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DESIGN OF I-SHAPED BEAMS AND COLUMNS WITH DIAPHRAGM BRACING

by

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July 1974

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DESIGN OF I-SHAPED BEAMS AND COLUMNS WITH DIAPHRAGM BRACING

Key Words: Beams (structural); Buildings; Bracing; Cold formed panels; Columns (structural); Design; Diaphragm; Shear strength.

Abstract

Cold-formed steel panels often are used as wall sheathing, roof decking or floor decking in steel framed buildings or pre-engineered metal buildings. Diaphragms formed by interconnecting these panels have considerable in-plane shear resistance, and can be utilized as bracing against buckling for individual members of a steel frame. For wall columns the diaphragm may be either directly attached or connected to girts which in turn are connected to the columns. A procedure is presented for the design of I-section beams and columns with diaphragm or diaphragm-girt bracing. The procedure is based on the ultimate load capacity of fully braced members, utilizing a conservative estimate of the shear strength and shear rigidity of the diaphragm. Design examples are included. The utilization of existing wall, floor or roof diaphragms as bracing for individual beams and columns can eliminate the need for other types of bracing, and/or reduce required member sizes. Thus it contributes to economical design.

Summary

A procedure is presented for the design of I-section beams and columns with diaphragm or diaphragm-girt bracing, utilizing the shear strength and rigidity of diaphragms formed by interconnecting cold formed steel panels. Design examples are included.

INTRODUCTION

Cold formed steel panels often are used as wall sheathing, roof decking or floor decking in steel framed buildings or pre-engineered metal buildings such as shown in Fig. 1. These panels carry loads normal to their plane by virtue of their bending strength. In addition, diaphragms formed by interconnecting these panels can resist in-plane shear deformations. Because of this shear resistance, such diaphragms are used as wind bracing for low rise buildings, as shear elements in folded plate and hyperbolic paraboloid construction, and as load distributing elements in portal frame buildings. (5,7,9,17,19) Another use of this diaphragm action is as bracing against buckling for individual members of a steel frame. This report deals only with the latter; that is, diaphragm bracing of individual columns and beams. For wall columns the diaphragm may be either directly attached or connected to girts which, in turn, are connected to the columns. These diaphragms are present in any event as wall, roof or floor, and therefore are available at no extra cost. If properly utilized, they can eliminate the need for other types of bracing and thus contribute to economical design.

Extensive research has been conducted at Cornell University and elsewhere to determine the increased load carrying capacity of beams and columns due to diaphragm or diaphragm-girt bracing. (1,3,8,10,11,15,21) Recommendations are made herein for the design of I-shaped members with such bracing. The bracing requirements are not a linear function of applied load; therefore, the design procedure is based on the ultimate

load capacity of the beams or columns, utilizing a conservative estimate of the strength and rigidity of the diaphragm. This is in contrast to most design procedures for other uses of diaphragms, which are usually formulated in terms of allowable load.

Effectiveness of diaphragm bracing or diaphragm-girt bracing in preventing lateral buckling depends on its two fundamental characteristics: (1) rigidity and (2) strength. Usually, it is not economical to provide anything less than "full" bracing, where full bracing is defined as bracing such that any increase in rigidity or strength of the diaphragm will not cause any significant increase in the load-carrying capacity of the braced members.⁽¹⁸⁾ For this reason, and for greater simplicity, this design procedure is limited to "fully" braced beams and columns. The procedure is based on analyses of I-section beams under uniform moment, and I-section columns under axial load. These analyses have been substantiated by tests of 35 diaphragm-braced assemblies as reported in References 1, 3 and 10.

Information regarding the load-carrying capacity of I-beams and columns with less than "full" bracing can be obtained from References 1 and 15. The capacity of channel and Z-section beams subjected to uniform moment is discussed in Reference 1. Cantilever beams and channel and Z-section beams subjected to loads in the plane of the web are discussed in Reference 8; and wall studs of these cross sections are covered in Reference 15.

CRITERIA FOR DESIGN

Design criteria are established herein for the following problems:

1. I-section beams continuously braced by a shear diaphragm on the compression flanges, where "continuous bracing" indicates that the diaphragm is connected directly to the member at short intervals;
2. I-section beams continuously braced by a shear diaphragm on the tension flanges;
3. Axially loaded I-section columns continuously braced by shear diaphragms on both flanges;
4. Axially loaded I-section columns continuously braced by a shear diaphragm on one flange only; and
5. Axially loaded I-section columns braced by girts which, in turn, are braced by a shear diaphragm.

The longitudinal ribs of the panels must be perpendicular to the member they are bracing, which is the usual case.

Behavior of Diaphragm-Braced Beams and Columns

Columns with equal bracing connected directly to both flanges (that is, symmetric bracing) tend to deflect laterally under load without twisting. Beams or columns with continuous bracing on one flange only tend to twist as well as deflect laterally. The diaphragm in these cases provides continuous restraint against (1) lateral movement in the plane of the diaphragm, and (2) twist of the member. In contrast, diaphragm-girt bracing provides these two restraints to a

column only at the points of attachment of the girts. In any case, due to these restraints the torsional flexural buckling moment of a beam or the buckling load of a column can be considerably increased. To evaluate the behavior of diaphragm-braced beams and columns, it is necessary to know the nature and the magnitude of the restraints available.

Shear Characteristics of a Diaphragm

The two important parameters which characterize a diaphragm are its shear stiffness (or, conversely, its flexibility) and shear strength. Considerable progress has been made recently in developing theory to predict these parameters. (5,6,12,14,20) As an alternative, the stiffness and strength of a diaphragm can be determined from the load-deflection curve obtained from a simple beam or cantilever shear test (Fig. 2) as described in Reference 9. This load-deflection relationship is generally not linear. Furthermore, in such tests, two nominally identical diaphragms may give considerably different load-deflection relationships at the higher loads--say, beyond 80% of ultimate load. Therefore, in this discussion, the shear stiffness G'_d and average shear strain γ_d at 80% of ultimate shear load are taken as the basic characteristics of the diaphragm. Shear stiffness G'_d is defined herein as

$$G'_d = \frac{(0.8 P_{ult}/b)}{(\Delta_d/a)} \quad (1)$$

where P_{ult} is the ultimate shear load in the diaphragm test, kips

Δ_d is the deflection at $0.8 P_{ult}$, in.

- a is the dimension of the shear diaphragm perpendicular to the test load direction, in.
- b is the dimension of the shear diaphragm parallel to the test load direction, in.

Eq. 1 indicates that the shear stiffness G'_d is in units of force per unit length. The use of $0.8 P_{ult}$ reflects the ultimate strength approach.

If the shear stiffness of a diaphragm is known, then the maximum shear strain that can be sustained by the diaphragm is a measure of its shear strength; that is, the characteristic shear strength is the product of the shear stiffness and shear strain. The shear strain γ_d at $0.8 P_{ult}$ is taken here as the measure of available shear strength and is given by

$$\gamma_d = \frac{\Delta_d}{a} \quad (2)$$

To insure that diaphragm failure will not precede member failure, it is proposed for design purposes to assume that reliable values of shear strain γ_{dr} and shear stiffness G'_{dr} are equal to γ_d and $2/3 G'_d$, respectively. Thus, from Eqs. 1 and 2,

$$G'_{dr} = 2/3 G'_d = \frac{0.53 P_{ult}/b}{\Delta_d/a} \quad (3)$$

and

$$\gamma_{dr} = \gamma_d = \frac{\Delta_d}{a} \quad (4)$$

A graphical representation of actual test values and proposed design values of shear stiffness and shear deflection are shown in Fig. 2. The type and spacing of fasteners used in a diaphragm test should be

the same as those used in connecting the diaphragms to the beams or columns in the actual structure. The panel lengths and purlin spacing to be used in a cantilever test (or in any analytical procedure) to simulate the actual structure are given below. As a conservative simplification, the bending rigidity of the diaphragm, which tends to prevent rotation of the member to which it is attached, is neglected in this design procedure.

1. Diaphragms continuously bracing beams or columns

The deflected position of the structure in this case is shown in Fig. 3a. It can be observed from Fig. 3a that the full length of each panel is under uniform shear. The length of panel to be used in a cantilever diaphragm test is the same as the length of the panel used in the structure, as shown in Fig. 3b. Purlin spacing in the test should be the same as the spacing of the beams or columns to be braced.

2. Diaphragms in a column-girt-diaphragm assembly

A typical deflected position of the diaphragm in a column-girt-diaphragm assembly is shown in Fig. 4b. It is seen from Fig. 4b that only a part of the length of the panel equal to the spacing of girts is under uniform shear. Therefore, the length of the panels to be used in a shear diaphragm cantilever test should be the same as the spacing of girts in the column-girt-diaphragm assembly, as shown in Fig. 4c. No intermediate purlins should be used in the test. Perimeter framing and fasteners should simulate the corresponding portion of the actual structure.

A simple beam shear test may be conducted instead of a

cantilever test, making proper choice of the panel length and spacing of the framing members. (9)

When a diaphragm-braced beam or column in a structure is to be analyzed, it is more convenient in the computations to use a reliable shear rigidity Q_{dr} of the entire portion of diaphragm contributing to the support of the member, rather than the unit shear stiffness G'_{dr} . The reliable shear rigidity Q_{dr} is defined as

$$Q_{dr} = G'_{dr} w = 2/3 G'_d w \quad (5a)$$

or, using Eq. 3,

$$Q_{dr} = \frac{0.53 P_{ult} w/b}{\Delta_d/a} \quad (5b)$$

where w is the dimension of the diaphragm, perpendicular to the longitudinal axis of the member, which contributes to the support of the member being analyzed. For example, in the case of floor beams braced by a diaphragm (Fig. 5), the end beams can be assumed to be supported by the diaphragm of dimension w equal to half the spacing of beams, and the intermediate beams are supported by the dimension of the diaphragm w equal to the full spacing of the beams. It can be observed from Eq. 5b that Q_{dr} is in units of force per unit shear strain (force/radian).

Bending Stiffness and Strength of a Girt

The performance of a girt also can be characterized by its bending stiffness and strength, with due consideration of the rigidity

of the girt-to-column connection. If the connection between girts and columns is fully rigid, the bending rigidity of the girt offers calculable restraint against twist of the column, at the point of attachment to the column. To compute the twist restraint, refer to the deflected position of the column-girt-diaphragm assembly shown in Fig. 6. For a rigid connection, the elastic restraining moment on the column per unit twist of the column, m , can be computed as

$$m = \frac{2 (6EI_g)}{w} \quad (6)$$

where I_g is the strong axis moment of inertia of the girt and E is Young's modulus. If the girt-to-column connection is effectively "pinned", then $m = 0$.

The strength of a girt can be designated by the bending slope at the column, θ_d , when the ends of a girt between two successive columns are subjected to equal and opposite moments, M_{yg} (Fig. 6b), where M_{yg} is the yield moment of the girt. The slope can be computed as

$$\theta_d = \frac{M_{yg} w}{6EI_g} \quad (7)$$

Initial Imperfections

The required strength of any type of bracing is a function of the initial imperfections of the load-carrying member. The pattern of initial deflections along the length of an imperfect beam or column is here assumed affine to the buckling pattern of the beam or column to obtain a conservative estimate of deflections under load. For

example, in the case of a continuously braced beam or column, with ends simply supported laterally or hinged, the buckling pattern is a half sine wave, and the initial deflection pattern is therefore also assumed as a half sine wave. The amplitude of the initial lateral deflection of the centroidal axis of a beam or column, E_o , is taken as the tolerance limit in sweep specified in the AISC Manual (Ref. 13, p 1-127). Hence, E_o is usually of the form

$$E_o = \frac{1''}{8} \times \frac{\text{length of member in feet}}{\text{specified integer}} \quad (8)$$

In addition, in the case of columns, an accidental eccentricity is considered by multiplying the initial lateral deflection by two in the design equations. Based on limited available information, the amplitude of the initial twist, F_o , is arbitrarily taken equal to 0.000667 radian per foot of length; that is,

$$F_o = 0.000667 \text{ rad./ft.} \times \text{length of member in feet} \quad (9)$$

Additional Deflections

Because of the initial lateral deflection or twist of a beam or column, additional lateral deflections and twist occur under applied load. The pattern of additional deflections along the length of a member is the same as the buckling pattern. These additional deflections cause shear forces in the diaphragm bracing. Also, because of these additional deflections, girts bend in the case of a column-girt-diaphragm assembly if the twist restraint $m \neq 0$ and if the column buckles in a torsional flexural mode.

Magnitudes of the additional deflections, maximum shear strain in the diaphragm, and maximum bending slope of the girts can be calculated using the design formulae given subsequently.

Factors of Safety

For rolled steel beams and columns the factors of safety as given in the AISC Specification⁽¹³⁾ are used. The slenderness ratio for computing the column safety factor depends on the buckling mode. For cold formed steel members the factors of safety as given in the AISI Specification⁽¹⁶⁾ are used.

Definition of "Full Bracing"

As stated earlier, "full bracing" is defined as bracing such that any increase in rigidity or strength of the diaphragm will not cause any significant increase in the load carrying capacity of the braced members. The implications of this definition will be discussed later for specific cases.

GENERAL DESIGN PROCEDURE

Briefly stated, the design procedure¹ is as follows: First, assume the member to be fully braced, and select a section of required load capacity. Then, select a diaphragm of specific panel dimensions, fastener type and fastener spacing, and check to see that its rigidity and strength are adequate to provide full bracing for the member.

Alternatively, one could start with a given member and diaphragm bracing system, and use an analysis procedure based on the equations herein to calculate the ultimate load capacity and allowable load.

The detailed design procedure is outlined below:

1. Choose a trial member for the design.
2. Assume that the member is fully braced, and compute its load capacity, P_{fb} .
3. Compute the safe load, P_s :

$$P_s = \frac{\text{load capacity of fully braced member}}{\text{factor of safety or load factor}} = \frac{P_{fb}}{F.S.} \quad (10)$$

If P_s is greater than and close to the required design load, proceed to check whether the bracing is adequate; otherwise, repeat the procedure from Step 1.

4. Compute Q_{id} , the shear rigidity required to fully brace an ideal member. The actual rigidity required to brace a real member will be greater than this. (18)

5. Select a trial diaphragm. If $Q > Q_{id}$, the diaphragm rigidity may be adequate for full bracing; proceed with Step 6. If $Q \leq Q_{id}$, full bracing cannot be achieved with this diaphragm; a more rigid diaphragm must be chosen, and this step repeated. (Alternatively, see Refs. 1 and 15 for beams and columns with less than full bracing.)
6. Compute the maximum shear strain, γ_{max} , in the diaphragm. If the reliable shear strain $\gamma_{dr} \geq \gamma_{max}$, the diaphragm strength is adequate for full bracing; proceed with Step 7 if applicable. If $\gamma_{dr} < \gamma_{max}$, a stronger diaphragm is necessary for full bracing; repeat the procedure from Step 5.
7. This step applies only for diaphragm-girt bracing. In most cases of full bracing there is no bending of the girts. However, if the flexural restraint $m \neq 0$ and failure is in the torsional flexural mode, the strength of the girts has to be checked. Compute θ_d and θ_{max} . If the computed bending slope of the girts θ_{max} is less than θ_d , the bracing provided by the girts in combination with the diaphragm is adequate to fully brace the column. If the girts are not strong enough, choose a stronger section for the girts and repeat the procedure from Step 5.

P_{fb} , γ_{max} , θ_{max} , Q_{id} and θ_d are computed from equations given herein; whereas Q_{dr} and γ_{dr} can be obtained from the load-deflection relationship of a shear diaphragm test or analysis.

Conventional Design Formulae

The yield moment of a beam, and the strong and weak axis buckling loads of a column, can be obtained in any rational manner, including multiplying the allowable load by the known safety factor.

I-SECTION BEAMS BRACED BY A SHEAR DIAPHRAGM
ON THE COMPRESSION FLANGES^(1,10)

Figs. 7a and 7b show the possible modes of failure of beams with diaphragm bracing on the compression flanges. In Fig. 7a the diaphragm rigidity and strength are not adequate to prevent lateral buckling of the beams. In Fig. 7b the diaphragm is adequate, and the beams fail by yielding. Full bracing in this case is defined as that which has adequate rigidity and strength to prevent lateral buckling until the beam yields. Thus, M_y , the yield moment, is taken as the moment capacity of a fully braced beam, M_{fb} :

$$M_{fb} = M_y \quad (11)$$

The torsional flexural buckling moment, M_{cr} , of an ideal beam with diaphragm bracing on the compression flange is⁽¹⁰⁾

$$M_{cr} = \sqrt{[EI_y \left(\frac{n\pi}{L}\right)^2 + Q][EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2]} + Qe \quad (12)$$

where EI_y is the weak axis bending rigidity

EC_w is the warping rigidity

GJ is the torsional rigidity

e is the distance between the center of gravity of the beam and the plane of the diaphragm

and $n = 1$ or 2 for ends simply supported or fixed, respectively, against lateral bending

The cross bending rigidity of the diaphragm is neglected in Eq. 12 and in all subsequent expressions.

The shear rigidity, Q_{id} , required for an ideal beam to attain

the "fully" braced moment $M_{fb} = M_y$ can be obtained by substituting M_y for M_{cr} in Eq. 12 and solving for Q , resulting in Eq. 13 shown in Table 1. A simplified and conservative expression for M_{cr} is given by

$$M_{cr} = 2 Qe \quad (14)$$

Then, with $M_{cr} = M_y$, a simple and conservative estimate for Q_{id} is obtained from Eq. 14 as

$$Q_{id} = M_y/2e \quad (15)$$

Conservative estimates of the amplitudes of additional lateral deflection of the centroidal axis (C) and twist (D) at moment M_y are given by Eqs. 16 and 17 in Table 1. The amplitude of the additional lateral deflection in the plane of the diaphragm is

$$C_1 = C + eD \quad (18)$$

and the maximum shear strain in the diaphragm is

$$\gamma_{max} = C_1 \frac{\pi}{L} \quad (19)$$

These expressions are used in Steps 1 through 6 of the design procedure to design a diaphragm-braced I-beam in Example No. 1.

I-SECTION BEAMS BRACED BY A SHEAR DIAPHRAGM
ON THE TENSION FLANGES^(1,10)

The possible failure modes for beams braced by diaphragms on the tension flanges are indicated in Fig. 8. The figure shows (a) vertical deflection, lateral deflection and twist of the cross section; (b) vertical deflection and twist; and (c) vertical deflection only. The buckling moment of the beam reaches M_{ϕ_e} (buckling moment of the beam with the centroidal axis of the tension flange as the fixed axis of rotation) asymptotically as the shear rigidity approaches infinity. In general, even for a very small increase in the moment capacity beyond $0.9 M_{\phi_e}$, a very large increase in shear rigidity is needed. Therefore, if a beam with diaphragm bracing on the tension flange buckles in a torsional-flexural mode (Figs. 8a and 8b) the buckling moment of the "fully" braced beam is arbitrarily taken as $0.9 M_{\phi_e}$. (Any other percentage could be used in similar fashion.) A fully braced beam may also fail by yielding (Fig. 8c). Hence, the moment capacity of a beam "fully" braced on its tension flanges is the smaller of these two values; that is,

$$M_{fb} = \text{Min.} (0.9 M_{\phi_e}, M_y) \quad (20)$$

The buckling moment M_{ϕ_e} is given by Eq. 21 in Table 2, and Eq. 22 gives the shear rigidity (Q_{id}) required for an ideal beam to be fully braced.

Amplitudes of additional lateral deflection of the centroidal axis (C) and twist (D) at moment M_{fb} of a fully braced beam are obtained

from Eqs. 23 and 24. The amplitude of the additional lateral deflection in the plane of the diaphragm (C_1) is

$$C_1 = C - eD \quad (25)$$

and γ_{\max} is obtained from Eq. 19. The design procedure is the same as described for beams with diaphragm bracing on the compression flange.

Bracing on the tension flange is, of course, less efficient than compression flange bracing, as can be demonstrated by the mathematical expressions cited above.

AXIALLY LOADED I-SECTION COLUMNS CONTINUOUSLY BRACED BY
SHEAR DIAPHRAGMS ON BOTH FLANGES^(1,10)

Diaphragm-braced columns in this case may buckle in one of the two modes shown in Fig. 9. The bracing is defined as "full" if its rigidity and strength are adequate to prevent weak-axis buckling of the columns (Fig. 9a) so that they buckle about their strong axis (Fig. 9b). Torsional-flexural buckling is not a failure mode for I-section columns with symmetrical diaphragm bracing. The buckling load of a "fully" braced column, P_{fb} , is therefore $P_{crx,L}$, the strong axis buckling load of the column of length, L .

$$P_{fb} = P_{crx,L} \quad (26)$$

The shear rigidity (Q_{id}) required for an ideal column to attain full bracing is given by Eq. 27 in Table 3, where E^* is the modulus corresponding to the average stress level (σ) of the column at P_{fb} . If $\sigma < \sigma_p$, $E^* = E$. But, if $\sigma > \sigma_p$,⁽⁴⁾

$$E^* = E \frac{(\sigma_y - \sigma) \sigma}{(\sigma_y - \sigma_p) \sigma_p} \quad (28)$$

Amplitude of the additional lateral deflection, C , of the centroidal axis of the column at load $P_{crx,L}$ is obtained from Eq. 29. For symmetrically braced columns no rotation is assumed, and the lateral deflection in the plane of the diaphragm is equal to the deflection at the centroidal axis, $C_1 = C$. The design follows the general procedure; the final step is to check the strength of the diaphragm ($\gamma_{max} < \gamma_{dr}$) using Eq. 19.

AXIALLY LOADED I-SECTION COLUMNS CONTINUOUSLY BRACED BY
A SHEAR DIAPHRAGM ON ONE FLANGE ONLY ^(1,10)

Diaphragm-braced columns in this case may buckle in one of the modes shown in Fig. 10; that is, torsional-flexural buckling or flexural buckling about the strong axis. The buckling load of the column approaches P_{ϕ_e} (the buckling load of the column with the centroidal axis of one of the flanges as the fixed axis of rotation) asymptotically as the shear rigidity Q approaches infinity. In general, even for a very small increase in load beyond about $0.9 P_{\phi_e}$, a very large increase in shear rigidity is needed. Therefore, if a column buckles in the torsional-flexural mode, as in Fig. 10a, the buckling load of the "fully" braced column is arbitrarily taken as $0.9 P_{\phi_e}$. The buckling load of a fully braced column is the smaller of the two values; that is,

$$P_{fb} = \text{Min.} (0.9 P_{\phi_e}, P_{crx,L}) \quad (31)$$

The buckling load P_{ϕ_e} is given by Eqs. 32a and 32b in Table 4 for the elastic and inelastic range, respectively. For an I-section, the polar moment of inertia I_p in Eqs. 32a and 32b is

$$I_p = I_x + I_y \quad (33)$$

The shear rigidity Q_{id} required for an ideal column to be fully braced is given by Eq. 34 in Table 4, where E^* is obtained from Eq. 28 and

$$G^* = G \frac{E^*}{E} \quad (35)$$

Amplitudes of additional lateral deflection of the centroidal axis (C) and twist (D) at the buckling load are given by Eqs. 36 and 37, respectively, in Table 4. The amplitude of the additional lateral deflection C_1 in the plane of the diaphragm is given by Eq. 25, and the maximum shear strain is obtained from Eq. 19.

AXIALLY LOADED I-SECTION COLUMNS WITH
DIAPHRAGM-GIRT BRACING^(1,3)

A typical column-girt-diaphragm assembly is shown in Fig. 11a. If "full" bracing is provided, the column may buckle in one of three modes: (1) flexural buckling about its strong axis, Fig. 11b, (2) torsional-flexural buckling, Fig. 11c, or (3) flexural buckling about its weak axis between successive girts, Fig. 11d; Therefore, the buckling load of such a fully-braced column is the smallest of these three values; that is,

$$P_{fb} = \text{Min.} (P_{crx,L}, P_{cry,\ell}, 0.9 P_{\phi e}) \quad (38)$$

where $P_{cry,\ell}$ is the weak axis buckling load of a column of length ℓ .

The design formulae given in this report are for columns with "hinged" ends; that is, the ends are flexurally hinged, and warping is unrestrained. Design equations for the various cases are given in Table 5; values of the required coefficients K_1 through K_4 appear in Table 6 for modes $i = 1 \dots j$, where j is the number of intermediate girts.

If the girt-column connection is fully flexible ($m = 0$), a fully braced column usually buckles in the torsional-flexural mode, but there is no bending of the girts. On the other hand, if the girt-column connection is rigid, the column usually buckles flexurally rather than by twisting, and again the girts do not bend. Therefore, the strength of the girts need not be checked in most cases of full bracing. Strength of the girts has to be checked only where $m \neq 0$

and the column buckles in a torsional-flexural mode.

If the column buckles in a torsional-flexural mode and $m \neq 0$, the maximum bending slope of the girts, θ_{\max} , is given by the twist of the column at the girt which is at or nearest the midheight of the column. Therefore,

$$\theta_{\max} = D, 0.866 D, \text{ or } D \quad (44)$$

for columns with 1, 2 or 3 intermediate girts, respectively.

DESIGN EXAMPLES

Example No. 1 - Beams Braced by a Diaphragm on Their Compression Flanges

Design an intermediate floor beam to span 20 ft and to carry a uniform live load and superimposed dead load of 550 lb/ft. The beams are 6 ft apart and are braced by a deck whose shear characteristics determined from tests are: $G'_d = 4.235$ kips/in. and $\gamma_d = 0.0045$ rad. Ends of the beams are considered simply supported laterally. Use ASTM A36 steel.

Solution: Assume that the beam is fully braced and that its dead load will be about 20 lb/ft.

$$M = \frac{wl^2}{8} = \frac{(550 + 20)(20)^2(12)}{8} = 342 \text{ kip-in.}$$

For a factor of safety of 1.67,

$$M_{\text{req'd}} = 1.67 \times 342 = 571 \text{ kip-in.}$$

$$\text{Req'd } S_x = M_{\text{req'd}} \div F_y = 571 \div 36 = 15.9 \text{ in.}^3$$

$$\text{Choose W10 x 17, } S_x = 16.2 \text{ in.}^3, \quad d = 10.12 \text{ in.}$$

$$\text{Fully braced moment } M_{\text{fb}} = M_y = S_x \sigma_y = 16.2 \times 36.0 = 583 \text{ kip-in.}$$

$$> M_{\text{req'd}}; \quad \therefore \text{OK}$$

Proceed to check whether the bracing is adequate.

Diaphragm Rigidity:

$$\text{Eq. 15: } Q_{id} = \frac{M_y}{2e} = \frac{583}{2(10.12 \div 2)} = 57.6 \text{ kips}$$

$$\begin{aligned} \text{Eq. 5a: } Q_{dr} &= G'_{dr} w = 2/3 G'_d w \\ &= 2/3 (4.235)(72) = 203.3 \text{ kips} > 57.6 \text{ kips} \end{aligned}$$

\therefore diaphragm rigidity may be adequate for full bracing.

Diaphragm Strength:

Eq. 8: Assumed initial deflection, $E_o = 1/8 \times 20/5 = 0.5''$

Eq. 9: Assumed initial twist, $F_o = 0.000667 \times 20$
 $= 0.01334 \text{ rad.}$

$$e = d/2 = 5.06''$$

Eq. 16: Additional deflection, $C = -0.1038''$

Eq. 17: Additional twist, $D = 0.0648 \text{ rad.}$

Eq. 18: Deflection of braced flange, $C_1 = 0.2242''$

Eq. 19: Maximum shear strain, $\gamma_{\max} = 0.00294 \text{ rad.}$

$$< \gamma_{dr} = \gamma_d = 0.0045 \text{ rad.}$$

\therefore diaphragm strength is adequate for "full" bracing. The W10 x 17 beam is fully braced and can safely carry a uniform load of 550 lb/ft.

Example No. 2 - Columns Braced by a Diaphragm on One Flange Only

Determine the size of an intermediate column of a side wall to support an axial load of 106 kips. Columns are 12 ft high, spaced at 6 ft intervals, and are continuously braced on one flange by a light gage steel diaphragm whose shear characteristics are $G'_d = 12.5 \text{ kips/in.}$ and $\gamma_d = 0.0045$. The ends of the column are assumed to be flexurally hinged, with warping unrestrained. Use ASTM A36 steel, $\sigma_y = 36 \text{ ksi}$, $\sigma_p = 18 \text{ ksi}$, $E = 29,000 \text{ ksi}$, $G = 11,500 \text{ ksi}$.

Solution: From Table 4, $P_{fb} = \text{Min.} (P_{crx,L}, 0.9 P_{\phi e})$. Using tables in the AISC Manual or other design aid as a guide, try W6 x 25.

Buckling Loads: $\frac{L}{r_x} = \frac{144}{2.69} = 53.5 < C_c = 126.1$

Ref. 13: $P_{crx,L} = 7.35 \times 36 \left[1 - \frac{1}{2} \left(\frac{53.5}{126.1} \right)^2 \right] = 237.5$ kips

Eq. 32b: $n = 1$, $P_{\phi_e} = 229.3$ kips

$0.9 P_{\phi_e} = 0.9 \times 229.3 = 206.4$ kips

$\therefore P_{fb} = \text{Min.} (237.5, 206.4)$

$P_{fb} = 0.9 P_{\phi_e} = 206.4$ kips

Factor of Safety:

$L/r_y = 144 \div 1.53 = 94.1 < 126.1 = C_c$

Ref. 13: $F.S. = \frac{5}{3} + \frac{3}{8} \left(\frac{94.1}{126.1} \right) - \frac{1}{8} \left(\frac{94.1}{126.1} \right)^3 = 1.89$

Safe axial load on column if fully braced = $\frac{206.4}{1.89}$

= 109.2 kips > 106 kips, \therefore OK

Check to see whether the bracing is "full".

Diaphragm Rigidity:

$\sigma = \frac{P_{fb}}{A} = \frac{206.4}{7.35} = 28.08$ ksi

Eq. 28: $\sigma_p = 1/2 \sigma_y$, $E^* = 19,900$ ksi

Eq. 35: $G^* = 7,890$ ksi

Eq. 34: $n = 1$, $Q_{id} = 52.0$ kips

Eq. 5a: $Q_{dr} = 2/3 G_d^1 w = 2/3 (12.5)(72) = 600$ kips > 52.0 kips

\therefore diaphragm rigidity may be adequate for full bracing.

Diaphragm Strength:

Eq. 8: Assumed initial sweep, $E_o = \frac{1}{8} \times \frac{12}{10} = 0.15''$

Eq. 9: Assumed initial twist, $F_o = .000667 \times 12 = 0.008$ rad.

Eq. 36: Additional deflection, $C = 0.41''$

Eq. 37: Additional twist, $D = 0.088$ rad.

Eq. 25: Deflection of braced flange, $C_1 = 0.13''$

Eq. 19: Maximum shear strain in diaphragm,

$$\gamma_{\max} = 0.0029 \text{ rad.} < \gamma_d = \gamma_{dr} = 0.0045 \text{ rad.}$$

\therefore diaphragm strength is adequate. The column is fully braced and can safely carry a design axial load of 106 kips.

Example No. 3 - Columns with Diaphragm-Girt Bracing

Determine the size of intermediate I-section columns spaced at 19'-4" intervals, and braced by girts, as in Fig. 12. The girts are braced by a standard corrugated diaphragm. Assume the ends of the columns are hinged, with warping unrestrained.

Length of column, $L = 12'-4''$

Spacing of girts = 6'-2''

Diaphragm stiffness, $G'_d = 6.47$ kips/in.

Diaphragm shear strain, $\gamma_d = 0.0069$ rad.

Twist restraint, $m = 4650$ k-in./rad.

Axial load on column = 220 kips

Use ASTM A572 Grade 50 steel, $F_y = 50$ ksi, $E = 29,000$ ksi,

$G = 11,500$ ksi, $\sigma_p = 25$ ksi

Solution: Try W12 x 31

Buckling Loads: $\frac{L}{r_x} = \frac{148}{5.12} = 28.9 < C_c = 107.0$

Ref. 13: $P_{crx,L} = 9.13 \times 50 \left[1 - \frac{1}{2} \left(\frac{28.9}{107} \right)^2 \right] = 440$ kips

$$\frac{\ell}{r_y} = \frac{74}{1.54} = 48.0 < C_c = 107.0$$

Ref. 13: $P_{cry,\ell} = 9.13 \times 50 \left[1 - \frac{1}{2} \left(\frac{48.0}{107} \right)^2 \right] = 410$ kips

$$\begin{aligned} \text{Eq. 40b: } P_{\phi_e} &= P_y = A F_y = 456.5 \text{ kips} \\ 0.9 P_{\phi_e} &= 411 \text{ kips} \end{aligned}$$

$$\begin{aligned} \text{Eq. 38: } P_{fb} &= \text{Min. } (440; 410; 411) \\ P_{fb} &= P_{\text{cry}, \ell} = 410 \text{ kips} \end{aligned}$$

Factor of Safety:

$$\text{Ref. 13: } F.S. = \frac{5}{3} + \frac{3}{8} \left(\frac{48}{107} \right) - \frac{1}{8} \left(\frac{48}{107} \right)^3 = 1.82$$

$$\text{Required ultimate strength} = 220 \times 1.82 = 400 \text{ kips}$$

$$< 410 \text{ kips} = P_{fb} \quad \therefore \text{OK}$$

Diaphragm Rigidity:

$$\sigma = \frac{P_{fb}}{A} = \frac{410}{9.13} = 44.9 \text{ ksi}$$

$$\text{Eq. 28: } E^* = 10,625 \text{ ksi}$$

$$\text{Eq. 35: } G^* = 4,210 \text{ ksi}$$

$$\begin{aligned} \text{Eq. 41: } \text{With } i = 1, K_1 &= .250, K_2 = .810, K_3 = .405 \\ Q_{id} &= 413 \text{ kips} \end{aligned}$$

$$\begin{aligned} \text{Eq. 5a: } Q_{dr} &= \frac{2}{3} G_d^* w = \frac{2}{3} (6.47)(232) = 1000 \text{ kips} \\ &> 413 \text{ kips. } \therefore \text{diaphragm rigidity may be adequate} \\ &\text{for full bracing.} \end{aligned}$$

Diaphragm Strength:

$$\text{Eq. 8: } E_o = \frac{1}{8} \times \frac{12.33}{10} = 0.154''$$

$$\text{Eq. 45: } C_1 = 0.251''$$

$$\text{Eq. 46: } \text{With } K_4 = 1.0, \gamma_{\text{max}} = 0.0034 < \gamma_{dr} = 0.0069$$

\therefore diaphragm strength is adequate for full bracing.

