = 5.692/2 = 2.846$ in.

$b_1 \quad = 5.692/[3-(-1)] = 1.423$ in.

$b_1+b_2 = 4.269$ in. > compression portion of the web = 2.846 in.

thus b_1+b_2 shall be taken as 2.846 in.. This verifies the assumption that the web is fully effective.

Full section properties are the same as that used in the determination of $\phi_b M_n$ since the section is fully effective.

$I_x \quad = 3.183$ in.^{4}

$S_e \quad = 1.061$ in.^{3}

$M \quad = f \times S_e = 29.35 \times 1.061$

$\quad = 31.14$ kips-in. $= M_s$ OK

Thus, use $I_x = 3.183$ in.^{4} for deflection determination.

EXAMPLE 4.2 Z-SECTION W/STIFFENED FLANGES (ASD)

Use the data given in Example 4.1 (Figure 4.1) to determine the allowable moment, M_a, by using the Allowable Stress Design (ASD) method on the basis of initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable moment. Use Type 301 stainless steel, 1/4-Hard: F_y = 50 ksi.

Solution:

1. Calculation of the allowable moment, M_a:

   The effective section properties calculated by the ASD method are the same as those determined in Example 4.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   Ω = 1.85 (Safety Factor stipulated in Table E of the Standard)

   M_n = 53.05 kips-in. (obtained from Example 4.1)

   $M_a = M_n/\Omega$ (Eq. E-1)

   $= 53.05/1.85$

   $= 28.68$ kips-in.

2. Calculation of the effective moment of inertia for deflection determination at the allowable moment, M_a:

   For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 4.1 for the LRFD method, except that the computed

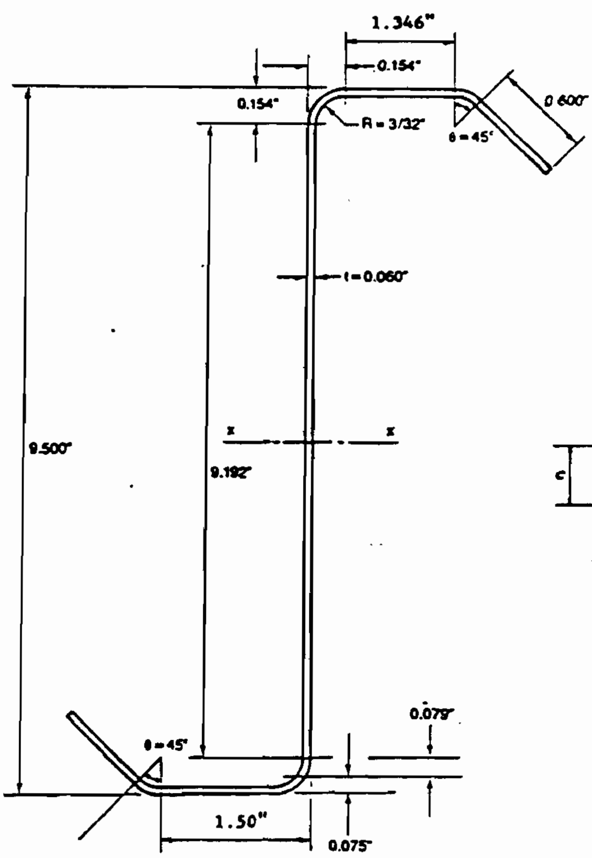
M (= fxS_e) should be equal to M_a.

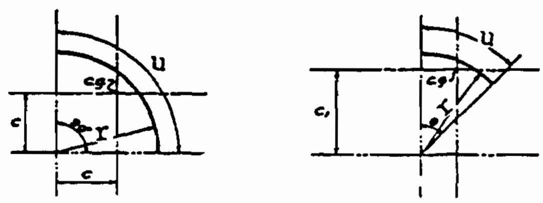
From the results of Example 4.1, it can be seen that by using a stress of f=29.35 ksi, the computed S_e = 1.061 in.^{3} which is based on the fully effective section. If the assumed stress is equal to f=27.03 ksi, the effective section modulus is also determined by the full section properties, i.e., S_e = 1.061 in.^{3}. This will give fxS_e = 28.68 kips-in., which is equal to M_a

Therefore, the computed I_x = 3.183 in^4 of the full section properties is used for deflection determination.

EXAMPLE 5.1 DEEP Z-SECTION w/STIFFENED FLANGES (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi_b M_n$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use Type 301 stainless steel, 1/4-Hard. Assume dead load to live load ratio D/L = 1/5 and 1.2D+1.6L governs the design.





Corner Line Element

Figure 5.1 Section for Example 5.1

Given:

1. Section: 9.5" x 1.500" x 0.060" Z-section with stiffened flanges.
2. Compression flange braced against lateral buckling.

Solution:

1. Calculation of the design flexural strength, $\phi_b M_n$:

a. Properties of 90° corners:

$r = R + t/2 = 3/32 + 0.060/2 = 0.124$ in.

Length of arc, $u = 1.57r = 1.57 \times 0.124 = 0.195$ in.

Distance of c.g. from center of radius,

$c = 0.637r = 0.637 \times 0.124 = 0.079$ in.

b. Properties of 135° corners:

$r = R + t/2 = 3/32 + 0.060/2 = 0.124$ in.

Length of arc, $u = (45^{\circ}/180^{\circ})(3.14)r = 0.785r = 0.785 \times 0.124$

$= 0.097$ in.

Distance of c.g. from center of radius,

$c_1 = r \sin\theta/\theta = (0.124 \times \sin 45^{\circ})/0.785 = 0.112$ in.

c. Computation of I_x, S_e, and M_n:

For the first approximation, assume a compression stress of $f = F_y = 50$ ksi (yield strength in longitudinal compression as given in Table A1 of the Standard) in the top fiber of the section and that the web is fully effective.

Compression flange:

$w = 1.346$ in.

$w/t = 1.346/0.060 = 22.43$

$$S = 1.28\sqrt{E_o/f} \qquad \text{(Eq. 2.4-1)}$$

$E_o = 27000$ ksi (Table A4 of the Standard)

$S = 1.28\sqrt{27000/50} = 29.74$

$S/3 = 10.36 < w/t = 24.52 < S = 31.09$

$$I_a = 399t^4\{[(w/t)/S]-0.33\}^3 \qquad \text{(Eq. 2.4.2-6)}$$

$$= 399(0.060)^4[(22.43/29.74)-0.33]^3$$

$= 0.000395 \text{ in.}^4$

$d = 0.600 \text{ in.}, \ d/t = 0.600/0.060 = 10$

$D = d + 0.154\tan(\theta/2) = 0.600 + 0.154\tan(45^\circ/2) = 0.664 \text{ in.}$

$I_s = d^3 t \sin^2\theta/12$ (Eq. 2.4-2)

$= (0.600)^3(0.060)\sin^2(45^\circ)/12 = 0.000540 \text{ in.}^4$

$I_s/I_a = 0.000540/0.000395 = 1.367$

$D/w = 0.664/1.346 = 0.493, \ 0.25 < D/w = 0.493 < 0.80$

$k = [4.82 - 5(D/w)](I_s/I_a)^n + 0.43 \leq 5.25 - 5(D/w)$ (Eq. 2.4.2-9)

$n = 1/2$

$[4.82 - 5(0.493)](1.367)^{1/2} + 0.43 = 3.183$

$5.25 - 5(0.493) = 2.785 < 3.183$

use $k = 2.995$

Since $I_s > I_a$ and $D/w < 0.8$, the stiffener is not considered as a simple lip.

$w/t = 22.43 < 90$ OK (Section 2.1.1-(1)-(i))

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{2.785})(22.43)\sqrt{50/27000} = 0.608 < 0.673$

$b = w$ (Eq. 2.2.1-1)

$= 1.346$ in. (i.e. compression flange fully effective)

Compression (upper) stiffener:

$k = 0.50$ (unstiffened compression element)

$d/t = 10.00$

f conservatively taken equal to 50 ksi as in top compression fiber.

$\lambda = (1.052/\sqrt{0.50})(10.00)\sqrt{50/27000} = 0.640 < 0.673$

Therefore,

$d'_s \quad = d = 0.600$ in.

$d_s \quad = d'_s(I_s/I_a) \leq d'_s$ (Eq. 2.4.2-11)

Since $I_s/I_a = 1.367 > 1.000$

$d_s \quad = d'_s = 0.600$ in. (i.e. compression stiffener is fully effective)

Thus, one concludes that the section is fully effective.

$y_{cg} \quad = 9.5/2 = 4.750$ in. (from symmetry)

It follows that a compression stress of f=50 ksi will govern as assumed.

To check if web is fully effective (Section 2.2.2):

$f_1 \quad = [(4.750-0.154)/4.750]x50 = 48.38$ ksi(compression)

$f_2 \quad = -48.38$ ksi(tension)

$\Psi \quad = f_2/f_1 = -48.38/48.38 = -1.000$

$k \quad = 4+2(1-\Psi)^3+2(1-\Psi)$

$\quad = 4+2[1-(-1)]^3+2[1-(-1)]$

$\quad = 24.000$

$h \quad = w = 9.192$ in., $h/t = w/t = 9.192/0.060 = 153.20$

$h/t \quad = 153.20 < 200$ OK (Section 2.1.2-(1))

$\lambda \quad = (1.052/\sqrt{24})(153.20)\sqrt{48.38/27000} = 1.393 > 0.673$

$\rho \quad = [1-(0.22/\lambda)]/\lambda$ (Eq.2.2.1-3)

$\quad = [1-(0.22/1.393)]/1.393 = 0.604$

$b_e \quad = \rho w$ (Eq. 2.2.1-2)

$\quad = 0.604 \times 9.192 = 5.552$ in.

$b_2 \quad = b_e/2$ (Eq. 2.2.2-2)

$\quad = 5.552/2 = 2.776$ in.

$b_1 \quad = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 5.552/[3-(-1)] = 1.388$ in.

The effective widths of web, b_1 and b_2, are defined in Figure 2 of the Standard.

$b_1+b_2 = 1.388 + 2.776 = 4.164$ in.

Compression portion of the web = $y_{cg} - 0.154$

$= 4.750 - 0.154$

$= 4.596$ in.

Since $b_1+b_2 = 4.164$ in. < 4.596 in., it follows that the web is not fully effective. Hence $y_{cg} = 4.750$ as assumed.

The procedure to determine the location of the neutral axis (N.A.) based on partially effective web is iterative. We start with $y_{cg} = 4.750$ in. and from Figure 2 of the Standard, scale b_1, b_2 already computed with respect to $y_{cg} = 4.750$ in. Then we proceed to compute a new N.A. and hence b_1+b_2. If (b_1+b_2) is the same as before, the solution stabilizes and the location of N.A. is calculated according to this (b_1+b_2). If (b_1+b_2) differ than before, one reiterates in the same manner until b_1+b_2 stabilizes.

Thus, for the first iteration, the web is divided into three segments: $b_1 = 1.388$ in., ineffective portion of web, and $b_2(=2.776)+4.750-0.154 = 7.372$ in.. Thus the ineffective portion of web = 9.192-1.388-7.372 = 0.432 in..

The compression flange and stiffener remain fully effective since nothing is altered in their calculations.

Effective section properties about x-axis:

Element	L (in.)	y Distance from Top Fiber (in.)		Ly (in.^{2})
b_1	1.388	0.154+(1.388/2)	= 0.848	1.177
$b_2+(9.5-y_{cg})-0.154$	7.372	9.5-0.154-(7.372/2)	= 5.660	41.726
Compression flange	1.346		0.030	0.040
Compression stiffener	0.600	$0.154-0.124\cos45^\circ$ $+(0.600/2)\sin45^\circ$	= 0.278	0.167
Top 90° corner	0.195		0.075	0.015
Top 135° corner	0.097	0.154-0.112	= 0.042	0.004
Bottom 135° corner	0.097	9.5-(0.154-0.112)	= 9.458	0.917
Bottom 90° corner	0.195	9.5-0.075	= 9.425	1.838
Bottom stiffener	0.600	9.5-0.278	= 9.222	5.533
Tension flange	1.346	9.5-(0.060/2)	= 9.470	12.747
Sum	13.236			64.164

y_{cg} = Ly/L = 64.164/13.236

= 4.848 in. (measured from top compression fiber)

f_1 = [(4.848-0.154)/4.848](50) = 48.41 ksi(compression)

f_2 = -[(9.5-4.848-0.154)/4.848](50) = -46.39 ksi(tension)

ψ = -46.39/48.41 = -0.958

$k = 4+2[1-(-0.958)]^3+2[1-(-0.958)]$

= 22.929

$\lambda = (1.052/\sqrt{22.929})(153.20)\sqrt{48.41/27000} = 1.425 > 0.673$

$\rho = [1-(0.22/1.425)]/1.425 = 0.593$

b_e = 0.593 x 9.192 = 5.451 in.

b_2 = 5.451/2 = 2.726 in.

b_1 = 5.451/[3-(-0.958)] = 1.377 in.

b_1+b_2 = 4.103 in. = 4.164 in. Therefore, need to reiterate.

For the second iteration:

$b_1 = 1.377$ in.

$b_2+(9.5-y_{cg})-0.154 = 2.726+9.5-4.848-0.154 = 7.224$ in.

Ineffective portion of web = 9.192-1.377-7.224 = 0.591 in.

Effective section properties about x-axis:

Element	L (in.)	y Distance from Top Fiber (in.)	Ly ($in.^2$)
b_1	1.377	0.843	1.161
$b_2+(9.5-y_{cg})-0.154$	7.224	5.734	41.422
Compression flange	1.346	0.030	0.040
Compression stiffener	0.600	0.278	0.167
Top 90° corner	0.195	0.075	0.015
Top 135° corner	0.097	0.042	0.004
Bottom 135° corner	0.097	9.458	0.917
Bottom 90° corner	0.195	9.425	1.838
Bottom stiffener	0.600	9.222	5.533
Tension flange	1.346	9.470	12.747
Sum	13.077		63.844

$y_{cg} = 65.844/13.077 = 4.882$ in. (measured from top compression fiber)

$f_1 = [(4.882-0.154)/4.882](50) = 48.42$ ksi

$f_2 = -[(9.5-4.882-0.154)/4.882](50) = -45.72$ ksi

$\Psi = -45.72/48.42 = -0.944$

$k = 4+2[1-(-0.944)]^3+2[1-(-0.944)] = 22.580$

$\lambda = (1.052/\sqrt{22.580})(153.20)\sqrt{48.42/27000} = 1.436 > 0.673$

$\rho = [1-(0.22/1.436)]/1.436 = 0.590$

b_e = 0.590 x 9.192 = 5.423 in.

b_2 = 5.423/2 = 2.712 in.

b_1 = 5.423/[3-(-0.946)] = 1.374 in.

b_1+b_2 = 4.086 in. = 4.103 in. Therefore, need to reiterate.

For the third iteration:

b_1 = 1.374 in.

$b_2+(9.5-y_{cg})-0.154$ = 2.712+9.5-4.882-0.154 = 7.176 in.

Ineffective portion of web = 9.192-1.374-7.176 = 0.642 in.

Effective section properties about x-axis:

L = 13.026 in.

Ly = 63.736 in.^{2}

y_{cg} = 63.736/13.026 = 4.893 in.

f_1 = [(4.893-0.154)/4.893](50) = 48.43 ksi

f_2 = -[(9.5-4.893-0.154)/4.893](50) = -45.50 ksi

ψ = -45.50/48.43 = -0.940

k = $4+2[1-(-0.940)]^3+2[1-(-0.940)]$ = 22.483

λ = $(1.052/\sqrt{22.483})(153.20)\sqrt{48.43/27000}$ = 1.440 > 0.673

ρ = [1-(0.22/1.440)]/1.440 = 0.588

b_e = 0.588 x 9.192 = 5.405 in.

b_2 = 5.405/2 = 2.703 in.

b_1 = 5.405/[3-(-0.940)] = 1.372 in.

b_1+b_2 = 4.075 in. = 4.086 in. Therefore, need to reiterate.

For the fourth iteration:

b_1 = 1.372 in.

$b_2+(9.5-y_{cg})-0.154 = 2.703+9.5-4.893-0.154 = 7.156$ in.

Ineffective portion of web = 9.192-1.372-7.156 = 0.664 in.

Effective section properties about x-axis:

$L = 13.004$ in.

$Ly = 63.689$ in.^{2}

$y_{cg} = 63.689/13.004 = 4.898$ in.

$f_1 = [(4.898-0.154)/4.898](50) = 48.43$ ksi

$f_2 = -[(9.5-4.898-0.154)/4.898](50) = -45.41$ ksi

$\Psi = -45.41/48.43 = -0.938$

$k = 4+2[1-(-0.938)]^3+2[1-(-0.938)] = 22.434$

$\lambda = (1.052/\sqrt{22.434})(153.20)\sqrt{48.43/27000} = 1.441 > 0.673$

$\rho = [1-(0.22/1.441)]/1.441 = 0.588$

$b_e = 0.588 \times 9.192 = 5.405$ in.

$b_2 = 5.405/2 = 2.703$ in.

$b_1 = 5.405/[3-(-0.938)] = 1.373$ in.

$b_1+b_2 = 4.076$ in. close enough to 4.075 in.

Thus, the solution stabilizes.

Hence we now compute the location of N.A. and moment of inertia using $b_1 = 1.373$ in. and $b_2 = 2.703$ in.

Effective section properties about x-axis:

Element	L (in.)	y Distance from Top Fiber (in.)	Ly ($in.^2$)	Ly^2 ($in.^3$)	I'_1 About Own Axis ($in.^3$)
b_1	1.373	0.841	1.155	0.971	0.216
$b_2+(9.5-y_{cg})-0.154$	7.151	5.771	41.268	238.160	30.473
Compression flange	1.346	0.030	0.040	0.001	--
Compression stiffener	0.600	0.278	0.167	0.046	0.009
Top 90° corner	0.195	0.075	0.015	0.001	--
Top 135° corner	0.097	0.042	0.004	--	--
Bottom 135° corner	0.097	9.458	0.917	8.677	--
Bottom 90° corner	0.195	9.425	1.838	17.322	--
Bottom stiffener	0.600	9.222	5.533	51.027	0.009
Tension flange	1.346	9.470	12.747	120.710	--
Sum	13.000		63.684	436.915	30.707

Distance from top fiber to x-axis is

y_{cg} = 63.684/13.000 = 4.899 in.

Since the distance from top compression fiber to the neutral axis is greater than one half the beam depth (= 4.750 in.), a compression stress of 50 ksi will govern as assumed.

I'_x $= Ly^2+I'_1-Ly^2_{cg}$

$= 436.915 + 30.707 - 13.000(4.899)^2$

$= 155.619$ in.^{3}

Actual $I_x = I'_x t$

$= 155.619 \times 0.060$

$= 9.337$ in.^{4}

S_e $= I_x/y_{cg}$

$= 9.337/4.899$

$= 1.906$ in.^{3}

$M_n = S_e F_y$ (Eq. 3.3.1.1-1)

$= 1.906 \times 50$

$= 95.30$ kips-in.

d. The design flexural strength, $\phi_b M_n$, based on initiation of yielding is determined as follows: (Section 3.3.1.1(1))

$\phi_b = 0.90$ (for section with stiffened compression flanges)

$\phi_b M_n = 0.90 \times 95.30 = 85.77$ kips-in.

2. Calculation of the effective moment of inertia for deflection determination at the service moment M_s:

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

$$\phi_b M_n = 1.2M_{DL} + 1.6M_{LL}$$
$$= [1.2(M_{DL}/M_{LL})+1.6]M_{LL}$$
$$= [1.2(1/5)+1.6]M_{LL}$$
$$= 1.84M_{LL}$$
$$M_{LL} = \phi_b M_n/1.84 = 85.77/1.84 = 46.61 \text{ kips-in.}$$
$$M_s = M_{DL} + M_{LL}$$
$$= (1/5+1)M_{LL}$$
$$= 1.2(46.61) = 55.93 \text{ kips-in.}$$

where

M_{DL} = Moment determined on the basis of nominal dead load

M_{LL} = Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive stress f under this service moment M_s. Knowing f, one proceeds as usual to obtain S_e and checks to see if ($f \times S_e$) is equal to M_s as it should. If not, reiterate until one obtains the desired level of accuracy. (Section 2.2.1-(2))

a. For the first iteration, assume a stress of f = 30 ksi in the top and bottom fibers of the section and that the web is fully effective.

Compression flange:

$$S = 1.28\sqrt{27000/30} = 38.40$$

$$S/3 = 12.80 < w/t = 22.43 < S = 38.40$$

$$I_a = 399(0.060)^4[(22.43/38.40)-0.33]^3$$

$$= 0.000085 \text{ in.}^4$$

$$I_s/I_a = 0.000540/0.000085 = 6.353$$

$$k = [4.82-5(0.493)](6.353)^{1/2}+0.43 = 6.366 > 2.785$$

Use k = 2.785

For deflection determination, the value of E_r, reduced modulus of elasticity determined by using Eq. (2.2.1-7), is substituted for E_o in Eq. (2.2.1-4).

For a compression and tension stresses of f = 30 ksi, the corresponding E_{sc} and E_{st} values for Type 301 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:

E_{sc} = 22650 ksi, E_{st} = 26900 ksi

$E_r = (E_{sc}+E_{st})/2$ (Eq. 2.2.1-7)

= (22650+26900)/2 = 24775 ksi

$\lambda = (1.052/\sqrt{2.785})(22.43)\sqrt{30/24775} = 0.492 < 0.673$

b_d = 1.346 in. (i.e. compression flange fully effective)

Compression (upper) stiffener:

f can be conservatively taken equal to 30 ksi as used in the top compression fiber.

$\lambda = (1.052/\sqrt{0.50})(10.00)\sqrt{30/24775} = 0.518 < 0.673$

therefore, d'_s = 0.600 in.

Since I_s/I_a = 6.353 > 1.0, it follows that $d_s = d'_s$ = 0.600 in. (i.e. compression stiffener fully effective).

Thus, section is fully effective (since web was assumed fully effective).

y_{cg} = 9.5/2 = 4.750 in. (from symmetry)

To check if web is fully effective:

$f_1 = [(4.750-0.154)/4.750](30) = 29.03$ ksi

f_2 = -29.03 ksi

ψ = -29.03/29.03 = -1.000

k = 24.000

For a compression and tension stresses of f=29.03 ksi, it is found that the vlues of E_{sc} and E_{st} are equal to 23305 ksi and 26950 ksi, respectively.

$E_r = (E_{sc}+E_{st})/2$ (Eq. 2.2.1-7)

$= (23305+26950)/2 = 25130$ ksi

$\lambda = (1.052/\sqrt{24})(153.20)\sqrt{29.03/25130} = 1.118 > 0.673$

$\rho = [1-(0.22/1.118)]/1.118 = 0.718$

$b_e = 0.718 \times 9.192 = 6.600$ in.

$b_2 = 6.600/2 = 3.300$ in.

$b_1 = 6.600/[3-(-1)] = 1.650$ in.

Compression portion of the web = $y_{cg} - 0.154$

$= 4.750 - 0.154 = 4.596$ in.

$b_1+b_2 = 4.950$ in. > 4.596 in.

Thus b_1+b_2 shall be taken as 4.596 in. This verifies the assumption that the web is fully effective.

Full section properties about x-axis:

Element	L (in.)	y Distance from Centerline of Section (in.)	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Web	9.192	--	--	64.722
Stiffeners	2 x 0.600 = 1.200	4.472	23.999	0.018
90^o corners	2 x 0.195 = 0.390	4.675	8.524	--
135^o corners	2 x 0.097 = 0.194	4.708	4.300	--
Flanges	2 x 1.346 = 2.692	4.720	59.973	--
Sum			96.796	64.740

$I'_x = Ly^2 + I'_1$

$= 96.796 + 64.740 = 161.536$ in.^{3}

$I_x = 161.536(0.060) = 9.692$ in.^{4}

$S_e = I_x/y_{cg} = 9.692/4.750 = 2.040$ in.^{3}

$M = f \times S_e = 30 \times 2.040$

$= 61.20$ kips-in. not equal to $M_s = 55.93$ kips-in.

Thus, need to reiterate.

However, one sees that we need to assume a smaller stress than 30 ksi and since the section was fully effective for f = 30 ksi, it will be fully effective for f < 30 ksi.

Thus $S_e = 2.040$ in.^{3}

Therefore, the correct f at $M_s = M_s/S_e = 55.93/2.040$

$= 27.42$ ksi. and $I_x = 9.692$ in.^{4} for deflection determination.

Remark:

It was clearly seen that in the calculation of $\Phi_b M_n$, the assumption of the web being fully effective was not true. However, it would be interesting to see the percentage of error if one neglected the partial effectiveness of the web and proceeded with the assumption of a fully effective web.

To demonstrate: neglect the partial effectiveness of the web in the first approximation in the calculation of $\Phi_b M_n$.

Thus the whole section is fully effective. Full section properties about x-axis (from part 2):

I_x = 9.692 in.^{4}

S_e = 2.040 in.^{3}

$\Phi_b M_n$ = 0.90(2.040x50) = 91.80 kips-in.

% error = (91.80-85.77)/85.77 x100% = 7.03%

Since the percentage of error is small, one could rationalize that in practical cases to get a first-hand quick answer one could assume the web being fully effective.

EXAMPLE 5.2 DEEP Z-SECTION W/STIFFENED FLANGES (ASD)

Use the data given in Example 5.1 (Figure 5.1) to determine the allowable moment, M_a, by using the Allowable Stress Design (ASD) method on the basis of initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable moment. Use Type 301 stainless steel, 1/4-Hard: F_y = 50 ksi.

Solution:

1. Calculation of the allowable moment, M_a:

   The effective section properties calculated by the ASD method are the same as those determined in Example 5.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   Ω = 1.85 (Safety Factor stipulated in Table E of the Standard)

   M_n = 95.30 kips-in. (obtained from Example 5.1)

   $$M_a = M_n/\Omega \qquad \text{(Eq. E-1)}$$
   $$= 95.30/1.85$$
   $$= 51.51 \text{ kips-in.}$$

2. Calculation of the effective moment of inertia for deflection determination at the allowable moment, M_a:

   For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 5.1 for the LRFD method, except that the computed moment M (= fxS_e) should be equal to M_a.

From the results of Example 5.1, it can be seen that by using a a stress of f=30 ksi, the computed S_e = 2.040 in.^{3} which is based on the fully effective section. If one assumes a smaller stress of f=25.25 ksi, the effective section modulus will also be determined on the basis of its full cross section, i.e., S_e = 2.040 in.^{3} Therefore, fxS_e = 25.25x2.040 = 51.51 kips-in., which is equal to M_a determined above.

Therefore, the computed I_x = 9.692 in^4 obtained from the full section properties can be used for deflection determination.

EXAMPLE 6.1 HAT SECTION (LRFD)

(Complete Flexural Design)

By using the Load and Resistance Factor Design (LRFD) method, check the adequacy of the hat section given in Figure 6.1 for bending moment, shear, web crippling, and deflection. Use Type 316 stainless steel, 1/4-Hard. Assume dead load to live load ratio D/L = 1/5 and 1.2D+1.6L governs the design.

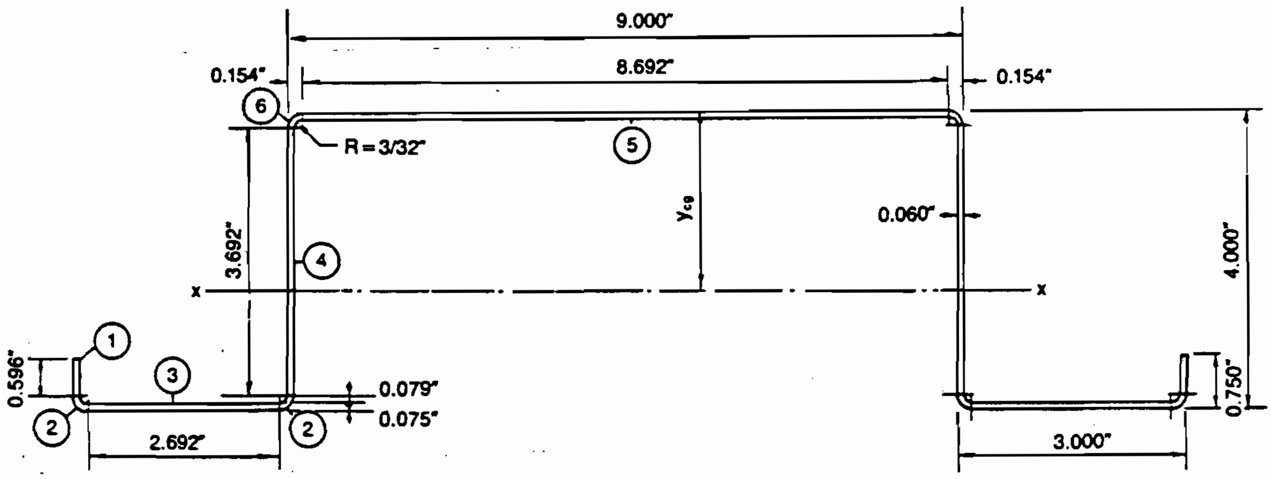


Figure 6.1 Section for Example 6.1

Given:

1. Section: Hat section, as shown in sketch.
2. Span length: L = 8 ft., with simple supports, no overhang, and 6-in. support bearing lengths.
3. Nominal Loading: Live = 250 lb/ft.; Dead = 20 lb/ft.

Solution:

1. Properties of 90° corners:

   Corner Radius, r = R + t/2 = 3/32 + 0.060/2 = 0.124 in.

Length of arc, $u = 1.57r = 1.57 \times 0.124 = 0.195$ in.

Distance of c.g. from center of radius,

$c = 0.637r = 0.637 \times 0.124 = 0.079$ in.

The moment of inertia, I', of corner about its own centroidal axis is negligible.

2. Nominal Section Strength, M_n (Section 3.3.1.1)

a. Procedure I - Based on Initiation of Yielding

Computation of I_x, S_e, and M_n: (first approximation)

* Assume a compressive stress of $f = F_y = 50$ ksi in the top fiber of the section. (See Table A1 of the Standard for yield strength.)

* Also assume web is fully effective.

Element 4:

$h/t = 3.692/0.060 = 61.53 < (h/t)_{max} = 200$ OK (Section 2.1.2-(1))

Assumed fully effective

Element 5:

$w/t = 8.692/0.060 = 144.9 < 400$ OK (Section 2.1.1-(1)-(ii))

$k \quad = 4$

$\lambda \quad = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

E_o is equal to 27000 ksi, which is obtained from Table A4 of the Standard.

$\lambda \quad = (1.052/\sqrt{4})(144.9)\sqrt{50/27000} = 3.280 > 0.673$

$\rho \quad = [1-(0.22/\lambda)]/\lambda$ (Eq. 2.2.1-3)

$= [1-(0.22/3.280)]/3.280 = 0.284$

$b \quad = \rho w$ (Eq. 2.2.1-2)

$= 0.284 \times 8.692$

$= 2.469$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.²)	Ly² (in.³)	I'_1 About Own Axis (in.³)
1	2 x 0.596 = 1.192	3.548	4.229	15.005	0.035
2	4 x 0.195 = 0.780	3.925	3.062	12.016	--
3	2 x 2.692 = 5.384	3.970	21.375	84.857	--
4	2 x 3.692 = 7.384	2.000	14.768	29.536	8.388
5	2.469	0.030	0.074	0.002	--
6	2 x 0.195 = 0.390	0.075	0.029	0.002	--
Sum	17.599		43.537	141.418	8.423

The distance from the top fiber to the neutral axis is

$y_{cg} = Ly/L = 43.537/17.599 = 2.474$ in.

Since the distance from top compression fiber to the neutral axis, y_{cg}, is greater than one half the beam depth, a compressive stress of F_y will govern as assumed.

$I'_x \quad = Ly^2 + I'_1 - Ly^2_{cg}$

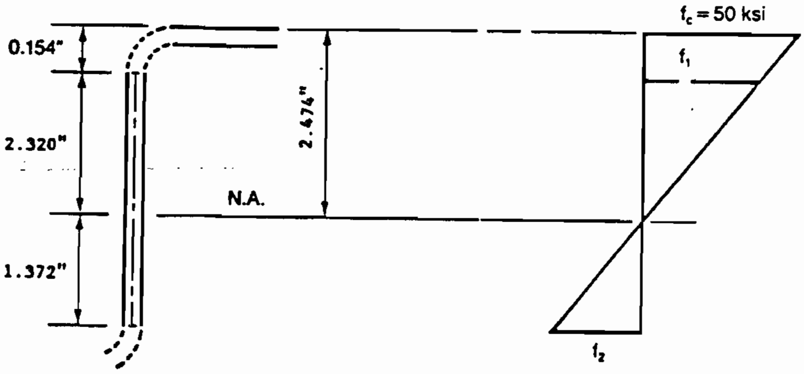
$= 141.418 + 8.423 - 17.599(2.474)^2$

$= 42.12$ in.³

Actual $I_x = tI'_x$

$= (0.060)(42.12) = 2.53 \text{ in.}^4$

Check Web



$f_1 = (2.320/2.474)(50) = 46.89$ ksi(compression)

$f_2 = -(1.372/2.474)(50) = -27.73$ ksi(tension)

$\Psi = f_2/f_1 = -27.73/46.89 = -0.591$

$k = 4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

$= 4+2[1-(-0.591)]^3+2[1-(-0.591)]$

$= 15.24$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}\,, \quad f = f_1$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{15.24})(61.53)\sqrt{46.89/27000} = 0.691 > 0.673$

$\rho = [1-(0.22/\lambda)]/\lambda$ (Eq. 2.2.1-3)

$= [1-(0.22/0.691)]/0.691 = 0.986$

$b_e = \rho w$ (Eq. 2.2.1-2)

$= 0.986 \times 3.692$

$= 3.640$ in.

$b_2 = b_e/2$ (Eq. 2.2.2-2)

$= 3.640/2 = 1.820$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 3.640/[3-(-0.591)] = 1.014$ in.

$b_1+b_2 = 1.014 + 1.820 = 2.834$ in. > 2.320 in. (compression portion of web, see sketch shown above)

Therefore, web is fully effective.

$S_e = I_x/y_{cg}$

$= 2.53/2.474$

$= 1.02$ in.^{3}

$M_n = S_eF_y$ (Eq. 3.3.1.1-1)

$= (1.02)(50)$

$= 51.0$ kips-in.

b. Procedure II - Based on Inelastic Reserve Capacity

$\lambda_1 = (1.11/\sqrt{F_y/E_o})$ (Eq. 3.3.1.1-2)

$= (1.11/\sqrt{50/27000}) = 25.79$

$\lambda_2 = (1.28/\sqrt{F_y/E_o})$ (Eq. 3.3.1.1-3)

$= (1.28/\sqrt{50/27000}) = 29.74$

$w/t = 8.692/0.06 = 144.9$

For $w/t > \lambda_2$, $C_y = 1$

Maximum compressive strain $= C_ye_y = e_y$

Therefore, the nominal moment, M_n is the same as the M_n determined by procedure I because the compression flange will yield first.

3. Design Flexural Strength, ϕ_bM_n (Section 3.3.1)

$\phi_b = 0.90$ (for section with stiffened compression flanges)

$\phi_bM_n = 0.90\text{x}51.0 = 45.9$ kips-in.

The factored load combination is as follows:

$w_u \quad = 1.2w_{DL} + 1.6w_{LL} = 1.2(0.02)+1.6(0.25) = 0.424$ kips/ft.

Maximum required flexural strength for a simply supported beam is

$M_u \quad = w_u L^2/8 = 0.424(8)^2(12)/8$

$\quad = 40.70$ kips-in. $< \phi_b M_n = 45.9$ kips-in. OK

4. Strength for Shear Only (Section 3.3.2)

The required shear strength at any section shall not exceed the design shear strength $\phi_v V_n$:

$\phi_v \quad = 0.85$

$V_n \quad = 4.84E_o t^3(G_s/G_o)/h$ (Eq. 3.3.2-1)

$v_n \quad = V_n/(ht)$

$\quad = 4.84E_o(G_s/G_o)/(h/t)^2$

In the determination of the shear strength, it is necessary to select a proper value of G_s/G_o for the assumed stress from Table A12 or Figure A9 of the Standard. For the first approximation, assume a shear stress of $v=F_y/2=25$ ksi and the corresponding value of G_s/G_o is equal to 0.888. Thus,

$h/t \quad = 3.692/0.060 = 61.53$

$v_n \quad = 4.84(27000)(0.888)/(61.53)^2$

$\quad = 30.7$ ksi > assumed stress $v=25$ ksi NG

For a second approximation, assume a stress of $f=28.82$ ksi and its corresponding value of G_s/G_o is 0.836.

$v_n \quad = 4.84(27000)(0.836)/(61.53)^2$

$\quad = 28.85$ ksi = assumed stress OK

Therefore, the total shear strength, V_n, for hat section is

$V_n \quad = (2 \text{ webs})(v_n)(ht)$

$= 2(28.85)(3.692 \times 0.060)$

$= 12.78$ kips

The design shear strength is determined as follows:

$\phi_v V_n = 0.85(12.78) = 10.85$ kips

$\phi_v V_n < 2(0.95 F_{yv} ht) = 2(0.95 \times 42 \times 3.692 \times 0.06) = 17.68$ kips OK

(The shear yield strength, F_{yv}, is obtained from Table A1 of the Standard.)

Maximum Required Shear Strength = Reaction

$V_u = w_u L/2 = 0.424(8)/2 = 1.70 \text{ k} < \phi_v V_n = 10.85 \text{ k}$ OK

5. Web Crippling Strength for End Reaction (Section 3.3.4)

$R/t = (3/32)/0.06 = 1.563 < 6$ OK

$h/t = 3.692/0.06 = 61.53 < 200$

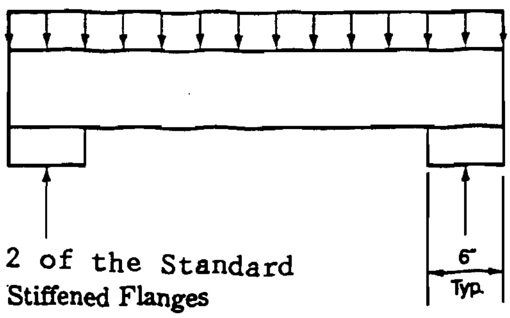


Table 2 of the Standard
Stiffened Flanges

$P_n = t^2 C_3 C_4 C_\theta [331-0.61(h/t)][1+0.01(N/t)]$ (Eq. 3.3.4-1)

$C_3 = (1.33-0.33k)k$ (Eq. 3.3.4-12)

$k = F_y/33 = 50/33 = 1.515$ (Eq. 3.3.4-21)

$C_3 = [1.33-0.33(1.515)](1.515) = 1.257$

$C_4 = (1.15-0.15R/t) \leq 1.0$ but not less than 0.50 (Eq. 3.3.4-13)

$(1.15-0.15R/t) = [1.15-0.15(1.563)] = 0.916 \leq 1.0$ OK

> 0.50 OK

$C_4 \quad = 0.916$

$C_\theta \quad = 0.7+0.3(\theta/90)^2$ (Eq. 3.3.4-20)

$= 0.7+0.3(90/90)^2 = 1.0$

$P_n \quad = (0.06)^2(1.257)(0.916)(1.0)[331-0.61(61.53)]$

$\times [1+0.01(6/0.06)] = 2.43$ k/web

$P_n \quad = (2 \text{ webs})(2.43 \text{ k/web}) = 4.86$ k

$\phi_w \quad = 0.70$

$\phi_w P_n = 0.70(4.86) = 3.40$ k

Reaction = 1.70 k < $\phi_w P_n$ = 3.40 k OK

6. Deflection Determination at Service Moment M_s

Find I_{eff} at $M_s = wL^2/8 = 0.27(8)^2(12)/8 = 25.92$ kips-in.

Computation of I_{eff}, first approximation

* Assume a stress of $f = 0.6F_y = 30$ ksi in the top and bottom fibers of the section.

* Also assume web is fully effective.

Element 5:

For deflection determination, the value of E_r, reduced modulus of elasticity determined by using Eq. (2.2.1-7), is substituted for E_o in Eq. (2.2.1-4). For a compression and tension stresses of f = 30 ksi, the corresponding E_{sc} and E_{st} values for Type 316 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:

$E_{sc} = 22650$ ksi, $\quad E_{st} = 26900$ ksi

$E_r = (E_{sc}+E_{st})/2$ (Eq. 2.2.1-7)

$= (22650+26900)/2 = 24775$ ksi

Thus, for compression flange (Element 5):

$\lambda = (1.052/\sqrt{4})(144.9)\sqrt{30/24775} = 2.652 > 0.673$ (Eq. 2.2.1-4)

$\rho = [1-(0.22/2.652)]/2.652 = 0.346$ (Eq. 2.2.1-3)

$b_d = \rho w$ (Eq. 2.2.1-6)

$= 0.346(8.692) = 3.007$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
1	2 x 0.596 = 1.192	3.548	4.229	15.005	0.035
2	4 x 0.195 = 0.780	3.925	3.062	12.016	--
3	2 x 2.692 = 5.384	3.970	21.375	84.857	--
4	2 x 3.692 = 7.384	2.000	14.768	29.536	8.388
5	3.007	0.030	0.090	0.003	--
6	2 x 0.195 = 0.390	0.075	0.029	0.002	--
Sum	18.137		43.553	141.419	8.423

The distance from the top fiber to the neutral axis is

$y_{cg} = Ly/L = 43.553/18.137 = 2.401$ in.

$I'_{eff} = Ly^2 + I'_1 - Ly^2_{cg}$

$= 141.419 + 8.423 - 18.137(2.401)^2$

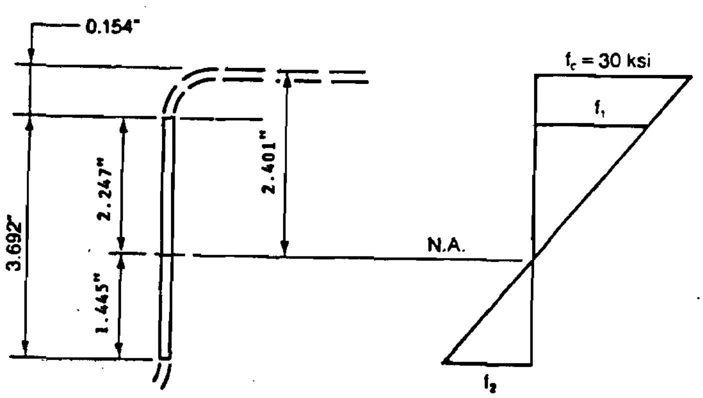
$= 45.29$ in.^{3}

Actual $I_{eff} = tI'_{eff}$

$= (0.060)(45.29) = 2.72$ in.^{4}

Check Web

* Should be fully effective



f_1 = (2.247/2.401)(30) = 28.08 ksi(compression)

f_2 = -(1.445/2.401)(30) = -18.05 ksi(tension)

$\Psi = f_2/f_1 = -18.05/28.08 = -0.643$

$k = 4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

$= 4+2[1-(-0.643)]^3+2[1-(-0.643)]$

$= 16.16$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}$, $f = f_1$ (Eq. 2.2.1-4)

For a compression stress of f_1 = 28.08 ksi and a tension stress of f_2 = 18.05 ksi, the values of E_{sc} and E_{st} are found as follows: E_{sc} = 24000 ksi, E_{st} = 27000 ksi.

$E_r = (24000+27000)/2$ (Eq. 2.2.1-7)

$= 25500$ ksi

$\lambda = (1.052/\sqrt{16.16})(61.53)\sqrt{28.08/25500} = 0.534 < 0.673$

$b = w$ (Eq. 2.2.1-1)

b_e = 3.692 in.

$b_2 = b_e/2$ (Eq. 2.2.2-2)

= 3.692/2 = 1.846 in.

$b_1 \quad = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 3.692/[3-(-0.643)] = 1.007$ in.

$b_1+b_2 = 1.013 + 1.846 = 2.859$ in. > 2.216 in. (compression portion of web, see the sketch shown above)

Therefore, web is fully effective.

$S_{eff} \quad = I_{eff}/y_{cg} = 2.72/2.401 = 1.13 \text{ in.}^3$

$M \quad = S_{eff}(0.6F_y)$

$= (1.13)(30)$

$= 33.9$ kips-in.

To determine I_{eff} at $M_s = 25.92$ kips-in., an approximation is used by extrapolating the following values:

(1) $M = 51.00$ kips-in., $I = 2.53 \text{ in.}^4$

(2) $M = 33.90$ kips-in., $I = 2.72 \text{ in.}^4$

(3) $M = 25.92$ kips-in., $I = ?$

$$(25.92-33.9)/(I-2.72) = (33.9-51.0)/(2.72-2.52)$$

$$-7.98 = -90.00(I-2.72)$$

$$0.0887 = I-2.72$$

$$I = 2.81 \text{ in.}^4$$

Use $I = 2.81 \text{ in.}^4$ in deflection calculations.

Deflection $= 5wL^4/384E_oI$

EXAMPLE 6.2 HAT SECTION (ASD)

Rework Example 6.1 by using the Allowable Stress Design (ASD) method.

Solution:

1. Calculation of the allowable moment, M_a:

   The effective section properties calculated by the ASD method are the same as those determined in Example 6.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   Ω = 1.85 (Safety Factor stipulated in Table E of the Standard)

   M_n = 51.0 kips-in. (obtained from Example 6.1)

   $M_a = M_n/\Omega$ (Eq. E-1)

   $= 51.0/1.85$

   $= 27.57$ kips-in.

   The maximum applied moment, $M_{max} = wL^2/8$

   $M_{max} = (0.25+0.02)(8)^2(12")/8 = 25.92$ kips-in. < 27.57 kips-in. OK

2. Strength for Shear Only.

   The nominal shear strength at the section was calculated in Example 6.1.(4) as follows:

   V_n = (2 webs)(v_n)(ht)

   $= 2(28.85)(3.692\text{x}0.060)$

   $= 12.78$ kips

   The allowable shear strength is determined as follows:

   $V_a = V_n/\Omega = 12.78/1.85$

   $V_a = 6.91$ kips < $2\text{x}(0.95F_{yv}\text{ht})/1.64 = 11.35$ kips, Use $V_a = 6.91$ kips

Maximum Shear Force = Reaction

$V_u = wL/2 = 0.27(8)/2 = 1.08 \text{ k} < V_a = 6.91$ kips OK

3. Web Crippling Strength.

The nominal web crippling strength was determined in Example 6.1 as follows:

$P_n = (0.06)^2(1.257)(0.916)(1.0)\ 331-0.61(61.53)$

$\quad \times\ 1+0.01(6/0.06) = 2.43$ k/web

$P_n = (2 \text{ webs})(2.43 \text{ k/web}) = 4.86$ k

$\Omega = 2.0$ (for single web)

$P_a = P_n/\Omega$

$\quad = 4.86/2.0 = 2.43$ kips

Reaction = 1.08 kips < $P_a = 2.43$ kips OK

4. Deflection Determination at Allowable Moment M_a

For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 8.1 for the LRFD method, except that the computed momemt M (= fxS_e) should be equal to M_a.

