01 Aug 1973

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A research project sponsored by the
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PREFACE

This report was originally a thesis presented to the Faculty of the Graduate School of Cornell University for the degree of Doctor of Philosophy, for conferment in May 1973.

The research project covered by this report was sponsored by the American Iron and Steel Institute.

The author wishes to express his deepest gratitude to Professors George Winter and Teoman Peköz, Project Directors. Their suggestions, criticism and guidance made this work possible.

The valuable cooperation of the Sheet Committees of the American Iron and Steel Institute and of Dr. Albert L. Johnson of the same institute is gratefully acknowledged.

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NOMENCLATURE

(Nomenclature for the Design Procedure and the Computer Programs are included in Chapter 6 and Appendix 4, respectively.)

A = cross-sectional area

a = dimension of the web (centerline dimensions)

b = dimension of the falnge (centerline dimensions)

c = dimension of the lip (centerline dimensions)

C_1 = amplitude of deflection in the x-direction (first term)

C_n = amplitude of deflection in the x-direction (general term)

C_w = warping constant

D = strain energy of the diaphragm

D_{nn} = matrix defined by Eq. (28a)

D_s = shear strain energy of the diaphragm

D_F = rotational strain energy

D_1 = amplitude of deflection in the y-direction (first term)

D_n = amplitude of deflection in the y-direction (general term)

d = overall dimension of web (depth of section)

d_1, d_2 = distance between the shear center and the top and bottom of the section, respectively

E = modulus of elasticity

E^* = inelastic modulus

E_t = tangent modulus

E_1 = amplitude of rotation of the column (first term)

E_n = amplitude of rotation of the column (general term)

F = rotational restraint by diaphragm bracing

F' = rotational restrain at $0.8P_{ult}$

F_r = reliable rotational restraint = $\frac{2}{3}F'$ (equivalent to F used in the governing equations and the theory)

F.S. = factor of safety (= 1.92)

G = shear modulus

G' = shear stiffness at $0.8P_{ult}$

G_r' = reliable shear stiffness = $\frac{2}{3}G'$

h_x, h_y = distances from point of constraint to the centroid (Ref. 3)

I_p = polar moment of inertia about shear center

I_x = moment of inertia with respect to x-axis

I_y = moment of inertia with respect to y-axis

I_{xy} = product of inertia with respect to x- and y-axes

I_{x1} = moment of inertia about the major axis

I_{y1} = moment of inertia about the minor axis

J = St. Venant torsion constant

$K = 1 - x_o^2/r_o^2$

K_ϕ = torsional modulus of elastic support (Ref. 3)

$K_1, K_2, \ldots K_{12}$ = constants accounting for different end conditions (see Table 1)

ℓ = span of the cantilever in the testing procedure of the rotational restraint F

L = length of the column

M_F = transverse moment applied to unit length of the diaphragm during testing

M_{ult} = ultimate moment applied to the diaphragm in the testing for the rotational restraint F

n = number of half-sine waves into which the column may buckle, or the nth term in the series

P = buckling load

P_x = Euler buckling load about the x-axis (strong axis buckling)

P_y = Euler buckling load about the y-axis

P_{xy} = defined by Eq. (25c)

P_ϕ = torsional buckling load

P_{x1} = Euler buckling load about the major axis of inertia

P_{y1} = Euler buckling load about the minor axis of inertia

P_{ult} = ultimate load in cantilever test

P'_ϕ = defined by Eq. (141b) (see also 14b)

P'_y = defined by Eq. (152)

P_{all} = allowable load on the stud

P_o = specified load on the stud (Section 6.3B)

P_a = inelastic buckling load

P_{cr} = critical buckling load

P_{crf} = buckling load between the fasteners

P_r = load capacity

Q = shear rigidity of the diaphragm bracing

Q_A = shape factor of the column

Q_r = reliable shear rigidity of the diaphragm

Q_I, Q_{II} = defined in Section X.6.3 of Appendix 6

R = defined by Eqs. (148) for channel section and (153) for z-section

r_o^2 = I_p/A

s = fastener spacing

U = strain energy of column

u = displacement of the shear center along the x-axis

u_o = initial imperfection in the x-direction

u_t = total displacement in the x-direction

u_D = displacement in the plane of the diaphragm

u_N = displacement of point N in the x-direction (Ref. 3)

v = displacement of the shear center along the y-axis

v_o = initial imperfection in the y-direction

v_t = total displacement in the y-direction

w = width of the diaphragm contributing to the bracing of one column

W = potential energy of the applied loads

x_o, y_o = distance between the centroid and shear center along the x- and y-axis, respectively

ϕ_{max} = calculated value of rotation of the column

λ = trial reduction factor less than 1.0

Π = total potential energy in a system

ϕ = rotation of the cross-section

ϕ_B = rotation due to cross bending of the diaphragm

ϕ_d = design rotational capacity of the diaphragm at $0.8P_{ult}$

ϕ_D = rotation caused by local deformation at the fastener location

ϕ_s = rotation due to deformation of the flange with respect to the web

ϕ_o = initial imperfection of the column

ϕ_{total} = total rotation of the column

α = factor used in the charts

$\alpha(z)$ = rate of change of deflection with respect to z-coordinates

σ = unit axial stress

σ_p = proportional limit stress

σ_y = yield stress

Δ_d = shear deflection at $0.8P_{ult}$ (cantilever test)

γ_d = design shear strain (at $0.8P_{ult}$)

γ_{max} = calculated value of shear strain in the diaphragm

Δ, Δ_o = defined by Eq. (69)

Δ_B = elastic deflection of the diaphragm due to bending in a beam type action

Δ_D = deflection due to local deformation of the diaphragm at the fastener location

Δ_S = deflection due to deformation of the flange with respect to the web

ABSTRACT

Lateral bracing has a significant effect on increasing the buckling load of compression members. In the case of wall stud construction, such bracing is provided by wallboards directly attached to the stud along its length and results in increasing the load carrying capacity significantly. The objective of this investigation is to study the behavior of singly symmetric sections braced by shear diaphragms and to apply the theoretical findings verified by experimental results to the design of wall studs.

In the present investigation the shear rigidity as well as the rotational restraint of the diaphragm are considered. Using an energy approach, general solutions are obtained for the cases of bracing on one or both sides. Solutions for channel, Z and I-sections are derived as special cases from the general solution.

Depending on the relative magnitudes of the diaphragm and column characteristics, higher buckling modes, associated with buckling in more than one half-sine wave, may govern the behavior of the stud. Results of numerical investigations indicate that in some cases of sections braced on one side only, higher buckling modes are as low as 50% of the critical buckling load computed by considering one half-sine wave only. On the other hand, higher buckling modes do not govern the behavior of sections braced on both sides with diaphragms whose characteristics are within the range of wall stud applications.

The shear rigidity as well as the rotational restraint of the diaphragm required for prediction of the failure load of the braced stud are determined experimentally using a variety of wallboard materials and fastener spacings.

The proposed design procedure is based on the ultimate load capacity of the column, utilizing a conservative estimate of the shear rigidity and rotational restraint of the wallboards acting as bracing diaphragms. The design procedure is applicable to buckling in the elastic and the inelastic domain. Beyond the elastic limit load, the influence of diaphragm bracing is less pronounced and high values of shear rigidity and rotational restraint would be needed to maintain the stability of the stud.

Based on the suggested design procedure, four computer programs are prepared for design of wall studs. Design aids in the form of charts and approximate formulas are provided to facilitate the use of the governing equations in predicting the critical buckling load.

Tests conducted on a total of 11 double-column assemblies of cold-formed steel sections with diaphragms on one or both sides have shown satisfactory agreement with the theoretical results. This indicates that the proposed design approach appears to be reliable.

Chapter 1

INTRODUCTION

1.1 Statement of the Problem

Lateral bracing can be used to eliminate the buckling of a compression member about its weak axis and thus increase the buckling load. Such bracing may be provided by diaphragms directly attached to the member along its length, typically wall sheathings attached to steel studs.

Previous research on diaphragm braced columns developed at Cornell had dealt only with doubly symmetric I-sections as they are used in conventional construction. This investigation intended to extend and generalize the theory on the stability of diaphragm-braced columns of symmetrical cross-sections to include columns made of singly symmetric and point symmetric sections, such as channels and zee-sections.

The goal of this investigation is to apply the results of the present investigation to wall-studs in order to modify the design approach of Section 5.1, Wall Studs, of the current "Specification for the Design of Cold-Formed Steel Structural Members,"(1) This specification was developed from some of the earliest work carried out at Cornell for the American Iron and Steel Institute some 25 years ago. While these provisions have remained essentially unchanged since the first or second edition of the specifications, they have two shortcomings:

a) The formulas of the present provision are based on the assumption that the collateral wall material furnishes an elastic extensional medium (spring supports) bracing the flexible stud to rigid parts of the structure or immovable objects considered fixed in space, such as a braced bay or a shear wall. In many cases the bracing of studs is provided in a manner different from that considered in the analysis and a different diaphragm action ensues. When the wall stud undergoes detrimental types of deformations at critical loads, the diaphragm resistance to distortion is maintained by its in-plane shear rigidity, rather than a spring type support. This type of diaphragm behavior in braced systems has been thoroughly investigated in the intervening years. With this development as the background it seems necessary to develop a different set of criteria for the stability of a braced stud against buckling.

b) The provision is limited to wall sheathing attached to both faces of the stud and gives no guidance to the frequent case of wall sheathing attached to one face only.

1.2 General

Cold-formed steel studs in walls or load-carrying partitions constitute the load carrying element in this type of light construction. The main function of the wall sheathing is that of enclosure, but it can also serve as a bracing system for the studs. Among the commonly used types of wall material are gypsum board, vegetable fiberboard and tempered board. Such materials, when used with steel studs, provide a practical, quickly erected, economical framing system for interior and exterior load bearing walls. Factory produced units of these composite walls are expedient to the recent developments of industrialized buildings and modular housing because they offer the use of one component system throughout the building.

The function of the bracing diaphragm in a system of two identically braced columns is to resist the forces which occur when the members deflect laterally under the action of the critical loads. The diaphragm in such a deformed state may be assumed to be in a state of pure shear, with elements of the diaphragm in a direction transverse to the members remaining mutually parallel during deformation. In wall stud construction, studs are essentially identical and such an assumption is practically valid.

Collateral or sheathing wall materials, often referred to as diaphragms, resist in-plane translation and rotation of the cross-section of the stud by virtue of their shear rigidity and rotational restraint, respectively. These properties of the diaphragm vary substantially for different types of materials, and the types of fasteners and their spacing used to connect the diaphragm to the stud. Failure of diaphragms in this type of construction is generally due to localized bearing followed by piling up of diaphragm material at the fastener location, as in the case of gypsum boards. Another type of failure is the tearing of the diaphragm material at the fastener location, as in the case of Celotex boards. Such failure is referred to as connection failure and generally is the primary cause of buckling of the braced stud. Therefore, properly fastened diaphragms are vital to stability and safety of the structure. Winter[2] indicates in a publication about light-gage (thin-walled) steel structures for buildings that tests show the insensitivity of welded steel diaphragms to cyclic loading from wind or earthquakes, whereas screw-connected diaphragms may be weakened by reversed loading of substantial magnitude. This observation lends itself to the case of wall studs braced with non-weldable diaphragms and it might be worthwhile to suggest the use of proper adhesives as substitutes for/or in addition to screw fasteners. The idea became evident to the writer during the execution of the test program for the present investigation. Testing its feasibility, however, is beyond the scope of this work.

Channel sections are the only wall studs available in most manufacturers' catalogues, and it seems that zee-sections are not commonly used. There is no apparent reason why such a limitation would be imposed by the manufacturers. The present investigation has shown that the zee-section, when braced, can sustain larger loads than channel sections of the same geometric dimensions. Moreover zee-sections, when nested, are more convenient and economical to transport than channel sections. Such reasons are sufficient to encourage the use of zee-sections in wall-stud construction.

1.3 Review of Related Literature

The stability of axially loaded columns has been a favorite subject for theoretical and experimental research since Euler derived his column formula in 1744. The major facts about column behavior are well known to all engineers interested in fundamental concepts. Research work is still continuing on many details, however, to refine the analysis of the buckling loads for the purpose of safety and economy.

It was not until the early part of the twentieth century, however, that methods and design techniques aimed at increasing the load carrying capacity of the column became widely applicable. One of these methods is to restrain the column against buckling in the weak direction. In such cases the column is capable of carrying buckling loads as high as the buckling load of the next buckling mode, provided that the possibility of yielding and local buckling are eliminated. This will result in considerable economy, especially when the restraining elements exist in the structure for other functional needs.

The concept of elastic restraints, well know as elastic foundations, was introduced in 1867 by Winkler. Further development of the theory was made by Timoshenko[3] for the buckling of a bar on an elastic foundation. In his analysis he argued that if there are many equally spaced elastic supports of equal rigidity, then their action on the buckled bar can be replaced by the action of a continuous elastic medium. Assuming a general expression for the displacement and using an energy method approach, he arrived at a simple formula similar to the Euler formula, except that a reduced length substitutes for the actual length of the bar.

In 1940, Vlasov[4] presented the governing differential equations of combined torsional and flexural buckling of a thin-walled beam embedded in an elastic medium. He also noted that in general the integration of these equations is a very difficult mathematical problem. Despite Vlasov's comment, Timoshenko found that if the ends of the bar are simply supported, the substitution of assumed functions of displacements into the differential equations lead to a cubic equation for the critical load.

Using Vlasov's previously mentioned equations, Timoshenko (3) investigated the buckling of a bar with a prescribed axis of rotation. In such a case the elastic foundation provides infinite rigidity against translation of the bar cross-section, while rotation is elastically restrained. Likewise he solved the case of a bar with a prescribed plane of deflection.

Based on Wagner and Kappus theories, Goodier[(5)] in 1941 investigated the behavior of columns which are torsionally weak. He also extended the analysis to the case of a bar of arbitrary cross-section attached to a perfectly flexible but inextensible sheet and he concluded that the attachment of a bar to a sheet will usually increase the critical buckling load of the bar, a typical conclusion to all of the previously mentioned cases. It is of interest to note that Pincus[(6)] found that the load increase based on elastic supports is generally small compared to the contribution of the bracing diaphragm acting as a shear-restraint medium.

It appears that the investigations previously mentioned are in the interest of aircraft design and not meant to be directly applicable to building design. It was not until 1947 when Green and Winter[(7)] presented a method, based on extensible type supports, for the design of light gage steel columns in wall-braced panels. Formulas are given which completely specify the necessary characteristics of the wall material and attachment to prevent failure of the stud in the plane of the wall. Methods of testing the wall material to determine the modulus of support are also included. The method is extended to different cases of bracing and some details are revised in Ref. 8. In fact the provisions of Section 5.1, Wall Sutds[1], are based on the results of the investigation in Refs. 7 and 8.

Winter[9] gave a method to determine the magnitude of the expected lateral force at buckling and to establish a lower limit on two characteristics of the lateral support, namely strength and rigidity, in order to provide full bracing to the column. Full bracing as defined is equivalent to immovable lateral supports. In a discussion to Ref. 9, Larsen[10] extended Winter's analysis to shear-type lateral supporting media with the diaphragm continuously connected to the column. It follows that the restraining force at any point along the column is a function of the rate of change of the deflection at that point and not the deflection itself.

Pincus[6,11] developed a theory predicting the failure load of elastic members continuously braced by diaphragms. Two types of diaphragm behavior are assumed: a) spring-bed supports and b) shear-resistant supports. It is concluded that the first, occurring rather uncommonly, produces a relatively small increase over the unsupported failure load. On the other hand, the shear-restrained support, found in many practical cases, may produce an n-fold increase over the buckling load of the unbraced column. From the general energy expression for a beam-column derived by F. Bleich[12], Pincus obtained a theoretical solution to the problem of a concentrically loaded I-section column braced by a shear diaphragm either on both sides or on one side of the section. The theoretical results are compared to eight tests of hot-rolled I-section columns braced with corrugated steel sheets.

Errera[13,14] corrected and modified some of the solutions presented by Pincus for the I-section column. Both Errera and Pincus adopted the double beam shear test to determine the shear rigidity of the diaphragm. In Ref. 13, it is noted that columns with an enforced axis of rotation are capable of carrying a higher load than columns not constrained in that manner.

Apparao[15,16] investigated the behavior of hot-rolled I-section columns braced with girts which in turn are braced with corrugated steel sheets and extended the analysis to the inelastic range. Jointly with Errera[17] a design recommendation for diaphragm-braced beams and symmetrical I-section columns was suggested. References 6, 11, 13, 14, 15, 16 and 17 have utilized the shear rigidity of the diaphragm but neglected its rotational restraint, with the justification that the buckling loads thus obtained are on the conservative side. Their solution is valid only for hinged and fixed end conditions, with mixed end conditions not considered in the analysis.

Dooley[18] presented a solution for the problem of an axially loaded symmetrical I-section column attached at finite intervals to sheeting rails and shear-stiff cladding. The supporting elements provide a total restraint against translation in the plane of the sheeting rails and an elastic restraint against rotation of the cross-section. He found that the column has adopted an instability trend towards torsional failure about the attached flange and that this may be analyzed by representing the restraint as continuous. In another paper, Dooley[19] extended the analysis to columns of nonsymmetrical I-sections with a restrained axis of twist under doubly eccentric load. Dooley's investigation[18] is similar to Apparao's[15] except that the solution of Ref. 18 does not permit translation of the cross-section, and in addition the initial imperfections of the column are neglected.

Horne[20] used a similar approach to that of Refs. 18 and 19 to obtain the more general solution for a column subjected to axial load together with uniform moment about the major axis. The buckling conditions are derived for an I-section column supported laterally by uniformly spaced side-rails, which provide rigid lateral supports and elastic torsional restraints. It has been stated that if the column buckles between the consecutive supports the lateral supports are fully effective and can be defined as "complete lateral supports".

1.4 Scope of the Investigation

In contrast to Sections 1, 2 and 3 of this chapter which serve to introduce the problem as well as the subject of diaphragm-braced columns in general, it is the aim of this section to outline the structure of the investigation itself.

Chapter 2 represents the basic theory of stability of diaphragm-braced columns. Most of the relations and expressions used in the main body of the thesis are derived and explained in this chapter.

Chapter 3 serves the purpose of checking the theoretical result against known solutions of special cases of Timoshenko [(3)]. Two examples show that the general equations of stability derived in this investigation can be used to obtain solutions of special cases.

Chapter 4 gives the results of several attempts to simplify the governing equations. Approximate formulas and charts for the cases of two-sided bracing are presented and their use is illustrated in Examples 1 and 2 of Appendix 1.

The experimental investigation of diaphragm-braced wall-studs is presented in Chapter 5. Comparison between experimental and theoretical results are included in Table 3.

Chapter 6 presents the proposed design procedure for elastic and inelastic analysis, as well as the collection of all equations that are needed in the design. Three practical examples to illustrate the proposed design procedure are given in Appendix 1.

A suggested computer program as well as its flow chart is included in Appendix 4. The program has been prepared for the cases of I, channel and zee-sections to serve as a design tool.

Chapter 2

THEORY OF DIAPHRAGM-BRACED COLUMNS

2.1 Basic Assumptions

Since we are dealing with a composite structure consisting of a load carrying member and a supporting diaphragm, the assumptions concerning each part of the composite structure will be reviewed independently. Regarding the column:

a) The member is prismatic and its cross-sections remain undeformed during buckling. This assumption has been considered with the rise of the theory of thin-walled members[3,4], and (up to now) no disagreement regarding its validity in practical situations has been noticed in the existing literature. Recently the effect of deformation of the corss-sections in their own planes has been considered by Wittrick[50], Goldberg et al[29], and Ghobarah[30]. This trend in the analysis aimed to investigate the overall and local buckling behavior and it is apparent that the interaction between the two exists. However, Peköz[34] in a discussion of the same assumption noted that for members of dimensions such that column behavior is predominant, the theory of torsional-flexural buckling provides relatively simple and accurate solutions.

b) Longitudinal axial strains due to axial load and shearing strains due to shear and warping of the cross-section are neglected.

c) Deformations are small with respect to the dimensions of the cross-sections (linearized problem).

d) Loads are applied statically at the centroid.

e) There are no initial imperfections (This will be considered later.).

f) The material is assumed to be linearly elastic. Modification of the results to account for the inelastic case is considered in Chapter 5.

Concerning the diaphragm, the following is considered:

a) The behavior of the diaphragm remains elastic until failure.

b) Compatibility of displacements is maintained between the column and the diaphragm.

c) Applied loads are sustained by the column alone; contribution of the diaphragm is neglected.

2.2 Method of Solution

The solution constitutes deriving the relationship between the critical buckling load of the column (P_{cr}) and both the shear rigidity (Q) and rotational restraint (F) of the diaphragm. Hence, P_{cr} can be determined if Q and F are known or values of Q and F may be calculated so that a certain load P_{cr} can be sustained by the column.

Considering a general cross-sectional shape of the column, the solution is derived separately for the following two cases:

a) Columns braced on both sides (Fig. 1).

b) Columns braced on one side only (Fig. 2).

The buckled shape of the column when the critical load is reached involves three generalized displacements, u, v and ϕ, of the shear center (Fig. 3). Accounting for these displacements in the analysis will add to the complexity of the solution as well as to the resulting governing equations. Considering that our goal is to find a solution to one of the simple structural problems, namely wall-studs, simple displacement functions are therefore utilized whenever possible. The energy approach offers the means of approximate solution in the cases in which the exact solution becomes too difficult or is not practicable. Another advantage of using this approach is noted by Winter[28] and Galambos[22], and emphasizes that fortunately the energy concepts are not very sensitive to variations of the deflected shape, and so we can expect reasonable results if we use an approximation of the deflected shape of the member.

2.3 Formulation of the Problem by the Energy Method

An energy principle in conjunction with the Rayleigh-Ritz method is used to obtain an approximate solution to the problem. The method is based on the principle that the total potential of the system must be a minimum if the system is to be in static equilibrium[21]. The total potential Π for the system of the diaphragm-braced columns is composed of the strain energy of the column U, the strain energy of the diaphragm D and the potential of the applied loads W, that is

$$\Pi = U + D + W \tag{1}$$

In mathematical terms the condition of equilibrium is expressed as

$$\delta\Pi = 0 \tag{2}$$

This states that for equilibrium the first variation of the total potential must vanish. Equation (1) can be used with the methods of the calculus of variations to obtain the governing differential equations. However, no direct solution can be found from these differential equations and on having a solution we face too unwieldy expressions. As an alternative to a direct solution of the governing differential equations, the Rayleigh-Ritz method is applied to the expression of the total potential energy to obtain a set of homogeneous simultaneous algebraic equations. These equations are expressed in terms of a set of indeterminate parameters of assumed displacements. The nontrivial solution of these equations determines the critical buckling load of the column. References 21, 22 and 32 indicate that the first variation of the total potential energy is not too sensitve to variations of the deflected shape and we can expect reasonable results if we use an approximate deflected shape of the columns, taking into consideration that the assumed deflected shape satisfied the end conditions of the column.

2.3.1 General Energy Expressions

In order to obtain a solution in a general form, it is necessary to express the total potential of the braced-column in terms of general parameters. Equation (1) states that

$$\Pi = U + D + W$$

where U = strain energy of the column

D = strain energy of the diaphragm

W = potential energy of the applied loads

(The form of the expression of each of the above terms will be considered below.)

2.3.2 Strain Energy of the Column (U)

In seeking a general solution, it was necessary to express the strain energy of the column in terms of parameters more general than those considered in Bleich's[12] energy expression which has been used in previous investigations[6,13,15].

Bleich selected as a system of coordinates X and Y, the principal axes of inertia with the centroid of the cross-section as the origin. Such a consideration tends to complicate the formulation of the energy expression in the case of the diaphragm-braced zee-sections. This appeared to be the reason that in Ref. 15, differential equations based on equilibrium consideration have been derived wherever Bleich's expression was not applicable. Also the same reason has been mentioned in conversation with N. Celebi[24].

In this investigation it has been found convenient to abandon the principal axes and take the x and y coordinates through the shear center, parallel and normal to the bracing diaphragm. For this purpose an energy expression developed by Goodier[5] is employed. Goodier, in 1941, by extending the ideas of Wagner[25,26], simplified Kappus'[27] theory and presented a simpler expression of the potential energy in which the X and Y axes are in any arbitrary position passing through the shear center of the cross-section.