Girts: Because the column buckles flexurally ($P_{fb} = P_{\text{cry}, \ell}$), there is no bending of the girts.

Therefore, the diaphragm-girt bracing is "full" bracing, and the W12 x 31 column can safely carry a load of 220 kips.

SUMMARY AND CONCLUSIONS

A procedure is presented for the design of I-section beams and columns with diaphragm or diaphragm-girt bracing. The procedure is based on the ultimate load capacity of fully braced members, utilizing a conservative estimate of the shear strength and shear rigidity of the diaphragm. Design examples are included. The utilization of existing wall, floor or roof diaphragms as bracing for individual beams and columns can eliminate the need for other types of bracing and/or reduce required member sizes. Thus it contributes to economical design.

ACKNOWLEDGMENTS

This paper is a result of an investigation sponsored at Cornell University by the American Institute of Steel Construction and the American Iron and Steel Institute, and is based on a similar earlier report.⁽²⁾ The cooperation of each of the cognizant research committees is appreciated. The research initially was under the general direction of G. P. Fisher, F. ASCE, and later George Winter, Hon. M. ASCE; their contributions are gratefully acknowledged.

TABLE 1 - EQUATIONS FOR BEAMS WITH DIAPHRAGM
BRACING ON THE COMPRESSION FLANGES (1,10)

$$M_{fb} = M_y \quad (11)$$

$$Q_{id} = \frac{M_y^2 - EI_y \left(\frac{n\pi}{L}\right)^2 [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ]}{e^2 EI_y \left(\frac{n\pi}{L}\right)^2 + 2M_y e + EC_w \left(\frac{n\pi}{L}\right)^2 + GJ} \quad (13)$$

or, conservatively,

$$Q_{id} = M_y / 2e \quad (15)$$

$$C = \frac{F_o Q e^2 + E_o (M_y - Qe)}{(2Qe - M_y)} \quad (16)$$

$$D = \frac{E_o Q + F_o (M_y - Qe)}{(2Qe - M_y)} \quad (17)$$

where E_o and F_o are obtained from Eqs. 8 and 9, respectively.

C_1 and γ_{max} are computed from Eqs. 18 and 19, respectively.

TABLE 2 - EQUATIONS FOR BEAMS WITH DIAPHRAGM
BRACING ON THE TENSION FLANGES^(1,10)

$$M_{fb} = \text{Min.} (M_y, 0.9 M_{\phi e}) \quad (20)$$

$$M_{\phi e} = \frac{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2}{2e} \quad (21)$$

$$Q_{id} = \frac{M_{fb}^2 - EI_y \left(\frac{n\pi}{L}\right)^2 [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ]}{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2 - 2e M_{fb}} \quad (22)$$

$$C = \frac{M_{fb} F_o [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2] + M_{fb} E_o (M_{fb} + Qe)}{[EI_y \left(\frac{n\pi}{L}\right)^2 + Q][EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2] - (M_{fb} + Qe)^2} \quad (23)$$

$$D = \frac{M_{fb} E_o [EI_y \left(\frac{n\pi}{L}\right)^2 + Q] + M_{fb} F_o (M_{fb} + Qe)}{[EI_y \left(\frac{n\pi}{L}\right)^2 + Q][EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2] - (M_{fb} + Qe)^2} \quad (24)$$

where E_o and F_o are obtained from Eqs. 8 and 9, respectively.

$$C_1 = C - eD \quad (25)$$

$$\gamma_{\max} = C_1 \frac{\pi}{L} \quad (19)$$

TABLE 3 - EQUATIONS FOR AXIALLY LOADED I-SECTION COLUMNS
BRACED BY SHEAR DIAPHRAGMS ON BOTH FLANGES (1, 10)

$$P_{fb} = P_{crx,L} \quad (26)$$

$$Q_{id} = P_{crx,L} - E^* I_y \left(\frac{n\pi}{L}\right)^2 \quad (27)$$

where $E^* = E$ or is obtained from Eq. 28 if $\sigma > \sigma_p$.

$$C = \frac{2 P_{crx,L} E_o}{E^* I_y \left(\frac{n\pi}{L}\right)^2 + Q - P_{crx,L}} \quad (29)$$

where E_o is obtained from eq. 8.

$$C_1 = C \quad (30)$$

$$\gamma_{max} = C_1 \frac{\pi}{L} \quad (19)$$

TABLE 4 - EQUATIONS FOR AXIALLY LOADED I-SECTION COLUMNS
BRACED BY SHEAR DIAPHRAGMS ON ONE FLANGE ONLY^(1,10)

$$P_{fb.} = \text{Min.} (P_{crx,L}, 0.9 P_{\phi e}) \quad (31)$$

$$\text{Elastic Range } P_{\phi e} = \frac{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2}{\left(\frac{I_p}{A} + e^2\right)} \quad (32a)$$

$$\text{Inelastic Range } P_{\phi e} = A \left[\sigma_y - \frac{A(\sigma_p)^2 \left(\frac{I_p}{A} + e^2\right)}{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2} \right] \quad (32b)$$

$$\text{where } I_p = I_x + I_y \quad (33)$$

$$Q_{id} = \frac{- [E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J - P_{fb} \frac{I_p}{A}] [E^* I_y \left(\frac{n\pi}{L}\right)^2 - P_{fb}]}{[E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J - P_{fb} \frac{I_p}{A}] + e^2 [E^* I_y \left(\frac{n\pi}{L}\right)^2 - P_{fb}]} \quad (34)$$

where E^* and G^* are obtained from Eqs. 28 and 35, respectively.

$$C = \frac{P_{fb} \left\{ [2E_o] [E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J + Qe^2 - P_{fb} \frac{I_p}{A}] + Qe \frac{I_p}{A} F_o \right\}}{\text{Det.}} \quad (36)$$

$$D = P_{fb} \frac{[E^* I_y \left(\frac{n\pi}{L}\right)^2 + Q - P_{fb}] F_o \frac{I_p}{A} + 2E_o Qe}{\text{Det.}} \quad (37)$$

where

$$\text{Det.} = [E^* I_y \left(\frac{n\pi}{L}\right)^2 + Q - P_{fb}] [E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J - P_{fb} \frac{I_p}{A}] + Qe^2 [E^* I_y \left(\frac{n\pi}{L}\right)^2 - P_{fb}]$$

and E_o and F_o are obtained from Eqs. 8 and 9, respectively.

C_1 and Y_{max} are obtained from Eqs. 25 and 19, respectively.

TABLE 5 - EQUATIONS FOR AXIALLY LOADED I-SECTION
COLUMNS WITH DIAPHRAGM-GIRT BRACING^(1,3)

$$P_{fb} = \text{Min.} (P_{crx,L}, 0.9 P_{\phi e}, P_{cry,l}) \quad (38)$$

If $m = 0$

Elastic Range

$$P_{\phi e} = \frac{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2}{\frac{P}{A} + e^2} \quad (39a)$$

Inelastic Range

$$P_{\phi e} = A \left[\sigma_y - \frac{A \sigma_p^2 \left(\frac{P}{A} + e^2\right)}{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2} \right] \quad (39b)$$

with $n = 1$ in Eqs. 39a and 39b

If $m \neq 0$

$$P_{\phi e} = \text{Min.} \left[\frac{E^* C_w \left(\frac{\pi}{L}\right)^2 + G^* J + e^2 E^* I_y \left(\frac{\pi}{L}\right)^2 + K_3 m \ell}{\left(\frac{P}{A} + e^2\right)}, P_y \right] \quad (40a)$$

or $P_{\phi e} = \text{Min.} \left[\frac{K_3 m \ell}{\left(\frac{P}{A} + e^2\right)}, P_y \right]$ (conservatively) (40b)

If $P_{fb} = P_{crx,L}$

$$Q_{id} = \frac{- (K_1 P^* - P_{fb}) (a^* - P_{fb} \frac{P}{A})}{K_2 \left[e^2 (K_1 P^* - P_{fb}) + (a^* - P_{fb} \frac{P}{A}) \right]} \quad (41)$$

where $P^* = \frac{\pi^2 E^* I_y}{L^2}$

$$a^* = K_1 E^* C_w \left(\frac{\pi}{L}\right)^2 + G^* J + K_3 m \ell$$

and K_1, K_2 and K_3 correspond to one of the modes,

TABLE 5 (cont'd)

$i = 1, \dots, j$, where j is the number of intermediate girts, and i is the mode number which gives the maximum value of Q_{id} in Eq. 41

$$C_1 = \frac{P_{fb} \left[2E_o \left\{ \left(a^* - P_{fb} \frac{l}{A} \right) + e^2 (K_1 P^* - P_{fb}) \right\} - e (K_1 P^* - P_{fb}) \left\{ 2E_o + \frac{l}{A} F_o \right\} \right]}{(K_1 P^* - P_{fb}) \left(a^* - P_{fb} \frac{l}{A} \right) + K_2 Q \left\{ e^2 (K_1 P^* - P_{fb}) + \left(a^* - P_{fb} \frac{l}{A} \right) \right\}} \quad (42)$$

$$D = \frac{P_{fb} \left[\left\{ 2E_o e + \frac{l}{A} F_o \right\} \left\{ K_1 P^* - P_{fb} + K_2 Q \right\} - e \left\{ K_1 P^* - P_{fb} \right\} 2E_o \right]}{(K_1 P^* - P_{fb}) \left(a^* - P_{fb} \frac{l}{A} \right) + K_2 Q \left\{ e^2 (K_1 P^* - P_{fb}) + \left(a^* - P_{fb} \frac{l}{A} \right) \right\}} \quad (43)$$

where K_1 , K_2 and K_3 in Eqs. 42 and 43 correspond to the first mode, $i = 1$.

If $P_{fb} = 0.9 P_{\phi e}$

Q_{id} : Use Eq. 41 above, but constants K_1 , K_2 and K_3 correspond to the first mode.

C_1 : Eq. 42 above

D : Eq. 43 above

If $m \neq 0$, $\theta_{max} = D, 0.866 D$, or D (44)
for columns with 1, 2 or 3 intermediate girts, respectively.

If $P_{fb} = P_{cry, L}$

Q_{id} : Eq. 41, same as for $P_{fb} = P_{crx, L}$

$$C_1 = \frac{2P_{fb} E_o}{K_1 P^* - P_{fb} + K_2 Q} \quad (45)$$

where K_1 , K_2 and K_3 correspond to the first mode.

TABLE 5 (cont'd)

For all failure modes

$$\gamma_{\max} = K_4 \frac{c_1}{\ell} \quad (46)$$

where K_4 is from Table 6.

TABLE 6 - CONSTANTS $K_1, K_2, K_3, K_4^{(1,2)}$

| | K_1 | K_2 | K_3 | K_4 |
|-----------------------------|--------|-------|-------|-------|
| <u>1 Intermediate Girt</u> | | | | |
| Mode: $i = 1$ | 0.250 | 0.810 | 0.405 | 1.000 |
| <u>2 Intermediate Girts</u> | | | | |
| Mode: $i = 1$ | 0.111 | 0.912 | 0.912 | 0.866 |
| $i = 2$ | 0.444 | 0.684 | 0.228 | 0.866 |
| <u>3 Intermediate Girts</u> | | | | |
| Mode: $i = 1$ | 0.0625 | 0.950 | 1.621 | 0.707 |
| $i = 2$ | 0.250 | 0.810 | 0.405 | 0.707 |
| $i = 3$ | 0.562 | 5.53 | 0.180 | 0.707 |

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APPENDIX II - NOMENCLATURE

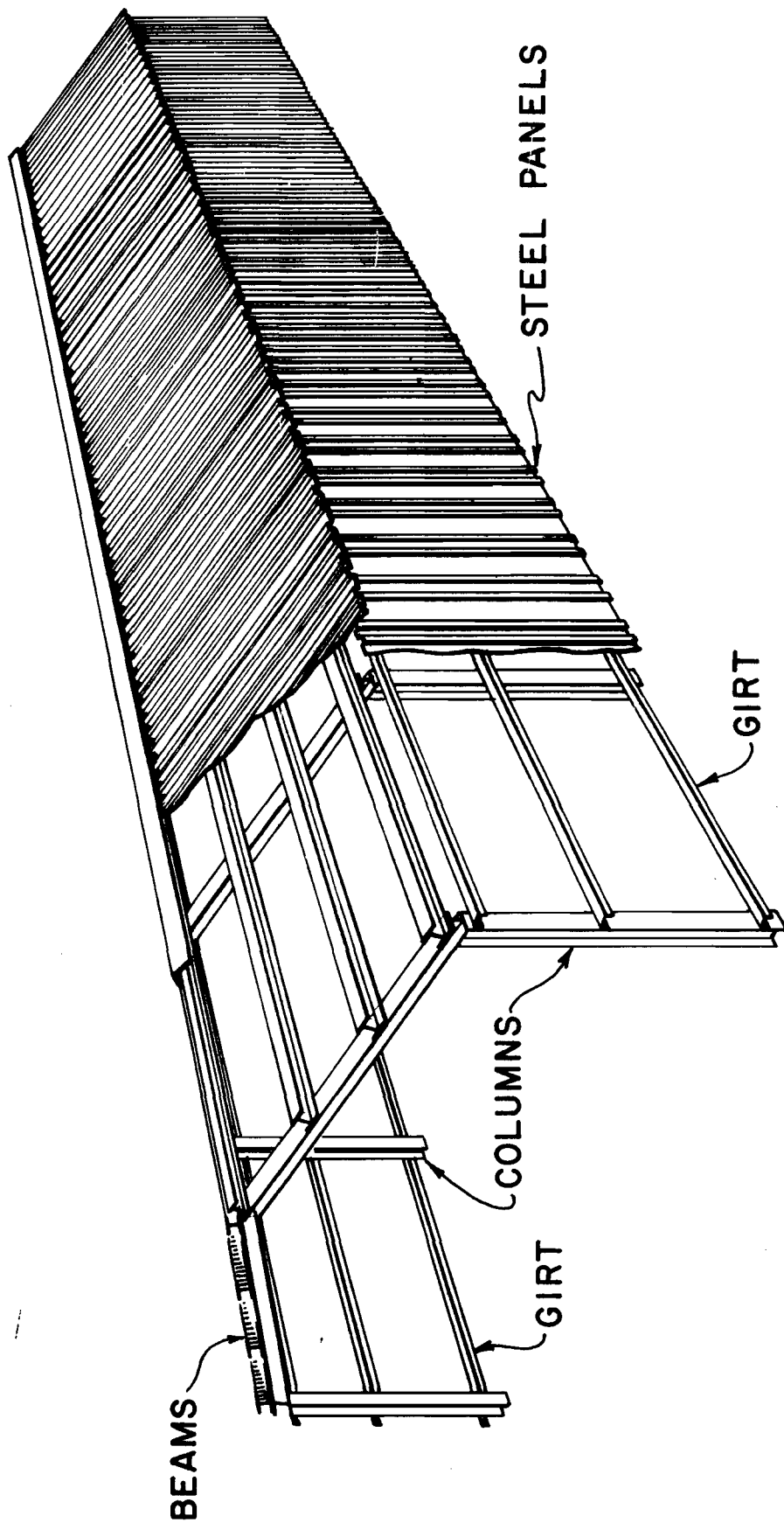
- A ----- cross sectional area, in.²
- a ----- dimension of shear diaphragm perpendicular to test load direction, in.
- a* ----- $= K_1 E^* C_w \left(\frac{\pi}{\ell}\right)^2 + G^* J + K_3 m \ell$, kip-in.²
- b ----- dimension of shear diaphragm parallel to test load direction, in.
- C ----- amplitude of additional lateral deflection of centroidal axis, in.
- C₁ ----- amplitude of additional lateral deflection in the plane of the diaphragm, in.
- C_w ----- warping constant of a section, in.⁶
- D ----- amplitude of additional twist of a member, radians
- E ----- modulus of elasticity, ksi
- E₀ ----- amplitude of initial lateral deflection of the centroidal axis of a member, in.
- E* ----- inelastic modulus, ksi
- e ----- distance between center of gravity of a member and the plane of the diaphragm, in.
- F₀ ----- amplitude of initial twist of a member, radians
- G ----- shear modulus, ksi
- G* ----- inelastic shear modulus, ksi
- G_d¹ ----- shear stiffness at 0.8 of ultimate load of diaphragm, kips/in.
- G_{dr}¹ ----- design value of shear stiffness, kips/in.
- I_p ----- polar moment of inertia, in.⁴
- I_x, I_y ----- moments of inertia of a section about X- and Y-axes, respectively, in.⁴
- I_g ----- moment of inertia of a girt about the bending axis, in.⁴
- i ----- mode number

- j ----- number of intermediate girts
 J ----- torsional constant of a section, in.⁴
 K_1, K_2, K_3, K_4 --- constants
 k ----- effective length factor
 L ----- length of member, in.
 ℓ ----- spacing of girts, in.
 M_{fb} ----- moment capacity of a "fully" braced beam, kip-in.
 M_{cr} ----- lateral torsional-flexural buckling moment of a diaphragm-braced beam, kip-in.
 M_p ----- plastic moment of a beam, kip-in.
 M_y ----- yield moment of a beam, kip-in.
 M_{yg} ----- yield moment of a girt, kip-in.
 $M_{\phi e}$ ----- buckling moment of a beam with the centroidal axis of the tension flange as a fixed axis of rotation, kip-in.
 m ----- elastic restraining moment on the column at a girt, kip-in. per radian
 P_{fb} ----- load capacity of a "fully" braced column, kips
 P_{ult} ----- ultimate shear load of a diaphragm from a test, kips
 $P_{crx,L}, P_{cry,L}$ - strong axis and weak axis buckling loads, respectively, of a column of length L , kips
 $P_{cry,\ell}$ ----- weak axis buckling load of a column of length ℓ , kips
 $P_{\phi e}$ ----- buckling load of a column with the centroidal axis of one of its flanges as the fixed axis of rotation, kips
 P_s ----- safe load on a member, kips
 P_y ----- $\sigma_y A$, kips
 P^* ----- $\frac{\pi^2 E^* I_y}{\ell^2}$, kips
 Q_{dr} ----- design value of shear rigidity, kips per radian
 Q ----- shear rigidity of diaphragm, kips/radian, or kips

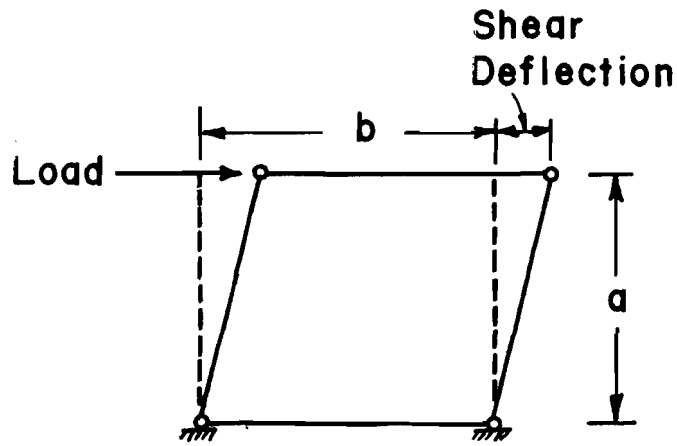
- Q_{id} ----- shear rigidity required for the "full" bracing of an ideal member, kips/radian
- r_x, r_y ----- radii of gyration of the section about X- and Y-axes, respectively, in.
- u, u_1 ----- additional deflections in the directions of X and X_1 axes, respectively, in.
- v ----- additional deflection in the direction of Y-axis, in.
- w ----- width of diaphragm contributing to the support of one member, in.
- X, X_1, Y ----- coordinate axes
- β ----- twist of the member, radians
- γ_{dr} ----- design value of diaphragm shear strain, radians
- γ_{max} ----- maximum shear strain in the diaphragm, radians
- Δ_d ----- shear deflection of a diaphragm at $0.8 P_{ult}$, radians
- θ_d ----- $= \frac{M_{yg} w}{6EI_{xg}}$ = bending slope of a girt at yield moment, radians
- θ_{max} ----- computed maximum bending slope of a girt, radians
- σ ----- average axial stress in a column, ksi
- σ_y ----- yield stress, ksi
- σ_p ----- proportional limit stress, ksi

LIST OF FIGURES

| <u>Figure No.</u> | <u>Title</u> |
|-----------------------|---|
| 1. | A Metal Building |
| 2. | Load-Deflection Relationship of a Shear Diaphragm Obtained from a Cantilever Test |
| 3. | Deflected Position of a Diaphragm in a Diaphragm- Braced Beam or Column Assembly, and in a Shear Diaphragm Test |
| 4. | Original and Deflected Positions of a Diaphragm in a Column-Girt-Diaphragm Assembly |
| 5. | Diaphragm Dimension w for an End Member or an Intermediate Member |
| 6. | Deflected Position of a Column-Girt-Diaphragm Assembly and Moment Diagram for the Girt |
| 7. | Modes of Failure of Beams Braced with a Diaphragm on the Compression Flanges |
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| 9. | Buckling Modes of Columns with Diaphragm Bracing on Both Flanges |
| 10. | Buckling Modes of Columns with Diaphragm Bracing on One Flange Only |
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| 12. | A Column-Girt-Diaphragm Assembly |



METAL BUILDING



CANTILEVER SHEAR DIAPHRAGM TEST ARRANGEMENT

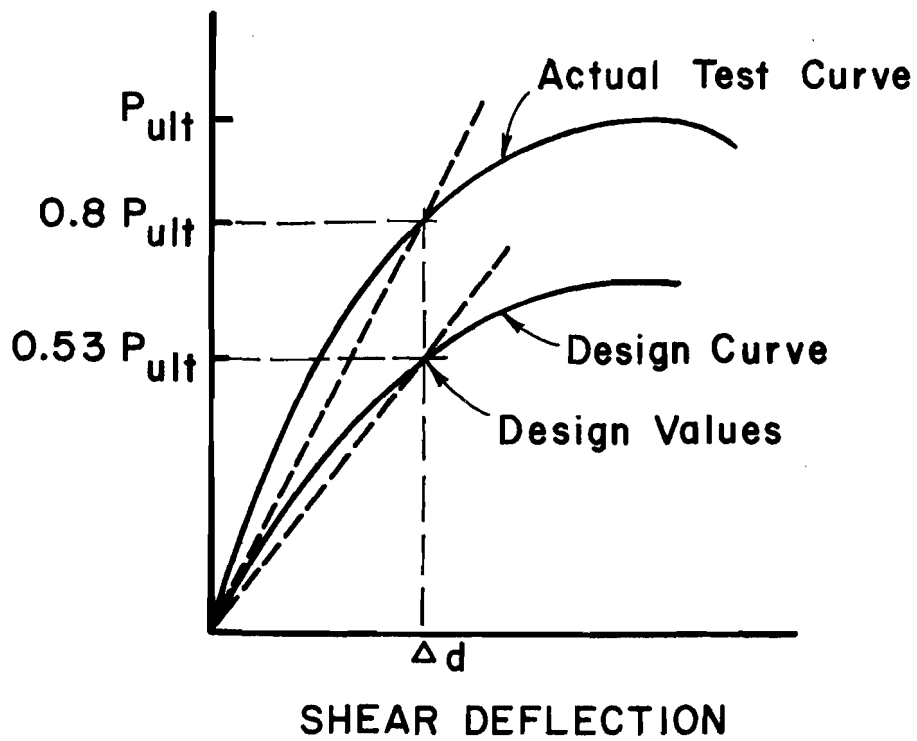
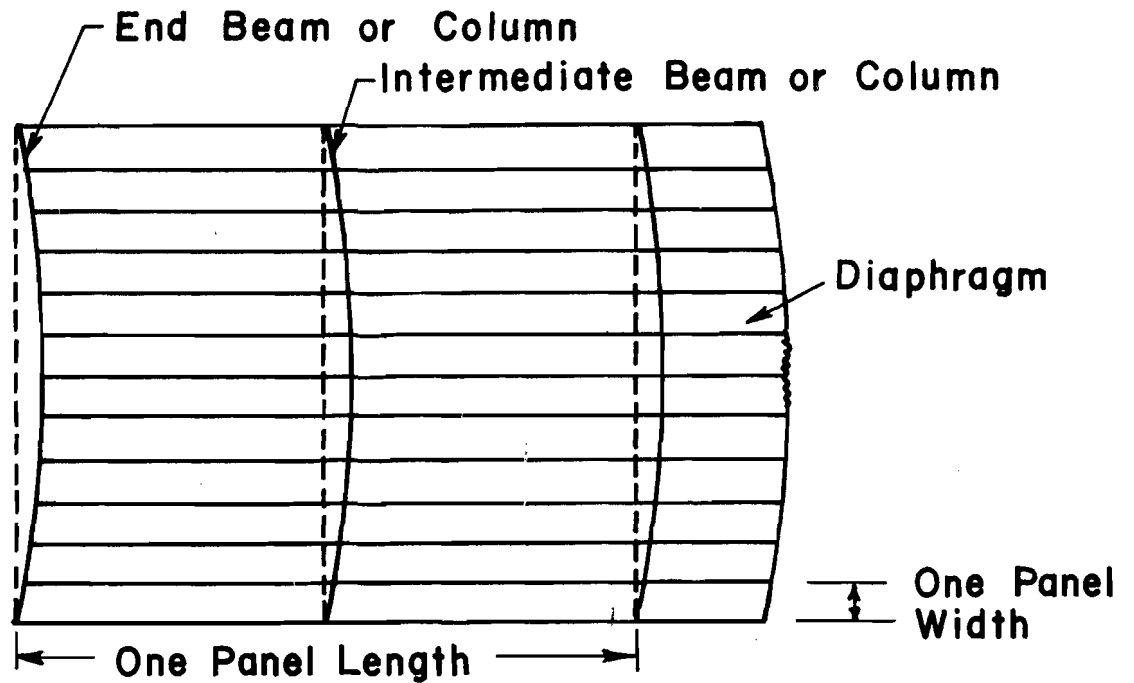
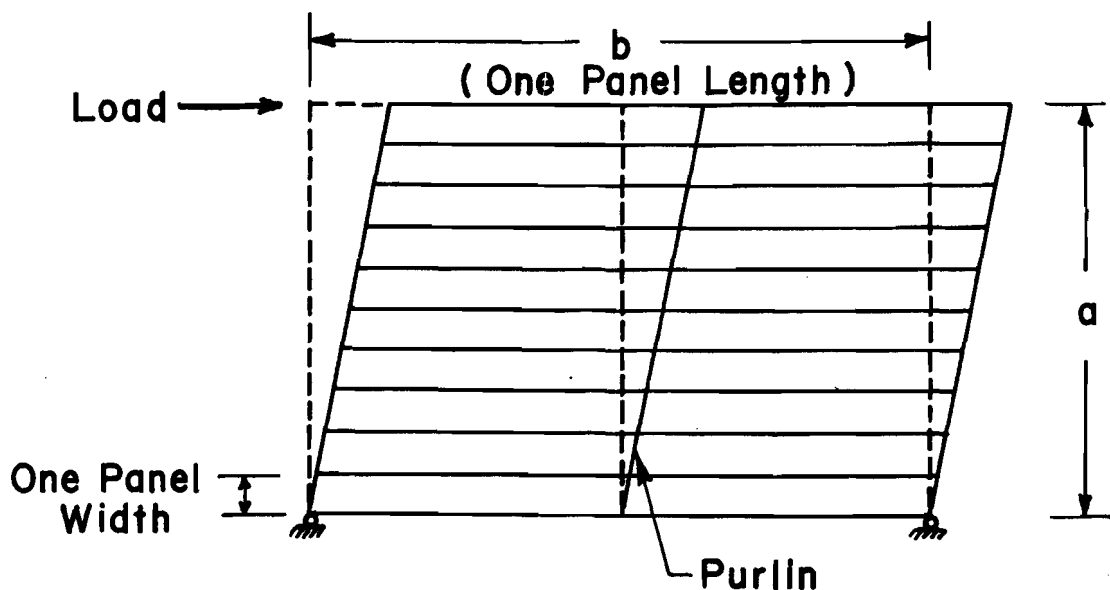


FIG. 2 LOAD-DEFLECTION RELATIONSHIP OF A SHEAR DIAPHRAGM OBTAINED FROM A CANTILEVER TEST



(a) PLAN VIEW FOR BEAMS; ELEVATION FOR COLUMNS

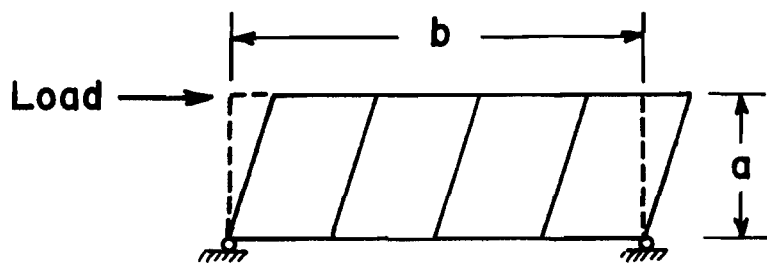
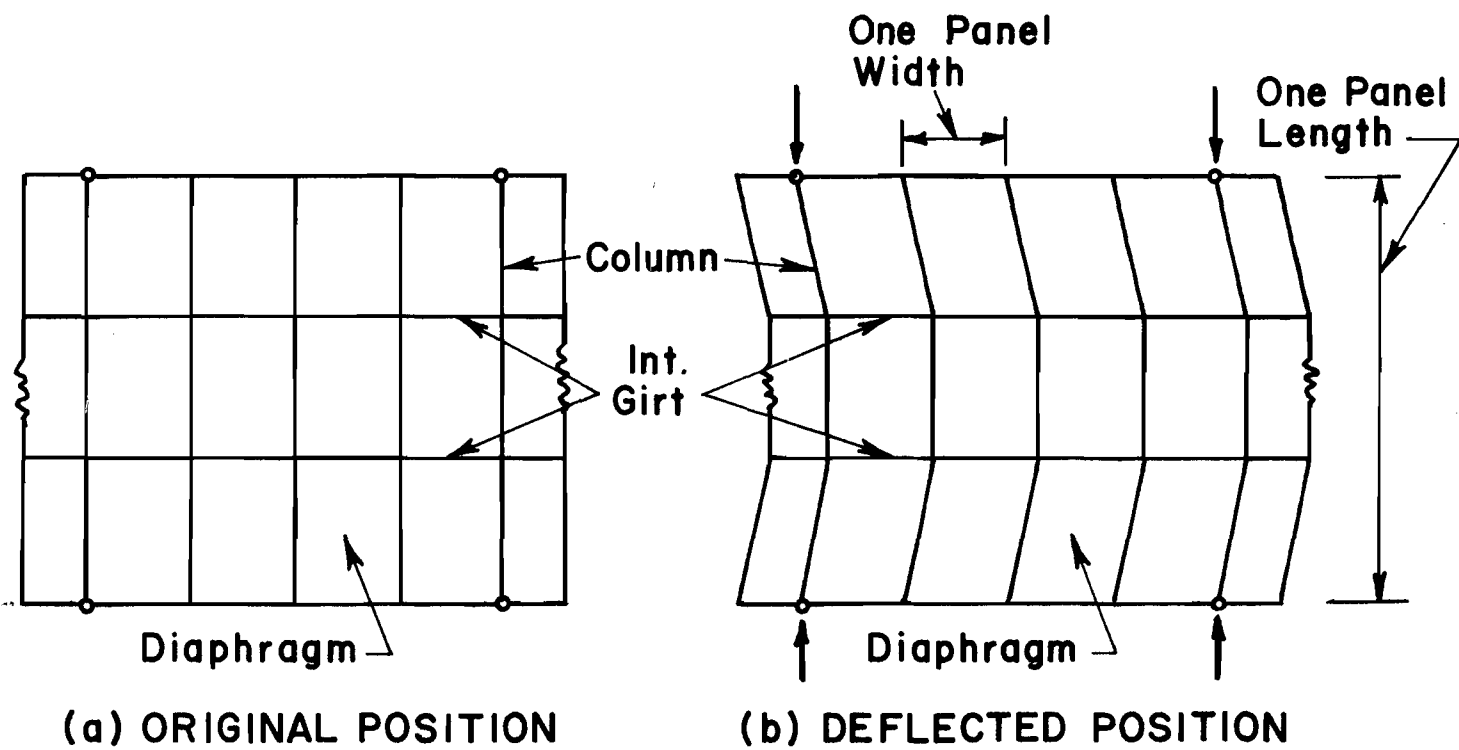