From the results of Example 6.1, it can be seen that to determine the moment of inertia I_{eff} at $M_a = 25.57$ kips-in., an approximation can be used by extrapolating the following values:

(1) M = 51.00 kips-in., I = 2.53 in.^{4}

(2) M = 33.90 kips-in., I = 2.72 in.^{4}

(3) $M = 25.57$ kips-in., $I = ?$

$$(25.57-33.9)/(I-2.72) = (33.9-51.0)/(2.72-2.53)$$

$$I = 2.81 \text{ in.}^4$$

Use $I = 2.81$ in.^{4} in deflection calculations.

(Deflection = $5wL^4/384E_oI$)

EXAMPLE 7.1 HAT SECTION w/INTERMEDIATE STIFFENER (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi_b M_n$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Use Type 316 stainless steel, 1/4-Hard. Compare structural economy of this section with an almost identical section without an intermediate stiffener computed in Example 6.1.

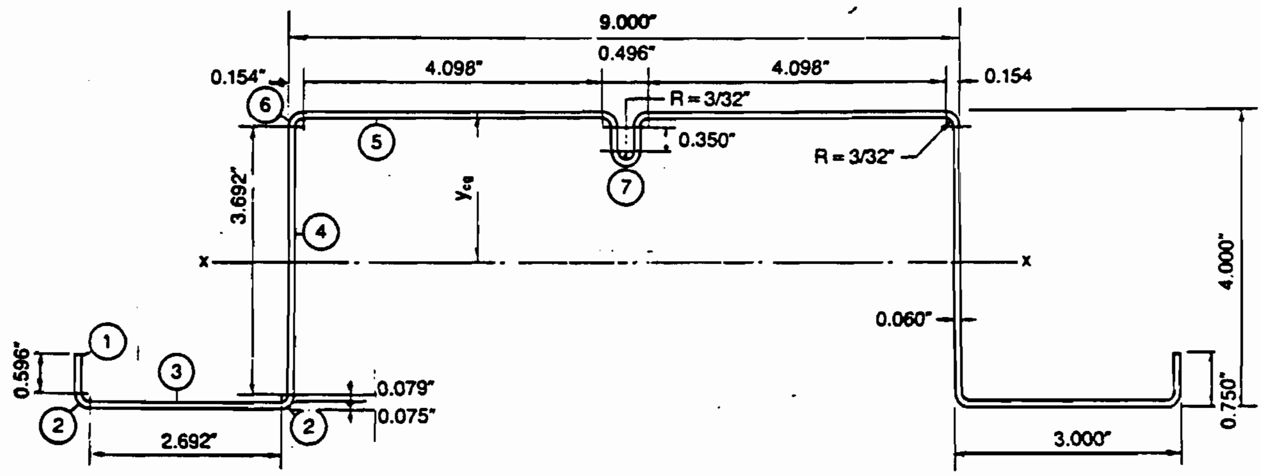


Figure 7.1 Section for Example 7.1

Given:

1. Section: Hat section, as shown in sketch.
2. Dead load to live load ratio D/L = 1/5 and 1.2D+1.6L governs the design.

Solution:

1. Properties of 90° corners:

   Corner Radius, r = R + t/2 = 3/32 + 0.060/2 = 0.124 in.

   Length of arc, u = 1.57r = 1.57 x 0.124 = 0.195 in.

Distance of c.g. from center of radius,

$c = 0.637r = 0.637 \times 0.124 = 0.079$ in.

The moment of inertia, I', of corner about its own centroidal axis is negligible.

2. Nominal Section Strength, M_n (Section 3.3.1.1)

Computation of I_x, S_e, and M_n for the first approximation:

* Assume a compressive stress of $f = F_y = 50$ ksi in the top fiber of the section. (See Table A1 of the Standard for yield strength values.)
* Also assume web is fully effective.

Element 4:

$h/t = 3.692/0.060 = 61.53 < 200$ OK (Section 2.1.2-(1))

Assumed fully effective

Element 5:

$E_o = 27000$ ksi (Table A4 of the Standard)

$$S = 1.28\sqrt{E_o/f} \quad \text{(Eq. 2.4-1)}$$

$$= 1.28\sqrt{27000/50} = 29.74$$

$b_o/t = 8.692/0.060 = 144.9 < 400$ OK (Section 2.1.1-(1)-(ii))

$3S = 3(29.74) = 89.22$

For $b_o/t > 3S$ (Case III)

$$I_a = t^4\{[128(b_o/t)/S]-285\} \quad \text{(Eq. 2.4.1-9)}$$

$$= (0.06)^4\{[128(144.9)/29.74]-285\} = 0.004038 \text{ in.}^4$$

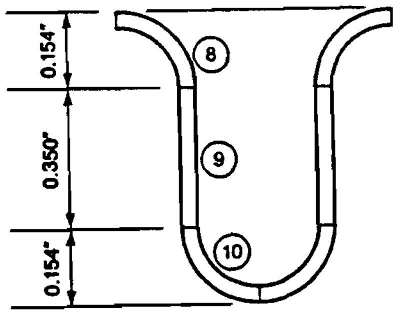
Determine full section properties of stiffener 7:

All inner radii = 3/32

$r = R + t/2 = 3/32+0.060/2 = 0.124$ in.

$u = 1.57r = 1.57(0.124) = 0.195$ in.

$c = 0.637r = 0.637(0.124) = 0.079$ in.



Element	L Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
8	2 x 0.195 = 0.390	0.075	0.0293	0.0022	--
9	2 x 0.350 = 0.700	0.329	0.2303	0.0758	0.0071
10	2 x 0.195 = 0.390	0.583	0.2274	0.1326	--
Sum	1.480		0.4870	0.2106	0.0071

Distance from top fiber to the neutral axis is

$y_{cg} = Ly/L = 0.4870/1.480 = 0.329$ in.

Total area of section, $Lt = (1.480)(0.060) = 0.0888$ in.^{2}

$$I'_s = Ly^2 + I'_1 - Ly^2_{cg}$$

$$= 0.2106 + 0.0071 - 1.480(0.329)^2$$

$$= 0.0575 \text{ in.}^3$$

Actual $I_s = tI'_s$

$$= (0.060)(0.0575) = 0.00345 \text{ in.}^4$$

Reduced Area of Stiffener

Element 9:

Stiffened element, $k = 4$

$f = F_y = 50$ ksi

$w/t = 0.350/0.060 = 5.83 < 400$ OK (Section 2.1.1-(1)-(ii))

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{4})(5.83)\sqrt{50/27000} = 0.132 < 0.673$

$b = w$ (Eq. 2.2.1-1)

$= 0.350$ in. (fully effective)

$A'_s = Lt = 0.0888$ in.^{2}

$A_s = A'_s(I_s/I_a) \leq A'_s$ (Eq. 2.4.1-11)

$= 0.0888(0.00345/0.00439)$

$= 0.0888(0.7859)$

$= 0.0698$ in.^{2} $< A'_s$ OK

$L_s = (A_s/t) = (0.0698/0.060) = 1.163$ in.

Continuing with element 5:

$k = 3(I_s/I_a)^{1/3}+1 \leq 4$ (Eq. 2.4.1-10)

$= 3(0.7859)^{1/3}+1 = 3.768 < 4$ OK

$w/t = 4.098/0.060 = 68.30$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{3.768})(68.30)\sqrt{50/27000} = 1.593 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/1.593)/1.593 = 0.541$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.541(4.098) = 2.217$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly ($in.^2$)	Ly^2 ($in.^3$)	I'_1 About Own Axis ($in.^3$)
1	2 x 0.596 = 1.192	3.548	4.229	15.005	0.035
2	4 x 0.195 = 0.780	3.925	3.062	12.016	--
3	2 x 2.692 = 5.384	3.970	21.375	84.857	--
4	2 x 3.692 = 7.384	2.000	14.768	29.536	8.388
5	2 x 2.217 = 4.434	0.030	0.133	0.004	--
6	2 x 0.195 = 0.390	0.075	0.029	0.002	--
7	Stiffener 1.163	0.329	0.383	0.126	0.058
Sum	20.727		43.979	141.546	8.481

The distance from the top fiber to the neutral axis is

y_{cg} = Ly/L = 44.018/21.035 = 2.093 in.

Since the distance from the top compression fiber to the neutral axis is greater than one half the beam depth, a compressive stress of F_y will govern as assumed.

$$I'_x = Ly^2 + I'_1 - Ly^2_{cg}$$

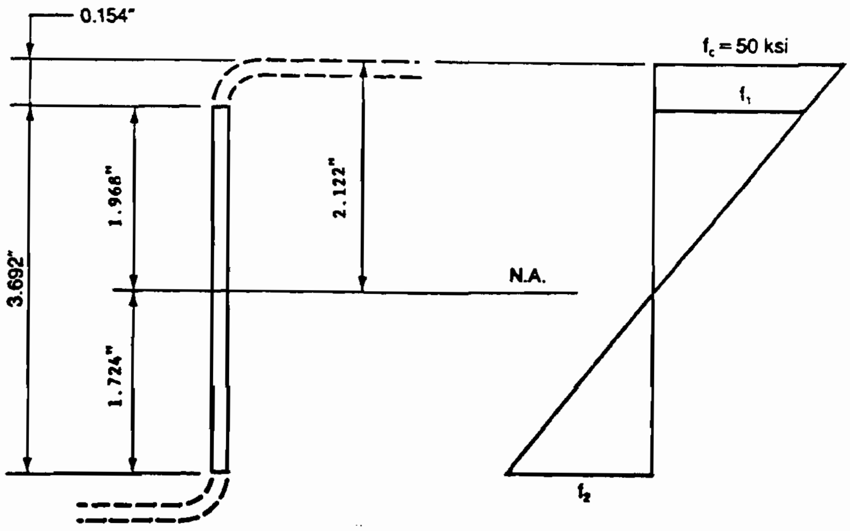
$$= 141.546 + 8.481 - 20.727(2.122)^2$$

$$= 56.70 \text{ in.}^3$$

$$\text{Actual } I_x = tI'_x$$

$$= (0.060)(56.70) = 3.40 \text{ in.}^4$$

Check Web



$f_1 = (1.968/2.122)(50) = 46.37$ ksi(compression)

$f_2 = -(1.724/2.122)(50) = -40.62$ ksi(tension)

$\Psi = f_2/f_1 = -40.62/46.37 = -0.876$

$k = 4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

$= 4+2[1-(-0.876)]^3+2[1-(-0.876)]$

$= 20.96$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$, $f = f_1$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{20.96})(61.53)\sqrt{46.37/27000} = 0.586 < 0.673$

$b = w$ (Eq. 2.2.1-1)

$b_e = 3.692$ in.

$b_2 = b_e/2$ (Eq. 2.2.2-2)

$= 3.692/2 = 1.846$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 3.692/[3-(-0.876)] = 0.953$ in.

$b_1+b_2 = 0.953 + 1.846 = 2.799$ in. $>$ 1.939 in. (compression portion of web, see sketch shown above)

Therefore, web is fully effective.

$S_e = I_x/y_{cg}$

$= 3.40/2.122$

$= 1.60 \text{ in.}^3$

$M_n = S_eF_y$ (Eq. 3.3.1.1-1)

$= (1.60)(50)$

$= 80.0$ kips-in.

3. Design Flexural Strength, Φ_bM_n (Section 3.3.1)

$\Phi_b = 0.90$ (for section with stiffened compression flanges)

$\Phi_bM_n = 0.90 \text{x} 80.0 = 72.00$ kips-in.

4. Deflection Determination at Servive Moment M_s

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

$$\begin{aligned}
\Phi_bM_n &= 1.2M_{DL} + 1.6M_{LL} \\
&= [1.2(M_{DL}/M_{LL})+1.6]M_{LL} \\
&= [1.2(1/5)+1.6]M_{LL} \\
&= 1.84M_{LL}
\end{aligned}$$

$M_{LL} = \Phi_bM_n/1.84 = 72.00/1.84 = 39.13$ kips-in.

$$\begin{aligned}
M_s &= M_{DL} + M_{LL} \\
&= (1/5+1)M_{LL} \\
&= 1.2(39.13) = 46.96 \text{ kips-in.}
\end{aligned}$$

where

M_{DL} = Moment determined on the basis of nominal dead load

M_{LL} = Moment determined on the basis of nominal live load

Find I_{eff} at M_s = 46.96 kips-in.

Computation of I_{eff}, first approximation

* Assume a stress of $f = 0.6F_y$ = 30 ksi in the top and bottom fibers of the section.
* Web is fully effective, because it was fully effective at a higher stress gradient.
* Element 9 of the stiffener, which was fully effective at f = 50 ksi will also be fully effective at f = 30 ksi.

Element 5:

$S = 1.28\sqrt{E_o/f}$, $f = 30$ (Eq. 2.4-1)

$= 1.28\sqrt{27000/30} = 38.40$

$b_o/t = 144.9$

$3S = 3(38.40) = 115.20$

For $b_o/t > 3S$ (Case III)

$I_a = t^4\{[128(b_o/t)/S]-285\}$ (Eq. 2.4.1-9)

$= (0.06)^4\{[128(144.9)/38.40]-285\} = 0.002566$ in.^{4}

$I_s = 0.00345$ in.^{4}

$k = 3(I_s/I_a)^{1/3}+1 \leq 4$ (Eq. 2.4.1-10)

$= 3(0.00345/0.002566)^{1/3}+1 = 4.311 > 4$

$k = 4$

$w/t = 68.30$

For deflection determination, the value of E_r, reduced modulus of elasticity determined by using Eq. (2.2.1-7), is substituted for E_o in Eq. (2.2.1-4). For a compression and tension stresses of f = 30 ksi, the corresponding E_{sc} and E_{st} values for

Type 316 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:

$E_{sc} = 22650$ ksi, $\quad E_{st} = 26900$ ksi

$$E_r = (E_{sc}+E_{st})/2 \qquad \text{(Eq. 2.2.1-7)}$$

$$= (22650+26900)/2 = 24775 \text{ ksi}$$

Thus, for compression flange (Element 5):

$$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}, \quad f = 30 \text{ ksi} \qquad \text{(Eq. 2.2.1-4)}$$

$$= (1.052/\sqrt{4})(68.30)\sqrt{30/24775} = 1.250 > 0.673$$

$$\rho = (1-0.22/\lambda)/\lambda \qquad \text{(Eq. 2.2.1-3)}$$

$$= (1-0.22/1.250)/1.250 = 0.659$$

$$b = \rho w \qquad \text{(Eq. 2.2.1-2)}$$

$$= 0.659(4.098) = 2.701 \text{ in.}$$

Stiffener, Element 7:

$$A_s = A'_s(I_s/I_a) \leq A'_s \qquad \text{(Eq. 2.4.1-11)}$$

$$= 0.0888(0.00345/0.002566)$$

$$= 0.133 \text{ in.}^2 > A'_s$$

$$A_s = A'_s = 0.0888 \text{ in.}^2$$

$$L_s = A_s/t = 0.0888/0.060 = 1.480 \text{ in.}$$

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly ($in.^2$)	Ly^2 ($in.^3$)	I'_1 About Own Axis ($in.^3$)
1	2 x 0.596 = 1.192	3.548	4.229	15.005	0.035
2	4 x 0.195 = 0.780	3.925	3.062	12.016	--
3	2 x 2.692 = 5.384	3.970	21.375	84.857	--
4	2 x 3.692 = 7.384	2.000	14.768	29.536	8.388
5	2 x 2.701 = 5.402	0.030	0.162	0.005	--
6	2 x 0.195 = 0.390	0.075	0.029	0.002	--
7	Stiffener 1.480	0.329	0.487	0.160	0.058
Sum	22.012		44.112	141.581	8.481

Distance from top fiber to the neutral axis is

$y_{cg} = Ly/L = 44.112/22.012 = 2.004$ in.

$I'_{eff} = Ly^2 + I'_1 - Ly^2_{cg}$

$= 141.581 + 8.481 - 22.012(2.004)^2$

$= 61.66 \text{ in.}^3$

Actual $I_{eff} = tI'_{eff}$

$= (0.060)(61.66) = 3.70 \text{ in.}^4$

$S_{eff} = I_{eff}/y_{cg} = 3.70/2.004 = 1.85 \text{ in.}^3$

$M = S_{eff}(0.6F_y)$

$= (1.85)(30)$

$= 55.5 \text{ kips-in.} > M_s = 46.96$ kips-in. NG

Computation of I_{eff}: second approximation by extrapolating the following data to obtain the stress value

(1) $M = 80.00$ kips-in., $f = F_y = 50$ ksi

(2) $M = 55.50$ kips-in., $f = 0.6F_y = 30$ ksi

(3) $M = 46.96$ kips-in., $f = ?$

$$(f-30)/(30-50) = (46.96-55.5)/(55.5-80.0)$$

$$f = 23.03 \text{ ksi}$$

* Compressive stress of f = 23.03 ksi in the top fiber of section

* Web is fully effective

* Element 9 of stiffener is fully effective

Element 5:

$S = 1.28\sqrt{E_o/f}$, $f = 23.03$ ksi (Eq. 2.4-1)

$= 1.28\sqrt{27000/23.03} = 43.83$

$b_o/t = 144.9$

$3S = 3(43.83) = 131.49$

For $b_o/t > 3S$ (Case III)

$I_a = t^4\{[128(b_o/t)/S]-285\}$ (Eq. 2.4.1-9)

$= (0.06)^4\{[128(144.9)/43.83]-285\} = 0.00179$ in.^{4}

$I_s = 0.00345$ in.^{4}

$k = 3(I_s/I_a)^{1/3}+1 \leq 4$ (Eq. 2.4.1-10)

Since $I_s/I_a > 1$, $k = 4$

$w/t = 68.30$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}$, $f = 23.03$ ksi (Eq. 2.2.1-4)

For a compression and tension stresses of f= 23.03 ksi, it is founf that the values of E_{sc} and E_{st} are equal to 26390 ksi and 27000 ksi, respectively.

$E_r = (26390+27000)/2$ (Eq. 2.2.1-7)

$= 26695$ ksi

$\lambda = (1.052/\sqrt{4})(68.30)\sqrt{23.03/26695} = 1.055 > 0.673$

$\rho \quad = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$\quad = (1-0.22/1.055)/1.055 = 0.750$

$b \quad = \rho w$ (Eq. 2.2.1-2)

$\quad = 0.750(4.098) = 3.074$ in.

Stiffener, Element 7:

$A_s \quad = A'_s(I_s/I_a) \leq A'_s$ (Eq. 2.4.1-11)

Since $I_s/I_a > 1$

$A_s \quad = A'_s = 0.0888 \text{ in.}^2$

$L_s \quad = A_s/t = 0.0888/0.060 = 1.480$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
1	2 x 0.596 = 1.192	3.548	4.229	15.005	0.035
2	4 x 0.195 = 0.780	3.925	3.062	12.016	--
3	2 x 2.692 = 5.384	3.970	21.375	84.857	--
4	2 x 3.692 = 7.384	2.000	14.768	29.536	8.388
5	2 x 3.074 = 6.148	0.030	0.184	0.006	--
6	2 x 0.195 = 0.390	0.075	0.029	0.002	--
7	Stiffener 1.480	0.329	0.487	0.160	0.058
Sum	22.758		44.134	141.582	8.481

Distance from top fiber to the neutral axis is

$y_{cg} \quad = Ly/L = 44.134/22.758 = 1.939$ in.

$I'_{eff} \quad = Ly^2 + I'_1 - Ly^2_{cg}$

$\quad = 141.582 + 8.481 - 22.758(1.939)^2$

$= 64.50 \text{ in.}^3$

Actual $I_{eff} = tI'_{eff}$

$= (0.060)(64.50) = 3.87 \text{ in.}^4$

$S_{eff} = I_{eff}/y_{cg} = 3.87/1.939 = 2.00 \text{ in.}^3$

$M = (2.00)(23.03) = 46.06$ kips-in. close to M_s OK

Use $I = 3.87 \text{ in.}^4$ in deflection calculations

5. Comparison of sections with and without intermediate stiffeners.

Hat Section	Total Area (in.^2)	Design Flexural Strength (kips-in.)
Without Stiffener	1.43	45.90
With Stiffener	1.49	72.00

Increase in weight = (1.49-1.43)/1.43 x100% = 4.2%

Increase in moment capacity = (72.00-45.90)/45.90 x100% = 56.9%

EXAMPLE 7.2 HAT SECTION W/INTERMEDIATE STIFFENER (ASD)

Rework Example 7.1 by using the Allowable Stress Design (ASD) method.

Solution:

1. Calculation of the allowable moment, M_a:

The effective section properties calculated by the ASD method are the same as those determined in Example 7.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

Ω = 1.85 (Safety Factor stipulated in Table E of the Standard)

M_n = 80.0 kips-in. (obtained from Example 7.1)

$M_a = M_n/\Omega$ (Eq. E-1)

$= 80.0/1.85$

$= 43.24$ kips-in.

2. Deflection Determination at Allowable Moment M_a

For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 7.1 for the LRFD method, except that the computed moment M (= fxS_e) should be equal to M_a.

Computation of I_{eff}: assume that

* A stress of f = 20.50 ksi in the top and bottom fibers of section
* Web is fully effective
* Element 9 of stiffener is fully effective

Element 5:

$S \quad = 1.28\sqrt{E_o/f}, \quad f = 20.50$ ksi (Eq. 2.4-1)

$\quad = 1.28\sqrt{27000/20.50} = 46.45$

$b_o/t = 144.9$

$3S \quad = 3(46.45) = 139.35$

For $b_o/t > 3S$ (Case III)

$I_a \quad = t^4\{[128(b_o/t)/S]-285\}$ (Eq. 2.4.1-9)

$\quad = (0.06)^4\{[128(144.9)/46.45]-285\} = 0.00148$ in.^{4}

$I_s \quad = 0.00345$ in.^{4}

$k \quad = 3(I_s/I_a)^{1/3}+1 \leq 4$ (Eq. 2.4.1-10)

Since $I_s/I_a > 1$, $k = 4$

$w/t \quad = 68.30$

$\lambda \quad = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}$, $f = 20.50$ ksi (Eq. 2.2.1-4)

For a compression and tension stresses of f= 20.50 ksi, it is found that the values of E_{sc} and E_{st} are equal to 26900 ksi and 27000 ksi, respectively.

$E_r \quad = (26900+27000)/2$ (Eq. 2.2.1-7)

$\quad = 26950$ ksi

$\lambda \quad = (1.052/\sqrt{4})(68.30)\sqrt{20.50/26950} = 0.991 > 0.673$

$\rho \quad = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$\quad = (1-0.22/0.991)/0.991 = 0.785$

$b \quad = \rho w$ (Eq. 2.2.1-2)

$\quad = 0.785(4.098) = 3.217$ in.

Stiffener, Element 7:

$A_s \quad = A'_s(I_s/I_a) \leq A'_s$ (Eq. 2.4.1-11)

Since $I_s/I_a > 1$

$A_s = A'_s = 0.0888 \text{ in.}^2$

$L_s = A_s/t = 0.0888/0.060 = 1.480 \text{ in.}$

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
1	2 x 0.596 = 1.192	3.548	4.229	15.005	0.035
2	4 x 0.195 = 0.780	3.925	3.062	12.016	--
3	2 x 2.692 = 5.384	3.970	21.375	84.857	--
4	2 x 3.692 = 7.384	2.000	14.768	29.536	8.388
5	2 x 3.217 = 6.434	0.030	0.193	0.006	--
6	2 x 0.195 = 0.390	0.075	0.029	0.002	--
7	Stiffener 1.480	0.329	0.487	0.160	0.058
Sum	23.044		44.143	141.582	8.481

Distance from top fiber to the neutral axis is

$y_{cg} = Ly/L = 44.143/23.044 = 1.916 \text{ in.}$

$I'_{eff} = Ly^2 + I'_1 - Ly^2_{cg}$

$= 141.582 + 8.481 - 23.044(1.916)^2$

$= 65.47 \text{ in.}^3$

Actual $I_{eff} = tI'_{eff}$

$= (0.060)(65.47) = 3.93 \text{ in.}^4$

$S_{eff} = I_{eff}/y_{cg} = 3.93/1.916 = 2.05 \text{ in.}^3$

$M = (2.05)(20.50) = 42.03$ kips-in. close to M_a OK

Use I = 3.93 in.^{4} in deflection calculations

EXAMPLE 8.1 I-SECTION W/UNSTIFFENED FLANGES (LRFD)

By using the Load and Resistance Factor Design (LRFD) criteria, determine the design flexural strength of an I-section (Fig. 8.1) used as a simply supported beam. Assume that the span length is 8 ft. with laterally braced at both ends and midspan and that the beam carries uniform load. Use Type 301, 1/4-Hard, stainless steel.

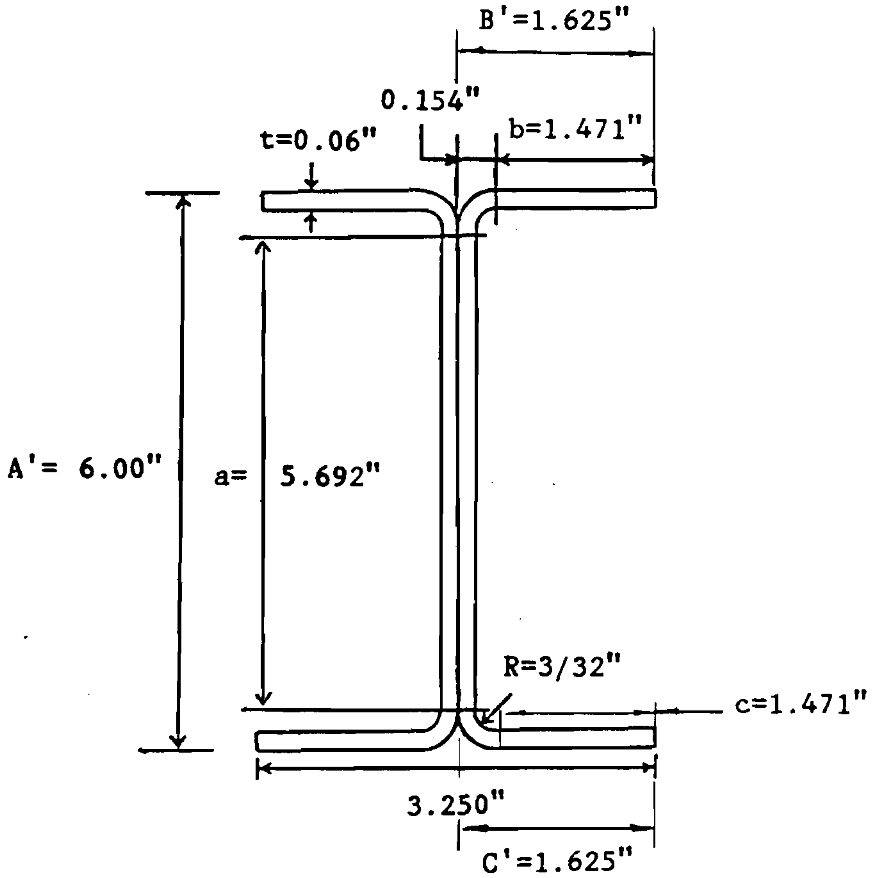


Figure 8.1 Section for Example 8.1

Solution:

1. Nominal section strength (Section 3.3.1.1).

   a. Procedure I - based on initiation of yielding

      For this I-section, the elastic section modulus of the effective section, S_e, based on initiation of yielding can be obtained from Example 1.1 for a channel section. Therefore,

      $S_e = 2x(0.711) = 1.422 \text{ in}^3$

$M_n \quad = S_e F_y$ (Eq. 3.3.1.1-1)

$= 1.422 \times 50 = 71.10$ kips-in.

b. Procedure II - based on inelastic reserve capacity

Since the member is subjected to lateral bcukling, therefore this provision does not apply in this example. Then,

$(M_n)_1 \quad = 71.10$ kips-in.

$\phi_b \quad = 0.85$

$\phi_b(M_n)_1 = 0.85 \times 71.10 = 60.44$ kips-in.

2. Lateral buckling strength (Section 3.3.1.2).

The following equations used for computing the sectional properties for I-section without lips are adopted from the Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

Basic parameters used for calculating the section properties of an I-section without lips:

$r \quad = R+t/2 = 3/32+0.060/2 = 0.124$ in.

From the sketch a = 5.692 in., b = 1.471 in., c = 1.471 in.,

A' = 6.0 in., B' = 1.625 in., C' = 1.625 in.,

$\alpha = 1.00$ (For I-section)

$\bar{a} \quad = A'-(t/2+\alpha t/2) = 6.0-(0.060/2+0.060/2) = 5.94$ in.

$\bar{b} \quad = B'- t/2 = 1.625-0.06/2 = 1.595$ in.

$\bar{c} \quad = \alpha(C'-t/2) = 1.625-0.06/2 = 1.595$ in.

$u \quad = 1.57r = 1.57 \times 0.124 = 0.195$ in.

$x \quad = a/2 = 2.97$ in.

a. Area:

$$A = t[2a+2b+2u+\alpha(2c+2u)] = t[2a+2b+2c+4u]$$

$$= 0.06(2x5.692+2x1.471+2x1.471+4x0.195)$$

$$= 1.083 \text{ in.}^2$$

b. Moment of inertia about x-axis:

$$I_x = 2t\{a(a/2+r)^2+0.0833a^3+0.358r^3+\alpha[c(a+2r)^2$$

$$+u(a+1.637r)^2+0.149r^3]\}-A(x)^2$$

$$= 2x0.06[5.692(5.692/2+0.124)^2+0.0833(5.692)^3$$

$$+0.358(0.124)^3+1.471(5.692+2x0.124)^2$$

$$+0.195(5.692+1.637x0.124)^2+0.149(0.124)^3]-1.083(2.97)^2$$

$$= 5.357 \text{ in.}^4$$

c. Moment of inertia about y-axis:

$$I_y = 2t\{b(b/2+r+t/2)^2+0.0833b^3+u(0.363r+t/2)^2+0.149r^3$$

$$+\alpha[c(c/2+r+t/2)^2+0.0833b^3+u(0.363r+t/2)^2+0.149r^3]\}$$

$$= 2t[b(b/2+r+t/2)^2+c(c/2+r+t/2)^2+2x0.0833b^3+2u(0.363r+t/2)^2$$

$$+2x0.149r^3]$$

$$= 2x0.06[1.471(1.531/2+0.124)^2+1.471(1.531/2+0.124)^2$$

$$+2x0.0833x(1.471)^3+2x0.195(0.363x0.124+0.06/2)^2$$

$$+2x0.149x(0.124)^3]$$

$$= 0.343 \text{ in.}^4$$

Therefore,

$$S_f = I_x/y_{cg} = 5.357/3.0 = 1.786 \text{ in}^3$$

$$C_b = 1.75+1.05(M_1/M_2)+0.3(M_1/M_2)^2$$

$$= 1.75+1.05(0/M_{max})+0.3(0/M_{max})^2$$

$= 1.75 < 2.3$

$I_{yc} = I_y/2 = 0.343/2 = 0.172 \text{ in.}^4$

$M_c = \pi^2 E_o C_b (E_t/E_o) d I_{yc}/L^2$ (Eq. 3.3.1.2-2)

$M_n = S_c(M_c/S_f)$ (Eq. 3.3.1.2-1)

$= S_c f$

$f = M_c/S_f$

$= (1/1.786)(\pi^2 x27000x1.75x6x0.172/(4x12)^2)(E_t/E_o)$

$= 116.95(E_t/E_o)$

In the determination of the lateral buckling stress, it is necessary to select a proper ratio of E_t/E_o from Table A10 or Figure A7 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of f=32 ksi. From Table A10, the corresponding value of E_t/E_o is found to be equal to 0.42. Thus,

$f_1 = 116.95x0.42$

$= 49.12$ ksi > assumed stress f=32 ksi

Because the computed stress is larger than the assumed value, the further successive approximation is needed.

Assume f=38.5 ksi, and

$E_t/E_o = 0.33$

$f_1 = 116.95x0.33$

$= 38.49$ ksi = assumed stress f=38.5 ksi OK

Therefore,

$f = M_c/S_f = 38.47$ ksi

Properties of 90^o corners:

$r = R + t/2 = 3/32 + 0.060/2 = 0.124$ in.

Length of arc, u = 1.57r = 1.57 x 0.124 = 0.195 in.

Distance of c.g. from center of radius,

c = 0.637r = 0.637 x 0.124 = 0.079 in.

Determination of elastic section modulus of the effective section calculated at a stress of f = 38.47 ksi in the extreme compression fiber (assume the webs are fully effective):

Compression flange: k = 0.50 (unstiffened compression element)

w/t = 1.471/0.06 = 24.52 < 50 OK (Section 2.1.1-(1)-(iii))

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{0.50})(24.52)\sqrt{38.47/27000} = 1.377 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/1.377)/1.377 = 0.610$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.610 \times 1.471$

$= 0.897$ in.

Effective section properties about x axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Webs	11.384	3.000	34.152	102.456	30.736
Upper Corners	0.390	0.075	0.029	0.002	--
Lower Corners	0.390	5.925	2.310	13.691	--
Compression Flanges	1.794	0.030	0.054	0.002	--
Tension Flanges	2.942	5.970	17.564	104.856	--
Sum	16.900		54.109	221.007	30.736

Distance from top fiber to x-axis is

$y_{cg} = 54.109/16.90 = 3.202$ in.

Since the distance of top compression fiber from neutral axis is greater than one half the beam depth, a compression stress of $f = 38.47$ ksi will govern.

To check if webs are fully effective (Section 2.2.2):

$f_1 = [(3.202-0.154)/3.202]\text{x}38.47 = 36.62$ ksi(compression)

$f_2 = -[(2.798-0.154)/3.202]\text{x}38.47 = -31.77$ ksi(tension)

$\Psi = f_2/f_1 = -31.77/36.62 = -0.868$

$k = 4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

$= 4+2[1-(-0.868)]^3+2[1-(-0.868)]$

$= 20.772$

$h = w = 5.692$ in., $h/t = w/t = 5.692/0.06 = 94.87$

$h/t = 54.47 < 200$ OK (Section 2.1.2-(1))

$\lambda = (1.052/\sqrt{20.772})(54.47)\sqrt{36.62/27000} = 0.463 < 0.673$

$b_e = w$ (Eq. 2.2.1-1)

$= 5.692$ in.

$b_2 = b_e/2$ (Eq. 2.2.2-2)

$= 5.692/2 = 2.846$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 5.692/[3-(-0.868)] = 1.472$ in.

Compression portion of the web calculated on the basis of the effective section $= y_{cg} - 0.154 = 3.202 - 0.154 = 3.048$ in.

Since $b_1+b_2 = 4.318$ in. > 3.048 in., b_1+b_2 shall be taken as 3.048 in.. This verifies the assumption that the webs are fully effective.

$I'_x \quad = Ly^2+I'_1-Ly^2_{cg}$

$= 221.007 + 30.736 - 16.90(3.202)^2$

$= 78.471$ in.^{3}

Actual $I_x = I'_x t$

$= 78.471 \times 0.06$

$= 4.708$ in.^{4}

$S_c \quad = I_x/y_{cg}$

$= 4.708/3.202$

$= 1.470$ in.^{3}

Therefore,

$(M_n)_2 \quad = S_c f = 1.470 \times 38.47$

$= 56.55$ kips-in.

$\Phi_b \quad = 0.85$

$\Phi_b(M_n)_2 \quad = 0.85 \times 56.55$

$= 48.07$ kips-in. $< \Phi_b(M_n)_1 = 60.44$ kips-in.

Therefore, $\Phi_b M_n = 48.07$ kips-in. (i.e., lateral buckling strength controls).

EXAMPLE 8.2 I-SECTION W/UNSTIFFENED FLANGES (ASD)

By using the Allowable Stress Design (ASD) method, rework Example 8.1 to determine the allowable bending strength of the I-section (Fig. 8.1).

Solution:

1. Nominal section strength

$M_n = S_e F_y$

$= 1.422 \times 50 = 71.10$ kips-in. (see Example 8.1)

$(M_n)_1 = 71.10$ kips-in.

Allowable bending strength

$\Omega = 1.85$

$(M_a)_1 = 71.10/1.85 = 38.43$ kips-in.

2. Lateral buckling strength

$M_n = S_c(M_c/S_f)$

$= S_c f$

$f = M_c/S_f = 38.47$ ksi

$S_c = 1.470$ in^3

(For detailed calculations, see Example 8.1)

$(M_n)_2 = 1.470 \times 38.47 = 56.55$ kips-in.

Allowable lateral buckling strength

$\Omega = 1.85$

$(M_a)_2 = 56.55/1.85 = 30.57$ kips-in.

Therefore, $M_a = 30.57$ kips-in. (i.e., lateral buckling controls)

EXAMPLE 9.1 CHANNEL W/LATERAL BUCKLING CONSIDERATION (LRFD)

Complete Flexural Design,
Unstiffened Compression Flange

By using the LRFD criteria, check the adequacy of a channel section (Fig. 9.1) to be used as a flexural member and to support a nominal live load of 200 lb/ft. and a nominal dead load of 40 lb/ft. Assume that the beam is continuous over three 10 ft. spans with 6 in. and 3 1/2 in. bearing lengths at interior and exterior supports, respectively. Also assume that, for each span, the compression flange is braced at the center and a quarter point of span, and $K_x = K_y = 1.0$. Use Type 304, 1/4-Hard, stainless steel.

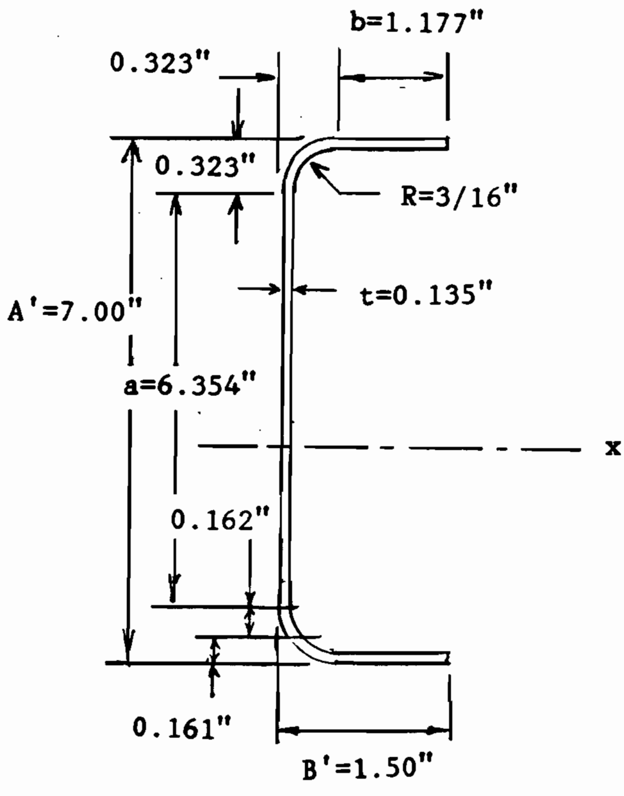


Figure 9.1 Section for Example 9.1

Solution:

1. Nominal section strength, M_n (Section 3.3.1.1).

a. Procedure I - based on initiation of yielding

Properties of 90^{o} corners:

$r = R + t/2 = 3/16 + 0.135/2 = 0.255$ in.

Length of arc, $u = 1.57r = 1.57 \times 0.255 = 0.40$ in.

Distance of c.g. from center of radius,

$c = 0.637r = 0.637 \times 0.255 = 0.162$ in.

Computation of I_x, S_e, and M_n:

For the first approximation, assume a compression stress of $f = F_y = 50$ ksi (yield strength in longitudinal compression, Table A1 of the Standard Specification) in the top fiber of the section and that the web is fully effective.

Compression flange: $k = 0.50$ (for unstiffened compression element, see Section 2.3.1 of the Standard)

$w/t = 1.177/0.135 = 8.72 < 50$ OK (Section 2.1.1-(1)-(iii))

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

The initial modulus of elasticity, E_o, for Type 304 stainless steel is obtained from Table A4 of the Standard, i.e., $E_o = 27000$ ksi.

$\lambda = (1.052/\sqrt{0.50})(8.72)\sqrt{50/27000} = 0.558 < 0.673$

$b = w$ (Eq. 2.2.1-1)

$= 1.177$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Web	6.354	3.500	22.239	77.837	21.378
Upper Corner	0.400	0.161	0.064	0.010	--
Lower Corner	0.400	6.839	2.736	18.709	--
Compression Flange	1.177	0.068	0.080	0.005	--
Tension Flange	1.177	6.933	8.160	56.574	--
Sum	9.508		33.279	153.135	21.378

Distance from top fiber to x-axis is

y_{cg} = 33.279/9.508 = 3.500 in.

Since the distance from top compression fiber to the neutral axis is equal to one half the beam depth, a compression stress of 50 ksi will govern as assumed (i.e., initial yield is in compression).

To check if web is fully effective (Section 2.2.2):

f_1 = [(3.500-0.323)/3.500]x50 = 45.39 ksi(compression)

f_2 = -[(3.500-0.323)/3.500]x50 = -45.39 ksi(tension)

Ψ = f_2/f_1 = -45.39/45.39 = -1.00

$k = 4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

$= 4+2[1-(-1.00)]^3+2[1-(-1.00)]$

= 24.00

h = w = 6.354 in., h/t = w/t = 6.354/0.135 = 47.07

$h/t = 47.07 < 200$ OK (Section 2.1.2-(1))

$\lambda = (1.052/\sqrt{24.0})(47.07)\sqrt{45.39/27000} = 0.414 < 0.673$

$b_e = 6.354$ in.

$b_2 = b_e/2$ (Eq. 2.2.2-2)

$= 6.354/2 = 3.177$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 6.354/[3-(-1.0)] = 1.589$ in.

The effective widths, b_1 and b_2, of web are defined in Figure 2 of the Standard.

$b_1+b_2 = 1.589 + 3.177 = 4.766$ in.

Compression portion of the web calculated on the basis of the effective section $= y_{cg} - 0.154 = 3.50 - 0.323 = 3.177$ in.

Since $b_1+b_2 = 4.766$ in. > 3.177 in., b_1+b_2 shall be taken as 3.177 in.. This verifies the assumption that the web is fully effective.

$I'_x = Ly^2+I'_1-Ly^2_{cg}$

$= 153.135 + 21.378 - 9.508(3.50)^2$

$= 58.04$ in.^{3}

Actual $I_x = I'_x t$

$= 58.04 \times 0.135$

$= 7.835$ in.^{4}

$S_e = I_x/y_{cg}$

$= 7.835/3.50$

$= 2.239$ in.^{3}

$(M_n)_1 = S_e F_y$ (Eq. 3.3.1.1-1)

$= 2.239 \times 50$

$= 111.95$ kips-in.

b. Procedure II - based on inelastic reserve capacity

For unstiffened compression element, $C_y = 1$.

Maximum compressive strain $= C_y e_y = e_y$.

Therefore, the nominal ultimate moment, M_n, is the same as the $(M_n)_1$ determined by Procedure I because the compression flange will yield first.

2. Lateral buckling strength, M_n (Section 3.3.1.2).

The following equations used for computing the sectional properties for channel with no lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

a. Basic parameters used for calculating the sectional properties:

(For a Channel with no lips)

$r = R+t/2 = 3/16+0.135/2 = 0.255$ in.

From the sketch, $A' = 7.0$ in., $B' = 1.50$ in.

$\alpha = 0.0$ (For sections with no lips)

$a = A'-(2r+t)$

$= 7.0-(2\text{x}0.255+0.135) = 6.355$ in.

$\bar{a} = A'-t = 7-0.135 = 6.865$ in.

$b = B'-[r+t/2+\alpha(r+t/2)] = 1.5-(0.255+0.135/2) = 1.177$ in.

$\bar{b} = B'-(t/2+\alpha t/2) = 1.5-0.135/2 = 1.433$ in.

$u = 1.57r = 1.57 \text{ x } 0.255 = 0.40$ in.

b. Area:

$A = t[a+2b+2u]$

$= 0.135(6.355+2x1.177+2x0.40)$

$= 1.284 \text{ in.}^2$

c. Moment of inertia about x-axis:

$I_x = 2t[0.0417a^3+b(a/2+r)^2+u(a/2+0.637r)^2+0.149r^3]$

$= 2x0.135[0.0417(6.355)^3+1.177(6.355/2+0.255)^2$

$+0.4(6.355/2+0.637x0.255)^2+0.149(0.255)^3]$

$= 7.839 \text{ in.}^4$

d. Distance bwtween centroid and web centerline:

$\overline{x} = (2t/A)[b(b/2+r)+u(0.363r)]$

$= (2x0.135/1.284)[1.177(1.177/2+0.255)+0.4(0.363x0.255)]$

$= 0.217 \text{ in.}$

e. Moment of inertia about y-axis:

$I_y = 2t[b(b/2+r)^2+0.0833b^3+0.356r^3] - A(\overline{x})^2$

$= 2x0.135[1.177(1.177/2+0.255)^2+0.0833(1.177)^3+0.356(0.255)^3]$

$-1.284(0.217)^2$

$= 0.204 \text{ in.}^4$

f. Distance between shear center and web centerline:

$m = [\bar{b}t/(12I_x)][3\bar{b}(\bar{a})^2]$

$= [1.433x0.135/(12x7.839)][3x1.433x(6.865)^2]$

$= 0.417 \text{ in.}$

g. Distance between centroid and shear center:

$x_o = -(\bar{x}+m) = -(0.217+0.417) = -0.634$ in.

h. St. Venant torsion constant:

$J = (t^3/3)[a+2b+2u]$

$= [(0.135)^3/3][6.355+2x1.177+2x0.4]$

$= 0.0078$ in.^{4}

i. Warping Constant:

$C_w = (t\bar{a}^2\bar{b}^3/12)\ (3\bar{b}+2\bar{a})/(6\bar{b}+\bar{a})$

$= [0.135(6.865)^2(1.433)^3/12]$

$\quad x\ (3x1.433+2x6.865)/[6(1.433)+6.865]$

$= 1.819$ in.^{6}

j. Radii of gyration:

$r_x = \sqrt{(I_x/A)} = \sqrt{(7.839/1.284)} = 2.47$ in.

$r_y = \sqrt{(I_y/A)} = \sqrt{(0.204/1.284)} = 0.40$ in.

$r_o^2 = r_x^2+r_y^2+x_o^2$

$= (2.47)^2+(0.40)^2+(-0.634)^2$

$= 6.662$ in.^{2}

$r_o = 2.581$ in.

Therefore, for determining the lateral buckling stress:

$M_n = S_c(M_c/S_f)$ Eq. (3.3.1.2-1)

where M_c is the critical moment calculated in accordance with Eq. (3.3.1.2-4) of the Standard.

$S_f = I_x/y_{cg} = 7.836/3.5 = 2.239$ in^3

$C_b = 1.75+1.05(M_1/M_2)+0.3(M_1/M_2)^2$

$= 1.75+1.05(-0.0063/0.10)+0.3(-0.0063/0.10)^2 = 1.685 < 2.3$

where M_1 and M_2 are determined from the moment diagram at the interior support.