Consider the general case of a column of any cross-section and an arbitrary set of axes X, Y, Z passing through the shear center as shown in Fig. 3. The strain energy of the column in terms of generalized displacements u, v of the shear center and rotation ϕ of the column section is given by[5]

$$U = \frac{1}{2}EI_y\int_0^L u''^2 dZ + EI_{xy}\int_0^L u''v''dZ + \frac{1}{2}EI_x\int_0^L v''^2 dZ$$

$$+ \frac{1}{2}EC_w\int_0^L \phi''^2 dZ + \frac{1}{2}GJ\int_0^L \phi'^2 dZ \tag{3}$$

where I_x, I_y are moments of inertia and I_{xy} is the product of inertia about the centroidal axes (X and Y) parallel and normal to the diaphragm.

2.3.3 Strain Energy of the Diaphragm

The strain energy of the diaphragm consists of two parts:

a) Shear strain energy, due to shear deformations in the plane of the diaphragm as a result of the component of lateral deflection of the column in the plane of the diaphragm.

The shear strain energy associated with one column as given in Ref. 13 is:

$$D_s = \frac{1}{2}\int_0^L Q[\alpha(Z)]^2 dZ \tag{4}$$

where Q = shear rigidity of the diaphragm contributing to the support of the column

$\alpha(Z)$ = lateral slope in the plane of the sheet (rate of change of deflection with respect to z-coordinate)

b) Rotational strain energy, due to the transverse rotation of the diaphragm at the location of the attachments during rotation of the column.

Figure 4 shows the original and final position of a braced section after rotating an angle ϕ. Such rotation imposes on the diaphragm a transverse moment acting at the diaphragm-column attachments. However, in the analysis it is assumed that this transverse restraining moment is continuous along the column length rather than being concentrated at the location of the attachments. In practical applications, the distance between the attachments compared to the column length and diaphragm dimensions justifies such an assumption. The same idealization is considered by Dooley[18, 19] for columns restrained at finite intervals against rotation by shear-stiff cladding. A similar idealization is considered by Winter et al [7] for columns braced with wall panels and by Timoshenko[3] for buckling of bars on elastic foundations, by replacing the action of spaced lateral supports with the actions of a continuous elastic medium.

Most commonly used diaphragms exhibit a certain amount of resistance to rotation, depending on the type of diaphragm and diaphragm-column attachment used. Such resistance provides rotational bracing to the column. The rotation of the diaphragm and the column consists of three parts:

1) ϕ_D due to local deformation at the fastener.
2) ϕ_B due to cross bending of the diaphragm.
3) ϕ_S due to deformation of the flange with respect to the web.

Hence, the total angle of rotation ϕ is equal to

$$\phi_{total} = \phi_D + \phi_B + \phi_S \qquad \text{(see Fig. 19)}$$

Depending on the location of the screw on the flange, a force in opposite direction than shown on Fig. 19 may lead to a larger ϕ_{total}.

It will be shown later (in the discussion of test results, Section 6.3.2) that the resistance of the diaphragm to local deformation at the fastener location is the major contributor to the diaphragm rotational restraint, especially for wall materials used in wall-studs applications.

The rotational restraint coefficient, F, is obtained experimentally since the local deformations cannot be determined analytically. The value of F should be based on the larger value of ϕ_{total} (giving a smaller value of F).

Denoting F as the rotational restraint coefficient of the diaphragm contributing to the bracing of one column, in units of moment per unit length of diaphragm per radian, and ϕ the angle of rotation of the column cross-section, then the transverse moment, M_F, applied to unit length of the diaphragm during twisting of the column section is equal to:

$$M_F = F \cdot \phi$$

The work done in rotating an element of unit length dZ is

$$\Delta D_F = \frac{1}{2} M_F \cdot \phi$$

Hence, integrating over the full length of the column, the total rotational strain energy associated with one column is

$$D_F = \frac{1}{2}\int_0^L M_F \cdot \phi dZ$$

or

$$D_F = \frac{1}{2}\int_0^L F \cdot \phi^2 dZ \qquad (5)$$

Adding Eqs. (4) and (5), the total strain energy of the diaphragm is

$$D = D_S + D_F$$

$$D = \frac{1}{2}\int_o^L Q[\alpha(Z)]^2 dZ + \frac{1}{2}\int_o^L F\cdot\phi^2 dZ \tag{6}$$

2.3.4 Potential Energy of Applied Loads (W)

Potential energy of applied loads during bending and twisting of the member is given in Ref. 5 as

$$W = -\sigma\{\frac{A}{2}\int_o^L u'^2 dZ + \frac{A}{2}\int_o^L v'^2 dZ + \frac{1}{2}I_p\int_o^L \phi'^2 dZ$$

$$+ Ax_o\int_o^L v'\phi' dZ - Ay_o\int_o^L u'\phi' dZ\}$$

$$= -\frac{1}{2}\int_o^L P(u'^2 + v'^2 + r_o^2\phi'^2 + 2x_o v'\phi' - 2y_o u'\phi')dZ \tag{7}$$

2.3.5 Total Potential of A System (Π)

By substitution of Eqs. (3), (6) and (7) into Eq. (1), the general expression of the total potential energy for a column of general shape is

$$\Pi = \frac{1}{2}\int_o^L \{EI_y u''^2 + 2EI_{xy}u''v'' + EI_x v''^2 + EC_w\phi''^2 + GJ\phi'^2$$

$$- P(u'^2 + v'^2 + r_o^2\phi'^2 + 2x_o v'\phi' - 2y_o u'\phi')$$

$$+ Q[\alpha(Z)]^2 + F\cdot\phi^2\}dZ \tag{8}$$

2.3.6 Total Potential of A System Braced on Both Sides

The general model utilized in the analysis as well as some of the column sections commonly used in structural application, and dealt with in the present investigation, are shown in Fig. 1. The model consists of a column of a general shaped section braced with identical diaphragms on both sides. These diaphragms exhibit shear rigidity Q and rotational restraint F and both properties are determined experimentally.

Consider the general displaced position of the cross-section as shown in Fig. 6, that is, translations u and v as well as rotation ϕ. Then the shear strain energy D_S as given by Eq. (4) is:

$$D_S = \frac{1}{2}\int_0^L Q[\alpha(Z)]^2 dZ$$

where Q and $\alpha(Z)$ are as previously defined. To account for two diaphragms, the above equation takes the form

$$D_S = \frac{1}{2}\int_0^L \frac{Q}{2}[\alpha_1(Z)]^2 dZ + \frac{1}{2}\int_0^L \frac{Q}{2}[\alpha_2(Z)]^2 dZ \qquad (9)$$

where $\alpha_1(Z)$, $\alpha_2(Z)$ are the rates of change of the lateral displacement with respect to Z in the plane of the bottom and top diaphragms, respectively.

From Fig. 6 it can be shown that the lateral displacement of the bottom diaphragm equals $(u - \phi dZ_2)$, hence

$$\alpha_1(Z) = \frac{d}{dZ}(u - \phi d_2) = u' - \phi' d_2 \qquad (10)$$

Similarly, for the top diaphragm,

$$\alpha_2(Z) = u' + \phi' d_1 \qquad (11)$$

where d_1 and d_2 are the distances from the shear center to the top and bottom diaphragms, respectively.

Substitution of Eqs. (10) and (11) into Eq. (9) yields

$$D_S = \frac{1}{2}\int_0^L Q[u'^2 + \phi'^2(\frac{d_1^2 + d_2^2}{2}) + u'\phi'(d_1 - d_2)]dZ \qquad (12)$$

Hence the total strain energy of the diaphragm D is:

$$D = D_S + D_F$$

$$= \frac{1}{2}\int_0^L \{Q[u'^2 + \phi'^2(\frac{d_1^2 + d_2^2}{2}) + u'\phi'(d_1 - d_2)] + F\cdot\phi^2\}dZ \qquad (13)$$

Using Eq. (13) to modify Eq. (8) to account for the case of two sided bracing, and considering the sign convention of Fig. 6, then the total potential energy of a system braced on both sides is:

$$\Pi = \frac{1}{2}\int_0^L \{EI_y u''^2 + 2EI_{xy}u''v'' + EI_x v''^2 + EC_w\phi''^2 + GJ\phi'^2$$

$$- P(u'^2 + v'^2 + r_o^2\phi'^2 - 2x_o v'\phi' + 2y_o u'\phi')$$

$$+ Q[u'^2 + \phi'^2(\frac{d_1^2 + d_2^2}{2}) + u'\phi' (d_1 - d_2)] + F\cdot\phi^2\}dZ \qquad (14)$$

2.3.7 Total Potential of A System Braced on One Side

Following the same procedure considered in the previous section and noticing in Fig. 7 that

$$u_D = u - \phi d_2$$

then $$\alpha(Z) = u_D' = u' - \phi' d_2 \qquad (15)$$

Substitution of Eq. (15) into Eq. (9) yields the total strain energy of the diaphragm D, which is

$$D = D_S + D_F$$

$$= \frac{1}{2}\int_0^L \{Q(u'^2 + \phi'^2 d_2^2 - 2u'\phi' d_2) + F\cdot\phi^2\} dZ \tag{16}$$

Hence, from Eqs. (8) and (16), the total potential energy of a system braced on one side is:

$$\Pi = \frac{1}{2}\int_0^L \{EI_y u''^2 + 2EI_{xy}u''v'' + EI_x v''^2 + EC_w\phi''^2 + GJ\phi'^2$$

$$- P(u'^2 + v'^2 + r_o^2\phi'^2 - 2x_o v'\phi' + 2y_o u'\phi')$$

$$+ Q(u'^2 + \phi'^2 d_2^2 - 2u'\phi' d_2) + F\cdot\phi^2\} dZ \tag{17}$$

2.4 General Solution

Assuming that a column with hinged ends may buckle in a number of half-waves of sinusoidal function, and considering similar shapes of the displacement functions (with different amplitudes) in the x and y-directions as well as the rotation of the column sections, then the displacements u, v and ϕ (Fig. 3) can be represented by the following infinite series (Assumed functions with different shapes are considered in Appendix 5.):

$$u = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi Z}{L} \tag{18a}$$

$$v = \sum_{n=1}^{\infty} D_n \sin\frac{n\pi Z}{L} \tag{18b}$$

$$\phi = \sum_{n=1}^{\infty} E_n \sin\frac{n\pi Z}{L} \tag{18c}$$

where n is the number of terms considered in the solution (n = 1,2,3,...). C_n, D_n, E_n is a set of indeterminate parameters which represents the amplitude of deflections and rotation.

These assumed displacements satisfy the column end conditions,

$$u = v = \phi = 0 \quad \text{for } Z = 0,L \tag{19a}$$

$$u'' = v'' = \phi'' = 0 \quad \text{for } Z = 0,L \tag{19b}$$

That is, the ends of the column are simply supported. For fixed end conditions the following infinite series may be assumed for the displacements u, v, ϕ,

$$u = \sum_{n=1}^{\infty} C_n\left(1 - \cos\frac{2n\pi Z}{L}\right) \tag{20a}$$

$$v = \sum_{n=1}^{\infty} D_n\left(1 - \cos\frac{2n\pi Z}{L}\right) \tag{20b}$$

$$\phi = \sum_{n=1}^{\infty} E_n\left(1 - \cos\frac{2n\pi Z}{L}\right) \tag{20c}$$

$(n = 1,2,3,\ldots)$

These displacement functions satisfy the column end conditions

$$u = v = \phi = 0 \quad \text{for } Z = 0,L \tag{21a}$$

$$u' = v' = \phi' = 0 \quad \text{for } Z = 0,L \tag{21b}$$

The solution of the case of two sided bracing is obtained by substitution of the assumed displacement function equations (18) into the expression of the total potential energy equation (14), and applying the Rayleigh-Ritz method, which requires that $\delta\Pi = 0$, which in the present case becomes:

$$\frac{\partial \Pi}{\partial C_n} = 0, \quad \frac{\partial \Pi}{\partial D_n} = 0, \quad \frac{\partial \Pi}{\partial E_n} = 0 \tag{22}$$

$(n = 1,2,3 \ldots)$

Equations (22) lead to a set of $3 \times n$ simultaneous algebraic equations in C_n, D_n and E_n $(n = 1,2,3\ldots)$. In matrix form these equations take the form

$$\underset{3n \times 3n}{\begin{bmatrix} D_{11} & & & & 0 & \\ & D_{22} & & & & \\ & & D_{33} & & & \\ & & & - & & \\ & 0 & & & - & \\ & & & & & D_{nn} \end{bmatrix}} \underset{3n \times 1}{\begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \cdot \\ \cdot \\ \Delta_n \end{Bmatrix}} = 0 \tag{23}$$

where

$$D_{nn} = \underset{3 \times 3}{\begin{bmatrix} (n^2P_y - P + Q) & n^2 P_{xy} & Q(\frac{d_1 - d_2}{2}) - Py_o \\ n^2 P_{xy} & n^2 P_x - P & Px_o \\ Q(\frac{d_1 - d_2}{2}) - Py_o & Px_o & r_o^2(P_\phi - P) + Q(\frac{d_1^2 + d_2^2}{2}) + \frac{F}{n^2}\frac{L^2}{\pi^2} \end{bmatrix}} \tag{24}$$

$(n = 1,2,3,\ldots)$

$$P_x = \frac{\pi^2 EI_x}{L^2} \tag{25a}$$

$$P_y = \frac{\pi^2 EI_y}{L^2} \tag{25b}$$

$$P_{xy} = \frac{\pi^2 EI_{xy}}{L^2} \tag{25c}$$

$$P_{\phi} = \frac{1}{r_o^2} \left(n^2 E C_w \frac{\pi^2}{L^2} + GJ\right) \tag{26}$$

$$\Delta_n = \begin{Bmatrix} C_n \\ D_n \\ E_n \end{Bmatrix} \tag{27}$$

$(n = 1,2,3 \ldots)$

Expanding Eq. (23), then

$$[D_{11}]\{\Delta_1\} = 0 \tag{28a}$$

$$[D_{22}]\{\Delta_2\} = 0 \tag{28b}$$

$$\begin{matrix} \cdot \\ \cdot \\ \cdot \end{matrix}$$

$$[D_{nn}]\{\Delta_n\} = 0 \tag{28n}$$

Hence, the $3 \times n$ simultaneous equations (23) can be segregated into n uncoupled sets of equations (28). Each of these sets contains, in general, 3 coupled equations.

Physically, this means that each of these sets, obtained for a certain value of n, corresponds to a certain buckling mode. Hence, n buckling loads can be obtained, the smallest value of which represents the critical buckling load of the system. This observation implies that Eqs. (18) may be replaced by the following simpler displacement functions without any effect on the final result:

$$u = C_n \sin\frac{n\pi Z}{L} \tag{29a}$$

$$v = D_n \sin\frac{n\pi Z}{L} \tag{29b}$$

$$\phi = E_n \sin\frac{n\pi Z}{L} \tag{29c}$$

($n = 1,2,3,\ldots$)

Therefore it is concluded that for a column with both ends hinged, the critical loads obtained from Eqs. (18) and (29) are identical and that this conclusion is valid for the case of one sided bracing as well. For fixed end conditions, upon substitution of Eq. (20) into the expression of the total potential energy equation (14) and following the same procedure of the hinged ends case, it has been found that the set of $3 \times n$ simultaneous algebraic equations represented by Eq. (23) are coupled. Hence, this differs from the case of hinged ends; n independent buckling modes will not occur. It follows that the simplification introduced in the case of a hinged ends column, replacing Eqs. (18) by Eqs. (29), cannot be achieved in this case. This is so for one sided bracing as well. A similar conclusion is valid for the cases of end conditions other than hinged or fixed (see end conditions listed in Table 1) with bracing on one or both sides of the column.

2.4.1 General Equation of Stability of A Two Sides Braced Column with Hinged Ends

Using the matrix given by Eq. (24), then the general form of the equation of stability of a system braced on both sides with column ends hinged is:

$$\begin{bmatrix} P_y-P+Q & P_{xy} & Q(\frac{d_1-d_2}{2})-Py_o \\ P_{xy} & P_x-P & Px_o \\ Q(\frac{d_1-d_2}{2})-Py_o & Px_o & r^2(P_\phi-P)+Q(\frac{d_1^2+d_2^2}{2})+\frac{F}{n^2}\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_n \\ D_n \\ E_n \end{Bmatrix} = 0 \quad (35)$$

where $n = 1,2,3...$

$$P_x = n^2\pi^2 EI_x/L^2 \quad (36a)$$

$$P_y = n^2\pi^2 EI_y/L^2 \quad (36b)$$

$$P_{xy} = n^2\pi^2 EI_{xy}/L^2 \quad (36c)$$

$$P_\phi = \frac{1}{r_o^2}(n^2 EC_w \frac{\pi^2}{L^2} + GJ) \quad (36d)$$

2.4.2 General Equations of Stability of A One Side Braced Column with Hinged Ends

Equation (17) gives the total potential of a column of a general shaped section braced on one side. By substitution of the assumed displacement function equations (18) or (29) into Eq. (17) and following the procedure for determining Eq. (35), outlined in Sections 2.4 and 2.4.1, the stability equation of a column braced on one side with ends hinged is given by the following:

$$\begin{bmatrix} P_y-P+Q & P_{xy} & -Py_o-Qd_2 \\ P_{xy} & P_x-P & Px_o \\ -Py_o-Qd_2 & Px_o & \frac{1}{r_o^2}\,(P_\phi-P)+Qd_2^2+\frac{F}{n^2}\,\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_n \\ D_n \\ E_n \end{Bmatrix} = 0 \tag{38}$$

where n = 1, 2, 3....and P_x, P_y, P_{xy}, P_ϕ are given by Eqs. (36).

2.4.3 P_{cr} of Particular Column Sections with Hinged Ends

Eqs. (35) and (38) will be used to derive the governing equations of the following cases:

a) Channel section braced on both sides.

b) Z-section braced on both sides.

c) Channel section braced on one side.

d) Z-section braced on one side.

The solution is given in terms of n, where n = 1, 2, 3.... The critical buckling load P_{cr} is the smallest value of P obtained from the governing equations for sufficient numbers of n. References 12 and 22 indicate that considering small values of n, that is, n = 1, 2, 3...., is sufficient to determine the smallest buckling load. However, this may not always be the case, and hence enough values of n should be tried until the smallest value of P is obtained.

For a particular cross-section the critical buckling load of the column will be derived from Eq. (35) or (38), by substituting for the geometric terms appearing in the general solution, those of the particular cross-section under consideration.

2.4.3.1 P_{cr} for Channel Section Braced on Both Sides (Hinged Ends)

For channel sections, $y_o = 0$

$$d_1 = d_2 = d/2$$

$$I_{xy} = 0, \qquad \text{hence } P_{xy} = 0.$$

Sunstituting the above parameters into Eq. (35) yields

$$\begin{bmatrix} P_y-P+Q & 0 & 0 \\ 0 & P_x-P & Px_o \\ 0 & Px_o & r_o^2(P_\phi-P+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_n \\ D_n \\ E_n \end{Bmatrix} = 0 \tag{39}$$

where n = 1,2,3,... and P_x, P_y, P_{xy}, P_ϕ are given by Eqs. (36). Notice that n is included in these parameters.

For a nontrivial solution of Eq. (39), the determinant of the coefficient matrix of C_n, D_n, E_n must vanish, hence

$$|D_{nn}| = 0 \tag{40}$$

then $$(P_y-P+Q)\{(P_x-P)[r_o^2(P_\phi-P)+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2}]-(Px_o)^2\} = 0 \tag{41}$$

Therefore two solutions are possible; these are

$$P_y - P + Q = 0 \tag{42}$$

and $$(P_x - P)[r_o^2(P_\phi - P) + Q\frac{d^2}{4} + \frac{F}{n^2}\frac{L^2}{\pi^2}] - (Px_o)^2 = 0 \tag{43}$$

Arranging terms of Eqs. (42) and (43) yields

$$P = P_y + Q \tag{44}$$

$$P^2(r_o^2-x_o^2)-P(r_o^2P+r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2})+P_x(r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2}) = 0 \tag{45}$$

Equation (44) characterizes the behavior of the column in the flexural mode and it can be seen that n = 1 gives the lowest buckling load. Equation (45) represents the torsional-flexural mode, and n must be chosen so that the buckling load thus obtained is minimum. Both modes are possible depending on the values of Q and F (see Fig. 8).

For a particular column with specific end conditions the terms P_y, P_x, P_ϕ represent the different possible buckling modes of the unbraced column and can be calculated from Eqs. (36) for chosen values of n.

Also the geometric parameters r_o^2 and x_o^2 are known from the section's dimensions. Therefore for a column braced with a diaphragm of known Q and F, the values of the buckling loads P can be calculated from Eqs. (44) and (45). The lowest value of P determined from both equations will give the critical buckling load of the column.

If Q = 0 and F = 0, that is, an unbraced column, and n = 1 then Eqs. (44) and (45) reduce to the same equations derived by Winter and Chajes[31]. Also the determinant in Eq. (39) will be the same as that of Timoshenko on page 333[3] and Eq. 20 of Peköz[32].

2.4.3.2 <u>P_{cr} for a Z-section Braced on Both Sides (Hinged Ends)</u>

For a Z-section,

$$y_o = 0$$

$$x_o = 0$$

$$d_1 = d_2 = d/2$$

Substituting these parameters into Eq. (17) yields:

$$\begin{bmatrix} P_y-P+Q & P_{xy} & 0 \\ P_{xy} & P_x-P & 0 \\ 0 & 0 & r_o^2(P_\phi-P)+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_n \\ D_n \\ E_n \end{Bmatrix} = 0 \qquad (46)$$

Solving for eigenvalues by setting the determinant of the coefficient matrix of C_n, D_n, E_n equal to zero and following the same procedure considered in the case of the channel section (2.4.3.1), then two solutions are possible:

$$P = P \quad + \frac{1}{r_o^2} \left(Q \frac{d^2}{4} + \frac{F}{n^2} \frac{L^2}{\pi^2}\right) \qquad (47)$$

and $$P^2 - P(P_x + P_y + Q) + (P_xP_y + P_xQ - P_{xy}^2) = 0 \qquad (48)$$

Equation (47) represents the increased torsional buckling load of the column. Since for point-symmetrical shapes under concentric loading, the torsional buckling rarely governs the mode of failure; Eq. (47) represents an upper bound to the expected buckling load obtained from Eqs. (47) and (48). Equation (48) governs the behavior of the column in the flexural mode. It is of interest to note that the rotational restraint of the diaphragm has no influence on the buckling load. This can be seen from Eq. (48), since F does not appear in the governing equation.

For a particular column with Q = 0 and end conditions hinged or fixed, it can be shown from Eq. (48) that $P_{cr} = P_{y1}$, where P_{y1} is the Euler buckling load about the axis of least moment of inertia. For other end conditions it will be proved, in Section 2.6A.3, that the Z-section column will not buckle about the axis of least moment of inertia and subsequently, a governing equation with various end conditions (listed in Table 1) will be given.

2.4.3.3 P_{cr} for a Channel Section Braced on One Side (Hinged Ends)

For a channel section, $y_o = 0$

$$d_2 = d/2$$

$$I_{xy} = 0 \qquad \text{hence } P_{xy} = 0$$

Substituting these parameters into Eq. (38) yields

$$\begin{bmatrix} P_y-P+Q & 0 & -Q\frac{d}{2} \\ 0 & P_x-P & Px_o \\ -Q\frac{d}{2} & Px_o & \frac{1}{r_o^2}(P_\phi-P)+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_n \\ D_n \\ E_n \end{Bmatrix} = 0 \qquad (49)$$

$(n = 1,2,3,\ldots)$

For nontrivial solutions of Eq. (49), the value of the determin ant of the coefficient matrix of C_n, D_n, E_n must vanish. Evaluating this determinant, the following third order polynomial is obtained:

$$P^3(r_o^2-x_o^2)-P^2[r_o^2P_x+r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2}+(P_y+Q)(r_o^2-x_o^2)]$$

$$+P[P_x(r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2})+(P_y+Q)(r_o^2P_x+r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2})$$

$$-(Q\frac{d}{2})^2]-(P_y+Q)[P_x(r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]+P_x(Q\frac{d}{2})^2 = 0 \qquad (50)$$

The smallest root of Eq. (50), determined by considering sufficient values of n, gives the critical buckling load.