(b) CANTILEVER SHEAR DIAPHRAGM TEST ARRANGEMENT

----- Original Position

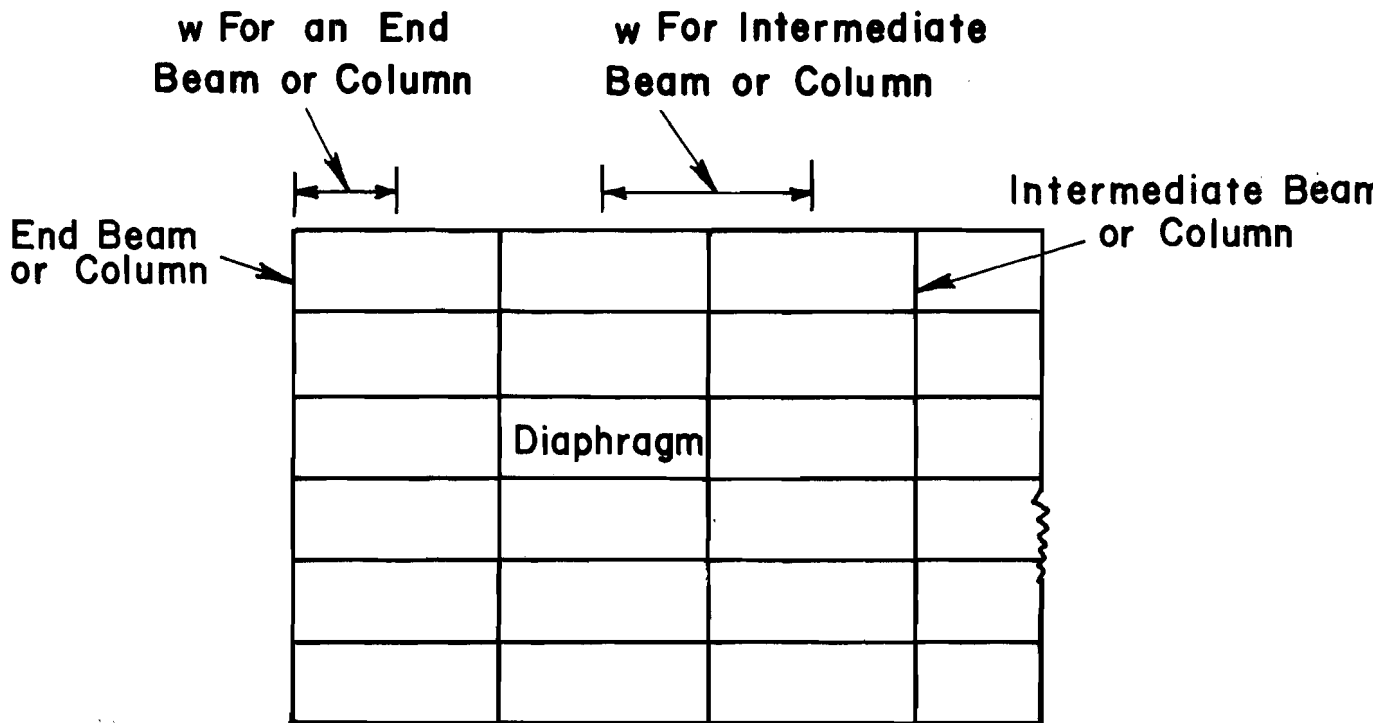
———— Deflected Position

FIG. 3 DEFLECTED POSITION OF A DIAPHRAGM IN A DIAPHRAGM-BRACED BEAM OR COLUMN ASSEMBLY, AND IN A SHEAR DIAPHRAGM TEST



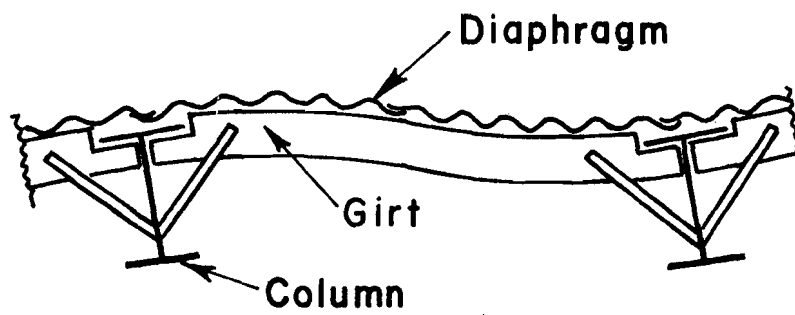
(c) CANTILEVER SHEAR DIAPHRAGM TEST ARRANGEMENT

FIG. 4 ORIGINAL AND DEFLECTED POSITIONS OF A DIAPHRAGM IN COLUMN-GIRT-DIAPHRAGM ASSEMBLY

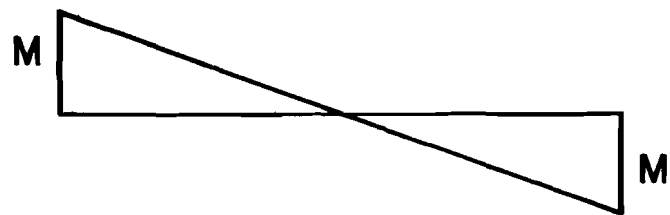


PLAN VIEW FOR BEAMS; ELEVATION FOR COLUMNS

FIG. 5 DIAPHRAGM DIMENSION w FOR AN END MEMBER OR AN INTERMEDIATE MEMBER

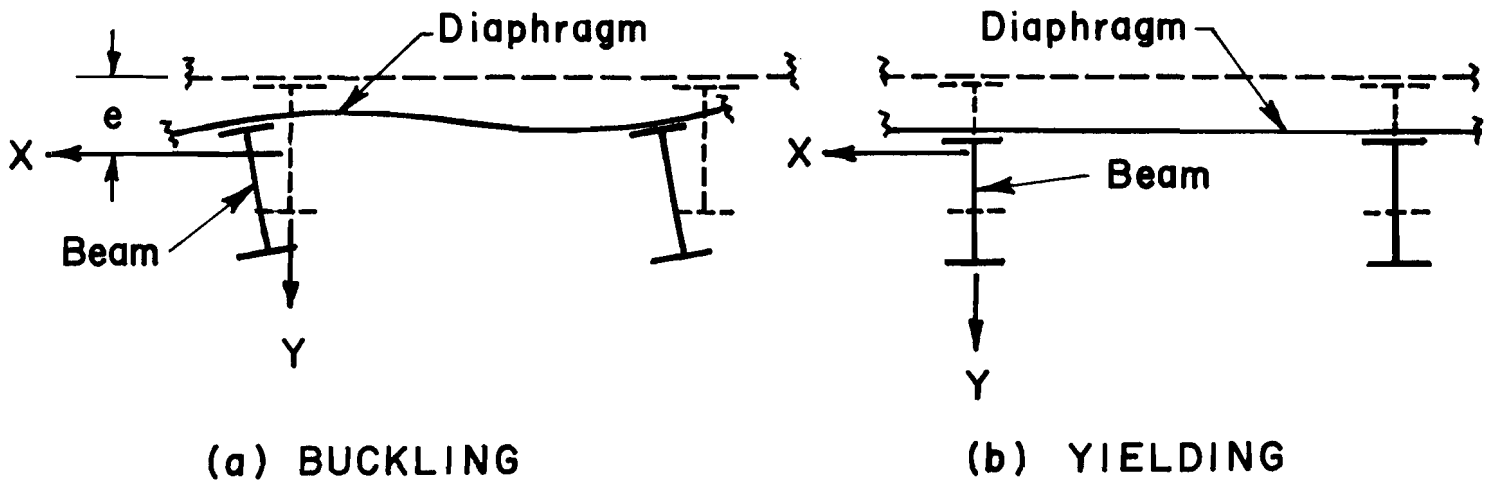


(a) DEFLECTED POSITION



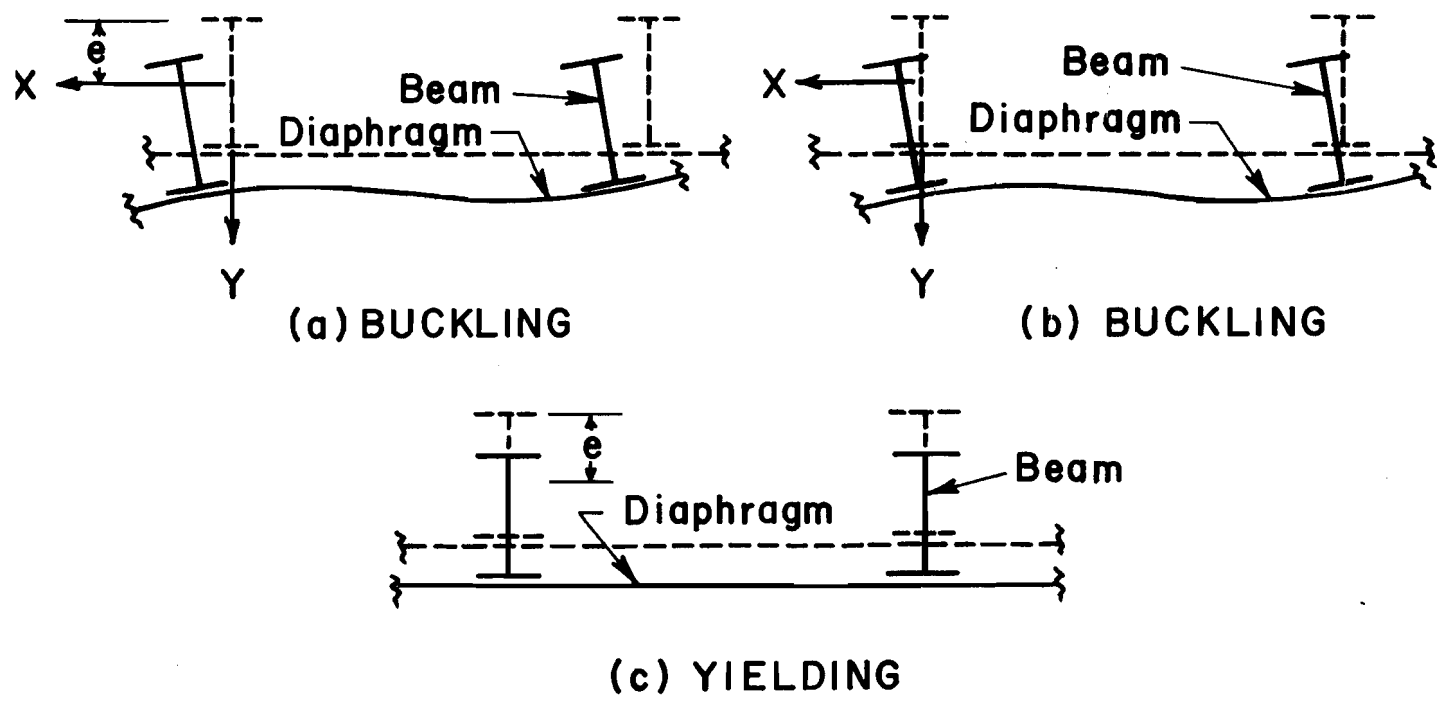
(b) MOMENT DIAGRAM FOR THE GIRT

FIG. 6 DEFLECTED POSITION OF A COLUMN-GIRT-DIAPHRAGM ASSEMBLY AND MOMENT DIAGRAM FOR THE GIRT



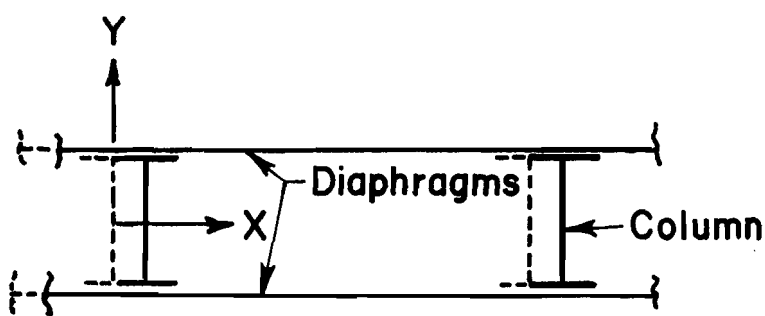
----- Original Position ——— Deflected Position

FIG. 7 MODES OF FAILURE OF BEAMS BRACED WITH A DIAPHRAGM ON THE COMPRESSION FLANGES

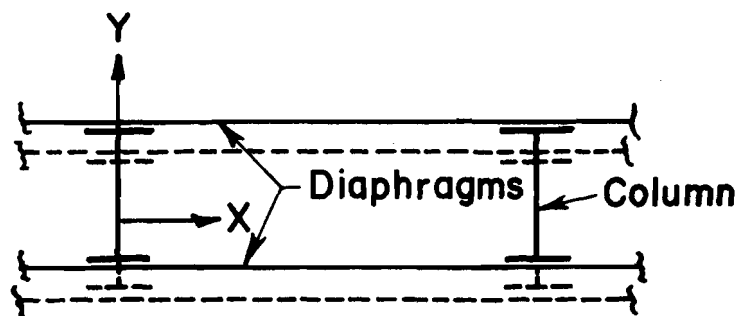


----- Original Position ——— Deflected Position

FIG. 8 MODES OF FAILURE OF BEAMS BRACED WITH A DIAPHRAGM ON THE TENSION FLANGES



(a) WEAK-AXIS BUCKLING

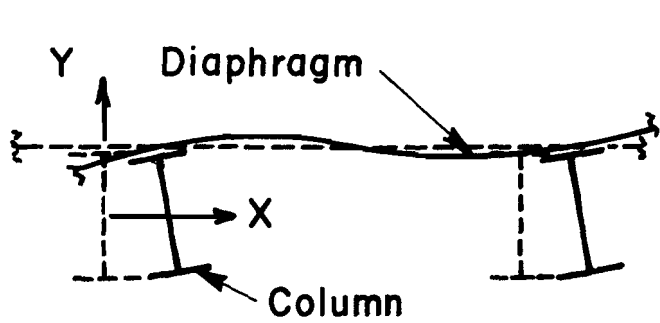


(b) STRONG-AXIS BUCKLING

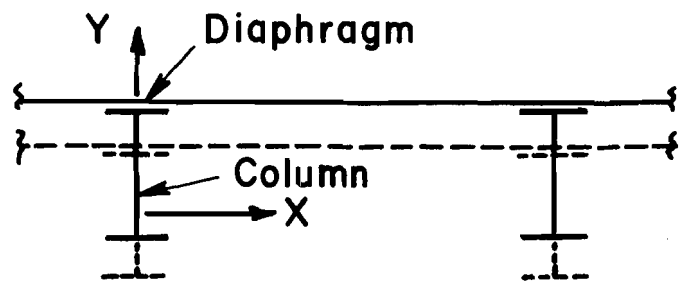
----- Original Position

———— Deflected Position

FIG. 9 BUCKLING MODES OF COLUMNS WITH DIAPHRAGM BRACING ON BOTH FLANGES



(a) TORSIONAL FLEXURAL BUCKLING

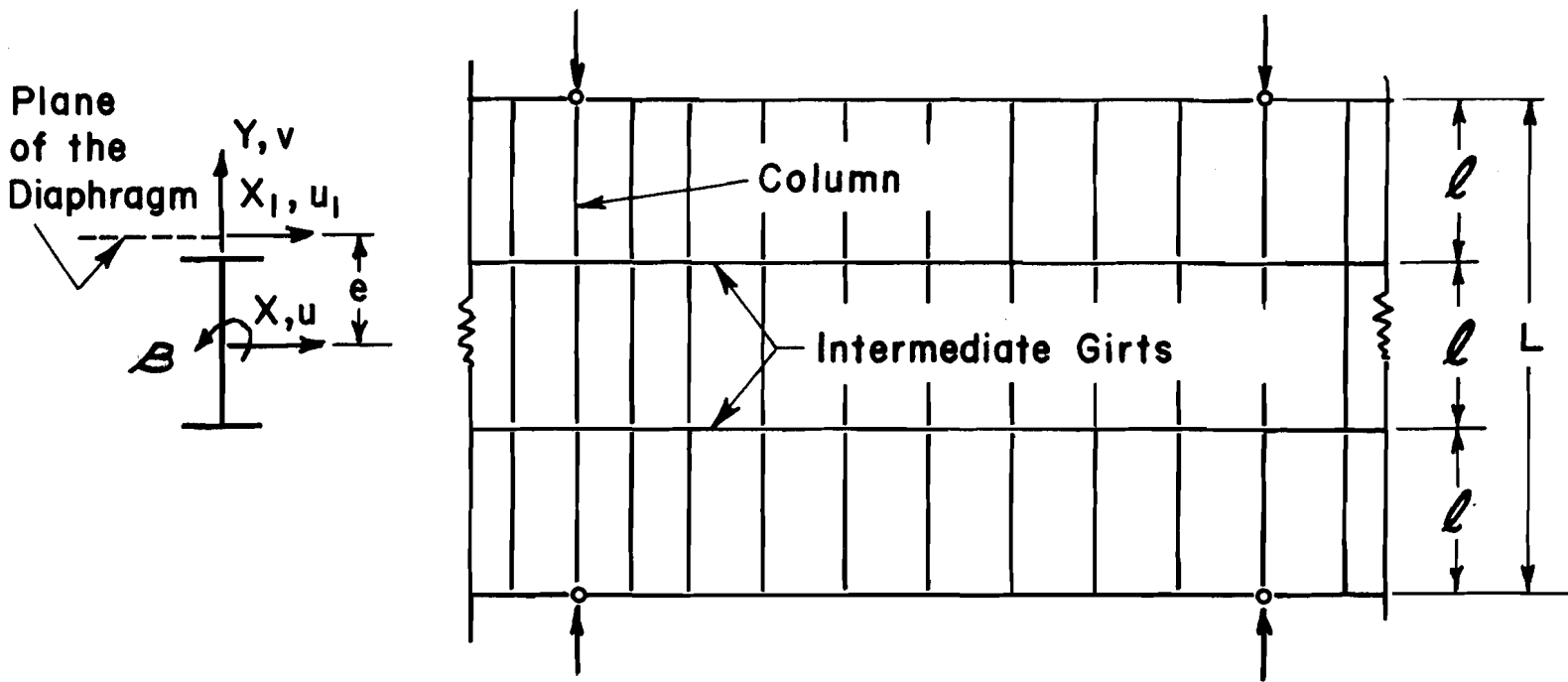


(b) FLEXURAL BUCKLING

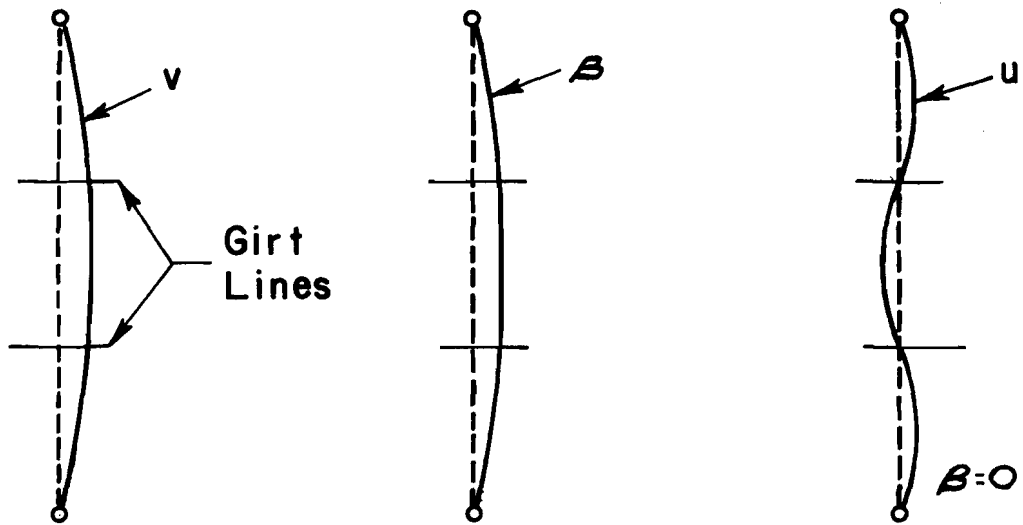
----- Original Position

———— Deflected Position

FIG. 10 BUCKLING MODES OF COLUMNS WITH DIAPHRAGM BRACING ON ONE FLANGE ONLY



(a) COLUMN-GIRT-DIAPHRAGM ASSEMBLY



(b) FLEXURAL
BUCKLING
ABOUT X-AXIS

(c) TORSIONAL
FLEXURAL
BUCKLING

(d) FLEXURAL
BUCKLING
ABOUT WEAK AXIS

FIG. 11 BUCKLING MODES OF A "FULLY-BRACED"
COLUMN WITH DIAPHRAGM-GIRT BRACING

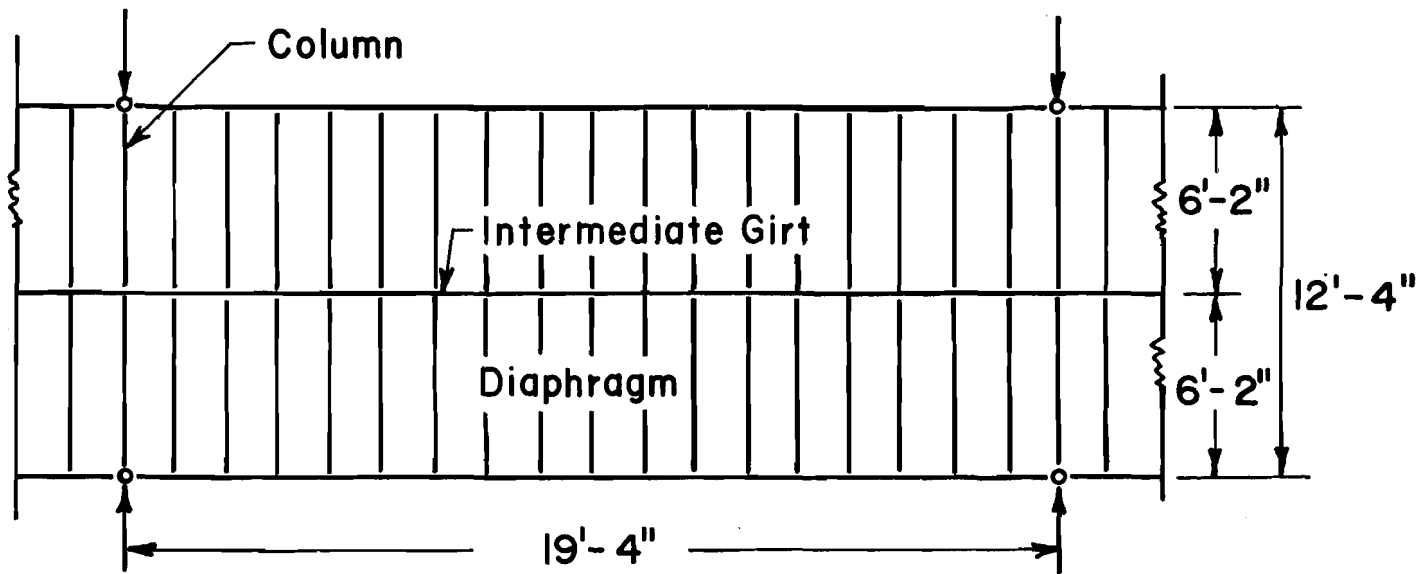


FIG. 12 A COLUMN-GIRT-DIAPHRAGM ASSEMBLY
(EXAMPLE NO. 3)

DESIGN OF I-SHAPED BEAMS AND COLUMNS WITH DIAPHRAGM BRACING

by

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April 1975

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DESIGN OF I-SHAPED BEAMS AND COLUMNS WITH DIAPHRAGM BRACING

Key Words: Beams (structural); Buildings; Bracing; Cold formed panels; Columns (structural); Design; Diaphragm; Shear strength.

Abstract

Cold-formed steel panels often are used as wall sheathing, roof decking or floor decking in steel framed buildings or pre-engineered metal buildings. Diaphragms formed by interconnecting these panels have considerable in-plane shear resistance, and can be utilized as bracing against buckling for individual members of a steel frame. For wall columns the diaphragm may be either directly attached or connected to girts which in turn are connected to the columns. A procedure is presented for the design of I-section beams and columns with diaphragm or diaphragm-girt bracing. The procedure is based on the ultimate load capacity of fully braced members, utilizing a conservative estimate of the shear strength and shear rigidity of the diaphragm. Design examples are included. The utilization of existing wall, floor or roof diaphragms as bracing for individual beams and columns can eliminate the need for other types of bracing, and/or reduce required member sizes. Thus it contributes to economical design.

Summary

A procedure is presented for the design of I-section beams and columns with diaphragm or diaphragm-girt bracing, utilizing the shear strength and rigidity of diaphragms formed by interconnecting cold formed steel panels. Design examples are included.

INTRODUCTION

Cold formed steel panels often are used as wall sheathing, roof decking or floor decking in steel framed buildings or pre-engineered metal buildings such as shown in Fig. 1. These panels carry loads normal to their plane by virtue of their bending strength. In addition, diaphragms formed by adequately interconnecting these panels can resist in-plane shear deformations, and thus act as bracing against buckling for individual columns and beams. For wall columns the diaphragm may be either directly attached or connected to girts which, in turn, are connected to the columns.

Extensive research has been conducted at Cornell University and elsewhere to determine the increased load carrying capacity of I-, channel- and Z-section beams and columns due to diaphragm or diaphragm-girt bracing. (1,3,8,10,11,15,16,21) Recommendations are made herein for the design of I-shaped members with such bracing. The bracing requirements are not a linear function of applied load; therefore, the design procedure is based on the ultimate load capacity of the beams or columns, utilizing a conservative estimate of the strength and rigidity of the diaphragm.

Effectiveness of diaphragm bracing or diaphragm-girt bracing in preventing lateral buckling depends on its two fundamental characteristics: (1) rigidity and (2) strength. Usually, it is not economical to provide anything less than "full" bracing, where full

bracing is defined as bracing such that any increase in rigidity or strength of the diaphragm will not cause any significant increase in the load-carrying capacity of the braced members.⁽¹⁹⁾ For this reason, and for greater simplicity, this design procedure is limited to "fully" braced beams and columns. The procedure is based on analyses of I-section beams under uniform moment, and I-section columns under axial load. These analyses have been substantiated by tests of 35 diaphragm-braced assemblies as reported in References 1, 3, 10 and 15.

CRITERIA FOR DESIGN

Design criteria are established herein for the following problems:

1. I-section beams continuously braced by a shear diaphragm on the compression or tension flanges, where "continuous bracing" indicates that the diaphragm is connected directly to the member at short intervals;
2. Axially loaded I-section columns continuously braced by shear diaphragms on one or both flanges; and
3. Axially loaded I-section columns braced by girts which, in turn, are braced by a shear diaphragm.

If panels with longitudinal ribs are used, the ribs must be perpendicular to the member they are bracing, which is the usual case.

Columns with equal bracing connected directly to both flanges (that is, symmetric bracing) tend to deflect laterally under load without twisting. Beams or columns with continuous bracing on only one flange tend to twist as well as deflect laterally. The diaphragm in these cases can provide continuous restraint against (1) lateral movement in the plane of the diaphragm, and (2) twist of the member. In contrast, diaphragm-girt bracing provides these two restraints to a column only at the points of attachment of the girts. To evaluate the behavior of diaphragm-braced beams and columns, it is necessary to know the nature and the magnitude of the restraints available.

Shear Characteristics of a Diaphragm

Considerable progress has been made recently in developing theory to predict the shear stiffness (or, conversely, flexibility) and shear strength of a diaphragm assembly. (5,6,7,12,14,18,20) As an alternative, these characteristics can be determined from the load-deflection curve obtained from a simple beam or cantilever shear test (Fig. 2) as described in Reference 9. This load-deflection relationship is generally not linear. Furthermore, in such tests two nominally identical diaphragms may give considerably different load-deflection relationships at the higher loads--say, beyond 80% of ultimate load. Therefore, in this discussion, the shear stiffness G'_d and average shear strain γ_d at 80% of ultimate shear load are taken as the basic characteristics of the diaphragm. Shear stiffness G'_d is defined herein as

$$G'_d = \frac{(0.8 P_{ult}/b)}{(\Delta_d/a)} \quad (1)$$

where P_{ult} is the ultimate shear load in the diaphragm test, kips

Δ_d is the deflection at $0.8 P_{ult}$, in.

a is the dimension of the shear diaphragm perpendicular to the test load direction, in.

and b is the dimension of the shear diaphragm parallel to the test load direction, in.

If the shear stiffness of a diaphragm is known, then the maximum shear strain that can be sustained by the diaphragm is a measure of its shear strength. The shear strain γ_d at $0.8 P_{ult}$ is

taken here as the measure of available shear strength, and is given by

$$\gamma_d = \frac{\Delta_d}{a} \quad (2)$$

To insure that diaphragm failure will not precede member failure, it is proposed for design purposes to assume that reliable values of shear strain γ_{dr} and shear stiffness G'_{dr} are equal to γ_d and $2/3 G'_d$, respectively. Thus, from Eqs. 1 and 2,

$$G'_{dr} = 2/3 G'_d = \frac{0.53 P_{ult}/b}{\Delta_d/a} \quad (3)$$

and

$$\gamma_{dr} = \gamma_d = \frac{\Delta_d}{a} \quad (4)$$

A graphical representation of actual test values and proposed design values of shear stiffness and shear deflection are shown in Fig. 2. The type and spacing of fasteners is very important, and those used in a diaphragm test should be the same as those used in connecting the diaphragms to the beams or columns in the actual structure. The panel lengths and purlin spacing to be used in a cantilever test (or in any analytical procedure) to simulate the actual structure are given below. As a conservative simplification, the bending rigidity of the diaphragm, which tends to prevent rotation of the member to which it is attached, is neglected in this design procedure.

1. Diaphragms continuously bracing beams or columns

The deflected position of the structure in this case is shown in Fig. 3a, where the full length of each panel is under uniform

shear. The length of panel to be used in a cantilever diaphragm test is the same as that used in the structure, as shown in Fig. 3b. Purlin spacing in the test should be the same as the spacing of the beams or columns to be braced.

2. Diaphragms in a column-girt-diaphragm assembly

A typical deflected position of the diaphragm in a column-girt-diaphragm assembly is shown in Fig. 4b, where it is seen that only a part of the length of the panel equal to the spacing of girts is under uniform shear. Therefore, the length of the panels to be used in a shear diaphragm cantilever test should be the same as the spacing of girts in the column-girt-diaphragm assembly, as shown in Fig. 4c. No intermediate purlins should be used in the test. Perimeter framing and fasteners should simulate the corresponding portion of the actual structure.

A simple beam shear test may be conducted instead of a cantilever test, making proper choice of the panel length and spacing of the framing members. (9)

When a diaphragm-braced beam or column in a structure is to be analyzed, it is more convenient in the computations to use a reliable shear rigidity Q_{dr} of the entire portion of diaphragm contributing to the support of the member, rather than the unit shear stiffness G'_{dr} . Using Eq. 3, the reliable shear rigidity Q_{dr} is expressed as

$$Q_{dr} = G'_{dr} w = 2/3 G'_d w = \frac{0.53 P_{ult} w/b}{\Delta_d/a} \quad (5)$$

where w is the dimension of the diaphragm, perpendicular to the longitudinal axis of the member, which contributes to the support of the member being analyzed. For example, in the case of floor beams braced by a diaphragm (Fig. 5), the end beams can be assumed to be supported by the diaphragm of dimension w equal to half the spacing of beams, and the intermediate beams are supported by the dimension of the diaphragm w equal to the full spacing of the beams.

Bending Stiffness and Strength of a Girt

The performance of a girt also can be characterized by its bending stiffness and strength, with due consideration of the rigidity of the girt-to-column connection. If the connection between girts and columns is fully rigid, the bending rigidity of the girt offers calculable restraint against twist of the column, at the point of attachment to the column (see Fig. 6). For a rigid connection, the elastic restraining moment per unit twist of the column, m , can be computed as

$$m = \frac{2 (6EI_g)}{w} \quad (6)$$

where I_g is the strong axis moment of inertia of the girt and E is Young's modulus. If the girt-to-column connection is effectively "pinned", then $m = 0$.

The strength of a girt can be designated by the bending slope at the column, θ_d , when the ends of a girt between two successive columns are subjected to equal and opposite moments, M_{yg}

(Fig. 6b), where M_{yg} is the yield moment of the girt. The slope can be computed as

$$\theta_d = \frac{M_{yg} w}{6EI_g} \quad (7)$$

Initial Imperfections and Additional Deflections

The required strength of any type of bracing is a function of the initial imperfections of the load-carrying member. The pattern of initial deflections along the length of an imperfect beam or column is here assumed affine to the buckling pattern to obtain a conservative estimate of deflections under load. The amplitude of the initial lateral deflection of the centroidal axis of a beam or column, E_o , is taken as the tolerance limit in sweep specified in the AISC Manual (Ref. 13, p 1-127). Hence, E_o is usually of the form

$$E_o = \frac{1''}{8} \times \frac{\text{length of member in feet}}{\text{specified integer}} \quad (8)$$

In addition, in the case of columns, an accidental eccentricity is considered by multiplying the initial lateral deflection by two in the design equations. Based on limited available information, the amplitude of the initial twist, F_o , is arbitrarily taken equal to 0.000667 radian per foot of length; that is,

$$F_o = 0.000667 \text{ rad./ft} \times \text{length of member in feet} \quad (9)$$

Because of the initial lateral deflection or twist of a beam or column, additional lateral deflections and twist occur under

applied load. These additional deflections cause shear forces in the diaphragm bracing, and may also cause bending of the girts in a column-girt-diaphragm assembly. Magnitudes of the additional deflections, maximum shear strain in the diaphragm, and maximum bending slope of the girts can be calculated using the design formulae given subsequently.

Factors of Safety

For rolled steel beams and columns the factors of safety as given in the AISC Specification⁽¹³⁾ are used. The slenderness ratio for computing the column safety factor depends on the buckling mode. For cold formed steel members the factors of safety as given in the AISI Specification⁽¹⁷⁾ are used.

GENERAL DESIGN PROCEDURE

Briefly stated, the design procedure is as follows: First, assume the member to be fully braced, and select a section of required load capacity. Then, select a diaphragm of specific panel dimensions, fastener type and fastener spacing, and check to see that its rigidity and strength are adequate to provide full bracing for the member.

The detailed design procedure is outlined below:

1. Choose a trial member for the design.
2. Assume that the member is fully braced, and compute its load capacity, P_{fb} .
3. Compute the safe load, P_s :

$$P_s = \frac{\text{load capacity of fully braced member}}{\text{factor of safety or load factor}} = \frac{P_{fb}}{F.S.} \quad (10)$$

If P_s is greater than and close to the required design load, proceed to check whether the bracing is adequate; otherwise, repeat the procedure from Step 1.

4. Compute Q_{id} , the shear rigidity required to fully brace an ideal member. The actual rigidity required to brace a real member will be greater than this. (19)
5. Select a trial diaphragm. If $Q > Q_{id}$, the diaphragm rigidity may be adequate for full bracing; proceed with Step 6. If $Q \leq Q_{id}$, full bracing cannot be achieved with this diaphragm; a more rigid diaphragm must be chosen, and this step repeated.

(Alternatively, see Refs. 1 and 16 for beams and columns with less than full bracing.)

6. Compute the maximum shear strain, γ_{max} , in the diaphragm. If the reliable shear strain $\gamma_{dr} \geq \gamma_{max}$, the diaphragm strength is adequate for full bracing; proceed with Step 7 if applicable. If $\gamma_{dr} < \gamma_{max}$, a stronger diaphragm is necessary for full bracing; repeat the procedure from Step 5.
7. This step applies only for diaphragm-girt bracing. In most cases of full bracing there is no bending of the girts. However, if the flexural restraint $m \neq 0$ and failure is in the torsional-flexural mode, the strength of the girts has to be checked. Compute θ_d and θ_{max} . If the computed bending slope of the girts θ_{max} is less than θ_d , the bracing provided by the girts in combination with the diaphragm is adequate to fully brace the column. If the girts are not strong enough, choose a stronger section for the girts and repeat the procedure from Step 5.

P_{fb} , γ_{max} , θ_{max} , Q_{id} and θ_d are computed from equations given herein; whereas Q_{dr} and γ_{dr} can be obtained from the load-deflection relationship of a shear diaphragm test or analysis. The yield moment of a beam, and the strong and weak axis buckling loads of a column, can be obtained in any rational manner, including multiplying the allowable load by the known safety factor.

**I-SECTION BEAMS BRACED BY A SHEAR DIAPHRAGM
ON THE COMPRESSION FLANGES^(1,10)**

Figs. 7a and 7b show the possible modes of failure of beams with diaphragm bracing on the compression flanges. Full bracing in this case is defined as that which has adequate rigidity and strength to prevent lateral buckling until the beam yields. Thus, M_y , the yield moment, is taken as the moment capacity of a fully braced beam, M_{fb} (Eq. 11, Table 1).