$M_c = C_b r_o A \sqrt{\sigma_{ey}\sigma_t}$ (Eq. 3.3.1.2-4)

where

$\sigma_{ey} = [(\pi^2 E_o)/(K_y L_y/r_y)^2](E_t/E_o)$ (Eq. 3.4.3-3)

$\sigma_t = [1/(Ar_o^2)][G_o J+(\pi^2 E_o C_w)/(K_t L_t)^2](E_t/E_o)$ (Eq. 3.4.2-1)

Therefore,

$\sigma_{ey} = [(\pi^2 x27000)/(1.0x2.5x12/0.40)^2]\ (E_t/E_o)$

$= 47.14\ (E_t/E_o)$

$\sigma_t = [1/(1.284x6.662)][(10500x0.0078+\pi^2 x27000x1.819/(1.0x2.5x12)^2]$

$x(E_t/E_o)$

$= 72.54\ (E_t/E_o)$

$M_n = S_c(M_c/S_f)$ Eq. (3.3.1.2-1)

$= S_c f$

where,

$f = M_c/S_f$

$= (1/2.239)(1.685x2.581x1.284x\ \sqrt{47.14x72.54})(E_t/E_o)$

$= 145.84(E_t/E_o)$ ksi

In the determination of the lateral buckling stress, it is necessary to select a proper ratio of E_t/E_o from Table A10 or Figure A7 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of f=32 ksi. From Table A10, the corresponding value of E_t/E_o is found to be equal to 0.42. Thus,

$f_1 = 145.84 \times 0.42$

$= 61.25$ ksi > assumed stress $f=32$ ksi

Because the computed stress is larger than the assumed value, the further successive approximation is needed. After several trials, assume $f=42.12$ ksi, and

$E_t/E_o = 0.2888$

$f_1 = 145.84 \times 0.2888$

$= 42.12$ ksi = assumed stress $f=42.12$ ksi OK

Therefore,

$f = M_c/S_f = 34.50$ ksi

It is noted that from the calculation of Part 1(a), the section is fully effective for $f=F_y=50$ ksi. Therefore, for the lateral buckling stress of $f=42.12$ ksi, the section will also be fully effective.

Thus,

$(M_n)_2 = S_c f = 2.239 \times 42.12 = 94.30$ kips-in.

3. Design flexural strength, $\phi_b M_n$

Based on the above calculations, the lateral buckling stress $(M_n)_2$ is less than the nominal section strength $(M_n)_1$. Therefore, lateral buckling governs the design.

$M_n = 94.30$ kips-in.

$\phi_b = 0.85$

$\phi_b M_n = 0.85 \times 94.30 = 80.16$ kips-in.

This value can be used for both positive and negative bending.

$w_u = 1.2 w_{DL} + 1.6 w_{LL} = 1.2(0.04)+1.6(0.20) = 0.368$ kips/ft.

For a continuous beam over three equal spans, the maximum bending moment is negative and occurs over the interior supports. It is given by

$$M_u = 0.100w_uL^2 = 0.100(0.368)(10)^2(12)$$

$$= 44.16 \text{ kips-in.} < \phi_bM_n = 80.16 \text{ kips-in. OK}$$

4. Strength for Shear Only (Section 3.3.2)

The required shear strength at any section shall not exceed the design shear strength ϕ_vV_n:

$$\phi_v = 0.85$$

$$V_n = 4.84E_ot^3(G_s/G_o)/h \qquad \text{(Eq. 3.3.2-1)}$$

$$v_n = V_n/(ht) \quad \text{(in terms of design shear stress)}$$

$$= 4.84E_o(G_s/G_o)/(h/t)^2$$

In the determination of the shear strength, it is necessary to select a proper value of G_s/G_o for the assumed stress from Table A12 or Figure A9 of the Standard. For the first approximation, assume a shear stress of $v=F_y/2=25$ ksi and the corresponding value of G_s/G_o is equal to 0.888. Thus,

$$h/t = 6.354/0.135 = 47.07$$

$$v_n = 4.84(27000)(0.888)/(47.07)^2$$

$$= 52.38\text{ksi} > \text{assumed stress } v=25 \text{ ksi NG}$$

For a second approximation, assume a stress of f=38.30 ksi and its corresponding value of G_s/G_o is 0.648.

$$v_n = 4.84(27000)(0.648)/(47.07)^2$$

$$= 38.24 \text{ ksi} \cong \text{assumed stress (close enough) OK}$$

Therefore, the total shear strength, V_n, for hat section is

$$V_n = (2 \text{ webs})(v_n)(ht)$$

$= 2(38.24)(6.354x0.135)$

$= 32.80$ kips

The design shear strength is determined as follows:

$\phi_v V_n = 0.85(32.80) = 27.88$ kips

$\phi_v V_n < 2(0.95F_{yv}ht) = 2(0.95x42x6.354x0.135) = 34.23$ kips OK

(The shear yield strength, F_{yv}, is obtained from Table A1 of the Standard.)

The maximum required shear strength is given by

$V_u = 0.600w_uL$

$= (0.600)(0.368)(10) = 2.21$ kips $< \phi_v V_n = 27.88$ kips OK

5. Strength for combined bending and shear (Section 3.3.3).

At the interior supports there is a combination of web bending and web shear:

$\phi_b M_n = 80.16$ kips-in. $\quad M_u = 44.16$ kips-in.

$\phi_v V_n = 27.88$ kips $\quad V_u = 2.21$ kips

For unreinforced webs

$(M_u/\phi_b M_n)^2+(V_u/\phi_v V_n)^2 \le 1.0$ (Eq. 3.3.3-1)

$(44.16/80.16)^2+(2.21/27.88)^2 = 0.31 < 1.0$ OK

6. Web crippling strength (Section 3.3.4).

$R/t = (3/16)/0.135 = 1.389 < 6$ OK

$h/t = 6.354/0.135 = 47.07 < 200$ OK

$N/t = 3.0/0.135 = 22.22 < 210$ OK (at end support)

$N/t = 6.0/0.135 = 44.44 < 210$ OK (at interior support)

Table 2 of the Standard applies:

For end reactions: (Eq. 3.3.4-2)

For interior reactions: (Eq. 3.3.4-4)

$k = F_y/33 = 50/33 = 1.515$ (Eq. 3.3.4-21)

$C_1 = (1.22-0.22k)k$ (Eq. 3.3.4-10)

$= [1.22-0.22(1.515)](1.515) = 1.343$

$C_2 = (1.06-0.06R/t)$ (Eq. 3.3.4-11)

$= [1.06-0.06(1.389)] = 0.977 < 1.0$ OK

$C_3 = (1.33-0.33k)k$ (Eq. 3.3.4-12)

$= [1.33-0.33(1.515)](1.515) = 1.258$

$C_4 = (1.15-0.15R/t) \leq 1.0$ but not less than 0.50 (Eq. 3.3.4-13)

$1.15-0.15R/t = 1.15-0.15(1.389) = 0.942 \leq 1.0$ OK

> 0.50 OK

$C_4 = 0.942$

$C_\theta = 0.7+0.3(\theta/90)^2$ (Eq. 3.3.4-20)

$= 0.7+0.3(90/90)^2 = 1.0$

For end reaction:

$P_n = t^2C_3C_4C_\theta[217-0.28(h/t)]\,[1+0.01(N/t)]$ (Eq. 3.3.4-2)

$= (0.135)^2(1.258)(0.942)(1.0)[217-0.28(47.07)]$

$\times [1+0.01(22.22)] = 5.38$ kips

$\phi_w = 0.70$

$\phi_w P_n = 0.70(5.38) = 3.77$ kips

End reaction is given by

$R = 0.400w_uL$

$= (0.400)(0.368)(10) = 1.47 \text{ kips} < \phi_w P_n = 3.77$ kips OK

For interior reaction:

$P_n = t^2C_1C_2C_\theta[538-0.74(h/t)]\,[1+0.007(N/t)]$ (Eq. 3.3.4-4)

$= (0.135)^2(1.343)(0.977)(1.0)[538-0.74(47.07)]$

$\times [1+0.007(44.44)] = 15.79$ kips

ϕ_w = 0.70

$\phi_w P_n$ = 0.70(15.79) = 11.05 kips

Interior reaction is given by

$R = 1.10 w_u L$

$= (1.10)(0.368)(10) = 4.05$ kips $< \phi_w P_n = 11.05$ kips OK

7. Combined bending and web crippling strength (Section 3.3.5).

At the interior supports there is a combination of web bending and web crippling:

$\phi_b M_n$ = 80.16 kips-in. $\quad M_u$ = 44.16 kips-in.

$\phi_w P_n$ = 11.05 kips $\quad R$ = 4.05 kips

For shapes having single unreinforced webs:

$$1.07(R/\phi_w P_n)+(M_u/\phi_b M_n) \le 1.42 \qquad \text{(Eq. 3.3.5-1)}$$

$$1.07(4.05/11.05)+(44.16/80.16) = 0.943 < 1.42 \text{ OK}$$

8. Deflection due to service live load.

From the result of sectional properties calculated in item (1) of this example, the section is fully effective at F_y = 50 ksi.

$S_x = S_e$ = 2.239 in.^{3}

Therefore, for any stress f which is less than F_y = 50 ksi, the section will be fully effective, i.e.,

I_x = 7.835 in.^{4}

This value can be used for deflection determination.

The maximum deflection occurs at a distance of 0.446L from the exterior supports. It is given by

$$\Delta = 0.0069 w L^4/(E_o I_x)$$

Thus, the live load deflection is calculated as follows:

$$\Delta = 0.0069(0.20)(10)^4(12)^3/(27000x7.835)$$

$$= 0.113 \text{ in.}$$

The live load deflection is limited to 1/240 of the span, i.e.,

L/240 = 10x12/240 = 0.5 in. > 0.113in. OK

From the above calculations, it can be concluded that the section is adequate.

EXAMPLE 9.2 CHANNEL W/LATERAL BUCKLING CONSIDERATION (ASD)

By using the ASD method, rework Example 9.1 for the same given data.

Solution:

1. Nominal section strength, M_n

For detailed calculations see Example 9.1.

$(M_n)_1 = S_e F_y$

$= 2.239x50$

$= 111.95$ kips-in.

2. Lateral buckling strength, M_n

For detailed calculations see Example 9.1.

$(M_n)_2 = S_c f$

$= 2.239x42.12$

$= 94.30$ kips-in.

3. Allowable bending strength, M_a

$M_n = 94.30$ kips-in. (based on lateral buckling strength)

$\Omega = 1.85$

$M_a = 94.30/1.85 = 50.97$ kips-in.

This value can be used for both positive and negative bending.

$w = w_{DL} + w_{LL} = 0.04+0.20 = 0.24$ kips/ft.

For a continuous beam over three equal spans, the maximum bending moment is negative and occurs over the interior supports. It is given by

$M = 0.100wL^2 = 0.100(0.24)(10)^2(12)$

$= 28.80$ kips-in. $< M_a = 50.97$ kips-in. OK

4. Strength for Shear Only

The required shear strength at any section shall not exceed the allowable shear strength V_a:

V_n = (2 webs)(v_n)(ht)

= 2(38.24)(6.354x0.135) (See Example 9.1 for v_n)

= 32.80 kips

The allowable shear strength is determined as follows:

Ω = 1.85

V_a = 32.80/1.85 = 17.73 kips

V_a shall be less than the allowable shear yielding strength, i.e.,

$V_a < 2(F_{yv}ht)/1.64 = 43.94$ kips OK

(The safety factor used for shear yielding is 1.64, and the shear yield strength, F_{yv}, is obtained from Table A1 of the Standard.)

The maximum required shear strength is given by

V = 0.600wL

= (0.600)(0.24)(10) = 1.44 kips < V_a = 17.73 kips OK

5. Strength for combined bending and shear

At the interior supports there is a combination of web bending and web shear:

M_a = 50.97 kips-in. M = 28.80 kips-in.

V_a = 17.73 kips V = 1.44 kips

For unreinforced webs

$(M/M_a)^2+(V/V_a)^2 \leq 1.0$

$(28.80/50.97)^2+(1.44/17.73)^2 = 0.326 < 1.0$ OK

6. Web crippling strength

See Example 9.1 for detailed calculations.

For end reaction:

$P_n = t^2 C_3 C_4 C_\theta [217-0.28(h/t)] [1+0.01(N/t)]$

$= (0.135)^2(1.258)(0.942)(1.0)[217-0.28(47.07)]$

$\times [1+0.01(22.22)] = 5.38$ kips

$\Omega = 2.00$

$P_a = 5.38/2.0 = 2.69$ kips

End reaction is given by

$R = 0.400wL$

$= (0.400)(0.240)(10) = 0.96$ kips $< P_a = 2.69$ kips OK

For interior reaction:

$P_n = t^2 C_1 C_2 C_\theta [538-0.74(h/t)] [1+0.007(N/t)]$

$= (0.135)^2(1.343)(0.977)(1.0)[538-0.74(47.07)]$

$\times [1+0.007(44.44)] = 15.79$ kips

$\Omega = 2.00$

$P_a = 15.79/2.0 = 7.90$ kips

Interior reaction is given by

$R = 1.10wL$

$= (1.10)(0.240)(10) = 2.64$ kips $< P_a = 7.90$ kips OK

7. Combined bending and web crippling strength

At the interior supports there is a combination of web bending and web crippling:

$M_a = 50.97$ kips-in. $\quad M = 28.80$ kips-in.

$P_a = 7.90$ kips $\quad R = 2.64$ kips

For shapes having single unreinforced webs:

$1.07(R/P_a)+(M/M_a) \le 1.42$

$1.07(2.64/7.90)+(28.80/50.97) = 0.923 < 1.42$ OK

8. Deflection due to service live load.

From the result of sectional properties calculated in item (1) of Example 9.1, the section is fully effective at $F_y = 50$ ksi. Therefore, for a stress f=42.12 ksi which is less than $F_y = 50$ ksi, the section will be fully effective, i.e.,

$I_x = 7.835 \text{ in.}^4$

This value can be used for deflection determination.

The maximum deflection occurs at a distance of 0.446L from the exterior supports. It is given by

$\Delta = 0.0069wL^4/(E_oI_x)$

Thus, the live load deflection is calculated as follows:

$\Delta = 0.0069(0.20)(10)^4(12)^3/(27000x7.835)$

$= 0.113$ in.

The live load deflection is limited to 1/240 of the span, i.e.,

L/240 = 10x12/240 = 0.5 in. > 0.113in. OK

From the above calculations, it can be concluded that the section is adequate.

EXAMPLE 10.1 HAT SECTION USING INELASTIC RESERVE CAPACITY (LRFD)

(Inelastic Reserve Capacity)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi_b M_n$. Use Type 301 stainless steel, annealed.

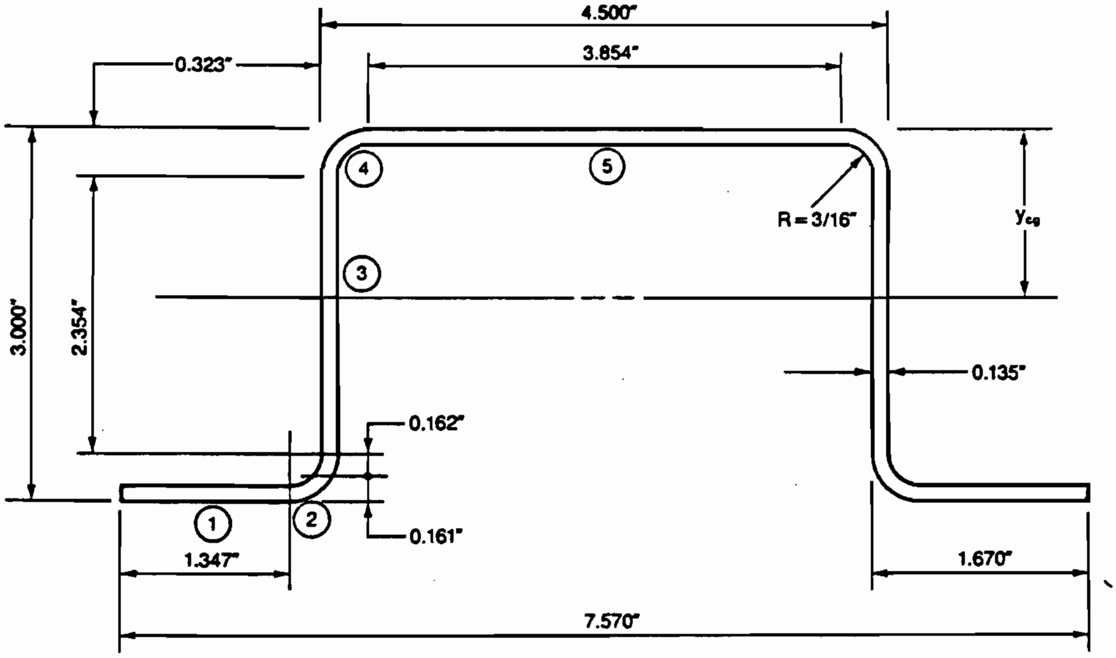


Figure 10.1 Section for Example 10.1

Given:

1. Section: Hat section, as shown in sketch.
2. Top flange continuously supported.
3. Span = 8 ft., simply supported.

Solution:

1. Properties of 90° corners:

   Corner Radius, $r = R + t/2 = 3/16 + 0.135/2 = 0.255$ in.

   Length of arc, $u = 1.57r = 1.57 \times 0.255 = 0.400$ in.

   Distance of c.g. from center of radius,

   $c = 0.637r = 0.637 \times 0.255 = 0.162$ in.

I' of corner about its own centroidal axis = $0.149r^3$

$= 0.149(0.255)^3 = 0.003$ in.^{3}. This is negligible.

2. Nominal Section Strength (Section 3.3.1.1)

a. Procedure I - Based on Initiation of Yielding

Computation of I_x, S_e, and M_n for the first approximation:

* Assume a compressive stress of $f = F_{yc} = 28$ ksi (yield strength in longitudinal compression, see Table A1 of the Standard) in the top fiber of the section.

* Assume web is fully effective.

Element 3:

$h/t = 2.354/0.135 = 17.44 < 200$ OK (Section 2.1.2-(1))

Assumed fully effective

Element 5:

$w/t = 3.854/0.135 = 28.55 < 400$ OK (Section 2.1.1-(1)-(ii))

$k = 4$

$$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o} \qquad \text{(Eq. 2.2.1-4)}$$

$E_o = 27000$ ksi is obtained from Table A4 of the Standard.

$\lambda = (1.052/\sqrt{4})(28.55)\sqrt{28/28000} = 0.475 < 0.673$

$$b = w \qquad \text{(Eq. 2.2.1-1)}$$

$= 3.854$ in. (Fully effective)

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly ($in.^2$)	Ly^2 ($in.^3$)	I'_1 About Own Axis ($in.^3$)
1	2 x 1.347 = 2.694	2.933	7.902	23.175	--
2	2 x 0.400 = 0.800	2.839	2.271	6.448	--
3	2 x 2.354 = 4.708	1.500	7.062	10.593	2.174
4	2 x 0.400 = 0.800	0.161	0.129	0.021	--
5	3.854	0.068	0.262	0.018	--
Sum	12.856		17.626	40.255	2.174

The distance from the top fiber to the neutral axis is

y_{cg} = Ly/L = 17.626/12.856 = 1.371 in.

$(3.000-y_{cg})/y_{cg}$ = (3.0-1.371)/1.371 = 1.188

$1.188xF_{yc}$ = 33.264 ksi > F_{yt} = 30 ksi NG

(F_{yt} is the yield strength in longitudinal tension, see Table A1 of the Standard.)

Since the computed stress in tension flange is larger than the specified yield strength, F_{yt} = 30 ksi, the compression stress of F_{yc} will not govern as assumed. The actual compressive stress will be less than F_{yc} and so the flange will still be fully effective. The tension flange will yield first. Section properties will not change.

Therefore,

$$I'_x = Ly^2 + I'_1 - Ly^2_{cg}$$

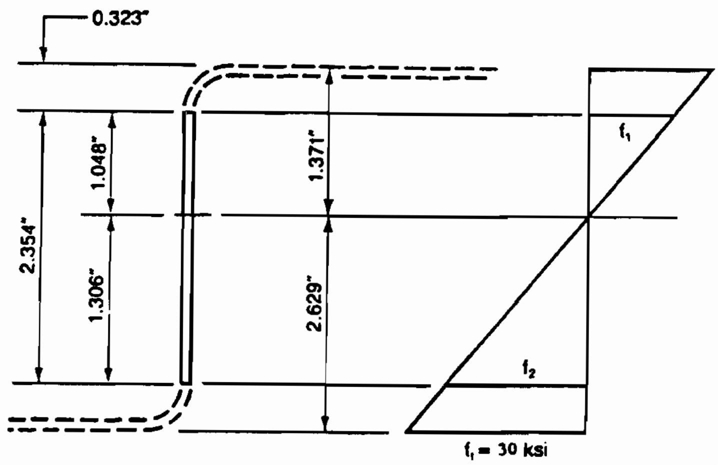
$$= 40.255 + 2.174 - 12.856(1.371)^2$$

$$= 18.26 \text{ in.}^3$$

Actual $I_x = tI'_x$

$= (0.135)(18.26) = 2.47 \text{ in.}^4$

Check Web



Assume a stress of f=30 ksi at the bottom of tension fiber.

$f_1 \quad = (1.048/1.629)(30) = 19.30$ ksi(compression)

$f_2 \quad = -(1.306/1.629)(30) = -24.05$ ksi(tension)

$\Psi \quad = f_2/f_1 = -24.05/19.30 = -1.246$

$k \quad = 4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

$= 4+2[1-(-1.246)]^3+2[1-(-1.246)]$

$= 31.15$

$\lambda \quad = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}\ , \quad f = f_1$ (Eq. 2.2.1-4)

For annealed Type 301 stainless steel, E_o value is equal to 28000 ksi, which is given in Table A4 of the Standard.

$\lambda \quad = (1.052/\sqrt{31.15})(17.44)\sqrt{19.30/28000} = 0.086 < 0.673$

$b \quad = w$ (Eq. 2.2.1-1)

$b_e \quad = 2.354$ in.

$b_2 \quad = b_e/2$ (Eq. 2.2.2-2)

$= 2.354/2 = 1.177$ in.

$b_1 \quad = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 2.354/[3-(-1.246)] = 0.554$ in.

$b_1+b_2 = 0.554 + 1.177 = 1.731$ in. $>$ 1.048 in. (compression portion of web, see the sketch shown above)

Therefore, web is fully effective.

$S_e \quad = I_x/(d-y_{cg}) = 2.47/(3-1.371) = 1.516$ in.^{3}

$M_n \quad = S_e F_y$ (Eq. 3.3.1.1-1)

$= (1.516)(30)$

$= 45.48$ kips-in.

b. Procedure II - Based on Inelastic Reserve Capacity

$\lambda_1 = (1.11/\sqrt{F_{yc}/E_o})$ (Eq. 3.3.1.1-2)

$= (1.11/\sqrt{28/28000}) = 35.10$

$\lambda_2 = (1.28/\sqrt{F_{yc}/E_o})$ (Eq. 3.3.1.1-3)

$= (1.28/\sqrt{28/28000}) = 40.48$

$w/t = 28.55$

For $w/t < \lambda_1 = 35.10$

$C_y = 3.0$

Compute location of e_y on strain diagram, the summation of longitudinal forces should be zero.

Refer to equations from Reck, Pekoz, and Winter, "Inelastic Strength of Cold-Formed Steel Beams," Journal of the Structural Division, November 1975, ASCE.

Distance from neutral axis to the outer compression fiber, y_c:

$t \quad = 0.135$ in.

$b_t = 2(1.670) = 3.340$ in.

$b_c = 4.500$ in.

$d = 3.000$ in.

$y_c = (1/4)(b_t - b_c + 2d)$

$= (1/4)[3.340-4.500+2(3.000)] = 1.210$ in.

$y_p = y_c/C_y$

$= 1.21/3.0 = 0.403$ in.

$y_t = d - y_c$

$= 3.000-1.210 = 1.790$ in.

$y_{cp} = y_c - y_p$

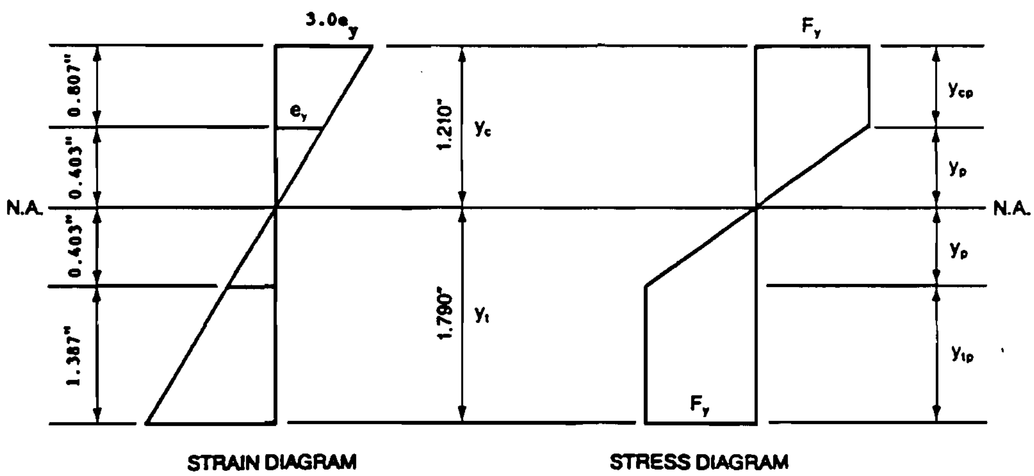
$= 1.210-0.403 = 0.807$ in.

$y_{tp} = y_t - y_p$

$= 1.790-0.403 = 1.387$ in.

Summing moments of stresses in component plates:

$$M_n = F_y t\{b_c y_c + 2y_{cp}[y_p + (y_{cp}/2)] + (4/3)y^2_p + 2y_{tp}[y_p + (y_{tp}/2)] + b_t y_t\}$$



$M_n = 28(0.135)\{4.500(1.210)+2(0.807)[0.403+(0.807/2)]$

$+(4/3)(0.403)^2+2(1.387)\ 0.403+(1.387/2)\ +3.340(1.790)$

$M_n = 60.42$ kips-in.

M_n shall not exceed $1.25S_eF_y = 1.25(45.48) = 56.85$ kips-in.

Therefore,

$M_n = 1.25S_eF_y = 56.85$ kips-in.

The inelastic reserve capacity is used in this example because the following conditions are met: (Section 3.3.1.1(2))

1) Member is not subject to twisting, lateral, torsional, or torsional-flexural buckling.
2) The effect of cold-forming is not included in determining the yield point, F_y.
3) The ratio of depth of the compressed portion of the web to its thickness does not exceed λ_1,

   $(1.210-0.323)/0.135 = 6.57 < \lambda_1 = 35.10$ OK
4) The shear force does not exceed $0.35F_y$ times the web area, h x t.

   This still needs to be checked for a complete design.
5) The angle between any web and the vertical does not exceed 30^o.

3. Design Flexural Strength, ϕ_bM_n

$\phi_b = 0.90$ (for section with stiffened compression flanges)

$\phi_bM_n = 0.90 \times 56.85 = 51.17$ kips-in.

EXAMPLE 10.2 HAT SECTION USING INELASTIC RESERVE CAPACITY (ASD)

Rework Example 10.1 by using the Allowable Stress Design (ASD) method.

Solution:

Calculation of the allowable moment, M_a:

The effective section properties calculated by the ASD method are the same as those determined in Example 10.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

$$M_a = M_n/\Omega \qquad \text{(Eq. E-1)}$$

Ω = 1.85 (Safety Factor stipulated in Table E of the Standard)

The nominal section strength based on inelastic reserve capacity is as follows:

M_n = 56.85 kips-in. (obtained from Example 10.1)

$M_a = M_n/\Omega$

$M_a = 56.85/1.85$

$= 30.73$ kips-in.

EXAMPLE 11.1 DECK SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi_b M_n$, based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the service moment. Compute the factored uniform load, w_u, as controlled either by bending or deflection. Use Type 201 stainless steel, 1/4-Hard. Assume dead load to live load ratio D/L = 1/5 and 1.2D + 1.6L governs the design.

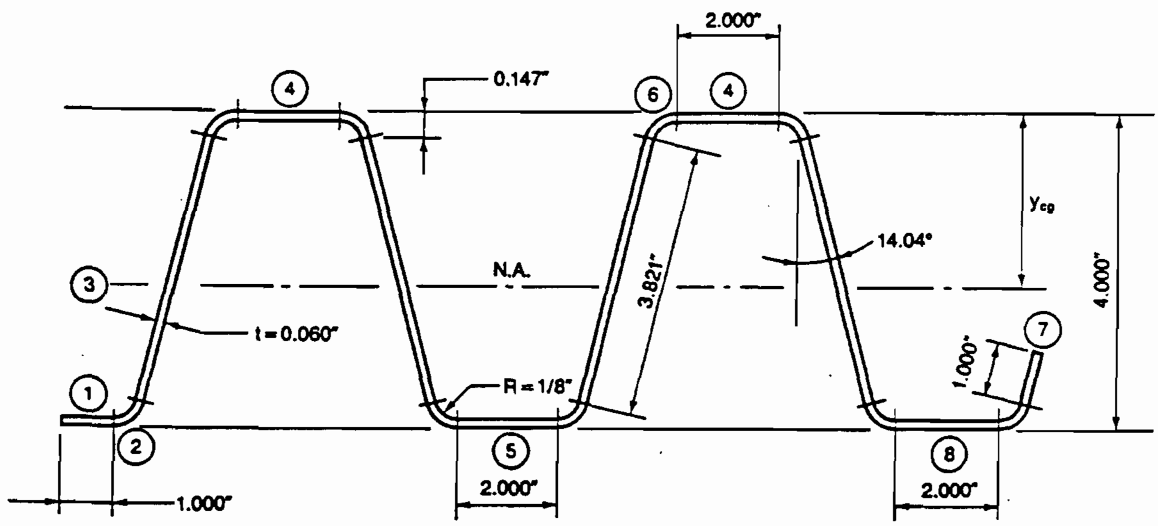
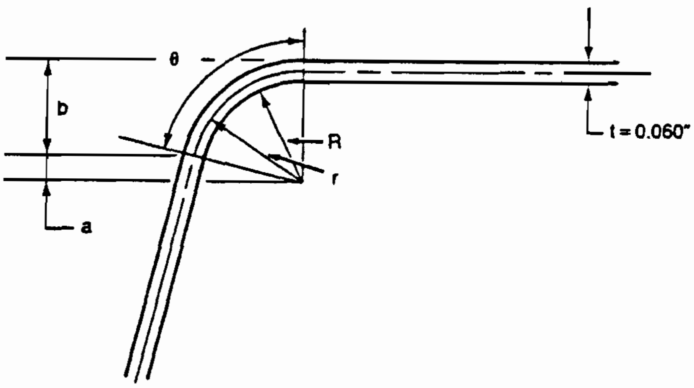


Figure 11.1 Section for Example 11.1

Given:

1. Section: Deck section, as shown in sketch.
2. Deck is continuous over three 10'-0" spans.
3. Deflection due to service live load is to be limited to 1/240 of the span.

Corner Properties:



$\theta = 75.96^{\circ}$

$R = 1/8''$

$r = 0.155''$

$a = r \sin(90^{\circ} - 75.96^{\circ})$

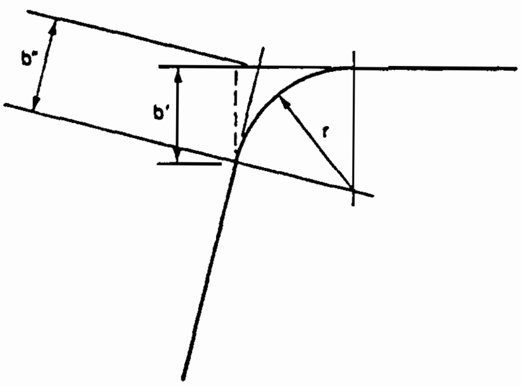
$= 0.155'' \sin 14.04^{\circ}$

$= 0.0376''$

$b = t/2 + r - a$

$= 0.060''/2 + 0.155'' - 0.0376''$

$= 0.147''$



$b' = b - t/2$

$= 0.147'' - 0.060''/2$

$= 0.117''$

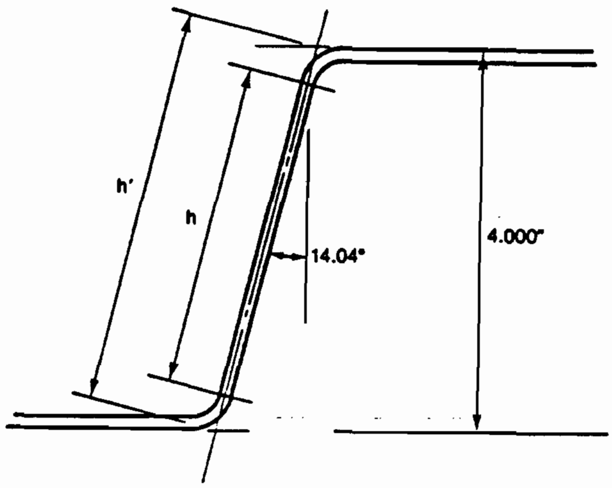
$b'/b'' = \cos(90^{\circ} - 75.96^{\circ})$

$b'' = b'/\cos 14.04^{\circ}$

$= 0.117''/\cos 14.04^{\circ}$

$= 0.121''$

Flat portion of web



$h' = 4.000''/\cos 14.04^{\circ}$

$= 4.123''$

$h = h' - 2b'' - t/\cos 14.04^{\circ}$

$= 4.123'' - 2(0.121'')$

$- 0.06''/\cos 14.04^{\circ}$

$= 3.819''$

<u>Solution</u>:

1. Full Section Properties:

Elements 2 and 6:

Corner Radius, $r = R + t/2 = 1/8 + 0.060/2 = 0.155$ in.

Angle, $\theta = 75.96^{\circ} = 1.326$ rad

Length of arc, $u = \theta r = 1.326(0.155) = 0.206$ in.

Distance of c.g. from center of radius,

$c_1 = r \sin\theta/\theta = 0.155(\sin 1.326)/1.326 = 0.113$ in.

The moment of inertia, I'_1, of arc element about its own centroidal axis is negligible.

Element 3:

$l = 3.819$ in.

$\theta = 14.04^{\circ}$

$\cos\theta = 0.9701$

$I'_1 = (\cos^2\theta l^3)/12 = [(0.9701)^2(3.819)^3]/12 = 4.368$ in.^{3}

Element 7:

$l = 1.000$ in.

$\theta = 14.04^{\circ}$

$\cos\theta = 0.9701$

$I'_1 = (\cos^2\theta l^3)/12 = [(0.9701)^2(1)^3]/12 = 0.0784$ in.^{3}

Distance from top fiber to the centroid of full section is

$y = 4-0.147-(1.000/2)\cos 14.04^{\circ} = 3.368$ in.

2. Section Modulus for Load Determination - Based on Initiation of Yielding

Since the effective design width of flat compressive elements is a function of stress, iteration is required.

Computation of I_x, S_e, and M_n for the first approximation:

* Assume a compressive stress of $f = F_y = 50$ ksi in the top fiber of the section. (See Table A1 of the Standard for F_y value.)
* Assume web is fully effective.

Element 3:

$h/t = 3.819/0.060 = 63.65 < 200$ OK (Section 2.1.2-(1))

Assumed fully effective

Element 4:

$w/t = 2.000/0.060 = 33.33 < 400$ OK (Section 2.1.1-(1)-(ii))

$k = 4$

$\lambda \;= (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}\,,\; f = F_y$ (Eq. 2.2.1-4)

From Table A4 of the Standard, E_o value is equal to 27000 ksi in longitudinal compression for Type 201, 1/4-Hard, stainless steel.

$\lambda \;= (1.052/\sqrt{4})(33.33)\sqrt{50/27000} = 0.754 > 0.673$

$\rho \;= (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$\;= (1-0.22/0.754)/0.754 = 0.939$

$b \;= \rho w$ (Eq. 2.2.1-2)

$\;= 0.939 \times 2.000$

$\;= 1.878$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
1	1.000	3.970	3.970	15.761	--
2	5 x 0.206 = 1.030	3.928	4.046	15.892	--
3	4 x 3.819 = 15.276	2.000	30.552	61.104	17.472
4	2 x 1.878 = 3.756	0.030	0.113	0.003	--
5 & 8	2 x 2.000 = 4.000	3.970	15.880	63.044	--
6	4 x 0.206 = 0.824	0.072	0.059	0.004	--
7	1.000	3.368	3.368	11.343	0.078
Sum	26.886		57.988	167.151	17.550

The distance from the top fiber to the neutral axis is

$y_{cg} = Ly/L = 57.988/26.886 = 2.157$ in.

Since the distance from the top compression fiber to the neutral axis is greater than one half of the deck depth, a compressive stress of F_y will govern as assumed.

$$I'_x = Ly^2 + I'_1 - Ly^2_{cg}$$

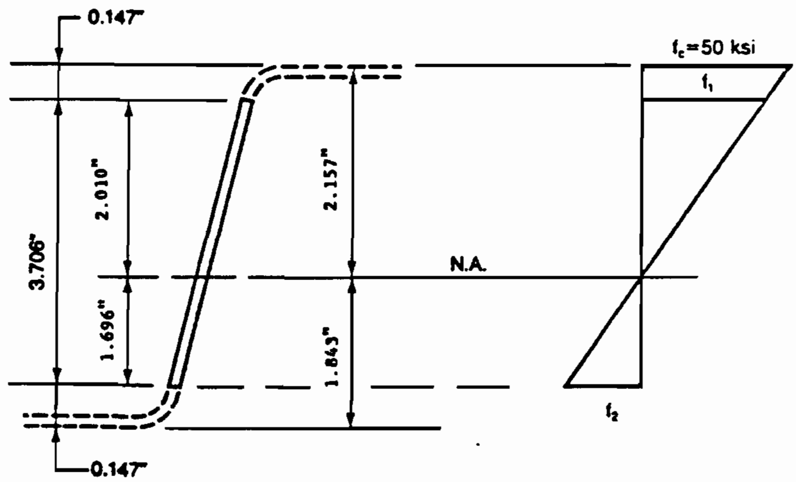
$$= 167.151 + 17.550 - 26.886(2.157)^2$$

$$= 59.61 \text{ in.}^3$$

$$\text{Actual } I_x = tI'_x$$

$$= (0.060)(59.61) = 3.58 \text{ in.}^4$$

Check Web



$$f_1 = (2.010/2.157)(50) = 46.59 \text{ ksi(compression)}$$

$$f_2 = -(1.696/2.157)(50) = -39.31 \text{ ksi(tension)}$$

$$\Psi = f_2/f_1 = -39.31/46.59 = -0.844$$

$$k = 4+2(1-\Psi)^3+2(1-\Psi) \qquad \text{(Eq. 2.2.2-4)}$$

$$= 4+2\left[1-(-0.844)\right]^3+2\left[1-(-0.844)\right]$$

$$= 20.23$$

$$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}\text{, } f = f_1 \qquad \text{(Eq. 2.2.1-4)}$$

$$= (1.052/\sqrt{20.23})(63.65)\sqrt{46.59/27000} = 0.618 < 0.673$$

$$b = w \qquad \text{(Eq. 2.2.1-1)}$$

$$b_e = 3.819 \text{ in.}$$

$$b_2 = b_e/2 \qquad \text{(Eq. 2.2.2-2)}$$

$= 3.819/2 = 1.910$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 3.819/[3-(-0.844)] = 0.993$ in.

$b_1+b_2 = 0.993 + 1.910 = 2.903$ in. > 2.002 in. (compression portion of web, see the sketch shown above.)

Therefore, web is fully effective.

$S_e = I_x/y_{cg}$

$= 3.58/2.157$

$= 1.66$ in.^{3}

$M_n = S_e F_y$ (Eq. 3.3.1.1-1)

$= (1.66)(50)$

$= 83.0$ kips-in.

$\phi_b = 0.90$ (for section with stiffened compression flanges)

$\phi_b M_n = 0.90 \times 83.0 = 74.7$ kips-in.

3. Moment of Inertia for Deflection Determination - Positive Bending

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

$\phi_b M_n = 1.2M_{DL} + 1.6M_{LL}$

$= [1.2(M_{DL}/M_{LL})+1.6]M_{LL}$

$= [1.2(1/5)+1.6]M_{LL}$

$= 1.84M_{LL}$

$M_{LL} = \phi_b M_n/1.84 = 74.70/1.84 = 40.60$ kips-in.

$M_s = M_{DL} + M_{LL}$

$= (1/5+1)M_{LL}$

$= 1.2(40.60) = 48.72$ kips-in.

where

M_{DL} = Moment determined on the basis of nominal dead load

M_{LL} = Moment determined on the basis of nominal live load

Computation of I_{eff} for the first approximation:

* Assume a stress of f = 28.66 ksi in the top and bottom fibers of the section.
* Since the web was fully effective at a higher stress gradient, it will be fully effective at this stress level.

Element 4:

$w/t = 33.33$

$k = 4$

For deflection determination, the value of E_r, reduced modulus of elasticity determined by using Eq. (2.2.1-7), is substituted for E_o in Eq. (2.2.1-4). For a compression and tension stresses of f = 28.66 ksi, the corresponding E_{sc} and E_{st} values for Type 201 stainless steel are obtained from Table A2 or Figure A1 of the Standard as follows:

$E_{sc} = 23550$ ksi, $E_{st} = 26970$ ksi

$$E_r = (E_{sc}+E_{st})/2 \qquad \text{(Eq. 2.2.1-7)}$$

$$= (23550+26970)/2 = 25260 \text{ ksi}$$

Thus, for compression flange (Element 4):

$$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}$$

$$= (1.052/\sqrt{4})(33.33)\sqrt{28.66/25260} = 0.591 < 0.673$$

$b_d = w$ (Eq. 2.2.1-5)

$= 2.000$ in. (Fully effective)

Note: All elements are fully effective.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
1	1.000	3.970	3.970	15.761	--
2	5 x 0.206 = 1.030	3.928	4.046	15.892	--
3	4 x 3.819 = 15.276	2.000	30.552	61.104	17.472
4	2 x 2.000 = 4.000	0.030	0.120	0.004	--
5 & 8	2 x 2.000 = 4.000	3.970	15.880	63.044	--
6	4 x 0.206 = 0.824	0.072	0.059	0.004	--
7	1.000	3.368	3.368	11.343	0.078
Sum	27.130		57.995	167.152	17.550

The distance from the top fiber to the neutral axis is

$y_{cg} = Ly/L = 57.995/27.130 = 2.138$ in.

Since the distance from the top compression fiber to the neutral axis is greater than one half the deck depth, the compressive stress of 28.66 ksi will govern as assumed.

$$I'_{eff} = Ly^2 + I'_1 - Ly^2_{cg}$$

$$= 167.152 + 17.550 - 27.130(2.138)^2$$

$$= 60.69 \text{ in.}^3$$

$$\text{Actual } I_{eff} = tI'_{eff}$$

$$= (0.060)(60.69) = 3.64 \text{ in.}^4$$

$S_{eff} = I_{eff}/y_{cg} = 3.64/2.138 = 1.70$ in.^{3}

$M = S_{eff}(28.66)$

$= (1.70)(28.66)$

$= 48.72$ kips-in. $= M_s$ OK

Thus, use $I_{eff} = 3.64$ in.^{4} for deflection calculations.

4. Section Modulus for Load Determination - Negative Bending (Based on Initiation of Yielding)

Following a similar procedure as in positive bending.

Computation of I_x, S_e and M_n for the first approximation:

* Assume a compressive stress of $f = F_y = 50$ ksi in the bottom fiber of the section.

* Assume web is fully effective.

Element 3:

$h/t = 3.819/0.060 = 63.65 < 200$ OK (Section 2.1.2-(1))

Assumed fully effective

Element 1:

$w/t = 1.000/0.060 = 16.67 < 50$ OK (Section 2.1.1-(1)-(iii))

$k = 0.50$

$\lambda = (1.052/\sqrt{0.50})(16.67)\sqrt{50/27000} = 1.067 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/1.067)/1.067 = 0.744$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.744 \times 1.000$

$= 0.744$ in.

Element 5:

Same as element 4 in positive bending case.

$b \;= 1.878$ in.

Element 8:

$w/t = 2.000/0.060 = 33.33 < 50$ OK (Section 2.1.1-(1)-(iii))

$S \;= 1.28\sqrt{E_o/f}$ (Eq. 2.4-1)

$\;= 1.28\sqrt{27000/50} = 29.74$

For $w/t > S$

$I_a \;= t^4\{[115(w/t)/S]+5\}$ (Eq. 2.4.2-13)

$\;= (0.060)^4\{[115(33.33)/29.74]+5\}$

$\;= 0.00174$ in.^{4}

$I_s \;= d^3 t\sin^2\theta/12$

$\;= (1.000)^3(0.060)(\sin 75.96^o)^2/12 = 0.00471$ in.^{4}

$D \;= 1.000+0.185\tan(75.96^o/2) = 1.144$ in.

$D/w = 1.144/2.000 = 0.572$

For $0.25 < D/w < 0.80$

$k \;= [4.82-5(D/w)](I_s/I_a)^{1/3}+0.43 \leq 5.25-5(D/w)$ (Eq. 2.4.2-9)

$[4.82-5(0.572)](0.00471/0.00174)^{1/3}+0.43 = 3.162$

$5.25-5(0.572) = 2.390 < 3.162$

$k \;= 2.390$

$\lambda \;= (1.052/\sqrt{2.390})(33.33)\sqrt{50/27000} = 0.976 > 0.673$

$\rho \;= (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$\;= (1-0.22/0.976)/0.976 = 0.794$

$b \;= \rho w$ (Eq. 2.2.1-2)

$\;= 0.794(2.000) = 1.588$ in.

Element 7:

I_s = 0.00471 in.^{4} (calculated previously)

I_a = 0.00174 in.^{4} (calculated previously)

d = 1.000 in.

Assume a maximum stress in element, $f = F_y = 50$ ksi, although it will be actually less.

$k = 0.50$

$w/t = 1.000/0.060 = 16.67 < 50$ OK (Section 2.1.1-(1)-(iii))

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{0.50})(16.67)\sqrt{50/27000} = 1.067 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/1.067)/1.067 = 0.744$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.744(1.000) = 0.744$ in.

$d'_s = 0.744$ in.

$d_s = d'_s(I_s/I_a) \leq d'_s$ (Eq. 2.4.2-11)

Since $I_s/I_a > 1$

$d_s = d'_s = 0.744$ in.

$I'_1 = (d_s)^3\sin^2\theta/12 = (0.744)^3(\sin 75.96^\circ)^2/12 = 0.032$ in.^{3}

The distance from top fiber to the centroid of the reduced section is

$y = 4-0.147-(0.744/2)\cos 14.04^\circ = 3.492$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly ($in.^2$)	Ly^2 ($in.^3$)	I'_1 About Own Axis ($in.^3$)
1	0.744	3.970	2.954	11.726	--
2	5 x 0.206 = 1.030	3.928	4.046	15.892	--
3	4 x 3.819 = 15.276	2.000	30.552	61.104	17.472
4	2 x 2.000 = 4.000	0.030	0.120	0.004	--
5	1.878	3.970	7.456	29.599	--
6	4 x 0.206 = 0.824	0.072	0.059	0.004	--
7	0.744	3.492	2.598	9.072	0.020
8	1.588	3.970	6.304	25.028	--
Sum	26.084		54.089	152.429	17.504

The distance from top fiber to the neutral axis is (see sketch below)

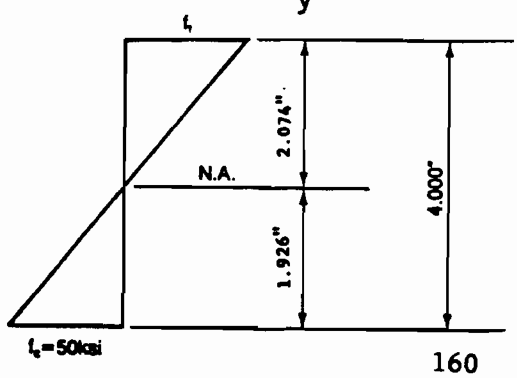
y_{cg} = Ly/L = 54.089/26.084 = 2.074 in.