2.4.3.4 P_{cr} for a Z-section Braced on One Side (Hinged Ends)

For zee-sections,

$$y_o = 0$$

$$x_o = 0$$

$$d_2 = d/2$$

Substituting these parameters into Eqs. (38) yields

$$\begin{bmatrix} P_y-P+Q & P_{xy} & -Q\frac{d}{2} \\ P_{xy} & P_x-P & 0 \\ -Q\frac{d}{2} & 0 & r_o^2(P_\phi-P)+Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_n \\ D_n \\ E_n \end{Bmatrix} = 0 \qquad (51)$$

Solving for the eigenvalues by setting the determinant of the coefficient matrix of C_n, D_n, E_n equal to zero and by evaluating the resulting determinant, the following third order polynomial is obtained:

$$P^3-P^2\,[P_x+P_y+Q+P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]$$

$$+P\{(P_y+Q)P_x-P_{xy}^2+(P_y+Q+P_x)[P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]-\frac{1}{r_o^2}(Q\frac{d}{2})^2\}$$

$$-[(P_y+Q)P_x-P_{xy}^2][P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]+\frac{1}{r_o^2}P_x(Q\frac{d}{2})^2 = 0 \qquad (52)$$

For a particular column and known values of Q and F, Eq. (52)

gives three values of buckling loads P for each value of n; the lowest value of P determines the critical buckling load P_{cr}.

2.5 Discussion of Cases with End Conditions Other Than Hinged

Based on assumed displacements in the form of an infinite series, the solution of the hinged end column is given in Section 2.4. Other end conditions (Table 1) are not considered for reasons which will be apparent in this section. However, by using the first term of the series it is possible to obtain a simple solution for the cases of these end conditions, provided that higher buckling modes are not critical.

It has been shown in Section 2.4 that in the case of a column with hinged ends, equations of the assumed displacements (18) can be replaced by Eqs. (29) without any change in the final result. This is because uncoupled modes of buckling, corresponding to each value of n, exist. The uncoupling of the modes rest chiefly on the orthogonality relations which exist between the terms of the assumed function. However, this is not the case for column with end conditions other than hinged, for example, fixed, or may be represented by the following geometrical condition (see also Table 1):

$$u = v = \phi = 0 \quad \text{at } Z = 0,L$$

$$u'' = v' = \phi' = 0 \quad \text{at } Z = 0,L$$

In such a case, upon using combinations of the assumed displacement functions chosen from Eqs. (18) and (20) to satisfy the above relations, it has been found that the set of algebraic equations resulting form minimizing the energy are coupled.

Therefore if n terms of the series are considered in the solution then the size of the matrix in Eq. (23) will be $3n \times 3n$. Hence, the requirement that the determinant of the coefficient matrix of C_n, D_n, E_n ($n = 1,2,3,\ldots$) must vanish for a nontrivial solution results in a polynomial of the $3n^{th}$ order. The smallest root of this polynomial gives the critical buckling load of the column. It is of importance to note that in our case the elements of the determinant are not all numerals; it contains eigenvalues added to and multiplied by numbers. These eigenvalues are not all on the diagonal of the matrix; some are off the diagonal. In other words it is impractical to evaluate such a determinant in order to arrive at a polynomial. Also the determinant is not in the known form of the eigenvalue problem which is written as

$$|A - \lambda I| = 0$$

Hence the problem may be classified as a polynomial equation of the $3n^{th}$ order.

Briefly, it can be stated that it is not a standard problem. The IBM Library Subroutines do not include direct aids to handle such a problem. A reference to a method published in an article by Jenkins and Traub[47] has been suggested by Cornell's Department of Computer Science.

Another approach to solve the problem is to assume a trial eigenvalue and then, after substituting the trial value in the determinant, check whether or not the latter vanished. Hence, the solution, though difficult, can be obtained provided that the entries of the stability matrix can be generated. To obtain these entries for $n = 3$ is quite involved and impractical, let alone the cases of $n > 3$ or if initial imperfections are considered.

The intent is to derive a design procedure for the simple case of wall-stud applications, for which the hinged end conditions simulate with reasonable conservative approximation the actual structure. Therefore the solution of cases with end conditions other than hinged and $n > 1$ are not of substantial importance to the development of the design procedure. On the other hand the close agreement between the test results of 11 double-column assemblies with end conditions $u'' = v' = \phi' = 0$ and $u = v = \phi = 0$ at $Z = 0, L$ and the predicted failure load based on $n = 1$ shows that higher buckling modes are not likely to govern. Hence the cases of end conditions listed in Table 1 will be given in the next section only for $n = 1$. The same conclusion has been considered by Pincus(6), Errera(13) and Dooley(18) in similar investigations despite the relatively simpler problems considered by them.

2.6 Solution by Considering Only the First Term of the Series

As an alternative design tool, the following closed form solutions will be derived using only the first term of the series, Eqs. (18) and (20). The solution is derived for a column with the general end conditions listed in Table 1, i.e. hinged, fixed and mixed. The following cases are considered for a column of a general shaped cross-section:

a) Column braced on both sides.

b) Columns braced on one side only.

Then particular cases of columns of channel and zee-sections will be derived as special cases of the general solution.

In order to obtain a general solution which accounts for the influence of the column end conditions on the buckling loads, coefficients K_i ($i = 1,2,\ldots,12$) are introduced in the resulting equations. Numerical values of K_i are listed in Table 1. These coefficients are calculated for each case of different end conditions by using the proper combinations of the following assumed displacement functions.

$$\left.\begin{array}{ll} \underline{\text{End Condition}} & \underline{\text{Displacement Function}} \\ u = u'' = 0 \text{ at } Z = 0,L & u = C_1 \sin\frac{\pi Z}{L} \\ u = u' = 0 \text{ at } Z = 0,L & u = (C_1/2)(1 - \cos\frac{2\pi Z}{L}) \\ v = v'' = 0 \text{ at } Z = 0,L & v = D_1 \sin\frac{\pi Z}{L} \\ v = v' = 0 \text{ at } Z = 0,L & v = (D_1/2)(1 - \cos\frac{2\pi Z}{L}) \\ \phi = \phi'' = 0 \text{ at } Z = 0,L & \phi = E_1 \sin\frac{\pi Z}{L} \\ \phi = \phi' = 0 \text{ at } Z = 0,L & \phi = (E_1/2)(1 - \cos\frac{2\pi Z}{L}) \end{array}\right\} \quad (53)$$

C_1, D_1, E_1 are amplitudes of deflections of u, v and , respectively. It has been found convenient to use $\frac{C_1}{2}$, $\frac{D_1}{2}$ and $\frac{E_1}{2}$ for fixed end conditions rather than C_1, D_1, E_1 as commonly used. This has no influence on the final results.

2.6A Equation of Stability of Columns Braced on Both Sides with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

The equation of stability is derived by substitution of the assumed displacement functions, chosen from Eqs. (53), into the expression of the total potential energy equation (14) and then applying the Rayleigh-Ritz method. This will result in three homogeneous simultaneous equations in C_1, D_1, E_1. These three equations are arranged in matrix form to give the following equation which describes the stability of the system in general form.

$$\begin{bmatrix} P_y-P+Q & P_{xy} & -K_5y_oP+K_6Q(d_1-d_2) \\ P_{xy} & P_x-P & K_7Px_o \\ -K_5y_oP+K_6Q(d_1-d_2) & K_7Px_o & r_o^2(P_\phi-P)+Q(\frac{d_1^2+d_2^2}{2})+K_8F\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_1 \\ D_1 \\ E_1 \end{Bmatrix} = 0 \tag{54}$$

where

$$P_x = K_1\pi^2EI_x/L^2 \tag{55a}$$

$$P_y = K_2\pi^2EI_y/L^2 \tag{55b}$$

$$P_{xy} = K_3\pi^2EI_{xy}/L^2 \tag{55c}$$

$$P = \frac{1}{r_o^2}(K_4EC_w\frac{\pi^2}{L^2} + GJ) \tag{55d}$$

K_1, K_2, K_3, K_4 ... K_8 are coefficients corresponding to different end conditions of the column and their values are given in Table 1

2.6A.1 Critical Buckling Loads of Particular Sections

Equation (54) will be used to determine the critical buckling load P_{cr}, of a certain column, as a function of the shear rigidity Q and the rotational restraint F. Two particular sections will be considered, namely channel and z-sections. It will be shown that the case of the I-section previously investigated[6,13] can be derived as a special case of the general solution.

For a particular cross-section the critical buckling load of the column will be determined by substituting for the geometric terms appearing in the general solution, those of the particular cross-section under consideration.

2.6A.2 P_{cr} for Channel Section Columns Braced on Both Sides with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

Applying the same Procedure of Section 2.4.3.1 to Eq. (54) the following governing equations are obtained:

$$P = P_y + Q \tag{56}$$

$$P^2(r_o^2-K_7^2x_o^2)-P(r_o^2P_x+r_o^2P_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})+P_x(r_o^2P_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2}) = 0 \tag{57}$$

The smallest value of P obtained from both Eqs. (56) and (57) give the critical buckling load P_{cr}. Equations (56) characterize the behavior of the column in the flexural mode. The occurrence of any of these modes is possible depending on the values of Q and F (see Fig. 8).

2.6A.3 P_{cr} for Z-section Columns Braced on Both Sides with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

Similarly, the following equations are obtained from Eq. (54), and P_{cr} is given by the smallest value of P determined from both equations:

$$P = P_\phi + \frac{1}{r_o^2}\left(Q\frac{d^2}{4} + K_8 F \frac{L^2}{\pi^2}\right) \tag{58}$$

$$P^2 - P(P_x + P_y + Q) + (P_xP_y + P_xQ - P_{xy}^2) = 0 \tag{59}$$

For the same reasoning given in Section 2.4.3.2, regarding the validity of Eq. (59) only, it is concluded that Eq. (58) does not govern since the torsional buckling mode for point-symmetrical sections rarely governs the failure mode of the column. Graphical representation of Eqs. (58) and (59) is shown on Fig. 9.

If Q = 0, that is, an unbraced column, an important and interesting result is obtained which, so far as the writer knows, hasn't been mentioned in the available literature: The Z-section column with mixed end conditions can only buckle about an axis in between the least axis of inertia and the web, and that such axis need not be located to calculate P_{cr} which is obtainable as a special case of Eq. (59).

It is well known that a Z-section column hinged at both ends in the x and y-axes (i.e. concentrically point-supported) will buckle about the axis of least inertia y_1, and the buckling load is given by the Euler equation:

$$P_{cr} = \frac{\pi^2 EIy_1}{(kL)^2} \quad \text{where } k = 1.0$$

If both ends are fixed in the x and y-directions the same equation applies except that k = 0.5. Now the question to be asked is what would be the buckling load if the end condition about the x-axis differs from that along the y-axis.

The answer to that question is given in a very approximate manner by A. Pflüger[33]. He investigated the buckling of a zee-section with hinged end conditions only and stated that other end conditions can be taken into account by a suitable reduction of the column length. No guidance to the proposed reduction is given and it appears that such consideration is left to the designer.

However, the answer to the problem can be obtained by considering Eq. (59) and letting Q = 0, hence

$$P^2 - P(P_x + P_y) + P_xP_y - P_{xy}^2 = 0 \qquad (60)$$

where

$$P_x = K_1\pi^2EI_x/L^2$$

$$P_y = K_2\pi^2EI_y/L^2$$

$$P_{xy} = K_3\pi^2EI_{xy}/L^2$$

From Table 1,

if $u'' = v'' = 0$	then $K_1 = K_2 = K_3 = 1.0$
$u' = v' = 0$	then $K_1 = K_2 = K_3 = 4.0$
$u' = v'' = 0$	then $K_1 = 4.0$, $K_2 = 1.0$, $K_3 = 0.849$
$u'' = v' = 0$	then $K_1 = 1.0$, $K_2 = 4.0$, $K_3 = 0.849$

Depending on the end condition about the x and y-axes the buckling load can be calculated from Eq. (60) and the appropriate K_1, K_2, K_3 values. Physically this means that the column will buckle about a new axis between the y and the y_1-axes at which the section will have a new value of moment of inertia larger than I_{y1} and smaller than I_y. However, there is no need to locate that new axis and calculate a new moment of inertia since Eq. (60) suffices

2.6A.4 Verification of P_{cr} for I-sections Braced on Both Sides with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

The behavior of I-section columns braced on both sides, including twist, has been investigated by Errera(13). It will be shown here that his equation 32 can be obtained from the general solution (Eq. 54) derived in this investigation.

For I-sections $y_o = x_o = 0$

$$d_1 = d_2 = d/2$$

$$I_{xy} = 0 \qquad \text{hence } P_{xy} = 0$$

Substitution of these parameters into Eq. (54) yields

$$\begin{bmatrix} P_y-P+Q & 0 & 0 \\ 0 & P_x-P & 0 \\ 0 & 0 & r_o^2(P_\phi-P)+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_1 \\ D_1 \\ E_1 \end{Bmatrix} = 0 \qquad (61)$$

Omitting the possibility of strong axis buckling (this is true for I-sections only) and replacing r_o^2P by the equivalent form in Ref. 13, then the determinant of the coefficient matrix of C_1, D_1, E_1 of Eq. (61) is identical to Eq. (32) of Ref. 13. Note that K_8 is equal to 1.0 for a hinged end column; other cases of mixed end conditions have not been considered in previous investigations.

2.6B Equations of Stability of Columns Braced on One Side Only with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

Following the same procedure given in Section 2.6A except using Eq. (17) instead of (14), the following equation results:

$$\begin{bmatrix} P_y-P+Q & P_{xy} & -K_5Py_o-K_6Qd_2 \\ P_{xy} & P_x-P & K_7Px_o \\ -K_5Py_o-K_6Qd_2 & K_7Px_o & r_o^2(P_\phi-P)+Qd_2^2+K_8F\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_1 \\ D_1 \\ E_1 \end{Bmatrix} = 0 \qquad (62)$$

where P_x, P_y, P_{xy}, P_ϕ are as defined by Eq. (55), and K_1, K_2, ..., K_8 are coefficients corresponding to different boundary conditions of the ends of the column and their values are given in Table 1.

2.6B.1 Critical Buckling Loads of Particular Sections

Equation (62) will be used to determine the critical buckling loads P_{cr} of certain columns as a function of Q and F. Two particular sections will be considered, namely channel sections and Z-sections, and it will be shown that the I-section previously investigated[13] can be derived as a special case of the general solution.

For a column of a particular cross-section the critical buckling load will be determined by substituting for the geometric terms appearing in the general solution, those belonging to the particular cross-section under consideration.

2.6B.2 P_{cr} for Channel Sections Braced on One Side with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

Considering Eq. (62) and following the same procedure of Section 2.4.3.3, the following equation characterizes the behavior of a column with any end conditions:

$$P^3(r_o^2-K_7^2x_o^2)-P^2[r_o^2P_x+r_o^2P_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2}+(P_y+Q)(r_o^2-K_7^2x_o^2)]$$

$$+P[P_x(r_o^2P_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})+(P_y+Q)(r_o^2P_x+r_oP_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})$$

$$-(K_6Q\frac{d}{2})^2]-(P_y+Q)[P_x(r_o^2P_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})]+P_x(K_6Q\frac{d}{2})^2 = 0 \qquad (63)$$

Equation (63) characterizes the behavior of channel section columns braced with diaphragms on one side only. For a particular column and specific end conditions all parameters (except Q and F) which form the coefficients in Eq. (63) are known. Hence for known values of Q and F, the smallest root of Eq. (63) gives the critical buckling load P_{cr}. Graphical representation of Eq. (63) is shown on Fig. 10.

2.6B.3 P_{cr} of Z-sections Braced on One Side with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

Considering Eq. (62) and the same procedure of Section 2.4.3.4, the following equation results:

$$P^3-P^2[P_x+P_y+P_\phi+Q+\frac{1}{r_o^2}(Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})]$$

$$+P\{(P_y+Q)P_x-P_{xy}^2+(P_y+Q+P_x)[P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})]-\frac{1}{r_o^2}(K_6Q\frac{d}{2})^2\}$$

$$-[(P_y+Q)P_x-P_{xy}^2][P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})]+\frac{1}{r_o^2}P_x(K_6Q\frac{d}{2})^2 = 0 \qquad (64)$$

As has been previously explained, all parameters of the equation are known except Q and F. For a particular column and known values of Q and F, Eq. (64) gives three values of the buckling load; the lowest value determines the critical buckling load P_{cr}. Graphical representation of this equation is shown on Fig. 11.

2.6B.4 Verification of P_{cr} for I-section Braced on One Side with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

The solution of a symmetric I-section braced on one side only is given in Ref. 13; it will be shown here that Equation 62 of Ref. 13 can be obtained as a special case from the general solution equation (62) derived in this investigation.

For I-sections

$$y_o = x_o = 0$$

$$d_1 = d_2 = \frac{d}{2}$$

$$I_{xy} = 0 \qquad \text{hence } P_{xy} = 0$$

Substitution of these parameters into Eq. (62) yields

$$\begin{bmatrix} P_y - P + Q & 0 & -K_6 Q\frac{d}{2} \\ 0 & P_x - P & 0 \\ -K_6 Q\frac{d}{2} & 0 & r_o^2(P_\phi - P) + Q\frac{d^2}{4} + K_8 F\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_1 \\ D_1 \\ E_1 \end{Bmatrix} = 0 \qquad (65)$$

For the hinged ends column considered in Ref. 13, $K_8 = K_6 = 1.0$, hence by rearranging rows and columns of the determinant of the coefficient matrix of the parameters C_1, D_1, E_1 an equation identical to Equation 62 of Ref. 13 results.

2.7 Load-Deflections Relationships of an Imperfect Column

The governing equations given in the previous section, 2.6, are derived for a perfect column. In the absence of a disturbing moment the column remains straight for any value of $P < P_{cr}$. When P_{cr} is reached the column undergoes displacements of indeterminate magnitudes. That is, the slightest disturbance will suffice to cause an indefinitely large deflection.

Real columns exhibit unavoidable initial imperfections which are the primary cause of deflections and/or rotation prior to the state of instability of the column. These deflections and rotation increase nonlinearly with increasing load and rapidly become very large, and result in failure as P_{cr} is approached. In a diaphragm-braced column, such deflection in the plane of the diaphragm and rotation of the column are resisted by the in-plane shear rigidity and rotational restraint of the diaphragm, respectively. When the diaphragm fails to resist certain values of the increasing deflection and/or rotation, failure of the whole assembly occurs. As a result, the capacity load P_r of the column will be less than P_{cr} calculated on the basis of an ideal column. This behavior has been realized by Winter(9) as he indicates that the minimum rigidities calculated for full bracing of ideal columns are not sufficient to achieve full bracing of real, i.e. imperfect columns. In an early design recommendation(17) a value of $P_r = 0.9P_{cr}$ has been suggested, hence deflections and rotation at this load level are calculated in order to check that the diaphragm is adequate for the load P_r to be reached. Details of checking the diaphragm adequacy will be given in Chapter 5. However, in this investigation it has been found that the use of $P_r = 0.9P_{cr}$ is not mandatory, since in some cases economical design can be achieved by values of P_r above or below $0.9P_{cr}$. Hence, in general the load capacity P_r is equal to

$$P_r = \lambda P_{cr}$$

where λ is a trial reduction factor less than 1.0. The value of λ is decided upon by starting with a trial value of λ, then calculating the corresponding deflections and rotation, and hence checking the diaphragm adequacy. If the diaphragm is not adequate, than a new value of λ will be tried and the checking repeated until the diaphragm adequacy is ensured. The last value of λ multiplied by P_{cr} gives the load capacity of the column, P_r.

It has been suggested(17) that for a conservative estimate of the additional deflections, the pattern of initial deflections along the length of an imperfect column is assumed affine to the buckling shape of the perfect column. Assumed values of amplitudes of the initial imperfections may be obtained from recognized specifications or to be measured from the actual structure, since the current Specification for the Design of Cold-Formed Steel Structural Members(1) has no guidance to how much tolerance limit in sweep and initial twist should be considered.

Load-deflections relationships as well as amplitude of additional deflections are derived in the next sections for the following cases by considering the first buckling mode, i.e. n = 1 and general end conditions listed in Table 1 (i.e. fixed, hinged or mixed end conditions):

a) Diaphragm bracing on both sides.

b) Diaphragm bracing on one side only.

2.7A Sections Braced on Both Sides with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

Typical column sections considered are:

·Channel sections

·Z-sections

2.7A.1 Method of Solution

The total potential energy for a perfect column (Eq. 14) is modified to account for the initial imperfections by considering the following modified displacements:

$$u_t = u + u_o$$
$$v_t = v + v_o$$
$$\phi_t = \phi + \phi_o$$

where u_t = total displacement in the x-direction

u = additional displacement in the x-direction

u_o = initial imperfection in the x-direction

Similar subscripts are adopted for v, ϕ. Hence, the total potential energy of an imperfect column becomes:

$$\Pi = \frac{1}{2}\int_o^L \{EI_y u''^2 + 2EI_{xy}u''v'' + EI_x v''^2 + EC_w \phi''^2 + GJ\phi'^2$$

$$- P[u_t'^2 - u_o'^2) + (v_t'^2 - v_o'^2) + \frac{I_p}{A}(\phi_t'^2 - \phi_o'^2) \qquad \text{(contd.)}$$

$$- 2x_o(v_t'\phi_t' - v_o'\phi_o') + 2y_o(u_t'\phi_t' - u_o'\phi_o')]$$

$$+ Q[u'^2 + \phi'^2(\frac{d_1^2 + d_2^2}{2}) + u'\phi'(d_1 - d_2)] + F\cdot\phi^2\}dZ \qquad (66)$$

Assumed Forms of the Initial Imperfections (n = 1)

The load-deflections relationships are derived for columns with general end conditions listed in Table 1 (i.e. hinged, fixed or mixed), for example, $u' = v'' = \phi' = 0$ at $Z = 0,L$. Different forms corresponding to different end conditions are represented by the following equations:

End Condition	Displacement Function	
$u_o = u_o'' = 0$ at $Z = 0,L$	$u_o = C_o \sin\frac{\pi Z}{L}$	
$u_o = u_o' = 0$ at $Z = 0,L$	$u_o = (C_o/2)(1 - \cos\frac{2\pi Z}{L})$	
$v_o = v_o'' = 0$ at $Z = 0,L$	$v_o = D_o \sin\frac{\pi Z}{L}$	
$v_o = v_o' = 0$ at $Z = 0,L$	$v_o = (D_o/2)(1 - \cos\frac{2\pi Z}{L})$	(67)
$\phi_o = \phi_o'' = 0$ at $Z = 0,L$	$\phi_o = E_o \sin\frac{\pi Z}{L}$	
$\phi_o = \phi_o' = 0$ at $Z = 0,L$	$\phi_o = (E_o/2)(1 - \cos\frac{2\pi Z}{L})$	

C_o, D_o, E_o are the amplitudes of additional deflections. Subscript "o" indicates initial imperfections.

General Form

In order to obtain a general solution which will also account for the influence of the end conditions coefficients K_9, K_{10}, K_{11}, K_{12} are introduced. These coefficients are calculated for each case of the different boundary conditions listed in Table 1 by using the proper combination of the assumed displacement functions given by Eqs. (53) and (67).

Following a procedure similar to that of Section 2.6A, i.e. using the energy equation (66) together with the assumed displacements of initial and additional deflections (Eqs. 53, 67), the following equation is obtained.

$$\begin{bmatrix} P_y-P+Q & P_{xy} & -K_5y_oP+K_6Q(d_1-d_2) \\ P_{xy} & P_x-P & K_7Px_o \\ -K_5y_oP+K_6Q(d_1-d_2) & K_7Px_o & r_o^2(P_\phi-P)+Q(\frac{d_1^2+d_2^2}{2})+K_8F\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_1 \\ D_1 \\ E_1 \end{Bmatrix} = P \begin{bmatrix} C_o+K_{12}E_oy_o \\ D_o-K_9x_oE_o \\ r_o^2E_o-K_{10}x_oD_o+K_{11}y_oC_o \end{bmatrix} \quad (68)$$

or

$$[D]\{\Delta\} = P[\Delta_o] \quad (69)$$

where C_o, D_o, E_o = amplitudes of initial imperfections

C_1, D_1, E_1 = amplitudes of additional deflections

K_1, K_2, ..., K_{12} are coefficients accounting for different conditions and their values are listed in Table 1.

The load-displacement relationship can be found from Eq. (68) by solving for C_1, D_1, E_1, hence

$$\{\Delta\} = P[D]^{-1}[\Delta_o] \quad (70)$$

If $C_o = D_o = E_o = 0$, Eq. (68) becomes identical to Eq. (54).

Formulas of amplitudes of the additional deflections C_1, D_1, E_1 will be found for the cases of channel and zee-sections.