The torsional-flexural buckling moment, M_{cr} , of an ideal beam with diaphragm bracing on the compression flange is⁽¹⁰⁾

$$M_{cr} = \sqrt{[EI_y \left(\frac{n\pi}{L}\right)^2 + Q] [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2]} + Qe \quad (12)$$

where EI_y is the weak axis bending rigidity

EC_w is the warping rigidity

GJ is the torsional rigidity

e is the distance between the center of gravity of the beam and the plane of the diaphragm

and $n = 1$ or 2 for ends simply supported or fixed, respectively, against lateral bending

The cross bending rigidity of the diaphragm is neglected in Eq. 12 and in all subsequent expressions.

The shear rigidity, Q_{id} , required for an ideal beam to attain the "fully" braced moment $M_{fb} = M_y$ can be obtained by substituting M_y for M_{cr} in Eq. 12 and solving for Q , resulting in Eq. 13 shown in Table 1. A simplified and conservative expression for M_{cr} is given by

$$M_{cr} = 2 Q_e \quad (14)$$

Then, with $M_{cr} = M_y$, a simple and conservative estimate for Q_{id} is obtained, see Eq. 15, Table I. Conservative estimates of the amplitudes of additional lateral deflection of the centroidal axis (C) and twist (D) at moment M_y are given by Eqs. 16 and 17. The amplitude of the additional lateral deflection in the plane of the diaphragm is

$$C_1 = C + eD \quad (18)$$

and the maximum shear strain in the diaphragm is

$$\gamma_{max} = C_1 \frac{\pi}{L} \quad (19)$$

These expressions are used in Steps 1 through 6 of the design procedure to design a diaphragm-braced I-beam in Example No. 1

I-SECTION BEAMS BRACED BY A SHEAR DIAPHRAGM
ON THE TENSION FLANGES (1,10)

The possible failure modes for beams braced by diaphragms on the tension flanges are indicated in Fig. 8. The buckling moment of the beam reaches M_{ϕ_e} (buckling moment of the beam with the centroidal axis of the tension flange as the fixed axis of rotation) asymptotically as the shear rigidity approaches infinity. In general, even for a very small increase in the moment capacity beyond $0.9 M_{\phi_e}$, a very large increase in shear rigidity is needed. Therefore, if a beam with diaphragm bracing on the tension flange buckles in a torsional-flexural mode (Figs. 8a and 8b) the buckling moment of the "fully" braced beam is arbitrarily taken as $0.9 M_{\phi_e}$. (Any other percentage could be used in similar fashion.) A fully braced beam may also fail by yielding (Fig. 8c). Hence, the moment capacity of a beam "fully" braced on its tension flanges is the smaller of these two values, as indicated by Eq. 20, Table 2. Other expressions required in the design procedure also are given in Table 2. Bracing on the tension flange is, of course, less efficient than compression flange bracing. This situation may occur, for example, under wind uplift loadings.

AXIALLY LOADED I-SECTION COLUMNS CONTINUOUSLY BRACED BY
SHEAR DIAPHRAGMS ON BOTH FLANGES^(1,10)

Diaphragm-braced columns in this case may buckle in one of the two modes shown in Fig. 9. The bracing is defined as "full" if its rigidity and strength are adequate to prevent weak-axis buckling of the columns (Fig. 9a) so that they buckle about their strong axis (Fig. 9b), and P_{fb} is therefore equal to $P_{crx,L}$, the strong axis buckling load of the column of length, L (Eq. 26, Table 3).

The shear rigidity (Q_{id}) required for an ideal column to attain full bracing is given by Eq. 27 in Table 3, where E^* is the modulus corresponding to the average stress level (σ) of the column at P_{fb} . If $\sigma < \sigma_p$ (the proportional limit stress), $E^* = E$. But, if $\sigma > \sigma_p$,⁽⁴⁾

$$E^* = E \frac{(\sigma_y - \sigma) \sigma}{(\sigma_y - \sigma_p) \sigma_p} \quad (28)$$

Other equations required in the design procedure are given in Table 3.

AXIALLY LOADED I-SECTION COLUMNS CONTINUOUSLY BRACED BY
A SHEAR DIAPHRAGM ON ONE FLANGE ONLY ^(1,10)

Diaphragm-braced columns in this case may buckle in one of the modes shown in Fig. 10. The buckling load of the column approaches P_{ϕ_e} (the buckling load of the column with the centroidal axis of one of the flanges as the fixed axis of rotation) asymptotically as the shear rigidity Q approaches infinity. In general, even for a very small increase in load beyond about $0.9 P_{\phi_e}$, a very large increase in shear rigidity is needed. Therefore, if a column buckles in the torsional-flexural mode, as in Fig. 10a, the buckling load of the "fully" braced column is arbitrarily taken as $0.9 P_{\phi_e}$. The buckling load of a fully braced column is the smaller of the two values, Eq. 31, Table 4. The buckling load P_{ϕ_e} is given by Eqs. 32a and 32b in Table 4 for the elastic and inelastic range, respectively. For an I-section, the polar moment of inertia, I_p , in Eqs. 32a and 32b is

$$I_p = I_x + I_y \quad (33)$$

The shear rigidity, Q_{id} , required for an ideal column to be fully braced is given by Eq. 34 in Table 4, where E^* is obtained from Eq. 28 and

$$G^* = G \frac{E^*}{E} \quad (35)$$

Other equations used in the design procedure also are given in Table 4.

AXIALLY LOADED I-SECTION COLUMNS WITH
DIAPHRAGM-GIRT BRACING^(1,3)

A typical column-girt-diaphragm assembly is shown in Fig. 11a. If "full" bracing is provided, the column may buckle in one of three modes shown in Fig. 11, and the buckling load of such a fully-braced column is the smallest of the three values, as in Eq. 38, Table 5, where $P_{cry, \ell}$ is the weak axis buckling load of a column of length ℓ .

The design formulae given in this report are for columns with "hinged" ends; that is, the ends are flexurally hinged, and warping is unrestrained. Design equations for the various cases are given in Table 5; values of the required coefficients K_1 through K_4 appear in Table 6 for modes $i = 1 \dots j$, where j is the number of intermediate girts. Strength of the girts has to be checked only when $m \neq 0$ and the column buckles in a torsional-flexural mode.

DESIGN EXAMPLES

Example No. 1 - Beams Braced by a Diaphragm on Their Compression Flanges

Design an intermediate floor beam to span 20 ft and to carry a uniform live load and superimposed dead load of 550 lb/ft. The beams are 6 ft apart and are braced by a deck whose shear characteristics determined from tests are: $G'_d = 4.235$ kips/in. and $\gamma_d = 0.0045$ rad. Ends of the beams are considered simply supported laterally. Use ASTM A36 steel.

Solution: Assume that the beam is fully braced and that its dead load will be about 20 lb/ft.

$$M = \frac{wl^2}{8} = \frac{(550 + 20)(20)^2(12)}{8} = 342 \text{ kip-in.}$$

For a factor of safety of 1.67,

$$M_{\text{req'd}} = 1.67 \times 342 = 571 \text{ kip-in.}$$

$$\text{Req'd } S_x = M_{\text{req'd}} \div \sigma_y = 571 \div 36 = 15.9 \text{ in.}^3$$

$$\text{Choose W10 x 17, } S_x = 16.2 \text{ in.}^3, \quad d = 10.12 \text{ in.}$$

$$\text{Fully braced moment } M_{\text{fb}} = M_y = S_x \sigma_y = 16.2 \times 36.0 = 583 \text{ kip-in.}$$

$$> M_{\text{req'd}}; \quad \therefore \text{OK}$$

Proceed to check whether the bracing is adequate.

Diaphragm Rigidity:

$$\text{Eq. 15: } Q_{\text{id}} = \frac{M_y}{2e} = \frac{583}{2(10.12 \div 2)} = 57.6 \text{ kips}$$

$$\begin{aligned} \text{Eq. 5: } Q_{\text{dr}} &= G'_{\text{dr}} w = \frac{2}{3} G'_d w \\ &= \frac{2}{3} (4.235)(72) = 203.3 \text{ kips} > 57.6 \text{ kips} \end{aligned}$$

\therefore diaphragm rigidity may be adequate for full bracing.

Diaphragm Strength:

Eq. 8: Assumed initial deflection, $E_o = 1/8 \times 20/5 = 0.5''$

Eq. 9: Assumed initial twist, $F_o = 0.000667 \times 20$
 $= 0.01334 \text{ rad.}$

$$e = d/2 = 5.06''$$

Eq. 16: Additional deflection, $C = -0.1038''$

Eq. 17: Additional twist, $D = 0.0648 \text{ rad.}$

Eq. 18: Deflection of braced flange, $C_1 = 0.2242''$

Eq. 19: Maximum shear strain, $\gamma_{\max} = 0.00294 \text{ rad.}$

$$< \gamma_{dr} = \gamma_d = 0.0045 \text{ rad.}$$

\therefore diaphragm strength is adequate for "full" bracing. The W10 x 17 beam is fully braced and can safely carry a uniform load of 550 lb/ft.

Example No. 2 - Columns with Diaphragm-Girt Bracing

Determine the required size of intermediate I-section columns 12'-4" long, spaced at 19'-4" intervals, carrying an axial load of 220 kips each, and braced by one line of girts at midheight. The girts are braced by a standard corrugated diaphragm. Assume the ends of the columns are hinged, with warping unrestrained.

Spacing of girts = 6'-2"

Diaphragm stiffness, $G_d' = 6.47$ kips/in.

Diaphragm shear strain, $\gamma_d = 0.0069$ rad.

Twist restraint, $m = 4650$ k-in./rad.

Use ASTM A572 Grade 50 steel, $\sigma_y = 50$ ksi, $E = 29,000$ ksi,
 $G = 11,500$ ksi, $\sigma_p = 25$ ksi

Solution: Try W12 x 31

Buckling Loads: $\frac{L}{r_x} = \frac{148}{5.12} = 28.9 < C_c = 107.0$

Ref. 13: $P_{crx,L} = 9.13 \times 50 \left[1 - \frac{1}{2} \left(\frac{28.9}{107} \right)^2 \right] = 440$ kips

$$\frac{\ell}{r_y} = \frac{74}{1.54} = 48.0 < C_c = 107.0$$

Ref. 13: $P_{cry,\ell} = 9.13 \times 50 \left[1 - \frac{1}{2} \left(\frac{48.0}{107} \right)^2 \right] = 410$ kips

Eq. 40b: $P_{\phi_e} = P_y = A F_y = 456.5$ kips

$$0.9 P_{\phi_e} = 411 \text{ kips}$$

Eq. 38: $P_{fb} = \text{Min.} (440; 410; 411)$

$$P_{fb} = P_{cry,\ell} = 410 \text{ kips}$$

Factor of Safety:

Ref. 13: $F.S. = \frac{5}{3} + \frac{3}{8} \left(\frac{48}{107} \right) - \frac{1}{8} \left(\frac{48}{107} \right)^3 = 1.82$

Required ultimate strength = $220 \times 1.82 = 400$ kips

< 410 kips = P_{fb} \therefore OK

Diaphragm Rigidity:

$$\sigma = \frac{P_{fb}}{A} = \frac{410}{9.13} = 44.9 \text{ ksi}$$

Eq. 28: $E^* = 10,625$ ksi

Eq. 35: $G^* = 4,210$ ksi

Eq. 41: With $i = 1$, $K_1 = .250$, $K_2 = .810$, $K_3 = .405$
 $Q_{id} = 413$ kips

Eq. 5: $Q_{dr} = 2/3 G'_d w = 2/3 (6.47)(232) = 1000$ kips
 > 413 kips. \therefore diaphragm rigidity may be
adequate for full bracing.

Diaphragm Strength:

Eq. 8: $E_o = \frac{1}{8} \times \frac{12.33}{10} = 0.154''$

Eq. 45: $C_1 = 0.251''$

Eq. 46: With $K_4 = 1.0$, $\gamma_{max} = 0.0034 < \gamma_{dr} = 0.0069$
 \therefore diaphragm strength is adequate for full
bracing.

Girts:

Because the column buckles flexurally ($P_{fb} = P_{cry, \ell}$), there
is no bending of the girts.

Therefore, the diaphragm-girt bracing is "full" bracing,
and the W12 x 31 column can safely carry a load of 220 kips.

SUMMARY AND CONCLUSIONS

A procedure is presented for the design of I-section beams and columns with diaphragm or diaphragm-girt bracing. The procedure is based on the ultimate load capacity of fully braced members, utilizing a conservative estimate of the shear strength and shear rigidity of the diaphragm. Design examples are included. The utilization of existing wall, floor or roof diaphragms as bracing for individual beams and columns can eliminate the need for other types of bracing and/or reduce required member sizes. Thus it contributes to economical design.

ACKNOWLEDGMENTS

This paper is a result of an investigation sponsored at Cornell University by the American Institute of Steel Construction and the American Iron and Steel Institute, and is based on a similar earlier report.⁽²⁾ The cooperation of each of the cognizant research committees is appreciated. The research initially was under the general direction of G. P. Fisher, F. ASCE, and later George Winter, Hon. M. ASCE; their contributions are gratefully acknowledged.

TABLE 1 - EQUATIONS FOR BEAMS WITH DIAPHRAGM
BRACING ON THE COMPRESSION FLANGES^(1,10)

$$M_{fb} = M_y = S_x \sigma_y \quad (11)$$

$$Q_{id} = \frac{M_y^2 - EI_y \left(\frac{n\pi}{L}\right)^2 [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ]}{e^2 EI_y \left(\frac{n\pi}{L}\right)^2 + 2M_y e + EC_w \left(\frac{n\pi}{L}\right)^2 + GJ} \quad (13)$$

or, conservatively,

$$Q_{id} = M_y / 2e \quad (15)$$

$$C = \frac{F_o Q e^2 + E_o (M_y - Qe)}{(2Qe - M_y)} \quad (16)$$

$$D = \frac{E_o Q + F_o (M_y - Qe)}{(2Qe - M_y)} \quad (17)$$

where E_o and F_o are obtained from Eqs. 8 and 9, respectively.

C_1 and γ_{max} are computed from Eqs. 18 and 19, respectively.

TABLE 2 - EQUATIONS FOR BEAMS WITH DIAPHRAGM
BRACING ON THE TENSION FLANGES (1,10)

$$M_{fb} = \text{Min.} (M_y, 0.9 M_{\phi e}) \quad (20)$$

$$M_{\phi e} = \frac{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2}{2e} \quad (21)$$

$$Q_{id} = \frac{M_{fb}^2 - EI_y \left(\frac{n\pi}{L}\right)^2 [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ]}{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2 - 2e M_{fb}} \quad (22)$$

$$C = \frac{M_{fb} F_o [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2] + M_{fb} E_o (M_{fb} + Qe)}{[EI_y \left(\frac{n\pi}{L}\right)^2 + Q][EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2] - (M_{fb} + Qe)^2} \quad (23)$$

$$D = \frac{M_{fb} E_o [EI_y \left(\frac{n\pi}{L}\right)^2 + Q] + M_{fb} F_o (M_{fb} + Qe)}{[EI_y \left(\frac{n\pi}{L}\right)^2 + Q][EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2] - (M_{fb} + Qe)^2} \quad (24)$$

where E_o and F_o are obtained from Eqs. 8 and 9, respectively.

$$C_1 = C - eD \quad (25)$$

$$\gamma_{\max} = C_1 \frac{\pi}{L} \quad (19)$$

TABLE 3 - EQUATIONS FOR AXIALLY LOADED I-SECTION COLUMNS
BRACED BY SHEAR DIAPHRAGMS ON BOTH FLANGES^(1,10)

$$P_{fb} = P_{crx,L} \quad (26)$$

$$Q_{id} = P_{crx,L} - E^* I_y \left(\frac{n\pi}{L}\right)^2 \quad (27)$$

where $E^* = E$ or is obtained from Eq. 28 if $\sigma > \sigma_p$.

$$C = \frac{2 P_{crx,L} E_o}{E^* I_y \left(\frac{n\pi}{L}\right)^2 + Q - P_{crx,L}} \quad (29)$$

where E_o is obtained from Eq. 8.

$$C_1 = C \quad (30)$$

$$\gamma_{max} = C_1 \frac{\pi}{L} \quad (19)$$

TABLE 4 - EQUATIONS FOR AXIALLY LOADED I-SECTION COLUMNS
BRACED BY SHEAR DIAPHRAGMS ON ONE FLANGE ONLY (1,10)

$$P_{fb} = \text{Min.} (P_{crx,L}, 0.9 P_{\phi e}) \quad (31)$$

Elastic Range

$$P_{\phi e} = \frac{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2}{\left(\frac{I_p}{A} + e^2\right)} \quad (32a)$$

Inelastic Range

$$P_{\phi e} = A \left[\sigma_y - \frac{A(\sigma_p)^2 \left(\frac{I_p}{A} + e^2\right)}{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2} \right] \quad (32b)$$

where $I_p = I_x + I_y$ (33)

$$Q_{id} = \frac{- [E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J - P_{fb} \frac{I_p}{A}] [E^* I_y \left(\frac{n\pi}{L}\right)^2 - P_{fb}]}{[E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J - P_{fb} \frac{I_p}{A}] + e^2 [E^* I_y \left(\frac{n\pi}{L}\right)^2 - P_{fb}]} \quad (34)$$

where E^* and G^* are obtained from Eqs. 28 and 35, respectively.

$$C = \frac{P_{fb} \left\{ [2E_o] [E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J + Qe^2 - P_{fb} \frac{I_p}{A}] + Qe \frac{I_p}{A} F_o \right\}}{\text{Det.}} \quad (36)$$

$$D = P_{fb} \frac{[E^* I_y \left(\frac{n\pi}{L}\right)^2 + Q - P_{fb}] F_o \frac{I_p}{A} + 2E_o Qe}{\text{Det.}} \quad (37)$$

where

$$\text{Det.} = [E^* I_y \left(\frac{n\pi}{L}\right)^2 + Q - P_{fb}] [E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J - P_{fb} \frac{I_p}{A}] + Qe^2 [E^* I_y \left(\frac{n\pi}{L}\right)^2 - P_{fb}]$$

and E_o and F_o are obtained from Eqs. 8 and 9, respectively.

C_1 and γ_{\max} are obtained from Eqs. 25 and 19, respectively.

TABLE 5 - EQUATIONS FOR AXIALLY LOADED I-SECTION
COLUMNS WITH DIAPHRAGM-GIRT BRACING^(1,3)

$$P_{fb} = \text{Min.} (P_{crx,L}, 0.9 P_{\phi e}, P_{cry,l}) \quad (38)$$

If $m = 0$

Elastic Range

$$P_{\phi e} = \frac{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2}{\frac{I_p}{A} + e^2} \quad (39a)$$

Inelastic Range

$$P_{\phi e} = A \left[\sigma_y - \frac{A \sigma_p^2 \left(\frac{I_p}{A} + e^2\right)}{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2} \right] \quad (39b)$$

with $n = 1$ in Eqs. 39a and 39b

If $m \neq 0$

$$P_{\phi e} = \text{Min.} \left[\frac{E^* C_w \left(\frac{\pi}{L}\right)^2 + G^* J + e^2 E^* I_y \left(\frac{\pi}{L}\right)^2 + K_3 m l}{\left(\frac{I_p}{A} + e^2\right)}, P_y \right] \quad (40a)$$

or $P_{\phi e} = \text{Min.} \left[\frac{K_3 m l}{\left(\frac{I_p}{A} + e^2\right)}, P_y \right]$ (conservatively) (40b)

If $P_{fb} = P_{crx,L}$

$$Q_{id} = \frac{-(K_1 P^* - P_{fb})(a^* - P_{fb} \frac{I_p}{A})}{K_2 \left[e^2 (K_1 P^* - P_{fb}) + (a^* - P_{fb} \frac{I_p}{A}) \right]} \quad (41)$$

where $P^* = \frac{\pi^2 E^* I_y}{l^2}$

$$a^* = K_1 E^* C_w \left(\frac{\pi}{l}\right)^2 + G^* J + K_3 m l$$

and K_1 , K_2 and K_3 correspond to one of the modes,

TABLE 5 (cont'd)

$i = 1, \dots, j$, where j is the number of intermediate girts, and i is the mode number which gives the maximum value of Q_{id} in Eq. 41

$$C_1 = \frac{P_{fb} \left[2E_o \left\{ \left(a^* - P_{fb} \frac{l}{A} \right) + e^2 (K_1 P^* - P_{fb}) \right\} - e (K_1 P^* - P_{fb}) \left\{ 2eE_o + \frac{l}{A} F_o \right\} \right]}{(K_1 P^* - P_{fb}) \left(a^* - P_{fb} \frac{l}{A} \right) + K_2 Q \left\{ e^2 (K_1 P^* - P_{fb}) + \left(a^* - P_{fb} \frac{l}{A} \right) \right\}} \quad (42)$$

$$D = \frac{P_{fb} \left[\left\{ 2E_o e + \frac{l}{A} F_o \right\} \left\{ K_1 P^* - P_{fb} + K_2 Q \right\} - e \left\{ K_1 P^* - P_{fb} \right\} 2E_o \right]}{(K_1 P^* - P_{fb}) \left(a^* - P_{fb} \frac{l}{A} \right) + K_2 Q \left\{ e^2 (K_1 P^* - P_{fb}) + \left(a^* - P_{fb} \frac{l}{A} \right) \right\}} \quad (43)$$

where K_1 , K_2 and K_3 in Eqs. 42 and 43 correspond to the first mode, $i = 1$.

If $P_{fb} = 0.9 P_{\phi e}$

Q_{id} : Use Eq. 41 above, but constants K_1 , K_2 and K_3 correspond to the first mode.

C_1 : Eq. 42 above

D : Eq. 43 above

If $m \neq 0$, $\theta_{max} = D, 0.866 D$, or D (44)
for columns with 1, 2 or 3 intermediate girts, respectively.

If $P_{fb} = P_{crx, L}$

Q_{id} : Eq. 41, same as for $P_{fb} = P_{crx, L}$

$$C_1 = \frac{2P_{fb} E_o}{K_1 P^* - P_{fb} + K_2 Q} \quad (45)$$

where K_1 , K_2 and K_3 correspond to the first mode.

TABLE 5 (cont'd)

For all failure modes

$$\gamma_{\max} = K_4 \frac{c_1}{l} \quad (46)$$

where K_4 is from Table 6.

TABLE 6 - CONSTANTS $K_1, K_2, K_3, K_4^{(1,2)}$

| | K_1 | K_2 | K_3 | K_4 |
|-----------------------------|--------|-------|-------|-------|
| <u>1 Intermediate Girt</u> | | | | |
| Mode: $i = 1$ | 0.250 | 0.810 | 0.405 | 1.000 |
| <u>2 Intermediate Girts</u> | | | | |
| Mode: $i = 1$ | 0.111 | 0.912 | 0.912 | 0.866 |
| $i = 2$ | 0.444 | 0.684 | 0.228 | 0.866 |
| <u>3 Intermediate Girts</u> | | | | |
| Mode: $i = 1$ | 0.0625 | 0.950 | 1.621 | 0.707 |
| $i = 2$ | 0.250 | 0.810 | 0.405 | 0.707 |
| $i = 3$ | 0.562 | 5.53 | 0.180 | 0.707 |

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APPENDIX II - NOMENCLATURE

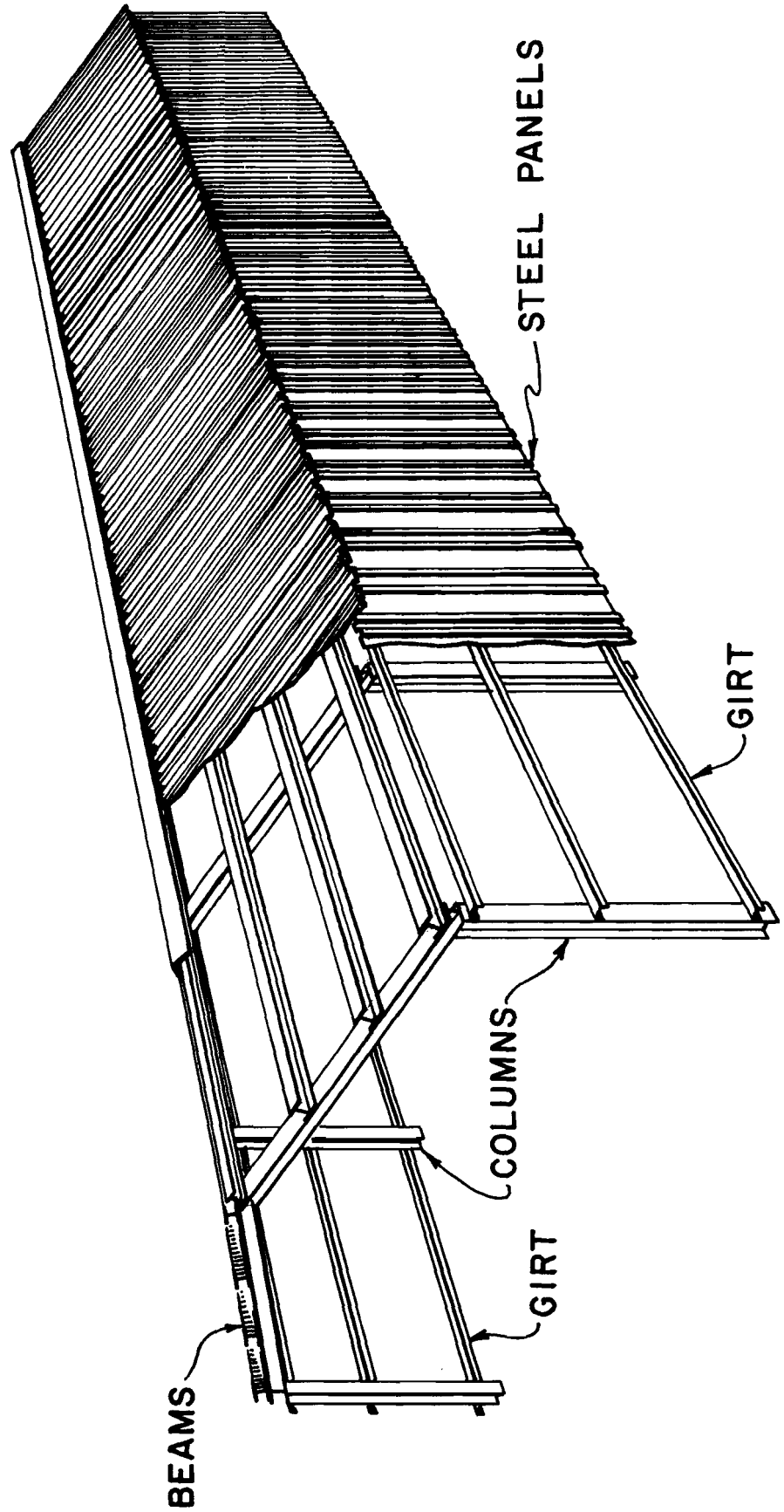
- A ----- cross sectional area, in.²
- a ----- dimension of shear diaphragm perpendicular to test load direction, in.
- a* ----- = $K_1 E^* C_w \left(\frac{\pi}{l}\right)^2 + G^* J + K_3 m l$, kip-in.²
- b ----- dimension of shear diaphragm parallel to test load direction, in.
- C ----- amplitude of additional lateral deflection of centroidal axis, in.
- C₁ ----- amplitude of additional lateral deflection in the plane of the diaphragm, in.
- C_w ----- warping constant of a section, in.⁶
- D ----- amplitude of additional twist of a member, radians
- E ----- modulus of elasticity, ksi
- E₀ ----- amplitude of initial lateral deflection of the centroidal axis of a member, in.
- E* ----- inelastic modulus, ksi
- e ----- distance between center of gravity of a member and the plane of the diaphragm, in.
- F₀ ----- amplitude of initial twist of a member, radians
- G ----- shear modulus, ksi
- G* ----- inelastic shear modulus, ksi
- G_d['] ----- shear stiffness at 0.8 of ultimate load of diaphragm, kips/in.
- G_{dr}['] ----- design value of shear stiffness, kips/in.
- I_p ----- polar moment of inertia, in.⁴
- I_x, I_y ----- moments of inertia of a section about X- and Y-axes, respectively, in.⁴
- I_g ----- moment of inertia of a girt about the bending axis, in.⁴
- i ----- mode number

- j ----- number of intermediate girts
 J ----- torsional constant of a section, in.⁴
 K_1, K_2, K_3, K_4 --- constants
 k ----- effective length factor
 L ----- length of member, in.
 ℓ ----- spacing of girts, in.
 M_{fb} ----- moment capacity of a "fully" braced beam, kip-in.
 M_{cr} ----- lateral torsional-flexural buckling moment of a diaphragm-braced beam, kip-in.
 M_p ----- plastic moment of a beam, kip-in.
 M_y ----- yield moment of a beam, kip-in.
 M_{yg} ----- yield moment of a girt, kip-in.
 $M_{\phi e}$ ----- buckling moment of a beam with the centroidal axis of the tension flange as a fixed axis of rotation, kip-in.
 m ----- elastic restraining moment on the column at a girt, kip-in. per radian
 P_{fb} ----- load capacity of a "fully" braced column, kips
 P_{ult} ----- ultimate shear load of a diaphragm from a test, kips
 $P_{crx,L}, P_{cry,L}$ - strong axis and weak axis buckling loads, respectively, of a column of length L , kips
 $P_{cry,\ell}$ ----- weak axis buckling load of a column of length ℓ , kips
 $P_{\phi e}$ ----- buckling load of a column with the centroidal axis of one of its flanges as the fixed axis of rotation, kips
 P_s ----- safe load on a member, kips
 P_y ----- $\sigma_y A$, kips
 P^* ----- $\frac{\pi^2 E^* I_y}{\ell^2}$, kips
 Q_{dr} ----- design value of shear rigidity, kips per radian
 Q ----- shear rigidity of diaphragm, kips/radian, or kips

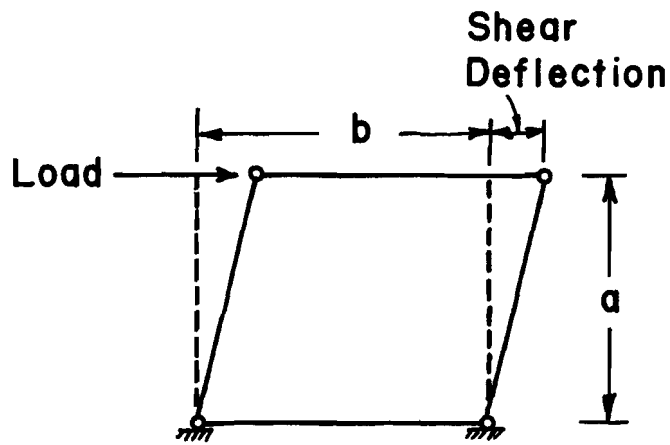
- Q_{id} ----- shear rigidity required for the "full" bracing of an ideal member, kips/radian
- r_x, r_y ----- radii of gyration of the section about X- and Y-axes, respectively, in.
- u, u_1 ----- additional deflections in the directions of X and X_1 axes, respectively, in.
- v ----- additional deflection in the direction of Y-axis, in.
- w ----- width of diaphragm contributing to the support of one member, in.
- X, X_1, Y ----- coordinate axes
- β ----- twist of the member, radians
- γ_{dr} ----- design value of diaphragm shear strain, radians
- γ_{max} ----- maximum shear strain in the diaphragm, radians
- Δ_d ----- shear deflection of a diaphragm at $0.8 P_{ult}$, radians
- θ_d ----- $= \frac{M_{yg} w}{6EI_{xg}}$ = bending slope of a girt at yield moment, radians
- θ_{max} ----- computed maximum bending slope of a girt, radians
- σ ----- average axial stress in a column, ksi
- σ_y ----- yield stress, ksi
- σ_p ----- proportional limit stress, ksi

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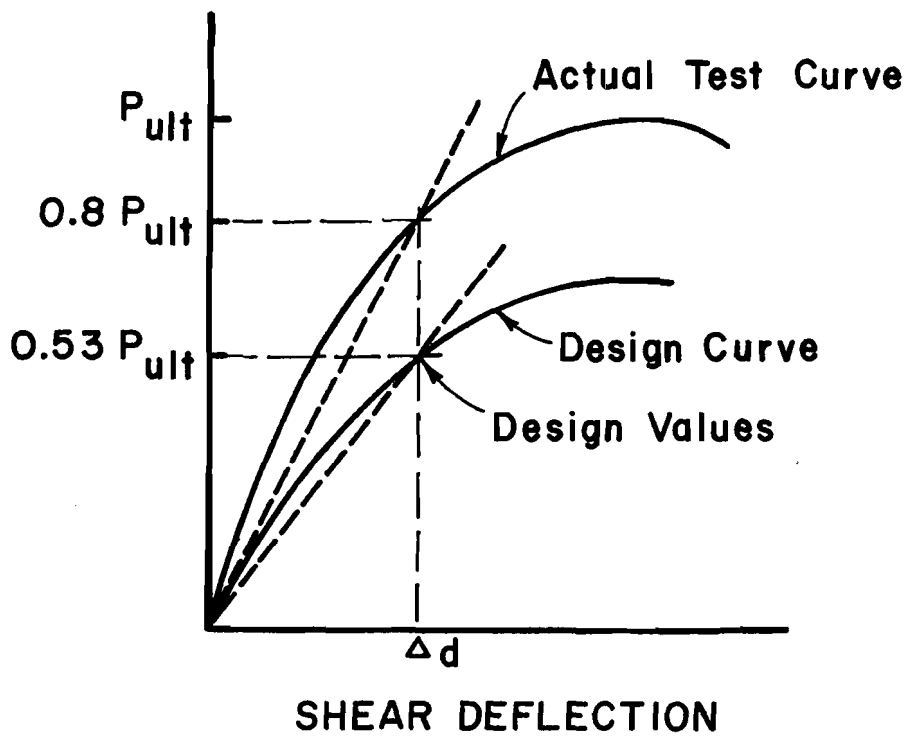
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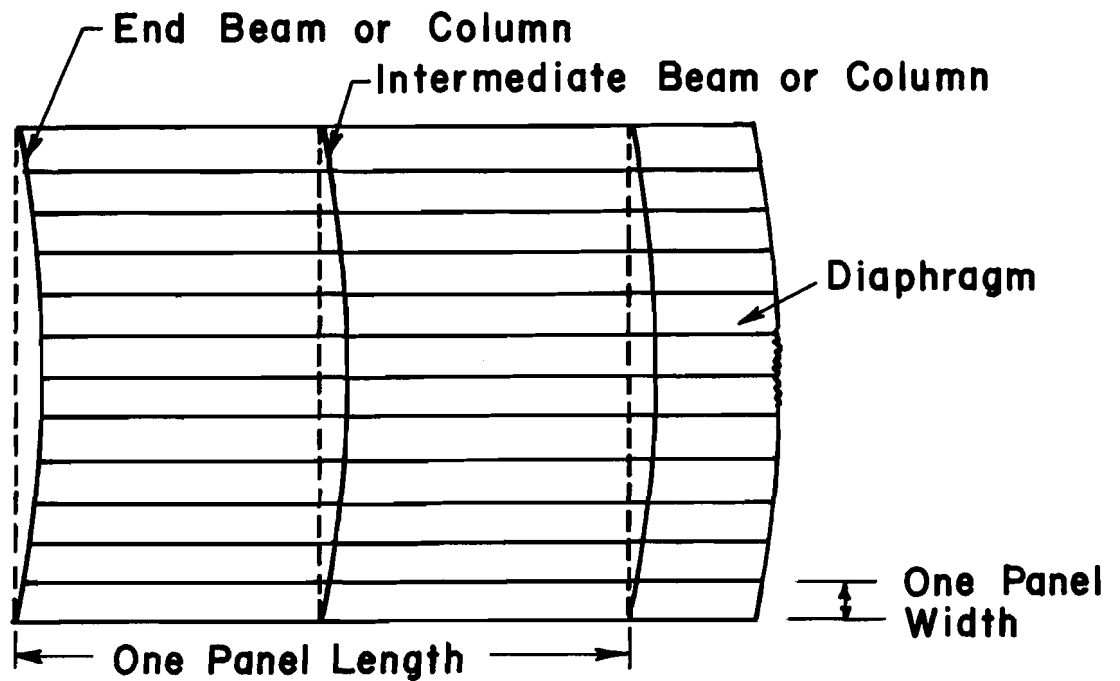
METAL BUILDING