The corresponding tension stress can be computed as follows:

$y_{cg}/(4.00-y_{cg})$ = 2.074/(4.00-2.074) = 1.077

$1.077 \times F_{yc}$ = 1.077 x 50 = 53.85 ksi < F_{yt} = 75 ksi OK

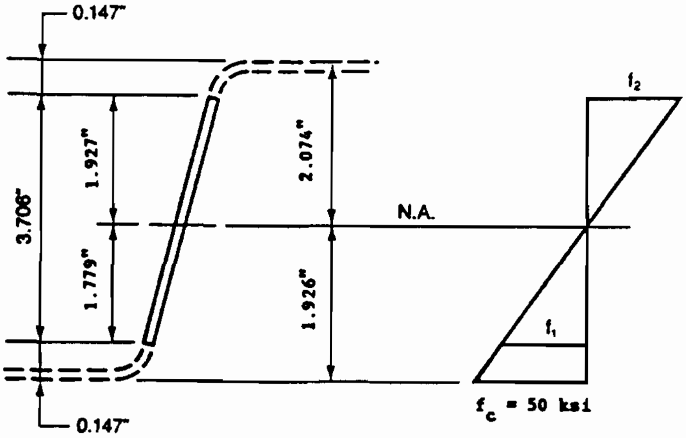
Because the distance of the top fiber from the neutral axis is greater than one half the deck depth, and also because the computed tension stress is less than the specified value, the compressive stress of $f=F_y$ will govern as assumed.



f_t = (2.074/1.926)(50)

= 53.85 ksi < F_{yt} OK

Check Web:



f_1 = (1.779/1.926)(50) = 46.18 ksi(compression)

f_2 = -(1.927/1.926)(50) = -50.03 ksi(tension)

Ψ = f_2/f_1 = -50.03/46.18 = -1.083

k = $4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

= $4+2[1-(-1.083)]^3+2[1-(-1.083)]$

= 26.24

λ = $(1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$, f = f_1 (Eq. 2.2.1-4)

= $(1.052/\sqrt{26.24})(63.65)\sqrt{46.18/27000}$ = 0.541 < 0.673

b = w (Eq. 2.2.1-1)

b_e = 3.819 in.

b_2 = $b_e/2$ (Eq. 2.2.2-2)

= 3.819/2 = 1.910 in.

b_1 = $b_e/(3-\Psi)$ (Eq. 2.2.2-1)

= 3.819/[3-(-1.083)] = 0.935 in.

b_1+b_2 = 0.935 + 1.910 = 2.845 in. > 1.763 in. (compression portion of web, see sketch shown above)

Therefore, web is fully effective.

Check Element 7:

Assume the maximum stress in element, $f = 46.18$ ksi

$k = 0.50$

$w/t = 16.67$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{0.50})(16.67)\sqrt{46.18/27000} = 1.026 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/1.026)/1.026 = 0.766$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.766(1.000) = 0.766$ in.

$d'_s = 0.766$ in.

$d_s = d'_s(I_s/I_a) \leq d'_s$ (Eq. 2.4.2-11)

Since $I_s/I_a > 1$

$d_s = d'_s = 0.766$ in.

$I'_1 = (d_s)^3\sin^2\theta/12 = (0.766)^3(\sin 75.96^o)^2/12 = 0.035$ in.^{3}

The distance from top fiber to the centroid of the reduced section is

$y = 4-0.147-(0.766/2)\cos 14.04^o = 3.481$ in.

Determine section properties, but only the properties of element 7 have changed

$\Delta L = 0.766-0.744 = 0.022$ in.

$\Delta Ly = (0.766)(3.481)-2.598 = 0.068$ in.^{2}

$\Delta Ly^2 = 0.766(3.481)^2-9.072 = 0.210$ in.^{3}

$\Delta I'_1 = 0.035-0.032 = 0.003$ in.^{3}

Therefore,

$L = 26.084+0.022 = 26.106$ in.

$Ly = 54.089+0.068 = 54.097$ in.^{2}

$Ly^2 = 152.429+0.210 = 152.639$ in.^{3}

$I'_1 = 17.504+0.003 = 17.507$ in.^{3}

The distance from top fiber to the neutral axis is

$y_{cg} = Ly/L = 54.097/26.106 = 2.072$ in.

$f_t = (2.072/1.928)(50) = 53.73$ ksi $< F_{yt} = 75$ ksi OK

$I'_x = Ly^2 + I'_1 - Ly^2_{cg}$

$= 152.639 + 17.507 - 26.106(2.072)^2$

$= 58.07$ in.^{3}

Actual $I_x = tI'_x$

$= (0.060)(58.07) = 3.48$ in.^{4}

$S_e = I_x/(4.00-y_{cg})$

$= 3.48/(4.00-2.072)$

$= 1.80$ in.^{3}

$M_n = S_eF_y$ (Eq. 3.3.1.1-1)

$= (1.80)(50)$

$= 90.0$ kips-in.

$\Phi_b = 0.85$ (for section with unstiffened compression flanges)

$\Phi_bM_n = 0.85 \times 90.0 = 76.5$ kips-in.

5. Moment of Inertia for Deflection Determination - Negative Bending

The unfactored loads are used to determine the section properties for deflection determination. For a load combination of 1.2D+1.6L, the service moment can be determined as follows:

$\Phi_bM_n = 1.2M_{DL} + 1.6M_{LL}$

$= [1.2(M_{DL}/M_{LL})+1.6]M_{LL}$

$= [1.2(1/5)+1.6]M_{LL}$

$= 1.84M_{LL}$

$M_{LL} = \Phi_b M_n/1.84 = 76.50/1.84 = 41.58$ kips-in.

$M_s = M_{DL} + M_{LL}$

$= (1/5+1)M_{LL}$

$= 1.2(41.58) = 49.90$ kips-in.

Computation of I_{eff} for the first approximation:

* Assume a stress of f = 27 ksi in the top and bottom fibers of the section.
* Since the web was fully effective at a higher stress gradient, it will be fully effective at this stress level.

Element 1:

$w/t = 16.67$

$k = 0.50$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}$

For a compression and tension stresses of f=27 ksi, the values of E_{sc} and E_{st} are equal to 24550 ksi and 27000 ksi, respectively.

$E_r = (24550+27000)/2$ (Eq. 2.2.1-7)

$= 25775$ ksi

$\lambda = (1.052/\sqrt{0.50})(16.67)\sqrt{27/25775} = 0.803 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/0.803)/0.803 = 0.904$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.904 \times 1.000$

$= 0.904$ in.

Element 5:

$w/t = 33.33$

$k = 4$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}$

$= (1.052/\sqrt{4})(33.33)\sqrt{27/25775} = 0.567 < 0.673$

$b_d = w$ (Eq. 2.2.1-5)

$= 2.000$ (Fully effective)

Element 8:

$w/t = 33.33$

$S = 1.28\ E_o/f$ (Eq. 2.4-1)

$= 1.28\ 27000/27 = 40.48$

For $S/3 < w/t < S$,

$I_a = t^4 399\ \{[(w/t)/S]-0.33\}^3$ (Eq. 2.4.2-6)

$= (0.060)^4(399)[(33.33/40.48)-0.33]^3$

$= 0.000621$ in.^{4}

$I_s = 0.00471$ in.^{4} (calculated previously)

$I_s/I_a = 0.00471/0.000621 = 7.58 > 1$

$D = 1.144$ in. (calculated previously)

$D/w = 0.572$ (calculated previously)

For $0.25 < D/w < 0.80$

$k = [4.82-5(D/w)](I_s/I_a)^{1/2}+0.43 \leq 5.25-5(D/w)$ (Eq. 2.4.2-9)

Since $I_s/I_a > 1$

$k = 5.25-5(D/w) = 5.25-5(0.572) = 2.390$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}$

E_r = 25775 ksi for a stress of f=27 ksi.

$\lambda = (1.052/\sqrt{2.390})(33.33)\sqrt{27/25775} = 0.734 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/0.734)/0.734 = 0.954$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.954(2.000) = 1.908$ in.

Element 7:

$I_s/I_a > 1$

$d = 1.000$ in.

Assume the maximum stress in element, f = 27 ksi, although it will be actually less.

$k = 0.50$

$w/t = 16.67$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}$

$= (1.052/\sqrt{0.50})(16.67)\sqrt{27/25775} = 0.803 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/0.803)/0.803 = 0.904$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.904(1.000) = 0.904$ in.

$d'_s = 0.904$ in.

$d_s = d'_s(I_s/I_a) \leq d'_s$ (Eq. 2.4.2-11)

Since $I_s/I_a > 1$

$d_s = d'_s = 0.904$ in.

$I'_1 = (d_s)^3\sin^2\theta/12 = (0.904)^3(\sin 75.96^\circ)^2/12 = 0.058$ in.^{3}

The distance from top fiber to the centroid of the reduced section is

$y = 4-0.147-(0.904/2)\cos 14.04^\circ = 3.415$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
1	0.904	3.970	3.589	14.248	--
2	5 x 0.206 = 1.030	3.928	4.046	15.892	--
3	4 x 3.819 = 15.276	2.000	30.552	61.104	17.472
4	2 x 2.000 = 4.000	0.030	0.120	0.004	--
5	2.000	3.970	7.940	31.522	--
6	4 x 0.206 = 0.824	0.072	0.059	0.004	--
7	0.904	3.415	3.087	10.542	0.058
8	1.908	3.970	7.575	30.072	--
Sum	26.846		56.968	163.388	17.530

The distance from top fiber to the neutral axis is

$y_{cg} = Ly/L = 56.968/26.846 = 2.122$ in.

$I'_{eff} = Ly^2 + I'_1 - Ly^2_{cg}$

$= 163.388 + 17.530 - 26.846(2.122)^2$

$= 60.03$ in.^{3}

Actual $I_{eff} = tI'_{eff}$

$= (0.060)(60.03) = 3.60$ in.^{4}

$S_{eff} = I_{eff}/(d-y_{cg}) = 3.60/(4-2.122) = 1.92$ in.^{3}

$M = (1.92)(27) = 51.84$ ksi $> M_s = 49.90$ ksi N.G.

Computation of I_{eff} for the second approximation:

* Assume a stress of f=25.85 ksi in the top and bottom fibers of the section.

Element 1:

$w/t = 16.67$

$k \quad = 0.50$

$\lambda \quad = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}$

For a compression and tension stresses of f=25.85 ksi, the values of E_{sc} and E_{st} are equal to 25180 ksi and 27000 ksi, respectively.

$E_r \quad = (25180+27000)/2$ (Eq. 2.2.1-7)

$\quad = 26090$ ksi

$\lambda \quad = (1.052/\sqrt{0.50})(16.67)\sqrt{25.85/26090} = 0.781 > 0.673$

$\rho \quad = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$\quad = (1-0.22/0.781)/0.781 = 0.920$

$b \quad = \rho w$ (Eq. 2.2.1-2)

$\quad = 0.920 \times 1.000$

$\quad = 0.917$ in.

Element 5:

Fully effective at f = 27 ksi

It will also be fully effective at f = 25.85 ksi

$b = 2.000$ in.

Element 8:

$w/t = 33.33$

$S \quad = 1.28\sqrt{E_o/f}$ (Eq. 2.4-1)

$\quad = 1.28\sqrt{27000/25.85} = 41.37$

For $S/3 < w/t < S$,

$I_s/I_a > 1$ by observation from the first approximation

$D/w = 0.572$

Since $I_s/I_a > 1$

$k = 2.390$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}$

$= (1.052/\sqrt{2.390})(33.33)\sqrt{25.85/26090} = 0.714 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/0.714)/0.714 = 0.969$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.969 \times 2.000$

$= 1.938$ in.

Element 7:

$I_s/I_a > 1$

$d = 1.000$ in.

Assume a maximum stress in element, f = 25.85 ksi, although it will be actually less.

$k = 0.50$

$w/t = 16.67$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_r}$

$= (1.052/\sqrt{0.50})(16.67)\sqrt{25.85/26090} = 0.781 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/0.781)/0.781 = 0.920$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.920(1.000) = 0.920$ in.

$d'_s = 0.920$ in.

Since $I_s/I_a > 1$

$d_s = d'_s = 0.920$ in.

$I'_1 = (d_s)^3 \sin^2\theta/12 = (0.920)^3(\sin 75.96^{\circ})^2/12 = 0.061 \text{ in.}^3$

The distance from top fiber to the centroid of the reduced section is

$y = 4-0.147-(0.920/2)\cos 14.04^{o} = 3.407$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
1	0.920	3.970	3.652	14.500	--
2	5 x 0.206 = 1.030	3.928	4.046	15.892	--
3	4 x 3.819 = 15.276	2.000	30.552	61.104	17.472
4	2 x 2.000 = 4.000	0.030	0.120	0.004	--
5	2.000	3.970	7.940	31.522	--
6	4 x 0.206 = 0.824	0.072	0.059	0.004	--
7	0.920	3.407	3.134	10.679	0.061
8	1.938	3.970	7.694	30.545	--
Sum	26.908		57.197	164.250	17.533

The distance from top fiber to the neutral axis is

$y_{cg} = Ly/L = 57.197/26.908 = 2.126$ in.

$I'_{eff} = Ly^2 + I'_1 - Ly^2_{cg}$

$= 164.250 + 17.533 - 26.908(2.126)^2$

$= 60.16$ in.^{3}

Actual $I_{eff} = tI'_{eff}$

$= (0.060)(60.16) = 3.61$ in.^{4}

$S_{eff} = I_{eff}/(d-y_{cg}) = 3.61/(4-2.126) = 1.93$ in.^{3}

$M = (1.93)(25.85) = 49.90$ kips-in. $= M_s$ OK

6. Summary

Positive Bending : $\phi_b M_n$ = 74.7 kips-in.

I_{eff} = 3.64 in.^{4}

Negative Bending : $\phi_b M_n$ = 76.5 kips-in.

I_{eff} = 3.62 in.^{4}

7. Compute Factored Uniform Load

For a continuous deck over three equal spans, the maximum bending moment is negative and occurs over the interior supports. It is given by:

$M_u = 0.100 w_u L^2$

Therefore, the maximum factored uniform load is

$w_u = M_u/0.100L^2 = 76.5/0.100(10'\text{x}12''/1)^2 = 0.0531$ kips/in.

$w_u = 0.638$ kips/ft

The maximum deflection occurs at a distance of 0.446L from the exterior supports. It is given by:

$\Delta = 0.0069 w L^4/E_o I$

This deflection is limited to $\Delta = L/240$ for live load. Therefore, the maximum live load which will satisfy the deflection requirement is

$w_{LL} = E_o I/[240(0.0069)L^3] = 27000(3.64)/[240(0.0069)(10\text{x}12)^3]$

$= 0.0343$ kips/in.

$w_{LL} = 0.412$ kips/ft

$w_u = 1.2w_{DL}+1.6w_{LL}$

$= [1.2(w_{DL}/w_{LL})+1.6]w_{LL}$

$= [1.2(1/5)+1.6]w_{LL}$

$= 1.84w_{LL} = 1.84(0.412) = 0.742$ kips/ft > 0.638 kips/ft

Therefore, design flexural strength governs.

Factored Uniform Load = 0.638 kips/ft.

8. Check Shear Strength (Section 3.3.2)

The required shear strength at any section shall not exceed the design shear strength $\phi_v V_n$:

$\phi_v = 0.85$

$V_n = 4.84 E_o t^3 (G_s/G_o)/h$ (Eq. 3.3.2-1)

$v_n = V_n/(ht)$

$= 4.84 E_o (G_s/G_o)/(h/t)^2$

In the determination of the shear strength, it is necessary to select a proper value of G_s/G_o for the assumed stress from Table A12 or Figure A9 of the Standard. For the first approximation, assume a shear stress of v=27 ksi and the corresponding value of G_s/G_o is equal to 0.863. Thus,

$h/t = 3.819/0.060 = 63.65 < 200$ (Section 2.1.2 (1))

$v_n = 4.84(27000)(0.863)/(63.65)^2$

$= 27.82$ ksi > assumed stress f=27 ksi NG

For a second approximation, assume a stress of f=27.59 ksi and its corresponding value of G_s/G_o is 0.855.

$v_n = 4.84(27000)(0.855)/(63.65)^2$

$= 27.58$ ksi = assumed stress OK

Therefore, the total shear strength, V_n, for hat section is

$V_n = 4(v_n)(ht)$ (a total of 4 webs)

$= 4(27.58)(3.819 \mathrm{x} 0.060)$

$= 25.28$ kips

The design shear strength is determined as follows:

$\phi_v V_n = 0.85(25.28) = 21.49$ kips

$\phi_v V_n < 4(0.95F_{yv}ht) = 4(0.95x42x3.819x0.06) = 36.57$ kips OK

(The shear yield strength, F_{yv}, is obtained from Table A1 of the Standard.)

The maximum required shear strength is given by

$V_u = 0.600w_uL$

$= (0.600)(0.638)(10) = 3.83 \text{ kips} < (\phi_v V_n)_v = 21.49$ kips OK

9. Check Strength for Combined Bending and Shear (Section 3.3.3)

At the interior supports, there is a combination of web bending and web shear:

$\phi_b M_n = 76.5$ kips-in. $\quad M_u = 0.100w_uL^2$

$\phi_v V_n = 21.49$ kips $\quad V_u = 0.600w_uL$

For unreinforced webs

$$(M_u/\phi_b M_n)^2 + (V_u/\phi_v V_n)^2 \leq 1.0 \qquad \text{(Eq. 3.3.3-1)}$$

Solve for w_u:

$$\{[0.100w_u(10x12)^2]/76.5\}^2 + \{[0.600w_u(10x12)]/21.49\}^2 = 1.0$$

$$354.33w_u^2 + 16.16w_u^2 = 1.0$$

$$370.49w_u^2 = 1.0$$

$$w_u = 0.0520 \text{ kips/in.}$$

$$= 0.624 \text{ kips/ft.}$$

Factored Uniform Load = 0.627 kip/ft is determined for the case of combined bending and shear.

10. Check Web Crippling Strength (Section 3.3.4)

$h = 3.819$ in.

$t = 0.060$ in.

$h/t = 3.819/0.06 = 63.65 < 200$ OK

$R = 1/8$ in.

$R/t = 0.125/0.06 = 2.083 < 7$ OK

Let $N = 6$ in.

$N/t = 6/0.06 = 100 < 210$ OK

$N/h = 6/3.819 = 1.57 < 3.5$ OK

Table 2 of the Standard is used to check the web crippling requirements. For end reactions, use Eq. (3.3.4-2). For interior reaction, use Eq. (3.3.4-4).

$k = F_y/33 = 50/33 = 1.515$ (Eq. 3.3.4-21)

$C_1 = (1.22-0.22k)k$ (Eq. 3.3.4-10)

$= [1.22-0.22(1.515)](1.515) = 1.343$

$C_2 = (1.06-0.06R/t)$ (Eq. 3.3.4-11)

$= [1.06-0.06(2.083)] = 0.935 < 1.0$ OK

$C_3 = (1.33-0.33k)k$ (Eq. 3.3.4-12)

$= [1.33-0.33(1.515)](1.515) = 1.258$

$C_4 = (1.15-0.15R/t) \leq 1.0$ but not less than 0.50 (Eq. 3.3.4-13)

$(1.15-0.15R/t) = [1.15-0.15(2.083)] = 0.838 \leq 1.0$ OK

> 0.50 OK

$C_4 = 0.838$

$\theta = 75.96^{o}$

$C_\theta = 0.7+0.3(\theta/90)^2$ (Eq. 3.3.4-20)

$= 0.7+0.3(75.96/90)^2 = 0.914$

a) For end reaction:

$P_n = t^2C_3C_4C_\theta[217-0.28(h/t)][1+0.01(N/t)]$ (Eq. 3.3.4-2)

$= (0.06)^2(1.258)(0.838)(0.914)[217-0.28(63.65)]$

$\quad x[1+0.01(100)] = 1.38$ kips/web

Total P_n for section:

$P_n = (4 \text{ webs})(1.38 \text{ k/web}) = 5.52$ kips

$\phi_w = 0.70$

$\phi_w P_n = 0.70(5.52) = 3.86$ kips

End reaction is given by

$R = 0.400 w_u L$

$= (0.400)(0.627)(10) = 2.51 \text{ kips} < \phi_w P_n = 3.86$ kips OK

b) For interior reaction:

$P_n = t^2 C_1 C_2 C_\theta [538-0.74(h/t)][1+0.007(N/t)]$ (Eq. 3.3.4-4)

$= (0.06)^2(1.343)(0.935)(0.914)[538-0.74(63.65)]$

$\quad x[1+0.007(100)] = 3.45$ kips/web

Total P_n for section:

$P_n = (4 \text{ webs})(3.45 \text{ k/web}) = 13.80$ kips

$\phi_w = 0.70$

$\phi_w P_n = 0.70(13.80) = 9.66$ kips

Interior reaction is given by

$R = 1.10 w_u L$

$= (1.10)(0.627)(10) = 6.90 \text{ kips} < \phi_w P_n = 9.66$ kips OK

EXAMPLE 11.2 DECK SECTION (ASD)

Rework Example 11.1 by using the Allowable Stress Design (ASD) method to determine the allowable bending moment based on initiation of yielding. Also determine the effective moment of inertia for deflection determination at the allowable momnet. Compute the allowable uniform load as controlled either by bending or deflection.

Solution:

1. Element Properties:

   See section properties calculated in Example 11.1.

2. Section Modulus for Load Determination - Positive Bending (Based on Initiation of Yielding)

   The effective section properties calculated by the ASD method are the same as those determined in Example 11.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

   $M_a = M_n/\Omega$ (E-1)

   $M_n = 83.0$ kips-in.

   $\Omega = 1.85$

   $M_a = 83.0/1.85 = 44.87$ kips-in.

3. Moment of Inertia for Deflection Determination - Positive Bending

   For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 11.1 for the LRFD method, except that the computed moment M (= fxS_e) should be equal to M_a.

   From the results of Example 11.1.(3), it was found that for a compression stress of f=28.66 ksi, the section is fully effective.

Then, for an assumed stress of f=26.39 ksi (less than f = 28.66 ksi), the section will also be fully effective, i.e.,

$S_e = I_x/y_{cg} = 3.64/2.138 = 1.70 \text{ in.}^3$

$M = fS_e = 26.39x1.70 = 44.87 \text{ kips-in.} = M_a$ OK

Therefore, use $I_{eff} = 3.64 \text{ in.}^4$ for deflection calculation.

4. Section Modulus for Load Determination - Negative Bending (Based on Initiation of Yielding)

The effective section properties calculated by the ASD method are the same as those determined in Example 11.1 for the LRFD method. Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

$$M_a = M_n/\Omega \qquad \text{(E-1)}$$

$M_n = 90.0$ kips-in.

$\Omega = 1.85$

$M_a = 90.0/1.85 = 48.65$ kips-in.

5. Moment of Inertia for Deflection Determination - Negative Bending

For deflection determination on the basis of the ASD method, the effective moment of inertia is determined by the same procedures given in Example 11.1 for the LRFD method, except that the computed moment M (= fxS_e) should be equal to M_a.

For an assumed stress of f=25.20 ksi, it is found that the section modulus is likely to be the same as calculated in Example 11.1.(5), i.e.,

$S_e = I_x/y_{cg} = 3.61/(4.0-2.126) = 1.93 \text{ in.}^3$

$M = fS_e = 25.20x1.93 = 48.65 \text{ kips-in.} = M_a$ OK

Therefore, use $I_{eff} = 3.61 \text{ in.}^4$ for deflection calculation.

6. Summary

Positive Bending : M_a = 44.87 kips-in.

I_{eff} = 3.64 in.^{4}

Negative Bending : M_a = 48.65 kips-in.

I_{eff} = 3.61 in.^{4}

7. Compute Allowable Uniform Load

For a continuous deck over three equal spans, the maximum bending moment is negative and occurs over the interior supports. It is given by:

$$M_a = 0.100wL^2$$

Therefore, the maximum factored uniform load is

$$w = M_a/0.100L^2 = 44.87/0.100(10'\text{x}12''/1)^2 = 0.0312 \text{ kip/in.}$$

$$w = 0.374 \text{ kip/ft}$$

The maximum deflection occurs at a distance of 0.446L from the exterior supports. It is given by:

$$\Delta = 0.0069wL^4/E_oI$$

This deflection is limited to Δ = L/240 for live load. Therefore, the maximum live load which will satisfy the deflection requirement is

$$w_{LL} = E_oI/[240(0.0069)L^3] = 27000(3.64)/[240(0.0069)(10\text{x}12)^3]$$

$$= 0.0343 \text{ kip/in.}$$

$$w_{LL} = 0.412 \text{ kip/ft}$$

Therefore, allowable bending strength governs.

Allowable Uniform Load = 0.374 kip/ft.

8. Check Shear Strength

The required shear strength at any section shall not exceed the allowable shear strength V_a:

Ω = 1.85 (for single web)

$v_n = 4.84(27000)(0.855)/(63.65)^2$

$= 27.58$ ksi (from Example 11.1.(8))

Therefore, the total shear strength, V_n, for hat section is

$V_n = 4(v_n)(ht)$ (a total of 4 webs)

$= 4(27.58)(3.819x0.060)$

$= 25.28$ kips

The allowable shear strength is determined as follows:

$V_a = V_n/\Omega = 25.28/1.85 = 13.66$ kips

$< 4(F_{yv}ht)/1.64 = 4(42x3.819x0.06)/1.64 = 23.47$ kips OK

The maximum required shear strength is given by

$V = 0.600wL$

$= (0.600)(0.374)(10) = 2.24$ kips < 13.66 kips OK

. Check Strength for Combined Bending and Shear

At the interior supports, there is a combination of web bending and web shear:

$M_a = 48.65$ kips-in. $M = 0.100wL^2$

$V_a = 13.66$ kips $V = 0.600wL$

For unreinforced webs

$(M/M_a)^2 + (V/V_a)^2 \leq 1.0$

Solve for w:

$[0.100w(10x12)^2 /48.65]^2 + [0.600w(10x12) /13.66]^2 = 1.0$

$$876.11w^2+27.78w^2 = 1.0$$

$$903.89w^2 = 1.0$$

$$w = 0.0333 \text{ kip/in.}$$

$$= 0.399 \text{ kip/ft.}$$

Allowable Uniform Load = 0.399 kip/ft is determined for the case of combined bending and shear.

10. Check Web Crippling Strength

The nominal web crippling strengths are calculated in Example 11.1.(10) as follows:

a) For end reaction:

$$P_n = t^2C_3C_4C_\theta[217-0.28(h/t)][1+0.01(N/t)] \qquad \text{(Eq. 3.3.4-2)}$$

$$= (0.06)^2(1.258)(0.838)(0.914)[217-0.28(63.65)]$$

$$\times[1+0.01(100)] = 1.38 \text{ kips/web}$$

Total P_n for section:

P_n = (4 webs)(1.38 k/web) = 5.52 kips

Ω = 2.0 (for single web)

$P_a = P_n/\Omega = 5.52/2.0 = 2.76$ kips

End reaction is given by

$R = 0.400wL$

$= (0.400)(0.374)(10) = 1.50 \text{ kips} < P_a = 2.76$ kips OK

b) For interior reaction:

$$P_n = t^2C_1C_2C_\theta[538-0.74(h/t)][1+0.007(N/t)] \qquad \text{(Eq. 3.3.4-4)}$$

$$= (0.06)^2(1.343)(0.935)(0.914)[538-0.74(63.65)]$$

$$\times[1+0.007(100)] = 3.45 \text{ kips/web}$$

Total P_n for section:

P_n = (4 webs)(3.45 k/web) = 13.80 kips

$\Omega \quad = 2.0$

$P_a \quad = P_n/\Omega = 13.8/2.0 = 6.90$ kips

Interior reaction is given by

$R \quad = 1.10wL$

$\quad = (1.10)(0.374)(10) = 4.11$ kips $< P_a = 6.90$ kips OK

EXAMPLE 12.1 CYLINDRICAL TUBULAR SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design flexural strength, $\phi_b M_n$, for the section shown in Figure 12.1. Use Type 301, 1/4-Hard stainless steel.

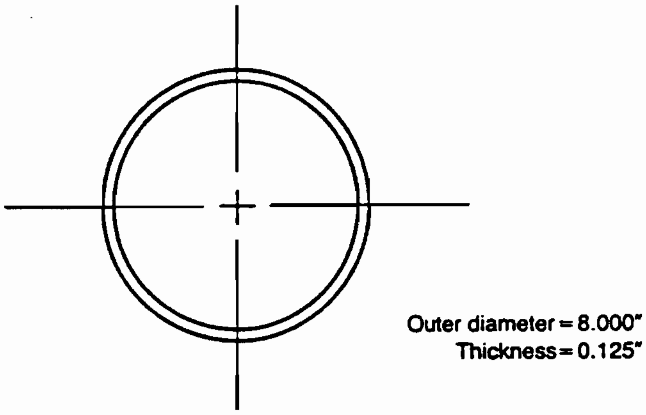


Figure 12.1 Section for Example 12.1

Solution:

Ratio of outside diameter to wall thickness,

$D/t = 8.000/0.125 = 64.00$

$0.881E_o/F_y = 0.881(27000/50) = 475.7$

Since $D/t < 0.881E_o/F_y$, the ASCE Specification can be used.

The design requirement for cylindrical tubular members is based on Section 3.6.1 of the Standard.

Because $0.112E_o/F_y = 0.112(27000/50) = 60.48$ and

$0.112E_o/F_y < D/t < 0.881E_o/F_y$,

$M_n = K_c F_y S_f$ (Eq. 3.6.1-2)

where

$S_f = \pi\left[(O.D.)^4-(I.D.)^4\right]/32(O.D.)$

$= \pi\left[(8)^4-(7.75)^4\right]/32(8)$

$= 5.995$ in.^{3}

$K_c = (1-C)(E_o/F_y)/[(8.93-\lambda_c)(D/t)] + 5.882C/(8.93-\lambda_c)$ (Eq. 3.6.1-3)

$C = F_{pr}/F_y$

$\lambda_c = 3.048C$

From Table A17 of the Standard, the ratio of F_{pr}/F_y is equal to 0.5 in longitudinal compression for Type 301, 1/4-Hard stainless steel.

Therefore,

$K_c = (1-0.5)(27000/50)/[(8.93-3.048x0.5)(64.0)]$

$+(5.882x0.5)/(8.93-3.048x0.5)$

$= 0.967$

$M_n = 0.967(50)(5.995)$

$= 289.86$ kips-in.

$\Phi_b = 0.90$

$\Phi_b M_n = 0.90 \times 289.86 = 260.90$ kips-in.

EXAMPLE 12.2 CYLINDRICAL TUBULAR SECTION (ASD)

Rework Example 12.1 by using the Allowable Stress Design (ASD) method.

Solution:

Calculation of the allowable moment, M_a:

The effective section properties calculated by the ASD method are the same as those determined in Example 12.1 for the LRFD method.

Therefore, the allowable moment can be determined in accordance with Appendix E of the Standard as follows:

Ω = 1.85 (Safety Factor stipulated in Table E of the Standard)

M_n = 289.86 kips-in. (obtained from Example 12.1)

$M_a = M_n/\Omega$ (Eq. E-1)

= 289.86/1.85

= 156.7 kips-in.

M_{max} = 25.92 kips-in. < 27.57 kips-in. OK

EXAMPLE 13.1 FLANGE CURLING (LRFD)

By using the LRFD criteria, determine the amount of curling for the compression flange of the channel section used in Example 1.1.

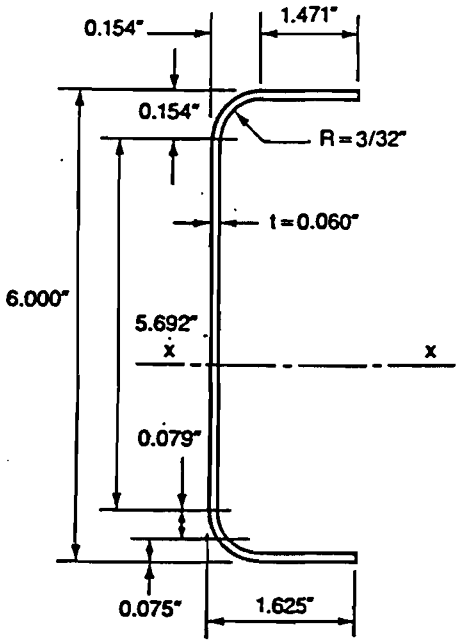


Figure 13.1 Section for Example 13.1

Solution:

1. Determination of the design flexural strength, $\phi_b M_n$:

   The elastic section modulus of the effective section, S_e, calculated with the extreme compression or tension fiber at F_y is determined in Example 1.1.

   $S_e = 0.711$ in.^{3}

   $M_n = S_e F_y$ (Eq. 3.3.1.1-1)

   $= 0.711 \times 50 = 35.55$ kips-in.

   $\phi_b = 0.85$

$\phi_b M_n = 0.85 \times 35.55 = 30.22$ kips-in.

2. Determination of the average stress in compression flange, f_{av}, at the service moment M_s:

$$\phi_b M_n = 1.2M_{DL} + 1.6M_{LL}$$
$$= [1.2(M_{DL}/M_{LL})+1.6]M_{LL}$$
$$= [1.2(1/5)+1.6]M_{LL}$$
$$= 1.84M_{LL}$$
$$M_{LL} = \phi_b M_n/1.84 = 30.22/1.84 = 16.42 \text{ kips-in.}$$
$$M_s = M_{DL} + M_{LL}$$
$$= (1/5+1)M_{LL}$$
$$= 1.2(16.42) = 19.70 \text{ kips-in.}$$

where

M_{DL} = Moment determined on the basis of nominal dead load

M_{LL} = Moment determined on the basis of nominal live load

The procedure is iterative: one assumes the actual compressive stress f under this service moment M_s. Knowing f, proceeds as usual to obtain S_e and checks to see if ($f \times S_e$) is equal to M_s as it should. If not, reiterate until one obtains the desired level of accuracy.

For the first approximation, assume a compression stress of $f = 25$ ksi in the top fiber of the section and that the web is fully effective.

Compression flange: $k = 0.50$ (for unstiffened compression element, see Section 2.3.1)

$w/t = 1.471/0.060 = 24.52 < 50$ OK (Section 2.1.1-(1)-(iii))

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

The initial modulus of elasticity, E_o, for Type 301 stainless steel is obtained from Table A4 of the Standard, i.e., $E_o = 27000$ ksi.

$\lambda = (1.052/\sqrt{0.50})(24.52)\sqrt{25/27000} = 1.110 > 0.673$

$\rho = [1-(0.22/\lambda)]/\lambda$ (Eq. 2.2.1-3)

$= [1-(0.22/1.110)]/1.110 = 0.722$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.722 \times 1.471$

$= 1.062$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Web	5.692	3.000	17.076	51.228	15.368
Upper Corner	0.195	0.075	0.015	0.001	--
Lower Corner	0.195	5.925	1.155	6.846	--
Compression Flange	1.062	0.030	0.032	0.001	--
Tension Flange	1.471	5.970	8.782	52.428	--
Sum	8.615		27.060	110.504	15.368

Distance from top fiber to x-axis is

$y_{cg} = 27.060/8.615 = 3.141$ in.

Since the distance from top compression fiber to the neutral axis is greater than one half the beam depth, a compression stress of 25 ksi will govern as assumed (i.e., initial yield is in compression).

To check if web is fully effective (Section 2.2.2):

$f_1 = [(3.141-0.154)/3.141]x25 = 23.77$ ksi(compression)

$f_2 = -[(2.859-0.154)/3.141]x25 = -21.53$ ksi(tension)

$\Psi = f_2/f_1 = -21.53/23.77 = -0.906$

$k = 4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

$= 4+2[1-(-0.906)]^3+2[1-(-0.906)]$

$= 21.660$

$h = w = 5.692$ in., $h/t = w/t = 5.692/0.060 = 94.87$

$h/t = 94.87 < 200$ OK (Section 2.1.2-(1))

$\lambda = (1.052/\sqrt{21.66})(94.87)\sqrt{23.77/27000} = 0.636 > 0.673$

$b_2 = b_e/2$ (Eq. 2.2.2-2)

$= 5.692/2 = 2.846$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 5.692/[3-(-0.906)] = 1.457$ in.

The effective widths, b_1 and b_2, of web are defined in Figure 2 of the Standard.

$b_1+b_2 = 1.457 + 2.846 = 4.303$ in.

Compression portion of the web calculated on the basis of the effective section = $y_{cg} - 0.154 = 3.141 - 0.154 = 2.987$ in.

Since $b_1+b_2 = 4.303$ in. > 2.987 in., b_1+b_2 shall be taken as 2.987 in. This verifies the assumption that the web is fully effective.

$I'_x = Ly^2 + I'_1 - Ly^2_{cg}$

$= 110.504 + 15.368 - 8.615(3.141)^2$

$= 40.877 \text{ in.}^3$

Actual $I_x = I'_x t$

$= 40.877 \times 0.060$

$= 2.453 \text{ in.}^4$

$S_e = I_x / y_{cg}$

$= 2.453/3.141$

$= 0.781 \text{ in.}^3$

$M_n = S_e F_y$ (Eq. 3.3.1.1-1)

$= 0.781 \times 25$

$= 19.53$ kips-in. $= M_s = 19.70$ kips-in. (close enough)

Therefore,

$f_{av} = f\,(b/w) = 25.0 \times (1.062/1.471) = 18.05$ ksi

3. Determination of the curling of the compression flange, c_f.

$w_f = 1.625 - 0.06 = 1.565$ in.

$w_f = \sqrt{0.061tdE/f_{av}}\,\sqrt[4]{(100c_f/d)}$ (Eq. 2.1.1-1)

$1.565 = \sqrt{0.061(0.06)(6)(27000)/18.05}\,\sqrt[4]{100c_f/6}$

$= 5.731\,\sqrt[4]{16.67c_f}$

$\sqrt[4]{16.67c_f} = 1.565/5.731$

$16.67c_f = (1.565/5.731)^4$

$c_f = (1.565/5.731)^4/16.67 = 0.00033$ in.

EXAMPLE 13.2 FLANGE CURLING (ASD)

Rework Example 13.1 by using the ASD method.

Solution:

1. Determination of the allowable bending moment, M_a:

The nominal bending strength, M_n, is obtained from Example 13.1 as follows:

$M_n = S_e F_y = 0.711 \times 50 = 35.55$ kips-in.

Therefore, the allowalbe moment:

$\Omega = 1.85$

$M_a = 35.55/1.85 = 19.22$ kips-in.

2. Determination of the average stress in compression flange, f_{av}, at the allowable moment M_a:

Assume that a compression stress of f=25 ksi in the top fiber of the section and that the web is fully effective. Therefore, from the calculation of Example 13.1.(2):

$M = S_e f = 0.781x25$

$= 19.53$ kips-in. $= M_a = 19.22$ kips-in. (close enough)

Therefore,

$f_{av} = f\ (b/w) = 25.0x(1.062/1.471) = 18.05$ ksi

3. Determination of the curling of the compression flange, c_f.

$w_f = 1.625 - 0.06 = 1.565$ in.

$t = 0.06$

$d = 6$

$$1.565 = \sqrt{0.061(0.06)(6)(27000)/18.05}\ \sqrt[4]{100c_f/6}$$

$$= 5.731\ \sqrt[4]{16.67c_f}$$

$$\sqrt[4]{16.67c_f} = 1.565/5.731$$

$$16.67c_f = (1.565/5.731)^4$$

$$c_f = (1.565/5.731)^4/16.67 = 0.00033 \text{ in.}$$

EXAMPLE 14.1 SHEAR LAG (LRFD)

For the tubular section shown in Fig. 14.1, determine the design flexural strength, $\phi_b M_n$, if the member is to be used as a simply supported beam and to carry a concentrated load at midspan. Assume that the span length is 3 ft. and the section material is Type 316, 1/4-Hard, stainless steel.

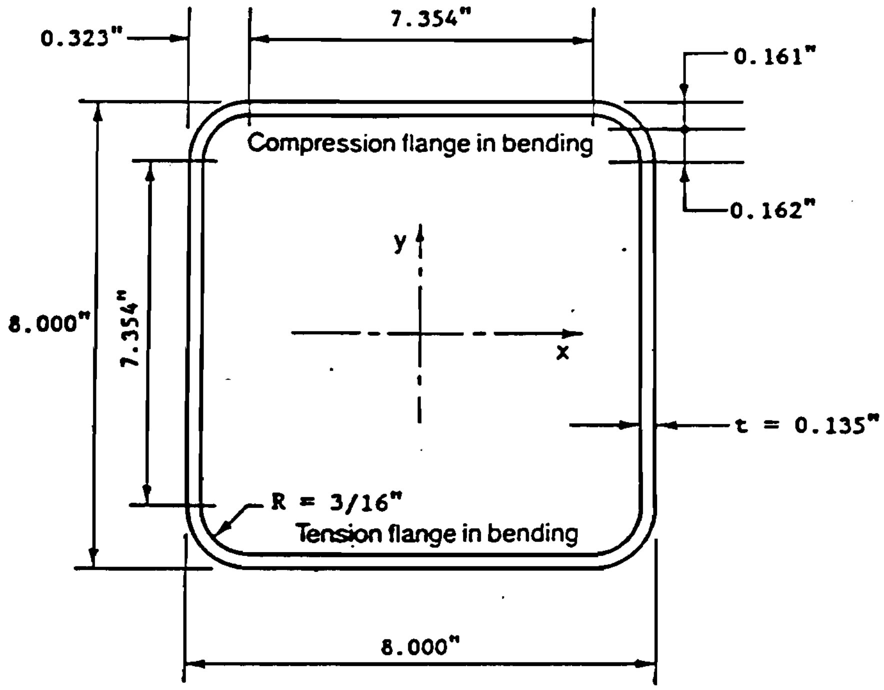


Figure 14.1 Section for Example 14.1

Solution:

1. Determination of the nominal moment, M_n, based on initiation of yielding (Section 3.3.1.1).

Properties of 90° corners:

$r = R + t/2 = 3/16 + 0.135/2 = 0.255$ in.

Length of arc, $u = 1.57r = 1.57 \times 0.255 = 0.400$ in.

Distance of c.g. from center of radius,

$c = 0.637r = 0.637 \times 0.255 = 0.162$ in.

Computation of I_x:

For the first approximation, assume a compression stress of $f = F_y = 50$ ksi in the compression flange, and that the webs are fully effective.

Compression flange: $k = 4.00$ (stiffened compression element supported by a web on each longitudinal edge)

$w/t = 7.354/0.135 = 54.47 < 400$ OK (Section 2.1.1-(1)-(ii))

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{4.00})(54.47)\sqrt{50/27000} = 1.233 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/1.233)/1.233 = 0.666$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.666 \times 7.354$

$= 4.898$ in.

Effective section properties about x axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Webs	14.708	4.000	58.832	235.328	66.286
Upper Corners	0.800	0.161	0.129	0.021	--
Lower Corners	0.800	7.839	6.271	49.160	--
Compression Flange	4.898	0.068	0.333	0.023	--
Tension Flange	7.354	7.933	58.339	462.806	--
Sum	28.560		123.904	747.338	66.286

Distance from top fiber to x-axis is

y_{cg} = 123.904/28.560 = 4.338 in.

Since the distance of top compression fiber from neutral axis is greater than one half the beam depth, a compression stress of 50 ksi will govern as assumed (i.e., initial yielding is in compression).

To check if webs are fully effective (Section 2.2.2):

$f_1 = [(4.338-0.323)/4.338]x50 = 46.28$ ksi(compression)

$f_2 = -[(3.662-0.323)/4.338]x50 = -38.49$ ksi(tension)

$\Psi = f_2/f_1 = -38.49/46.28 = -0.832$

$k = 4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

$= 4+2[1-(-0.832)]^3+2[1-(-0.832)]$

$= 19.961$

$h = w = 7.354$ in., $h/t = w/t = 7.354/0.135 = 54.47$

$h/t = 54.47 < 200$ OK (Section 2.1.2-(1))

$\lambda = (1.052/\sqrt{19.961})(54.47)\sqrt{46.28/27000} = 0.531 < 0.673$

$b_e = w$ (Eq. 2.2.1-1)

$= 7.354$ in.

$b_2 = b_e/2$ (Eq. 2.2.2-2)

$= 7.354/2 = 3.677$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 7.354/[3-(-0.843)] = 1.914$ in.

Compression portion of the web calculated on the basis of the effective section = $y_{cg} - 0.323 = 4.338 - 0.323 = 4.015$ in.

Since $b_1+b_2 = 5.591$ in. > 4.015 in., b_1+b_2 shall be taken as 4.015 in.. This verifies the assumption that the webs are fully effective.

$I'_x = Ly^2+I'_1-Ly^2_{cg}$

$= 747.338 + 66.286 - 28.560(4.338)^2$

$= 276.175$ in.^{3}

Actual $I_x = I'_x t$

$= 276.175 \times 0.135$

$= 37.284$ in.^{4}

$S_e = I_x/y_{cg}$

$= 37.284/4.338$

$= 8.595$ in.^{3}

$M_n = S_eF_y = 8.595 \times 50$

$= 429.75$ kips-in.

2. Determination of the nominal moment, M_n, based on shear lag consideration (Section 2.1.1(3)).

w_f = (8-2x0.135)/2 = 3.865 in.

L/w_f = 3x12/3.865 = 9.314 < 30

Because the L/w_f ratio is less than 30, and the member carries a concentrated load, consideration for shear lag is needed.

From Table 1 of the Standard:

L/w_f = 10, effective design width/actual width = 0.73

L/w_f = 8, effective design width/actual width = 0.67

L/w_f = 9.314, effective design width/actual width = ?

$$(10-9.314)/(9.314-8) = (0.73-x)/(x-0.67)$$

$$0.686(x-0.67) = 1.314(0.73-x)$$

$$x = 0.709$$

Therefore, the effective design widths of compression and tension flanges between webs are

0.709(8-2x0.135) = 5.481 in.

b = 5.481-2R = 5.481-2(3/16) = 5.106 in.

Because of symmetry and assume webs are fully effective,

y_{cg} = 4.000 in.

Effective section properties about x-axis:

L = 28.560-4.898-7.354+5.106x2 = 26.520 in.