2.7A.2 Amplitudes of Deflections of a Channel Section Braced on Both Sides with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

For channel sections $y_o = 0$

$$I_{xy} = 0$$

$$d_1 = d_2 = \frac{d}{2} \quad \text{hence } P_{xy} = 0$$

Formulas of amplitudes of additional deflections are obtained by substituting the above listed parameters into Eq. (68) and replacing P by P_r where P_r, as defined in Section 2.7, is the reduced critical buckling load. The critical buckling load in the case of a channel section is the smallest load obtained from Eqs. (56) and (57). Solving the matrix equation (70) for C_1, D_1, E_1 the amplitudes of the additional deflections, the following formulas are obtained:

$$C_1 = \frac{P_r C_o}{\text{Det.}}\{(P_x - P_r)[r_o^2(P_\phi - P_r) + Q\frac{d^2}{4} + K_8 F\frac{L^2}{\pi^2}] - (K_7 P_r x_o)^2\} \qquad (71)$$

$$D_1 = \frac{P_r}{\text{Det.}}(P_y - P_r + Q) \; [r_o^2(P_\phi - P_r) + Q\frac{d^2}{4} + K_8 F\frac{L^2}{\pi^2}](D_o - K_9 x_o E_o)$$
$$-K_7 P_r x_o(r_o^2 E_o - K_{10} x_o D_o) \qquad (72)$$

$$E_1 = \frac{P_r}{\text{Det.}}(P_y - P_r + Q)\{(P_x - P_r)(r_o E_o - K_{10} x_o D_o) - K_7 P_r x_o(D_o - K_9 x_o E_o)\} \quad (73)$$

$$\text{Det} = (P_y - P_r + Q)\{(P_x - P_r)[r_o(P_\phi - P_r) + Q\frac{d^2}{4} + K_8 F\frac{L^2}{\pi^2}] - (K_7 P_r x_o)\} \quad (74)$$

2.7A.3 Amplitudes of Deflections of a Z-section Braced on Both Sides with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

For zee-sections $x_o = y_o = 0$

and $d_1 = d_2 = \frac{d}{2}$

by substituting these parameters into Eq. (68) and following the same procedure for the channel sections except that P_{cr} is obtained from Eq. (59); then the formulas of the amplitudes of additional deflections are given by:

$$C_1 = \frac{P_r}{Det.}[r_o^2(P_\phi-P_r)+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2}][C_o(P_x-P_r)-D_oP_{xy}] \qquad (75)$$

$$D_1 = \frac{P_r}{Det.}[r_o^2(P_\phi-P_r)+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2}][D_o(P_y-P_r+Q)-C_oP_{xy}] \qquad (76)$$

$$E_1 = \frac{P_rE_or_o^2}{Det.}[(P_y-P_r+Q)(P_x-P_r)-P_{xy}^2] \qquad (77)$$

where $Det = [(P_y-P_r+Q)(P_x-P_r)-P_{xy}^2][r_o^2(P_\phi-P)+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2}]$ (78)

2.7B Sections Braced on One Side with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

Equations of additional deflections for cases of n > 1 are given in Chapter 5 for the case of hinged end columns only. Typical column sections considered herein are channel and zee-sections.

2.7B.1 Method of Solution

The total potential energy for perfect column equation (34) is modified to account for the initial imperfection by considering the following displacements:

$$u_t = u + u_o$$
$$v_t = v + v_o$$
$$\phi_t = \phi + \phi_o$$

where u_t, u, u_o, v_t, ... are defined in Section 2.7A.1. Hence the total potential energy of the imperfect column becomes:

$$\Pi = \frac{1}{2}\int_0^L \{EI_y u''^2 + 2EI_{xy}u''v'' + EI_x v''^2 + EC_w\phi''^2 + GJ\phi'^2$$

$$-P(u_t'^2 - u_o'^2) + (v_t'^2 - v_o'^2) + \frac{I_p}{A}(\phi_t'^2 - \phi_o'^2) - 2x_o(v_t'\phi_t' - v_o'\phi_o')$$

$$+2y_o(u_t'\phi_t' - u_o'\phi_o')] + Q(u'^2 + \phi'^2 d_2^2 - 2u'\phi' d_2) + F\cdot\phi^2\}dZ \qquad (79)$$

Following a procedure similar to that of Section 2.7A.1, i.e. substituting assumed displacements, chosen from Eqs. (53) and (67) into Eq. (79), and minimizing the resulting energy expression according to the Rayleigh-Ritz method, then the following stability equation of the imperfect column is obtained.

$$\begin{bmatrix} P_y-P+Q & P_{xy} & -K_5y_oP-K_6Qd \\ P_{xy} & P_x-P & K_7Px_o \\ -K_5y_oP-K_6Qd_2 & K_7Px_o & r_o^2(P_\phi-P)+Qd_2^2+K_8F\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_1 \\ D_1 \\ E_1 \end{Bmatrix} = P \begin{bmatrix} C_o+K_{12}E_oy_o \\ D_o-K_9x_oE_o \\ r_oE_o-K_{10}x_oD_o \\ +K_{11}y_oC_o \end{bmatrix} \qquad (80)$$

or

$$[D]\{\Delta\} = P[\Delta_o] \qquad (81)$$

where C_o, D_o, E_o = amplitudes of initial deflections

C_1, D_1, E_1 = amplitudes of additional deflections

P_x, P_y, P_{xy}, P_ϕ are defined by Eqs. (55)

K_1, K_2, K_3, ..., K_{12} are coefficients accounting for different end conditions and their values are listed in Table 1.

The load-displacement relationship can be found from Eq. (81) by solving for C_1, D_1, E_1, hence

$$P[D]^{-1} = [\Delta_o] \tag{82}$$

If $C_o = D_o = E_o = 0$, Eq. (81) becomes identical to Eq. (62). Formulas of amplitudes of the additional deflections C_1, D_1, E_1 will be found for the cases of channel and Z-sections.

2.7B.2 Amplitudes of Deflections of Channel Section Braced on One Side with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

For channel sections $y_o = 0$

$$I_{xy} = 0$$

$$d_2 = \frac{d}{2} \qquad \text{hence } P_{xy} = 0$$

Formulas for amplitudes of additional deflections are obtained by substituting the above parameters into Eq. (80) and replacing P by P_r, where P_r, as defined in Section 2.7, is the load capacity of the column. Solving the matrix equation (82) for C_1, D_1, E_1 the amplitudes of additional deflections, the following formulas are obtained:

$$C_1 = \frac{P_r}{\text{Det.}}\{C_o(A_3A_5-A_4^2)+A_4A_2(D_o-K_9x_oE_o)-A_3A_2(r_o^2E_o-K_{10}x_oD_o)\} \tag{83}$$

$$D_1 = \frac{P_r}{\text{Det.}}\{C_oA_4A_2+(A_1A_5+A_2^2)(D_o-K_9x_oE_o)-A_1A_4(r_o^2E_o-K_{10}x_oD_o)\} \tag{84}$$

$$E_1 = \frac{P_r}{\text{Det.}}\{-C_oA_2A_3-A_1A_4(D_o-K_9x_oE_o)+A_1A_3(r_o^2E_o-K_{10}x_oD_o)\} \tag{85}$$

where

$$\text{Det} = A_1(A_3A_5 - A_4^2) - A_3A_2^2 \tag{86}$$

$$A_1 = P_y - P_r + Q$$

$$A_2 = -K_6Q\frac{d}{2}$$

$$A_3 = P_x - P_r$$

$$A_4 = K_7 P_r x_o$$

$$A_5 = r_o^2(P_\phi - P_r) + Q\,\frac{d^2}{4} + K_8 F\,\frac{L^2}{\pi^2}$$

2.7B.3 Amplitudes of Deflections of a Z-section Braced on One Side with Hinged, Fixed or Other End Conditions Listed in Table 1 and n = 1

For Z-sections $x_o = y_o = 0$

and $d_2 = \frac{d}{2}$

By substituting these parameters into Eq. (80) and following the same procedure for the channel section, then the formulas of the amplitudes of additional deflections are given by:

$$C_1 = \frac{P_r}{\text{Det.}}\{C_oA_1A_5 - D_oA_2A_5 - E_or_o^2A_3A_4\} \tag{87}$$

$$D_1 = \frac{P_r}{\text{Det.}}\{-C_oA_2A_5 + D_o(A_1A_5 - A_3^2) + E_or_o^2A_2A_3\} \tag{88}$$

$$E_1 = \frac{P_r}{\text{Det.}}\{-C_oA_3A_4 + D_oA_3A_2 + E_or_o^2(A_1A_4 - A_2^2)\} \tag{89}$$

where
$$\text{Det} = A_1A_4A_5 - A_2^2A_5 - A_3^2A_4 \tag{90}$$

$$A_1 = P_y - P_r + Q$$

$$A_2 = P_{xy}$$

$$A_3 = -K_6 Q\frac{d}{2}$$

$$A_4 = P_x - P_r$$

$$A_5 = r_o^2(P_\phi - P_r) + Q\,\frac{d^2}{4} + K_8F\,\frac{L^2}{\pi^2}$$

2.8 Amplitudes of Deflections of Columns with Hinged Ends, $n = 1,2,3,\ldots$

For columns with both ends hinged the displacements u, v and ϕ are represented by Eqs. (29) of Section 2.4. These equations are:

$$u = C_n \sin\frac{n\pi Z}{L} \tag{29a}$$

$$v = D_n \sin\frac{n\pi Z}{L} \tag{29b}$$

$$\phi = E_n \sin\frac{n\pi Z}{L} \tag{29c}$$

It has been suggested(17) that for a conservative estimate of the additional deflections, the pattern of initial deflections along the length of the column is assumed affine to the buckling shape of the perfect column; therefore the initial imperfections u_o, v_o and ϕ_o may be represented by the following functions:

$$u_o = C_o \sin\frac{n\pi Z}{L} \tag{90a}$$

$$v_o = D_o \sin\frac{n\pi Z}{L} \tag{90b}$$

$$\phi_o = E_o \sin\frac{n\pi Z}{L} \tag{90c}$$

Following the method of solution of Section 2.7A.1, equations of the amplitudes of deflections are derived by considering the energy expressions given by Eqd. (66) and (79), and the displacement functions given by Eqs. (29) and (90). The cases of channel and zee-sections braced on one side or on both sides are considered and the result is given in the following sections.

In general, the parameters P_x, P_y, P_{xy} and P_ϕ used in the following equations are given by Eqs. (36). Note that n is included in these equations.

2.8.1 Amplitudes of Deflections of a Channel Section Braced on Both Sides (Hinged Ends)

$$C_1 = \frac{P_r C_o}{Det.}\{(P_x-P_r)[r_o^2(P_\phi-P_r)+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2}]-(Px_o)^2\} \tag{91}$$

$$D_1 = \frac{P_r}{Det.}(P_y-P_r+Q)\ [r_o^2(P_\phi-P_r)+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2}](D_o-x_oE_o)$$

$$-P_r x_o(r_o^2E_o-K_{10}x_oD_o) \tag{92}$$

$$E_1 = \frac{P_r}{Det.}(P_y-P_r+Q)\{(P_x-P_r)(r_o^2E_o-x_oD_o)-P_rx_o(D_o-x_oE_o)\} \tag{93}$$

$$Det = (P_y-P_r+Q)\{(P_x-P_r)[r_o^2(P_\phi-P_r)+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2}]-(P_rx_o)^2\} \tag{94}$$

2.8.2 Amplitudes of Deflections of a Z-section Braced on Both Sides (Hinged Ends)

$$C_1 = \frac{P_r}{Det.}[r_o^2(P_\phi-P_r)+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2}][C_o(P_x-P_r)-D_oP_{xy}] \tag{95}$$

$$D_1 = \frac{P_r}{Det.}[r_o^2(P_\phi-P_r)+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2}][D_o(P_y-P_r+Q)-C_oP_{xy}] \tag{96}$$

$$E_1 = \frac{P_rE_or_o^2}{Det.}[(P_y-P_r+Q)(P_x-P_r)-P_{xy}^2] \tag{97}$$

where $$Det = [(P_y-P_r+Q)(P_x-P_r)-P_{xy}^2][r_o^2(P_\phi-P_r)+Q\frac{d^2}{4}+\frac{F}{n^2}\frac{L^2}{\pi^2}] \tag{98}$$

2.8.3 Amplitudes of Deflections of Channel Sections Braced on One Side (Hinged Ends)

$$C_1 = \frac{P_r}{Det.}\{C_o(A_3A_5-A_4^2)+A_4A_2(D_o-x_oE_o)-A_3A_2(r_o^2E_o-x_oD_o)\} \quad (99)$$

$$D_1 = \frac{P_r}{Det.}\{C_oA_4A_2+(A_1A_5+A_2^2)(D_o-x_oE_o)-A_1A_4(r_o^2E_o-x_oD_o)\} \quad (100)$$

$$E_1 = \frac{P_r}{Det.}\{-C_oA_2A_3-A_1A_4(D_o-x_oE_o)+A_1A_3(r_o^2E_o-x_oD_o)\} \quad (101)$$

where

$$Det = A_1(A_3A_5 - A_4^2) - A_3A_2^2 \quad (102)$$

$$A_1 = P_y - P_r + Q$$

$$A_2 = -Q\frac{d}{2}$$

$$A_3 = P_x - P_r$$

$$A_4 = P_rx_o$$

$$A_5 = r_o^2(P_\phi - P_r) + Q\,\frac{d^2}{4} + \frac{F}{n^2}\,\frac{L^2}{\pi^2}$$

2.8.4 Amplitudes of Deflections of Z-section Braced on One Side (Hinged Ends)

$$C_1 = \frac{P_r}{Det.}\{C_oA_4A_5-D_oA_2A_5-E_or_o^2A_3A_4\} \quad (103)$$

$$D_1 = \frac{P_r}{Det.}\{-C_oA_2A_5+D_o(A_1A_5-A_3^2)+E_or_o^2A_2A_3\} \quad (104)$$

$$E_1 = \frac{P_r}{Det.}\{-C_oA_3A_4+D_oA_3A_2+E_or_o^2(A_1A_4-A_2^2)\} \quad (105)$$

where

$$Det = A_1A_4A_5 - A_2^2A_5 - A_3^2A_4 \quad (106)$$

$$A_1 = P_y - P_r + Q$$

$$A_2 = P_{xy}$$

$$A_3 = -Q\frac{d}{2}$$

$$A_4 = P_x - P_r$$

$$A_5 = r_o^2(P_\phi - P_r) + Q\,\frac{d^2}{4} + \frac{F}{n^2}\,\frac{L^2}{\pi^2}$$

2.9 Summary of the Governing Equations of a Perfect Column

The following summarizes the governing equations for the four cases considered in the present investigation. These equations are obtained as special cases from the general solution which is based on assumed displacements represented by

I) $n = 1,2,3,\ldots$

II) $n = 1$

I) GOVERNING EQUATIONS ($n = 1,2,3,\ldots$)

The following equations are valid for columns with hinged ends only, where P_x, P_y, P_{xy}, P_ϕ are given by Eqs. (36).

1) Channel sections braced on both sides

$$P = P_y + Q \tag{44}$$

$$P^2(r_o^2-x_o^2)-P(r_o^2P_x+r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})+P_x(r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2}) = 0 \tag{45}$$

P_{cr} is the smallest value of P obtained from Eq. (44) or from Eq. (45) by choosing n which minimizes the resulting roots.

2) Z-sections braced on both sides

P_{cr} is the smallest root obtained from the following equation,

$$P^2 - P(P_x + P_y + Q) + (P_xP_y + P_xQ - P_{xy}^2) = 0 \qquad (48)$$

3) Channel section braced on one side

P_{cr} is the smallest root of the following equation provided that n is chosen to minimize these roots:

$$P^3(r_o^2-x_o^2)-P^2[r_o^2P_x+r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2}+(P_y+Q)(r_o^2-x_o^2)$$

$$+P[P_x(r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})+(P_y+Q)(r_o^2P_x+r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})-(Q\frac{d}{2})^2]$$

$$-(P_y+Q)[P_x(r_o^2P_\phi+Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]+P_x(Q\frac{d}{2})^2 = 0 \qquad (50)$$

4) Z-sections braced on one side

P_{cr} is the smallest root of the following equation provided that n is chosen to minimize these roots:

$$P^3-P^2[(P_x+P_y+Q+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]+P\{(P_y+Q)P_x-P_{xy}^2$$

$$+(P_y+Q+P_x)[P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]-(Q\frac{d}{2})^2\}-[P_y+Q)P_x$$

$$-P_{xy}^2][P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]+\frac{1}{r_o^2}P_x(Q\frac{d}{2})^2 = 0 \qquad (52)$$

II) GOVERNING EQUATIONS (n = 1)

The following equations are valid for columns with general end conditions (Table 1) where P_x, P_y, P_{xy}, P_ϕ are given by Eqs. (55).

1) Channel sections braced on both sides

P_{cr} is the smallest value of P obtained from the following two equations:

$$P = P_y + Q \tag{56}$$

$$P^2(r_o^2-K_7^2x_o^2)-P(r_o^2P_x+r_o^2P_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})+P_x(r_o^2P_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})= 0 \tag{57}$$

2) Z-sections braced on both sides

P_{cr} is the smallest root of the quadratic equation,

$$P^2 - P(P_x + P_y + Q) + (P_xP_y + P_xQ - P_{xy}^2) = 0 \tag{59}$$

3) Channel sections braced on one side

P_{cr} is the smallest root of the cubic equation,

$$\begin{aligned}&P^3(r_o^2-K_7^2x_o^2)-P^2[r_o^2P_x+r_o^2P_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2}+(P_y+Q)(r_o^2-K_7^2x_o^2)\\&+P[P_x(r_o^2P_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})+(P_y+Q)(r_o^2P_x+r_o^2P_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})-(K_6Q\frac{d}{2})^2]\\&-P_x(P_y+Q)(r_o^2P_\phi+Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})+P_x(K_6Q\frac{d}{2})^2 = 0\end{aligned} \tag{63}$$

4) Z-sections braced on one side

P_{cr} is the smallest root of the cubic equation,

$$\begin{aligned}&P^3-P^2[P_x+P_y+P_\phi+Q+\frac{1}{r_o^2}(Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})]\\&+P\{(P_y+Q)P_x-P_{xy}^2+(P_y+Q+P_x)[P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})]-\frac{1}{r_o^2})K_6Q\frac{d}{2})^2\}\\&-[(P_y+Q)P_x-P_{xy}^2][P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+K_8F\frac{L^2}{\pi^2})]+\frac{1}{r_o^2}P_x(K_6Q\frac{d}{2})^2 = 0\end{aligned} \tag{64}$$

Chapter 3

CHECKING THE THEORETICAL RESULTS

This chapter serves two purposes:

1) To check the validity of the stability equations for special cases of known solutions by Timoshenko in which constraints are imposed on some components of the generalized displacements.

2) To clarify any possible misconception when using the stability equations to derive solutions of special cases, so that a correct and well-conditioned mathematical model of the structure exists.

3.1 General

The previous chapter presents the theory and the general equations of stability (35, 38, 54 and 62) from which governing equations of specific cases are derived. At that stage, it was not necessary to involve the reader in details of the potential energy concepts and how a special case must be conditioned so that the solution can be derived from the general formulas.

Despite the limited size of published information about these details it is scattered in many references and most of it is not related to the subject matter. The method of solution derived in this chapter is assembled from more than one source; it is inevitably indebted to all other sources, the work by Gallagher[35], Green[36] and Rubenstein[37] having principal aid.

3.2 Constraints

In our case constraints are induced on the system by limiting the freedom of sections between the column end to undergo displacements or rotation. Typical examples would be the cases of fixed axis of rotation and prescribed plane of deflection. Such constraints result from relationships among displacements which can always be presented by constraint equations. If constraint exists, these equations will relate not only the displacements involved but also the force components.

Equations (35) and (54), and (38) and (62), or (68) and (80) with initial imperfections, have been derived for a column of general cross-section braced on both sides or on one side, respectively. On deriving these equations generalized displacements u, v, ϕ were considered. Therefore these general solutions are directly applicable to cases in which the three displacements are possible. Hence, the solution of a special case is possible by direct substitution for the geometric terms appearing in the general solution, those belonging to the particular column cross-section. However, if any of these displacements u, v, ϕ are restrained then direct solution from the general formulas is not possible without pre-conditioning of the case under consideration.

3.3 Effect of Constraint on the Energy Solution

In general two main steps are involved in the energy solution. The first step is to derive the expression of the total potential energy which is a quadratic form in the displacements. The second step is to minimize the energy expression by differentiating. Hence we start with a quadratic form and end with a linear one. It is now apparent that application of the constraints to the energy expression before differentiating will enable the minimization of the actual energy of the system and hence a correct answer can be obtained. However, if the constraints are applied after differentiating, a false answer is expected. Further explanation of this reasoning is given by Gallagher[35] in Fig. 12. The line AB represents a constraint and the curve ABC represents the potential energy. Clearly the constraint prevents the minimum from occurring at the point predicted by first variation Π_p.

It is important to note that the above conclusion does not apply for systems in which u, v and ϕ may occur independently, that is, uncoupled. For example, the three buckling modes, about the y-axis, about the x-axis and twist, of an I-section column braced on both sides with shear diaphragms are uncoupled and u, v and occur independently. If one or more of these displacements are limited to zero, then by definition, there is no constraint, since relation between the displacements does not exist. Thus the general solution can be used directly.

3.4 Methods of Solution

Two methods of solution are possible:

1) A direct solution is to introduce these constraints in the energy expression before differentiating, so that the solution to a specific problem may be obtained.

2) A short cut to the solution of a special case may be obtained from the general solution (no constraints) by trans-

formation of coordinates and condensation of the original matrix. The term condensation refers here to the contraction in size of a system of equations by elimination of certain degrees of freedom.

The first method of solution has been used extensively in Chapter 2, hence no reference to it will be included in this chapter. The second method will be explained in detail and more than one example will be solved for illustration.

3.5 Solution by Matrix Condensation

Consider the general solution with initial imperfection, given by Eqs. (68) or (80) in matrix notations. Then

$$[D]\{\Delta\} = \{P\Delta_o\} \tag{107}$$

where $\{\Delta\}$ is the generalized displacement vector

$\{\Delta_o\}$ is the initial imperfection, scalar

Now certain constraints are imposed on a group of the displacements $\{\Delta\}$ and it is required to derive the condensed matrix $[\bar{D}]$ after including the effect of these constraints. This requires transformation of the degrees of freedom and in order to develop such transformation, we divide the degrees of freedom into two groups, $\{\Delta_1\}$ and $\{\Delta_2\}$, where $\{\Delta_1\}$ is the constrained part. Hence by partitioning of Eq. (107), then

$$\left[\begin{array}{c|c} D_{11} & D_{12} \\ \hline D_{21} & D_{22} \end{array}\right] \left\{\begin{array}{c} \Delta_1 \\ \hline \Delta_2 \end{array}\right\} = \left\{\begin{array}{c} P_1\Delta_{01} \\ \hline P_2\Delta_{02} \end{array}\right\} \tag{108}$$

As has been defined in Section 3.2, constraints result from relationships among displacements; hence introducing the equations of constraints, then

$$[G_1 \;\; G_2]\begin{Bmatrix}\Delta_1\\ \Delta_2\end{Bmatrix} = \{0\} \tag{109}$$

where $[G_1]$ and $[G_2]$ are geometric terms relating $\{\Delta_1\}$ to $\{\Delta_2\}$. Solving Eq. (109) for $\{\Delta_1\}$, then

$$\{\Delta_1\} = -[G_1]^{-1}[G_2]\{\Delta_2\} \tag{110}$$

Nothing that $\{\Delta_2\} = [I]\{\Delta_2\}$, then we can write the following transformation equation,

$$\begin{Bmatrix}\Delta_1\\ \hline \Delta_2\end{Bmatrix} = \begin{bmatrix}-G_1^{-1}G_2\\ \hline I\end{bmatrix}\{\Delta_2\} \tag{111}$$

or

$$\begin{Bmatrix}\Delta_1\\ \Delta_2\end{Bmatrix} = [T]\{\Delta_2\} \tag{112}$$

where the transformation matrix $[T] = \begin{bmatrix}-G_1^{-1}G_2\\ \hline I\end{bmatrix}$ (113)

The intent is to remove these degrees of freedom $\{\Delta_1\}$ from the potential energy, from which Eq. (107) is derived, by the use of a condensation scheme. The potential energy in partitioning form is[35]:

$$\Pi_p = \frac{1}{2}\lfloor\Delta_1 \;\; \Delta_2\rfloor\left[\begin{array}{c|c}D_{11} & D_{12}\\ \hline D_{21} & D_{22}\end{array}\right]\begin{Bmatrix}\Delta_1\\ \hline \Delta_2\end{Bmatrix} - \lfloor\Delta_1 \;\; \Delta_2\rfloor\begin{Bmatrix}P_1\Delta_{01}\\ \hline P_2\Delta_{02}\end{Bmatrix} \tag{114}$$

Substituting Eq. (112) into Eq. (114), and then minimizing Π_p by differentiating with respect to $\{\Delta_2\}$, and then equating the result to zero,

$$[\bar{D}]\{\Delta_2\} = [T]^T\{P\Delta_o\} \tag{115}$$

with
$$[\bar{D}] = [T]^T[D][T] \tag{116}$$

In Eq. (113), let
$$-[G_1]^{-1}[G_2] = [\Gamma] \tag{117}$$

Then the equality takes the form

$$\bar{D} = \Gamma^T D_{11}\Gamma + \Gamma^T D_{12} + D_{21}\Gamma + D_{22} \tag{118}$$

which is the condensed matrix after imposing the constraints.