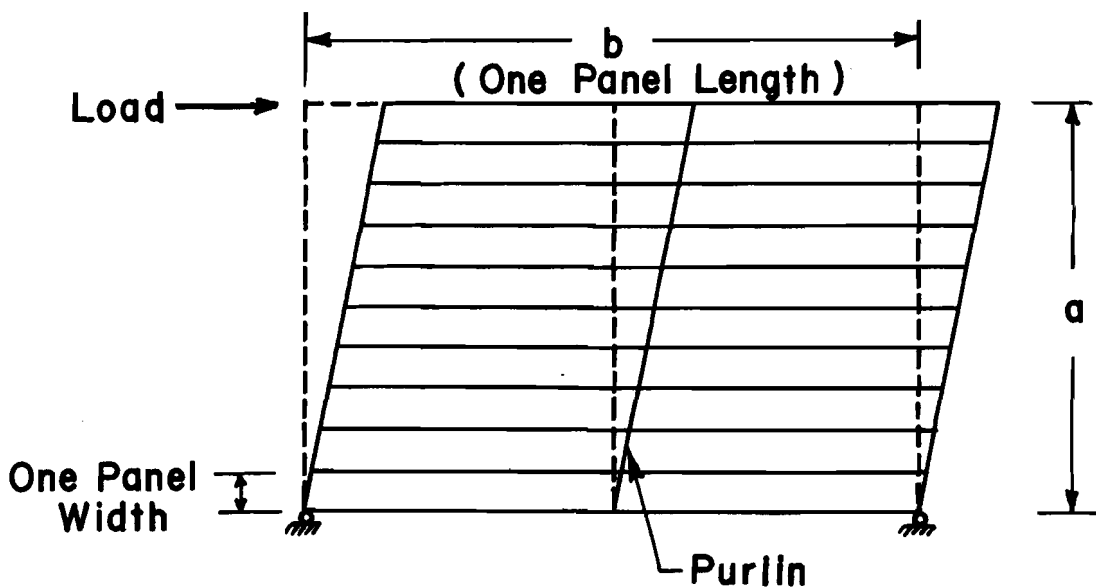
**CANTILEVER SHEAR DIAPHRAGM
TEST ARRANGEMENT**



**FIG. 2 LOAD-DEFLECTION RELATIONSHIP OF A SHEAR
DIAPHRAGM OBTAINED FROM A CANTILEVER TEST**



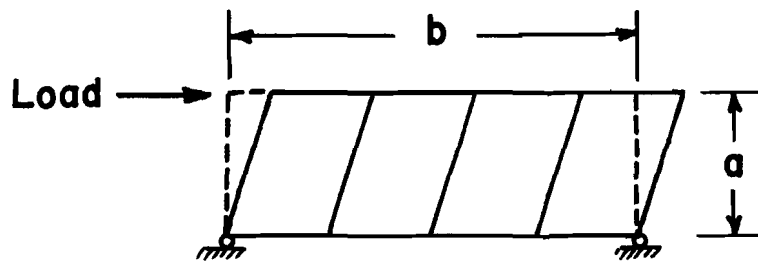
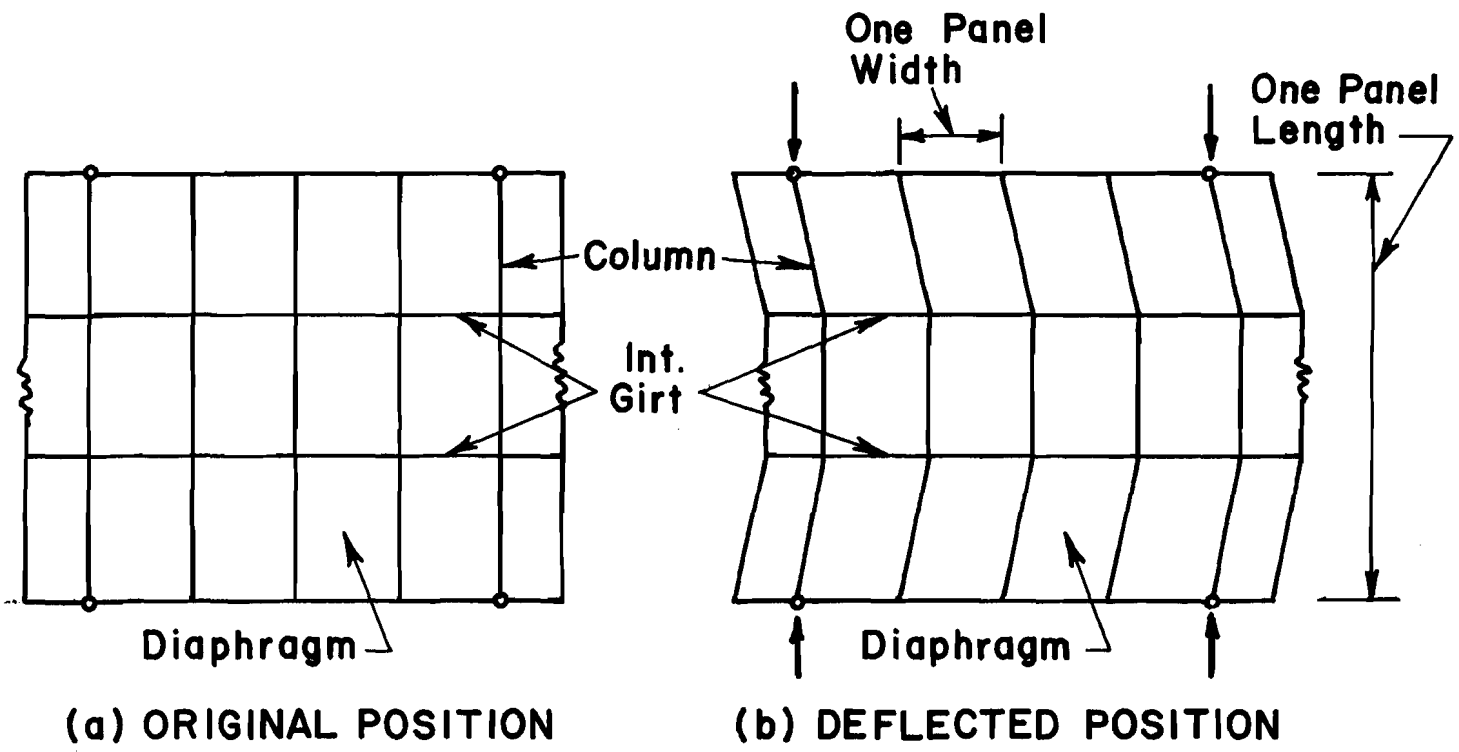
(a) PLAN VIEW FOR BEAMS; ELEVATION FOR COLUMNS



(b) CANTILEVER SHEAR DIAPHRAGM TEST ARRANGEMENT

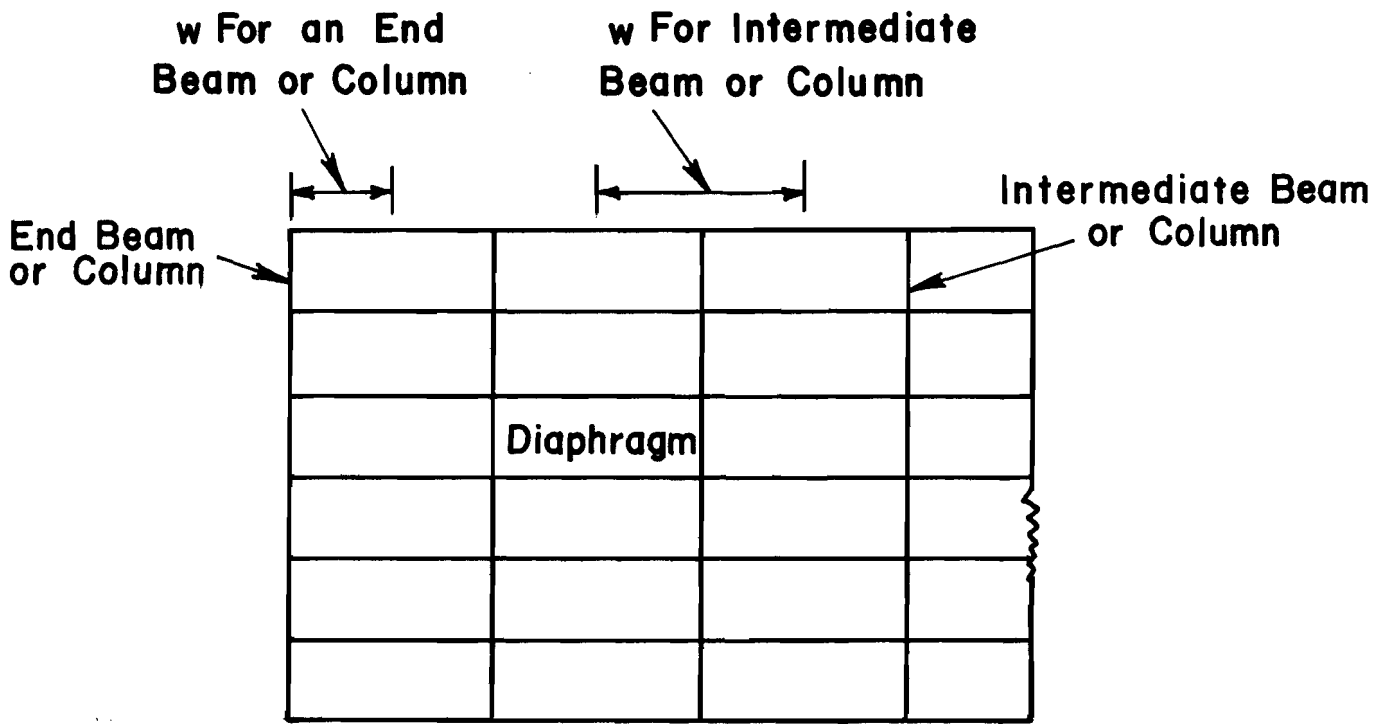
----- Original Position ——— Deflected Position

FIG. 3 DEFLECTED POSITION OF A DIAPHRAGM IN A DIAPHRAGM-BRACED BEAM OR COLUMN ASSEMBLY, AND IN A SHEAR DIAPHRAGM TEST



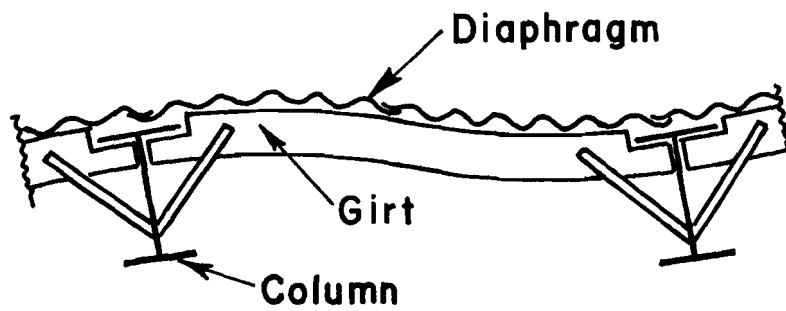
(c) CANTILEVER SHEAR DIAPHRAGM TEST ARRANGEMENT

FIG. 4 ORIGINAL AND DEFLECTED POSITIONS OF A DIAPHRAGM IN COLUMN-GIRT-DIAPHRAGM ASSEMBLY

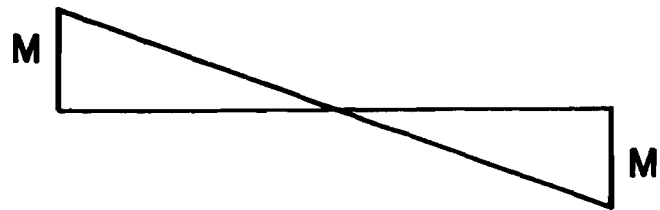


PLAN VIEW FOR BEAMS; ELEVATION FOR COLUMNS

FIG. 5 DIAPHRAGM DIMENSION w FOR AN END MEMBER OR AN INTERMEDIATE MEMBER

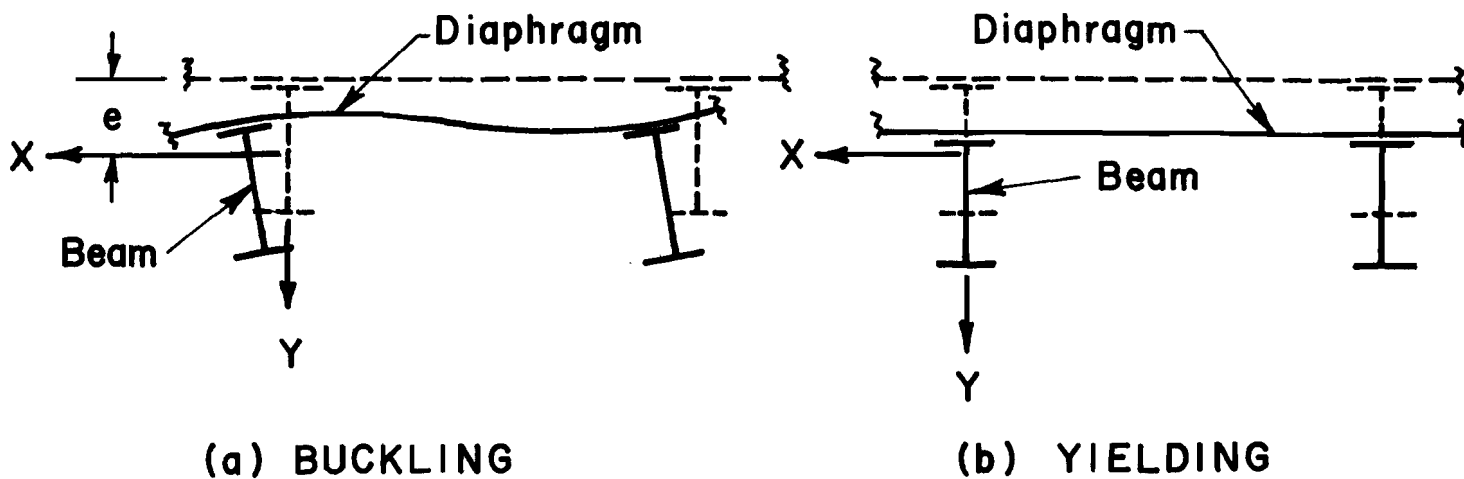


(a) DEFLECTED POSITION



(b) MOMENT DIAGRAM FOR THE GIRT

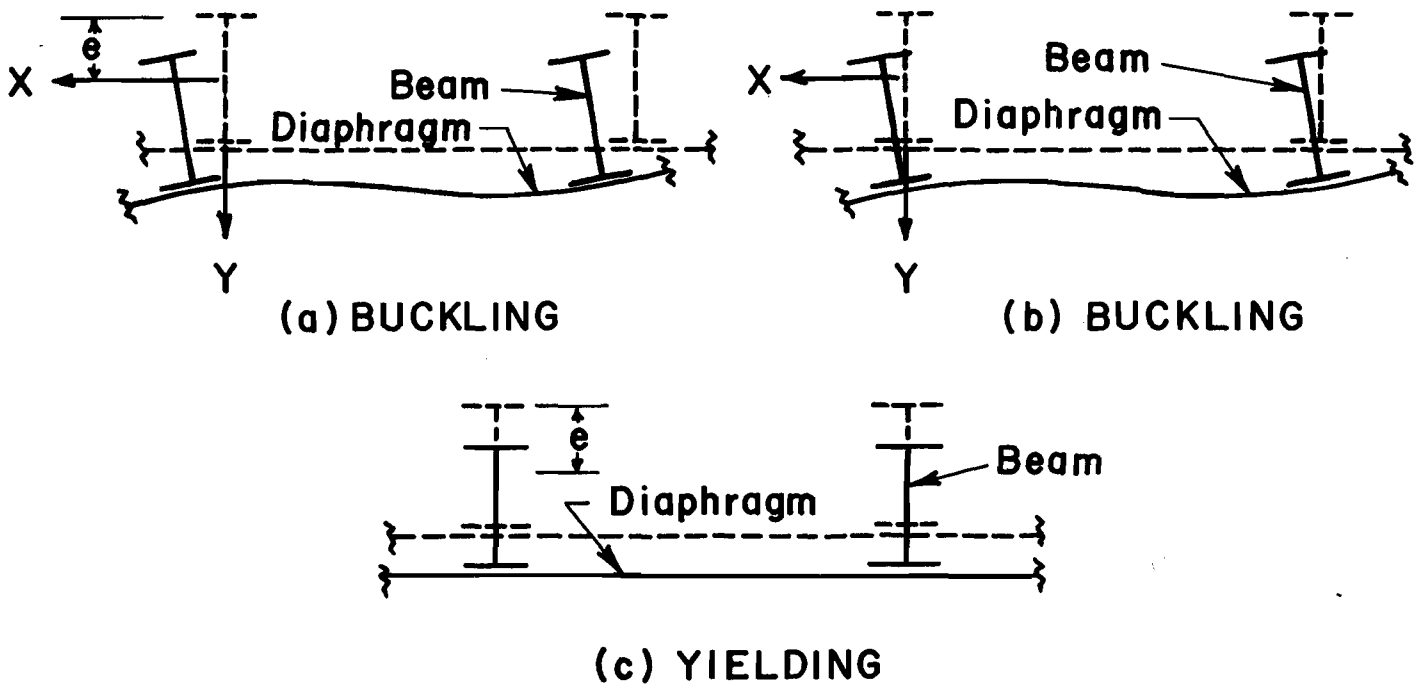
FIG. 6 DEFLECTED POSITION OF A COLUMN-GIRT-DIAPHRAGM ASSEMBLY AND MOMENT DIAGRAM FOR THE GIRT



----- Original Position

—— Deflected Position

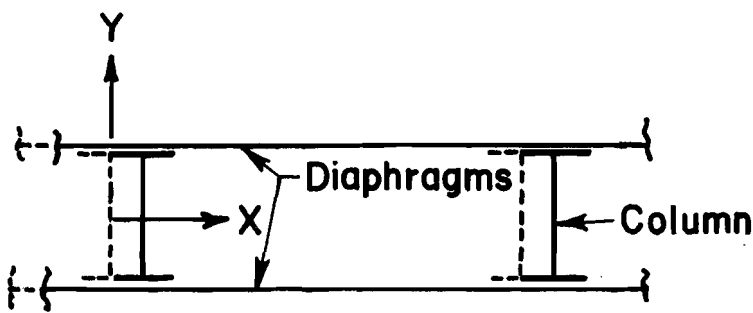
FIG. 7 MODES OF FAILURE OF BEAMS BRACED WITH A DIAPHRAGM ON THE COMPRESSION FLANGES



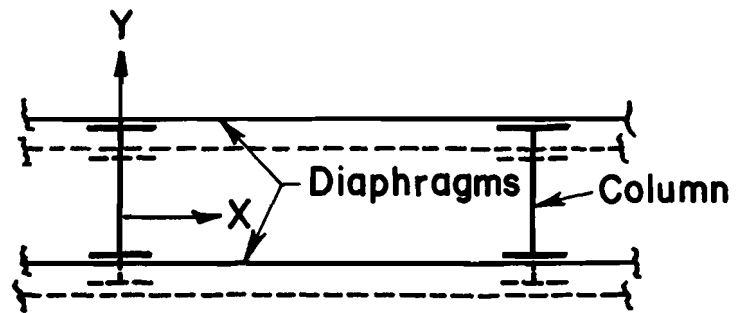
----- Original Position

———— Deflected Position

FIG. 8 MODES OF FAILURE OF BEAMS BRACED WITH A DIAPHRAGM ON THE TENSION FLANGES



(a) WEAK-AXIS BUCKLING

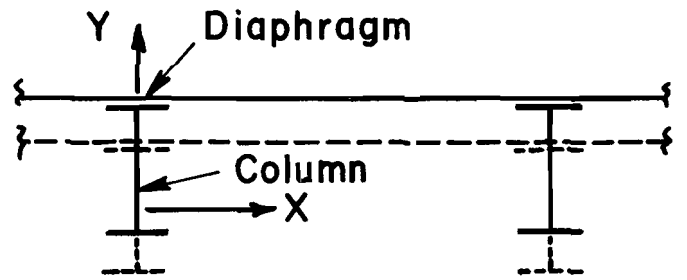
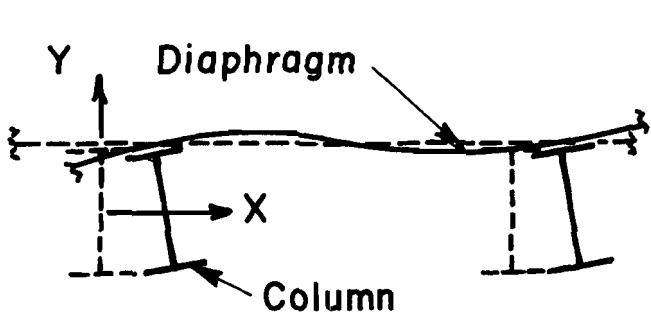


(b) STRONG-AXIS BUCKLING

----- Original Position

———— Deflected Position

FIG. 9 BUCKLING MODES OF COLUMNS WITH DIAPHRAGM BRACING ON BOTH FLANGES



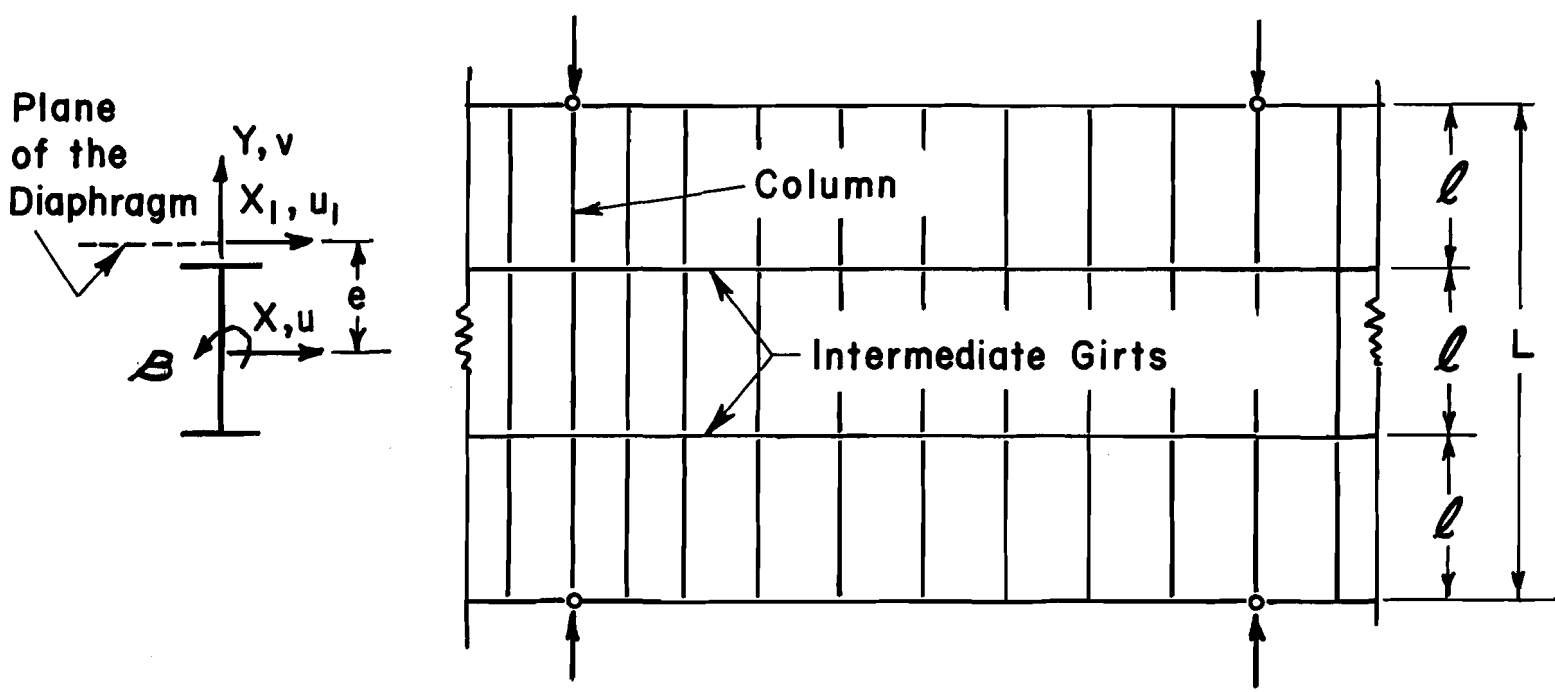
(a) TORSIONAL FLEXURAL BUCKLING

(b) FLEXURAL BUCKLING

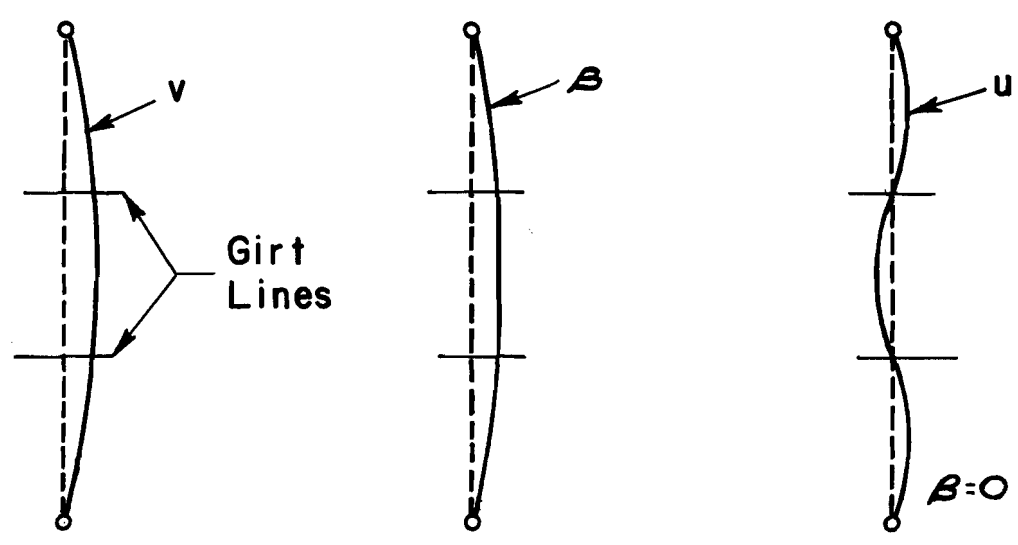
----- Original Position

———— Deflected Position

FIG. 10 BUCKLING MODES OF COLUMNS WITH DIAPHRAGM BRACING ON ONE FLANGE ONLY



(a) COLUMN - GIRT - DIAPHRAGM ASSEMBLY



(b) FLEXURAL BUCKLING ABOUT X-AXIS

(c) TORSIONAL FLEXURAL BUCKLING

(d) FLEXURAL BUCKLING ABOUT WEAK AXIS

FIG. 11 BUCKLING MODES OF A "FULLY-BRACED" COLUMN WITH DIAPHRAGM-GIRT BRACING

DESIGN OF I-SHAPED BEAMS WITH DIAPHRAGM BRACING

by

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and

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INTRODUCTION

Cold formed steel panels often are used as wall sheathing, roof decking or floor decking in steel framed buildings. These panels carry loads normal to their plane by virtue of their bending strength. In addition, diaphragms formed by adequately interconnecting these panels can resist in-plane shear deformations. Because of this shear resistance, such diaphragms are used as wind bracing for low rise buildings, as shear elements in folded plate and hyperbolic paraboloid construction, and as load distributing elements in portal frame buildings.^(4,6,8,17,19) Another use of this diaphragm action is as bracing against buckling for individual members of a steel frame, which is the subject of this report. Design recommendations are presented herein for diaphragm bracing of I-shaped beams to prevent buckling. The diaphragms are present in any event as part of the roof or floor, and therefore are available at no extra cost. If properly utilized, they can eliminate the need for other types of bracing and thus contribute to economical design.

Extensive research has been conducted at Cornell University and elsewhere to determine the increased load-carrying capacity of beams and columns due to diaphragm bracing.^(1,3,7,9,10,14,15,21) This report gives the basis for the proposed design procedure, describes the specific steps and presents a design example.

Usually, it is not economical to provide anything less than "full" bracing for a member, where this is defined as bracing such that any increase in rigidity or strength of the diaphragm will cause no significant increase in the load-carrying capacity of the braced

members.⁽¹⁸⁾ For this reason, and for greater simplicity, this design procedure is limited to "fully braced" beams. The procedure is based on analyses of I-section beams under uniform moment, and has been substantiated by tests of diaphragm-braced assemblies as reported in References 1 and 9.

Information regarding the load-carrying capacity of I-beams with less than "full" bracing can be obtained from Reference 1, which also discusses the capacity of channel and Z-section beams subjected to uniform moment. Cantilever beams and channel and Z-section beams subjected to loads in the plane of the web are discussed in Reference 7.

CRITERIA FOR DESIGN

Design criteria are established herein for I-section beams continuously braced by a shear diaphragm on the compression or tension flanges, where "continuous bracing" indicates that the diaphragm is connected directly to the member at short intervals. If panels with longitudinal ribs are used, the ribs must be perpendicular to the member they are bracing. The bracing requirements are not a linear function of the applied load; therefore, the design procedure is based on the ultimate load capacity of the beams, utilizing a conservative estimate of the strength and rigidity of the diaphragm. This is in contrast to most design procedures for other uses of diaphragms, which are usually formulated in terms of allowable load.

Beams with continuous bracing tend to twist as well as deflect laterally under load. The diaphragm can provide restraint against both of these motions, and due to these restraints the torsional flexural buckling moment of a beam can be considerably increased. As a conservative simplification, the bending rigidity of the diaphragm, which tends to prevent rotation of the member to which it is attached, is neglected in this design procedure. Thus, only the shear characteristics of the diaphragm are considered.

Shear Characteristics of a Diaphragm

Considerable progress has been made recently in developing methods to predict the two important parameters which characterize a diaphragm assembly: the shear stiffness (or conversely, flexibility)

and shear strength. (4,5,6,8,11,13,20) Tabulated values for specific assemblies are given in some of these references, and in proprietary literature of panel and fastener manufacturers. As an alternative, these characteristics can be determined from the load-deflection curve obtained from a simple beam or cantilever shear test (Fig. 1) as described in Reference 8. The resulting load-deflection relationship is generally not linear; a typical test curve is shown in the figure. Furthermore, in such tests, two nominally identical diaphragms may give considerably different load-deflection relationships at higher loads, say beyond 80% of ultimate load. The shear stiffness at 80% of ultimate shear load is denoted as G_d^i . To insure that diaphragm failure will not precede beam failure, it is proposed to use only 2/3 of G_d^i as a reliable design value of shear stiffness, G_{dr}^i . Thus

$$G_{dr}^i = 2/3 G_d^i = 2/3 \times \frac{0.8 P_{ult}/b}{\Delta_d/a} = \frac{0.53 P_{ult}/b}{\Delta_d/a} \quad (1)$$

where P_{ult} is the ultimate shear load in the diaphragm test, kips

Δ_d is the deflection at $0.8 P_{ult}$, in.

a is the dimension of the shear diaphragm perpendicular to the test load direction, in., and

b is the dimension of the shear diaphragm parallel to the test load direction, in.

Eq. 1 indicates that G_{dr}^i is in units of force per unit length.

If the shear stiffness of a diaphragm is known, then the maximum shear strain that can be sustained by a diaphragm is a measure of its shear strength; that is, the shear strength is the product of the shear stiffness and shear strain. The reliable design value of shear strain, γ_{dr} , to be used as a measure of shear strength is taken here equal to the

shear strain at $0.8 P_{ult}$, γ_d :

$$\gamma_{dr} = \gamma_d = \frac{\Delta_d}{a} \quad (2)$$

A graphical representation of proposed design values of shear stiffness and shear deflection are shown in Fig. 1.

The type and spacing of fasteners is very important, and those used in a diaphragm test should be the same as those used in connecting the diaphragms to the beams in the actual structure. The deflected position of a beam continuously braced by a diaphragm is shown in Fig. 2a, where the full length of each panel is under uniform shear. The length of panel to be used in a cantilever diaphragm test (or in any analytical procedure) is the same as that used in the structure, as shown in Fig. 2b. Purlin spacing in the test should be the same as the spacing of the beams to be braced. A simple beam shear test may be conducted instead of a cantilever test, making proper choice of the panel length and spacing of the framing members. (8)

When a diaphragm-braced beam in a structure is to be analyzed, it is more convenient in the computations to use a reliable shear rigidity Q_{dr} of the entire portion of diaphragm contributing to the support of the member, rather than the unit shear stiffness G'_{dr} . Using Eq. 1, the reliable shear rigidity Q_{dr} is expressed as

$$Q_{dr} = G'_{dr} w = \frac{2}{3} G'_d w = \frac{0.53 P_{ult} w/b}{\Delta_d/a} \quad (3)$$

where w is the dimension of the diaphragm, perpendicular to the longitudinal axis of the member, which contributes to the support of the member

being analyzed. For example, in the case of floor beams braced by a diaphragm (Fig. 3), the end beams can be assumed to be supported by the diaphragm of dimension w equal to half the spacing of beams, and the intermediate beams are supported by the dimension of the diaphragm w equal to the full spacing of the beams. It can be observed from Eq. 3 that Q_{dr} is in units of force per unit shear strain (force/radian).