Ly^2 = 747.338-0.023-462.806+5.106$(0.068)^2$+5.106$(7.933)^2$

= 605.866 in.^{3}

I'_1 = 66.286 in.^{3}

To check if webs are fully effective:

f_1 = $[(4.000-0.323)/4.000]x50 = 45.96$ ksi

f_2 = -45.96 ksi

Ψ = $-45.96/45.96 = -1.000$

k = $4+2[1-(-1.000)]^3+2[1-(-1.000)] = 24.000$

λ = $(1.052/\sqrt{24.0})(54.47)\sqrt{45.96/27000} = 0.483 < 0.673$

b_e = 7.354 in.

b_2 = 7.354/2 = 3.677 in.

b_1 = $7.354/[3-(-1.000)]$ = 1.839 in.

Compression portion of the web calculated on the basis of the effective section = 4.000 - 0.323 = 3.677 in..

Since b_1+b_2 = 5.516 in. > 3.677 in., b_1+b_2 shall be taken as 3.677 in.. This verifies the assumption that the webs are fully effective.

I'_x = $605.866 + 66.286 - 26.520(4.000)^2$

= 247.832 in.^{3}

Actual I_x = 247.832 x 0.135

= 33.457 in.^{4}

S_e = 33.457/4.000 = 8.364 in.^{3}

M_n = 8.364 x 50

= 418.20 kips-in. < 429.75 kips-in. (initial yielding)

3. Determination of the design flexural strength, $\phi_b M_n$.

M_n = 418.20 kips-in.

ϕ_b = 0.90

$\phi_b M_n$ = 0.90 x 418.20 = 376.38 kips-in.

EXAMPLE 14.2 SHEAR LAG (ASD)

Rework Example 14.1 to determine the allowable bending moment for the tubular section.

Solution:

1. Determination of the nominal moment, M_n, based on initiation of yielding

$M_n = S_eF_y = 8.595 \times 50$

$= 429.75$ kips-in. (from Example 14.1)

2. Determination of the nominal moment, M_n, based on shear lag consideration.

$w_f = (8-2x0.135)/2 = 3.865$ in.

$L/w_f = 3x12/3.865 = 9.314 < 30$

Because the L/w_f ratio is less than 30, and the member carries a concentrated load, consideration for shear lag is needed.

$S_e = 33.457/4.000 = 8.364$ in.^{3} (from Example 14.1)

$M_n = 8.364 \times 50$

$= 418.20$ kips-in. $<$ 429.75 kips-in. (initial yielding)

3. Determination of the allowable bending strength, M_a.

$M_n = 418.20$ kips-in.

$\Omega = 1.85$

$M_a = 418.20/1.85 = 226.05$ kips-in.

EXAMPLE 15.1 C-SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for C-section as shown in Figure 15.1. Use Type 304 stainless steel, 1/4-Hard.

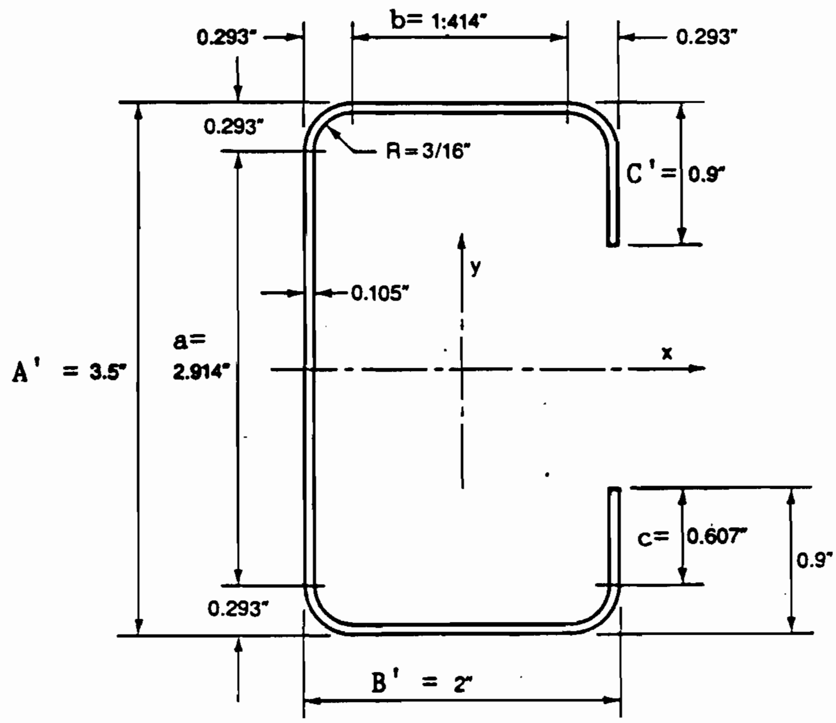


Figure 15.1 Section for Example 15.1

Given:

1. Section: 3.5" x 2.0" x 0.105" channel with stiffened flanges.

2. $K_xL_x = K_yL_y = K_tL_t = 6$ ft.

Solution:

The following equations used for computing the sectional properties for channel with lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

1. Basic parameters used for calculating the section properties:

$r = R+t/2 = 3/16+0.105/2 = 0.240$ in.

From the sketch $a = 2.914$ in., $b = 1.414$ in., $c = 0.607$ in.,

$A' = 3.5$ in., $B' = 2.0$ in., $C' = 0.9$ in.,

$\alpha = 1.00$ (Since the section has lips)

$\bar{a} = A'-t = 3.5-0.105 = 3.395$ in.

$\bar{b} = B'-[t/2+\alpha t/2] = B'-t = 2-0.105 = 1.895$ in.

$\bar{c} = \alpha[C'-t/2] = C'-t/2 = 0.9-0.105/2 = 0.848$ in.

$u = 1.57r = 1.57 \times 0.240 = 0.377$ in.

2. Area:

$$A = t[a+2b+2u+\alpha(2c+2u)] = t[a+2b+2c+4u]$$

$$= 0.105[2.914+2\times1.414+2\times0.607+4\times0.377]$$

$$= 0.889 \text{ in.}^2$$

3. Moment of inertia about x-axis:

$$I_x = 2t\{0.0417a^3+b(a/2+r)^2+u(a/2+0.637r)^2+0.149r^3$$

$$+\alpha[0.0833c^3+(c/4)(a-c)^2+u(a/2+0.637r)^2+0.149r^3]\}$$

$$= 2t[0.0417a^3+b(a/2+r)^2+2u(a/2+0.637r)^2+0.298r^3$$

$$+0.0833c^3+(c/4)(a-c)^2]$$

$$= 2\times0.105[0.0417(2.914)^3+1.414(2.914/2+0.240)^2$$

$$+2\times0.377(2.914/2+0.637\times0.240)^2+0.298(0.240)^3$$

$$+0.0833(0.607)^3+(0.607/4)(2.914-0.607)^2]$$

$$= 1.657 \text{ in.}^4$$

4. Distance from centroid of section to centerline of web:

$$\bar{x} = (2t/A)\{b(b/2+r)+u(0.363r)+\alpha[u(b+1.637r)+c(b+2r)]\}$$

$$= [(2x0.105)/0.889]\{1.414(1.414/2+0.240)+0.377(0.363x0.240)$$

$$+0.377(1.414+1.637x0.240)+0.607(1.414+2x0.240)\}$$

$$= 0.757 \text{ in.}$$

5. Moment of inertia about y-axis:

$$I_y = 2t\{b(b/2+r)^2+0.0833b^3+0.356r^3+\alpha[c(b+2r)^2$$

$$+u(b+1.637r)^2+0.149r^3]\}-A(\bar{x})^2$$

$$= 2x0.105\{1.414(1.414/2+0.240)^2+0.0833(1.414)^3$$

$$+0.356(0.240)^3+0.607(1.414+2x0.240)^2$$

$$+0.377(1.414+1.637x0.240)^2+0.149(0.240)^3\}-0.889(0.757)^2$$

$$= 0.524 \text{ in.}^4$$

6. Distance from shear center to centerline of web:

$$m = (\bar{b}t/12I_x)[6\bar{c}(\bar{a})^2+3\bar{b}(\bar{a})^2-8(\bar{c})^3]$$

$$= [(1.895x0.105)/(12x1.657)][6x0.848(3.395)^2$$

$$+3x1.895(3.395)^2-8(0.848)^3]$$

$$= 1.194 \text{ in.}$$

7. Distance from centroid to shear center:

$$x_o = -(\bar{x}+m) = -(0.757+1.194)$$

$$= -1.951 \text{ in.}$$

8. St. Venant torsion constant:

$$J = (t^3/3)[a+2b+2u+\alpha(2c+2u)]$$

$$= [(0.105)^3/3][2.914+2x1.414+4x0.377+2x0.607]$$

$$= 0.003266 \text{ in.}^4$$

9. Warping Constant:

$$C_w = (t^2/A)\{[\bar{x}A(\bar{a})^2]/t[(\bar{b})^2/3+m^2-m\bar{b}]+(A/3t)[(m)^2(\bar{a})^3$$
$$+(\bar{b})^2(\bar{c})^2(2\bar{c}+3\bar{a})]-(I_x m^2/t)(2\bar{a}+4\bar{c})+[m(\bar{c})^2/3][8(\bar{b})^2(\bar{c})$$
$$+2m(2\bar{c}(\bar{c}-\bar{a})+\bar{b}(2\bar{c}-3\bar{a}))]+[(\bar{b})^2(\bar{a})^2/6][(3\bar{c}+\bar{b})(4\bar{c}+\bar{a})-6(\bar{c})^2]$$
$$-[m^2(\bar{a})^4]/4\}$$

$$= [(0.105)^2/0.889]\{[0.757\text{x}0.889\text{x}(3.395)^2]/0.105[(1.895)^2/3$$
$$+(1.194)^2-1.194\text{x}1.895]+0.889/(3\text{x}0.105)[(1.194)^2(3.395)^3$$
$$+(1.895)^2(0.848)^2(2\text{x}0.848+3\text{x}3.395)]$$
$$-[1.657\text{x}(1.194)^2]/0.105(2\text{x}3.395+4\text{x}0.848)$$
$$+[1.194(0.848)^2]/3\ 8(1.895)^2(0.848)$$
$$+2\text{x}1.194(2\text{x}0.848(0.848-3.395)+1.895(2\text{x}0.848-3\text{x}3.395))]$$
$$+(1.895)^2(3.395)^2/6][(3\text{x}0.848+1.895)(4\text{x}0.848+3.395)$$
$$-6(0.848)^2]-[(1.194)^2(3.395)^4/4]\}$$
$$= 2.050 \text{ in.}^6$$

10. Radii of gyration:

$$r_x = \sqrt{(I_x/A)} = \sqrt{(1.657/0.889)} = 1.365 \text{ in.}$$
$$r_y = \sqrt{(I_y/A)} = \sqrt{(0.524/0.889)} = 0.768 \text{ in.}$$
$$(K_yL_y)/r_y = (6\text{x}12)/0.768 = 93.75 < 200$$
$$r_o^2 = r_x^2+r_y^2+x_o^2 = (1.365)^2+(0.768)^2+(-1.951)^2$$
$$= 6.259 \text{ in.}^2$$

11. Torsional-flexural constant:

$$\beta = 1-(x_o/r_o)^2 \qquad \text{(Eq. 3.4.3-4)}$$
$$= 1-(-1.951)^2/6.259$$

$= 0.392$

12. Determination of F_n: (Section 3.4 of the Standard)

For this singly symmetric section (x-axis is the axis of symmetry), F_n shall be taken as the smaller of either (Eq. 3.4.1-1) or (Eq. 3.4.3-1):

a. For Flexural Buckling:

$$(F_n)_1 = (\pi^2 E_t)/(K_y L_y/r_y)^2 \quad \text{(Eq. 3.4.1-1)}$$

In the determination of the flexural buckling stress, it is necessary to select a proper value of E_t from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of f=20 ksi. From Table A13, the corresponding value of E_t is found to be equal to 27000 ksi. Thus,

$$(F_n)_1 = (\pi^2 x27000)/(93.75)^2$$

$$= 30.32 \text{ ksi} > \text{assumed stress } f=20 \text{ ksi}$$

Because the computed stress is larger than the assumed value, the further successive approximation is needed.

Assume f=22.7 ksi, and

$$E_t = 20250 \text{ ksi}$$

$$(F_n)_1 = (\pi^2 x20250)/(93.75)^2$$

$$= 22.74 \text{ ksi} = \text{assumed stress } f=22.7 \text{ ksi} \quad \text{OK}$$

Alternatively, the tangent modulus E_t can be determined by using the Modified Ramberg-Osgood equation as given in Appendix B of the Standard as follows:

$$E_t = (E_o F_y)/[F_y+0.002nE_o(f/F_y)^{n-1}] \quad \text{(Eq. B-2)}$$

From Table B in the Standard, the coefficient n is equal to 4.58 for Type 304, 1/4-Hard stainless steel. Thus, for an assumed compression stress of f = 23.1 ksi,

$$E_t = (27000x50)/[50+0.002x4.58x27000x(23.1/50)^{3.58}]$$
$$= 20584 \text{ ksi}$$

Therefore,

$$(F_n)_1 = (\pi^2 x20584)/(93.75)^2$$
$$= 23.11 \text{ ksi} = \text{assumed stress } f=23.1 \text{ ksi} \quad \text{OK}$$

It is found that for this example, the flexural buckling stress determined by using Eq. B-2 is apporximately 2 % larger than that by using the tabulated value.

b. For Torsional-Flexural Buckling:

$$(F_n)_2 = (1/2\beta)[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}] \qquad \text{(Eq. 3.4.3-1)}$$

where

$$\sigma_{ex} = [(\pi^2E_o)/(K_xL_x/r_x)^2](E_t/E_o) \qquad \text{(Eq. 3.4.3-3)}$$
$$\sigma_t = [1/(Ar_o^2)][G_oJ+(\pi^2E_oC_w)/(K_tL_t)^2](E_t/E_o) \qquad \text{(Eq. 3.4.2-1)}$$
$$G_o = 10500 \text{ ksi (Table A4 of the Standard)}$$

Similar to the determination of flexural buckling stress, the plasticity reduction factor of E_t/E_o depends on the assumed stress value. For the first approximation, assume a buckling stress of f=20 ksi. The value of E_t/E_o is found to be equal to 1.0, which is obtained from Table A10 or Figure A7 of the Standard. Thus,

$$\sigma_{ex} = [(\pi^2x27000)/(6x12/1.365)^2]x(1.0)$$
$$= 95.78 \text{ ksi}$$

$$\sigma_t = [1/(0.889x6.259)][10500x0.003266 + \pi^2x27000x2.05/(6x12)^2]x(1.0)$$
$$= 25.10 \text{ ksi}$$

Therefore,

$$(F_n)_2 = (1/2\beta)\left[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}\right] \quad \text{(Eq. 3.4.3-1)}$$

$$= [1/(2\text{x}0.392)]\left[(95.78+25.10) - \sqrt{(95.78+25.10)^2-4\text{x}0.392\text{x}95.78\text{x}25.10}\right]$$

$$= 21.37 \text{ ksi} > \text{assumed value } f=20 \text{ ksi}$$

For the second approximation, assume a stress of f=20.46 ksi, and

$E_t/E_o = 0.957$.

$(F_n)_2$ = 20.46 ksi = assumed value OK

The plasticity reduction factor E_t/E_o can be alternatively determined by using the Ramberg-Osgood equation given in the Appendix B of the Standard as follows:

$$E_t/E_o = F_y/\left[F_y+0.002nE_o(f/F_y)^{n-1}\right] \quad \text{(Eq. B-5)}$$

From Table B in the Standard, the coefficient n is equal to 4.58 for Type 304, 1/4-Hard stainless steel. Thus, for an assumed compression stress of f = 18.6 ksi,

$$E_t/E_o = 50/\left[50+0.002\text{x}4.58\text{x}27000\text{x}(18.6/50)^{3.58}\right]$$

$$= 0.875$$

Therefore,

$(F_n)_2$ = 21.37x(0.875)= 18.7 ksi = assumed value OK

(The lateral buckling stress determined by using Eq. B-5 is approximately 8.6 % less than that computed by using Table A10.)

Then, F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$.

F_n = 20.46 ksi (based on tabulated E_t/E_o value)

13. Determination of A_e:

Flanges:

$d \quad = 0.607$ in.

$I_s \quad = d^3t/12 = (0.607)^3(0.105)/12$

$\quad = 0.001957$ in.^{4}

$D \quad = 0.9$ in.

$w \quad = 1.414$ in.

$D/w \quad = 0.9/1.414 \approx 0.636 < 0.80$

$S \quad = 1.28\sqrt{E_o/f}, \quad f = F_n$ (Eq. 2.4-1)

The initial modulus of elasticity, E_o, for Type 301 stainless steel is obtained from Table A4 of the Standard, i.e., E_o = 27000 ksi.

$S \quad = 1.28\sqrt{27000/20.46} = 46.50$

$w/t \quad = 1.414/0.105 \approx 13.47 < S/3 = 15.50$ (Eq. 2.4.2-1)

$I_a \quad = 0$ (no edge stiffener needed) (Eq. 2.4.2-2)

$b \quad = w$ (Eq. 2.4.2-3)

$\quad = 1.414$ in. (flanges fully effective)

$w/t \quad = 13.47 < 90$ (Section 2.1.1-(1)-(i))

Web:

$w \quad = 2.914$ in., $k = 4.00$

$\lambda \quad = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}, \quad f = F_n$ (Eq. 2.2.1-4)

$\quad = (1.052/\sqrt{4})(2.914/0.105)\sqrt{20.46/27000}$

$\quad = 0.402 < 0.673$

$b \quad = w$ (Eq. 2.2.1-1)

$\quad = 2.914$ in. (web fully effective)

$w/t \quad = 2.914/0.105 \approx 27.75 < 400$ (Section 2.1.1-(1)-(ii))

Lips:

$d \quad = 0.607$ in.

$k \quad = 0.50$ (unstiffened compression element)

$d_s \quad = d'_s$ (Eq. 2.4.2-4)

$\lambda \quad = (1.052/\sqrt{0.50})(0.607/0.105)\sqrt{20.46/27000}$

$= 0.237 < 0.673$

$d'_s \quad = d = 0.607$ in., $d_s = 0.607$ in.

$d/t \quad = 5.78 < 50$ (Section 2.1.1-(1)-(iii))

Since flanges, web, and lips are fully effective, the effective area is the same as the full section area, i.e.,

$A_e \quad = A = 0.889 \text{ in.}^2$

14. Determination of $\phi_c P_n$: (Section 3.4 of the Standard)

$P_n \quad = A_e F_n$ (Eq. 3.4-1)

$= 0.889 \times 20.46$

$= 18.19$ kips

$\phi_c \quad = 0.85$

$\phi_c P_n \quad = 0.85 \times 18.19$

$= 15.46$ kips

EXAMPLE 15.2 C-SECTION (ASD)

Determine the allowable axial load for C-section used in Example 15.1.

Solution:

1. Basic parameters used for calculating the section properties:

   See Example 15.1 for section properties of C-section.

2. Determination of F_n

   The following is the result obtained from Example 15.1.

   a. For Flexural Buckling:

   $(F_n)_1 = (\pi^2 E_t)/(K_y L_y/r_y)^2$ (Eq. 3.4.1-1)

   $(F_n)_1 = (\pi^2 x20250)/(93.75)^2$ (E_t is based on Table A13)

   $= 22.74$ ksi

   b. For Torsional-Flexural Buckling:

   $(F_n)_2 = (1/2\beta)[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}]$ (Eq. 3.4.3-1)

   where

   $\sigma_{ex} = [(\pi^2 E_o)/(K_x L_x/r_x)^2](E_t/E_o)$ (Eq. 3.4.3-3)

   $\sigma_t = [1/(Ar_o^2)][G_o J+(\pi^2 E_o C_w)/(K_t L_t)^2](E_t/E_o)$ (Eq. 3.4.2-1)

   $G_o = 10500$ ksi (Table A4 of the Standard)

   $(F_{n})_2 = (1/2\beta)[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}]$ (Eq. 3.4.3-1)

   $= [1/(2x0.392)][(95.78+25.10)$

   $-\sqrt{(95.78+25.10)^2-4x0.392x95.78x25.10}]\ x(0.957)$

   $= 20.46$ ksi (controls) (E_t/E_o is based on Table A10)

   Then, F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$.

   $F_n = 20.46$ ksi

3. Determination of A_e:

   The effective area is the same as the full section area, i.e.,

   $A_e = A = 0.889$ in.^{2}

4. Determination of P_a:

$$P_n = A_e F_n \quad \text{(Eq. 3.4-1)}$$

$$= 0.889 \times 20.46$$

$$= 18.19 \text{ kips}$$

$$\Omega = 2.15$$

$$P_a = P_n/\Omega = 18.19/2.15$$

$$= 8.46 \text{ kips}$$

EXAMPLE 16.1 C-SECTION w/WIDE FLANGE (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for C-section as shown in Figure 16.1 Use Type 304 stainless steel, 1/4-Hard.

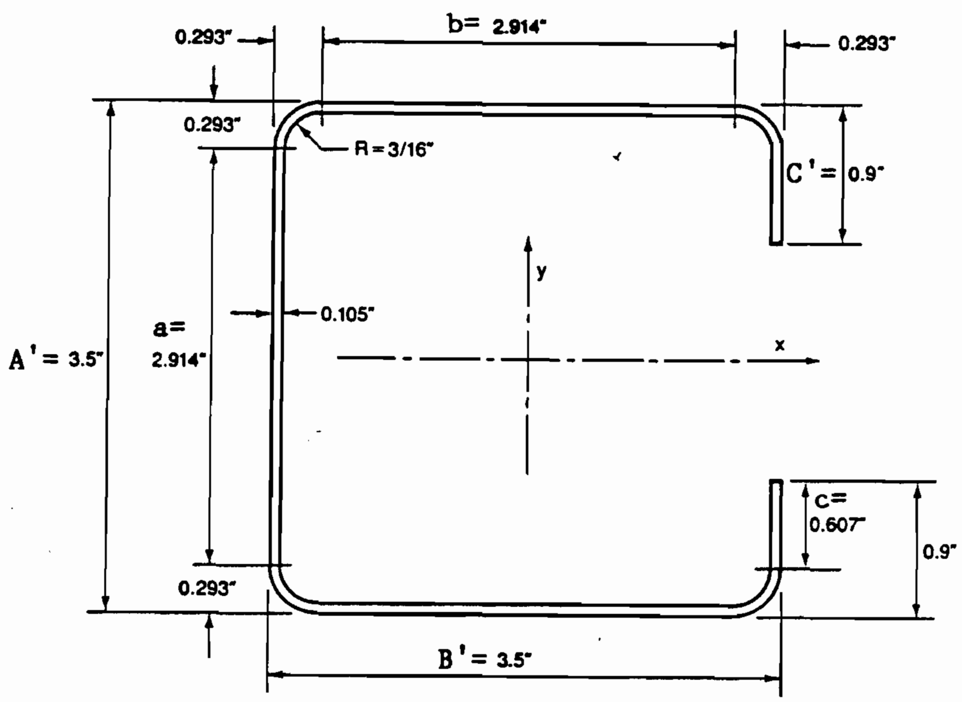


Figure 16.1 Section for Example 16.1

Given:

1. Section: 3.5" x 3.5" x 0.105" channel with stiffened flanges.
2. $K_xL_x = K_yL_y = K_tL_t = 6$ ft.

Solution:

The following equations used for computing the sectional properties for channel with lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

1. Basic parameters used for calculating the section properties:

$r = R+t/2 = 3/16+0.105/2 = 0.240$ in.

From the sketch $a = 2.914$ in., $b = 2.914$ in., $c = 0.607$ in.,

$A' = 3.5$ in., $B' = 3.5$ in., $C' = 0.9$ in.,

$\alpha = 1.00$ (for section has lips)

$\bar{a} = A'-t = 3.5-0.105 = 3.395$ in.

$\bar{b} = B'-t = 3.5-0.105 = 3.395$ in.

$\bar{c} = C'-t/2 = 0.9-0.105/2 = 0.848$ in.

$u = 1.57r = 1.57 \times 0.240 = 0.377$ in.

2. Area:

$$
\begin{aligned}
A &= t[a+2b+2c+4u] \\
&= 0.105[2.914+2x2.914+2x0.607+4x0.377] \\
&= 1.204 \text{ in.}^2
\end{aligned}
$$

3. Moment of inertia about x-axis:

$$
\begin{aligned}
I_x &= 2t[0.0417a^3+b(a/2+r)^2+2u(a/2+0.637r)^2+0.298r^3 \\
&\quad +0.0833c^3+(c/4)(a-c)^2] \\
&= 2x0.105[0.0417(2.914)^3+2.914(2.914/2+0.240)^2 \\
&\quad +2x0.377(2.914/2+0.637x0.240)^2+0.298(0.240)^3 \\
&\quad +0.0833(0.607)^3+(0.607/4)(2.914-0.607)^2] \\
&= 2.564 \text{ in.}^4
\end{aligned}
$$

4. Distance from centroid of section to centerline of web:

$$\bar{x} = (2t/A)[b(b/2+r)+u(0.363r)+u(b+1.637r)+c(b+2r)]$$

$= [(2\times0.105)/1.204][2.914(2.914/2+0.240)+0.377(0.363\times0.240)$

$+0.377(2.914+1.637\times0.240)+0.607(2.914+2\times0.240)]$

$= 1.445$ in.

5. Moment of inertia about y-axis:

$I_y = 2t[b(b/2+r)^2+0.0833b^3+0.505r^3+c(b+2r)^2$

$+u(b+1.637r)^2]-A(\bar{x})^2$

$= 2\times0.105[2.914(2.914/2+0.240)^2+0.0833(2.914)^3$

$+0.505(0.240)^3+0.607(2.914+2\times0.240)^2$

$+0.377(2.914+1.637\times0.240)^2]-1.204(1.445)^2$

$= 2.017$ in.^{4}

6. Distance from shear center to centerline of web:

$m = (\bar{b}t/12I_x)[6\bar{c}(\bar{a})^2+3\bar{b}(\bar{a})^2-8(\bar{c})^3]$

$= [(3.395\times0.105)/(12\times2.564)][6\times0.848(3.395)^2$

$+3\times3.395(3.395)^2-8(0.848)^3]$

$= 1.983$ in.

7. Distance from centroid to shear center:

$x_o = -(\bar{x}+m) = -(1.445+1.983)$

$= -3.428$ in.

8. St. Venant torsion constant:

$J = (t^3/3)[a+2b+2c+4u]$

$= [(0.105)^3/3][2.914+2\times2.914+2\times0.607+4\times0.377]$

$= 0.004424$ in.^{4}

9. Warping Constant:

$$C_w = (t^2/A)\{[\bar{x}A(\bar{a})^2/t][(\bar{b})^2/3+m^2-m\bar{b} +(A/3t)[(m)^2(\bar{a})^3$$
$$+(\bar{b})^2(\bar{c})^2(2\bar{c}+3\bar{a})]-(I_x m^2/t)(2\bar{a}+4\bar{c})+[m(\bar{c})^2/3][8(\bar{b})^2(\bar{c})$$
$$+2m(2\bar{c}(\bar{c}-\bar{a})+\bar{b}(2\bar{c}-3\bar{a}))]+[(\bar{b})^2(\bar{a})^2/6][(3\bar{c}+\bar{b})(4\bar{c}+\bar{a})-6(\bar{c})^2$$
$$-m^2(\bar{a})^4/4\}$$

$$= [(0.105)^2/1.204]\{[1.445x1.204x(3.395)^2/0.105][(3.395)^2/3$$
$$+(1.983)^2-1.983x3.395 +1.204/(3x0.105)[(1.983)^2(3.395)^3$$
$$+(3.395)^2(0.848)^2(2x0.848+3x3.395)]$$
$$-[2.564x(1.983)^2/0.105 (2x3.395+4x0.848)$$
$$+ 1.983(0.848)^2/3][8(3.395)^2(0.848)$$
$$+2x1.983(2x0.848(0.848-3.395)+3.395(2x0.848-3x3.395))]$$
$$+[(3.395)^2(3.395)^2/6][(3x0.848+3.395)(4x0.848+3.395)$$
$$-6(0.848)^2]-[(1.983)^2(3.395)^4/4]\}$$

$$= 7.572 \text{ in.}^6$$

10. Radii of gyration:

$$r_x = \sqrt{(I_x/A)} = \sqrt{(2.564/1.204)} = 1.459 \text{ in.}$$

$$r_y = \sqrt{(I_y/A)} = \sqrt{(2.017/1.204)} = 1.294 \text{ in.}$$

$$(K_yL_y)/r_y = (6x12)/1.294 = 55.64 < 200$$

$$r_o^2 = r_x^2+r_y^2+x_o^2 = (1.459)^2+(1.294)^2+(-3.428)^2$$
$$= 15.554 \text{ in.}^2$$

11. Torsional-flexural constant:

$$\beta = 1-(x_o/r_o)^2 \qquad \text{(Eq. 3.4.3-4)}$$
$$= 1-(-3.428)^2/15.554$$
$$= 0.244$$

12. Determination of F_n: (Section 3.4 of the Standard)

For this singly symmetric section (x-axis is the axis of symmetry), F_n shall be taken as the smaller of either (Eq. 3.4.1-1) or (Eq. 3.4.3-1):

a. For Flexural Buckling:

$(F_n)_1 = (\pi^2 E_t)/(K_y L_y/r_y)^2$ (Eq. 3.4.1-1)

In the determination of the flexural buckling stress, it is necessary to select a proper value of E_t from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of f=32.0 ksi. From Table A13, the corresponding value of E_t is found to be equal to 11300 ksi. Thus,

$(F_n)_1 = (\pi^2 x11300)/(55.64)^2$

$= 36.02$ ksi > assumed stress f=32.0 ksi

Because the computed stress is larger than the assumed value, further successive approximations are needed. For the second approximation, assume f=33.77 ksi, and

$E_t = 10600$ ksi

$(F_n)_1 = (\pi^2 x10600)/(55.64)^2$

$= 33.79$ ksi = assumed stress f=33.77 ksi OK

b. For Torsional-Flexural Buckling:

$(F_n)_2 = (1/2\beta)[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}]$ (Eq. 3.4.3-1)

where

$\sigma_{ex} = [(\pi^2 E_o)/(K_x L_x/r_x)^2](E_t/E_o)$ (Eq. 3.4.3-3)

$\sigma_t = [1/(Ar_o^2)][G_o J+(\pi^2 E_o C_w)/(K_t L_t)^2](E_t/E_o)$ (Eq. 3.4.2-1)

$G_o = 10500$ ksi (Table A4 of the Standard)

Similar to the determination of flexural buckling stress, the plasticity reduction factor of E_t/E_o used for determining the torsional-flexural buckling stress depends on the assumed stress value. For the first approximation, assume a buckling stress of f=20 ksi. The value of E_t/E_o is found to be equal to 1.0, which is obtained from Table A10 or Figure A7 of the Standard. Thus,

$$\sigma_{ex} = [(\pi^2 x 27000)/(6x12/1.459)^2]x(1.0)$$
$$= 109.42 \text{ ksi}$$

$$\sigma_t = [1/(1.204x15.554)][10500x0.004424 + \pi^2 x27000x7.572/(6x12)^2]\,x(1.0)$$
$$= 23.27 \text{ ksi}$$

Therefore,

$$(F_n)_2 = (1/2\beta)[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}] \qquad \text{(Eq. 3.4.3-1)}$$
$$= [1/(2x0.244)][(109.42+23.27)$$
$$-\sqrt{(109.42+23.27)^2-4x0.244x109.42x23.27}\,]$$
$$= 19.92 \text{ ksi}$$

Because the computed stress $(F_n)_2$ is less than the assumed value of f=20 ksi, the second approximation will be assumed that a stress of f=19.92 ksi and E_t/E_o = 1.0. Thus,

$(F_n)_2$ = 19.92 ksi OK

F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$.

F_n = 19.92 ksi

13. Determination of A_e:

Flanges:

d = 0.607 in.

$I_s = d^3t/12 = (0.607)^3(0.105)/12$

$= 0.001957 \text{ in.}^4$

$D = 0.9 \text{ in.}$

$w = 2.914 \text{ in. (for flange)}$

$D/w = 0.9/2.914 = 0.309 < 0.80$

$S = 1.28\sqrt{E_o/f}, \quad f = F_n$ (Eq. 2.4-1)

The initial modulus of elasticity, E_o, for Type 304 stainless steel is obtained from Table A4 of the Standard, i.e., $E_o = 27000$ ksi

$S = 1.28\sqrt{27000/19.92} = 47.12, \quad S/3 = 15.71$

$w/t = 2.914/0.105 = 27.75$

$S/3 < w/t < S$

$I_a = 399t^4\{[(w/t)/S]-0.33\}^3$ (Eq. 2.4.2-6)

$= 399(0.105)^4[(27.75/47.12)-0.33]^3$

$= 0.000842 \text{ in.}^4 < I_s = 0.001957 \text{ in.}^4$

$C_1 = 2-(I_s/I_a) \geq 1.0$ (Eq. 2.4.2-8)

$= 2-(0.001957/0.000842) = -0.32 < 1.0$

$C_1 = 1.0$

$C_2 = I_s/I_a \leq 1.0$ (Eq. 2.4.2-7)

$I_s/I_a = (0.001957/0.000842) = 2.32 > 1.0$

$C_2 = 1.0$

$0.25 < D/w = 0.309 < 0.8$

$k = [4.82-5(D/w)](I_s/I_a)^n+0.43 \leq 5.25-5(D/w)$ (Eq. 2.4.2-9)

$n = 1/2$

$[4.82-5(0.309)](0.001957/0.000842)^{1/2}+0.43 = 5.414$

$5.25-5(0.309) = 3.705 < 5.414$

$k = 3.705$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}, \quad f = F_n$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{3.705})(27.75)\sqrt{19.92/27000} = 0.412 < 0.673$

$b \quad = w$ (Eq. 2.2.1-1)

$= 2.914$ in. (flanges fully effective)

$w/t \quad = 27.75 < 90$ (Section 2.1.1-(1)-(i))

Web:

$w \quad = 2.914$ in., $k = 4.00$

$\lambda \quad = (1.052/\sqrt{4})(2.914/0.105)\sqrt{19.92/27000}$

$= 0.397 < 0.673$

$b \quad = w = 2.914$ in. (web fully effective)

$w/t \quad = 2.914/0.105 = 27.75 < 400$ (Section 2.1.1-(1)-(ii))

Lips:

$d \quad = 0.607$ in.

$k \quad = 0.50$ (unstiffened compression element)

$\lambda \quad = (1.052/\sqrt{0.50})(0.607/0.105)\sqrt{19.92/27000}$

$= 0.234 < 0.673$

$d'_s \quad = d = 0.607$ in.

$d_s \quad = d'_s(I_s/I_a) \leq d'_s$ (Eq. 2.4.2-11)

$= 0.607(2.32) = 1.408 > d'_s = 0.607$ in.

$d_s \quad = 0.607$ in. (Lip fully effective in computing the overall effective area)

$d/t \quad = 5.78$

Since flanges, web, and lips are fully effective, the effective area is the same as the full section area, i.e.,

$A_e \quad = A = 1.204 \text{ in.}^2$

14. Determination of $\phi_c P_n$: (Section 3.4 of the Standard)

$P_n \quad = A_e F_n$ (Eq. 3.4-1)

$= 1.204 \times 19.92$

$= 23.98$ kips

$\phi_c = 0.85$

$\phi_c P_n = 0.85 \times 23.98$

$= 20.38$ kips

EXAMPLE 16.2 C-SECTION w/WIDE FLANGE (ASD)

Determine the allowable axial load for C-section used in Example 16.1.

Solution:

1. Basic parameters used for calculating the section properties:

   See Example 16.1 for section properties of C-section.

2. Determination of F_n

   The following results are obtained from Example 16.1.

   a. For Flexural Buckling:

$$(F_n)_1 = (\pi^2 E_t)/(K_y L_y/r_y)^2 \qquad \text{(Eq. 3.4.1-1)}$$

$$(F_n)_1 = (\pi^2 x10600)/(55.64)^2$$

$$= 33.79 \text{ ksi}$$

   b. For Torsional-Flexural Buckling:

$$(F_n)_2 = (1/2\beta)\left[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}\right] \qquad \text{(Eq. 3.4.3-1)}$$

   where

$$\sigma_{ex} = \left[(\pi^2 E_o)/(K_x L_x/r_x)^2\right](E_t/E_o) \qquad \text{(Eq. 3.4.3-3)}$$

$$\sigma_t = 1/(Ar_o^2)\left[G_o J+(\pi^2 E_o C_w)/(K_t L_t)^2\right](E_t/E_o) \qquad \text{(Eq. 3.4.2-1)}$$

$$G_o = 10500 \text{ ksi (Table A4 of the Standard)}$$

$$(F_n)_2 = (1/2\beta)\left[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}\right] \qquad \text{(Eq. 3.4.3-1)}$$

$$= \left[1/(2x0.244)\right]\left[(109.42+23.27)\right.$$

$$\left.-\sqrt{(109.42+23.27)^2-4x0.244x109.42x23.27}\right]$$

$$= 19.92 \text{ ksi (control)}$$

   Then, F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$.

$$F_n = 19.92 \text{ ksi}$$

3. Determination of A_e:

   The effective area is the same as the full section area, i.e.,

$$A_e = A = 1.204 \text{ in.}^2 \text{ (from Example 16.1)}$$

4. Determination of P_a:

$$P_n = A_e F_n \quad \text{(Eq. 3.4-1)}$$

$$= 1.204 \times 19.92$$

$$= 23.98 \text{ kips}$$

$$\Omega = 2.15$$

$$P_a = P_n/\Omega = 23.98/2.15$$

$$= 11.15 \text{ kips}$$

EXAMPLE 17.1 I-SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine th design axial strength for the I-section as shown in Figure 17.1. Use Typ 409 stainless steel.

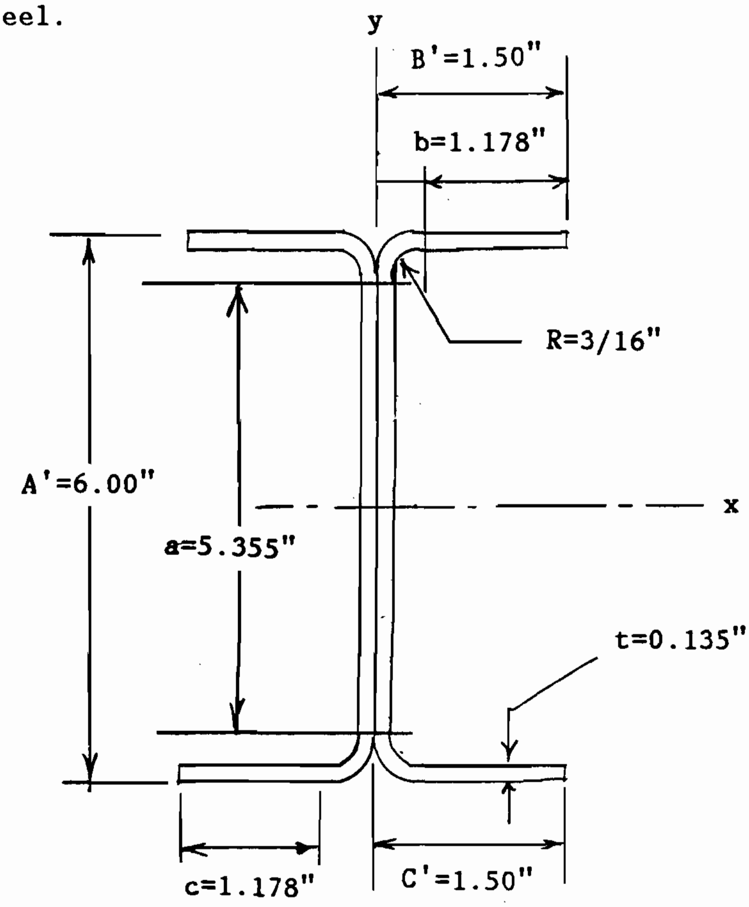


Figure 17.1 Section for Example 17.1

Given:

1. Section: 6.0" x 3.0" x 0.135" I-section with no lips.
2. $K_x L_x$ = 14 ft., $K_y L_y$ = 7.0 ft.

Solution:

The following equations used for computing the sectional properties for I-section with no lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

1. Basic parameters used for calculating the sectional properties:

$r \quad = R+t/2 = 3/16+0.135/2 = 0.255$ in.

From the sketch, $A' = 6.0$ in., $B' = C' = 1.5$ in.

$\alpha = 1.00$ (For I-section)

$$a = A'-[r+t/2+(r+t/2)]$$

$$= 6.0-(0.255+0.135/2+0.255+0.135/2) = 5.355 \text{ in.}$$

$$\bar{a} = A'-(t/2+\alpha t/2) = 6.0-0.135 = 5.865 \text{ in.}$$

$$b = c = B'-(r+t/2) = 1.5-(0.255+0.135/2) = 1.178 \text{ in.}$$

$$\bar{b} = \bar{c} = B'-t/2 = 1.5-0.135/2 = 1.433 \text{ in.}$$

$$u = 1.57r = 1.57 \times 0.255 = 0.40 \text{ in.}$$

2. Area:

$$A = t[2a+2b+2u+\alpha(2c+2u)]$$

$$= 0.135(2x5.355+2x1.178+2x0.40+2x1.178+2x0.4)$$

$$= 2.298 \text{ in.}^2$$

3. Moment of inertia about y-axis:

$$I_y = 2t\{b(b/2+r+t/2)^2+0.0833b^3+u(0.363r+t/2)^2+0.149r^3$$

$$+\alpha[c(c/2+r+t/2)^2+0.0833b^3+u(0.363r+t/2)^2+0.149r^3]\}$$

$$= 2tx2[b(b/2+r+t/2)^2+0.0833b^3+u(0.363r+t/2)^2+0.149r^3]$$

$$= 2x0.135x2[1.178(1.178/2+0.255+0.135/2)^2+0.0833(1.178)^3$$

$$+0.4(0.363x0.255+0.135/2)^2+0.149(0.255)^3]$$

$$= 0.609 \text{ in.}^4$$

4. Distance between centroid and flange centerline:

$$\bar{y} = \bar{a}/2 = 5.865/2 = 2.933 \text{ in.}$$

5. Moment of inertia about x-axis:

$$I_x = 2t\{0.358r^3+a(a/2+r)^2+0.0833a^3+\alpha[u(a+1.637r)^2 +0.149r^3+c(a+2r)^2]\}-A(y)^2$$

$$= 2x0.135[0.358(0.255)^3+5.355(5.355/2+0.255)^2+0.0833(5.355)^3 + 0.4(5.355+1.637x0.255)^2+0.149(0.255)^3+1.178(5.355+2x0.255)^2] -2.298(2.933)^2$$

$$= 10.66 \text{ in.}^4$$

6. Distance between shear center and flange centerline:

$$m = \bar{a}/2 = 2.933 \text{ in.}$$

7. Distance between centroid and shear center:

$$y_o = -(\bar{y}-m) = 0$$

8. St. Venant torsion constant:

$$J = (2t^3/3)[a+b+u+\alpha(u+c)]$$

$$= [2x(0.135)^3/3](5.355+1.178+0.4+0.4+1.178)$$

$$= 0.0140 \text{ in.}^4$$

9. Warping Constant:

$$C_w = (t\bar{a}^2/12)x\ 8(\bar{b})^3(\bar{c})^3/[(\bar{b})^3+(\bar{c})^3]$$

$$= (t\bar{a}^2/12)x4\bar{b}^3$$

$$= [0.135(5.865)^2/12]x4(1.433)^3$$

$$= 4.55 \text{ in.}^6$$

10. Radii of gyration:

$r_x = \sqrt{(I_x/A)} = \sqrt{(10.66/2.298)} = 2.154$ in.

$r_y = \sqrt{(I_y/A)} = \sqrt{(0.609/2.298)} = 0.515$ in.

$(K_xL_x)/r_x = (14x12)/2.514 = 66.83 < 200$

$(K_yL_y)/r_y = (7x12)/0.515 = 163.1 < 200$ (control)

$r_o^2 = r_x^2+r_y^2+y_o^2 = (2.154)^2+(0.515)^2+0$

$= 4.905$ in.^{2}

11. Determination of F_n: (Section 3.4 of the Standard)

For this doubly symmetric section (x-axis is the major axis), F_n shall be taken as the smaller of either (Eq. 3.4.1-1) or (Eq. 3.4.2-1):

a. For Flexural Buckling:

$(F_n)_1 = (\pi^2E_t)/(K_yL_y/r_y)^2$ (Eq. 3.4.1-1)

In the determination of the flexural buckling stress, it is necessary to select a proper value of E_t from Table A14 or Figure A12 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of f=8 ksi. From Table A14, the corresponding value of E_t is found to be equal to 27000 ksi. Thus,

$(F_n)_1 = (\pi^2 x27000)/(163.1)^2$

$= 10.02$ ksi > assumed stress f=8 ksi

Because the computed stress is larger than the assumed value, the further successive approximation is needed. After several trials, assume f=10.0 ksi, and

$E_t = 26900$ ksi

$(F_n)_1 = (\pi^2 x26900)/(163.1)^2$

$= 9.98$ ksi $\cong$ assumed stress f=10.0 ksi OK

Alternatively, the tangent modulus E_t can be determined by using the Modified Ramberg-Osgood equation as given in Appendix B of the Standard as follows:

$$E_t = (E_oF_y)/[F_y+0.002nE_o(f/F_y)^{n-1}] \qquad \text{(Eq. B-2)}$$

From Table B in the Standard, the coefficient n is equal to 9.7 for Type 409 stainless steel in longitudinal compression.

Thus, for an assumed compression stress of f = 10.0 ksi,

(F_y = 30 ksi, E_o = 27000 ksi)

$$E_t = (27000x30)/[30+0.002x9.7x27000x(10.0/30)^{8.7}]$$

$$= 26966 \text{ ksi}$$

Therefore,

$$(F_n)_1 = (\pi^2 x26966)/(163.1)^2$$

$$= 10.0 \text{ ksi} = \text{assumed stress f=10.0 ksi OK}$$

It is found that for this example, the flexural buckling stress determined by using Eq. B-2 is practically the same as that determoined by using the tabulated value.

b. For Torsional Buckling:

$$(F_n)_2 = [1/(Ar_o^2)][G_oJ+(\pi^2E_oC_w)/(K_tL_t)^2](E_t/E_o) \qquad \text{(Eq. 3.4.2-1)}$$

G_o = 10500 ksi (Table A4 of the Standard)

Similar to the determination of flexural buckling stress, the plasticity reduction factor of E_t/E_o depends on the assumed stress value. For the first approximation, assume a buckling stress of f=8 ksi. The value of E_t/E_o is found to be equal to 1.0, which is obtained from Table A11 or Figure A8 of the Standard. Thus,

$(F_n)_2$ = $[1/(2.298x4.905)][10500x0.014 + \pi^2x27000x4.555/(7x12)^2]x(1.0)$

= 28.3 ksi > 8 ksi NG

After several trials, assume a stress of f=19.65 ksi, and

E_t/E_o = 0.694.