It should be noted that if the initial imperfections Δ_o in Eq. (80) of Section 2.7B.1, from which Eq. (115) is derived, are equated to zero than the resulting equation is identical to the general solution without imperfection, Eq. (62), Section 2.6B. In other words the [D] matrix is the same. Hence, if in Eq. (115), $\Delta_o = 0$, then

$$[\bar{D}]\{\Delta_2\} = 0 \tag{118a}$$

is a valid transformation of Eq. (62), with $[\bar{D}]$ as given by Eq. (118).

Application of this method to two examples of constraints is presented in the next section.

3.6 Verification of the Stability Equation

In this section special cases of known solutions(3) are derived from the stability equations to examine their validity. The solution of these cases given in Chapter 2 is not known in the existing literature; however, Timoshenko(3) derived solutions of different cases with constraints. His solutions are derived for each case based on equilibrium considerations.

Only the general solution, Eq. (62), of columns braced on one side is considered since unsymmetrical sections braced on both sides have no similarity with any existing information. However, in Section 2.4A.6, it is shown that the solution[13] of a symmetrical I-section braced on both sides can be derived from the general solution, Eq. (54). Also in Section 2.4B.1 it is found that by substituting $Q = F = 0$ into Eq. (39), then the resulting expression is valid for unbraced sections and the results are compared to some known solutions.

In the following section cases with constraints solved by Timoshenko[3] are compared with solutions from the general solution (Eqs. 35, 62) by the method of condensation explained in Section 3.5.

3.6.1 Bar with a Prescribed Plane of Deflection[3]

In Ref. 3, top of p. 244, the following two equations are given (notations are changed for the purpose of comparison). These are:

$$(P_x - P)D_1 - [P_{xy}(y_o - h_y) - P_{xy}]E_1 = 0 \tag{119}$$

$$(-P_{xy}(y_o-h_y)+Px_o)D_1+[r_o^2(P_\phi-P)+P_y(y_o-h_y)^2+Py_o^2-Ph_y^2]E_1 = 0 \tag{120}$$

The following is considered in deriving these equations:

$$k_\phi = 0 \tag{121}$$

$$u_N = u + (y_o - h_y)\phi = 0 \tag{122}$$

Now it will be shown that Eqs. (119) and (120) can be derived as a special case of the general solution equation (62). By virtue of Eqs. (121) and (122), $Q = F = 0$; also $K_1 = K_2 = K_3 \ldots = K_8 = 1.0$, since the case considered is for a hinged end column. Therefore Eq. (62) takes the form

$$\left[\begin{array}{c|cc} P_y-P & P_{xy} & -Py_o \\ \hline P_{xy} & P_x-P & Px_o \\ -Py_o & Px_o & r_o^2(P_\phi-P) \end{array}\right] \left\{\begin{array}{c} C_1 \\ \hline D_1 \\ E_1 \end{array}\right\} = 0 \tag{123}$$

From Eq. (122), the constraint equation is:

$$[1 \mid 0 \quad (y_o-h_y)] \left\{\begin{array}{c} C_1 \\ \hline D_1 \\ E_1 \end{array}\right\} = \{0\}$$

Then $[G_1] = [1]$ and $[G_2] = [0 \quad (y_o-h_y)]$.

From Eq. (117),

$$[\Gamma] = -[G_1]^{-1}[G_2]$$

then

$$[\Gamma] = [0 \quad -(y_o-h_y)] \tag{124}$$

From Eqs. (118) and (123),

$$\Gamma^T D_{11} \Gamma = \begin{bmatrix} 0 & 0 \\ 0 & (P_y-P)(y_o-h_y)^2 \end{bmatrix} \tag{125}$$

$$\Gamma^T D_{12} = \begin{bmatrix} 0 & 0 \\ -P_{xy}(y_o-h_y) & Py_o(y_o-h_y) \end{bmatrix} \tag{126}$$

$$D_{21}\Gamma = \begin{bmatrix} 0 & -P_{xy}(y_o-h_y) \\ 0 & Py_o(y_o-h_y) \end{bmatrix} \tag{127}$$

$$D_{22} = \begin{bmatrix} P_x-P & Px_o \\ Px_o & r_o^2(P_\phi-P) \end{bmatrix} \tag{128}$$

By Eq. (118),

$$\bar{D} = \Gamma^T D_{11}\Gamma + \Gamma^T D_{12} + D_{21}\Gamma + D_{22}$$

Then adding Eqs. (125) and (128), and substituting for $\bar{D}$ in Eq. (118a) gives

$$\begin{bmatrix} P_x-P & Px_o-P_{xy}(y_o-h_y) \\ Px_o-P_{xy}(y_o-h_y) & r_o^2(P_\phi-P)+P_y(y_o-h_y)^2+Py_o^2-Ph_y^2 \end{bmatrix} \begin{Bmatrix} D_1 \\ E_1 \end{Bmatrix} = 0$$

which is identical to Equations 119 and 120 of Timoshenko.

3.6.2 Bar with Prescribed Axis of Rotation

Reference 3, p. 240, Equation (5-56):

The critical buckling load of a hinged end column is derived, based on the following:

$$k_x = k_y = 0 \tag{129}$$

$$x_o = y_o = 0 \tag{130}$$

$$u + (y_o - h_y)\phi = 0 \tag{131a}$$

$$v - (x_o - h_x)\phi = 0 \tag{131b}$$

Adopting our notation, Eq. (5-56)[3] is written in the form

$$P_{cr} = \frac{P_y h_y^2 + P_x h_x^2 + r_o^2 P_\phi + \frac{F}{n^2}\frac{L^2}{\pi^2}}{h_x^2 + h_y^2 + r_o^2} \tag{132}$$

Using the general solution equation (38) and the method outlined in Section 3.5, it will be shown that solutions typical of Eq. (132) can be obtained.

By the virtue of Eq. (129) all terms of Q in Eq. (38) vanish; also Eqs. (131) imply that the constraints are applied to the components u and v. Hence for a hinged end column, Eq. (38) takes the form

$$\left[\begin{array}{cc|c} P_y - P & P_{xy} & 0 \\ P_{xy} & P_x - P & 0 \\ \hline 0 & 0 & r_o^2(P_\phi - P) + \frac{F}{n^2}\frac{L^2}{\pi^2} \end{array}\right] \left\{\begin{array}{c} C_1 \\ D_1 \\ \hline E_1 \end{array}\right\} = 0 \tag{133}$$

From Eqs. (131) with $x_o = y_o = 0$, the constraint equations are:

$$\begin{bmatrix} 1 & 0 & -h_y \\ 0 & 1 & +h_x \end{bmatrix} \begin{Bmatrix} C_1 \\ D_1 \\ E_1 \end{Bmatrix} = \{0\}$$

Hence $$[G_1] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad [G_2] = \begin{bmatrix} -h_y \\ +h_x \end{bmatrix}$$

From Eq. (117)

$$\Gamma = -[G_1]^{-1}[G_2] = \begin{bmatrix} +h_y \\ -h_x \end{bmatrix} \tag{134}$$

Then, from Eqs. (133) and (134),

$$\Gamma^T D_{11}\Gamma = [(P_y-P)h_y^2-P_{xy}h_xh_y-P_{xy}h_xh_y+P_x-P)h_x^2] \tag{135}$$

$$\Gamma^T D_{12} = 0 \tag{136}$$

$$D_{21}\Gamma = 0 \tag{137}$$

$$D_{22} = r_o^2(P_\phi - P) + \frac{F}{n^2}\frac{L^2}{\pi^2} \tag{138}$$

Then by Eq. (118)

$$\overline{D} = \Gamma^T D_{11}\Gamma + \Gamma^T D_{12}\Gamma + D_{21}\Gamma + D_{22}$$

Adding Eqs. (135) and (138), and substituting for $\overline{D}$ in Eq. (118a) gives:

$$\left[(P_y-P)h_y^2-P_{xy}h_xh_y-P_{xy}h_xh_y+(P_x-P)h_x^2+r_o^2(P_\phi-P)+\frac{F}{n^2}\frac{L^2}{\pi^2}\right]\{E_1\} = \{0\} \tag{139}$$

For a bar with two planes of symmetry[3], $I_{xy} = 0$, that is, $P_{xy} = 0$. For nontrivial solutions, the coefficients of E_1 in Eq. (139) must vanish. Then

$$(P_y - P)h_y^2 + (P_x - P)h_x^2 + r_o^2(P_\phi - P) + \frac{F}{n^2}\frac{L^2}{\pi^2} = 0$$

Hence $$P_{cr} = \frac{P_y h_y^2 + P_x h_x^2 + r_o P_\phi + \frac{F}{n^2}\frac{L^2}{\pi^2}}{h_y^2 + h_x^2 + r_o^2} \tag{140}$$

Equation (140), derived from the general solution, is identical to Eq. (132) of Timoshenko.

Chapter 4

DESIGN SIMPLIFICATION OF THE GOVERNING EQUATIONS

4.1 General

The governing equations of channel and zee-section columns braced with diaphragms, as presented in Chapter 2, are too involved for design use, especially in the case of wall studs, with which the present investigation is concerned. Therefore an attempt is made to develop practical means for checking the critical buckling load of these cases. This chapter gives the results of this effort to simplify the use of these governing equations as well as a list of the methods used and comments on their applicability and efficiency, so that a record of the present state of knowledge will be available if future consideration of the problem should arise.

Two approaches were considered to develop simple design methods. These are:

1) Reducing the quadratic and cubic governing equations to linear approximate formulas within practical levels of approximation.

2) Preparing design charts to serve as design aids within practical ranges of the varying parameters involved.

By using the first approach, it was possible to obtain approximate formulas that give buckling loads within practical accuracy for sections braced on both sides. For sections braced on one side, more than one method has been used to derive several approximate formulas. However, none of these formulas yield acceptable approximate values of the buckling load. The loads obtained did not have any regular pattern; besides, unconservative values have been obtained in some cases. Therefore the use of the exact governing equations is recommended. In fact, once the various parameters (P_x, P_y, P_{xy}, P_ϕ) are calculated for a particular case, then solving the resulting cubic equation for the smallest root only is not a difficult problem, even without electronic computational facility. Numerical analysis methods offer several techniques to simplify the solution(41,51).

4.2 Higher Buckling Modes

It has been shown in Ref. 3 that bars with enforced axes of rotation or on an elastic foundation may buckle in a higher buckling mode, that is, buckle into a number of n half-sine waves, where n = 1,2,3... . Such a conclusion does not apply to the case of I-section columns with two-sided shear-type bracing, as can be seen from Eq. (32)(13):

$$P_{cr} = \frac{n^2\pi^2EI_y}{L^2} + Q$$

Obviously, the lowest value of P_{cr} is obtained for n = 1, regardless of the relative stiffness of the column and the diaphragm. However, if twisting is involved in the failure mode, then the number of half-sine waves depends on the relative magnitude of the flexural (and torsional) rigidity of the column, the shear rigidity and the rotational restraint of the diaphragm. These parameters are considered in a numerical investigation to examine the validity of higher buckling modes. The variations of these parameters are chosen within the practical range of wall studs construction. The following gives a summary as well as the results of considering such variations.

a) Studs braced on both sides

1) $n = 1,2,\ldots 10$

2) For channel sections $Q = 0, 10, 20, \ldots 100$

$F = 0.0, 0.01, 0.02, \ldots 0.2$

3) For zee-sections $Q = 0, 10, 20, \ldots 200$

(F does not influence the behavior of the column)

4) $L = 8, 12, 16$ feet

Practical values of Q and F do not exceed 90 and 0.08, respectively. Sections were chosen at random to cover the range from 2" to 6" sections, with form factors equal to or less than 1.0.

b) Studs braced on one side

1) n was considered up to 10 and then reduced to $n = 1,2, \ldots 5$ to save on the computational expenses, since higher buckling modes result by examining the first five terms. Also, values of Q considered herein differ from those considered in the above case (a), since it has been found that higher buckling modes are more likely to occur with combinations of small values of Q and large values of F.

2) $Q = 0, 20, \ldots 80$ and

$F = 0.0, 0.05, \ldots 0.2$

3) $L = 8, 12, 16$ feet

From the numerical investigation the following has been concluded:

1) For zee-sections braced on both sides, n = 1 gives the lowest buckling mode.

2) For channel sections braced on both sides, higher buckling modes occur only for combinations of very low values of Q and high values of F; for example, for a 6" channel 16-gage without lips, the higher buckling mode of 1.007 times the buckling mode corresponding to n = 1 for values of Q and F equal to 10 and 0.08, respectively. Such combinations of Q and F are not realistic for commonly used diaphragms. Sections with form factor equal to 1.0 (sections with small depths) do not show any tendency to buckle in a higher mode.

3) For sections braced on one side, the higher buckling mode governs in some cases. The ratio of the higher buckling mode to the buckling mode corresponding to n = 1 can be as low as 0.5 in some cases.

Hence it is concluded that within practical limits of Q and F higher buckling modes are not likely to occur for studs braced on both sides; therefore the governing equations as well as the additional deflections equations, derived in Chapter 2 by considering n = 1, are valid and will be considered in this chapter.

However, in the cases of sections braced on one side, the possibility of higher buckling modes should be investigated. The choice of the values of n to be tested depends on how accurate the result should be.

4.3 Approximate Formulas

It was possible to obtain approximate linear formulas for the exact governing equations (57) and (59) of channel and zee-sections, respectively. In addition, simple formulas for the torsional-flexural buckling of unbraced singly symmetrical sections[38] and unbraced zee-sections (Eq. 60) are introduced.

The following lists the method used in the attempts to simplify the exact solutions as well as the variables considered to check the numerical accuracy of the approximate formulas. Finally, the proposed formulas are listed together with comparison of the approximate to exact loads.

4.3.1 <u>Methods Used to Obtain Approximate Formulas</u>

Appendix 2 includes a brief description and comments on the efficiency of each method. Herein they are listed according to their applicability to the cases under consideration:

1) Newton-Raphson method
2) Secant method for polynomial roots
3) Binomial expansion
4) Approximation by a piecewise linear function
5) Negligible terms of quadratics and cubics
6) Method of split rigidity
7) Comparing the behavior of sections braced on both sides with those braced on one side

4.3.2 <u>List of Variables</u>

The approximate formulas give the buckling load in terms of the parameters P_x, P_y, P_{xy}, P_ϕ, Q and F. In order to numerically check the accuracy of the formulas, these parameters have been varied to cover a wide range of wall stud constructions. These ranges are:

1) <u>Stud length L</u>, varies from 8'-0" to 16'-0"

2) <u>Diaphragm characteristics Q and F</u>

Q varies from 0.0 to 100.0 kips

F varies from 0.0 to 0.100 k.in/in.rad.

Practical values of Q are in the range of 8 to 90 kips while for F the range is from 0.015 to 0.080 kips-in/in-rad.

3) <u>Stud cross-section</u>

Standard section, with and without lips, listed in the AISI Manual[46] and manufacturers' catalogues are considered. Depths of sections vary from 2" to 8". Material thicknesses considered are 0.036", 0.048", 0.06", 0.075" and 0.105". The following gives the number of different sections examined in each of the following cases:

a) 52 channel sections braced on both sides

b) 32 zee-sections braced on both sides

c) 4 channel sections braced on one side

d) 4 zee-sections braced on one side

e) 52 channel sections with NO BRACING

4) <u>Column end conditions</u>

It has been suggested that the case of a column with both ends hinged would be satisfactory, since it represents to a great extent the actual end conditions of the stud in that type of construction. Also, the calculated buckling load will be on the conservative side.

4.3.3 Approximate Formulas

The result of simplifying the governing equations yields approximate formulas expressing the buckling load of the column in terms of known parameters. These formulas are not meant to replace the original governing equations, but are made up only to simplify the design approach within acceptable ranges of approximation. Whenever accurate results are necessary the use of the governing equations is recommended. The accuracy of the formulas was checked by comparing the buckling load thus determined to the buckling load obtained from the exact governing equation. All numerical computations were done on an IBM 360/65 system at the Cornell Computing Center.

4.3.4 Channel Sections Braced on Both Sides

The exact buckling load is the smallest value of P obtained from Eqs. (56) and (57). Equation (56) is already simple and by using Newton's method, an approximate value of the smallest root of Eq. (57) is given by the following formula:

$$P = P' + \frac{P'}{\frac{P_x + P'_\phi}{P'k} - 2} \tag{141}$$

where
$$P' = P_x P'_\phi / (P_x + P'_\phi) \tag{141a}$$

$$P'_\phi = P_\phi + \frac{1}{r_o^2}\left(Q\,\frac{d^2}{4} + F\,\frac{L^2}{\pi^2}\right) \tag{141b}$$

$$k = 1 - \frac{x_o^2}{r_o^2} \tag{141c}$$

Equation (141) represents the torsional-flexural buckling mode. The flexural mode alone is given by Eq. (56) as

$$P = P_y + Q \tag{56}$$

Hence the approximate value of the critical buckling load is the smallest value of P calculated from Eqs. (56) and (141). The accuracy of the formula was checked numerically by calculating the buckling load of columns varying in length, cross-section and diaphragm characteristics within the list of variables of Section 4.3.2. The numerical computations show that the ratios of the approximate to exact values of P range between 1.0 and 0.939; meanwhile, all values of P are on the conservative side.

4.3.5 Zee-sections Braced on Both Sides

The exact critical buckling load is given by the smallest root of the quadratic equation (59). Using Newton's method to find the smallest root of that equation, the following approximate expression has been obtained:

$$P_{cr} = P_{y1} + P' \; 1 + \frac{1}{\frac{P_{x1} - P_{y1} + Q}{P'} - 2} \tag{142}$$

where $P' = \dfrac{Q(P_x - P_{y1})}{P_{x1} - P_{y1} + Q}$

and P_{x1}, P_{y1} are the Euler buckling loads about the minimum axes of inertia, respectively. The numerical computations of sections listed in the AISI Manual(46) show that the ratios of P_{APRX}/P_{EXACT} range from 1.0 to 0.922. No zee-sections were listed in any of the manufacturers' catalogues. It seems that zee-sections are not commonly used as wall studs. However, made up sections of the same dimensions as the channel sections listed in the catalogues were used to check the accuracy of the approximate formulas. The ratios of P_{APRX}/P_{EXACT} range between 1.0 and 0.892, except for some studs 8'-0" long and with Q =

80.0 kips, the range is between 1.0 and 0.84.

4.3.6 Torsional-Flexural Buckling of Singly Symmetrical Sections Without Bracing

The torsional-flexural buckling load of unbraced channel sections is given in Ref. 38 by the smallest root of a quadratic equation (see Eq. 143). It has been found that the critical buckling load can be obtained by simple formulas which proved to yield good approximation.

Applying Newton's method to find the smallest root of the exact equation(38),

$$P^2\left(1 - \frac{x_o^2}{r_o^2}\right) - P(P_\phi + P_x) + P_x P_\phi = 0 \qquad (143)$$

the following formulas then give the approximate values of P_{cr}:

$$P_{cr} = \frac{P_x K^3 - P_\phi}{(2K^2 - 1.0) - \frac{P_\phi}{P_x}} \qquad \text{for } P_x \geq P_\phi \qquad (144a)$$

and

$$P_{cr} = \frac{P_\phi K^3 - P_x}{(2K^2 - 1.0) - \frac{P_x}{P_\phi}} \qquad \text{for } P_\phi \geq P_x \qquad (144b)$$

where $K = 1 - (x_o^2/r_o^2)$.

The accuracy of the approximate solution has been checked numerically for 50 different channel sections and colum lengths varying from 8'-0" to 16'-0". The range of ratios of the approximate to the exact loads is between 1.0 and 0.962 while most of the ratios are very close to 1.0.

4.4 Solution of the Governing Equations by Design Charts

The calculations of the buckling loads of a column braced with diaphragms from the governing equations (57), (59), (63) and (64) require that the buckling loads of the unbraced column (P_x, P_y, P_{xy}, P_ϕ) should be known beforehand. Then P_{cr} is obtained by solving the resulting quadratic or cubic equation. Such computations for unsymmetrical sections are tedious and liable to errors arising from imporper transcribing of the numbers used. Therefore a graphical solution of the algebraic equations would be desirable.

Peköz[34] proposed charts to facilitate the computation of P_{cr} of unbraced channel section columns. It is possible to extend the idea to prepare charts for the following two cases of wall studs:

1) Channels braced with diaphragms on both sides

2) Zees braced with diaphragms on both sides

In regard to sections braced on one side only, the graphical solution seems to be impractical and would be impossible if the same approach considered for two sided bracing is applied because:

a) It is not possible to express P_{cr} explicitly as a function of the varying parameters (P_x, P_y ...) since the governing equation is cubic and such a step is necessary, as will be seen in the solution of the case of two sided bracing.

b) The presence of six parameters (P_x, P_y, P_{xy}, P_ϕ, Q and F) in addition to the possibility of higher buckling modes (n = 1,2,3...) and the geometric parameters (r_o^2 and x_o^2) would make it impractical to prepare charts since several charts should be available before the critical buckling load can be obtained. Design tables would also face the same obstacle. Therefore the use of the governing equations is recommended.

4.4.1 Design Charts for Channel Sections Braced on Both Sides

The governing equation (57) for hinged end columns takes the form:

$$P^2\left(1 - \frac{x_o^2}{r_o^2}\right) - P\left[P_x + P_\phi + \frac{1}{r_o^2}\left(Q\frac{d^2}{4} + F\frac{L^2}{\pi^2}\right)\right] + P_x\left(P_\phi + Q\frac{d^2}{4} + F\frac{L^2}{\pi^2}\right) = 0 \qquad (145)$$

Let
$$P'_\phi = P_\phi + \frac{1}{r_o^2}\left(Q\,\frac{d^2}{4} + F\,\frac{L^2}{\pi^2}\right) \qquad (146)$$

and
$$K = 1 - \frac{x_o^2}{r_o^2} \qquad (147)$$

Then the smallest root of Eq. (145) is given by the quadratic form:

$$P = \frac{(P_x + P_\phi) - [(P_x + P'_\phi)^2 - 4P_xP'_\phi K]^{1/2}}{2K}$$

Introduce the dimensionless parameter

$$R = \frac{P'_\phi}{P_x} \qquad (148)$$

Then
$$P = P_x\left(\frac{1 + R}{2K}\right)\left[1 - \left\{1 - \frac{4RK}{(1 + R)^2}\right\}^{1/2}\right]$$

Let
$$\alpha = \left(\frac{1 + R}{2K}\right)\left[1 - \left\{1 - \frac{4RK}{(1 + R)^2}\right\}^{1/2}\right] \qquad (149)$$

α is a function of the dimensionless parameters K and R. Hence

$$P = P_x\alpha \qquad (150)$$

The plots of K vs. b/a and c/a shown in Fig. 14 lead to the direct determination of α from the values of R given by Eq. (148), b/a and c/a. Therefore the smallest root of Eq. (57)

can be determined from Eq. (150) and the known α. Hence the critical buckling load P_{cr} is the smallest value of P obtained from Eq. (150) and $P = P_y + Q$. (56)

4.4.2 Design Charts for Zee-sections Braced on Both Sides

The critical buckling load P_{cr} is given by the smallest root of the governing equation (59), hence

$$P_{cr} = \frac{P_x + P_y + Q}{2}\left[1 - \left\{1 - \frac{4(P_xP_y + P_xQ - P_{xy}^2)}{P_x + P_y + Q}\right\}^{1/2}\right] \quad (151)$$

Let $$P_y' = P_y + Q \quad (152)$$

$$R = \frac{P_y'}{P_x} \quad (153)$$

and $$K = \left(\frac{P_{xy}}{P_x}\right)^2 \quad (154)$$

K can be expressed in terms of the dimensionless parameters c/a, b/a since $K = I_{xy}/I_x$.

Substitution of Eqs. (152), (153) and (154) into Eq. (151) yields

$$P_{cr} = P_x\left(\frac{1 + R}{2}\right)\left[1 - \left\{1 - \frac{4(R - K)}{(1 + R)^2}\right\}^{1/2}\right]$$

Let $$\alpha = \left(\frac{1 + R}{2}\right)\left[1 - \left\{1 - \frac{4(R - K)}{(1 + R)^2}\right\}^{1/2}\right] \quad (155)$$

Then $$P_{cr} = P_x\alpha \quad (156)$$

The plot of K vs. b/a and c/a shown in Fig. 15 leads to the direct determination of α from the values of R given by Eq. (153), b/a and c/a. Therefore P_{cr} can be obtained by Eq. (156).