Initial Imperfections and Additional Deflections

The required strength of any type of bracing is a function of the initial imperfections of the load-carrying member. The pattern of initial deflection along the length of an imperfect beam is here assumed similar to the buckling pattern to obtain a conservative estimate of deflections under load. For example, in the case of a continuously braced beam, with ends simply supported laterally or hinged, the buckling pattern is a half sine wave, and the initial deflection pattern is therefore also assumed as a half sine wave. The amplitude of the initial lateral deflection of the centroidal axis of a beam, E_o , is taken as the tolerance limit in sweep specified in the AISC Manual (Ref. 12, p 1-127). Hence,

$$E_o = \frac{1''}{8} \times \frac{\text{length of member in feet}}{\text{either 5 or 10, as specified}} \quad (4)$$

Based on limited available information, the amplitude of the initial twist, F_o , is arbitrarily taken equal to 0.000667 radian per foot of length; that is,

$$F_o = 0.000667 \text{ rad./ft} \times \text{length of member in feet} \quad (5)$$

Because of the initial lateral deflection or twist of a beam, additional lateral deflections and twist occur under applied load. The pattern of additional deflections along the length of a member is the same as the buckling pattern. These additional deflections cause shear forces in the diaphragm bracing. Magnitudes of the additional deflections and maximum shear strain in the diaphragm can be calculated using the design formulae given subsequently.

Factors of Safety

The factors of safety used for rolled steel beams and cold formed steel beams in this design procedure are the same as those used in the AISC Specification⁽¹²⁾ and AISI Specification,⁽¹⁶⁾ respectively.

GENERAL DESIGN PROCEDURE

Briefly stated, the design procedure is as follows: First, assume the member to be fully braced, and select a section of required load capacity. Then, select a diaphragm of specific panel dimensions, fastener type and fastener spacing, and check to see that its rigidity and strength are adequate to provide full bracing for the member. (Alternatively, one could start with a given member and diaphragm bracing system, and use an analysis procedure based on the equations herein to calculate the ultimate load capacity and allowable load.)

The detailed design procedure is outlined below:

1. Choose a trial member for the design.
2. Assume that the member is fully braced, and compute its moment capacity, M_{fb} .
3. Compute the safe moment, M_s :

$$M_s = \frac{\text{moment capacity of fully braced member}}{\text{factor of safety or load factor}} = \frac{M_{fb}}{F.S.} \quad (6)$$

If M_s is greater than and close to the required design load, proceed to check whether the bracing is adequate; otherwise, repeat the procedure from Step 1.

4. Compute Q_{id} , the shear rigidity required to fully brace an ideal member. The rigidity required to brace a real member always will be greater than this. (18)
5. Select a trial diaphragm. If $Q > Q_{id}$, the diaphragm rigidity may be adequate for full bracing; proceed with Step 6. If

$Q \leq Q_{id}$, full bracing cannot be achieved with this diaphragm; a more rigid diaphragm must be chosen, and this step repeated.

6. Compute the maximum shear strain, γ_{max} , in the diaphragm. If the reliable value of shear strain $\gamma_{dr} \geq \gamma_{max}$, the diaphragm strength is adequate for full bracing. If $\gamma_{dr} < \gamma_{max}$, a stronger diaphragm is necessary for full bracing; repeat the procedure from Step 5.

M_{fb} , γ_{max} , and Q_{id} are computed from equations given herein; whereas Q_{dr} and γ_{dr} can be obtained from the load-deflection relationship of a shear diaphragm test or analysis. The yield moment of a beam is

$$M_y = S_x F_y \quad (7)$$

I-SECTION BEAMS BRACED BY A SHEAR DIAPHRAGM
ON THE COMPRESSION FLANGES^(1,9)

Figs. 4a and 4b show the possible modes of failure of beams with diaphragm bracing on the compression flanges. In Fig. 4a the diaphragm rigidity and strength are not adequate to prevent lateral buckling of the beams. In Fig. 4b the diaphragm is adequate, and the beams fail by yielding. Full bracing in this case is defined as that which has adequate rigidity and strength to prevent lateral buckling until the beam yields. Thus, M_y , the yield moment, is taken as the moment capacity of a fully braced beam, M_{fb} :

$$M_{fb} = M_y \quad (8)$$

The torsional flexural buckling moment, M_{cr} , of an ideal beam with diaphragm bracing on the compression flange is⁽⁹⁾

$$M_{cr} = \sqrt{[EI_y \left(\frac{n\pi}{L}\right)^2 + Q][EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2]} + Qe \quad (9)$$

where EI_y is the weak axis bending rigidity

EC_w is the warping rigidity

GJ is the torsional rigidity

e is the distance between the center of gravity of the beam and the plane of the diaphragm

and $n = 1$ or 2 for ends simply supported or fixed, respectively, against lateral bending

The cross bending rigidity of the diaphragm is neglected in Eq. 9 and in all subsequent expressions.

The shear rigidity, Q_{id} , required for an ideal beam to attain

the "fully" braced moment $M_{fb} = M_y$ can be obtained by substituting M_y for M_{cr} in Eq. 9 and solving for Q , resulting in Eq. 10 shown in Table 1. A simplified and conservative expression for M_{cr} is given by

$$M_{cr} = 2 Qe \quad (11)$$

Then, with $M_{cr} = M_y$, a simple and conservative estimate for Q_{id} is obtained, see Eq. 12, Table 1. Conservative estimates of the amplitudes of additional lateral deflection of the centroidal axis (C) and twist (D) at moment M_y are given by Eqs. 13 and 14 in Table 1. The amplitude of the additional lateral deflection in the plane of the diaphragm (C_1) and the maximum shear strain in the diaphragm (γ_{max}) also are given in Table 1 as Equations 15 and 16, respectively. These expressions are used in Steps 1 through 6 of the design procedure to design a diaphragm-braced I-beam in the example given later.

I-SECTION BEAMS BRACED BY A SHEAR DIAPHRAGM
ON THE TENSION FLANGES (1,9)

The possible failure modes for beams braced by diaphragms on the tension flanges are indicated in Fig. 5. The figure shows (a) vertical deflection, lateral deflection and twist of the cross section; (b) vertical deflection and twist; and (c) vertical deflection only. The buckling moment of the beam reaches M_{ϕ_e} (buckling moment of the beam with the centroidal axis of the tension flange as the fixed axis of rotation) asymptotically as the shear rigidity approaches infinity, as shown in Fig. 6. In general, even for a very small increase in the moment capacity beyond $0.9 M_{\phi_e}$, a very large increase in shear rigidity is needed. Therefore, if a beam with diaphragm bracing on the tension flange buckles in a torsional-flexural mode (Figs. 5a and 5b) the buckling moment of the "fully" braced beam is arbitrarily taken as $0.9 M_{\phi_e}$. (Any other percentage could be used in similar fashion.) A fully braced beam may also fail by yielding (Fig. 5c). Hence, the moment capacity of a beam "fully" braced on its tension flanges is the smaller of these two values as indicated by Eq. 17, Table 2. Other expressions required in the design procedure also are given in Table 2. Bracing on the tension flange is, of course, less efficient than compression flange bracing. However, this situation may occur under wind uplift loadings or other design conditions.

DESIGN EXAMPLE

Beams Braced by a Diaphragm on Their Compression Flanges

Design an intermediate floor beam to span 20 ft and to carry a uniform live load and superimposed dead load of 550 lb/ft. The beams are 6 ft apart and are braced on their compression flanges by a deck whose shear characteristics determined from tests are: $G'_d = 4.235$ kips/in. and $\gamma_d = 0.0045$ rad. Ends of the beams are considered simply supported laterally. Use ASTM A36 steel.

Solution: Assume that the beam is fully braced and that its dead load will be about 20 lb/ft.

$$M = \frac{wL^2}{8} = \frac{(550 + 20)(20)^2(12)}{8} = 342 \text{ kip-in.}$$

For a factor of safety of 1.67,

$$M_{\text{req'd}} = 1.67 \times 342 = 571 \text{ kip-in.}$$

$$\text{Req'd } S_x = M_{\text{req'd}} \div F_y = 571 \div 36 = 15.9 \text{ in.}^3$$

$$\text{Choose W10} \times 17, \quad S_x = 16.2 \text{ in.}^3, \quad d = 10.12 \text{ in.}$$

$$\text{Fully braced moment } M_{fb} = M_y = S_x F_y = 16.2 \times 36.0 = 583 \text{ kip-in.}$$

$$> M_{\text{req'd}}; \quad \therefore \text{OK}$$

Proceed to check whether the bracing is adequate.

Diaphragm Rigidity:

$$\text{Eq. 12: } Q_{id} = \frac{M_y}{2e} = \frac{583}{2(10.12 \div 2)} = 57.6 \text{ kips}$$

$$\begin{aligned} \text{Eq. 3: } Q_{dr} &= G'_{dr} w = 2/3 G'_d w \\ &= 2/3 (4.235)(72) = 203.3 \text{ kips} > 57.6 \text{ kips} \end{aligned}$$

\therefore diaphragm rigidity may be adequate for full bracing.

Diaphragm Strength:

Eq. 4: Assumed initial deflection, $E_o = 1/8 \times 20/5 = 0.5''$

Eq. 5: Assumed initial twist, $F_o = 0.000667 \times 20$
 $= 0.01334 \text{ rad.}$

$$e = d/2 = 5.06''$$

Eq. 13: Additional deflection, $C = -0.1038''$

Eq. 14: Additional twist, $D = 0.0648 \text{ rad.}$

Eq. 15: Deflection of braced flange, $C_1 = 0.2242''$

Eq. 16: Maximum shear strain, $\gamma_{\max} = 0.00294 \text{ rad.}$

$$< \gamma_{dr} = \gamma_d = 0.0045 \text{ rad.}$$

∴ diaphragm strength is adequate for "full" bracing. The W10 x 17 beam is fully braced and can safely carry a uniform load of 550 lb/ft.

SUMMARY AND CONCLUSIONS

Cold formed steel panels often are used as wall sheathing, roof decking or floor decking in steel-framed buildings. Diaphragms formed by adequately interconnecting these panels can have considerable in-plane shear resistance. This shear resistance has a number of structural uses; for one, it can act as bracing against buckling for the individual members of a steel frame to which the diaphragm is attached. A procedure is presented for the design of I-section beams with diaphragm bracing on either the compression or tension flanges. The procedure is based on the ultimate load capacity of a fully braced beam, utilizing a conservative estimate of the shear strength and shear rigidity of the diaphragm. The basis for the proposed procedure, the specific design steps, and a design example are presented. The utilization of existing floor or roof diaphragms as bracing for individual beams can eliminate the need for other types of bracing and/or reduce required member sizes. Thus it contributes to economical design.

ACKNOWLEDGMENTS

This paper is a result of an investigation sponsored at Cornell University by the American Institute of Steel Construction and the American Iron and Steel Institute, and is based on an earlier Cornell report.⁽²⁾ The cooperation of each of the cognizant research committees is appreciated. The research initially was under the general direction of G. P. Fisher, F. ASCE, and later George Winter, Hon. M. ASCE; their contributions are gratefully acknowledged.

TABLE 1 - EQUATIONS FOR BEAMS WITH DIAPHRAGM
BRACING ON THE COMPRESSION FLANGES (1,9)

$$M_{fb} = M_y \quad (8)$$

$$Q_{id} = \frac{M_y^2 - EI_y \left(\frac{n\pi}{L}\right)^2 [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ]}{e^2 EI_y \left(\frac{n\pi}{L}\right)^2 + 2M_y e + EC_w \left(\frac{n\pi}{L}\right)^2 + GJ} \quad (10)$$

or, conservatively,

$$Q_{id} = M_y / 2e \quad (12)$$

$$C = \frac{F_o Q e^2 + E_o (M_y - Qe)}{(2Qe - M_y)} \quad (13)$$

$$D = \frac{E_o Q + F_o (M_y - Qe)}{(2Qe - M_y)} \quad (14)$$

where E_o and F_o are obtained from Eqs. 4 and 5, respectively.

$$C_1 = C + eD \quad (15)$$

$$Y_{max} = C_1 \frac{\pi}{L} \quad (16)$$

TABLE 2 - EQUATIONS FOR BEAMS WITH DIAPHRAGM
BRACING ON THE TENSION FLANGES (1,9)

$$M_{fb} = \text{Min.} (M_y, 0.9 M_{\phi e}) \quad (17)$$

$$M_{\phi e} = \frac{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2}{2e} \quad (18)$$

$$Q_{id} = \frac{M_{fb}^2 - EI_y \left(\frac{n\pi}{L}\right)^2 [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ]}{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2 - 2e M_{fb}} \quad (19)$$

$$C = \frac{M_{fb} F_o [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2] + M_{fb} E_o (M_{fb} + Qe)}{[EI_y \left(\frac{n\pi}{L}\right)^2 + Q] [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2] - (M_{fb} + Qe)^2} \quad (20)$$

$$D = \frac{M_{fb} E_o [EI_y \left(\frac{n\pi}{L}\right)^2 + Q] + M_{fb} F_o (M_{fb} + Qe)}{[EI_y \left(\frac{n\pi}{L}\right)^2 + Q] [EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + Qe^2] - (M_{fb} + Qe)^2} \quad (21)$$

where E_o and F_o are obtained from Eqs. 4 and 5, respectively.

$$C_1 = C - eD \quad (22)$$

$$Y_{\max} = C_1 \frac{\pi}{L} \quad (16)$$

APPENDIX I - REFERENCES

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APPENDIX II - NOMENCLATURE

| | |
|--------------|---|
| a | dimension of shear diaphragm perpendicular to test load direction, in. |
| b | dimension of shear diaphragm parallel to test load direction, in. |
| C | amplitude of additional lateral deflection of centroidal axis, in. |
| C_1 | amplitude of additional lateral deflection in the plane of the diaphragm, in. |
| C_w | warping constant of a section, in. ⁶ |
| D | amplitude of additional twist of a member, radians |
| E | modulus of elasticity, ksi |
| E_o | amplitude of initial lateral deflection of the centroidal axis of a member, in. |
| e | distance between center of gravity of a member and the plane of the diaphragm, in. |
| F_o | amplitude of initial twist of a member, radians |
| F_y | yield stress, ksi |
| G | shear modulus, ksi |
| G'_d | shear stiffness at 0.8 of ultimate load of diaphragm, kips/in. |
| G'_{dr} | design value of shear stiffness, kips/in. |
| J | torsional constant of a section, in. ⁴ |
| L | length of member, in. |
| M_{cr} | lateral torsional-flexural buckling moment of a diaphragm-braced beam, kip-in. |
| M_{fb} | moment capacity of a "fully" braced beam, kip-in. |
| M_o | buckling moment of an unbraced beam, kip-in. |
| M_s | safe moment of a beam, kip-in. |
| M_y | yield moment of a beam, kip-in. |
| $M_{\phi e}$ | buckling moment of a beam with the centroidal axis of the tension flange as a fixed axis of rotation, kip-in. |

| | |
|----------------|--|
| P_{ult} | ultimate shear load of a diaphragm from a test, kips |
| Q_{dr} | design value of shear rigidity, kips per radian |
| Q | shear rigidity of diaphragm, kips/radian, or kips |
| Q_{id} | shear rigidity required for the "full" bracing of an ideal member, kips/radian |
| w | width of diaphragm contributing to the support of one member, in. |
| X, Y | coordinate axes |
| γ_{dr} | design value of diaphragm shear strain, radians |
| γ_{max} | maximum shear strain in the diaphragm, radians |
| Δ_d | shear deflection of a diaphragm at $0.8 P_{ult}$, radians |

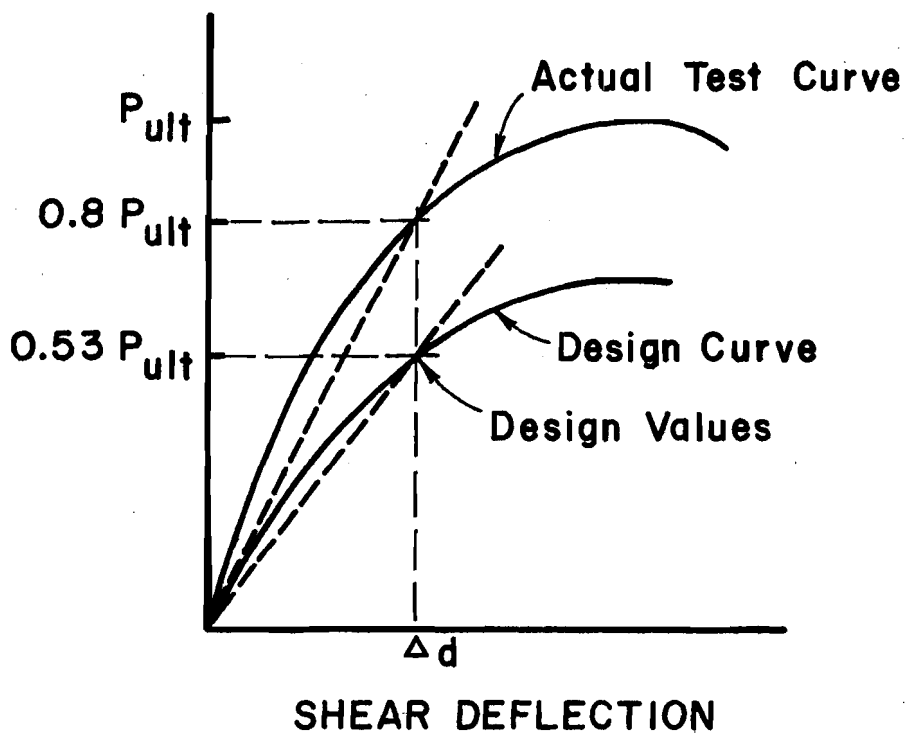
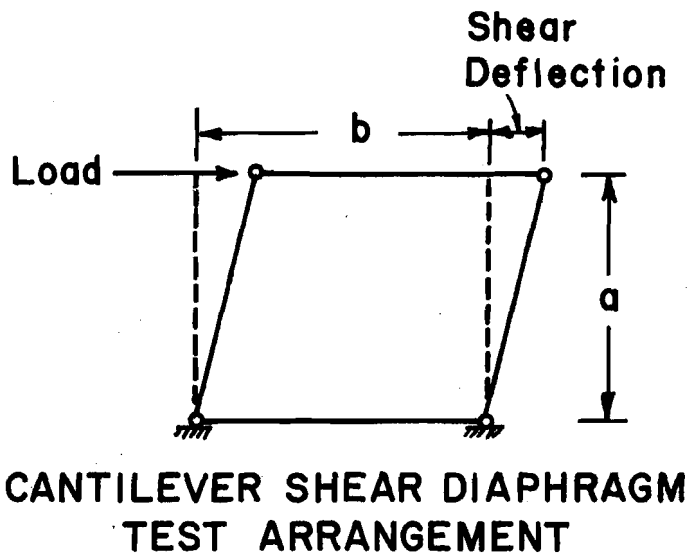
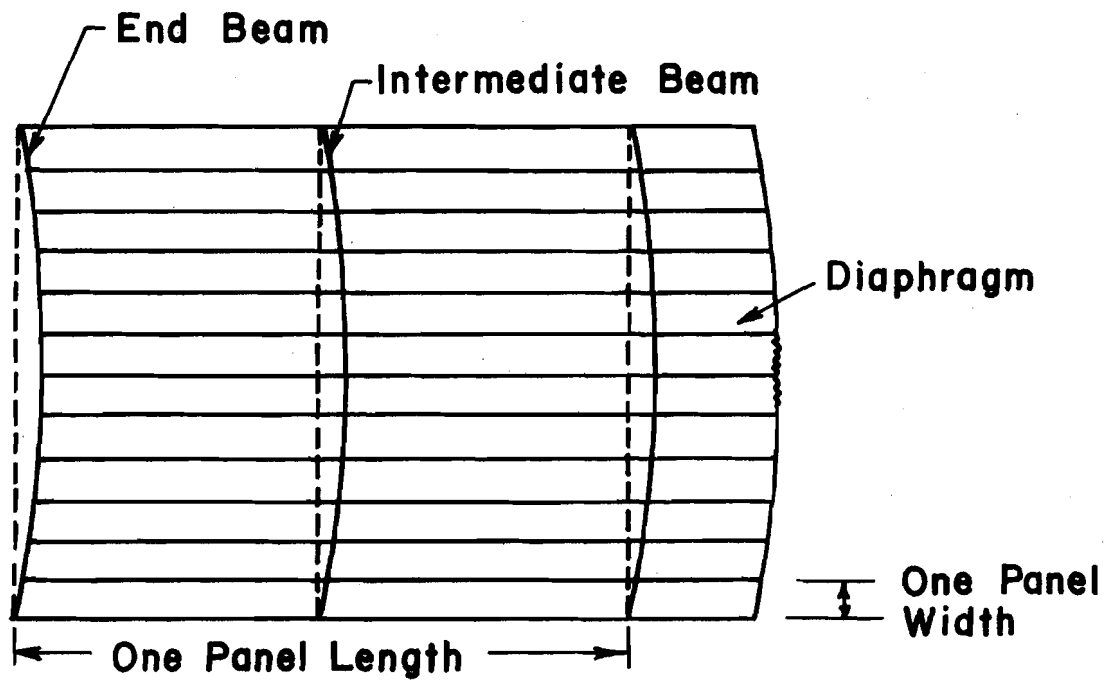
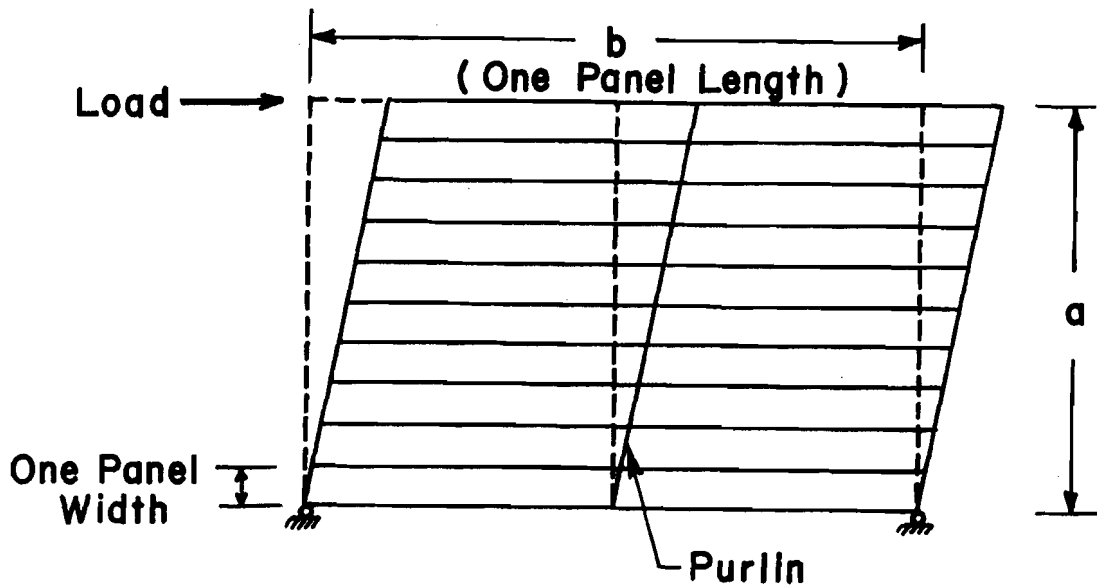


FIG. 1 LOAD-DEFLECTION RELATIONSHIP OF A SHEAR DIAPHRAGM OBTAINED FROM A CANTILEVER TEST



(a) PLAN VIEW

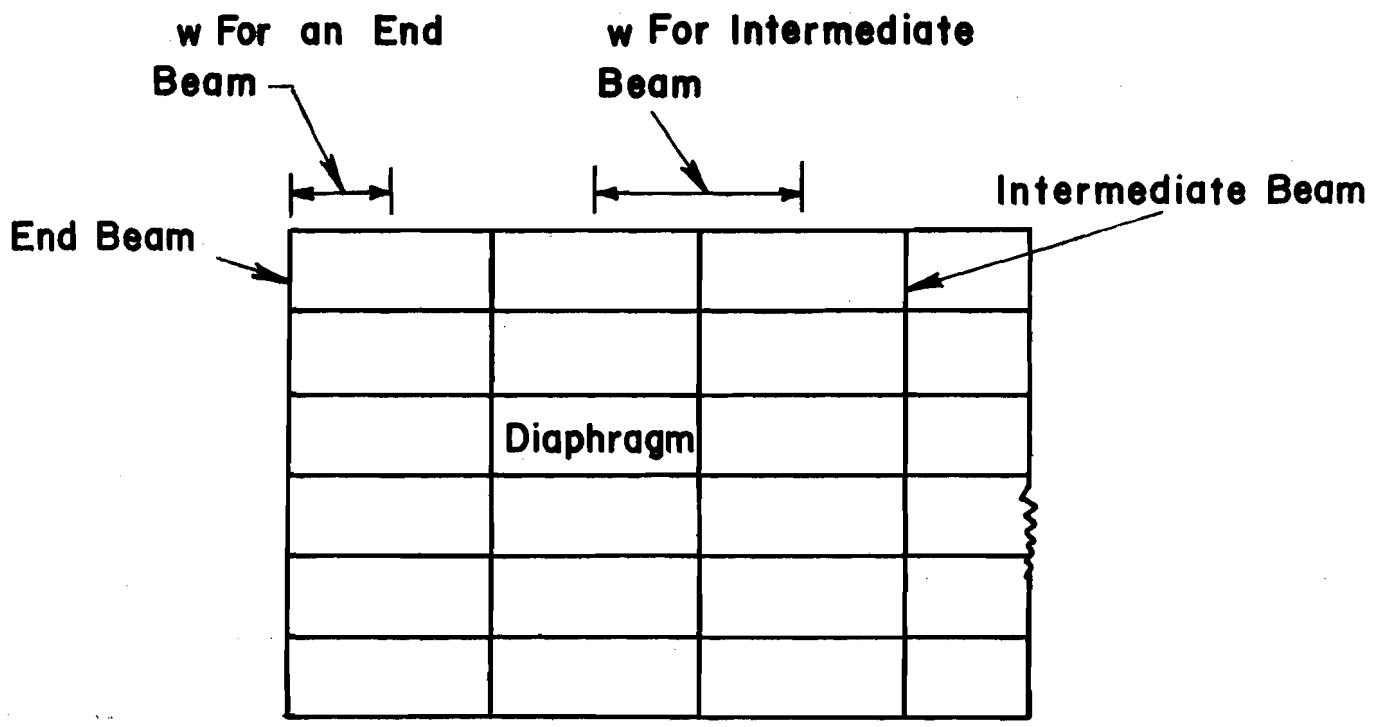


(b) CANTILEVER SHEAR DIAPHRAGM TEST ARRANGEMENT

----- Original Position

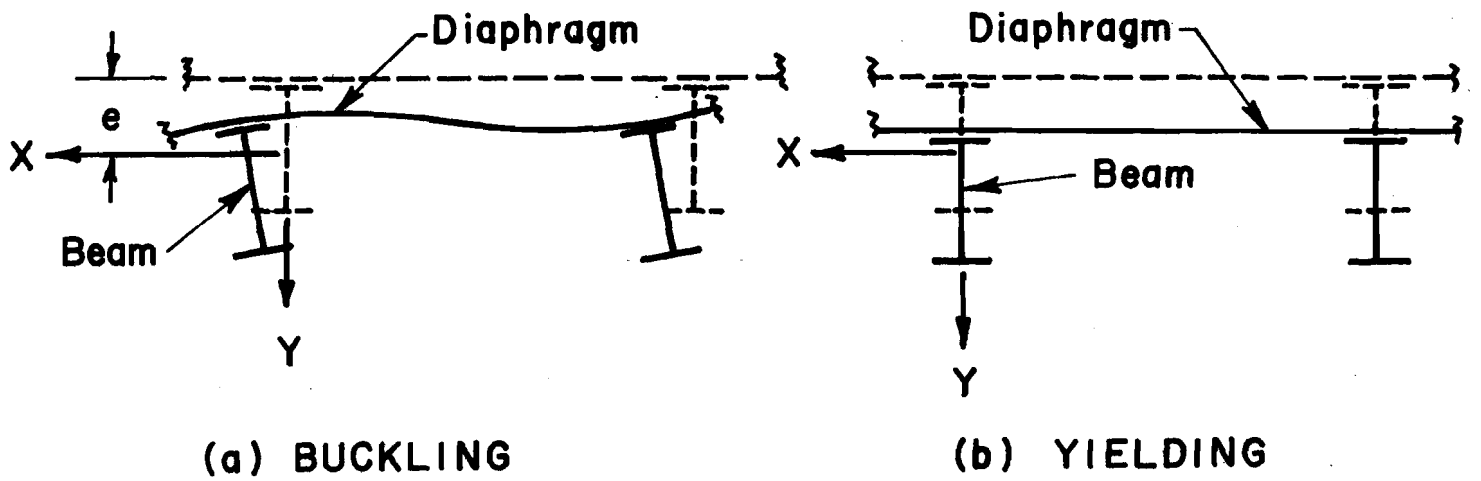
———— Deflected Position

FIG. 2 DEFLECTED POSITION OF A DIAPHRAGM IN A DIAPHRAGM-BRACED BEAM ASSEMBLY, AND IN A SHEAR DIAPHRAGM TEST



PLAN VIEW

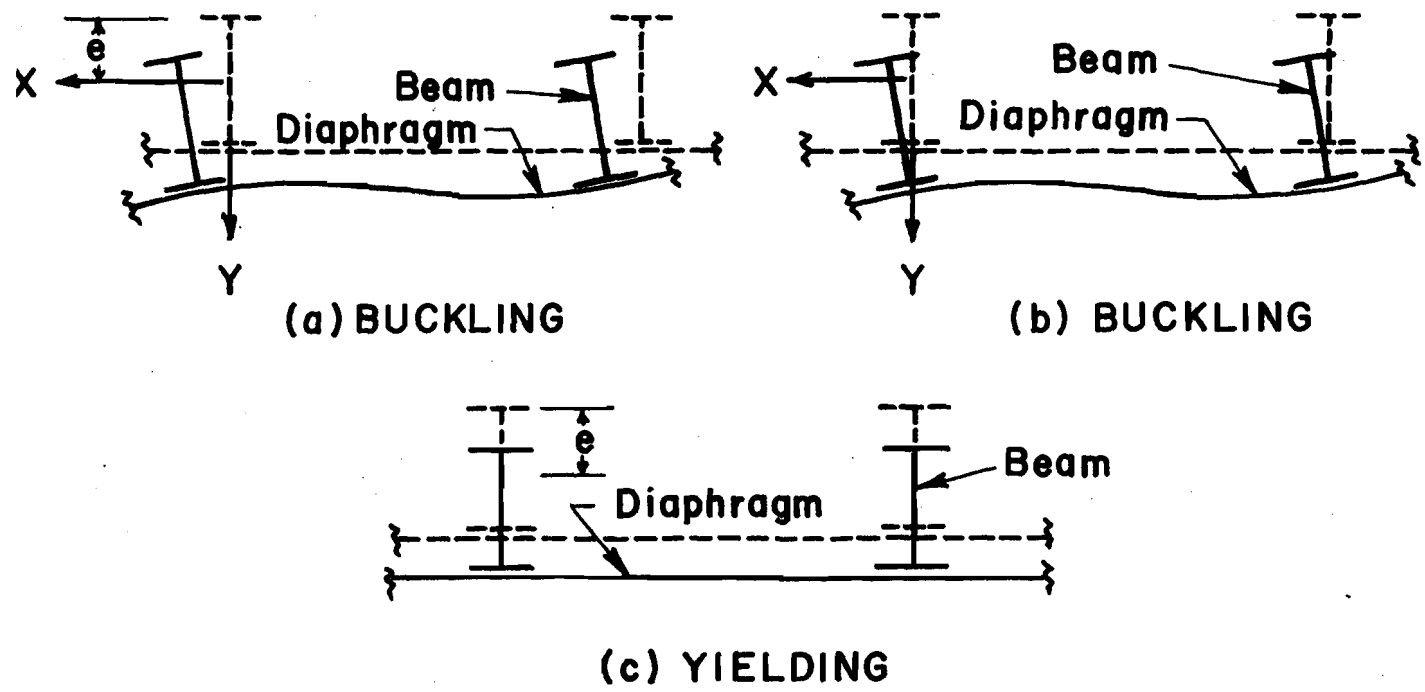
FIG. 3 DIAPHRAGM DIMENSION w FOR AN END MEMBER OR AN INTERMEDIATE MEMBER



----- Original Position

—— Deflected Position

FIG. 4 MODES OF FAILURE OF BEAMS BRACED WITH A DIAPHRAGM ON THE COMPRESSION FLANGES



----- Original Position ——— Deflected Position

FIG. 5 MODES OF FAILURE OF BEAMS BRACED WITH A DIAPHRAGM ON THE TENSION FLANGES

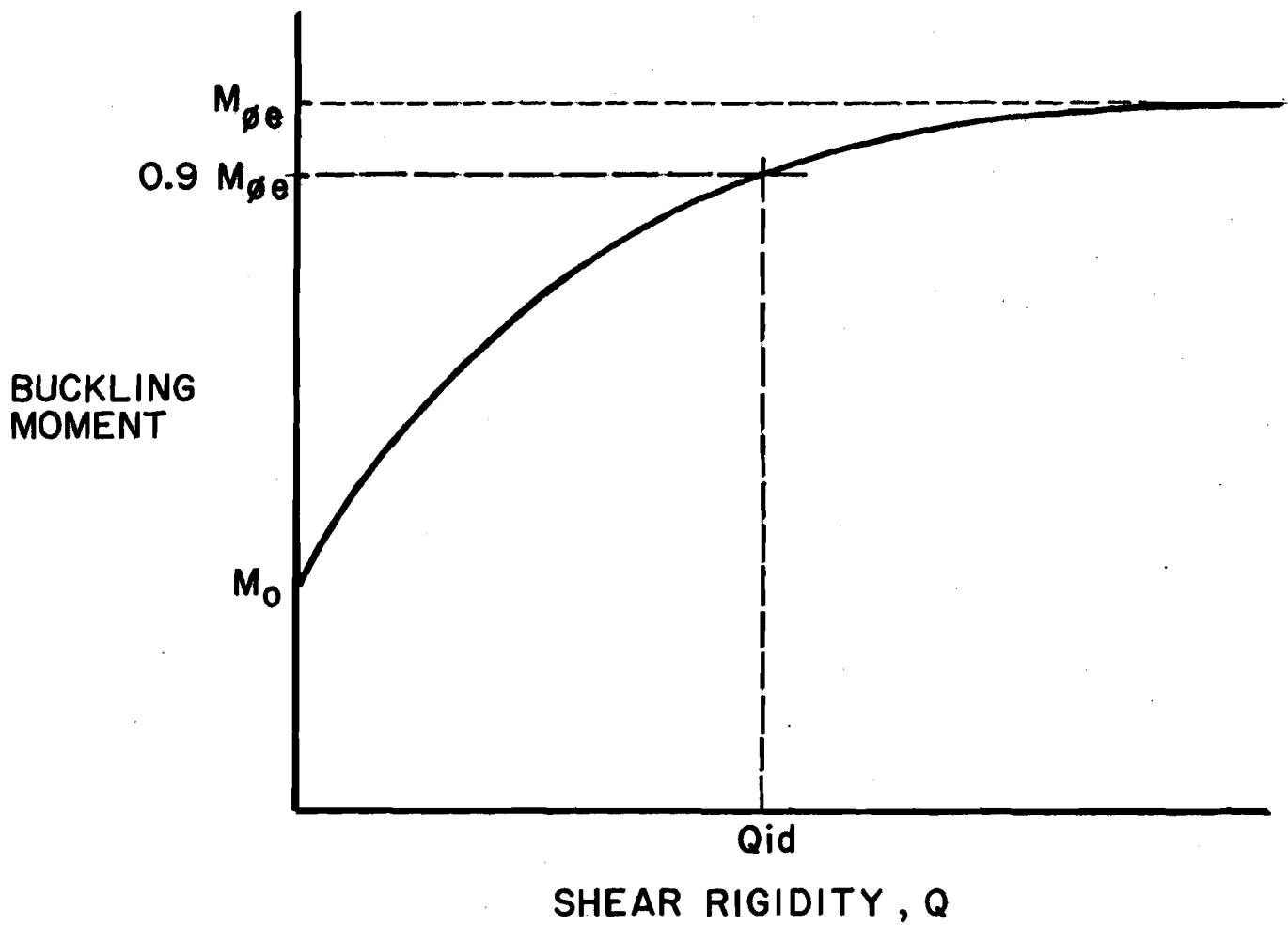


FIG. 6 RELATIONSHIP BETWEEN TORSIONAL FLEXURAL BUCKLING MOMENT AND SHEAR RIGIDITY FOR A BEAM WITH DIAPHRAGM BRACING ON THE TENSION FLANGE

DESIGN OF I-SHAPED COLUMNS WITH DIAPHRAGM BRACING

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DESIGN OF I-SHAPED BEAMS WITH DIAPHRAGM BRACING

Key Words: Beams (structural); Buildings; Bracing; Cold formed panels; Design; Diaphragm; Shear strength.