$(F_n)_2$ = $[1/(2.298x4.905)][10500x0.014 + \pi^2x27000x4.555/(7x12)^2]x(0.694)$

= 19.64 = assumed value OK

The plasticity reduction factor E_t/E_o can be alternatively determined by using the Ramberg-Osgood equation given in the Appendix B of the Standard as follows:

$$E_t/E_o = F_y/[F_y+0.002nE_o(f/F_y)^{n-1}] \qquad \text{(Eq. B-5)}$$

From Table B in the Standard, the coefficient n is equal to 9.7 for Type 409 stainless steel. Thus, for an assumed compression stress of f = 19.65 ksi,

E_t/E_o = 30/ 30+0.002x9.7x27000x$(19.65/30)^{8.7}$

= 0.694

Therefore,

$(F_n)_2$ = 28.3x(0.694)= 19.65 ksi = assumed value OK

The lateral buckling stress determined by using Eq. B-5 is practically the same as that computed by using Table A11.

Then, F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$.

F_n = 10.0 ksi

12. Determination of A_e:

Unstiffened Compression Flanges: (k=0.5)

$w/t \quad = 1.178/0.135 = 8.73 < 50$ (Eq. 2.4.2-1)

$\lambda \quad = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}, \quad f = F_n$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{0.5})(8.73)\sqrt{10.0/27000}$

$= 0.25 < 0.673$

$b \quad = w$ (Eq. 2.4.2-3)

$= 1.178$ in. (flanges fully effective)

Web: (Sec. 2.2.2-(2))

$w \quad = 5.355$ in.,

$w/t \quad = 5.355/0.135 = 39.67$

$k \quad = 4.0$

$\lambda \quad = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}, \quad f = F_n$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{4})(39.67)\sqrt{10.0/27000}$

$= 0.40 < 0.673$

$b \quad = w$ (Eq. 2.2.1-1)

$= 5.355$ in. (web fully effective)

Since flanges and webs are fully effective, the effective area is the same as the full section area, i.e.,

$A_e \quad = A = 2.298 \text{ in.}^2$

13. Determination of $\phi_c P_n$: (Section 3.4 of the Standard)

$P_n \quad = A_e F_n$ (Eq. 3.4-1)

$= 2.298 \times 10.0$

$= 22.98$ kips

$\phi_c \quad = 0.85$

The design axial strength is

$\phi_c P_n \quad = 0.85 \times 22.98$

$= 19.53$ kips

EXAMPLE 17.2 I-SECTION (ASD)

Determine the allowable axial load for the I-section used in Example 17.1.

Solution:

1. Basic parameters used for calculating the sectional properties:

   See Example 17.1 for calculation of sectional properties of the I-section.

2. Determination of F_n

   The following results are obtained from Example 17.1.

   a. For Flexural Buckling:

   $(F_n)_1 = (\pi^2 E_t)/(K_y L_y/r_y)^2$ (Eq. 3.4.1-1)

   $(F_n)_1 = (\pi^2 x 26900)/(163.1)^2$

   $= 10.0$ ksi (see Example 17.1 for E_t)

   b. For Torsional Buckling:

   $(F_n)_2 = [1/(Ar_o^2)][G_o J+(\pi^2 E_o C_w)/(K_t L_t)^2](E_t/E_o)$ (Eq. 3.4.2-1)

   $= [1/(2.298x4.905)][10500x0.014 + \pi^2 x27000x4.555/(7x12)^2](0.694)$

   $= 19.65$ ksi

   Then, F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$.

   $F_n = 10.0$ ksi

3. Determination of A_e:

   The effective area is the same as the full section area, i.e.,

   $A_e = A = 2.298$ in.^{2} (See Example 17.1)

4. Determination of P_a:

   $P_n = A_e F_n$ (Eq. 3.4-1)

   $= 2.298 \times 10.0$

$$= 22.98 \text{ kips}$$

$$\Omega = 2.15$$

The allowable axial load is

$$P_a = P_n/\Omega = 22.98/2.15$$

$$= 10.69 \text{ kips}$$

EXAMPLE 18.1 I-SECTION W/LIPS (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for the I-section as shown in Figure 18.1. Use Type 409 stainless steel.

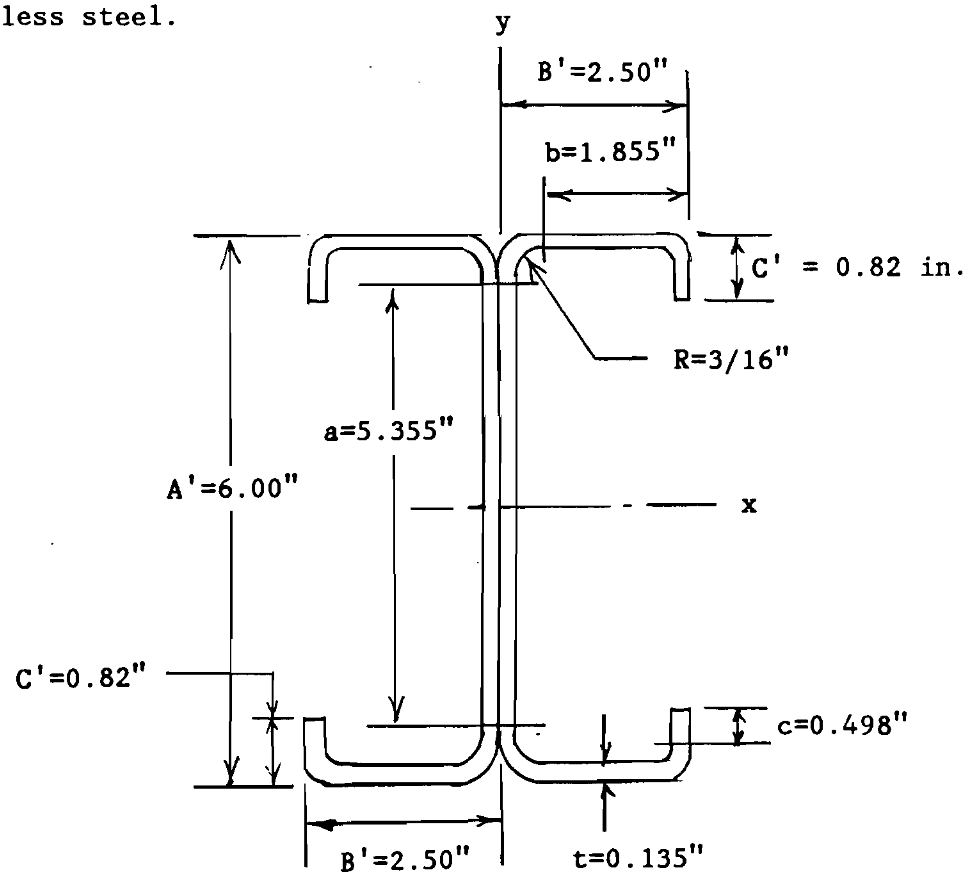


Figure 18.1 Section for Example 18.1

Given:

1. Section: 6.0" x 5.0" x 0.135" I-section with lips.
2. $K_x = K_y = 1.0$, $L_x = 12.0$ ft. and $L_y = 6.0$ ft.

Solution:

The following equations used for computing the sectional properties for I-section with lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

1. Basic parameters used for calculating the sectional properties:

(For a channel with lips)

$r = R+t/2 = 3/16+0.135/2 = 0.255$ in.

From the sketch, $A' = 6.0$ in., $B' = 2.5$ in., $C' = 0.82$ in.

$\alpha = 1.00$ (For sections with lips)

$$a = A'-(2r+t)$$

$$= 6.0-(2x0.255+0.135) = 5.355 \text{ in.}$$

$$\bar{a} = A'-t = 6-0.135 = 5.865 \text{ in.}$$

$$b = B'-[r+t/2+\alpha(r+t/2)] = 2.5-(2x0.255+0.135) = 1.855 \text{ in.}$$

$$\bar{b} = B'-(t/2+\alpha t/2) = 2.5-0.135 = 2.365 \text{ in.}$$

$$c = \alpha[C'-(r+t/2)] = 0.82-(0.255+0.135/2) = 0.498 \text{ in.}$$

$$\bar{c} = \alpha(C'-t/2) = 0.82-0.135/2 = 0.753 \text{ in.}$$

$$u = 1.57r = 1.57 \times 0.255 = 0.40 \text{ in.}$$

2. Area: (lipped I-section)

$$A = 2xt[a+2b+2u+\alpha(2c+2u)]$$

$$= 2x0.135[5.355+2x1.855+2x0.40+2x0.498+2x0.4]$$

$$= 3.148 \text{ in.}^2$$

3. Moment of inertia about x-axis: (lipped I-section)

$$I_x = 2x2t\{0.0417a^3+b(a/2+r)^2+u(a/2+0.637r)^2+0.149r^3$$

$$+\alpha[0.0833c^3+(c/4)(a-c)^2+u(a/2+0.637r)^2+0.149r^3]\}$$

$$= 2x2x0.135[0.0417(5.355)^3+1.855(5.355/2+0.255)^2$$

$$+0.4(5.355/2+0.637x0.255)^2+0.149(0.255)^3+0.0833(0.498)^3$$

$$+(0.498/4)(5.355-0.498)^2+0.4(5.355/2+0.637x0.255)^2$$

$+0.149(0.255)^3]$

$= 17.15 \text{ in.}^4$

4. Distance bwtween centroid and web centerline for a lipped channel:

$\bar{x} \quad = (2t/A)\{b(b/2+r)+u(0.363r)+\alpha[u(b+1.637r)+c(b+2r)]\}$

$= (2x0.135/1.574)[1.855(1.855/2+0.255)+0.4(0.363x0.255)$

$+0.4(1.855+1.637x0.255)+0.498(1.855+2x0.255)]$

$= 0.741 \text{ in.}$

5. Moment of inertia about y-axis:

For a channel with lips

$I_y' \quad = 2t\{b(b/2+r)^2+0.0833b^3+0.356r^3+\alpha[c(b+2r)^2$

$+u(b+1.637r)^2+0.149r^3]\}-A(x)^2$

$= 2x0.135[1.855(1.855/2+0.255)^2+0.0833(1.855)^3+0.356(0.255)^3$

$+0.498(1.855+2x0.255)^2+0.4(1.855+1.637x0.255)^2+0.149(0.255$

$-1.574(0.741)^2$

$= 1.292 \text{ in.}^4$

For lipped I-section

$I_y \quad = 2[I_y'+A(\bar{x}+t/2)^2]$

$= 2[1.292+1.574(0.741+0.135/2)^2] = 4.642 \text{ in.}^4$

6. Distance between shear center and y-axis:(lipped I-section)

$m \quad = 0$

7. Distance between centroid and shear center: (lipped I-section)

$x_o \quad = 0$

8. St. Venant torsion constant: (lipped I-section)

$$J = (2xt^3/3)[a+2b+2u+\alpha(2c+2u)]$$

$$= [2x(0.135)^3/3][5.355+2x1.855+2x0.4+2x0.498+2x0.4]$$

$$= 0.0191 \text{ in.}^4$$

9. Warping Constant: (lipped I-section)

$$C_w = (t\bar{b}^2/3)[(\bar{a})^2\bar{b}+3(\bar{a})^2\bar{c}+6\bar{a}(\bar{c})^2+4(\bar{c})^3]$$

$$= [0.135(2.365)^2/3][(5.865)^2(2.365)+3(5.865)^2(0.753)$$

$$+6(5.865)(0.753)^2+4(0.753)^3]$$

$$= 45.49 \text{ in.}^6$$

10. Radii of gyration: (lipped I-section)

$$r_x = \sqrt{(I_x/A)} = \sqrt{(17.15/3.148)} = 2.334 \text{ in.}$$

$$r_y = \sqrt{(I_y/A)} = \sqrt{(4.642/3.148)} = 1.214 \text{ in.}$$

$$(K_xL_x)/r_x = (12x12)/2.334 = 61.70 < 200 \text{ (control)}$$

$$(K_yL_y)/r_y = (6x12)/1.214 = 59.3 < 200$$

$$r_o^2 = r_x^2+r_y^2+x_o^2 = (2.334)^2+(1.214)^2+0$$

$$= 6.921 \text{ in.}^2$$

11. Determination of F_n: (Section 3.4 of the Standard)

For this doubly symmetric section (x-axis is the major axis), F_n shall be taken as the smaller of either (Eq. 3.4.1-1) or (Eq. 3.4.2-1):

a. For Flexural Buckling:

$$(F_n)_1 = (\pi^2E_t)/(K_xL_x/r_x)^2 \qquad \text{(Eq. 3.4.1-1)}$$

or the above equation can be written as follows:

$$(F_n)_1 = [(\pi^2E_o)/(K_xL_x/r_x)^2](E_t/E_o)$$

In the determination of the flexural buckling stress, it is necessary to select a proper value of (E_t/E_o) from Table A11 or Figure A8 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of f=20 ksi. From Table A11, the corresponding value of (E_t/E_o) is found to be equal to 0.66. Thus,

$$(F_n)_1 = [(\pi^2 x27000)/(61.7)^2]\ (0.66)$$

$$= 46.2 \text{ ksi} > \text{assumed stress f=20 ksi}$$

Because the computed stress is larger than the assumed value, the further successive approximation is needed. After several trials, assume f=23.50 ksi, and $(E_t/E_o) = 0.336$.

$$(F_n)_1 = [(\pi^2 x27000)/(61.7)^2]x0.336$$

$$= 23.52 \text{ ksi} = \text{assumed stress f=23.50 ksi} \quad \text{OK}$$

b. For Torsional Buckling:

$$(F_n)_2 = [1/(Ar_o^2)][G_oJ+(\pi^2E_oC_w)/(K_tL_t)^2](E_t/E_o) \qquad \text{(Eq. 3.4.2-1)}$$

G_o = 10500 ksi (Table A4 of the Standard)

Similar to the determination of flexural buckling stress, the plasticity reduction factor of E_t/E_o depends on the assumed stress value. For the first approximation, assume a buckling stress of f=24 ksi. The value of E_t/E_o is found to be equal to 0.29, which is obtained from Table A11 or Figure A8 of the Standard. Thus,

$$(F_n)_2 = [1/(3.148x6.921)][10500x0.0191+ \pi^2x27000x45.49/(6x12)^2]x(0.29)$$

$$= 33.79 \text{ ksi} > 24 \text{ ksi NG}$$

After several trials, assume a stress of f=25.43 ksi, and $E_t/E_o = 0.219$.

$(F_n)_2 = [1/(3.148x6.921)][10500x0.0191 + \pi^2 x27000x45.49/(6x12)^2]x(0.219)$

$= 25.46 \cong$ assumed value $= f = 25.43$ ksi OK

Then, F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$.

$F_n = 23.52$ ksi (based on flexural buckling)

12. Determination of A_e:

Flanges:

$d = 0.498$ in.

$I_s = d^3t/12 = (0.498)^3(0.135)/12$

$= 0.001389$ in.^{4}

$D = 0.82$ in.

$w = 1.855$ in.

$D/w = 0.82/1.855 = 0.442 < 0.80$

$S = 1.28\sqrt{E_o/f}$, $f = F_n$ (Eq. 2.4-1)

The initial modulus of elasticity, E_o, for Type 409 stainless steel is obtained from Table A5 of the Standard, i.e., $E_o = 27000$ ksi.

$S = 1.28\sqrt{27000/23.52} = 43.37$

$w/t = 1.855/0.135 = 13.74 < S/3 = 14.46$ (Eq. 2.4.2-1)

$I_a = 0$ (no edge stiffener needed) (Eq. 2.4.2-2)

$b = w$ (Eq. 2.4.2-3)

$= 1.855$ in. (flanges fully effective)

$w/t = 13.74 < 90$ (Section 2.1.1-(1)-(i))

Web:

$w = 5.355$ in., $k = 4.00$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$, $f = F_n$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{4})(5.355/0.135)\sqrt{23.52/27000}$

$= 0.616 < 0.673$

$b = w$ (Eq. 2.2.1-1)

$= 5.355$ in. (web fully effective)

$w/t = 5.355/0.135 = 39.67 < 400$ (Section 2.1.1-(1)-(ii))

Lips:

$d = 0.498$ in.

$k = 0.50$ (unstiffened compression element)

$d_s = d'_s$ (Eq. 2.4.2-4)

$\lambda = (1.052/\sqrt{0.50})(0.498/0.135)\sqrt{23.52/27000}$

$= 0.162 < 0.673$

$d'_s = d = 0.498$ in., $d_s = 0.498$ in.

$d/t = 3.69 < 50$ (Section 2.1.1-(1)-(iii))

Since flanges, web, and lips are fully effective, the effective area is the same as the full section area, i.e.,

$A_e = A = 3.148$ in.^{2}

14. Determination of $\phi_c P_n$: (Section 3.4 of the Standard)

$P_n = A_e F_n$ (Eq. 3.4-1)

$= 3.148 \times 23.52$

$= 74.04$ kips

$\phi_c = 0.85$

The design axial strength is

$\phi_c P_n = 0.85 \times 74.04$

$= 62.93$ kips

EXAMPLE 18.2 I-SECTION W/LIPS (ASD)

Determine the allowable axial load for the I-section used in Example 18.1.

Solution:

1. Basic parameters used for calculating the sectional properties:

   See Example 18.1 for calculation of sectional properties of the I-section.

2. Determination of F_n

   The following results are obtained from Example 18.1.

   a. For Flexural Buckling:

   $$(F_n)_1 = (\pi^2 E_o)/(K_x L_x/r_x)^2 (E_t/E_o)$$

   $$(F_n)_1 = [(\pi^2 x 27000)/(61.7)^2](0.336)$$

   $$= 23.52 \text{ ksi}$$

   b. For Torsional Buckling:

   $$(F_n)_2 = [1/(Ar_o^2)][G_o J + (\pi^2 E_o C_w)/(K_t L_t)^2](E_t/E_o) \qquad \text{(Eq. 3.4.2-1)}$$

   After several trials, assume a stress of f=25.43 ksi, and E_t/E_o = 0.2185.

   $$(F_n)_2 = [1/(3.148x6.921)][10500x0.0191 + \pi^2 x27000x45.49/(6x12)^2]x(0.219)$$

   $$= 25.46 \cong \text{assumed value OK}$$

   Then, F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$.

   $$F_n = 23.52 \text{ ksi}$$

3. Determination of A_e:

   The effective area is the same as the full section area, i.e.,

   $$A_e = A = 3.148 \text{ in.}^2$$

4. Determination of P_a:

   $$P_n = A_e F_n \qquad \text{(Eq. 3.4-1)}$$

$$= 3.148 \times 23.52$$

$$= 74.04 \text{ kips}$$

$$\Omega = 2.15$$

The allowable axial load is

$$P_a = P_n/\Omega = 74.04/2.15$$

$$= 34.44 \text{ kips}$$

EXAMPLE 19.1 T-SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for the T-section as shown in Figure 19.1. Use Type 304, 1/4-Hard stainless steel.

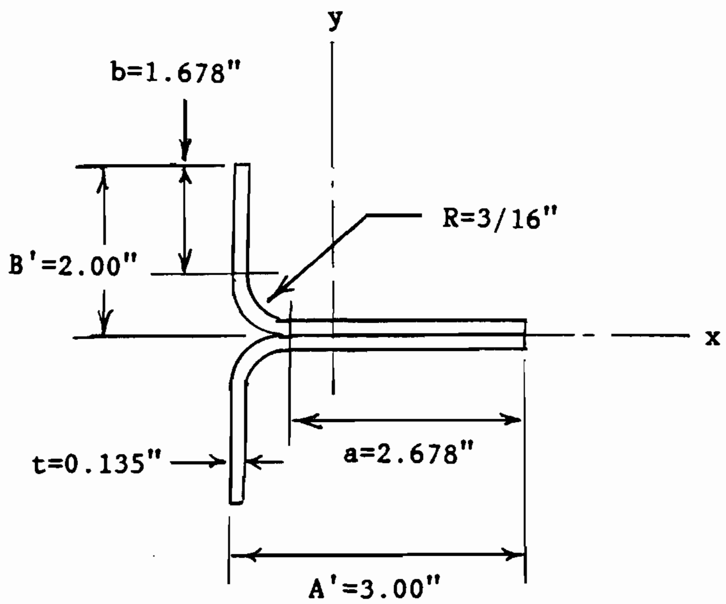


Figure 19.1 Section for Example 19.1

Given:

1. Section: as ahown.
2. $K_xL_x = K_yL_y = 8.0$ ft.

Solution:

The following equations used for computing the sectional properties for T-section are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

1. Basic parameters used for calculating the sectional properties:

$r = R+t/2 = 3/16+0.135/2 = 0.255$ in.

From the sketch, $A' = 3.0$ in., $B' = 2.0$ in.

$\alpha = 0.00$ (For T-section)

$a = A'-[r+t/2+\alpha(r+t/2)]$

$= 3.0-(0.255+0.135/2) = 2.678$ in.

$\bar{a} = A'-(t/2+\alpha t/2) = 3.0-0.135/2 = 2.933$ in.

$b = B'-(r+t/2) = 2.0-(0.255+0.135/2) = 1.678$ in.

$\bar{b} = B'-t/2 = 2.0-0.135/2 = 1.933$ in.

$u = 1.57r = 1.57 \times 0.255 = 0.40$ in.

2. Area:

$A = t(2a+2b+2u)$

$= 0.135(2x2.678+2x1.678+2x0.40) = 1.284$ in.^{2}

3. Moment of inertia about x-axis:

$I_x = 2t[b(b/2+r+t/2)^2+0.0833b^3+u(0.363r+t/2)^2$
$+0.149r^3]$

$= 2x0.135[1.678(1.678/2+0.255+0.135/2)^2+0.0833(1.678)^3$
$+0.4(0.363x0.255+0.135/2)^2+0.149(0.255)^3]$

$= 0.721$ in.^{4}

4. Distance between centroid and flange centerline:

$\bar{x} = (2t/A)[u(0.363r)+a(a/2+r)]$

$= (2x0.135/1.284)[0.4(0.363x0.255)+2.678(2.678/2+0.255)]$

$= 0.905$ in.

5. Moment of inertia about y-axis:

$$I_y = 2t\left[0.358r^3+a(a/2+r)^2+0.0833a^3\right] - A(\bar{x})^2$$

$$= 2x0.135\ 0.358(0.255)^3+2.678(2.678/2+0.255)^2+0.0833(2.678)^3$$

$$-1.284(0.905)^2$$

$$= 1.219 \text{ in.}^4$$

6. Distance between shear center and flange centerline:

$$m = \bar{a}\left\{1-(\bar{b})^3/\left[(\bar{b})^3+(\bar{c})^3\right]\right\}$$

$$= 2.933\left\{1-(1.678)^3/\left[(1.678)^3+0\right]\right\} = 0$$

7. Distance between centroid and shear center:

$$x_o = -(\bar{x}-m) = -0.905 \text{ in.}$$

8. St. Venant torsion constant:

$$J = (2xt^3/3)\left[a+b+u\right]$$

$$= \left[2x(0.135)^3/3\right]\left[2.678+1.678+0.4\right]$$

$$= 0.0078 \text{ in.}^4$$

9. Warping Constant:

$$C_w = 0$$

10. Radii of gyration:

$$r_x = \sqrt{(I_x/A)} = \sqrt{(0.721/1.284)} = 0.749 \text{ in.}$$

$$r_y = \sqrt{(I_y/A)} = \sqrt{(1.219/1.284)} = 0.974 \text{ in.}$$

$$(K_xL_x)/r_x = (8x12)/0.749 = 128.17 < 200 \text{ (control)}$$

$$(K_yL_y)/r_y = (8x12)/0.974 = 98.56 < 200$$

$$r_o^2 = r_x^2+r_y^2+x_o^2$$

$= (0.749)^2+(0.974)^2+(0.905)^2$

$= 2.329 \text{ in.}^2$

11. Torsional-flexural constant:

$\beta = 1-(x_o/r_o)^2$ (Eq. 3.4.3-4)

$= 1-(0.905)^2/2.329$

$= 0.648$

12. Determination of F_n: (Section 3.4 of the Standard)

For this singly symmetric section (x-axis is the axis of symmetric), F_n shall be taken as the smaller of either (Eq. 3.4.1-1) or (Eq. 3.4.3-1):

a. For Flexural Buckling:

$(F_n)_1 = (\pi^2 E_t)/(K_x L_x/r_x)^2$ (Eq. 3.4.1-1)

In the determination of the flexural buckling stress, it is necessary to select a proper value of E_t from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of f=20 ksi. From Table A13, the corresponding value of E_t is found to be equal to 27000 ksi. Thus,

$(F_n)_1 = (\pi^2 x27000)/(128.17)^2$

$= 16.2 \text{ ksi} < \text{assumed stress } f=20 \text{ ksi}$

Because the computed stress is less than the assumed value, flexural buckling is in the elastic region and therefore, no further approximation is needed. Thus,

$E_t = 27000 \text{ ksi}$

$(F_n)_1 = 16.2 \text{ ksi}$

b. For Torsional-Flexural Buckling:

$$(F_n)_2 = (1/2\beta)\left[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}\right] \quad \text{(Eq. 3.4.3-1)}$$

where

$$\sigma_{ex} = \left[(\pi^2E_o)/(K_xL_x/r_x)^2\right](E_t/E_o) \quad \text{(Eq. 3.4.3-3)}$$

$$\sigma_t = \left[1/(Ar_o^2)\right]\left[G_oJ+(\pi^2E_oC_w)/(K_tL_t)^2\right](E_t/E_o) \quad \text{(Eq. 3.4.2-1)}$$

G_o = 10500 ksi (Table A4 of the Standard)

Similar to the determination of flexural buckling stress, the plasticity reduction factor of E_t/E_o used for determining the torsional-flexural buckling stress depends on the assumed stress value. For the first approximation, assume a buckling stress of f=20 ksi. The value of E_t/E_o is found to be equal to 1.0, which is obtained from Table A10 or Figure A7 of the Standard. Thus,

$$\sigma_{ex} = \left[(\pi^2 x27000)/(128.17)^2\right]x(1.0)$$

$$= 16.2 \text{ ksi}$$

$$\sigma_t = \left[1/(1.284x2.329)\right]\left[10500x0.0078 + 0\right] x (1.0)$$

$$= 27.4 \text{ ksi}$$

Therefore,

$$(F_n)_2 = (1/2\beta)\left[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}\right] \quad \text{(Eq. 3.4.3-1)}$$

$$= \left[1/(2x0.648)\right]\left[(16.2+27.4) - \sqrt{(16.2+27.4)^2-4x0.648x16.2x27.4}\right]$$

$$= 12.5 \text{ ksi} < \text{assumed stress} = 20 \text{ ksi}$$

Because the computed stress $(F_n)_2$ is less than the assumed value of f=20 ksi, the second approximation will be assumed that a stress of f=12.5 ksi and E_t/E_o = 1.0. Thus,

$(F_n)_2 \quad = 12.5$ ksi OK

F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$. Thus,

$F_n \quad = 12.5$ ksi

13. Determination of A_e:

Flanges: (k = 0.5)

$w \quad = 1.678$ in.

$w/t \quad = 1.678/0.135 = 12.43$

$\lambda \quad = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}, \quad f = F_n$ (Eq. 2.2.1-4)

$\quad = (1.052/\sqrt{0.5})(12.43)\sqrt{12.5/27000}$

$\quad = 0.398 < 0.673$

$b \quad = w$

Stem: (k = 0.5)

$w \quad = 2.678$ in.

$w/t \quad = 2.678/0.135 = 19.84$

$\lambda \quad = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}, \quad f = F_n$ (Eq. 2.2.1-4)

$\quad = (1.052/\sqrt{0.5})(19.84)\sqrt{12.5/27000}$

$\quad = 0.635 < 0.673$

$b \quad = w$ (Eq. 2.2.1-1)

$\quad = 2.678$ in.

Since flanges and stem are fully effective, the effective area is the same as the full section area, i.e.,

$A_e \quad = A = 1.284 \text{ in.}^2$

14. Determination of $\phi_c P_n$: (Section 3.4 of the Standard)

$P_n \quad = A_e F_n$ (Eq. 3.4-1)

$= 1.284 \times 12.5$

$= 16.05$ kips

$\phi_c = 0.85$

The design axial strength is

$\phi_c P_n = 0.85 \times 16.05$

$= 13.64$ kips

EXAMPLE 19.2 T-SECTION (ASD)

Determine the allowable axial load for the T-section used in Example 19.1.

Solution:

1. Basic parameters used for calculating the sectional properties:

   See Example 19.1 for calculation of sectional properties of the T-section.

2. Determination of F_n

   The following results are obtained from Example 19.1.

   a) For Flexural Buckling:

   $(F_n)_1 = (\pi^2 E_t)/(K_x L_x/r_x)^2$ (Eq. 3.4.1-1)

   $(F_n)_1 = (\pi^2 x 26000)/(128.17)^2$

   $= 16.2$ ksi

   b) For Torsional-Flexural Buckling:

   $(F_n)_2 = (1/2\beta)\left[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}\,\right]$ (Eq. 3.4.3-1)

   where

   $\sigma_{ex} = \left[(\pi^2 E_o)/(K_x L_x/r_x)^2\right](E_t/E_o)$ (Eq. 3.4.3-3)

   $\sigma_t = \left[1/(A r_o^2)\right]\left[G_o J+(\pi^2 E_o C_w)/(K_t L_t)^2\right](E_t/E_o)$ (Eq. 3.4.2-1)

   $G_o = 10500$ ksi (Table A4 of the Standard)

   $(F_n)_2 = (1/2\beta)\left[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}\,\right]$ (Eq. 3.4.3-1)

   $= \left[1/(2x0.648)\right]\left[(16.2+27.4)\right.$

   $\left.-\sqrt{(16.2+27.4)^2-4x0.648x16.2x27.4}\,\right]x(1.0)$

   $= 12.5$ ksi (control)

   Then, F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$.

   $F_n = 12.5$ ksi

3. Determination of A_e:

The effective area is the same as the full section area, i.e.,

$A_e = A = 1.284$ in.^{2} (see Example 19.1)

4. Determination of P_a:

$P_n = A_e F_n$ (Eq. 3.4-1)

$= 1.284 \times 12.5$

$= 16.05$ kips

$\Omega = 2.15$

The allowable axial load is

$P_a = P_n/\Omega = 16.05/2.15$

$= 7.47$ kips

EXAMPLE 20.1 TUBULAR SECTION - SQUARE (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for section shown in Figure 20.1. Use Type 301 stainless steel, 1/4-Hard.

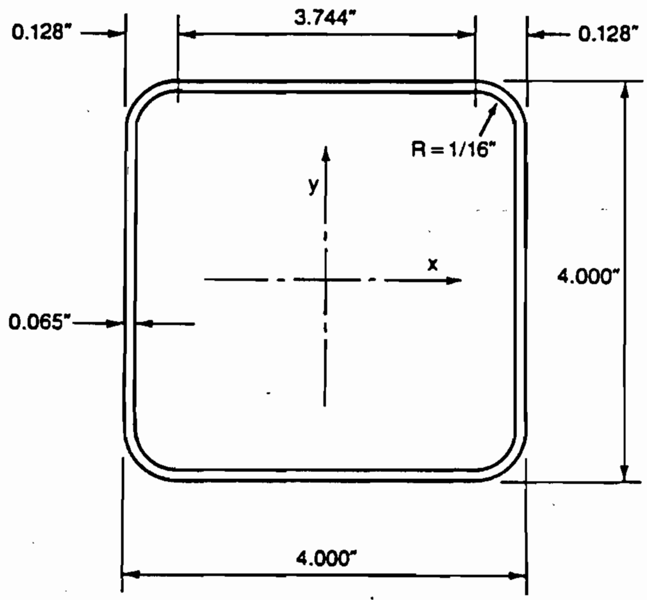


Figure 20.1 Section for Example 20.1

Given:

1. Section: 4.0" x 4.0" x 0.065" Square Tube.

2. $K_xL_x = K_yL_y = 10$ ft.

Solution:

1. Properties of 90^o Corners:

   $r = R + t/2 = 1/16 + 0.065/2 = 0.095$ in.

   Length of arc, $u = 1.57r = 1.57 \times 0.095 = 0.149$ in.

   Distance of c.g. from center of radius,

   $c = 0.637r = 0.637 \times 0.095 = 0.061$ in.

$I_x = I_y = I$ (doubly symmetric section)

Element	L (in.)	y Distance to Center of Section (in.)	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Flanges	2 x 3.744 = 7.488	2 - 0.065/2 = 1.968	29.001	--
Corners	4 x 0.149 = 0.596	3.744/2+0.061 = 1.933	2.227	--
Web	2 x 3.744 = 7.488	--	--	8.747
Sum	15.572		31.228	8.747

$w/t = 3.744/0.065 = 57.60 < 400$ (Section 2.1.1-(1)-(ii))

$A = Lt = 15.572x0.065 = 1.012$ in.^{2}

$I' = Ly^2+I'_1 = 31.228+8.747 = 39.975$ in.^{3}

$I = I't = 39.975x0.065 = 2.598$ in.^{4}

$r = \sqrt{I/A} = \sqrt{2.598/1.012} = 1.602$ in.

$KL/r = 10x12/1.602 = 74.91 < 200$ (Section 3.4-(5))

2. Since the square tube is a doubly symmetric closed section, provisions of Section 3.4.1 of the Standard apply, i.e., section is not subjected to torsional-flexural buckling.

$F_n = \pi^2E_t/(KL/r)^2$ (Eq. 3.4.1-1)

In the determination of the flexural buckling stress, it is necessary to select a proper value of E_t from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of f=24.0 ksi.

From Table A13, the corresponding value of E_t is found to be equal to 17000 ksi. Thus,

$F_n = (\pi^2 x17000)/(74.91)^2$

$= 29.90$ ksi > assumed stress f=24.0 ksi

Because the computed stress is larger than the assumed value, further successive approximations are needed. For the second approximation, assume f=26.33 ksi, and

$E_t = 14960$ ksi

$F_n = (\pi^2 x14960)/(74.91)^2$

$= 26.31$ ksi $\cong$ assumed stress OK

3. Determination of the Effective Width:

$k = 4.0$

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$, $f = F_n$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{4})(3.744/0.065)\sqrt{26.31/27000} = 0.946 > 0.673$

(Section not fully effective)

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/0.946)/0.946 = 0.811$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.811x3.744 = 3.036$ in.

$A_e = A-4(w-b)t$

$= 1.012-4(3.744-3.036)x0.065$

$= 0.828$ in.^{2}

4. Determination of the Design Axial Strength:

$P_n = A_e F_n$ (Eq. 3.4-1)

$= 0.828x26.31$

$= 21.80$ kips

$\phi_c = 0.85$

$\phi_c P_n = 0.85 \times 21.80 = 18.53$ kips

EXAMPLE 20.2 TUBULAR SECTION - SQUARE (ASD)

Determine the allowable axial load for tubular section used in Example 20.1.

Solution:

1. Basic parameters used for calculating the section properties:

   See Example 20.1 for section properties of tubular section.

2. Determination of F_n

   The following results are obtained from Example 20.1.

   For Flexural Buckling Only:

   $F_n = (\pi^2 E_t)/(K_y L_y/r_y)^2$ (Eq. 3.4.1-1)

   $F_n = (\pi^2 x14960)/(74.91)^2$

   $= 26.31$ ksi

3. Determination of A_e:

   The effective area is obtained from Example 20.1 as follows:

   $A_e = A = 0.828$ in.^{2}

4. Determination of P_a:

   $P_n = A_e F_n$ (Eq. 3.4-1)

   $= 0.828 \times 26.31$

   $= 21.80$ kips

   $\Omega = 2.15$

   $P_a = P_n/\Omega = 21.80/2.15$

   $= 10.14$ kips

EXAMPLE 21.1 TUBULAR SECTION - ROUND (LRFD)

By using the Load and Resistance Factor Design (LRFD) method, determine the design axial strength for the tubular section shown in Figure 21.1. Use Type 316 stainless steel, 1/4-Hard.

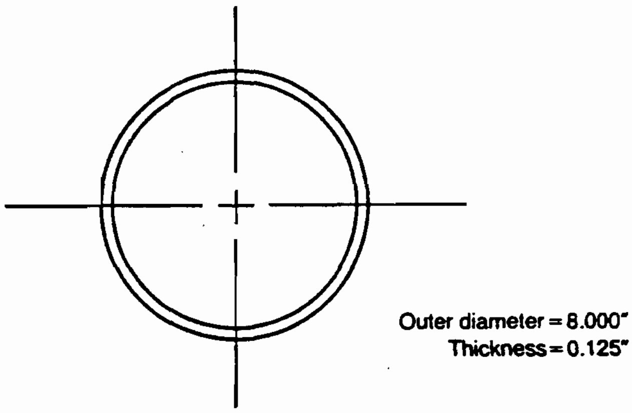


Figure 21.1 Section for Example 21.1

Given:

1. Section: Shown in sketch above.
2. Height: L = 10'-0", simply supported at each end.

Solution:

1. Full section properties:

$$I = (1/4)\pi[(O.R.)^4-(I.R.)^4]$$

$$= (1/4)\pi[(4)^4-(3.875)^4]$$

$$= 23.98 \text{ in.}^4$$

$$A = (1/4)\pi[(O.D.)^2-(I.D.)^2]$$

$$= (1/4)\pi[(8)^2-(7.75)^2]$$

$$= 3.093 \text{ in.}^2$$

$$r = \sqrt{I/A}$$

$$= \sqrt{23.98/3.093}$$

$= 2.784$ in.

2. Determination of Design Axial Strength:

Ratio of outside diameter to wall thickness,

$D/t = 8.000/0.125 = 64.00$

$D/t < 0.881E_o/F_y = 0.881(27000/50) = 475.7$ OK

The design axial strength, $\phi_c P_n$, for cylindrical tubular member is determined in accordance with Section 3.6.2 of the Standard as follows:

$\phi_c = 0.80$

$P_n = F_n A_e$ (Eq. 3.6.2-1)

$F_n = \pi^2 E_t/(KL/r)^2$ (Eq. 3.4.1-1)

where F_n is the flexural buckling stress determined according to Section 3.4.1 of the Standard.

$A_e = [1-(1-(E_t/E_o)^2)(1-A_o/A)]A$ (Eq. 3.6.2-2)

$A_o = K_c A$ (Eq. 3.6.2-3)

$K_c = (1-C)(E_o/F_y)/[(8.93-\lambda_c)(D/t)] + 5.882C/(8.93-\lambda_c)$ (Eq. 3.6.1-3)

$C = F_{pr}/F_y$

$\lambda_c = 3.048C$

From Table A17 of the Standard, the ratio of F_{pr}/F_y is equal to 0.5 in longitudinal compression for Type 301, 1/4-Hard stainless steel.

Therefore,

$K_c = (1-0.5)(27000/50)/[(8.93-3.048x0.5)(64.0)]$

$+(5.882x0.5)/(8.93-3.048x0.5)$

$= 0.967$

$A_o = 0.967A_e$

In the determination of the flexural buckling stress, it is necessary to select a proper value of E_t from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of f=40.0 ksi. From Table A13, the corresponding value of E_t is found to be equal to 8370 ksi. Thus,

$$F_n = (\pi^2 x8370)/(10x12/2.784)^2$$
$$= (\pi^2 x8370)/(43.10)^2$$
$$= 44.46 \text{ ksi} > \text{assumed stress } f=40.0 \text{ ksi}$$

Because the computed stress is larger than the assumed value, further successive approximations are needed.

Assume f=41.83 ksi, and

$$E_t = 7876 \text{ ksi}$$
$$F_n = (\pi^2 x7876)/(43.10)^2$$
$$= 41.84 \text{ ksi} = \text{assumed stress } f=41.83 \text{ ksi} \quad \text{OK}$$

For the compressive stress of F_n = 41.83 ksi, the corresponding value of E_t/E_o is equal to 0.292, which is obtianed from Table A10 of Figure A7 of the Standard. Therefore,

$$A_e = [1-(1-(E_t/E_o)^2)(1-A_o/A)]A$$
$$= [1-(1-(0.292)^2)(1-0.967)]A$$
$$= 0.97xA = 3.00 \text{ in.}^2$$

$$P_n = F_n A_e \qquad \text{(Eq. 3.6.2-1)}$$
$$= (41.83)(3.00)$$
$$= 125.50 \text{ kips}$$

$$\phi_c = 0.80$$
$$\phi_c P_n = 0.80 \text{ x } 125.50$$
$$= 100.40 \text{ kips}$$

EXAMPLE 21.2 TUBULAR SECTION - ROUND (ASD)

Determine the allowable axial load for tubular section used in Example 21.1.

Solution:

1. Basic parameters used for calculating the section properties:

   See Example 21.1 for section properties of tubular section.

2. Determination of F_n

   The following results are obtained from Example 21.1.

   $F_n = (\pi^2 E_t)/(K_y L_y/r_y)^2$ (Eq. 3.4.1-1)

   $F_n = (\pi^2 x 7876)/(43.10)^2$

   $= 41.84$ ksi

3. Determination of A_e:

   The effective area is obtained from Example 21.1 as follows:

   $A_e = 3.00$ in.^{2}

4. Determination of P_a:

   $P_n = A_e F_n$ (Eq. 3.4-1)

   $= 3.00 \times 41.83$

   $= 125.50$ kips

   $\Omega = 2.15$

   $P_a = P_n/\Omega = 125.50/2.15$

   $= 58.37$ kips

EXAMPLE 22.1 C-SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) criteria, check the adequacy of a channel section (Fig. 22.1) to be used as compression member which is subjected to eccentrically axial loads of P_{DL} = 0.35 kips and P_{LL} = 1.75 kips. Consider the following two loading cases: (A) axial loads are applied 2 in. to the left of the c.g. of the full section at both ends, (B) axial loads are applied 2 in. to the left and 4 in. above the c.g. of the full section at both ends. Assume that the effective length factors K_x = K_y = K_t = 1.0, and that the unbraced lengths L_x = L_y = L_t = 16 ft. Use Type 304, 1/4-Hard, stainless steel. Assume dead to live load ratio D/L = 1/5 and 1.2D+1.6L governs the design.

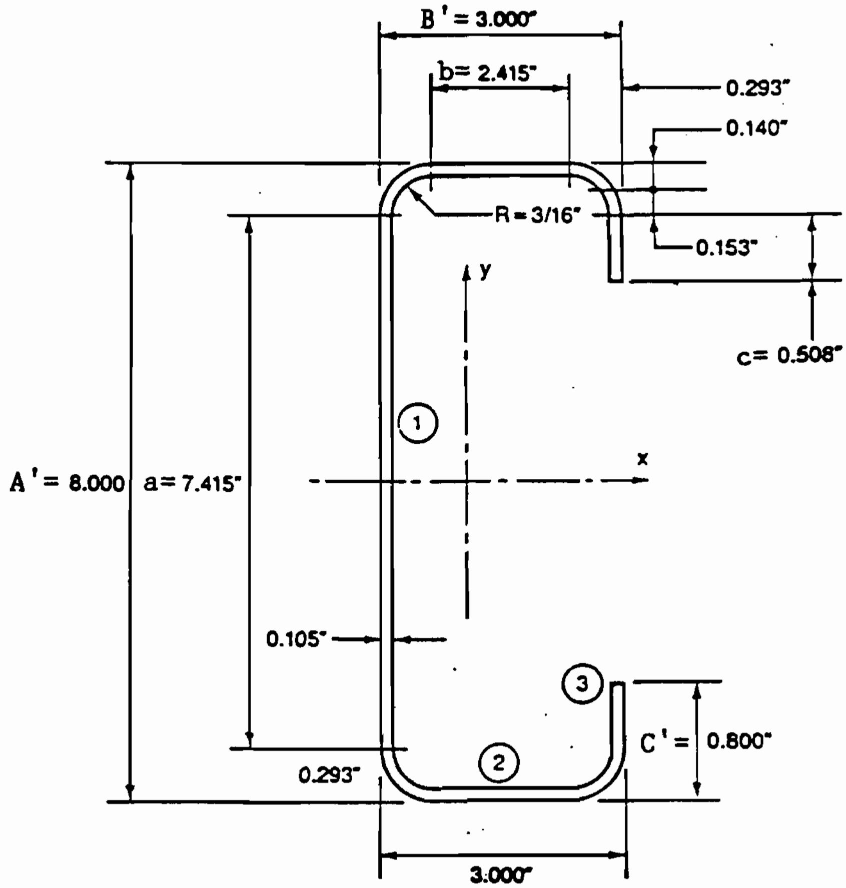


Figure 22.1 Section for Example 22.1

Solution: Part (A)

The following equations used for computing the sectional properties for channel with lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

1. Full section properties:

$r = R+t/2 = 3/16+0.105/2 = 0.240$ in.

$a = A'-(2r+t) = 8.000-(2x0.240+0.105) = 7.415$ in.

$\bar{a} = A'-t = 8.000-0.105 = 7.895$ in.

$b = B'-(2r+t) = 3.000-(2x0.240+0.105) = 2.415$ in.

$\bar{b} = B'-t = 3.000-0.105 = 2.895$ in.

$c = C'-(r+t/2) = 0.800-(0.240+0.105/2) = 0.508$ in.

$\bar{c} = C'-t/2 = 0.800-(0.105/2) = 0.748$ in.

$u = 1.57r = 1.57 \times 0.240 = 0.377$ in.