4.5 Summary of Simplified Equations and Graphical Aid for Design Use

A summary of approximate formulas that, in the writer's opinion, are practically accurate and simple to use are given in this section.

These formulas have been numerically examined over a wide range of various cross-sections, stud lengths and diaphragm properties. Unfortunately, despite the several attempts to simplify the unwieldy cubic equations, no satisfactory approximation has been achieved.

It should be noted that the proposed charts are for the boundary conditions $u'' = v'' = \phi'' = 0$ at both ends. However, they can be easily extended to other different boundary conditions by using the suitable K_i factors listed in Table 1 in the basic equations (145) and (151).

The following lists the proposed approximate formulas:

1) Channel sections braced on both sides:

$$P = P'\left[1 + \frac{1}{\frac{P_x + P'_\phi}{P'k} - 2}\right] \tag{141}$$

2) Zee-sections braced on both sides

$$P_{cr} = P_{y1} + P'\left[1 + \frac{1}{\frac{P_{x1} - P_{y1} + Q}{P'} - 2}\right] \tag{142}$$

3) Torsional-flexural buckling of <u>unbraced singly symmetric</u> sections

$$P_{cr} = \frac{P_x K^3 - P_\phi}{2K^2 - 1.0 - \frac{P_\phi}{P_x}} \qquad \text{for } P_x \geq P_\phi \tag{144a}$$

and $$P_{cr} = \frac{P_\phi K^3 - P_x}{2K^2 - 1.0 - \frac{P_x}{P_\phi}} \quad \text{for } P_\phi \geq P_x \qquad (144b)$$

As an alternative to Eqs. (145) and (151), design charts are proposed for channel and zee-sections braced on both sides and are plotted on Figs. 14 and 15, respectively.

For channel and zee-sections braced on one side, direct solution of the cubic equations (63 and 64) seems to be the only possible method, and is simple by computer subroutines.

Chapter 5

EXPERIMENTAL VERIFICATION OF THEORY

5.1 General

The purpose of this phase of the investigation is to verify experimentally the theoretically predicted failure loads developed in Chapter 2. Tests were conducted on a total of 11 double-column assemblies with diaphragms on one or both sides. The stud sections used in the tests and sketches of the test assemblies are shown in Table 4 and Fig. 18, respectively. A variety of wallboard materials were utilized in the testing program. They were tested separately to determine experimentally their shear and restraint characteristics. All studs were 12'-0" high, concentrically loaded and free to rotate about an axis parallel to the web while rotation was restrained about the centroidal axis perpendicular to the web. Rotation of the end sections about the column center line was restrained. Test results as well as the predicted failure loads of all assemblies are presented in Table 3, and Figs. 24 to 28. Predicted failure loads are based on $u = v = \phi = u'' = v' = \phi' = 0$ at each end consistent with the testing end conditions. Measured initial imperfections were used in computing displacements.

5.2 Materials Used

<u>Steel Studs</u>: Section type A listed in Table 4 is a 16-gage standard section commonly used in wall stud construction. All other sections listed are cold-formed from 12-gage hot-rolled sheets by a local fabricating company, according to pre-designed cross-sectional dimensions.

Types of Fasteners: Self-drilling, number 6, bugle head, 1" long, dry-wall screws were used in all tests. Spacing of the screws was selected so that an expected diaphragm shear stiffness could be obtained.

Wall Materials: The following lists the various types of wallboards used in the testing program.

1) 5/8" GYP. Boards
2) 3/8" GYP. Boards
3) 1/2" Homosote
4) 1/2" Celotex
5) 1/2" Impregnated Celotex
6) 1/2" Heavy Impregnated Celotex

5.3 Material Properties

Tests performed to determine the diaphragm characteristics as well as the mechanical properties of the steel stud are described in the following sections.

5.3.1 Diaphragm Shear Stiffness G' and Shear Strength γ_d

These quantities can be computed from the load-deflection curve obtained from a cantilever shear diaphragm test as described in the procedure of testing light gage steel diaphragms in shear (Ref. 48). The test set-up is shown in Fig. 21. The frame was made of 12-gage steel stud sections and was used for all tests. Centerline size of the frame was 24" x 24". The pinned connections of the frame members offered no resistance to frame deformations prior to attaching the wallboard. The diaphragms were fastened to the frame members with 1" long self-drilling screws spaced at the same spacing used in the stud assembly. Loads were applied in increments and deflections in the plane of the diaphragm were measured. The diaphragm shear deflection Δ is given by

$$\Delta = D_3 - [D_1 + \frac{a}{b}(D_2 + D_4)]$$

where D_1, D_2, D_3, D_4 are the readings of dials 1, 2, 3, 4, respectively, while a and b are the dimensions of the frame centerlines as shown in Fig. 21. The shear stiffness G' is defined as:

$$G' = \frac{0.8P_{ult}/b}{\Delta_d/a} \qquad (157)$$

where Δ_d is the shear deflection at $0.8P_{ult}$. If the shear stiffness G' is known, then the shear strain γ_d gives a measure of the strength of the diaphragm. Shear strain γ_d is given by:

$$\gamma_d = \frac{\Delta_d}{a} \qquad (158)$$

Equations (157) and (158) are used to compute the shear stiffness and shear strength of the different types of wallboard materials used in the testing program. The computed values are listed in Table 2. Load-deflection curves of these wallboards are shown in Fig. 22.

In general the shear characteristics of the diaphragm depend to a great extent on the fastener spacing, type of fastener mechanical properties as well as dimensions of the diaphragm and whether the diaphragm is fastened along two or four sides.

5.3.2 Rotational Restraint of the Diaphragm

Details of the test-set-up, as shown in Fig. 20, consist of a diaphragm fastened at one edge to a clamped stud section; the other edge was acted upon my a slowly increasing load. The span of the cantilever was half the distance between the studs. In our case the cantilever spanned 12". At the edge where the load was applied, a light stiffening timber strip was used to obtain a uniform deflection of the free end of the diaphragm. The fasteners were aligned at the flange centerline to simulate the position of the fasteners in the stud assembly. Such fastener location was kept the same in the tests since the fastener location influences the rotational restraint of the diaphragm.

The total deflection Δ_{total} of the free end of the diaphragm is equal to:

$$\Delta_{total} = \Delta_D + \Delta_B + \Delta_S \qquad \text{(see Fig. 19)}$$

where Δ_D = deflection due to local deformation of the diaphragm at the fastener location

Δ_B = elastic deflection of the diaphragm due to bending in a beam type action

Δ_S = deflection due to deformation of the flange with respect to the web

It has been found from the test results that the major part of Δ_{total} is mainly due to Δ_D caused by the local deformation of the diaphragm. In regard to Δ_B and Δ_S, the following had been considered:

1) Deflection Δ_S due to deformation of the flange with respect to the web was so small that it can be neglected without considerable error. For example, in the test of $\frac{1}{2}$" HOMOSOTE BOARDS, for which the highest moment was sustained, such deflection at $0.8M_{ult}$ was 0.06" while the total deflection $_{total}$ was equal to 1.9". Such results were expected, especially for wall materials used in wall-studs applications.

2) Elastic deflection were calculated by knowing experimentally the value of EI of a unit width of the diaphragm material at each stage of loading. These values of EI were obtained from a simple span flexural beam test made of the diaphragm material.

It is of interest to note that in up to about 20% of the ultimate loads in the flexural beam test of different diaphragms behavior was elastic and the values of the elastic moduli calculated for the tested diaphragms were:

1/2" Celotex	$E = 0.05 \times 10^6$ lb/in^2
1/2" Impregnated Celotex	$E = 0.076 \times 10^6$
1/2" Homosote	$E = 0.09 \times 10^6$
3/8" GYP.	$E = 0.275 \times 10^6$

The elastic deflections at $0.8M_{ult}$ were found to be 10%, 11% and 20% of the total deflection for 1/2" Celotex, 1/2" Homosote and 3/8" GYP. boards, respectively.

Therefore, since the elastic deflection Δ_B contributes a small part of the total deflection Δ_{total}, it has been concluded that no provision to calculate the elastic deflection separately would be necessary. Instead, the measured Δ_{total}, which includes Δ_B and Δ_S as well, would give an accurate means for calculating the rotation of the diaphragm ϕ_d and consequently the rotational restraint F.

Figure 20 shows the rotation of the diaphragm vs. the applied moment where the rotational restraint F' is given by:

$$F' = \frac{0.8M_{ult}}{\phi_d} \text{ lb.in/in.rad.} \tag{159}$$

where M_{ult} is in lb.in/in.

ϕ_d is the angle (in radians) of rotation of the diaphragm at $0.8M_{ult}$, and is equal to:

$$\phi_d = \frac{\Delta_{total}}{\ell} \tag{160}$$

(for large values of Δ_{total}, $\phi_d = \sin^{-1}(\frac{\Delta_{total}}{\ell})$), where

Δ_{total} = measured deflection at the free edge of the cantilever (in.)

ℓ = span of the cantilever (in.), ℓ = 12" in the tests

Values of F' and ϕ_d for different diaphragm materials are listed in Table 2. These correspond to the direction of force giving conservative values as discussed in Section 2.3.3.

5.3.3 Tension Coupon Tests

Standard tensile coupons from the web of section type A (see Fig. 23), taken from 3 different pieces showed average yield stress of 58.0 $\pm$ 0.2 ksi and elastic modulus E = 30.0 x 10^3 ksi.

Tensile coupon tests taken from the hot-rolled sheets from which all other sections are cold-formed showed an average yield stress 53.0 $\pm$ 0.4 ksi and modulus of elasticity of 29.6 x

10^3 ksi.

In both cases the proportional limit σ_p was above 70% of the yield stress.

5.4 Description of Tests

All test assemblies consisted of two equally loaded studs of channel or zee-sections with wallboards on one or on both sides.

The wallboards, forming a continuous bracing diaphragm, were attached to the studs with 1" long self-drilling screws at a selected spacing identical to that used in the cantilever shear test.

A 300 kip capacity universal hydraulic testing machine was used in all tests. The ends of each stud were welded to 3/4" base plates and the studs were individually supported on knife edges parallel to the web. Each stud rested on a 50 kip capacity hydraulic jack connected to a common supply to insure that the same load was applied to each stud throughout the test unaffected by minor variations in the individual length of the two columns.

A minimum of 16 dial gages reading 0.001 inch were used in each test to measure the column deflections as shown in Fig. 18. To avoid premature failure, the centering of the studs was repeated at increasingly higher loads up to about 2/3 of the predicted failure load.

The distribution of initial imperfections along the stud were measured, after attaching the diaphragm, by a transit. The maximum value of initial bow measured at the middle of the stud was 0.10" (i.e. about L/1500).

5.5 <u>Specimens' Design and Test Results</u>

Tested assemblies are classified into two groups:

a) Sections braced with diaphragms on both sides

b) Sections braced with diaphragms on one side only

For each of these groups and each type of section (channel or zee), the governing equations characterizing the column behavior are different. Consequently, sections with specific dimensions were needed to verify each of these different cases.

At the early stage of the investigation, 4 assemblies made of channel sections were tested (1A, 2A, 3A and 4A). These sections (type A, Table 4) were stock items of wall studs products. The test results of assemblies of channel sections braced on one side only are satisfactory and are in good agreement with the theory. However, columns braced on both sides failed due to sudden local buckling of the web and the results therefore do not relate to the overall buckling characterization (see Table 3). Thus it was found necessary to test sections proportioned so that local buckling and failure by yielding of the column material could not occur before overall buckling of the stud. These sections are classified in types B, C, D (see Table 4).

Diaphragm materials and fastener spacing were chosen so that only the desired mode of buckling would occur. For example, channel sections braced on both sides may buckle in the torsional-flexural buckling mode or in the flexural mode (see Fig. 8), depending on the value of Q of the bracing diaphragm.

It can be shown from the graph that diaphragms with large values of Q and F may force the stud to buckle in the torsional-flexural mode. The $2\frac{1}{2}''$ channel section type B with two sided bracing was designed to fail in the torsional-flexural mode while the 3"channel section type C braced on both sides was designed to buckle in the flexural mode. Assemblies 5B, 6C, 7C, listed in Table 3, corroborated such behavior of the channel sections (see Figs. 24, 25).

For zee-sections braced with diaphragms on both sides, only felxural buckling governs. Assemblies 8D and 9D were tested with two different diaphragms to verify such behavior, and the test results are shown in Fig. 26.

The behavior of channel and zee-sections braced on one side is characterized by torsional-flexural buckling only; therefore no special considerations were necessary in choosing the stud section and the diaphragm materials. Test results of these two cases are shown on Figs. 27 and 28, respectively.

Test results are listed with the predicted failure loads of all assemblies in Table 3. Figures 24 to 28 depict these results as well as the behavior of the stud as a function of Q and F. In general, all failure loads are in good agreement with the predicted loads.

5.6 <u>Interpretation of Test Results</u>

Test results considerably substantiate the theoretical findings. The failure loads rangs from 92% to 99% of the theoretical loads except for the 16 gage channel section type A tested in the early stage of the investigation, which failed at

85% of the theoretical buckling load.

Such a low value is related to excessive initial imperfections which had been noticed in that light section.

The failure loads are always less than the theoretical load, except for assembly 1A which failed by local buckling of the stud at 102% of the theoretical load. The theoretical loads are higher than the failure loads due to two main reasons:

1) The theory is based on an energy approach and the assumed deflected shape yields approximate critical loads higher than the rigorous critical load. Assuming a deflected shape that is not exactly as the actual one is equivalent to introducing restraints to the member which increase the calculated buckling load. Nevertheless, the comparison between failure and theoretical loads lead to reasonably satisfactory results.

2) The theoretical load is the load at which bifurcation of equilibrium occurs in a perfect column. An actual member, due to unavoidable imperfections of geometry and eccentricities of loading, does not exhibit this idealized behavior.

In fact, the difference between the tested loads and the theoretical loads is not significant in spite of the abovementioned reasons. This can be related to the following: in regard to the first reason, the Payleigh-Ritz method is used to obtain an approximate solution by direct substitution of assumed displacement functions into the total energy expression. Fortunately, the first variation of the total potential energy is not too sensitive to variations of the deflected shape and we can expect reasonable results if we use an approximate deflected shape of the column, making sure that such an assumed deflected shape satisfies the end conditions of the stud. As for the second reason concerning the imperfections of the column and its effect on the failure load, it has been found that these initial imperfections were quite small. This was due to the extreme case taken in fabricating these sections. For example, the maximum initial deflection in the plane of the diaphragm was found equal to:

$$\delta_{in} = \frac{L}{1500}$$

Such small initial imperfections in addition to the centering procedure of the stud during testing tend to closely idealize the condition of the stud.

Failure of the diaphragm due to connection failure is the primary mode of the overall column buckling. Two types of failure were observed:

a) Sudden failure generally occurs when torsional-flexural buckling is encountered. Local deformation appears at the end fastener before complete failure occurs; however, at failure the fasteners at the middle portion pull off from the diaphragm. This behavior can be noticed from photographs 1 and 2. These photographs belong to assembly 5B, which was designed to fail in torsional-flexural buckling. Referring to photograph 1, local deformation of the diaphragm at the end fastener was observed at a load equal to 20.5 k per stud. Upon further increase of the applied loads, local deformation of the diaphragm started at the location of the fastener next to the end one, as can be seen from the photograph. At that stage excessive rotation of the stud section was observed and finally, sudden failure of the stud at a load of 23.4 was accompanied by pulling off from the diaphragm at the middle portion (see photograph 2). Photograph 1 belongs to the stud which did not fail. Local deformation of the diaphragm at the end fasteners of each stud were identical prior to failure. Twisting, as shown in Fig. 31, before failure further indicates tendency for torsional flexural buckling.

The 97% of the theoretical loads achieved by this test is considered satisfactory. In addition, this indicates that the experimental procedure of determining the diaphragm characteristics (Q, F) which are used in calculating the theoretical load is reliable.

b) Slow failure, compared to the first type, usually occurred when flexural buckling governed. In this type of failure, only the end fastener was observed to be overstressed. The fasteners at the middle section of the column did not seem to be critical. Failure of the assembly was accompanied by tearing of the diaphragm material at the end fastener only. Figures 31, 32 depict the distinctive behavior of channel sections braced on both sides failing in the torsional-flexural mode and the flexural mode, respectively. Figure 31 shows both the experimental and theoretical displacements of the middle section of the stud in the plane of the diaphragm as well as the rotation of the same section. This figure represents test 5B in which both the diaphragm and the stud section were selected so that torsional-flexural buckling governs. The figure shows that the rotation of the stud is more critical than the deflection, especially when the buckling load is reached. The rotation becomes indefinitely large while the deflection has a finite value and this is why torsional-flexural buckling is unavoidable.

Contrary to the previous case, Fig. 32 shows that for flexural buckling the displacement of the middle section of the stud in the plane of the diaphragm is more critical than the rotation of the same section, and at the critical load the displacement becomes indefinitely large while the rotation has a real value. Therefore flexural buckling in the plane of the diaphragm is imminent. This type of failure occurred when assemblies 6C and 7C, which were designed to fail in flexural buckling, were tested.

Theoretical loads of the 12 gage studs were all within the proportional limit of the stress-strain curve of the virgin material, except for the zee-section braced on both sides. The theoretical loads for the latter has been corrected by taking into account the tangent modulus of elasticity E_t, measured from the tension coupon test results, at the buckling stress level. Substituting E_t for E and $G(\frac{E_t}{E})$ for G in the governing equations, the final load was obtained by iteration. These loads were slightly higher than those obtained by using formulas of Section 3.6.1 of the AISI Specification(1). For example, the inelastic theoretical loads for assembly 9D computed by iteration and by the AISI formula are 28.2 and 27.4 k, respectively. The difference, though expected, is not significant because the AISI formulas are based on a proportional limit equal to one-half the yield stress, and in the present investigation the proportional limit is 0.74 of the yield stress of the virgin material.

It is of interest to note that the zee-section braced on both sides did not show tendency to rotate; only displacements along the wall and perpendicular to the wall were observed. Such behavior has been predicted by the theory. On the other hand, displacement and rotation of considerable values were measured during testing of channel and zee-sections braced on one side only. The theory predicts in these cases indefinitely large values of displacement and rotation at the critical loads (see Figs. 33 and 34). This is contrary to what has been found in the case of sections braced on both sides, for which either one - displacement or rotation - becomes indefinitely large while the other has a real value. Hence, in the cases of sections braced on one side only, the rotational restraint of the diaphragm, F, is as important as the shear rigidity Q.

It should be noted that flexural buckling indicated for channel sections is about the centroidal axis parallel to the web. For channel sections, torsional flexural buckling load is always smaller than the flexural buckling load about the centroidal axis perpendicular to the web.

The agreement between the test and theoretical results is seen to be very satisfactory. This indicates that the design approach presented in the next chapter is expected to give reliable results.

Chapter 6

WALL STUDS DESIGN CRITERIA

The detailed method of analysis is presented below in Section 6.3 of this chapter. A collection of all the equations which are needed for the design, and which have been derived in various parts of this investigation, are included here with a new set of numbers in Section 6.4. Following these equations Section 6.6 gives a list of the new and the original number for each equation and its source when necessary. Section 6.7 contains a complete nomenclature for the design procedure. Complementary to this chapter are Appendices 1, 4 and 6. Design examples as well as complete design computer programs are given in Appendices 1 and 4, respectively. Appendix 5 provides a record of the reasoning behind, and justification for, the various parts of the design criteria.

6.1 Introduction

The design procedure suggested herein is based mainly on the theoretical results of Chapter 2. The procedure is formulated in a systematic step-by-step method of analysis, so that direct application of the theoretical findings would be facilitated. The reasoning behind, and justifications for, the various steps of analysis is given in Appendix 6. Based on the suggested procedure, Appendix 4 comprises four computer programs given as a design aid. These have been utilized in the solution of the design examples of Appendix 1.

It is not intended to formulate the findings of this chapter in design specification language. Rather, the suggested design procedure outlines rational and practical methods of design.

6.2 Limitation of the Procedure

The design procedure is limited to channel, zee and I-section studs hinged at both ends, subjected to axial concentric load and attached to wallboards as specified herein, forming a continuous diaphragm on one or both sides of the section.

In general, two design situations may arise in wall stud analysis. These will be handled separately under headings A and B:

A) Determining the allowable load of the stud if the diaphragm shear rigidity Q and rotational restraint F are known.

B) Finding Q and F so that the stud can sustain a specified allowable load.

In both cases, buckling loads in the elastic and inelastic domains are considered in the analysis.

6.3 Method of Analysis

(A) Allowable Load P is Required for Known Values of Q and F (Sections braced on one side or on both sides)

(1) Calculate the critical buckling load P_{cr} and the corresponding n (when applicable) of the perfect column based on the governing equations listed in Section 6.4.1; accordingly, compute $\sigma_{cr} = P_{cr}/A$. It should be noted that the fastener spacing is related to the value of n. The fasteners should be arranged according to Section 6.4.4.

(2) IF $\sigma_{cr} \leq 0.5 Q_A \sigma_y$,

then elastic buckling governs and hence follow steps

(2a) through (2f), otherwise go to step 3.

(2a) Consider a trial load $P_r = \lambda P_{cr}$ where $\lambda \leq 1.0$ is a trial reduction factor.

(2b) Consider a real column and introduce the initial imperfections C_o, D_o and E_o according to Flormulas 11. Then from equations of Section 6.4.2, calculate the deflection C_1 and the rotation E_1 at that particular load P_r and the corresponding n (when applicable). From this compute the maximum shear strain γ_{max} and the maximum rotation ϕ_{max} of the assembly according to Eqs. (12) and (13) or (17) and (18).

(2c) Check that the calculated γ_{max} and ϕ_{max} do not exceed γ_d and ϕ_d of the bracing diaphragm, respectively.

(2d) If such a condition is not met, then try a smaller λ and hence a smaller P_r, and repeat the analysis in steps (2b) and (2c) until the requirements are satisfied.

(2e) The load capacity of the stud for known Q and F is therefore given by the last trial value of P_r.

(2f) The allowable design load $P_{all} = P_r/F.S.$

(3) IF $\sigma_{cr} > 0.5 Q_A \sigma_y$

then inelastic buckling governs.

(3a) Calculate the inelastic buckling load P_a using Eq. (24) (AISI formula); compute $\sigma = P_a/A$ and determine the corresponding inelastic moduli E* and G* from Eqs. (25) and (27), respectively.

(3b) Find the initial imperfections C_o, D_o, E_o of the real column according to Formula 11. From the equations of Section 6.4.2, with $P_r = P_a$ and (when applicable) the value of n as obtained in step 1, calculate C_1 and E_1 based on the computed E* and G*. Next, calculate γ_{max} and ϕ_{max} from Eqs. (12) and (13) or (17) and (18).

(3c) Check that $\gamma_{max} \leq \gamma_d$ and $\phi_{max} \leq \phi_d$. If such requirements are not satisfied, then try a smaller load (i.e. $P_r = \lambda P_a$ where $\lambda < 1.0$).

(3d) Calculate the stress corresponding to the new trial load,

$$\sigma = P_a/A$$

Check whether $\sigma > 0.5Q_A\sigma_y$ and if so, calculate the respective E* and G*; otherwise use the elastic moduli E and G instead.

(3e) With the new trial load and the corresponding moduli, calculate γ_{max} and ϕ_{max} from the equations of Section 6.4.2 (as in step 3b).

(3f) Check that $\gamma_{max} \leq \gamma_d$ and $\phi_{max} \leq \phi_d$. Repeat the procedure until these requirements are met.

(3g) The value of the last trial load (P_r) represents the load capacity of the stud.

(3h) The allowable design load $P_{all} = P_r/F.S.$

(B) <u>Required Q and F if Allowable Load on the Stud P_{all} is Given</u>

(I) <u>Sections Braced on Both Sides</u>

(1) Calculate the required load capacity P_o where

$$P_o = P_{all} \times F.S.$$

(2) Check that:

$$P_o > P_{cr,UB}$$

$$P_o < P_{cr,x}$$

$$\text{and} \qquad P_o < P_{yield}$$

where $P_{cr,UB}$ = critical buckling load of unbraced stud

$P_{cr,x}$ = strong axis buckling load (perpendicular to the wall)

P_{yield} = yield load of the stud

If any of the above conditions are violated, change the stud cross-section for economical design.