Abstract

Cold-formed steel panels often are used as wall sheathing, roof decking or floor decking in steel framed buildings. Diaphragms formed by interconnecting these panels have considerable in-plane shear resistance, and can be utilized as bracing against buckling for individual members of a steel frame. The utilization of existing floor or roof diaphragms as bracing for individual beams can eliminate the need for other types of bracing, and/or reduce required member sizes, thus contributing to economical design. A procedure is presented for the design of I-section beams with diaphragm bracing on either the tension or compression flanges. The procedure is based on the ultimate load capacity of fully braced members, utilizing a conservative estimate of the shear strength and shear rigidity of the diaphragm. A design example is included.

Summary

A procedure is presented for the design of I-section beams with diaphragm bracing on the compression or tension flanges, utilizing the shear strength and rigidity of diaphragms formed by interconnecting cold formed steel panels. A design example is included.

LIST OF FIGURES

Figure
No.

- 1 Load-Deflection Relationship of a Shear Diaphragm Obtained from a Cantilever Test
- 2 Deflected Position of a Diaphragm in a Diaphragm-Braced Beam Assembly, and in a Shear Diaphragm Test
- 3 Diaphragm Dimension w for an End Member or an Intermediate Member
- 4 Modes of Failure of Beams Braced with a Diaphragm on the Compression Flanges
- 5 Modes of Failure of Beams Braced with a Diaphragm on the Tension Flanges
- 6 Relationship Between Torsional Flexural Buckling Moment and Shear Rigidity for a Beam with Diaphragm Bracing on the Tension Flange

INTRODUCTION

A design procedure for I-shaped beams braced by diaphragms was presented in an earlier publication.⁽⁸⁾ This report extends the method to I-shaped columns braced by diaphragms. These diaphragms can be formed, for example, by adequately interconnecting cold-formed steel panels which are often used as wall sheathing, roof decking or floor decking in steel framed buildings. The panels carry loads normal to their planes by virtue of their bending strength and, where suitably interconnected to form a diaphragm, also can develop considerable in-plane shear resistance. Several of the structural uses of the shear resistance of these diaphragms, such as wind bracing for low rise buildings, shear elements in folded plate and hyperbolic paraboloid construction, and load distributing elements in portal frame buildings, have been described elsewhere.^(5,6,14) This report discusses another use of diaphragm action; that is, as bracing to prevent buckling of individual columns of a steel frame.

Extensive research has been conducted at Cornell University and elsewhere to determine the increased load-carrying capacity of columns due to diaphragm bracing.^(1,3,7,10,11) This report gives the basis for the proposed design procedure, describes the specific steps, and presents design examples. While some of the algebraic expressions are lengthy, the procedure is straightforward and provides a rational analytic basis for design where no other is currently known.

Usually, it is not economical to provide anything less than "full" bracing for a member, where this is defined as bracing such that

any increase in rigidity or strength of the diaphragm will cause no significant increase in the load-carrying capacity of the braced members.⁽¹³⁾ For this reason, and in an effort toward simplicity, this design procedure is limited to "fully braced" columns. The procedure is based on analyses of I-section columns under axial load, and has been substantiated by tests of diaphragm-braced assemblies as reported in References 1, 3, 7, and 10. The behavior of diaphragm-braced columns of unsymmetrical section is discussed in Reference 11.

CRITERIA FOR DESIGN

This design procedure covers the following:

1. Axially loaded I-section columns continuously braced by shear diaphragms on both flanges, where "continuous bracing" indicates that the diaphragm is connected directly to the column at short intervals;
2. Axially loaded I-section columns continuously braced on one flange only; and
3. Axially loaded I-section columns braced by girts which, in turn, are braced by a shear diaphragm, as in most pre-engineered metal buildings such as shown in Fig. 1.

If panels with longitudinal ribs are used, the ribs must be perpendicular to the member they are bracing, which is the usual case.

Bracing requirements are not a linear function of the applied load; therefore, the design procedure is based on the ultimate load capacity of the columns, utilizing a conservative estimate of the strength and rigidity of the diaphragm. This is in contrast to most design procedures for other uses of diaphragms, which are usually formulated in terms of allowable load.

Columns with equal bracing connected directly to both flanges (that is, symmetric bracing) tend to deflect laterally under load without twisting, while columns with continuous bracing on only one flange tend

to twist as well as deflect laterally. The diaphragm in these cases provides continuous restraint against (1) lateral movement in the plane of the diaphragm, and (2) twist of the member. In contrast, diaphragm-girt bracing provides these two restraints to a column only at the points of attachment of the girts. In either case, due to these restraints the buckling load of a column can be considerably increased.

Shear Characteristics of a Diaphragm

Considerable progress has been made recently in developing methods to predict the two important parameters which characterize a diaphragm assembly: the shear stiffness (or conversely, flexibility) and shear strength. (5,6,8,14,15) Tabulated values for specific assemblies are given in some of these references, and in proprietary literature of panel and fastener manufacturers. As an alternative, these characteristics can be determined from the load-deflection curve obtained from a simple beam or cantilever shear test (Fig. 2) as described in Reference 6. The resulting load-deflection relationship is generally not linear; a typical test curve is shown in the figure. Furthermore, in such tests, two nominally identical diaphragms may give considerably different load-deflection relationships at higher loads--say, beyond 80% of ultimate load. The shear stiffness at 80% of ultimate shear load is denoted as G_d^1 . To insure that diaphragm failure will not precede beam failure, it is proposed to use only 2/3 of G_d^1 as a reliable design value of shear stiffness, G_{dr}^1 . Thus

$$G_{dr}^1 = (2/3)G_d^1 = 2/3 \times \frac{0.8 P_{ult}/b}{\Delta_d/a} = \frac{0.53 P_{ult}/b}{\Delta_d/a} \quad (1)$$

where P_{ult} is the ultimate shear load in the diaphragm test, kips

Δ_d is the deflection at $0.8 P_{ult}$, in.

a is the dimension of the shear diaphragm perpendicular to the test load direction, in., and

b is the dimension of the shear diaphragm parallel to the test load direction, in.

Eq. 1 indicates that G_{dr}^I is in units of force per unit length.

If the shear stiffness of a diaphragm is known, then the maximum shear strain that can be sustained by a diaphragm is a measure of its shear strength; that is, the shear strength is the product of the shear stiffness and shear strain. The reliable design value of shear strain, γ_{dr} , to be used as a measure of shear strength is taken here equal to the shear strain at $0.8 P_{ult}$, γ_d :

$$\gamma_{dr} = \gamma_d = \frac{\Delta_d}{a} \quad (2)$$

Fig. 2 shows a graphical representation of proposed design values of shear stiffness and shear deflection.

The type and spacing of fasteners is very important, and those used in a diaphragm test should be the same as those used in the actual structure. The panel lengths and purlin spacing to be used in a cantilever test (or in any analytical procedure) to simulate the actual structure are given below. As a conservative simplification, the bending rigidity of the diaphragm, which tends to prevent rotation of the member to which it is attached, is neglected in this design procedure.

1. Diaphragms continuously bracing columns

The deflected position of the structure in this case is shown

in Fig. 3a, where it can be observed that the full length of each panel is under uniform shear. The length of panel to be used in a cantilever diaphragm test is the same as the length of the panel used in the structure, as shown in Fig. 3b. Purlin spacing in the test should be the same as the spacing of the columns to be braced.

2. Diaphragms in a column-girt-diaphragm assembly

A typical deflected position of the diaphragm in a column-girt-diaphragm assembly is shown in Fig. 4b, where it is seen that only a part of the length of the panel equal to the spacing of girts is under uniform shear. Therefore, the length of the panels to be used in a shear diaphragm cantilever test should be the same as the spacing of girts in the column-girt-diaphragm assembly, as shown in Fig. 4c. No intermediate purlins should be used in the test. Perimeter framing and fasteners should simulate the corresponding portion of the actual structure.

A simple beam shear test may be conducted instead of a cantilever test, making proper choice of the panel length and spacing of the framing members. (6)

When a diaphragm-braced column in a structure is to be analyzed, it is more convenient in the computations to use a reliable shear rigidity Q_{dr} of the entire portion of diaphragm contributing to the support of the member, rather than the unit shear stiffness G'_{dr} . Using Eq. 1, the reliable shear rigidity Q_{dr} is expressed as

$$Q_{dr} = G'_{dr} w = (2/3)G'_d w = \frac{0.53 P_{ult} w/b}{\Delta_d/a} \quad (3)$$

where w is the dimension of the diaphragm, perpendicular to the longitudinal axis of the member, which contributes to the support of the member being braced. For example, in the case of columns braced directly by a diaphragm (Fig. 3), the end columns can be assumed to be supported by the diaphragm of dimension w equal to half the column spacing. The intermediate columns are supported by the dimension of the diaphragm w equal to the full column spacing. It can be observed from Eq. 3 that Q_{dr} is in units of force per unit shear strain (force/radian).

Bending Stiffness and Strength of a Girt

The performance of a girt also can be characterized by its bending stiffness and strength, with due consideration of the rigidity of the girt-to-column connection. If the connection between girts and columns is fully rigid, the bending rigidity of the girt offers calculable restraint against twist of the column, at the point of attachment to the column. To compute the twist restraint, refer to the deflected position of the column-girt-diaphragm assembly shown in Fig. 5. For a rigid connection, the elastic restraining moment on the column per unit twist of the column, m , can be computed as

$$m = \frac{2 (6EI_g)}{s} \quad (4)$$

where I_g is the strong axis moment of inertia of the girt, E is Young's modulus, and s is the column spacing. If the girt-to-column connection is effectively "pinned", then $m = 0$.

The strength of a girt can be designated by the bending slope at the column, θ_d , when the ends of a girt between two successive columns

are subjected to equal and opposite moments, M_{yg} (Fig. 5b), where M_{yg} is the yield moment of the girt. The slope can be computed as

$$\theta_d = \frac{M_{yg} s}{6EI_g} \quad (5)$$

Initial Imperfections and Additional Deflections

The required strength of any type of bracing is a function of the initial imperfections of the load-carrying member. The pattern of initial deflections along the length of an imperfect column is here assumed affine to the buckling pattern to obtain a conservative estimate of deflections under load. For example, in the case of a continuously braced column, with ends simply supported laterally or hinged, the buckling pattern is a half sine wave, and the initial deflection pattern is therefore also assumed as a half sine wave. The amplitude of the initial lateral deflection of the centroidal axis, E_o , is taken as the tolerance limit in sweep specified in the AISC Manual (Ref. 9, p 1-127). Hence

$$E_o = \frac{1''}{8} \times \frac{\text{length of member in feet}}{\text{either 5 or 10, as specified}} \quad (6)$$

In addition, an accidental eccentricity is considered by multiplying the initial lateral deflection by two in the design equations. Based on limited available information, the amplitude of the initial twist, F_o , is arbitrarily taken equal to 0.000667 radian per foot of length; that is,

$$F_o = 0.000667 \text{ rad./ft.} \times \text{length of member in feet} \quad (7)$$

Because of the initial lateral deflection or twist of a column, additional lateral deflections and twist occur under applied load. The

pattern of additional deflections along the length of a member is the same as the buckling pattern. These additional deflections cause shear forces in the diaphragm bracing. Also, because of these additional deflections, girts bend in the case of a column-girt-diaphragm assembly if the twist restraint $m \neq 0$ and if the column buckles in a torsional flexural mode. Magnitudes of the additional deflections, maximum shear strain in the diaphragm, and maximum bending slope of the girts can be calculated using the design formulae given subsequently.

Factors of Safety

The factors of safety used for rolled steel columns and cold-formed steel columns in this design procedure are the same as those used in the AISC Specification⁽⁹⁾ and AISI Specification,⁽¹²⁾ respectively.

GENERAL DESIGN PROCEDURE

Briefly stated, the design procedure is as follows: First, assume the member to be fully braced, and select a section of required load capacity. Then, select a diaphragm of specific panel dimensions, fastener type and fastener spacing, and check to see that its rigidity and strength are adequate to provide full bracing for the member.

The detailed design procedure is outlined below:

1. Choose a trial member for the design.
2. Assume that the member is fully braced, and compute its load capacity, P_{fb} .
3. Compute the safe load, P_s :

$$P_s = \frac{\text{load capacity of fully braced member}}{\text{factor of safety or load factor}} = \frac{P_{fb}}{F.S.} \quad (8)$$

If P_s is greater than and close to the required design load, proceed to check whether the bracing is adequate; otherwise, repeat the procedure from Step 1.

4. Compute Q_{id} , the shear rigidity required to fully brace an ideal member. The actual rigidity required to brace a real member will be greater than this. (13)

5. Select a trial diaphragm. If $Q > Q_{id}$, the diaphragm rigidity may be adequate for full bracing; proceed with Step 6. If $Q \leq Q_{id}$, full bracing cannot be achieved with this diaphragm; a more rigid diaphragm must be chosen, and this step repeated. (Alternatively, see Ref. 1 for columns with less than full bracing.)

6. Compute the maximum shear strain, γ_{\max} , in the diaphragm. If the reliable shear strain $\gamma_{dr} \geq \gamma_{\max}$, the diaphragm strength is adequate for full bracing; proceed with Step 7 if applicable. If $\gamma_{dr} < \gamma_{\max}$, a stronger or stiffer diaphragm is necessary for full bracing; repeat the procedure from Step 5.

7. This step applies only for diaphragm-girt bracing. In most cases of full bracing there is no bending of the girts. However, if the flexural restraint $m \neq 0$ and failure is in the torsional flexural mode, the strength of the girts has to be checked. Compute θ_d and θ_{\max} . If the computed bending slope of the girts θ_{\max} is less than θ_d , the bracing provided by the girts in combination with the diaphragm is adequate to fully brace the column. If the girts are not strong enough, choose a stronger section for the girts, and repeat the procedure from Step 5.

P_{fb} , γ_{\max} , θ_{\max} , Q_{id} and θ_d are computed from equations given herein; whereas, Q_{dr} and γ_{dr} can be obtained from the load-deflection relationship of a shear diaphragm test or analysis. The strong and weak axis buckling loads of a column can be obtained in any rational manner, including multiplying the allowable load by the known safety factor.

AXIALLY LOADED I-SECTION COLUMNS CONTINUOUSLY BRACED BY
SHEAR DIAPHRAGMS ON BOTH FLANGES (1,7)

Diaphragm-braced columns in this case may buckle in one of the two modes shown in Fig. 6. The bracing is defined as "full" if its rigidity and strength are adequate to prevent weak-axis buckling of the columns (Fig. 6a) so that they buckle about their strong axis (Fig. 6b). Torsional-flexural buckling is not a failure mode for I-section columns with symmetrical diaphragm bracing. The buckling load of a "fully" braced column, P_{fb} , is therefore $P_{crx,L}$, the strong axis buckling load of the column of length, L .

$$P_{fb} = P_{crx,L} \quad (9)$$

The shear rigidity (Q_{id}) required for an ideal column to attain full bracing is given by Eq. 10 in Table 1, where E^* is the modulus corresponding to the average stress level (σ) of the column at P_{fb} . If $\sigma < \sigma_p$, $E^* = E$. But, if $\sigma > \sigma_p$, (4)

$$E^* = E \frac{(\sigma_y - \sigma) \sigma}{(\sigma_y - \sigma_p) \sigma_p} \quad (11)$$

Amplitude of the additional lateral deflection, C , of the centroidal axis of the column at load $P_{crx,L}$ is obtained from Eq. 12 in Table 1. For symmetrically braced columns no rotation is assumed, and the lateral deflection in the plane of the diaphragm is equal to the deflection at the centroidal axis, $C_1 = C$. The design follows the general procedure; the final step is to check the strength of the diaphragm ($\gamma_{max} < \gamma_{dr}$) using

$$\gamma_{max} = C_1 \frac{\pi}{L} \quad (14)$$

AXIALLY LOADED I-SECTION COLUMNS CONTINUOUSLY BRACED BY
A SHEAR DIAPHRAGM ON ONE FLANGE ONLY (1,7)

Diaphragm-braced columns in this case may buckle in one of the modes shown in Fig. 7; that is, torsional-flexural buckling or flexural buckling about the strong axis. The buckling load of the column approaches P_{ϕ_e} (the buckling load of the column with the centroidal axis of one of the flanges as the fixed axis of rotation) asymptotically as the shear rigidity Q approaches infinity. In general, even for a very small increase in load beyond about $0.9 P_{\phi_e}$, a very large increase in shear rigidity is needed. Therefore, if a column buckles in the torsional-flexural mode, as in Fig. 7a, the buckling load of the "fully" braced column is arbitrarily taken as $0.9 P_{\phi_e}$. (Any other percentage could be used in similar fashion.) The buckling load of a fully braced column is the smaller of the two values; that is,

$$P_{fb} = \text{Min.} (0.9 P_{\phi_e}, P_{crx,L}) \quad (15)$$

The buckling load P_{ϕ_e} is given by Eqs. 16a and 16b in Table 2 for the elastic and inelastic range, respectively. For an I-section, the polar moment of inertia I_p in Eqs. 16a and 16b is

$$I_p = I_x + I_y \quad (17)$$

The shear rigidity Q_{id} required for an ideal column to be fully braced is given by Eq. 18 in Table 2, where E^* is obtained from Eq. 11 and

$$G^* = G \frac{E^*}{E} \quad (19)$$

Amplitudes of additional lateral deflection of the centroidal axis (C)

and twist (D) at the buckling load are given by Eqs. 20 and 21, respectively, in Table 2. The amplitude of the additional lateral deflection C_1 in the plane of the diaphragm is

$$C_1 = C - eD \quad (22)$$

and the maximum shear strain is obtained from Eq. 14. Example No. 1 illustrates the design procedure.

AXIALLY LOADED I-SECTION COLUMNS WITH
DIAPHRAGM-GIRT BRACING^(1,3)

A typical column-girt-diaphragm assembly is shown in Fig. 8a. If "full" bracing is provided, the column may buckle in one of three modes: (1) flexural buckling about its strong axis, Fig. 8b, (2) torsional-flexural buckling, Fig. 8c, or (3) flexural buckling about its weak axis between successive girts, Fig. 8d. Therefore, the buckling load of such a fully braced column is the smallest of these three values; that is,

$$P_{fb} = \text{Min.} (P_{crx,L}, P_{cry,\lambda}, 0.9 P_{\phi e}) \quad (23)$$

where $P_{cry,\lambda}$ is the weak axis buckling load of a column of length λ .

The design formulae given in this report are for columns with "hinged" ends; that is, the ends are flexurally hinged, and warping is unrestrained. Design equations for the various cases are given in Table 3; values of the required coefficients K_1 through K_4 appear in Table 4 for modes $i = 1 \dots j$, where j is the number of intermediate girts.

If the girt-column connection is fully flexible ($m = 0$), a fully braced column usually buckles in the torsional-flexural mode, but there is no bending of the girts. On the other hand, if the girt-column connection is rigid, the column usually buckles flexurally rather than by twisting, and again the girts do not bend. Therefore, strength of the girts has to be checked only where $m \neq 0$ and the column buckles in a torsional-flexural mode. The maximum bending slope of the girts, θ_{max} , is given by the twist of the column at the girt which is at or nearest the midheight of the column as indicated in Eq. 29. Example No. 2 illustrates the design procedure.

DESIGN EXAMPLES

Example No. 1 - Columns Braced by a Diaphragm on One Flange Only

Determine the size of an intermediate column of a side wall to support an axial load of 106 kips. Columns are 12 ft high, spaced at 6 ft intervals, and are continuously braced on one flange by a light gage steel diaphragm whose shear characteristics are $G'_d = 12.5$ kips/in. and $\gamma_d = 0.0045$. The ends of the column are assumed to be flexurally hinged, with warping unrestrained. Use ASTM A36 steel, $\sigma_y = 36$ ksi, $\sigma_p = 18$ ksi, $E = 29,000$ ksi, $G = 11,500$ ksi.

Solution: From Table 2, $P_{fb} = \text{Min.} (P_{crx,L}, 0.9 P_{\phi_e})$. Using tables in the AISC Manual or other design aid as a guide, try W6 x 25.

$$\text{Buckling Loads: } \frac{L}{r_x} = \frac{144}{2.69} = 53.5 < C_c = 126.1$$

$$\text{Ref. 9: } P_{crx,L} = 7.35 \times 36 \left[1 - \frac{1}{2} \left(\frac{53.5}{126.1} \right)^2 \right] = 237.5 \text{ kips}$$

$$\text{Eq. 16b: } n = 1, P_{\phi_e} = 229.3 \text{ kips}$$

$$0.9 P_{\phi_e} = 0.9 \times 229.3 = 206.4 \text{ kips}$$

$$\therefore P_{fb} = \text{Min.} (237.5, 206.4)$$

$$P_{fb} = 0.9 P_{\phi_e} = 206.4 \text{ kips}$$

Factor of Safety:

$$L/r_y = 144 \div 1.53 = 94.1 < 126.1 = C_c$$

$$\text{Ref. 9: } \text{F.S.} = \frac{5}{3} + \frac{3}{8} \left(\frac{94.1}{126.1} \right) - \frac{1}{8} \left(\frac{94.1}{126.1} \right)^3 = 1.89$$

$$\text{Safe axial load on column if fully braced} = \frac{206.4}{1.89}$$

$$= 109.2 \text{ kips} > 106 \text{ kips}, \therefore \text{OK}$$

Check to see whether the bracing is "full".

Diaphragm Rigidity:

$$\sigma = \frac{P_{fb}}{A} = \frac{206.4}{7.35} = 28.08 \text{ ksi}$$

Eq. 11: $\sigma_p > 1/2 \sigma_y$, $E^* = 19,900 \text{ ksi}$

Eq. 19: $G^* = 7,890 \text{ ksi}$

Eq. 18: $n = 1$, $Q_{id} = 52.0 \text{ kips}$

Eq. 3: $Q_{dr} = 2/3 G_d^i w = 2/3 (12.5)(72) = 600 \text{ kips} > 52.0 \text{ kips}$

\therefore diaphragm rigidity may be adequate for full bracing.

Diaphragm Strength:

Eq. 6: Assumed initial sweep, $E_o = \frac{1}{8} \times \frac{12}{10} = 0.15''$

Eq. 7: Assumed initial twist, $F_o = .000667 \times 12 = 0.008 \text{ rad.}$

Eq. 20: Additional deflection, $C = 0.41''$

Eq. 21: Additional twist, $D = 0.088 \text{ rad.}$

Eq. 22: Deflection of braced flange, $C_1 = 0.13''$

Eq. 14: Maximum shear strain in diaphragm,

$$\gamma_{\max} = 0.0029 \text{ rad.} < \gamma_d = \gamma_{dr} = 0.0045 \text{ rad.}$$

\therefore diaphragm strength is adequate. The column is fully braced and can safely carry a design axial load of 106 kips.

Example No. 2 - Columns with Diaphragm-Girt Bracing

Determine the size of intermediate I-section columns 12'-4" long, spaced at 19'-4" intervals, carrying an axial load of 220 kips each, and braced by one line of girts at midheight. The girts are braced by a standard corrugated diaphragm. Assume the ends of the columns are hinged, with warping unrestrained.

Spacing of girts = 6'-2"

Diaphragm stiffness, $G'_d = 6.47$ kips/in.

Diaphragm shear strain, $\gamma_d = 0.0069$ rad.

Twist restraint, $m = 4650$ k-in./rad.

Use ASTM A572 Grade 50 steel, $F_y = 50$ ksi, $E = 29,000$ ksi

$G = 11,500$ ksi, $\sigma_p = 25$ ksi.

Solution: Try W12 x 31

Buckling Loads: $\frac{L}{r_x} = \frac{148}{5.12} = 28.9 < C_c = 107.0$

Ref. 9: $P_{crx,L} = 9.13 \times 50 \left[1 - \frac{1}{2} \left(\frac{28.9}{107} \right)^2 \right] = 440$ kips

$$\frac{\lambda}{r_y} = \frac{74}{1.54} = 48.0 < C_c = 107.0$$

Ref. 9: $P_{cry,\lambda} = 9.13 \times 50 \left[1 - \frac{1}{2} \left(\frac{48.0}{107} \right)^2 \right] = 410$ kips

Eq. 25b: $P_{\phi e} = P_y = A F_y = 456.5$ kips

$$0.9 P_{\phi e} = 411 \text{ kips}$$

Eq. 23: $P_{fb} = \text{Min.} (440; 410; 411)$

$$P_{fb} = P_{cry,\lambda} = 410 \text{ kips}$$

Factor of Safety:

Ref. 9: $F.S. = \frac{5}{3} + \frac{3}{8} \left(\frac{48}{107} \right) - \frac{1}{8} \left(\frac{48}{107} \right)^3 = 1.82$

Required ultimate strength = $220 \times 1.82 = 400$ kips

< 410 kips = P_{fb} \therefore OK

Diaphragm Rigidity:

$$\sigma = \frac{P_{fb}}{A} = \frac{410}{9.13} = 44.9 \text{ ksi}$$

Eq. 11: $E^* = 10,625$ ksi

Eq. 19: $G^* = 4,210 \text{ ksi}$

Eq. 26: With $i = 1$, $K_1 = .250$, $K_2 = .810$, $K_3 = .405$

$Q_{id} = 413 \text{ kips}$

Eq. 3: $Q_{dr} = 2/3 G_d^1 w = 2/3 (6.47)(232) = 1000 \text{ kips}$

$> 413 \text{ kips}$. \therefore diaphragm rigidity may be adequate for full bracing.

Diaphragm Strength:

Eq. 6: $E_o = \frac{1}{8} \times \frac{12.33}{10} = 0.154''$

Eq. 30: $C_1 = 0.251''$

Eq. 31: With $K_4 = 1.0$, $\gamma_{\max} = 0.0034 < \gamma_{dr} = 0.0069$

\therefore diaphragm strength is adequate for full bracing.

Girts: Because the column buckles flexurally ($P_{fb} = P_{cry, \ell}$), there is no strong axis bending of the girts.

Therefore, the diaphragm-girt bracing is "full" bracing, and the W12 x 31 column can safely carry a load of 220 kips.

SUMMARY AND CONCLUSIONS

A procedure is presented for the design of I-section columns with diaphragm or diaphragm-girt bracing. The procedure is based on the ultimate load capacity of fully braced members, utilizing a conservative estimate of the shear strength and shear rigidity of the diaphragm. Design examples are included.

While the algebraic expressions are somewhat lengthy in certain cases, the procedure is straightforward and provides a rational analytic basis for design where no other is currently known to exist. Additional research or experience may lead to design simplifications.

ACKNOWLEDGMENTS

This paper is a result of an investigation sponsored at Cornell University by the American Institute of Steel Construction and the American Iron and Steel Institute, and is based on an earlier Cornell report.⁽²⁾ The cooperation of each of the cognizant research committees is appreciated. The research initially was under the general direction of G. P. Fisher, F. ASCE, and later George Winter, Hon. M. ASCE; their contributions are gratefully acknowledged.

TABLE 1 - EQUATIONS FOR AXIALLY LOADED I-SECTION COLUMNS
BRACED BY SHEAR DIAPHRAGMS ON BOTH FLANGES (1,7)

$$P_{fb} = P_{crx,L} \quad (9)$$

$$Q_{id} = P_{crx,L} - E^* I_y \left(\frac{n\pi}{L}\right)^2 \quad (10)$$

where $E^* = E$ or is obtained from Eq. 11 if $\sigma > \sigma_p$.

$$C = \frac{2 P_{crx,L} E_o}{E^* I_y \left(\frac{n\pi}{L}\right)^2 + Q - P_{crx,L}} \quad (12)$$

where E_o is obtained from Eq. 6.

$$C_i = C \quad (13)$$

$$\gamma_{max} = C_i \frac{\pi}{L} \quad (14)$$

TABLE 2 - EQUATIONS FOR AXIALLY LOADED I-SECTION COLUMNS
BRACED BY SHEAR DIAPHRAGMS ON ONE FLANGE ONLY^(1,7)

$$P_{fb} = \text{Min.} (P_{crx,L}, 0.9 P_{\phi e}) \quad (15)$$

Elastic Range

$$P_{\phi e} = \frac{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2}{\left(\frac{I_p}{A} + e^2\right)} \quad (16a)$$

Inelastic Range

$$P_{\phi e} = A \left[\sigma_y - \frac{A(\sigma_p)^2 \left(\frac{I_p}{A} + e^2\right)}{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2} \right] \quad (16b)$$

where

$$I_p = I_x + I_y \quad (17)$$

$$Q_{id} = \frac{- [E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J - P_{fb} \frac{I_p}{A}] [E^* I_y \left(\frac{n\pi}{L}\right)^2 - P_{fb}]}{[E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J - P_{fb} \frac{I_p}{A}] + e^2 [E^* I_y \left(\frac{n\pi}{L}\right)^2 - P_{fb}]} \quad (18)$$

where E^* and G^* are obtained from Eqs. 11 and 19, respectively.

$$C = \frac{P_{fb} \left\{ [2E_o] [E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J + Qe^2 - P_{fb} \frac{I_p}{A}] + Qe \frac{I_p}{A} F_o \right\}}{\text{Det.}} \quad (20)$$

$$D = P_{fb} \frac{[E^* I_y \left(\frac{n\pi}{L}\right)^2 + Q - P_{fb}] F_o \frac{I_p}{A} + 2E_o Qe}{\text{Det.}} \quad (21)$$

where

$$\text{Det.} = [E^* I_y \left(\frac{n\pi}{L}\right)^2 + Q - P_{fb}] [E^* C_w \left(\frac{n\pi}{L}\right)^2 + G^* J - P_{fb} \frac{I_p}{A}] + Qe^2 [E^* I_y \left(\frac{n\pi}{L}\right)^2 - P_{fb}]$$

and E_o and F_o are obtained from Eqs. 6 and 7, respectively.

C_1 and Y_{\max} are obtained from Eqs. 22 and 14, respectively.