Distance of corner's c.g. from center of radius = 0.637r

= 0.637(0.240) = 0.153 in.

$$A = t[a+2b+2c+4u] = 0.105[7.415+2x2.415+2x0.508+4x0.377]$$

$$= 1.551 \text{ in.}^2$$

$$I_x = 2t[0.0417a^3+b(a/2+r)^2+2u(a/2+0.637r)^2+0.298r^3$$
$$+0.0833c^3+(c/4)(a-c)^2]$$

$$= 2x0.105[0.0417(7.415)^3+2.415(7.415/2+0.240)^2$$
$$+2x0.377(7.415/2+0.637x0.240)^2+0.298(0.240)^3$$
$$+0.0833(0.508)^3+(0.508/4)(7.415-0.508)^2]$$

$$= 15.108 \text{ in.}^4$$

$$\bar{x} = (2t/A)[b(b/2+r)+u(0.363r)+u(b+1.637r)+c(b+2r)$$

$$= (2x0.105/1.551)\ 2.415(2.415/2+0.240)+0.377(0.363x0.240)$$
$$+0.377(2.415+1.637x0.240)+0.508(2.415+2x0.240)]$$

$$= 0.820 \text{ in.}$$

$$I_y = 2t[b(b/2+r)^2+0.0833b^3+0.505r^3+c(b+2r)^2 +u(b+1.637r)^2]-A(\bar{x})^2$$

$$= 2\text{x}0.105[2.415(2.415/2+0.240)^2+0.0833(2.415)^3 +0.505(0.240)^3+0.508(2.415+2\text{x}0.240)^2 +0.377(2.415+1.637\text{x}0.240)^2]-1.551(0.820)^2$$

$$= 1.786 \text{ in.}^4$$

$$m = (\bar{b}t/12I_x)[6\bar{c}(\bar{a})^2+3\bar{b}(\bar{a})^2-8(\bar{c})^3]$$

$$= [(2.895\text{x}0.105)/(12\text{x}15.108)][6\text{x}0.748(7.895)^2 +3\text{x}2.895(7.895)^2-8(0.748)^3]$$

$$= 1.371 \text{ in.}$$

$$x_o = -(\bar{x}+m) = -(0.820+1.371)$$

$$= -2.191 \text{ in.}$$

$$J = (t^3/3)\ a+2b+2c+4u$$

$$= [(0.105)^3/3][7.415+2\text{x}2.415+2\text{x}0.508+4\text{x}0.377]$$

$$= 0.005699 \text{ in.}^4$$

$$C_w = (t^2/A)\{[\bar{x}A(\bar{a})^2\ /t][(\bar{b})^2/3+m^2-m\bar{b}]+(A/3t)[(m)^2(\bar{a})^3 +(\bar{b})^2(\bar{c})^2(2\bar{c}+3\bar{a})]-(I_x m^2/t)(2\bar{a}+4\bar{c})+[m(\bar{c})^2/3][8(\bar{b})^2(\bar{c}) +2m(2\bar{c}(\bar{c}-\bar{a})+\bar{b}(2\bar{c}-3\bar{a}))]+[(\bar{b})^2(\bar{a})^2/6][(3\bar{c}+\bar{b})(4\bar{c}+\bar{a})-6(\bar{c})^2] -m^2(\bar{a})^4/4\}$$

$$= [(0.105)^2/1.551]\{[0.820\text{x}1.551\text{x}(7.895)^2/0.105][(2.895)^2/3 +(1.371)^2-1.371\text{x}2.895]+1.551/(3\text{x}0.105)[(1.371)^2(7.895)^3 +(2.895)^2(0.748)^2(2\text{x}0.748+3\text{x}7.895)] -[15.108\text{x}(1.371)^2/0.105](2\text{x}7.895+4\text{x}0.748) +[1.371(0.748)^2/3][8(2.895)^2(0.748) +2\text{x}1.371(2\text{x}0.748(0.748-7.895)+2.895(2\text{x}0.748-3\text{x}7.895))] +[(2.895)^2(7.895)^2/6][(3\text{x}0.748+2.895)(4\text{x}0.748+7.895)$$

$-6(0.748)^2] - [(1.371)^2(7.895)^4/4]$

$= 23.468 \text{ in.}^6$

$\beta_w = -\{0.0833[t\bar{x}(\bar{a})^3]+t(\bar{x})^3\bar{a}\}$

$= -\{0.0833[0.105 x 0.820(7.895)^3]+0.105(0.820)^3 x 7.895\}$

$= -3.987$

$\beta_f = (t/2)[(\bar{b}-\bar{x})^4-(\bar{x})^4]+[t(\bar{a})^2/4][(\bar{b}-\bar{x})^2-(\bar{x})^2]$

$= (0.105/2)[(2.895-0.820)^4-(0.820)^4]$

$+[0.105(7.895)^2/4][(2.895-0.820)^2-(0.820)^2]$

$= 6.894$

$\beta_1 = 2\bar{c}t(\bar{b}-x)^3+(2/3)t(\bar{b}-x)[(\bar{a}/2)^3-(\bar{a}/2-\bar{c})^3]$

$= 2x0.748x0.105(2.895-0.820)^3+(2/3)x0.105(2.895$

$-0.820)\{(7.895/2)^3-[(7.895/2)-0.748]^3\}$

$= 5.581$

$j = (1/2I_y)(\beta_w+\beta_f+\beta_1)-x_o$

$= [1/(2x1.786)](-3.987+6.894+5.581)-(-2.191)$

$= 4.567$

$r_x = \sqrt{I_x/A} = \sqrt{15.108/1.551} = 3.121 \text{ in.}$

$K_xL_x/r_x = 1(16x12)/3.121 = 61.52$

$r_y = \sqrt{I_y/A} = \sqrt{1.786/1.551} = 1.073 \text{ in.}$

$K_yL_y/r_y = 1(16x12)/1.073 = 178.94 < 200$ (Section 3.4-(5))

$r_o = \sqrt{r_x^2+r_y^2+x_o^2}$ (Eq. 3.3.1.2-9)

$= \sqrt{(3.121)^2+(1.073)^2+(-2.191)^2} = 3.961 \text{ in.}$

$\beta = 1-(x_o/r_o)^2$ (Eq. 3.4.3-4)

$= 1-(-2.191/3.961)^2 = 0.694$

2. Determination of ϕ_cP_n (Section 3.4):

Since the channel is singly symmetric, F_n shall be taken as the smaller of F_n calculated according to Section 3.4.1 or F_n calculated according to Section 3.4.2.

a. For Flexural Buckling:

$$(F_n)_1 = (\pi^2 E_t)/(K_y L_y/r_y)^2 \qquad \text{(Eq. 3.4.1-1)}$$

In the determination of the flexural buckling stress, it is necessary to select a proper value of E_t from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of f=20 ksi. From Table A13, the corresponding value of E_t is found to be equal to 27000 ksi. Thus,

$$(F_n)_1 = (\pi^2 x27000)/(178.94)^2$$

$$= 8.322 \text{ ksi} < \text{assumed stress } f=20 \text{ ksi}$$

Because the computed stress is less than the assumed value, no further approximation is needed. The section is subject to the elastic flexural buckling.

Therefore, $(F_n)_1 = 8.322$ ksi

b. For Torsional-Flexural Buckling:

$$(F_n)_2 = (1/2\beta)\left[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}\right] \qquad \text{(Eq. 3.4.3-1)}$$

where

$$\sigma_{ex} = \left[(\pi^2 E_o)/(K_x L_x/r_x)^2\right](E_t/E_o) \qquad \text{(Eq. 3.4.3-3)}$$

$$\sigma_t = \left[1/(Ar_o^2)\right]\left[G_o J+(\pi^2 E_o C_w)/(K_t L_t)^2\right](E_t/E_o) \qquad \text{(Eq. 3.4.2-1)}$$

G_o = 10500 ksi (Table A4 of the Standard)

Similar to the determination of flexural buckling stress, the plasticity reduction factor of E_t/E_o depends on the assumed stress value. For the first approximation, assume a buckling stress of f=20 ksi. The value of E_t/E_o is found to be equal to 1.0, which is obtained from Table A10 or Figure A7 of the Standard. Thus,

$$\sigma_{ex} = [(\pi^2 x 27000)/(16x12/3.121)^2]x(1.0)$$
$$= 70.41 \text{ ksi}$$

$$\sigma_t = [1/(1.551x15.69)][10500x0.005699 + \pi^2 x27000x23.468/(16x12)^2]x(1.0)$$
$$= 9.43 \text{ ksi}$$

Therefore,

$$F_{n2} = (1/2\beta)[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}\,] \qquad \text{(Eq. 3.4.3-1)}$$
$$= [1/(2x0.694)]\,[(70.41+9.43)$$
$$-\sqrt{(70.41+9.43)^2-4x0.694x70.41x9.43}\,]$$
$$= 9.024 \text{ ksi} < \text{assumed value } f=20 \text{ ksi} \quad \text{OK}$$

This section is subject to elastic torsional-flexural buckling, and $(F_n)_2$ = 9.024 ksi

Then, F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$.

F_n = 8.322 ksi

For element 1:

w = 7.415 in.

w/t = 7.415/0.105 = 70.62 < 400 OK (Section 21.1-(1)-(ii))

k = 4.0 (Since connected to two stiffened elements)

$$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o} \qquad \text{(Eq. 2.2.1-4)}$$
$$= (1.052/\sqrt{4.00})(70.62)\sqrt{8.322/27000}$$
$$= 0.652 < 0.673$$

$$b = w \qquad \text{(Eq. 2.2.1-1)}$$
= 7.415 in. (Element 1 fully effective)

For element 2:

$w \quad = 2.415$ in.

$w/t \quad = 2.415/0.105 = 23.00$

$S \quad = 1.28\sqrt{E_o/f}\ , \ f = F_n$ (Eq. 2.4-1)

$\quad = 1.28\sqrt{27000/8.322} = 72.91$

$S/3 \quad = 24.30$

$w/t \quad = 23.00 < S/3 = 24.30$

$b \quad = w$ (Eq. 2.4.2-3)

$\quad = 2.415$ in. (Element 2 fully effective)

For element 3:

$d \quad = 0.508$ in.

$d/t \quad = 0.508/0.105 = 4.84$

$k \quad = 0.50$ (unstiffened compression element)

$\lambda \quad = (1.052/\sqrt{0.50})(4.84)\sqrt{8.322/27000}$

$\quad = 0.126 < 0.673$

$d'_s \quad = d = 0.508$ in.

$d_s \quad = d'_s$ (Eq. 2.4.2-4)

$\quad = 0.508$ in. (Element 3 fully effective)

Thus the whole section is fully effective.

$A_e \quad = A = 1.551 \text{ in.}^2$

$P_n \quad = A_e F_n$ (Eq. 3.4-1)

$\quad = 1.551 \times 8.322$

$\quad = 12.91$ kips

$\phi_c \quad = 0.85$

$\phi_c P_n \quad = 0.85 \times 12.91$

$\quad = 10.97$ kips

3. $P_u = 1.2x0.35+1.6x1.75 = 3.22$ kips

$P_u/\phi_c P_n = 3.22/10.97 = 0.294 > 0.15$

Must check both interaction equations (Eq. 3.5-1) and (Eq. 3.5-2).

4. Determination of $\phi_c P_{no}$ (Section 3.4 for $F_n = F_y$):

For element 1:

$\lambda = (1.052/\sqrt{4.00})(70.62)\sqrt{50/27000} = 1.599 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/1.599)/1.599 = 0.539$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.539x7.415 = 4.000$ in.

For element 2:

$S = 1.28\sqrt{27000/50.0} = 29.74$

$S/3 = 9.91$

$S/3 = 9.91 < w/t = 23.00 < S = 29.74$

$I_a = 399t^4\{[(w/t)/S]-0.33\}^3$ (Eq. 2.4.2-6)

$= 399(0.105)^4[(23/29.74)-0.33]^3$

$= 0.004227$ in.^{4}

$I_s = d^3t/12 = (0.508)^3(0.105)/12$

$= 0.001147$ in.^{4}

$I_s/I_a = 0.001147/0.004227 = 0.271$

$D/w = 0.8/2.415 = 0.331$

$n = 1/2$

$k = [4.82-5(D/w)](I_s/I_a)^n+0.43 \leq 5.25-5(D/w)$ (Eq. 2.4.2-9)

$[4.82-5(0.331)](0.271)^{1/2}+0.43 = 2.078$

$5.25-5(0.331) = 3.595 > 2.078$

$k \quad = 2.078$

$\lambda \quad = (1.052/\sqrt{2.078})(23.00)\sqrt{50/27000} = 0.722 > 0.673$

$\rho \quad = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/0.722)/0.722 = 0.963$

$b \quad = \rho w$ (Eq. 2.2.1-2)

$= 0.963 \times 2.415 = 2.326$ in.

For element 3:

$\lambda \quad = (1.052/\sqrt{0.50})(4.84)\sqrt{50/27000} = 0.310 < 0.673$

$d'_s \quad = d = 0.508$ in.

$d_s \quad = d'_s(I_s/I_a) \leq d'_s$ (Eq. 2.4.2-11)

Since $I_s/I_a = 0.271 < 1.0$

$d_s \quad = 0.508(0.271) = 0.138$ in.

$A_e \quad = 1.551-0.105(7.415-4.000)-0.105(0.508-0.138)x2$

$-0.105(2.415-2.326)x2$

$= 1.096 \text{ in.}^2$

$P_{no} \quad = 1.096 \times 50 = 54.80$ kips

$\phi_c \quad = 0.85$

$\phi_c P_{no} \quad = 0.85 \times 54.80$

$= 46.58$ kips

5. Determination of M_{uy} (required flexural strength about y-axis): (M_{ux} = 0 since $e_y = 0$)

M_{uy} will be with respect to the centroidal axes of the effective section determined for the required axial strength alone.

$A_e \quad = 1.551 \text{ in.}^2$ under required axial strength alone

Since $A_e = A$, the centroidal axes for the effective section are the same as those for the full section. Therefore, e_x did not change.

M_{uy} = 3.22(2.00) = 6.44 kips-in. (Required Flexural Strength)

The interaction equations (Eq. 3.5-1) and (Eq. 3.5-2) reduce to the following:

$$P_u/\phi_c P_n + C_{my} M_{uy}/\phi_b M_{ny} \alpha_{ny} \leq 1.0 \qquad \text{(Eq. 3.5-1)}$$

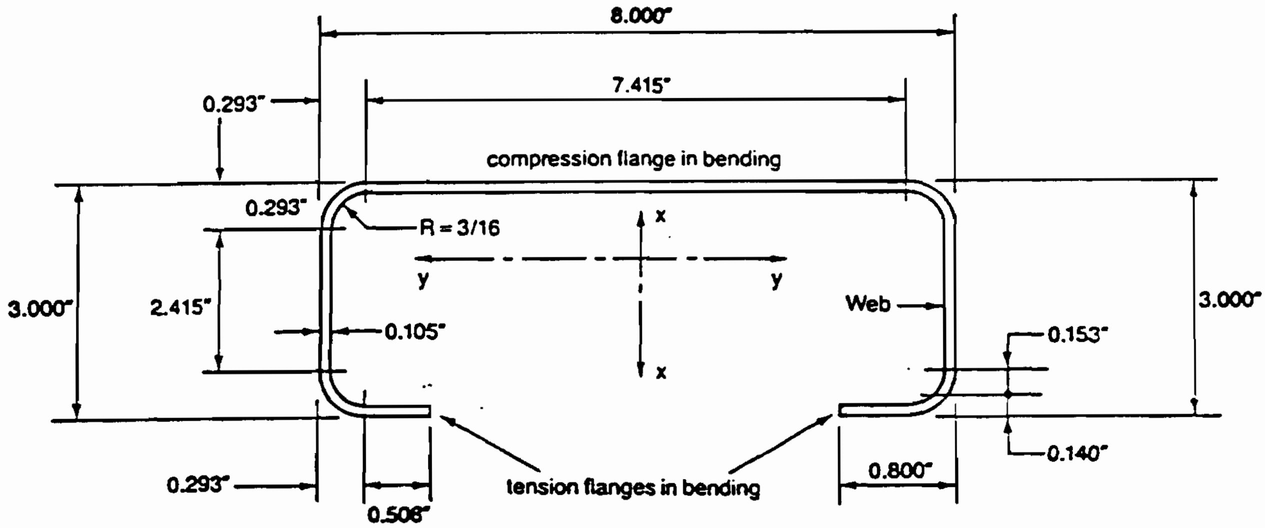
$$P_u/\phi_c P_{no} + M_{uy}/\phi_b M_{ny} \leq 1.0 \qquad \text{(Eq. 3.5-2)}$$

6. Determination of $\phi_b M_{ny}$ (Section 3.3.1):

$\phi_b M_{ny}$ shall be taken as the smaller of the design flexural strengths calculated according to sections 3.3.1.1 and 3.3.1.2:

a. Section 3.3.1.1: M_{ny} will be calculated on the basis of initiation of yielding.

Here it is evident that the initial yielding will not be in the compression flange, rather it will be in the tension flange.



The procedure is iterative: one assumes the actual compressive stress f under M_{ny}. Knowing f one proceeds as usual to obtain

x_{cg} (measured from top fiber) to neutral axis. Then one obtains $f = F_y\, x_{cg}/(3-x_{cg})$ and checks if it equals to the assumed value. If not, one reiterates by assuming another f until finally it checks. Then for this condition one obtains I_y and $M_{ny} = f(I_y/x_{cg}) = F_y\, I_y/(3-x_{cg})$.

For the first iteration assume a compressive stress f = 20 ksi in the top compression fibers and that the webs are fully effective.

Compression flange:

$k = 4.00$

$w/t = 7.415/0.105 = 70.62$

$\lambda = (1.052/\sqrt{4.00})(70.62)\sqrt{20/27000} = 1.011 > 0.673$

$\rho = [1-(0.22/1.011)]/1.011 = 0.774$

$b = 0.774 \times 7.415 = 5.739$ in.

To calculate effective section properties about y-axis:

Element	L Effective Length (in.)	x Distance from Top Fiber (in.)	Lx (in.²)	Lx² (in.³)	I'₁ About Own Axis (in.³)
Webs	2x2.415 = 4.830	1.500	7.245	10.868	2.347
Upper Corners	2x0.377 = 0.754	0.140	0.106	0.015	--
Lower Corners	2x0.377 = 0.754	2.860	2.156	6.167	--
Compression Flange	5.739	0.053	0.304	0.016	--
Tension Flanges	2x0.508 = 1.016	2.948	2.995	8.830	--
Sum	13.093		12.806	25.896	2.347

Distance from top fiber to y-axis is

x_{cg} = 12.806/13.093 = 0.978 in.

$f = F_y[x_{cg}/(3-x_{cg})]$

$= 50[0.978/(3.00-0.978)] = 24.18$ ksi > 20 ksi

need to do another iteration.

For the second iteration assume a compressive stress f = 25.50 ksi in the top compression fibers, and that the webs are fully effective.

Compression flange:

$\lambda = (1.052/\sqrt{4.0})(70.62)\sqrt{25.5/27000} = 1.142 > 0.673$

$\rho = [1-(0.22/1.142)]/1.142 = 0.707$

$b = 0.707 \times 7.415 = 5.242$ in.

Effective section properties about y-axis:

Element	L Effective Length (in.)	x Distance from Top Fiber (in.)	Lx ($in.^2$)	Lx^2 ($in.^3$)	I'_1 About Own Axis ($in.^3$)
Webs	2x2.415 = 4.830	1.500	7.245	10.868	2.347
Upper Corners	2x0.377 = 0.754	0.140	0.106	0.015	--
Lower Corners	2x0.377 = 0.754	2.860	2.156	6.167	--
Compression Flange	5.242	0.053	0.278	0.015	--
Tension Flanges	2x0.508 = 1.016	2.948	2.995	8.830	--
Sum	12.596		12.780	25.895	2.347

Distance from top fiber to y-axis is

x_{cg} = 12.780/12.596 = 1.015 in.

f = 50 [1.015/(3.00-1.015)] = 25.57 ksi (close enough)

Thus actual compressive stress f = 25.50 ksi

To check if the webs are fully effective (Section 2.2.2):

f_1 = [(1.015-0.293)/1.985](50) = 18.17 ksi(compression)

f_2 = - [(1.985-0.293)/1.985](50) = -42.62 ksi(tension)

Ψ = f_2/f_1 = -42.62/18.19 = -2.343

k = $4+2(1-\Psi)^3+2(1-\Psi)$ (Eq. 2.2.2-4)

= 4+2 [1-(-2.343)]³+2 [1-(-2.343)]

= 85.406

h = w = 2.415 in.

w/t = 2.415/0.105 = 23.00 < 200 OK (Section 2.1.2-(1))

λ = $(1.052/\sqrt{85.406})(23.00)\sqrt{18.19/27000}$ = 0.068 < 0.673

b_e = 2.415 in.

b_2 = $b_e/2$ (Eq. 2.2.2-2)

= 2.415/2 = 1.208 in.

b_1 = $b_e/(3-\Psi)$ (Eq. 2.2.2-1)

= 2.415/ [3-(-2.343)] = 0.452 in.

Compression portion of each web calculated on the basis of the effective section = x_{cg}-0.293 = 1.015-0.293 = 0.722 in.

Since b_1+b_2 = 1.660 in. > 0.722 in., b_1+b_2 shall be taken as 0.722 in.. This verifies the assumption that the web is fully effective.

I'_y = $Lx^2 + I'_1 - Lx^2_{cg}$

= 25.895 + 2.347 - 12.596(1.015)²

$= 15.265 \text{ in.}^3$

Actual $I_y = I'_y t$

$= 15.265(0.105) = 1.603 \text{ in.}^4$

$S_e = I_y/(3.000 - x_{cg})$

$= 1.603/(3.000-1.015)$

$= 0.808 \text{ in.}^3$

$M_{ny} = S_e F_y$ (Eq. 3.3.1.1-1)

$= 0.808(50)$

$= 40.40$ kips-in.

$\Phi_b = 0.90$

$\Phi_b M_{ny} = 0.90 \times 40.40 = 36.36$ kips-in.

b. Section 3.3.1.2: M_{ny} will be calculated on the basis of the lateral buckling strength. (y-axis is the axis of bending).

$M_n = S_c(M_c/S_f)$ (Eq. 3.3.1.2-1)

$M_c = C_s C_b A \sigma_{ex} \left[j + C_s\sqrt{j^2 + r_o^2(\sigma_t/\sigma_{ex})}\right]$ (Eq. 3.3.1.2-5)

where

$\sigma_{ex} = [(\pi^2 E_o)/(K_x L_x/r_x)^2](E_t/E_o)$ (Eq. 3.4.3-3)

$= 70.41 \times (E_t/E_o)$ ksi (from item 2.b of this example)

$\sigma_t = 1/(A r_o^2)[G_o J + (\pi^2 E_o C_w)/(K_t L_t)^2](E_t/E_o)$ (Eq. 3.4.2-1)

$= 9.43 \times (E_t/E_o)$ ksi (from item 2.b of this example)

$C_b = 1.75 + 1.05(M_1/M_2) + 0.3(M_1/M_2)^2$

$= 1.75 + 1.05(-1.0) + 0.3(-1.0)^2 = 1.0$

$C_s = 1.0$

$r_o = 3.961$ in.

$j = 4.567$

$M_c = 1.0 \times 1.0 \times (1.551)(70.41)[4.567$

$$+1.00\sqrt{(4.567)^2+(3.961)^2(9.43/70.41)}\,]$$

$$= 1022.0\ (E_t/E_o)\ \text{kips-in.}$$

$$M_n = S_c(M_c/S_f) \qquad \text{Eq. (3.3.1.2-1)}$$

$$M_n = S_c f$$

$$f = M_c/S_f = 1022.0(E_t/E_o)/2.046 = 499.5(E_t/E_o)\ \text{ksi}$$

In the determination of the lateral buckling stress, it is necessary to select a proper ratio of E_t/E_o from Table A10 or Figure A7 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of $f=F_y=50$ ksi. From Table A10, the corresponding value of E_t/E_o is found to be equal to 0.19. Thus,

$$f_1 = 499.5\text{x}0.19$$

$$= 94.9\ \text{ksi} > \text{assumed stress } f=50\ \text{ksi}$$

Because the computed stress is larger than the maximum yield strength, the lateral buckling stress shall be limited to 50 ksi. Therefore,

$$f = M_c/S_f = 50.0\ \text{ksi}$$

To calculate effective section properties to obtain S_c at a stress of 50.0 ksi, we assume that the webs are fully effective.

Compression flange:

$$\lambda = (1.052/\sqrt{4.00})(70.62)\sqrt{50.0/27000} = 1.599 > 0.673$$

$$\rho = [1-(0.22/1.599)]/1.599 = 0.539$$

$$b = 0.539 \times 7.415 = 3.997\ \text{in.}$$

Effective section properties about y-axis:

Element	L Effective Length (in.)	x Distance from Top Fiber (in.)	Lx (in.^{2})	Lx^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Webs	2x2.415 = 4.830	1.500	7.245	10.868	2.347
Upper Corners	2x0.377 = 0.754	0.140	0.106	0.015	--
Lower Corners	2x0.377 = 0.754	2.860	2.156	6.167	--
Compression Flange	3.997	0.053	0.212	0.011	--
Tension Flanges	2x0.508 = 1.016	2.948	2.995	8.830	--
Sum	11.351		12.714	25.891	2.347

Distance from top fiber to y-axis is

x_{cg} = 12.714/11.351 = 1.120 in.

To check if the webs are fully effective (Section 2.2.2):

f_1 = [(1.120-0.293)/1.120](50.0) = 36.92 ksi(compression)

f_2 = -[(1.880-0.293)/1.120](50.0) = -70.85 ksi(tension)

ψ = -70.85/36.92 = -1.919

k = $4+2[1-(-1.919)]^3+2[1-(-1.919)]$

= 59.581

λ = $(1.052/\sqrt{59.581})(23.00)\sqrt{36.92/27000}$ = 0.116 < 0.673

b_e = 2.415 in.

b_2 = 2.415/2 = 1.208 in.

b_1 = 2.415/[3-(-1.919)] = 0.491 in.

Compression portion of each web calculated on the basis of the effective section = 1.120-0.293 = 0.827 in.

Since b_1+b_2 = 1.699 in. > 0.827 in., b_1+b_2 shall be taken as 0.827 in.. This verifies the assumption that the web is fully effective.

I'_y = 25.891 + 2.347 - 11.351(1.120)^2

= 13.999 in.^{3}

Actual I_y = 13.999(0.105) = 1.470 in.^{4}

S_c = I_y/x_{cg} = 1.470/1.120 = 1.313 in.^{3}

M_{ny} = $M_c S_c/S_f$ (Eq. 3.3.1.2-1)

= 102.30(1.313)/2.046

= 65.65 kips-in.

ϕ_b = 0.85

$\phi_b M_{ny}$ = 0.85 x 65.65 = 55.80 kips-in.

$\phi_b M_{ny}$ shall be the smaller of 36.36 kips-in. and 55.80 kips-in.

Thus

$\phi_b M_{ny}$ = 36.36 kips-in.

7. C_{my} = 0.6-0.4(M_1/M_2) ≥ 0.4

M_1/M_2 = -1.00 (single curvature)

0.6-0.4(-1.00) = 1.00 > 0.4

C_{my} = 1.00

8. Determination of $1/\alpha_{ny}$:

ϕ_c = 0.85

P_E = $\pi^2 E_o I_y/(K_y L_y)^2$ (Eq. 3.5-5)

I_y = 1.786 in.^{4}

$K_y L_y$ = 1.0(16x12) = 192 in.

P_E = $[\pi^2(27000)(1.786)]/(192)^2$ = 12.91 kips

$1/\alpha_{ny} = 1/[1-P_u/(\phi_c P_E)]$ (Eq. 3.5-4)

$= 1/[1-3.22/(0.85x12.91)] = 1.415$

$\alpha_{ny} = 0.707$

9. Check interaction equations:

$P_u/\phi_c P_n + C_{my} M_{uy}/\phi_b M_{ny} \alpha_{ny} \leq 1.0$ (Eq. 3.5-1)

$3.22/10.970+1.00x6.44/(36.36x0.707) = 0.294+0.251$

$= 0.545 < 1.0$ OK

$P_u/\phi_c P_{no} + M_{uy}/\phi_b M_{ny} \leq 1.0$ (Eq. 3.5-2)

$3.22/46.58+6.44/36.36 = 0.069+0.177 = 0.246 < 1.0$ OK

Therefore the section is adequate for the applied loads.

Solution: Part (B)

1. Full section properties are the same as previously calculated in part (A.1).

2. $\phi_c P_n$ = 10.970 kips (calculated in part (A)).

3. $P_u/\phi_c P_n = 3.22/10.970 = 0.294 > 0.15$

Therefore the following interaction equations must be satisfied.

$P_u/\phi_c P_n + C_{mx} M_{ux}/\phi_b M_{nx} \alpha_{nx} + C_{my} M_{uy}/\phi_b M_{ny} \alpha_{ny} \leq 1.0$ (Eq. 3.5-1)

$P_u/\phi_c P_{no} + M_{ux}/\phi_b M_{nx} + M_{uy}/\phi_b M_{ny} \leq 1.0$ (Eq. 3.5-2)

4. $\phi_c P_{no}$ = 46.58 kips (calculated in part (A.4)).

5. Determination of M_{ux} (Section 3.5):

The centroidal x-axis is the same for both the full and effective sections.

$e_y = 4.000$ in.

$M_{ux} = P_u e_y = 3.22(4.000) = 12.88$ kips-in.

6. Determination of $\phi_b M_{nx}$ (Section 3.3.1):

$\phi_b M_{nx}$ shall be taken as the smaller of the design flexural strengths calculated according to Sections 3.3.1.1 and 3.3.1.2.

a. Section 3.3.1.1: M_{nx} will be calculated based on the initiation of yielding.

First approximation:

* Assume a compressive stress of $f = F_y = 50$ ksi in the top fiber of the section.

* Assume that the web is fully effective.

Compression flange:

$w = 2.415$ in.

$w/t = 2.415/0.105 = 23.00$

$S = 1.28\sqrt{E_o/f}$ (Eq. 2.4-1)

$= 1.28\sqrt{27000/50.0} = 29.74$

For $S/3 = 9.91 < w/t = 23.00 < S = 29.74$

$I_a = t^4 399\{[(w/t)/S]-0.33\}^3$ (Eq. 2.4.2-6)

$= (0.105)^4(399)[(23.00/29.74)-0.33]^3$

$= 0.004227$ in.^{4}

$I_s = d^3t/12$ (Eq. 2.4-2)

$= (0.508)^3(0.105)/12 = 0.001147$ in.^{4}

$I_s/I_a = 0.001147/0.004227 = 0.271$

$D \quad = 0.800$ in.

$D/w \quad = 0.800/2.415 = 0.331$

$w/t \quad = 23.00 < 50$ OK (Section 2.1.1-(1)-(iii))

For $0.25 < D/w = 0.331 < 0.8$

$k \quad = [4.82-5(D/w)](I_s/I_a)^{1/2}+0.43 \leq 5.25-5(D/w)$ (Eq. 2.4.2-9)

$[4.82-5(0.331)](0.344)^{1/2}+0.43 = 2.078$

$5.25-5(0.331) = 3.595$

$k \quad = 2.078$

$\lambda \quad = (1.052/\sqrt{2.078})(23.00)\sqrt{50.0/27000} = 0.722 > 0.673$

$\rho \quad = [1-(0.22/0.722)]/0.722 = 0.963$

$b \quad = 0.963 \times 2.415 = 2.326$ in.

Compression stiffener:

$d \quad = 0.508$ in.

$d/t \quad = 0.508/0.105 = 4.84$

$k \quad = 0.50$

Assume the maximum stress in element, $f = F_y = 50$ ksi although it will be actually less.

$\lambda \quad = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

$\quad = (1.052/\sqrt{0.50})(4.84)\sqrt{50.0/27000} = 0.310 < 0.673$

For $\lambda < 0.673$

$b \quad = w$ (Eq. 2.2.1-1)

$d'_s \quad = 0.508$ in.

$d_s \quad = d'_s(I_s/I_a) \leq d'_s$ (Eq. 2.4.2-11)

$\quad = 0.508(0.271)$

$\quad = 0.138$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly ($in.^2$)	Ly^2 ($in.^3$)	I'_1 About Own Axis ($in.^3$)
Compression Flange	2.326	0.053	0.123	0.007	--
Compression Stiffener	0.138	0.362	0.050	0.018	--
Compression Corners	2x0.377 = 0.754	0.140	0.106	0.015	--
Web	7.415	4.000	29.660	118.640	33.974
Tension Flange	2.415	7.948	19.194	152.557	--
Tension Stiffener	0.508	7.453	3.786	28.218	0.011
Tension Corners	2x0.377 = 0.754	7.860	5.926	46.582	--
Sum	14.310		58.845	346.037	33.985

Distance from neutral axis to top fiber,

y_{cg} = Ly/L = 58.845/14.310 = 4.112 in.

Since the distance from the neutral axis to the top compression fiber is greater than half the depth of the section, a compressive stress of F_y = 50 ksi governs as assumed.

$I'_x = Ly^2 + I'_1 - Ly^2_{cg}$

$= 346.037 + 33.985 - 14.310(4.112)^2$

$= 138.06 \text{ in.}^3$

Actual $I_x = tI'_x$

$= (0.105)(138.06) = 14.50 \text{ in.}^4$

Check Web

w/t = 7.415/0.105 = 70.62 < 200 OK (Section 2.1.2-(1))

f_1 = [(4.112-0.293)/4.112](50) = 46.44 ksi(compression)

f_2 = - (3.888-0.293)/4.112 (50) = -43.71 ksi(tension)

Ψ = f_2/f_1 = -43.71/46.46 = -0.941

k = 4+2[1-(-0.941)]³+2[1-(-0.941)]

= 22.51

λ = $(1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$ (Eq. 2.2.1-4)

= $(1.052/\sqrt{22.51})(70.62)\sqrt{46.44/27000}$ = 0.649 < 0.673

For λ < 0.673

b = w (Eq. 2.2.1-1)

b_e = 7.415 in.

b_2 = 7.415/2 = 3.708 in.

b_1 = 7.415/[3-(-0.941)] = 1.882 in.

b_1+b_2 = 1.882+3.708 = 5.590 in. > 3.785 in. (compression portion of web)

Therefore web is fully effective as assumed.

Check Compression Stiffener

Actual maximum stress in stiffener = 46.44 ksi

λ = $(1.052/\sqrt{0.50})(4.84)\sqrt{46.44/27000}$ = 0.299 < 0.673

For λ < 0.673

d'_s = 0.508 in.

Since I_s/I_a is unchanged

d_s = 0.138 in.

Conservative assumption OK

S_e = I_x/y_{cg} = 14.50/4.112 = 3.526 in.³

M_{nx} = S_eF_y (Eq. 3.3.1.1-1)

= (3.526)(50) = 176.30 kips-in.

Φ_b = 0.90

$\phi_b M_{nx}$ = 0.90 x 176.30 = 158.67 kips-in.

b. Section 3.3.1.2: M_{nx} will be calculated based on the lateral buckling strength.

For the full section:

I_x = 15.108 in.^{4}

y_{cg} = 4.000 in.

S_f = I_x/y_{cg} = 15.108/4.000 = 3.777 in.^{3}

M_y = $S_f F_y$

= 3.777(50) = 188.85 kips-in.

C_b = 1.00 (for members subject to combined axial load and bending moment)

r_o = 3.961 in.

A = 1.551 in.^{2}

σ_{ey} = $[\pi^2 E_o/(K_y L_y/r_y)^2](E_t/E_o)$

= $[\pi^2(27000)/(178.94)^2](E_t/E_o)$

= 8.322 (E_t/E_o) ksi

σ_t = 9.43(E_t/E_o) ksi (from part (A))

M_c = $C_b r_o A \sqrt{\sigma_{ey}\sigma_t}$ (Eq. 3.3.1.2-5)

= $(1.000)(3.961)(1.551)\sqrt{(8.322)(9.430)}(E_t/E_o)$

= 54.42 (E_t/E_o) kips-in.

Let f = M_c/S_f

= $54.42(E_t/E_o)/3.777$ = 14.41 (E_t/E_o) ksi

For the stress f less than 20 ksi, the plasticity reduction factor of E_t/E_o is equal to 1.0. The section is subject to elastic lateral buckling. Therefore,

M_c = 54.42 kips-in.

$f \quad = 14.41$ ksi

Determine S_c, the elastic section modulus of the effective section calculated at a stress of M_c/S_f in the extreme compression fiber.

For compression flange:

$w \quad = 2.415$ in.

$w/t \quad = 2.415/0.105 = 23.00$

$S \quad = 1.28\sqrt{E_o/f}$, $f = F_n$ (Eq. 2.4-1)

$S \quad = 1.28\sqrt{27000/14.41} = 55.41$

$S/3 = 18.47 < w/t = 23.00 < S = 55.41$

$I_a \quad = 399(0.105)^4[(23.00/55.41)-0.33]^3$

$\quad = 0.000030$ in.^{4}

$I_s \quad = 0.001147$ in.^{4}

$I_s/I_a = 0.001147/0.000030 = 38.23$

$[4.82-5(0.331)](38.23)^{1/2}+0.43 = 20.00 > 3.595$

$k \quad = 3.595$

$\lambda \quad = (1.052/\sqrt{3.595})(23.00)\sqrt{14.41/27000} = 0.295 < 0.673$

$b \quad = w = 2.415$ in. (compression flange fully effective)

For compression stiffener:

f is taken conservatively as 14.41 ksi as used in the top compression fiber.

$d/t \quad = 4.84$

$\lambda \quad = (1.052/\sqrt{0.50})(4.84)\sqrt{14.41/27000} = 0.166 < 0.673$

$d'_s \quad = d = 0.508$ in.

And since $I_s/I_a = 38.23 > 1.0$

$d_s = d'_s = 0.508$ in. (compression stiffener fully effective)

And since the web was fully effective at the stress $f = F_y = 50$ ksi, it will be fully effective for $f = 14.41$ ksi.

Thus the whole section is fully effective at $M_c/S_f = 15.71$ ksi

Therefore

$S_c = S_f = 3.777 \text{ in.}^3$

$M_{nx} = M_c S_c/S_f$

$= 54.42(3.777)/3.777$

$= 54.42$ kips-in.

$\phi_b = 0.85$

$\phi_b M_{nx} = 0.85 \times 54.42 = 46.26$ kips-in.

$\phi_b M_{nx}$ shall be the smaller of 158.67 kips-in. and 46.26 kips-in.

Therefore

$\phi_b M_{nx} = 46.26$ kips-in.

7. Determination of C_{mx} (Section 3.5):

$M_1/M_2 = -1.00$ (single curvature)

$C_{mx} = 0.6-0.4(-1.0) = 1.00 > 0.4$ OK

8. Determination of α_{nx} (Section 3.5):

$P_u = 3.22$ kips

$$P_E = \pi^2 E_o I_x/(K_x L_x)^2 \qquad \text{(Eq. 3.5-5)}$$

$= [\pi^2(27000)(15.108)]/[1(16)x12]^2 = 109.21$ kips

$\phi_c = 0.85$

$1/\alpha_{nx} = 1/[1-P_u/(\phi_c P_E)]$ (Eq. 3.5-4)

$= 1/[1-3.22/(0.85x109.21)] = 1.036$

$\alpha_{nx} = 0.965$

9. $M_{uy} = 6.44$ kips-in. (calculated in part (A.5))

10. $\phi_b M_{ny} = 36.36$ kips-in. (calculated in part (A.6))

11. $C_{my} = 1.0$ (calculated in part (A.7))

12. $\alpha_{ny} = 0.707$ (calculated in part (A.8))

13. Interaction equations (Section 3.5):

$P_u/\phi_c P_n + C_{mx} M_{ux}/\phi_b M_{nx}\alpha_{nx} + C_{my} M_{uy}/\phi_b M_{ny}\alpha_{ny} \leq 1.0$ (Eq. 3.5-1)

$3.22/10.970+1.0x12.88/(46.26x0.965)+1.0x6.44/(36.36x0.707)$

$0.294+0.289+0.251 = 0.834 < 1.0$ OK

$P_u/\phi_c P_{no} + M_{ux}/\phi_b M_{nx} + M_{uy}/\phi_b M_{ny} \leq 1.0$ (Eq. 3.5-2)

$3.22/46.58+12.88/46.26+6.44/36.36$

$0.069+0.278+0.177 = 0.524 < 1.0$ OK

Therefore the section is adequate for the applied loads.

EXAMPLE 22.2 C-SECTION (ASD)

Rework Example 22.1 by using the Allowable Stress Design (ASD) method to check the adequacy of a channel section (Fig. 22.1) to be used as compression member.

Solution: Part (A)

1. Full section properties:

   The section properties (A, I_x, etc.) are the same as those calculated in Example 22.1.(1).

2. Determination of P_a:

   The following results are obtained from Example 22.1.(2).

a) For Flexural Buckling:

$$(F_n)_1 = (\pi^2 E_t)/(K_y L_y/r_y)^2 \qquad \text{(Eq. 3.4.1-1)}$$

$$(F_n)_1 = (\pi^2 x27000)/(178.94)^2$$

$$= 8.322 \text{ ksi}$$

The section is subject to the elastic flexural buckling.

b) For Torsional-Flexural Buckling:

$$(F_n)_2 = (1/2\beta)\left[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}\right] \qquad \text{(Eq. 3.4.3-1)}$$

where

$$\sigma_{ex} = \left[(\pi^2 E_o)/(K_x L_x/r_x)^2\right](E_t/E_o) \qquad \text{(Eq. 3.4.3-3)}$$

$$\sigma_t = 1/(Ar_o^2)\left[G_o J+(\pi^2 E_o C_w)/(K_t L_t)^2\right](E_t/E_o) \qquad \text{(Eq. 3.4.2-1)}$$

$$G_o = 10500 \text{ ksi (Table A4 of the Standard)}$$

$$F_{n2} = (1/2\beta)\left[(\sigma_{ex}+\sigma_t)-\sqrt{(\sigma_{ex}+\sigma_t)^2-4\beta\sigma_{ex}\sigma_t}\right] \qquad \text{(Eq. 3.4.3-1)}$$

$$= \left[1/(2x0.694)\right]\left[(70.41+9.43)\right.$$

$$\left.-\sqrt{(70.41+9.43)^2-4x0.694x70.41x9.43}\right]$$

$$= 9.024 \text{ ksi}$$

This section is subject to elastic torsional-flexural buckling.

Then, F_n should be the smaller of $(F_n)_1$ and $(F_n)_2$.

$F_n \quad = 8.322$ ksi

Therefore,

$P_n \quad = A_eF_n = 1.551x8.322$

$\quad\quad = 12.91$ kips

$\Omega \quad = 2.15$

$P_a \quad = P_n/\Omega = 12.91/2.15 = 6.0$ kips

3. $P = 0.35+1.75 = 2.10$ kips

$P/P_a = 2.10/6.0 = 0.350 > 0.15$

Must check both interaction equations as follows:

$P/P_a+C_{mx}M_x/(M_{ax}\alpha_x)+C_{my}M_y/(M_{ay}\alpha_y) \leq 1.0$

$P/P_{ao}+M_x/M_{ax}+M_y/M_{ay} \leq 1.0$

4. Determination of P_{ao} (for $F_n = F_y$):

$A_e \quad = 1.096$ in.^{2}(from Example 19.1.(4))

$P_{no} \quad = 1.096 \times 50 = 54.80$ kips

$\Omega \quad = 2.15$

$P_{ao} \quad = P_{no}/\Omega = 54.8/2.15$

$\quad\quad = 25.49$ kips

5. Determination of M_y (required flexural strength about y-axis):

($M_x = 0$ since $e_y = 0$)

M_y will be with respect to the centroidal axes of the effective section determined for the required axial strength alone.

$A_e \quad = 1.551$ in.^{2} under required axial strength alone

Since $A_e = A$, the centroidal axes for the effective section are the same as those for the full section. Therefore, e_x did not change.

M_y = 2.10(2.00) = 4.20 kips-in. (Required Flexural Strength)

The interaction equations reduce to the following:

$P/P_a + C_{my}M_y/M_{ay}\alpha_y \leq 1.0$

$P/P_{ao} + M_y/M_{ayo} \leq 1.0$

6. Determination of M_{ay}

M_{ay} shall be taken as the smaller of the allowable flexural strengths calculated according to Sections 3.3.1.1 and 3.3.1.2:

a. Section 3.3.1.1: M_{ay} will be calculated on the basis of initiation of yielding.

S_e = 1.603/(3.000-1.015)

= 0.808 in.³ (from Example 22.1)

M_{ny} = S_eF_y (Eq. 3.3.1.1-1)

= 0.808(50)

= 40.40 kips-in.

Ω = 1.85

M_{ay} = M_{ny}/Ω = 40.40/1.85 = 21.84 kips-in.

b. Section 3.3.1.2: M_{ay} will be calculated on the basis of the lateral buckling strength. (y-axis is the axis of bending).

M_n = $S_c(M_c/S_f)$ (Eq. 3.3.1.2-1)

M_n = S_cf

f = M_c/S_f = 50.0 ksi

S_c = I_y/x_{cg} = 1.470/1.120 = 1.313 in.³

M_{ny} = M_cS_c/S_f (Eq. 3.3.1.2-1)

$= 50.0(1.313)$

$= 65.65$ kips-in.

$\Omega = 1.85$

$M_{ay} = M_{ny}/\Omega = 65.65/1.85 = 35.49$ kips-in.

M_{ay} shall be the smaller of 21.84 kips-in. and 35.49 kips-in.

Thus

$M_{ay} = 21.84$ kips-in.

7. $C_{my} = 0.6-0.4(M_1/M_2) \geq 0.4$

$M_1/M_2 = -1.00$ (single curvature)

$0.6-0.4(-1.00) = 1.00 > 0.4$

$C_{my} = 1.00$

8. Determination of $1/\alpha_{ny}$:

$\Omega = 2.15$

$P_{cr} = \pi^2 E_o I_y/(K_y L_y)^2$

$I_y = 1.786$ in.^{4}

$K_y L_y = 1.0(16x12) = 192$ in.

$P_{cr} = \pi^2(27000)(1.786)/(192)^2 = 12.91$ kips

$1/\alpha_{ny} = 1/[1-(\Omega_c P/P_{cr})]$

$= 1/[1-(2.15x2.1/12.91)] = 1/0.650$

$\alpha_{ny} = 0.650$

9. Check interaction equations:

$P/P_a + C_{my} M_y/M_{ay} \alpha_{ny} \leq 1.0$

$2.1/6.0+1.00x4.2/(21.84x0.650) = 0.350+0.296$

$= 0.646 < 1.0$ OK

$P/P_{ao}+M_y/M_{ay} \leq 1.0$

$2.1/25.49+4.2/21.84 = 0.082+0.192 = 0.274 < 1.0$ OK

Therefore the section is adequate for the applied loads.

Solution: Part (B)

1. Full section properties are the same as previously calculated in Part (A.1).
2. $P_a = 6.0$ kips (calculated in Part (A))
3. $P = 0.35+1.75 = 2.10$ kips

   $P/P_a = 2.10/6.0 = 0.350 > 0.15$

   Must check both interaction equations as follows:

   $P/P_a+C_{mx}M_x/(M_{ax}\alpha_x)+C_{my}M_y/(M_{ay}\alpha_y) \leq 1.0$

   $P/P_{ao}+M_x/M_{ax}+M_y/M_{ay} \leq 1.0$

4. Determination of $P_{ao} = 25.49$ kips (calculated in Part (A))
5. Determination of M_x

   The centroidal x-axis is the same for both the full and effective sections.

   $e_y = 4.00$ in.

   $M_x = 2.10(4.00) = 8.40$ kips-in. (Required Flexural Strength)

6. Determination of M_{ax}

   M_{ax} shall be taken as the smaller of the allowable flexural strengths calculated according to sections 3.3.1.1 and 3.3.1.2:

a. Section 3.3.1.1: M_{ax} will be calculated on the basis of initiation of yielding.

$S_e = 14.50/4.112$

$= 3.526 \text{ in.}^3$ (from Example 22.1 Part (A))

$M_{nx} = S_e F_y$ (Eq. 3.3.1.1-1)

$= 3.526(50)$

$= 176.30$ kips-in.

$\Omega = 1.85$

$M_{ax} = M_{nx}/\Omega = 176.30/1.85 = 95.30$ kips-in.

b. Section 3.3.1.2: M_{ax} will be calculated on the basis of the lateral buckling strength.

$M_n = S_c(M_c/S_f)$ (Eq. 3.3.1.2-1)

$M_n = S_c f$

$f = M_c/S_f = 54.42/3.777 = 14.41$ ksi

$S_c = S_e = 3.777 \text{ in.}^3$

$M_{nx} = M_c S_c/S_f$ (Eq. 3.3.1.2-1)

$= 14.41 \text{x} 3.777$

$= 54.42$ kips-in.

$\Omega = 1.85$

$M_{ax} = M_{nx}/\Omega = 54.42/1.85 = 29.42$ kips-in.

M_{ax} shall be the smaller of 95.30 kips-in. and 29.42 kips-in.

Thus

$M_{ax} = 29.42$ kips-in.