(3) IF $\sigma = \frac{P_o}{A} \leq 0.5 Q_A \sigma_y$,

then elastic behavior governs. Follow steps (3a) through (3c), otherwise go to step 4.

(3a) Substitute P_o for P in the governing equations of Section 6.4.1.1. Find the value of Q that satisfies the respective equation; except in the case of channel sections, the governing value of Q is the larger one obtained from Eq. (2) and Eq. (3) with F = 0. This furnishes a starting value for Q.

(3b) As a first trial increase the value of Q or Q and F, above that of step (3a). Then from the equations of Section 6.4.2 and 6.4.2.1, with $P_r = P_o$, calculate C_1 and E_1 and hence γ_{max} and ϕ_{max}.

(3c) Select from Diaphragm Catalogues or from diaphragm test results a suitable diaphragm for which the parameters Q, F, γ_d and ϕ_d are equal to or larger than calculated in step (3b). If such a diaphragm is not available then repeat step (3b) with larger Q or Q and F until a suitable diaphragm is obtained. Such a diaphragm will be adequate for bracing the stud for the given load.

(4) IF $\sigma = \frac{P_o}{A} > 0.5Q_A\sigma_y$,
then inelastic buckling governs.

(4a) The procedure is a trial and error method. Assume practical values of Q or Q and F and use the governing equations of Section 6.4.1 to find the elastic P_{cr}. Calculate the corresponding inelastic buckling load P_a from Eq. (24). If $P_a \geq P_o$ then proceed to the next step; otherwise try larger values of the diaphragm constants.

(4b) From Eqs. (25) and (27) calculate the inelastic moduli E* and G* corresponding to the stress $\sigma = P_o/A$. Then, from the equations of Sections 6.4.2 and 6.4.2.1 (with Q, F obtained from step (4a), $P_r = P_o$, E* and G*) calculate C_1, E_1, then γ_{max} and ϕ_{max}.

(4c) Select a diaphragm from Diaphragm Catalogues or diaphragm test results, for which Q, F, γ_d and ϕ_d are equal to or larger than their corresponding values calculated in step (4b). If such a diaphragm is not available then repeat the analysis starting with step (4a), until a suitable diaphragm can be obtained.

(II) <u>Sections Braced on One Side Only</u>

(1) Calculate the load capacity P_o from

$$P_o = P_{all} \times F.S.$$

(2) Check that

$$P_o > P_{cr,UB}$$

$$P_o < P_{cr,x}$$

and $P_o < P_{yield}$

If any of these conditions are not satisfied then change the stud cross-section for an economical design.

(3) IF $\sigma = \frac{P_o}{A} \leq 0.5 Q_A \sigma_y$,

elastic buckling governs. Follow steps (3a) through (3d); otherwise go to step 4.

(3a) Consider a sufficient number of n-values, where n is the number of half-sine waves into which the column may buckle. For conventional wall stud application, n = 1,2,3...6 commonly suffices. Assume a trial value of F and use the governing equations of Section 6.4.1 with $P = P_o$ to find the value of Q for each considered value of n. With the largest value of Q and the assumed F, use the governing equations of Section 6.4.1.2 to find P_{cr} and the corresponding n. The fastener spacing is related to the value of n. The fasteners should be arranged according to Section 6.4.4.

(3b) Check that $P_{cr} \leq P_o$; otherwise increase Q and F, and hence find P_{cr} and the corresponding n (as outlined in step 3a). Repeat until a calculated value of $P_{cr} \leq P_o$ can be found. Now, the output of this step is Q, F and n, that is, one knows the diaphragm properties and the critical buckling mode. It is left to check the daiphragm adequacy.

(3c) With $P_r = P_o$ and the values of Q, F and n found in step (3b), use the equations of Sections 6.4.2 and 6.4.2.2 to calculate C_1 and E_1, hence γ_{max} and ϕ_{max}.

(3d) Select a diaphragm from Diaphragm Catalogues or diaphragm test results, for which Q, F, γ_d and ϕ_d are equal to or larger than their corresponding values calculated in step (3c). If such a diaphragm is not available then try larger values of Q and F, and follow the method of analysis outlined in steps (3b) and (3c) until a suitable diaphragm can be obtained.

(4) IF $\sigma = \frac{P_o}{A} > 0.5Q_A\sigma_y$,

then inelastic behavior governs.

(4a) Consider a sufficient number of n-values, where n is the number of half-sine waves into which the column may buckle. For conventional wall stud application, considering n = 1,2,3...6 commonly suffices. The procedure from now on is a trial and error approach. Assume practical values of Q and F, then use the governing equations of Section 6.4.1 to find the elastic P_{cr} and the corresponding n. Calculate the inelastic buckling load P_a from Eq. (24). If $P_a > P_o$, proceed to the next step; otherwise try larger values of Q and F.

(4b) From Eqs. (25) and (27) calculate the inelastic moduli E* and G* corresponding to the stress $= P_o/A$.

Then, from the equations of Sections 6.4.2 and 6.4.2.2 (with Q, F, n obtained from step (4a), E^*, G^* and $P_r = P_o$) calculate C_1, E_1 then γ_{max} and ϕ_{max}.

(4c) Select from Diaphragm Catalogues or diaphragm test results a suitable diaphragm for which Q, F, γ_d and ϕ_d are at least equal to their respective values obtained in steps (4a) and (4b). If such a diaphragm is not available then increase the values of Q and F and follow the analysis outlined in steps (4a) and (4b), until a suitable diaphragm can be obtained.

6.4 Design Formulas

This section contains all the equations needed for the design. They were originally derived in various parts of this report. Here these equations are included under a new set of numbers, and for cross-referencing between the original and the new numbers, a list is given in Section 6.6 for this purpose.

6.4.1 The Governing Equations

In general the following parameters will be used in the subsequent equations:

$$P_x = n^2\pi^2 EI_x/L^2 \tag{1a}$$

$$P_y = n^2\pi^2 EI_y/L^2 \tag{1b}$$

$$P_{xy} = n^2\pi^2 EI_{xy}/L^2 \tag{1c}$$

$$P_\phi = \frac{1}{r_o^2}\left(n^2 EC_w \frac{\pi^2}{L^2} + GJ\right) \tag{1d}$$

where n = 1,2,3,... for sections braced on one side only
and n = 1 for sections braced on both sdies

$E = E^*$ and $G = G^*$ if inelastic behavior governs

6.4.1.1 Sections Braced on Both Sides

For the design equations below, the parameters P_x, P_y, P_ϕ and P_{xy} are computed from Eqs. (1) with n = 1. The critical buckling load P_{cr} is the smallest value of P calculated from the governing equation (or equations) for the section under investigation.

CHANNEL SECTION:

$$P = P_y + Q \tag{2}$$

$$P^2(r_o^2-x_o^2)-P(r_o^2P_x+r_o^2P_\phi+Q\frac{d^2}{4}+F\frac{L^2}{\pi^2})+P_x(r_o^2P_\phi+Q\frac{d^2}{4}+F\frac{L^2}{\pi^2}) = 0 \tag{3}$$

Z-SECTION:

$$P^2 - P(P_x + P_y + Q) + P_xP_y + P_xQ - P_{xy}^2 = 0 \tag{4}$$

I-SECTION:

$$P = P_y + Q \tag{5}$$

$$P = P_x \tag{6}$$

6.4.1.2 Sections Braced on One Side Only

The parameters P_x, P_y, P_ϕ and P_{xy} appearing in the following equations are calculated from Eqs. (1) with the value of n = 1,2,3,... . Usually for wall stud applications n = 1,2,3,... 6 will suffice to detect the governing buckling mode. Note that for I-sections P_x must be computed for n = 1 only (see reasoning in the conclusion of Appendix 5).

The smallest value of P obtained from the governing equation (or equations) for a particular section and for different values of n determines the critical buckling load P_{cr}.

CHANNEL SECTION:

$$P^3(1-\frac{x_o^2}{r_o^2})-P^2[P_x+P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{L^2}\,\frac{L^2}{\pi^2})+(P_y+Q)(1-\frac{x_o^2}{r_o^2})]$$

$$+P[P_x(P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})+(P_y+Q)(P_x+P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})-\frac{1}{r_o^2}(Q\frac{d}{2})^2]$$

$$-P_x(P_y+Q)[P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]+\frac{1}{r_o^2}P_x(Q\frac{d}{2})^2 = 0 \qquad (7)$$

Z-SECTION:

$$P^3-P^2\;[P_x+P_y+Q+P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]$$

$$+P\{(P_y+Q)P_x-P_{xy}^2+(P_y+Q+P_x)[P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]-\frac{1}{r_o^2}(Q\frac{d}{2})^2\}$$

$$-[(P_y+Q)P_x-P_{xy}^2][P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]+\frac{1}{r_o^2}P_\phi(Q\frac{d}{2})^2 = 0 \qquad (8)$$

I-SECTION:

$$P = P_x \qquad (9)$$

where $P_x = \pi^2EI/L^2$ (i.e. n = 1)

$$P^2-P[P_y+P_\phi+Q+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]$$

$$+(P_y+Q)[P_\phi+\frac{1}{r_o^2}(Q\frac{d^2}{4}+\frac{F}{n^2}\,\frac{L^2}{\pi^2})]-\frac{1}{r_o^2}(Q\frac{d}{2})^2 = 0 \qquad (10)$$

(n = 1,2,3,...)

6.4.2 <u>Equations of γ_{max} and ϕ_{max}</u>

In the equations below, the parameters P_x, P_y, P_ϕ and P_{xy}

are defined by Eqs. (1). The value of n corresponding to P_{cr} obtained in the previous section is used in the following equations wherever required. If inelastic behavior governs, then E and G employed in this section should be replaced by the E* and G* from Eqs. (25) and (27), below, for the stress level at which the deflections and rotations are computed (i.e. corresponding to $\sigma = P_r/A$).

Initial imperfections accounting for initial sweep plus accidental load eccentricities may be considered according to the following tentatively suggested formulas:

$$C_o = 2(L/700) \tag{11a}$$

$$D_o = L/700 \tag{11b}$$

$$E_o = 0.0006 \text{ rad. per foot of length} \tag{11c}$$

6.4.2.1 Sections Braced on Both Sides

The maximum shear strain $_{max}$ and maximum rotation $_{max}$ are computed according to the following formulas:

$$\gamma_{max} = \frac{\pi}{L}(C_1 + E_1 \frac{d}{2}) \tag{12}$$

$$\phi_{max} = E_1 \tag{13}$$

where C_1 and E_1 are absolute values calculated from the following equations for a particular section.

CHANNEL SECTION:

$$C_1 = \frac{P_r C_o}{P_y - P_r + Q} \tag{14a}$$

$$E_1 = \frac{P_r}{A_1}\{(P_x - P_r)(r_o^2 E_o - x_o D_o) - P_r x_o (D_o - x_o E_o)\} \tag{14b}$$

where $A_1 = (P_x - P_r)[r_o^2(P_\phi - P_r) + Q\frac{d^2}{4} + F\frac{L^2}{\pi^2}] - (P_r x_o)^2$

Z-SECTION:

$$C_1 = \frac{P_r[C_o(P_x - P_r) - D_o P_{xy}}{(P_y - P_r + Q)(P_x - P_r) - P_{xy}^2} \quad (15a)$$

$$E_1 = \frac{P_r E_o r_o^2}{r_o^2(P_\phi - P_r) + Q\frac{d^2}{4} + F\frac{L^2}{\pi^2}} \quad (15b)$$

I-SECTION:

$$C_1 = \frac{P_r C_o}{P_y - P_r + Q} \quad (16a)$$

$$E_1 = 0 \quad (16b)$$

6.4.2.2 Sections Braced on One Side Only

γ_{max} and ϕ_{max} are computed according to the following formulas:

$$\gamma_{max} = \frac{n\pi}{L}(C_1 - E_1\frac{d}{2}) \quad (17)$$

$$\phi_{max} = E_1 \quad (18)$$

where C_1 and E_1 are calculated from the following equations for a particular section.

CHANNEL SECTION:

$$C_1 = \frac{P_r}{Det.}\{\frac{C_o}{n}(A_3A_5 - A_4^2) + \frac{A_4A_2}{n}(D_o - x_oE_o) - \frac{A_3A_2}{n}(r_o^2E_o - x_oD_o)\} \quad (19a)$$

$$E_1 = \frac{P_r}{Det.}\{-\frac{C_o}{n}A_2A_3 - \frac{A_1A_4}{n}(D_o - x_oE_o) + \frac{A_1A_3}{n}(r_o^2E_o - x_oD_o)\} \quad (19b)$$

where $$Det. = A_1(A_3A_5 - A_4^2) - A_3A_2^2$$

$$A_1 = P_y - P_r + Q$$

$$A_2 = -Q\frac{d}{2}$$

$$A_3 = P_x - P_r$$

$$A_4 = P_r x_o$$

$$A_5 = r_o^2(P_\phi - P_r) + Q\,\frac{d^2}{4} + \frac{F}{n^2}\,\frac{L^2}{\pi^2}$$

Z-SECTION:

$$C_1 = \frac{P_r}{\text{Det.}}\{\frac{C_o}{n}A_4A_5 - \frac{D_o}{n}A_2A_5 - \frac{E_o}{n}r_o^2A_3A_4\} \qquad (20a)$$

$$E_1 = \frac{P_r}{\text{Det.}}\{-\frac{C_o}{n}A_3A_4 + \frac{D_o}{n}A_3A_2 + \frac{E_o}{n}r_o^2(A_1A_4 - A_2^2)\} \qquad (20b)$$

where $$\text{Det.} = A_1A_4A_5 - A_2^2A_5 - A_3^2A_4$$

$$A_1 = P_y - P_r + Q$$

$$A_2 = P_{xy}$$

$$A_3 = -Q\frac{d}{2}$$

$$A_4 = P_x - P_r$$

$$A_5 = r_o^2(P_\phi - P_r) + Q\,\frac{d^2}{4} + \frac{F}{n^2}\,\frac{L^2}{\pi^2}$$

I-SECTION:

$$C_1 = P_r(A_5\,\frac{C_o}{n} - r_o^2\,\frac{E_o}{n}\,A_2)/(A_1A_5 - A_2^2) \qquad (21a)$$

$$E_1 = P_r(A_2\,\frac{C_o}{n} - r_o^2\,\frac{E_o}{n}\,A_1)/(A_2^2 - A_1A_5) \qquad (21b)$$

where
$$A_1 = P_y - P_r + Q$$
$$A_2 = -Q\frac{d}{2}$$
$$A_5 = r^2(P_\phi - P_r) + Q\,\frac{d^2}{4} + \frac{F}{n^2}\,\frac{L^2}{\pi^2}$$

6.4.3 Inelastic and Local Buckling Behavior

The buckling stress $\sigma_{cr} = P_{cr}/A$ computed from the governing equations on the basis of the elastic theory may fall under one of the following conditions:

$$\sigma_{cr} \leq 0.5\sigma_y \qquad \text{(elastic buckling)} \tag{a}$$

or
$$\sigma_{cr} > 0.5\sigma_y \qquad \text{(inelastic buckling)} \tag{b}$$

If the first condition governs, then buckling occurs elastically and consequently the parameters P_{cr}, E and G involved in the governing equations need not be modified.

If the second condition governs, then inelastic buckling occurs, and hence the inelastic buckling load P_a may be determined by the AISI formula of Section 3.6.1.2, without a factor of safety, as follows:

$$P_a = A(\sigma_y - \frac{\sigma_y^2}{4\sigma_{cr}}) \tag{c}$$

The elastic modulus of elasticity of steel E and the shear modulus G may be replaced, when necessary, by the inelastic moduli E* and G*, where

$$E^* = E[\frac{\sigma(\sigma_y - \sigma)}{\sigma_p(\sigma_y - \sigma_p)}]$$

or
$$E^* = 4E\sigma(\sigma_y - \sigma)/\sigma_y^2 \tag{d}$$

in which σ is the average stress level corresponding to P_a; that is,

$$\sigma = P_a/A \tag{e}$$

and it is assumed that $\sigma_p = 0.5\sigma_y$.

The shear modulus in the inelastic range may be given as

$$G^* = G(E^*/E) \tag{f}$$

The effects which local buckling of thin-walled compression members can have in reducing the column strength is presented in Section 3.6.1 of the current AISI Specification by a form factor Q. To avoid confusion with the diaphragm rigidity Q, the former will here be designated as Q_A. If this form factor is less than 1.0 then replacing σ_y by $Q_A\sigma_y$ in Eqs. (a) through (f) will furnish design formulas which provide adequate safety against local buckling and account for cases in which combinations of overall and local buckling occur. Therefore these equations, respectively, take the forms:

$$\sigma_{cr} \leq 0.5Q_A\sigma_y \qquad \text{(elastic buckling)} \tag{22}$$

$$\sigma_{cr} > 0.5Q_A\sigma_y \qquad \text{(inelastic buckling)} \tag{23}$$

$$P_a = A(Q_A\sigma_y - \frac{Q_A^2\sigma_y^2}{4\sigma_{cr}}) \tag{24}$$

$$E^* = \frac{4E\ (Q_A\sigma_y - \sigma)}{Q_A^2\sigma_y^2} \tag{25}$$

$$\sigma_p = 0.5Q_A\sigma_y \tag{26}$$

$$G^* = G(E^*/E) \tag{27}$$

6.4.4 Diaphragm Characteristics and Fastener Arrangements

Obtainable from diaphragm test results (see Chapter 5) or from catalogues of diaphragm characteristics, whichever available.

Reliable value of shear rigidity:

$$Q_r = \frac{2}{3}G'w$$

Reliable value of rotational restraint:

$$F_r = \frac{2}{3}F'$$

For the purpose of simplifying the notations used in the design equations the subscript r used in the above expressions is omitted without changing the intended meaning of the parameters Q_r and F_r, hence

$$Q = \frac{2}{3}G'w \tag{28}$$

and

$$F = \frac{2}{3}F' \tag{29}$$

Design value of shear strain capacity of diaphragm:

$$\gamma_d = \Delta_{d/a} \tag{30}$$

Design value of rotational capacity of diaphragm:

$$\phi_d = \frac{\Delta_d}{w/2} \tag{31}$$

Influence of the Fastener Spacing

Buckling of diaphragm braced studs may occur in one or more half-sine waves (App. 6, Sec. X.6.3). In any case a minimum of three fasteners, one at each end of the wave and one at the middle are required so that diaphragm action is fully utilized. In other words, the fastener spacing may not exceed $\frac{L}{2n}$. In addition the spacing of fasteners between the end of a wave and that at the middle of the same wave must not exceed the fastener spacing used in the cantilever diaphragm test. However, in no case should the load carrying capacity of the stud exceed the buckling load computed on the basis of an unbraced column with effective buckling length equal to the spacing between the fasteners. Such an analysis may be made according to provisions of Section 3.6 (Axially Loaded Compression Members) of the current AISI Specification.

The procedure of checking the possibility of buckling between the fasteners is illustrated in the design examples of Appendix 1 as well as in the computer programs included in Appendix 4.

6.5 Design Aids

To simplify the use of the governing equations, two design aids are introduced. These are design charts and design computer programs.

6.5.1 Design Charts

Figures 14 and 15 are graphical solutions of the governing equations for channel and z-sections braced on both sides. The procedure for using these charts as well as details of the parameters involved are included in Sections 4.4.1 and 4.4.2. The use of these charts is also illustrated in the solved examples in Appendix 1.

6.5.2 Computer Programs

The design procedure given in Section 6.3 for channel, zee and I-sections braced on one or both sides has been programmed for the purpose of direct application to wall studs design. The use of these computer programs is recommended in the cases for which design charts are not provided and the governing equations are complicated, in particular, when higher buckling modes are involved. Detailed description of the features of computer programs as well as their listings are given in Appendix 4, and their use is illustrated in the design examples of Appendix 1.

6.6 List of Original and New Numbers of the Design Equations

The following provides for cross-referencing of equations of the design procedure listed in this chapter with their corresponding original equations included in the present report and in other references.

Design Equation Number (present chapter)	Corresponding Original Equation Number (Chapters 2, 5 and other references)
(1)	(36)
(2)	(56) with $K_1 = 1$
	or (44) $n = 1$
(3)	(57) with $K_2 = K_4 = K_8 = 1$
	or (45) $n = 1$
(4)	(59) with $K_1 = K_2 = K_3 = 1$
	or (48) $n = 1$
(5)	(61)
	or (34) Ref. 13
(6)	(61)
(7)	(50)
(8)	(52)
(9) and (10)	(38) with $x_o = y_o = 0$
	and $P_{xy} = 0$
(11)	Ref. 17, Section 2.2.4
(12) and (13)	Ref. 17, Section 4.7
(14)	(71), (73) and (74), $K_1=K_2...=K_{10} = 1$
(15)	(75), (77) and (78), $K_1=K_2...=K_{10} = 1$
(16)	Ref. 17, Section 4.4
(17) and (18)	Ref. 17, Section 4.7
(19)	(99), (101) and (102)

Design Equation Number	Corresponding Original Equation Number
(20)	(103), (105) and (106)
(21)	(99), (101) and (103) with $x_o = 0$ or from procedures of Section 2.8
(22), (23), (24), (26)	AISI Specification, Ref. 1, Section 3.6
(25) and (27)	Refs. 3, 13, 15, 38
(28) and (29)	Ref. 17
(30)	(158)
(31)	(160)

6.7 Nomenclature of the Design Procedure

a	dimension of shear diaphragm perpendicular to load direction in cantilever diaphragm test
A	cross-sectional area
C_o	amplitude of initial lateral deflection of the centroidal axis of the stud in the x-direction
C_1	amplitude of deflection in the x-direction
C_w	warping constant
d	overall dimension of the web (depth of section)
D_o	amplitude of initial lateral deflection of the centroidal axis of the stud in the y-direction
E	modulus of elasticity
E*	inelastic modulus defined by Eq. (25)
E_1	amplitude of twist of the stud
F	rotational restraint supplied by the diaphragm bracing (used in the governing equations and is equivalent to F_r)
F.S.	factor of safety (= 1.92)
F'	rotational restraint at $0.8P_{ult}$ (diaphragm test)
F_r	reliable rotational restraint of the diaphragm
G	shear modulus
G*	inelastic shear modulus
G'	shear stiffness at $0.8P_{ult}$ (diaphragm test)
I_x, I_y	moment of inertia of section about x- and y-axes (passing through the centroid), respectively
I_{xy}	product moment of inertia with respect to x- and y-axes

J	St. Venant torsion constant
L	length of stud
n	number of half-sine waves into which the column buckles
P	buckling load (used in the governing equations)
P_o	allowable load × factor of safety
P_a	inelastic buckling load
P_{all}	allowable load
P_{cr}	critical buckling load
$P_{cr,UB}$	critical buckling load of unbraced stud
$P_{cr,x}$	critical buckling load (perpendicular to the wall)
P_r	trial load capacity
P_x	Euler buckling load about the x-axis
P_y	Euler buckling load about the y-axis
P_{yield}	yield load of stud
P_{xy}	defined by Eq. (1c)
P_ϕ	torsional buckling load
Q	diaphragm shear rigidity (used in the governing equations and is equivalent to Q_r)
Q_r	reliable shear rigidity
r_o^2	I_p/A, where I_p is the polar moment of inertia about the shear center
x_o	distance between the centroid and shear center along the x-axis
w	width of diaphragm contributing to the bracing of one stud
σ	unit axial stress

σ_p	proportional limit stress
σ_{cr}	P_{cr}/A
σ_y	yield stress
Δ_d	deflection under load at $0.8P_{ult}$ in rotational capacity diaphragm test
γ_{max}	calculated shear strain in the diaphragm
ϕ_{max}	calculated rotation of the stud
γ_d	design shear strain at $0.8P_{ult}$ (diaphragm test)
ϕ_d	design rotational restraint capacity at $0.8P_{ult}$ (diaphragm test)
λ	trial reduction factor < 1.0

Chapter 7

RESULTS AND CONCLUSIONS

The stability of diaphragm braced columns of general shaped sections under concentric load in the elastic and inelastic domains has been investigated. Interest has been centered upon the derivation of the basic equations in general form. Hence, the solutions of special cases such as I, channel and zee-sections could be obtained from the general solution. The theoretical results are applied to the case of wall studs construction in order to modify the design approach of Section 5.1, Wall Studs, of the current Specification for the Design of Cold-Formed Steel Structural Members(1). The investigation has led to the following results and conclusions.