TABLE 3 - EQUATIONS FOR AXIALLY LOADED I-SECTION
COLUMNS WITH DIAPHRAGM-GIRT BRACING^(1,2,3)

$$P_{fb} = \text{Min.} (P_{crx,L}, 0.9 P_{\phi e}, P_{cry,l}) \quad (23)$$

If $m = 0$

Elastic Range

$$P_{\phi e} = \frac{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2}{\frac{I_p}{A} + e^2} \quad (24a)$$

Inelastic Range

$$P_{\phi e} = A \left[\sigma_y - \frac{A \sigma_p^2 \left(\frac{I_p}{A} + e^2\right)}{EC_w \left(\frac{n\pi}{L}\right)^2 + GJ + e^2 EI_y \left(\frac{n\pi}{L}\right)^2} \right] \quad (24b)$$

with $n = 1$ in Eqs. 24a and 24b

If $m \neq 0$

$$P_{\phi e} = \text{Min.} \left[\frac{E^* C_w \left(\frac{\pi}{L}\right)^2 + G^* J + e^2 E^* I_y \left(\frac{\pi}{L}\right)^2 + K_3 m l}{\left(\frac{I_p}{A} + e^2\right)}, P_y \right] \quad (25a)$$

or $P_{\phi e} = \text{Min.} \left[\frac{K_3 m l}{\left(\frac{I_p}{A} + e^2\right)}, P_y \right]$ (conservatively) (25b)

If $P_{fb} = P_{crx,L}$

$$Q_{id} = \frac{-(K_1 P^* - P_{fb}) \left(a^* - P_{fb} \frac{I_p}{A}\right)}{K_2 \left[e^2 (K_1 P^* - P_{fb}) + \left(a^* - P_{fb} \frac{I_p}{A}\right) \right]} \quad (26)$$

where $P^* = \frac{\pi^2 E^* I_y}{l^2}$

$$a^* = K_1 E^* C_w \left(\frac{\pi}{l}\right)^2 + G^* J + K_3 m l$$

and K_1, K_2 and K_3 correspond to one of the modes,

(continued)

TABLE 3 (cont'd)

$i = 1, \dots, j$, where j is the number of intermediate girts, and i is the mode number which gives the maximum value of Q_{id} in Eq. 26

$$C_1 = \frac{P_{fb} [2E_o \left\{ (a^* - P_{fb} \frac{l}{A}) + e^2 (K_1 P^* - P_{fb}) \right\} - e (K_1 P^* - P_{fb}) \left\{ 2eE_o + \frac{l}{A} F_o \right\}]}{(K_1 P^* - P_{fb}) (a^* - P_{fb} \frac{l}{A}) + K_2 Q \left\{ e^2 (K_1 P^* - P_{fb}) + (a^* - P_{fb} \frac{l}{A}) \right\}} \quad (27)$$

$$D = \frac{P_{fb} \left[\left\{ 2E_o e + \frac{l}{A} F_o \right\} \left\{ K_1 P^* - P_{fb} + K_2 Q \right\} - e \left\{ K_1 P^* - P_{fb} \right\} 2E_o \right]}{(K_1 P^* - P_{fb}) (a^* - P_{fb} \frac{l}{A}) + K_2 Q \left\{ e^2 (K_1 P^* - P_{fb}) + (a^* - P_{fb} \frac{l}{A}) \right\}} \quad (28)$$

where K_1 , K_2 and K_3 in Eqs. 27 and 28 correspond to the first mode, $i = 1$.

If $P_{fb} = 0.9 P_{\phi e}$

Q_{id} : Use Eq. 26 above, but constants K_1 , K_2 and K_3 correspond to the first mode.

C_1 : Eq. 27 above

D : Eq. 28 above

If $m \neq 0$, $\theta_{max} = D, 0.866 D$, or D (29)
for columns with 1, 2 or 3 intermediate girts, respectively.

If $P_{fb} = P_{cry, L}$

Q_{id} : Eq. 26, same as for $P_{fb} = P_{crx, L}$

$$C_1 = \frac{2P_{fb} E_o}{K_1 P^* - P_{fb} + K_2 Q} \quad (30)$$

where K_1 , K_2 and K_3 correspond to the first mode.

TABLE 3 (cont'd)

For all failure modes

$$\gamma_{\max} = K_4 \frac{c_1}{l} \quad (31)$$

where K_4 is from Table 4.

TABLE 4 - CONSTANTS $K_1, K_2, K_3, K_4^{(1,2)}$

| | K_1 | K_2 | K_3 | K_4 |
|-----------------------------|--------|-------|-------|-------|
| <u>1 Intermediate Girt</u> | | | | |
| Mode: $i = 1$ | 0.250 | 0.810 | 0.405 | 1.000 |
| <u>2 Intermediate Girts</u> | | | | |
| Mode: $i = 1$ | 0.111 | 0.912 | 0.912 | 0.866 |
| $i = 2$ | 0.444 | 0.684 | 0.228 | 0.866 |
| <u>3 Intermediate Girts</u> | | | | |
| Mode: $i = 1$ | 0.0625 | 0.950 | 1.621 | 0.707 |
| $i = 2$ | 0.250 | 0.810 | 0.405 | 0.707 |
| $i = 3$ | 0.562 | 5.53 | 0.180 | 0.707 |

APPENDIX I - REFERENCES

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APPENDIX II - NOMENCLATURE

- A ----- cross sectional area, in.²
- a ----- dimension of shear diaphragm perpendicular to test load direction, in.
- a* ----- = $K_1 E^* C_w \left(\frac{\pi}{l}\right)^2 + G^* J + K_3 m$, kip-in.²
- b ----- dimension of shear diaphragm parallel to test load direction, in.
- C ----- amplitude of additional lateral deflection of centroidal axis, in.
- C₁ ----- amplitude of additional lateral deflection in the plane of the diaphragm, in.
- C_w ----- warping constant of a section, in.⁶
- D ----- amplitude of additional twist of a member, radians
- E ----- modulus of elasticity, ksi
- E₀ ----- amplitude of initial lateral deflection of the centroidal axis of a member, in.
- E* ----- elastic or inelastic modulus, ksi
- e ----- distance between center of gravity of a member and the plane of the diaphragm, in.
- F₀ ----- amplitude of initial twist of a member, radians
- G ----- shear modulus, ksi
- G* ----- inelastic shear modulus, ksi
- G_d^l ----- shear stiffness at 0.8 of ultimate load of diaphragm, kips/in.
- G_{dr}^l ----- design value of shear stiffness, kips/in.
- I_p ----- polar moment of inertia, in.⁴
- I_x, I_y ----- moments of inertia of a section about X- and Y-axes, respectively, in.⁴
- I_g ----- moment of inertia of a girt about the bending axis, in.⁴
- i ----- mode number

- j ----- number of intermediate girts
- J ----- torsional constant of a section, in.⁴
- K_1, K_2, K_3, K_4 --- constants
- k ----- effective length factor
- L ----- length of member, in.
- λ ----- spacing of girts, in.
- m ----- elastic restraining moment on the column at a girt, kip-in. per radian
- P_{fb} ----- load capacity of a "fully" braced column, kips
- P_{ult} ----- ultimate shear load of a diaphragm from a test, kips
- $P_{crx,L}, P_{cry,L}$ - strong axis and weak axis buckling loads, respectively, of a column of length L, kips
- $P_{cry,\lambda}$ ----- weak axis buckling load of a column of length λ , kips
- $P_{\phi e}$ ----- buckling load of a column with the centroidal axis of one of its flanges as the fixed axis of rotation, kips
- P_s ----- safe load on a member, kips
- P_y ----- $\sigma_y A$, kips
- P^* ----- $\frac{\pi^2 E^* I_y}{2}$, kips
- Q_{dr} ----- design value of shear rigidity, kips per radian
- Q ----- shear rigidity of diaphragm, kips/radian, or kips
- Q_{id} ----- shear rigidity required for the "full" bracing of an ideal member, kips/radian
- r_x, r_y ----- radii of gyration of the section about X- and Y-axes, respectively, in.
- S ----- spacing of columns, in.
- u, u_1 ----- additional deflections in the directions of X and X_1 axes, respectively, in.
- v ----- additional deflection in the direction of Y-axis, in.
- w ----- width of diaphragm contributing to the support of one member, in.

X, X_1, Y ----- coordinate axes

β ----- twist of the member, radians

γ_{dr} ----- design value of diaphragm shear strain, radians

γ_{max} ----- maximum shear strain in the diaphragm, radians

Δ_d ----- shear deflection of a diaphragm at $0.8 P_{ult}$, radians

θ_d ----- $= \frac{M_{yg} w}{6EI_{xg}}$ = bending slope of a girt at yield moment, radians

θ_{max} ----- computed maximum bending slope of a girt, radians

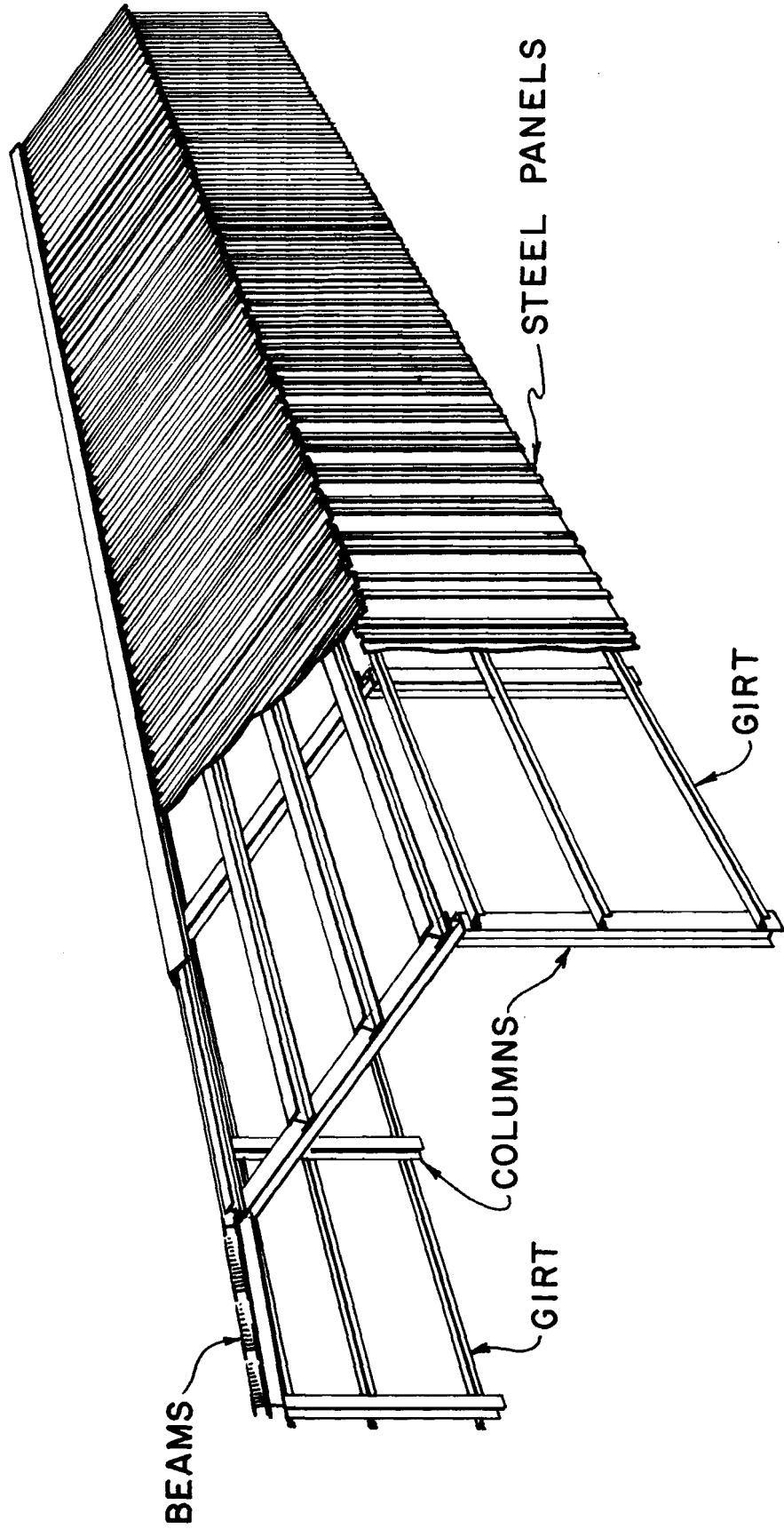
σ ----- average axial stress in a column, ksi

σ_y ----- yield stress, ksi

σ_p ----- proportional limit stress, ksi

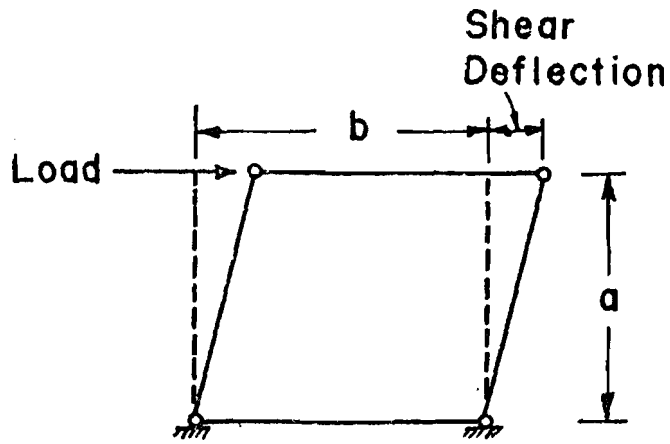
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METAL BUILDING





CANTILEVER SHEAR DIAPHRAGM
TEST ARRANGEMENT

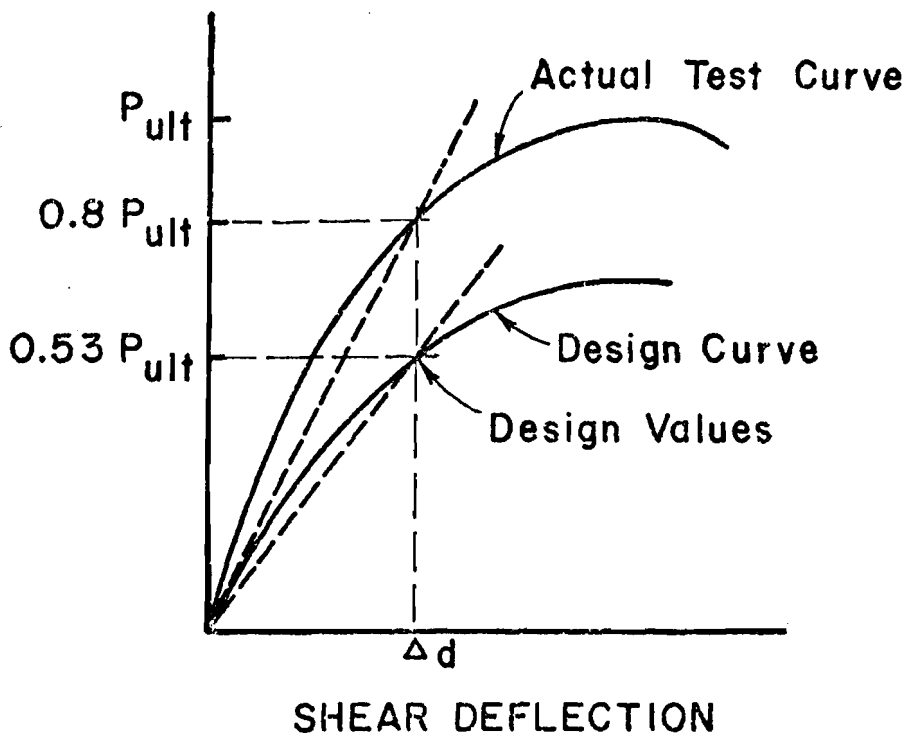
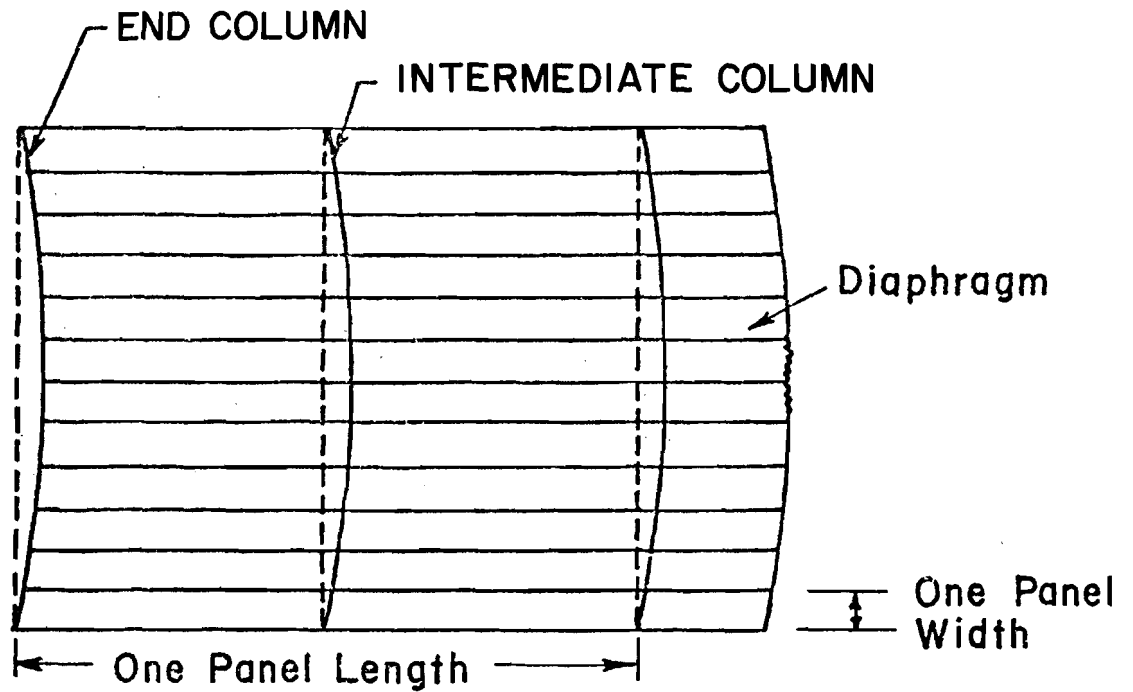
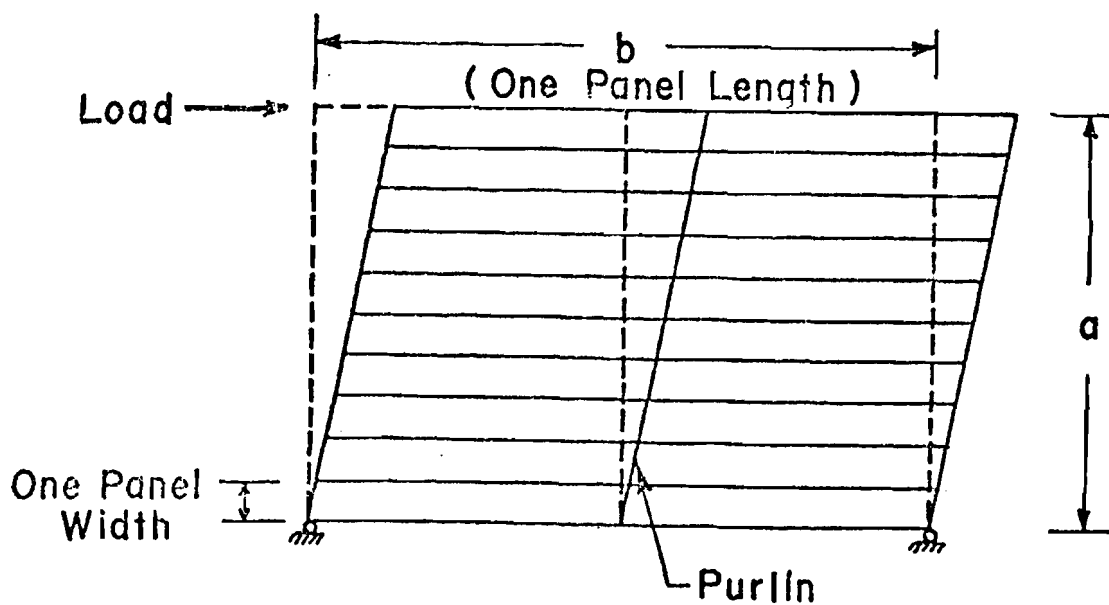


FIG. 2 LOAD-DEFLECTION RELATIONSHIP OF A SHEAR
DIAPHRAGM OBTAINED FROM A CANTILEVER TEST



(a) PLAN VIEW



(b) CANTILEVER SHEAR DIAPHRAGM TEST ARRANGEMENT

----- Original Position

———— Deflected Position

FIG. 3 DEFLECTED POSITION OF A DIAPHRAGM IN A DIAPHRAGM-BRACED COLUMN ASSEMBLY, AND IN A SHEAR DIAPHRAGM TEST

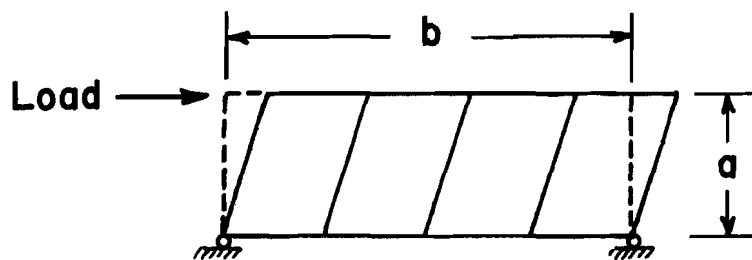
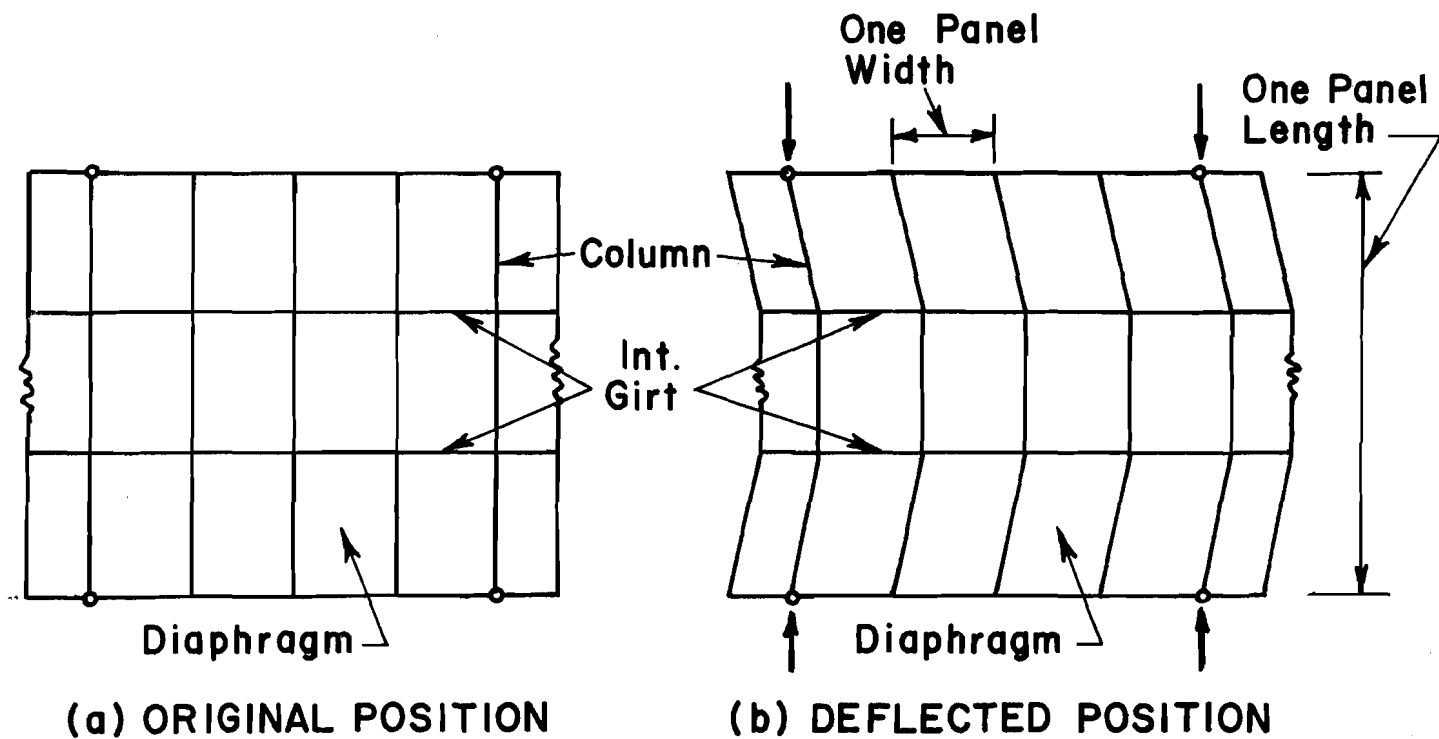
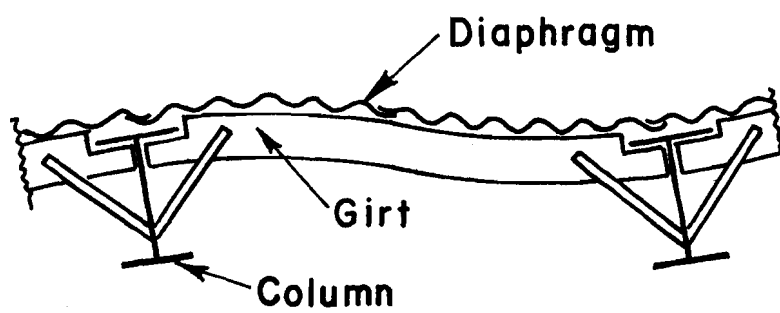
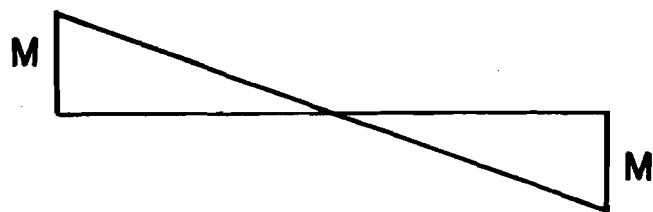


FIG. 4 ORIGINAL AND DEFLECTED POSITIONS OF A DIAPHRAGM IN COLUMN-GIRT-DIAPHRAGM ASSEMBLY

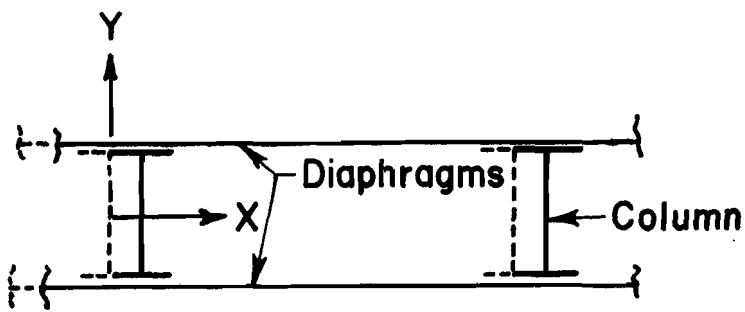


(a) DEFLECTED POSITION

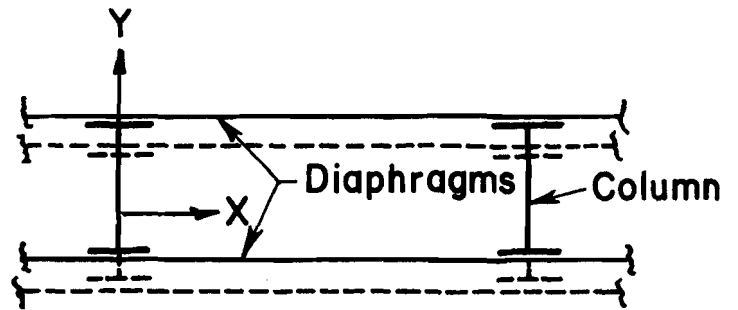


(b) MOMENT DIAGRAM FOR THE GIRT

FIG. 5 DEFLECTED POSITION OF A COLUMN-GIRT-DIAPHRAGM ASSEMBLY AND MOMENT DIAGRAM FOR THE GIRT



(a) WEAK-AXIS BUCKLING

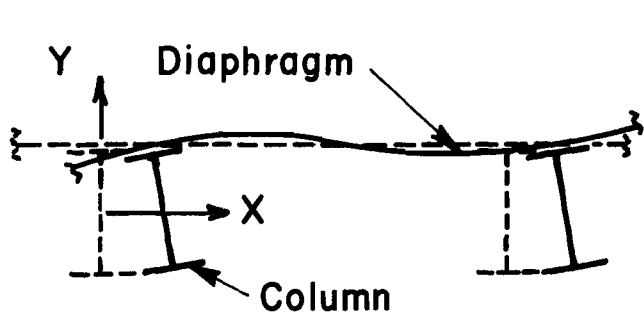


(b) STRONG-AXIS BUCKLING

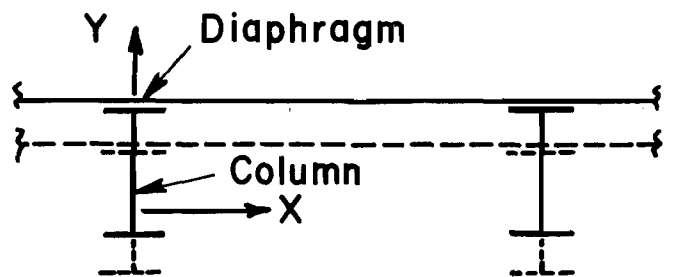
----- Original Position

———— Deflected Position

FIG. 6 BUCKLING MODES OF COLUMNS WITH DIAPHRAGM BRACING ON BOTH FLANGES



(a) TORSIONAL FLEXURAL BUCKLING

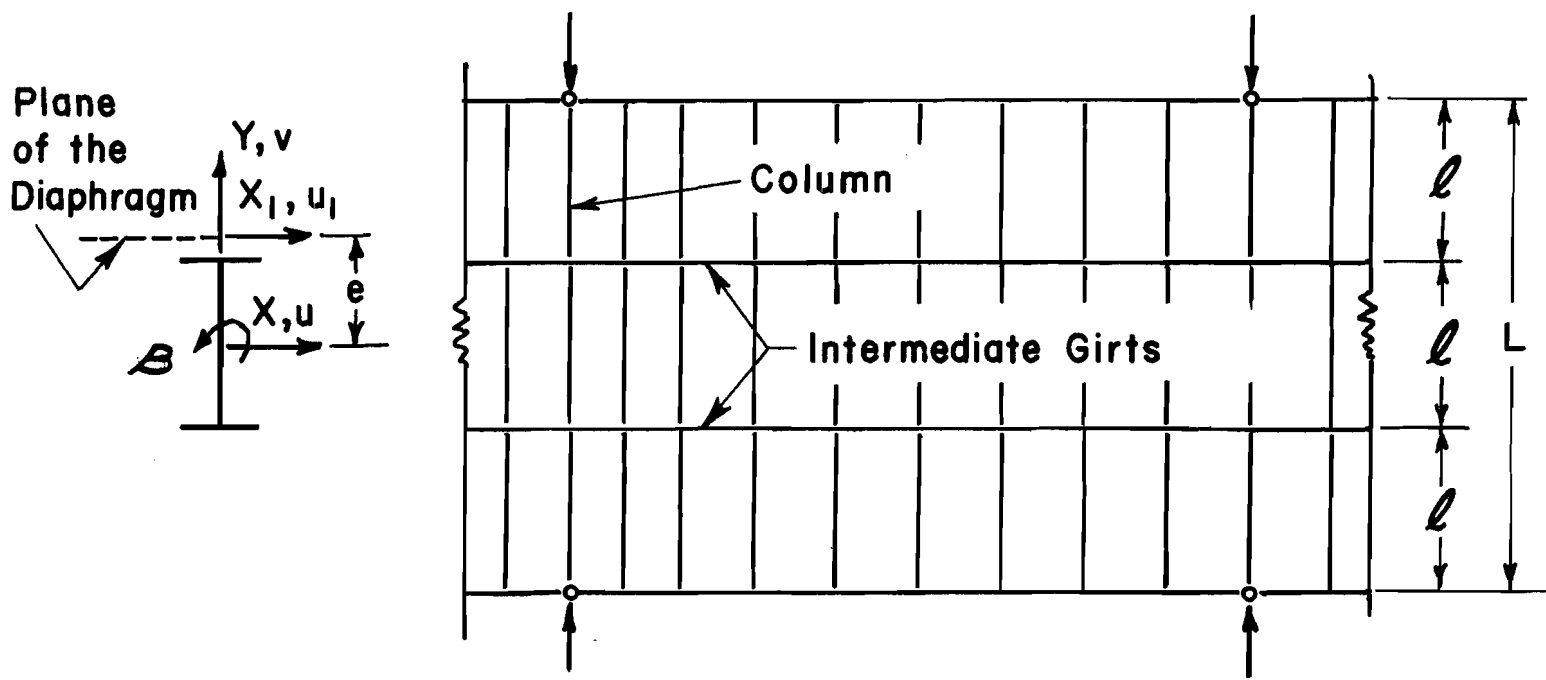


(b) FLEXURAL BUCKLING

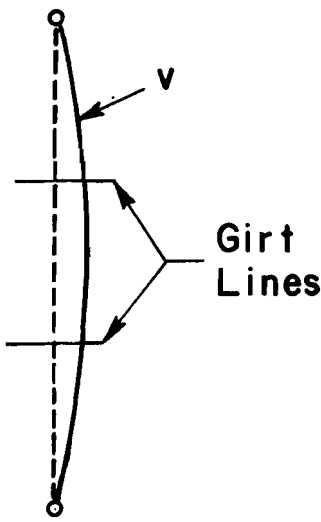
----- Original Position

———— Deflected Position

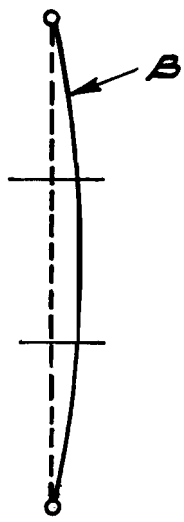
FIG. 7 BUCKLING MODES OF COLUMNS WITH DIAPHRAGM BRACING ON ONE FLANGE ONLY



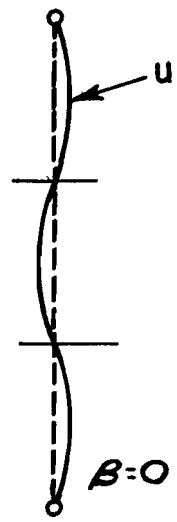
(a) COLUMN - GIRT - DIAPHRAGM ASSEMBLY



(b) FLEXURAL BUCKLING ABOUT X-AXIS



(c) TORSIONAL FLEXURAL BUCKLING



(d) FLEXURAL BUCKLING ABOUT WEAK AXIS

FIG. 8 BUCKLING MODES OF A "FULLY-BRACED" COLUMN WITH DIAPHRAGM-GIRT BRACING

DESIGN OF I-SHAPED COLUMNS WITH DIAPHRAGM BRACING

Key Words: Bracing; Buckling; Buildings; Columns; Diaphragms;
Shear Strength; Structural Engineering.

Abstract

A procedure is presented for the design of I-shaped columns braced by diaphragms. The diaphragms can be formed, for example, by interconnected cold-formed steel panels which are often used as wall sheathing for steel framed buildings. The diaphragms may be directly attached to one or both flanges of the columns, or connected to girts which in turn are connected to the columns. The procedure is based on the ultimate load capacity of fully braced members, utilizing a conservative estimate of the shear strength and shear rigidity of the diaphragm. Design examples are included. In many instances the existing wall sheathing provides adequate bracing against buckling. The method is similar to one proposed earlier for diaphragm-braced I-section beams.

Summary

A procedure is presented for the design of I-section columns braced directly by diaphragms, or by girts which in turn are braced by diaphragms. The method utilizes the shear strength and rigidity of diaphragms such as those formed by interconnecting cold-formed steel panels. Design examples are included.