7. $C_{mx} = 0.6-0.4(M_1/M_2) \geq 0.4$

M_1/M_2 = -1.00 (single curvature)

0.6-0.4(-1.00) = 1.00 > 0.4

C_{mx} = 1.00

8. Determination of $1/\alpha_{nx}$:

Ω = 2.15

$P_{cr} = \pi^2 E_o I_x/(K_x L_x)^2$

$= \pi^2(27000)(15.108)/(192)^2 = 109.21$ kips

$1/\alpha_{ny} = 1/[1-(\Omega_c P/P_{cr})]$

$= 1/[1-(2.15x2.1/109.21)] = 1/0.959$

$\alpha_{ny} = 0.650$

9. M_{uy} = 4.2 kips-in.

10. M_{ay} = 21.84 kips-in.

11. C_{my} = 1.0

12. α_{ny} = 0.650

13. Check interaction equations:

$P/P_a+C_{mx}M_x/(M_{ax}\alpha_{nx})+C_{my}M_y/(M_{ay}\alpha_{ny}) \leq 1.0$

2.1/6.0+1.00x8.4/(29.42x0.959)+1.00x4.2/(21.84x0.650)

= 0.350+0.298+0.296 = 0.944 < 1.0 OK

$P/P_{ao}+M_x/M_{ax}+M_y/M_{ay} \leq 1.0$

2.1/25.49+8.4/29.42+4.2/21.84 = 0.082+0.286+0.192 = 0.560 < 1.0 OK

Therefore the section is adequate for the applied loads.

EXAMPLE 23.1 TUBULAR SECTION (LRFD)

By using the Load and Resistance Factor Design (LRFD) criteria, check the adequacy of a tubular section (Fig. 23.1) to be used as compression member which is subjected to an eccentrically axial load. The service axial load is P = 15 kips. Consider the following loading case: the eccentricity of axial load at each end of member, e_y, is 4 in. and member is bent in single curvature about x-axis, and $e_x = 0$. Assume that the effective length factors $K_x = K_y = 1.0$, and that the unbraced lengths $L_x = L_y = 10$ ft. Use Type 304, 1/4-Hard, stainless steel. Assume dead to live load ration D/L=1/5 and 1.2D+1.6L gonerns the design.

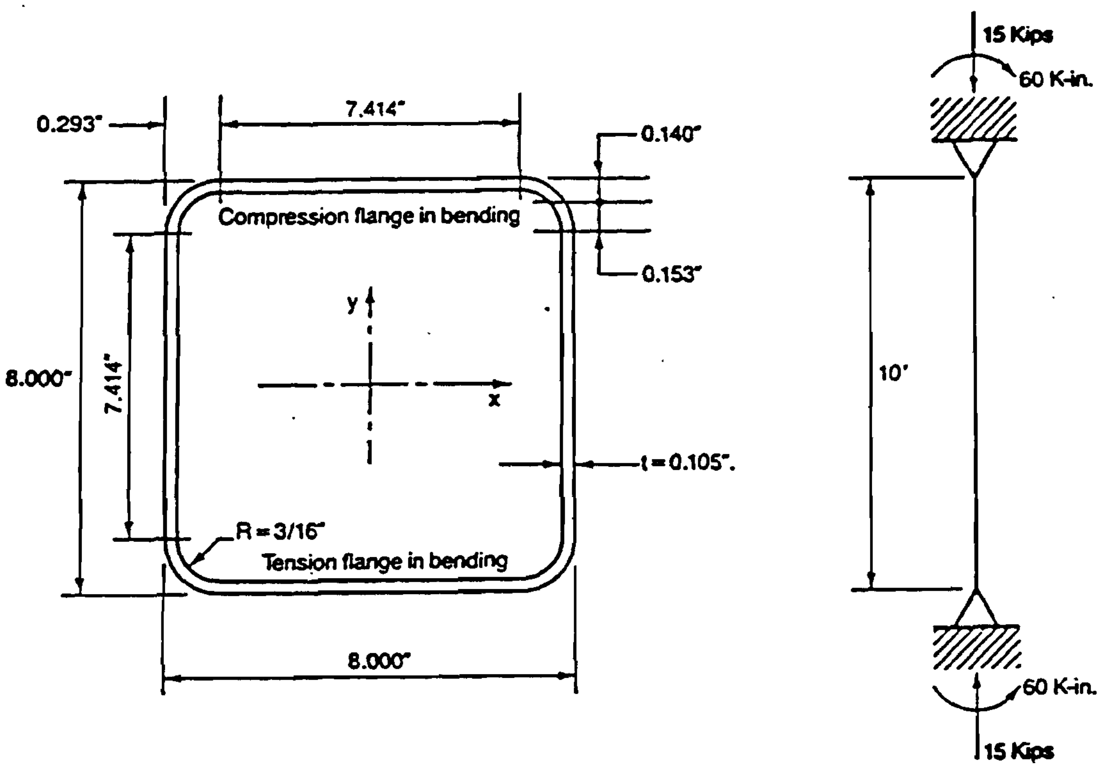


Figure 23.1 Section for Example 23.1

Solution:

1. Full section properties:

$r = R + t/2 = 3/16+0.105/2 = 0.240$ in.

Length of arc, $u = 1.57r = 1.57 \times 0.240 = 0.377$ in.

Distance of c.g. from center of radius,

$c = 0.637r = 0.637 \times 0.240 = 0.153$ in.

$I_x = I_y$ (doubly symmetric section)

Element	L (in.)	y Distance to Center of Section (in.)	Ly^2 ($in.^3$)	I'_1 About Own Axis ($in.^3$)
Flanges	2 x 7.414 = 14.828	3.948	231.120	--
Corners	4 x 0.377 = 1.508	3.860	22.469	--
Webs	2 x 7.414 = 14.828	--	--	67.921
Sum	31.164		253.589	67.921

$A = Lt = 31.164 \times 0.105 = 3.272 \text{ in.}^2$

$I' = Ly^2 + I'_1 = 253.589 + 67.921 = 321.510 \text{ in.}^3$

$I_x = I_y = I't = 321.510 \times 0.105 = 33.759 \text{ in.}^4$

$r_x = r_y = 33.759/3.272 = 3.212$ in.

$S_x = I_x/4.000 = 33.759/4.000 = 8.440 \text{ in.}^3$

$K_xL_x/r_x = 1.0(10 \times 12)/3.212 = 37.36 < 200$ OK (Section 3.4-(5))

2. Determination of $\phi_c P_n$ (Section 3.4):

Since the square tube is a doubly symmetric closed section, provisions of Section 3.4.1 apply, i.e., section is not subjected to torsional flexural buckling.

For Flexural Buckling:

$F_n = (\pi^2 E_t)/(K_y L_y/r_y)^2$ (Eq. 3.4.1-1)

In the determination of the flexural buckling stress, it is necessary to select a proper value of E_t from Table A13 or Figure A11 in the Standard for the assumed stress. For the first approximation, assume a compressive stress of f=44 ksi. From Table A13, the corresponding value of E_t is found to be equal to 7200 ksi. Thus,

$F_n = (\pi^2 x7200)/(37.36)^2$

$= 50.91$ ksi > assumed stress f=44 ksi NG

Because the computed stress is larger than the assumed value, further successive approximations are needed.

Assume f=46.66 ksi, and

$E_t = 6600$ ksi

$F_n = (\pi^2 x6600)/(37.36)^2$

$= 46.67$ ksi = assumed stress OK

$w = 7.414$ in.

$w/t = 7.414/0.105 = 70.61 < 400$ OK (Section 2.1.1-(1)-(ii))

$k = 4.00$ (Section 2.2.1-(1))

$\lambda = (1.052/\sqrt{k})(w/t)\sqrt{f/E_o}$, $f = F_n$ (Eq. 2.2.1-4)

$= (1.052/\sqrt{4.00})(70.61)\sqrt{46.44/27000} = 1.544 > 0.673$

$\rho = (1-0.22/\lambda)/\lambda$ (Eq. 2.2.1-3)

$= (1-0.22/1.544)/1.544 = 0.555$

$b = \rho w$ (Eq. 2.2.1-2)

$= 0.555x7.414 = 4.115$ in.

$A_e = A-4(w-b)t$

$= 3.272-4(7.414-4.115)(0.105) = 1.886$ in.^{2}

$P_n = A_e F_n$ (Eq. 3.4-1)

$= 1.886x46.66 = 88.00$ kips

$\Phi_c = 0.85$

$\Phi_c P_n = 0.85x88.00 = 74.80$ kips

3. $P_{DL}+P_{LL} = (P_{DL}/P_{LL}+1)P_{LL}$

$= (1/5+1)P_{LL} = 1.2P_{LL} = P$

$P_{LL} = P/1.2 = 15/1.2 = 12.5$ kips

$P_u = 1.2P_{DL}+1.6P_{LL}$

$= (1.2P_{DL}/P_{LL}+1.6)P_{LL}$

$= [1.2(1/5)+1.6](12.5) = 23$ kips

where

P_{DL} = Axial load determined on the basis of nominal dead load

P_{LL} = Axial load determined on the basis of nominal live load

$P_u/\Phi_c P_n = 23/74.80 = 0.307 > 0.15$

Must check both interaction equations (Eq. 3.5-1), (Eq. 3.5-2).

4. Determination of $\Phi_c P_{no}$ (Section 3.4 for $F_n = F_y$)

$\lambda = (1.052/\sqrt{4.00})(70.61)\sqrt{50.0/27000} = 1.544 > 0.673$

$\rho = (1-0.22/1.598)/1.598 = 0.540$

$b = 0.540x7.414 = 4.004$ in.

$A_e = 3.272-4(7.414-4.004)(0.105) = 1.840$ in.^{2}

$P_{no} = 1.840x50.00 = 92.00$ kips

$\Phi_c P_{no} = 0.85x92.00 = 78.20$ kips

5. Determination of M_{ux}, M_{uy} (Section 3.5):

Since the section is doubly symmetric, the centroidal axes of the effective section at $\phi_c P_n$ are the same as those of the full section.

$M_{ux} = P_u e_y = 23x4 = 92$ kips-in.

$M_{uy} = P_u e_x = 0$

Since $M_{uy} = 0$, the interaction equations (Eq. 3.5-1) and (Eq.3.5-2) reduce to the following :

$$P_u/\phi_c P_n + C_{mx} M_{ux}/\phi_b M_{nx} \alpha_{nx} \leq 1.0 \qquad \text{(Eq. 3.5-1)}$$

$$P_u/\phi_c P_{no} + M_{ux}/\phi_b M_{nx} \leq 1.0 \qquad \text{(Eq. 3.5-2)}$$

6. Determination of $\phi_b M_{nx}$ (Section 3.3.1):

$\phi_b M_{nx}$ shall be taken as the smaller of the design flexural strengths calculated according to Sections 3.3.1.1 and 3.3.1.2:

a. Section 3.3.1.1: M_{nx} will be calculated on the basis of initiation of yielding.

Computation of I_x:

For the first approximation, assume a compression stress of $f = F_y = 50$ ksi in the compression flange, and that the web is fully effective.

Compression flange: k = 4.00 (stiffened compression element supported by a web on each longitudinal edge)

w/t = 7.414/0.105 = 70.61 < 400 OK (Section 2.1.1-(1)-(ii))

$\lambda = (1.052/\sqrt{4.00})(70.61)\sqrt{50.0/27000} = 1.598 > 0.673$

$\rho = (1-0.22/1.598)/1.598 = 0.540$

$b = 0.540x7.414 = 4.004$ in.

Effective section properties about x-axis:

Element	L Effective Length (in.)	y Distance from Top Fiber (in.)	Ly (in.^{2})	Ly^2 (in.^{3})	I'_1 About Own Axis (in.^{3})
Webs	14.828	4.000	59.312	237.248	67.921
Upper Corners	0.754	0.140	0.106	0.015	--
Lower Corners	0.754	7.860	5.926	46.582	--
Compression Flange	4.004	0.053	0.212	0.011	--
Tension Flange	7.414	7.948	58.926	468.348	--
Sum	27.754		124.482	752.204	67.921

Distance from top fiber to x-axis is

y_{cg} = Ly/L = 124.482/27.754 = 4.485 in.

Since the distance of top compression fiber from neutral axis is greater than one half the section depth (i.e., 4.485 > 4.000), a compression stress of 50 ksi will govern as assumed (i.e., initial yielding is in compression).

To check if the web is fully effective (Section 2.2.2)

f_1 = [(4.485-0.293)/4.485](50) = 46.73 ksi(compression)

f_2 = - [(3.515-0.293)/4.485](50) = -35.92 ksi(tension)

Ψ = f_2/f_1 = -35.92/46.73 = -0.769

k = 4+2[1-(-0.769)]^3+2[1-(-0.769)]

= 18.610

h = w = 7.414 in., h/t = w/t = 7.414/0.105 = 70.61

h/t = 70.61 < 200 OK (Section 2.1.2-(1))

$\lambda = (1.052/\sqrt{18.610})(70.61)\sqrt{46.73/27000} = 0.716 > 0.673$

$\rho = (1-0.22/0.716)/0.716 = 0.968$

$b_e = 0.968x7.414 = 7.177$ in.

$b_2 = b_e/2$ (Eq. 2.2.2-2)

$= 7.177/2 = 3.589$ in.

$b_1 = b_e/(3-\Psi)$ (Eq. 2.2.2-1)

$= 7.177/[3-(-0.769)] = 1.904$ in.

Compression portion of the web calculated on the basis of the effective section = $y_{cg}-0.293 = 4.485-0.293 = 4.192$ in.

Since $b_1+b_2 = 5.493$ in. > 4.192 in., b_1+b_2 shall be taken as 4.192 in.

This verifies the assumption that the web is fully effective.

$I'_x = Ly^2 + I'_1 - Ly^2_{cg}$

$= 752.204 + 67.921 - 27.754(4.485)^2$

$= 261.847$ in.^{3}

Actual $I_x = tI'_x$

$= (0.105)(261.847) = 27.494$ in.^{4}

$S_e = I_x/y_{cg} = 27.494/4.485 = 6.130$ in.^{3}

$M_{nx} = S_eF_y$ (Eq. 3.3.1.1-1

$= (6.130)(50) = 306.50$ kips-in.

$\Phi_b = 0.90$

$\Phi_bM_{nx} = 0.90 \times 306.50 = 275.85$ kips-in.

b. Section 3.3.1.2: M_{nx} will be calculated on the basis of lateral buckling strength. However for this square tube (closed box-type member) the provisions of Section 3.3.1.2 do not apply.

Therefore

$\phi_b M_{nx} = 275.85$ kips-in.

7. $C_{mx} = 0.6-0.4(M_1/M_2)$

$M_1/M_2 = -(92/92) = -1.0$ (single curvature)

$0.6-0.4(M_1/M_2) = 0.6-0.4(-1.0) = 1.0$

8. Determination of $1/\alpha_{nx}$:

$\phi_c = 0.85$

$P_E = \pi^2 E_o I_x/(K_x L_x)^2$ (Eq. 3.5-5)

$I_x = 33.759$ in.^{4}

$K_x L_x = 1.0(10x12) = 120$ in.

$P_E = [\pi^2(27000)(33.759)]/(120)^2 = 624.73$ kips

$1/\alpha_{nx} = 1/(1-P_u/\phi_c P_E)$ (Eq. 3.5-4)

$= 1/[1-23/(0.85x624.73)] = 1.045$

$\alpha_{nx} = 0.957$

9. Check interaction equations:

$P_u/\phi_c P_n + C_{mx} M_{ux}/\phi_b M_{nx} \alpha_{nx} \leq 1.0$ (Eq. 3.5-1)

$23/74.80+1x92/(275.85x0.957) = 0.307+0.349 = 0.656 < 1.0$ OK

$P_u/\phi_c P_{no} + M_{ux}/\phi_b M_{nx} \leq 1.0$ (Eq. 3.5-2)

$23/78.20+92/275.85 = 0.294+0.334 = 0.628 < 1.0$ OK

Therefore the section is adequate for the applied loads.

EXAMPLE 23.2

Rework Example 23.1 by using the Allowable Stress Design (ASD) method to check the adequacy of a tubular section (Fig. 23.1) to be used as a compression member.

Solution

1. Full section properties are the same as those calculated in Example 23.1.

2. Determination of P_a

   The following results are obtained from Example 23.1.(2).

   $F_n = (\pi^2 x6600)/(37.36)^2$

   $= 46.67$ ksi

   $A_e = 1.886$ in.^{2}

   $P_n = F_nA_e = 46.67x1.886$

   $= 88.0$ kips

   $\Omega = 2.15$

   $P_a = P_n/\Omega = 88.0/2.15 = 40.93$ kips

3. $P = 15$ kips

   $P/P_a = 15.0/40.93 = 0.366 > 0.15$

   Must check both interaction equations as follows:

   $P/P_a + C_{mx}M_x/(M_{ax}\alpha_x) + C_{my}M_y/(M_{ay}\alpha_y) \leq 1.0$

   $P/P_{ao} + M_x/M_{ax} + M_y/M_{ay} \leq 1.0$

4. Determination of P_{ao}

   $P_{no} = A_eF_y$

$= 1.84 x 50 = 92.0$ kips

$\Omega = 2.15$

$P_{ao} = P_{no}/\Omega = 92.0/2.15 = 42.79$ kips

5. Determination of M_x and M_y

$e_y = 4.00$ in., $e_x = 0$

$M_x = 15.0(4.00) = 60.0$ kips-in. (Required Flexural Strength)

$M_y = 0$

6. Determination of M_{ax}

M_{ax} shall be taken as the smaller of the allowable flexural strengths calculated according to sections 3.3.1.1 and 3.3.1.2:

a. Section 3.3.1.1: M_{ax} will be calculated on the basis of initiation of yielding.

$S_e = 27.494/4.485$

$= 6.130$ in.^{3} (from Example 23.1)

$M_{nx} = S_e F_y$ (Eq. 3.3.1.1-1)

$= 6.130(50)$

$= 306.50$ kips-in.

$\Omega = 1.85$

$M_{ax} = M_{nx}/\Omega = 306.50/1.85 = 165.68$ kips-in.

b. Section 3.3.1.2: M_{ax} will be calculated on the basis of the lateral buckling strength. However for this square tube (close box-type member) the provision of Section 3.3.1.2 do not apply.

Therefore,

$M_{ax} = 165.68$ kips-in.

7. $C_{mx} = 0.6-0.4(M_1/M_2) \geq 0.4$

$M_1/M_2 = -1.00$ (single curvature)

$0.6-0.4(-1.00) = 1.00 > 0.4$

$C_{mx} = 1.00$

8. Determination of $1/\alpha_{nx}$:

$\Omega = 2.15$

$P_{cr} = \pi^2 E_o I_x/(K_x L_x)^2$

$= [\pi^2(27000)(33.759)]/(120)^2 = 624.73$ kips

$1/\alpha_{nx} = 1/[1-(\Omega_c P/P_{cr})]$

$= 1/[1-(2.15x2.1/624.73)] = 1/0.948$

$\alpha_{nx} = 0.948$

13. Check interaction equations:

15.0/40.93+1.00x60.0/(165.68x0.948)

= 0.366+0.382 = 0.748 < 1.0 OK

$P/P_{ao}+M_x/M_{ax} \leq 1.0$

15.0/42.79+60.0/165.68 = 0.351+0.362 = 0.713 < 1.0 OK

Therefore the section is adequate for the applied loads.

EXAMPLE 24.1 FLAT SECTION w/BOLTED CONNECTION (LRFD)

Determine the maximum design strength, ϕP_n, for the bolted connection shown in Fig. 24.1. Use two 1/2 in. diameter hot-finished, Type 316 bolts with washers under both bolt head and nut. The plates are Type 304, 1/16-Hard, stainless steel.

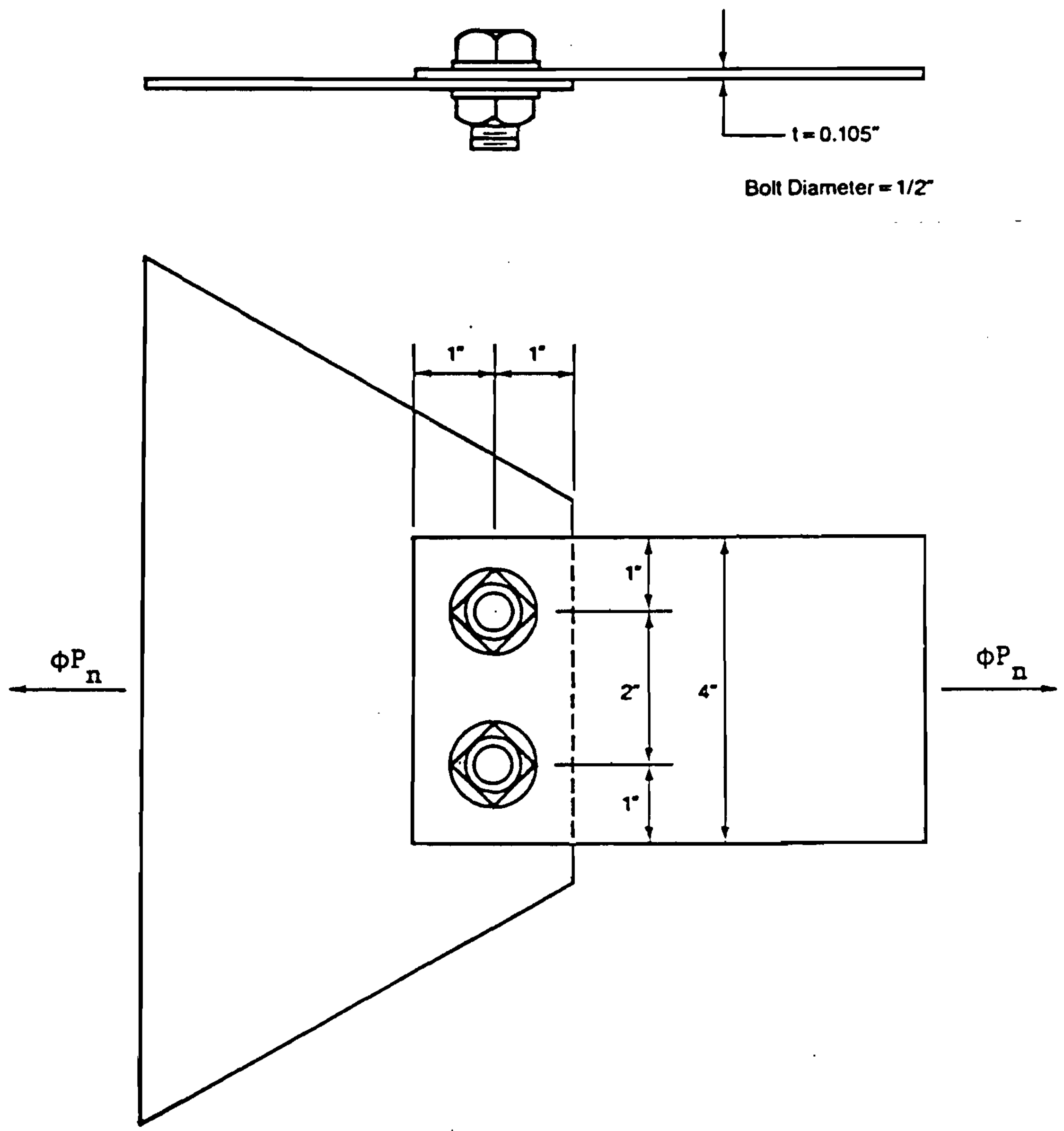


Figure 24.1 Bolted Connection for Example 24.1

Solution:

1. Design strength based on spacing and edge distance (Section 5.3.1)

$$P_n = teF_u \qquad \text{(Eq. 5.3.1-1)}$$

$e \quad = 1.0$ in.

$F_u \quad = 80$ ksi (from Table A16 of the Standard)

$P_n \quad = 0.105(1)(80) = 8.40$ kips/bolt

$\phi P_n = 0.7(2 \text{ bolts})(8.40 \text{ kips/bolt}) = 11.76$ kips

Distance between bolt hole centers must be greater than 3d.

3d = 3(0.5) = 1.5 in. < 2 in. OK

Distance between bolt hole center and edge of connecting member must be greater than 1.5d.

1.5d = 1.5(0.5) = 0.75 in. < 1 in. OK

2. Design strength based on tension on net section.

Required tension strength on net section of bolted connection shall not exceed $\phi_t T_n$ from Section 3.2:

A_n - based on Table 5

$A_n \quad = 0.105\ 4-2(1/2+1/16) \quad = 0.302 \text{ in.}^2$

$F_y \quad = 45$ (from Table A1 of the Standard)

$T_n \quad = A_n F_y$ (Eq. 3.2-1)

$\quad = (0.302)(45) = 13.59$ kips

$\phi_t \quad = 0.85$

$\phi_t T_n = 0.85(13.59) = 11.55$ kips

or ϕP_n from Section 5.3.2:

$P_n \quad = (1.0-r+2.5rd/s)F_u A_n \leq F_u A_n$ (Eq. 5.3.2-2)

where in this case:

$r \quad = 2(\phi P_n/2)/\phi P_n = 1$

$d \quad = 0.5$ in.

$s \quad = 2$ in.

$P_n \quad = [1.0-(1)+2.5(1)(0.5)/2](80)(0.302)$

$\quad = 15.10$ kips $< 80(0.302) = 24.16$ kips OK

$\phi \quad = 0.70$ for single shear connection

$\phi P_n = 0.70(15.10) = 10.57$ kips

Therefore, design strength based on tension on net section is 10.57 kips.

3. Design strength based on bearing (Section 5.3.3)

For single shear with washers under bolt head and nut, the design bearing strength ϕP_n is:

$\phi \quad = 0.65$

$P_n \quad = 2.00F_u dt = 2.00(80)(0.5)(0.105) = 8.4$ kips/bolt

$\phi P_n = 0.65(2 \text{ bolts})(8.4 \text{ kips/bolt}) = 10.92$ kips

4. Design strength based on bolt shear (Section 5.3.4)

$P_n \quad = A_b F_n$ (Eq. 5.3.4-1)

$A_b \quad = (\pi/4)(0.5)^2 = 0.196 \text{ in.}^2$

$F_n \quad = F_{nv} = 45$ ksi (Table 6, for no threads in shear plane)

$P_n \quad = (45)(0.196) = 8.82$ kips/bolt

$\phi \quad = 0.65$

$\phi P_n = 0.65(2 \text{ bolts})(8.82 \text{ kips/bolt}) = 11.47$ kips

5. Comparing the values from 1, 2, 3, and 4 above, the design tensile strength on the net section of the connected part controls and thus,

$\phi P_n = 10.57$ kips

EXAMPLE 24.2 FLAT SECTION w/BOLTED CONNECTION (ASD)

Rework Example 24.1 to determine the maximum allowable load, P_a

Solution:

1. Allowable load based on spacing and edge distance

P_n = 0.105(1)(80) = 8.40 kips/bolt (from Example 24.1.(1))

Ω = 2.40 (Table E of the Standard)

P_a = (2 bolts)(8.40 kips/bolt)/(2.40) = 7.0 kips

Distance between bolt hole centers must be greater than 3d.

3d = 3(0.5) = 1.5 in. < 2 in. OK

Distance between bolt hole center and edge of connecting member must be greater than 1.5d.

1.5d = 1.5(0.5) = 0.75 in. < 1 in. OK

2. Allowable load based on tension on net section.

Required tension strength on net section of bolted connection shall not exceed $\Phi_t T_n$ from Section 3.2:

$T_n = A_n F_y$ = (0.302)(45) = 13.59 kips (Example 24.1)

Ω = 1.85

T_a = (13.59)/1.85 = 7.35 kips

or P_n from Section 5.3.2:

$$P_n = (1.0-r+2.5rd/s)F_u A_n \leq F_u A_n$$

$$= [1.0-(1)+2.5(1)(0.5)/2](80)(0.302)$$

= 15.10 kips < 80(0.302) = 24.16 kips (Example 24.1)

Ω = 2.40

P_a = (15.10)/2.40 =6.29 kips

Therefore, allowable load based on tension on net section is 6.29 kips.

3. Allowable load based on bearing

   For single shear with washers under bolt head and nut, the design bearing strength ϕP_n is: (Example 24.1)

   $P_n = 2.00F_u dt = 2.00(80)(0.5)(0.105) = 8.4$ kips/bolt

   $\Omega = 2.40$

   $P_a = (2 \text{ bolts})(8.4 \text{ kips/bolt})/2.40 = 7.0$ kips

4. Allowable load based on bolt shear

   $P_n = A_b F_n$

   $= (45)(0.196) = 8.82$ kips/bolt (Example 24.1)

   $\Omega = 3.0$

   $P_a = (2 \text{ bolts})(8.82 \text{ kips/bolt})/3.0 = 5.88$ kips

5. Comparing the values from 1, 2, 3, and 4 above, the allowable load based on bolt shear strength controls and thus,

   $P_a = 5.88$ kips

EXAMPLE 25.1 FLAT SECTION w/LAP FILLET WELDED CONNECTION (LRFD)

Using the Load and Resistance Factor Design (LRFD) criteria, check to see if longitudinal fillet welded connection shown in Fig. 25.1 is adequate to transmit a factored load F = 4.5 kips. Assume that Type 301, 1/4-Hard, stainless steel sheet and E308 electrode are to be used.

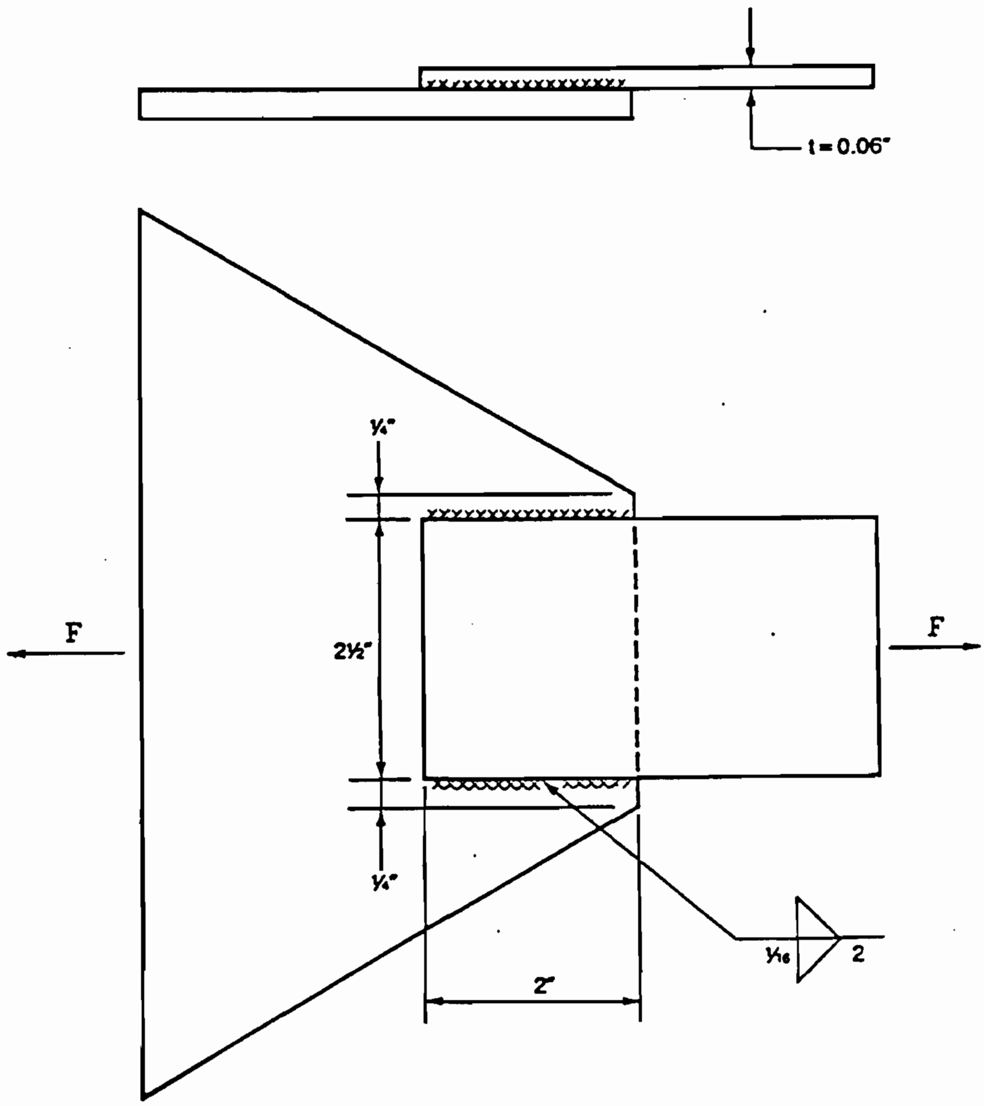


Figure 25.1 Welded Connection for Example 25.1

Solution:

1. Design Strength for Weld Sheet.

   $L/t = 2/0.06 = 33.33 > 30$

   For $L/t \geq 30$,

   $\phi = 0.55$

$P_n = 0.43tLF_{ua}$ (Eq. 5.2.2-2)

$= 0.43(0.06)(2)(90) = 4.64$ kips

(See Table A16 of the Standard for F_{ua} value.)

$\phi P_n = 0.55(4.64) = 2.55$ kips/weld

(2.55 kips/weld)(2 welds) = 5.1 kips > 4.5 kips OK

2. Design Strength for Weld Metal.

$\phi = 0.55$

$P_n = 0.75t_wLF_{xx}$ (5.2.2-3)

$t_w = 0.707(0.0625) = 0.044$ in.

$F_{xx} = 80$ ksi (from Table A15 of the Standard)

$P_n = 0.75\ (0.044)(2)(80) = 5.28$ kips

$\phi P_n = 0.55(5.28) = 2.90$ kips/weld

(2.90 kips/weld)(2 welds) = 5.80 kips > 4.5 kips OK

EXAMPLE 25.2 FLAT SECTION w/LAP FILLET WELDED CONNECTION (ASD)

Using the Allowable Stress Design (ASD) method, check to see if longitudinal fillet welded connection shown in Fig. 25.1 is adequate to transmit a total load F = 3.5 kips. Assume that Type 301, 1/4-Hard, stainless steel sheet and E308 electrode are to be used.

Solution:

1. Allowable load for Weld Sheet.

   $P_n = 0.43tLF_{ua}$ (Example 25.1)

   $= 0.43(0.06)(2)(90) = 4.64$ kips/weld

   $\Omega = 2.50$ (Table E of the Standard)

   $P_a = 4.64x2/2.50 = 3.71$ kips >3.5 kips OK

2. Allowable load for Weld Metal.

   $\Omega = 2.50$

   $P_n = 0.75\ (0.044)(2)(80) = 5.28$ kips/weld (Example 25.1)

   $P_a = 5.28x2/2.50 = 4.22$ kips > 3.5 kips OK

EXAMPLE 26.1 FLAT SECTION w/GROOVE WELDED CONNECTION IN BUTT JOINT (LRFD)

Determine the design tensile strength, ϕP_n, normal to the effective area of the groove welded connection as shown in Fig. 26.1. Use Type 304, annealed, stainless steel and E308 electrode.

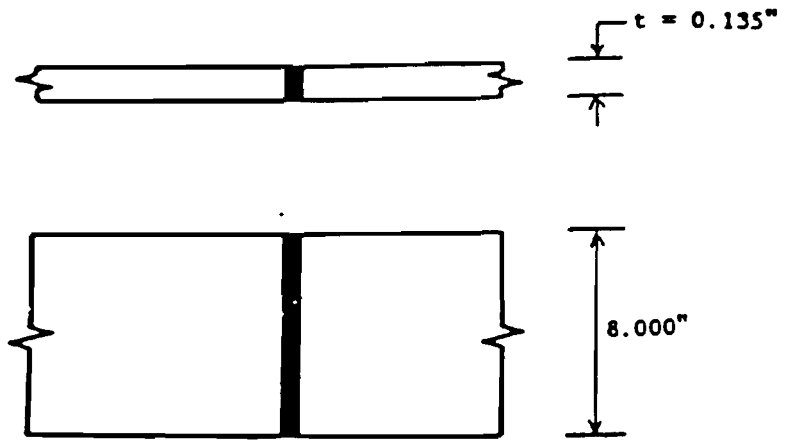


Figure 26.1 Welded Connection for Example 26.1

Solution:

Determination of the design tensile strength, ϕP_n, normal to the effective area provided that the effective throat equal to the thickness of the welded sheet. (Section 5.2.1).

$P_n = LtF_{ua}$ (Eq. 5.2.1-1)

F_{ua} = 75 ksi (Table A16 of the Standard)

F_{xx} = 80 ksi (Table A15 of the Standard)

The minimum tensile strength for weld metal is larger than that the base metal. OK

$$P_n = (8.000)(0.135)(75)$$

$$= 81.00 \text{ kips}$$

$$\phi = 0.60$$

$$\phi(P_n)_1 = 0.60 \times 81.00$$

$$= 48.60 \text{ kips}$$

EXAMPLE 26.2 FLAT SECTION w/GROOVE WELDED CONNECTION IN BUTT JOINT (ASD)

Rework Example 26.1 to determine the allowable tensile load, P_a, normal to the effective area of the groove welded connection.

Solution:

Determination of the allowable tensile load, P_a, normal to the effective area.

$P_n = (8.000)(0.135)(75)$

$= 81.00$ kips (Example 26.1)

$F_{xx} = 80$ ksi $> F_{ua} = 75$ ksi OK

$\Omega = 2.50$ (Table E of the Standard)

$(P_a)_1 = 81.00/2.50 = 32.4$ kips

EXAMPLE 27.1 BUILT-UP SECTION - CONNECTING TWO CHANNELS (LRFD)

By using the LRFD criteria, determine the maximum permissible longitudinal spacing of connectors joining two channels to form an I-section (Fig. 27.1) to be used as a compression member with unbraced length of 12 ft. Also design resistance welds connecting the two channels to form an I-section used as a beam with the following load, span, and support conditions: (a) Span: 10'-0", (b) Total uniformly distributed factored load including factored dead load: 0.520 kips per lin. ft., and (c) Length of bearing at end support: 3 in. Use Type 304, 1/4-Hard, stainless steel.

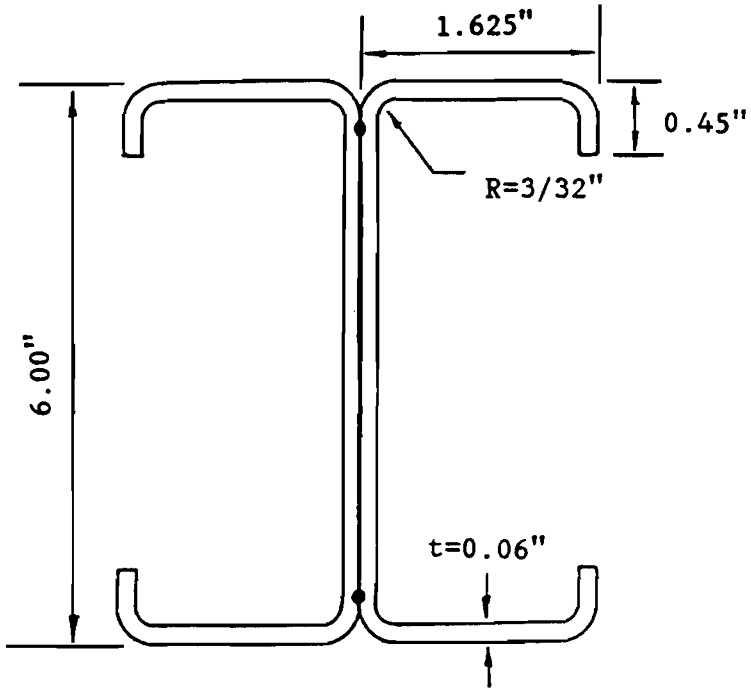


Figure 27.1 Section for Example 27.1

Solution:

1. Maximum longitudinal spacing of connectors for compression member

   Section 4.1.1(1) .

   For compression members, the maximum permissible longitudinal spacing of connectors is

   $$s_{max} = Lr_{cy}/(2r_I) \qquad \text{(Eq. 4.1.1-1)}$$

where

r_{cy} = radius of gyration of one channel about its centroidal axis parallel to web.

r_I = radius of gyration of I-section about axis perpendicular to direction in which buckling would occur for given conditions of end support and intermediate bracing.

The following equations used for computing the sectional properties for channel with lips are based on the information in Part III of Cold-Formed Steel Design Manual (1986), American Iron and Steel Institute, Washington, D.C.

Basic parameters used for calculating the section properties of a channel section with lips: (For parameter designations, see Fig. 22.1

$r = R+t/2 = 3/32+0.060/2 = 0.124$ in.

From the sketch a = 5.692 in., b = 1.317 in., c = 0.296 in.,

A' = 6.0 in., B' = 1.625 in., C' = 0.45 in.,

α = 1.00 (Since the section has lips)

$\bar{a} = A'-t = 6.0-0.060 = 5.94$ in.

$\bar{b} = B'-[t/2+\alpha t/2] = B'-t = 1.625-0.06 = 1.565$ in.

$\bar{c} = \alpha[C'-t/2] = C'-t/2 = 0.45-0.06/2 = 0.42$ in.

$u = 1.57r = 1.57 \times 0.124 = 0.195$ in.

a. Area:

$$A = t[a+2b+2u+\alpha(2c+2u)] = t[a+2b+2c+4u]$$

$$= 0.06[5.692+2x1.317+2x0.296+4x0.195]$$

$$= 0.582 \text{ in.}^2$$

b. Moment of inertia about x-axis:

$$I_x = 2t\{0.0417a^3+b(a/2+r)^2+u(a/2+0.637r)^2+0.149r^3$$

$$+\alpha[0.0833c^3+(c/4)(a-c)^2+u(a/2+0.637r)^2+0.149r^3]\}$$

$$= 2t[0.0417a^3+b(a/2+r)^2+2u(a/2+0.637r)^2+0.298r^3$$

$$+0.0833c^3+(c/4)(a-c)^2]$$

$$= 2x0.06[0.0417(5.692)^3+1.317(5.692/2+0.124)^2$$

$$+2x0.195(5.692/2+0.637x0.124)^2+0.298(0.124)^3$$

$$+0.0833(0.296)^3+(0.296/4)(5.692-0.296)^2]$$

$$= 2.976 \text{ in.}^4$$

c. Distance from centroid of section to centerline of web:

$$\bar{x} = (2t/A)\{b(b/2+r)+u(0.363r)+\alpha[u(b+1.637r)+c(b+2r)]\}$$

$$= [(2x0.06)/0.582][1.317(1.317/2+0.124)+0.195(0.363x0.124)$$

$$+0.195(1.317+1.637x0.124)+0.296(1.317+2x0.124)]$$

$$= 0.371 \text{ in.}$$

d. Moment of inertia about y-axis:

$$I_y = 2t\{b(b/2+r)^2+0.0833b^3+0.356r^3+\alpha[c(b+2r)^2$$

$$+u(b+1.637r)^2+0.149r^3]\}-A(\bar{x})^2$$

$$= 2x0.06[1.317(1.317/2+0.124)^2+0.0833(1.317)^3$$

$$+0.356(0.124)^3+0.296(1.317+2x0.124)^2$$

$$+0.195(1.317+1.637x0.124)^2+0.149(0.124)^3]-0.582(0.371)^2$$

$$= 0.181 \text{ in.}^4$$

e. Distance from shear center to centerline of web:

$$m = (\bar{b}t/12I_x)\ 6\bar{c}(\bar{a})^2+3\bar{b}(\bar{a})^2-8(\bar{c})^3$$

$$= [(1.565x0.06)/(12x2.976)][6x0.42(5.94)^2$$

$+3x1.565(5.94)^2-8(0.42)^3]$

$= 0.668$ in.

f. r_{cy}:

$r_{cy} = \sqrt{I_y/A} = \sqrt{0.181/0.582}$

$= 0.558$ in.

Based on the above information, the section properties of I-section composed of two channels can be determined as follows:

$I_x = 2x2.976 = 5.952$ in^4

$A = 2x0.582 = 1.164$ in^2

$r_x = \sqrt{I_x/A} = \sqrt{5.952/1.164} = 2.26$ in

$I_y = 2x\ 0.81+0.582x(0.371+0.06/2)^2 = 0.549$ in^2

$r_y = \sqrt{I_y/A} = \sqrt{0.549/1.164} = 0.687$ in $< r_x$

Therefore, $r_I = r_y = 0.687$ in

$s_{max} = (12x12)x0.558/(2x0.687) = 58.48$ in.

Therefore, the maximum spacing of connectors used for connecting these two channels as a compression member is 58 in.

2. Design resistance welds connecting the two channels to form an I-section used as a beam Section 4.1.1(2) .

a. Spacing of welds between end supports:

The maximum permissible longitudinal spacing of welds for a flexural member is

$$s_{max} = L/6 \qquad \text{(Eq. 4.1.1-2)}$$

$= 12x10/6 = 20$ in.

Maximum spacing is also limited by

$s_{max} = 2gT_s/(mq)$ (Eq. 4.1.1-3)

in which

g = 5.0 in. (assumed for 6 in. deep section)

T_s = 0.60x2.27x0.25 = 0.341 kips (Section 5.2.3)

m = 0.668 in. (from avobe-calculated value)

q = 3x0.520/12 = 0.130 kips per lin. in.

Therefore

s_{max} = 2x5.0x0.341/(0.668x0.130) = 39.27 in.

s_{max} = L/6 controls. Use a spacing of 20 in. throughout the span.

b. Strength of welds at end supports:

Since the weld spacing is larger than the bearing length of 3.0 in., the required design strength of the welds directly at the reaction is

$T_s = Pm/(2g)$ (Eq. 4.1.1-5)

= 0.520x5x0.668/(2x5) = 0.174 kips

which is less than 0.341 kips as provided. OK

EXAMPLE 27.2 BUILT-UP SECTION - CONNECTING TWO CHANNELS (ASD)

Rework Example 27.1 for the same given data by using the ASD method. Assume that the applied uniform load is 0.4 kips/ft for the I-section used as a beam.

Solution:

1. Maximum longitudinal spacing of connectors for compression member Section 4.1.1(1) .

   For compression members, the maximum permissible longitudinal spacing of connectors is

   $s_{max} = Lr_{cy}/(2r_I)$

   $= (12x12)x0.558/(2x0.687) = 58.48$ in.

   Refer to Example 27.1 for the section properties used to calculate s_{max} The maximum spacing of connectors used for connecting these two channels as a compression member is 58 in.

2. Design resistance welds connecting the two channels to form an I-section used as a beam Section 4.1.1(2) .

   a. Spacing of welds between end supports:

      The maximum permissible longitudinal spacing of welds for a flexural member is

      $s_{max} = L/6 = 12x10/6 = 20$ in.

      Maximum spacing is also limited by

      $s_{max} = 2gT_s/(mq)$

      in which

      g = 5.0 in. (assumed for 6 in. deep section)

      T_s = (0.25x2.27)/2.50 = 0.227 kips

      m = 0.668 in. (from Example 24.1)

$q = 3x0.40/12 = 0.10$ kips per lin. in.

Therefore

$s_{max} = 2x5.0x0.227/(0.668x0.10) = 33.98$ in.

$s_{max} = L/6$ controls. Use a spacing of 20 in. throughout the span.

b. Strength of welds at end supports:

Since the weld spacing is larger than the bearing length of 3.0 in., the required design strength of the welds directly at the reaction is

$$T_s = Pm/(2g)$$
$$= 0.40x5x0.668/(2x5) = 0.134 \text{ kips}$$

which is less than 0.227 kips as provided. OK