1) Considering the combined action of the shear rigidity Q and the rotational restraint F of the diaphragm, an energy approach is utilized to obtain the solution. For each of the two cases of diaphragm bracing, namely columns braced on both sides and on one side only, the solution has been derived separately. The general equation of stability for each case is given by Eqs. (35) and (38), respectively, for columns with hinged end conditions. These equations are based on an assumed displacement function in the form of an infinite series, Eq. (18). Hence equations of stability (39, 46, 49 and 51) for particular cases of channel and z-sections are obtained. The critical buckling load expressed as a function of Q and F for each particular section under the previously specified bracing

conditions is given by the governing equations (44, 45, 48, 50 and 52). It should be noted that in general, when these equations are used, the possibility of higher buckling modes should be investigated by considering a sufficient value of n, n = 1, 2,3,... .

2) Equation (23) is the stability equation in general terms for a column with hinged ends. This equation results from utilizing assumed displacement functions in an energy method of solution. These displacement functions are given in the form of infinite series (Eqs. 18). Since three displacements are encountered, then if n terms are considered in the solution Eq. (23) contains 3×n algebraic equations. It has been found that these equations yield n uncoupled modes of buckling and consequently, n different buckling loads can be calculated, the smallest load of which gives the critical buckling load of the column; the corresponding value of n determines the number of half-sine waves into which the column buckles. For example, if n = 1,2,...5 is considered, then 5 different modes are possible and 15 equations result. Each three of these equations forms an independent set of equations which characterizes one of the five buckling modes. The buckling load corresponding to each mode is determined by solving for the smallest root of the three simultaneous equations of each set. Such a property, uncoupled modes, introduced considerable simplification to the method of solution; that is, reduction in the number of equations to be solved simultaneously. However, it has been found that such a simplification does not apply to the cases of boun-

daries other than hinged, for example, fixed, or boundaries listed in Table 1. This is so because the equations forming the stability equation of these cases are all coupled and only one buckling mode occurs. Hence if 5 terms are considered, then 15 algebraic equations have to be solved simultaneously for the smallest root which determines the critical buckling load. Therefore it was possible with n = 1,2,3,... to derive governing equations for the case of hinged end columns only, since for cases other than hinged, deriving such equations tends to be impractical. However, for the latter cases, the method of solution outlined in Chapter 2 suffices if the need of considering such end conditions arises. On the other hand, the intent is to derive a design procedure for the simple case of wall stud application, for which the hinged end conditions simulate the actual structure with reasonable conservative approximation. Therefore interest has been focused on deriving governing equations needed only for the design procedure.

3) By considering only the first term of the series the governing equations (56, 57, 59, 63 and 64) are valid for all cases of columns with hinged, fixed and other end conditions (see Table 1). Values of the coefficients K_1, K_2, ... K_{12} which appear in these equations can be obtained from Table 1. These coefficients account for different types of end conditions. The equations are valid provided that the higher buckling modes are ruled out.

4) Higher buckling modes are conventionally associated with buckling in more than one half-wave, i.e. $n > 1$. In some

cases, depending on the relative magnitude of the diaphragm characteristics and the column stiffness, higher buckling modes govern the behavior of the column. Since considering such a possibility tends to complicate the design approach, a numerical investigation has been conducted to examine the validity of higher buckling modes. In the numerical investigation the variation of the diaphragm shear rigidity and its rotational restraint as well as the columns' flexural and torsional rigidities are chosen to be within the practical range of wall stud construction (see Section 4.2). The results indicate that higher buckling modes do not govern the behavior of studs of channel and zee-sections braced on both sides. Therefore, for these cases governing equations based on $n = 1$ are derived in Section 2.6. However, for sections braced on one side only, higher buckling modes are possible in some cases and such a possibility should always be considered. Hence, for these cases governing equations based on $n = 1,2,3,...$ are derived in Section 2.4. The solution of these equations can be facilitated by the use of the computer program suggested in Section 6.5.2 and documented in Appendix 4 (see also conclusion 8).

5) The governing equations derived in Chapter 2 are based on assumed displacement functions of similar shapes, i.e. the number of half-sine waves, n, simultaneously takes the same values in each of the displacement functions. Accordingly, higher buckling modes are investigated by considering sufficient values of n in the solution. However, different shapes of displacement functions ensue if n takes different values in

each of the displacement functions, for example, i, j, m (see Eq. 5.18 of Appendix 5). Higher buckling modes based on displacement functions of different shapes have been investigated in Appendix 5. It has been found that higher buckling modes resulting from assuming functions of different shapes do not govern the buckling behavior of all the cases considered except the case of an I-section braced on one side only (see conlcusion at the end of Appendix 5).

6) As one direct application of the governing equations (59) for the zee-section braced on both sides, Eq. (60) is obtained by setting Q = 0 in Eq. (59). Therefore Eq. (60) governs the behavior of unbraced zee-sections with hinged, fixed and mixed end conditions. This equation, though simple, has not been known before in available publications known to the writer. It should be noted that higher buckling modes are not critical in this case.

7) The stability equations given in Sections 2.4 and 2.6 are checked in Chapter 3 against cases of known solutions derived by Timoshenko[3].

The results indicate that the solution of these cases can be obtained as special cases from the general solution given by the stability equations. In addition, in Section 2.6A.4, it has been shown that the solution of I-section columns braced on both sides, derived by Errera[13], can be obtained as a special case of the stability equation (54). Further verifications of the theoretical results of this investigation has also been considered in Section 2.4.3.1. It has been shown that with Q =

0 and F = 0, the equations of Section 2.4.3.1 render equations derived for unbraced columns by Winter and Chajes[31], Timoshenko[3] and Peköz[32].

8) The use of the governing equations of sections braced on both sides can be simplified by the use of the approximate formulas and charts presented in Chapter 4. These formulas and charts are valid, with minor modification, for the cases of inelastic buckling as well. Examples 1 and 2 in Appendix 1 illustrate the use of these design aids in practical situations. Unfortunately, simplification of the governing equations of sections braced on one side only is not possible without considerable loss of accuracy. Therefore, the direct solution of the cubic equations given in Section 2.4 seems to be the only possible method and is simple by computer subroutines. Four programs (A1, A2, B1, B2) based on the suggested design procedure of Chapter 6 are prepared for the purpose of wall studs design. Listings of the programs and their flow charts as well as sample outputs of the design examples of Appendix 1 are included in Appendix 4. In programs A2 and B2, higher buckling modes can be examined by considering any desired number of possible buckling modes (n); hence, the smallest buckling load determined gives the critical buckling load of the stud. It should be noted that sufficient numbers of modes should be considered in the analysis; however, in most of the cases examined in this investigation, critical buckling modes occur in the second or third mode. Hence it is suggested that considering 6 modes as a first trial would suffice. The use of the computer

program in practical situations is demonstrated throughout the solutions of the design examples given in Appendix 1.

9) It is not intended to formulate the findings of this investigation in design specification language. Rather, the design approach presented in Chapter 6 outlines a rational and practical method of design. The proposed design criteria is based on the ultimate load capacity of the column, utilizing a conservative estimate of the shear rigidity and rotational restraint of the wallboards acting as bracing diaphragms. In this procedure the adequacy of the diaphragm is checked by comparing the computer values of shear strain γ_{max} and rotation ϕ_{max} to those provided by the tested diaphragm (γ_d, ϕ_d). Hence the diaphragm is adequate if the conditions that $\gamma_{max} \leq \gamma_d$ and $\phi_{max} \leq \phi_d$ are satisfied. The design procedure is valid for both elastic and inelastic ranges and examples to illustrate its use in practical problems are included in Appendix 1.

10) In general, two design situations may arise in wall stud analysis, namely determining the buckling load of the stud if the diaphragm shear rigidity Q and its rotational restraint F are known, while in the other situation the buckling load is given and Q and F are to be obtained. The proposed design procedure allows the analysis in both cases. In the first situation the design method utilizes the use of the diaphragm capacity that exists in the structure; in essence, no minimum requirement on Q and F is needed so that the diaphragm can be declared adequate. This differs from the method of analysis of a previous design criterion given in Ref. 17 which requires a

minimum value of Q defined as the shear rigidity required for full bracing. Such a requirement might not always be on the economical side. In this respect the proposed design procedure is favorable. Other differences between the proposed design procedure and that of Ref. 17 are: including the rotational restraint of the diaphragm, F, including unsymmetrical and point symmetrical sections in the analysis and in addition, the design criteria of Ref. 17 do not allow for the design situation in which the buckling load of the braced stud is required if a disphragm shear rigidity less than the shear rigidity required for full bracing should be used.

11) The results of the experimental investigation carried out indicate that the agreement between the tests and the theoretical results is satisfactory. This also indicates that the proposed design approach is expected to give reliable results.

12) Two important observations can be made from the test results:

a) The rotational restraint of the diaphragm is as important as its shear rigidity in providing for the stability of diaphragm-braced studs, especially if torsional-flexural buckling governs.

b) The use of adhesives as substitutes for/or in addition to the fasteners is recommended for cases in which cyclic loading from wind or earthquakes is possible.

13) The use of zee-sections in wall stud construction tends to be more economical than the use of channel sections of the same dimensions. This has been observed from the test re-

sults (see Figs. 29, 30) and from the solved Example 1 of Appendix 1. The gain in the case of sections braced on one side is more than that for the case of two-sided bracing. To the writer's knowledge, none of the available manufacturers' catalogues include zee-section studs. Unless there are certain practical and constructional reasons behind the uncommon use of the zee-section, their use should be recommended.

Appendix 1

DESIGN EXAMPLES

Three design examples to illustrate the use of the suggested design procedure, given in Chapter 6, are included herein. In the first example, Q and F are given and the unknown quantity is the buckling load, while in the second example, the buckling load is given and Q and F are to be obtained. These two design situations, often met in practice, are also considered in the third example.

The use of the following design aids have been demonstrated in the solutions:

1) Computer programs based on the governing equations listed in Section 6.4. Samples of the computer output and the programs are included in Appendix 4.
2) Design charts, presented in Chapter 4 and shows in Figs. 14 and 15.
3) Approximate formulas given in Chapter 4.

Throughout this Appendix reference has been made to the numbers of the design equations listed in Section 6.4 of Chapter 6, unless otherwise indicated.

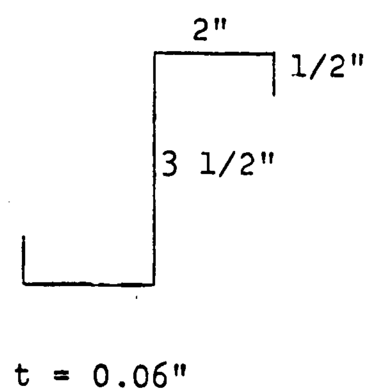
The design examples follow on the next pages.

<u>EXAMPLE 1</u> (bracing on both sides)

I) Calculate the ultimate and allowable loads of an 8'-0" wall stud made of $3\frac{1}{2}$" × 2" zee-section-16 gage, with both ends hinged. The stud is attached on both sides to $\frac{1}{2}$" GYPSUM WALLBOARDS which form a continuous diaphragm. Fasteners are spaced every 12" apart. Tests of the diaphragm have shown reliable shear rigidity and rotational restraint of 50 k and 0.06 k.in/in.rad., respectively, while γ_d = 0.011 in/in. and ϕ_d = 0.15 radians. The stud is cold-formed from high strength steel sheets with a yield stress of 50 ksi.

II) Replace the zee-section by a channel section of the same cross-sectional dimensions and compare the allowable loads in both cases.

<u>SOLUTION CASE (I). Z-SECTION:</u>



This section is listed in the AISI Manual[46] from which all geometrical properties can be obtained. The form factor of the section Q_A = 0.861 has been considered in the analysis.

The critical buckling load P_{cr}, based on elastic behavior, is obtained by the use of:

i) Computer program A1 which is based on the original governing equations.

ii) Approximate formulas.

iii) Design charts.

Using Eqs. (1) on an elastic basis and n = 1, the following parameters needed for the analysis are calculated:

$P_x = 32.15$ k $\quad$ $P_y = 15.93$ k $\quad$ $P_{xy} = 14.84$ k

$P_\phi = 11.93$ k $\quad$ $P_{y1} = 7.13$ k $\quad$ $P_{x1} = 40.95$ k

i) Design by the Use of the Governing Equations

With $Q = 60.0$ k

$F = 0.06$ k.in/in.rad.

and the above calculated parameters, solve Eq. (4) for the smallest value of P, hence

$$P_{cr} = 26.56 \text{ k}$$

See the computer output included in Appendix 4 for the value of the elastic critical buckling load.

ii) Design by the Use of Approximate Formulas (Chapter 4)

From Eq. (142a) of Chapter 4

$$P' = 15.0 \text{ k}$$

Then from Eq. (142)

$$P_{cr} = 26.38 \text{ k}$$

iii) P_{cr} from the Design Charts (Chapter 4)

From Fig. 15 and the following parameters

$$b/a = 0.565$$

$$c/a = 0.137$$

$$R = (P_y + Q)/P_x = 2.05$$

Then, from the charts $\alpha = 0.82$.

Therefore $P_{cr} = \alpha P_x = 26.5$ k.

The ratios of approximate to exact loads for cases (ii) and (iii) are 0.99 and 1.0, respectively.

Check the possibility of buckling between the fasteners

Compute the critical buckling load of the unbraced stud with the buckling length equal to the distance between the fasteners (s = 12.0"),

$$P_{crf} = \pi^2 I_{11}/s^2$$

where I_{11} is the moment of inertia about the minor axis. In the present example

$$I_{11} = 0.226 \text{ in}^4$$
$$s = 12.0 \text{ in}$$

then $P_{crf} = 456.52$ k. Since $P_{crf} >> P_{cr}$, buckling between the fasteners does not govern,

The output shown in Appendix 4 does not contain such details. However, details of the analysis are available from Program A1 if the control variable PRINT = 1 is used in the program instead of PRINT = 0.

Check inelastic behavior

So far, the critical loads, assuming elastic behavior, are calculated; now it is left to check whether or not our assumption is valid and hence to check the diaphragm adequacy.

Since the shape factor of the section $Q_A < 1.0$, then

$$\sigma_y = 0.861 \times 50 = 43.05 \text{ ksi}$$
$$\sigma_{cr} = 26.56/0.496 = 53.59$$

According to Eq. (23),

$$53.59 > 0.5 \times 43.05$$

Thus inelastic behavior governs, and P_{cr} should be limited to the inelastic buckling load P_a given by Eq. (24). Therefore

$$P_a = 17.05 \text{ k.}$$

<u>Check the diaphragm adequacy</u>

From Eq. (25) with $\sigma = 17.0/0.496 = 34.0$ ksi

$$E^* = 19000 \text{ ksi}$$

From Eq. (27)

$$G^* = 7250 \text{ ksi}$$

Then in the inelastic range,

$$P_x = 20.64 \text{ k} \qquad P_\phi = 7.66 \text{ k}$$

$$P_y = 10.23 \text{ k} \qquad P_{xy} = 9.53 \text{ k}$$

From Eqs. (11)

$$C_o = 0.274 \text{ in.}$$

$$D_o = 0.137 \text{ in.}$$

$$E_o = 0.002 \text{ rad.}$$

Try $P_r = P_a = 17.05$ (i.e. $\lambda = 1.0$). Then from Eqs. (15a) and (15b),

$$C_1 = 0.0853 \text{ in.}$$

$$E_1 = 0.0007 \text{ in.}$$

From Eqs. (12) and (13)

$$\gamma_{max} = 0.00283 < \gamma_d = 0.011 \qquad \text{O.K.}$$

and $$\phi_{max} = 0.0007 < \phi_d = 0.15 \qquad \text{O.K.}$$

Thus the diaphragm is adequate for bracing and the load capacity of the stud $P_r = 17.05$ k. By considering a factor of safety F.S. = 1.92, the allowable load ,

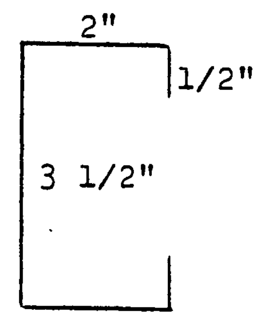
$$\underline{P_{all} = 17.05/1.92 = 8.88 \text{ k}}.$$

See the computer output in Appendix 4 for the value of of P_{all}. Other details may be obtained from Program A1, with PRINT = 1.

By including the effect of the wallboards, it was possible to increase the critical buckling load of the unbraced stud from $P_{y1} = 7.13$ to $P_r = 17.05$ k (i.e. 2.4 times).

SOLUTION CASE (II). CHANNEL SECTION

The section is listed in the AISI Manual[46]. The form factor Q_A = 0.861. The diaphragm bracing is the same as in (I).



t = 0.06"

Using Eqs. (1) on an elastic basis and n = 1, the following parameters are calculated.

$$P_x = 32.15 \text{ k} \qquad P_y = 8.77 \text{ k}$$

$$P_\phi = 5.13 \text{ k}$$

1) Design by the Use of the Governing Equations

From Eq. (2), $P = 41.14$ k

and the smallest root of Eq. (3), $P = 21.80 < 41.14$ k

Hence torsional-flexural buckling governs and the elastic critical buckling load

$$P_{cr} = 21.60 \text{ k}$$

See the computer output included in Appendix 4.

ii) Design by the use of Approximate Formulas (Chapter 4)

From Eq. (141), P_{cr} = 21.744

iii) Design by Charts (Chapter 4)

From Fig. 14 and $b/a = 0.565$

$$c/a = 0.137$$

$$R = 1.29$$

the factor $\alpha = 0.66$. Hence $P_{cr} = 0.66 \times 32.48 = 21.70$ k.

The ratios of approximate to exact loads in cases (ii) and (iii) are 0.996 and 0.99, respectively.

Check the possibility of buckling between the fasteners

Compute the critical buckling load of the unbraced stud with buckling length equal to the distance between the fasteners (s = 12.0"). Such a load is given by the smallest value of

a) Flexural buckling about the y-axis.

$$P_{crf} = \pi^2 EI_y/s^2 = 561.2 \text{ k}.$$

b) Torsional buckling.

Equation (5.35b) of Appendix 5 with Q = F = 0 indicates that coupling of torsional and flexural buckling modes is not possible. That is, P_x and P_ϕ are not coupled to give torsional-flexural buckling modes as expected in a usual situation of an unbraced column. Therefore,

$$P_{crf} = (GJ + \pi^2 EC_w/s^2)/r_o^2$$

$$= 245.0 < 561.2 \text{ k}$$

and torsional buckling governs the behavior of buckling between fasteners. However, since for the braced stud

$$P_{cr} = 21.6 < 245.0 \text{ k}$$

buckling between the fasteners is unlikely to occur.

Details of the above computations may be obtained from Program A1.

Check inelastic behavior

$$\sigma_y = 0.86 \times 50 = 43.05 \text{ ksi}$$

$$\sigma_{cr} = 21.6/0.496 = 43.75 \text{ ksi}$$

From Eq. (23)

$$43.75 > 0.5 \times 43.05$$

Then inelastic behavior governs, and therefore from Eq. (24)

$$P_a = 16.09 \text{ k.}$$

Check diaphragm adequacy

1st Trial ($\lambda = 1.0$)

$$P_r = \lambda P_a = 16.09 \text{ k.}$$

From Eq. (25) with $\sigma = 16.09/0.496 = 32.0$ ksi,

$$E^* = 21889.0 \text{ ksi}$$

From Eq. (27)

$$G^* = 8384.0 \text{ ksi}$$

Then in the inelastic range

$$P_x = 23.85 \text{ k} \qquad P_y = 6.5 \text{ k}$$

$$P_\phi = 3.82 \text{ k}$$

From Eqs. (11)

$$C_o = 0.274 \text{ in.}$$
$$D_o = 0.137 \text{ in.}$$
$$E_o = 0.002 \text{ rad.}$$

From Eqs. (14a) and (14b)

$$C_1 = 0.109 \text{ in.}$$
$$E_1 = 0.198 \text{ rad.}$$

From Eqs. (12) and (13)

$$\gamma_{max} = 0.0149 > 0.011 \qquad \text{N.G.}$$
$$\phi_{max} = 0.1984 > 0.15 \qquad \text{N.G.}$$

Thus the diaphragm is <u>not</u> adequate for bracing the stud so that a load capacity $P_r = 16.08$ k can be sustained. Therefore, it is necessary to reduce P_r and consider a new trial value.

<u>2nd Trial</u> ($\lambda = 0.98$)

$$P_r = 0.98 \times 16.086 = 15.765 \text{ k}$$

As before, the corresponding moduli are

$$E^* = 22766.0$$

$$G^* = 8720.58$$

Then $$P_x = 24.81 \text{ k} \qquad P_y = 6.77 \text{ k}$$

$$P_\phi = 3.96 \text{ k}$$

From Eqs. (14a) and (14b)

$$C_1 = 0.1055 \text{ in.}$$

$$E_1 = 0.1296 \text{ rad.}$$

From Eqs. (12) and (13)

$$\gamma_{max} = 0.0109 < 0.011 \qquad \text{O.K.}$$

$$\phi_{max} = 0.1296 < 0.15 \qquad \text{O.K.}$$

Since $\gamma_{max} < \gamma_d$

and $\phi_{max} < \phi_d$

Then for a load capacity

$$P_r = 15.765 \text{ k}$$

the diaphragm is adequate for bracing. The allowable load

$$\underline{P_{all} = 15.765/1.92 = 8.211 \text{ k}}.$$

See the computer output included in Appendix 4 for the value of P_{all}. Other details of the analysis are obtainable from Program A1 by letting the control variable PRINT = 1.

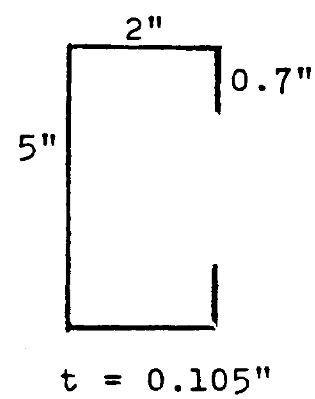
EXAMPLE 2 (bracing on both sides)

A wall is about 6 in. think, 12 ft. high, to be constructed of light gage cold-formed steel studs spaced at 24 in. and covered on both sides by wallboards. The studs are made of 5" channel sections-12 gage with both ends considered hinged and are subjected to equal critical concentric loads from a rigid roof beam. The studs are cold-formed from high strength steel sheets with a yield stress of 50 ksi.

It is required to specify the type of wall material and the type of fasteners to be used for each of the following loading:

(I) Allowable load on stud $P_{all} = 8.0$ k

(II) Allowable load on stud $P_{all} = 16.0$ k



The form factor of the section Q = 0.907. This example illustrates the design procedure outlined in Section 6.3 (part B, sections braced on both sides).

SOLUTION CASE (I). $P_{all} = 8.0$ k

With F.S. = 1.92, calculate the load capacity of the stud. Therefore

$$P_o = 8.0 \times 1.92 = 15.36 \text{ k}$$

Check that:

$$P_o > P_{cr,UB}$$

$$P_o < P_{cr,x}$$

and

$$P_o < P_{yield}$$

where $P_{cr,UB}$ = critical buckling load of unbraced stud

$P_{cr,x}$ = strong axis buckling load (perpendicular to the wall)

P_{yield} = yield load of stud

For the given stud, using Eqs. (1) (on an elastic basis and n = 1) or using the Computer Program B1, the following parameters are obtained (see sample of computer output in Appendix 4):

$$P_x = 56.55 \text{ k} \qquad P_y = 8.14 \text{ k}$$

$$P_\phi = 12.02 \text{ k}$$

Therefore, the following is computed:

$P_{cr,UB}$ = 8.14 k (Torsional-flexural buckling load = 10.1 k does not govern since flexural buckling load P_y is smaller.)

$P_{cr,x}$ = 56.55 k

$$P_{yield} = 1.048 \times 0.907 \times 50 = 47.0 \text{ k}$$

Comparing these values with P_c = 15.26 k, it can be seen that the above three requirements regarding $P_{cr,UB}$, $P_{cr,x}$ and P_{yield} are satisfied. Therefore the stud cross-section is satisfactory. The next step is to specify a suitable diaphragm and check the possibility of buckling between the fasteners.

Check inelastic behavior

The stress level at P_o = 15.36 is equal to

$$\frac{15.36}{1.048} = 15.0 \text{ ksi} < 0.5 \times 0.907 \times 50$$

Then according to Eq. (22), elastic behavior governs.

i) Design by the Use of the Governing Equations

From Eq. (2), $Q = 15.36 - 8.14 = 7.22$ kips.

From Eq. (3), torsional-flexural mode, setting $F = 0$, Q is found equal to

$$Q = 4.35 \text{ k}$$

This indicates that with $Q = 4.35$ k, the torsional-flexural buckling load equal to 15.36 k would occur provided that no lower buckling modes are preceded. However, as can be seen in Fig. 17, if a shear rigidity ($Q = 4.35$ k) is chosen, a flexural buckling mode will occur way before torsional-flexural buckling can take place. Such a flexural load is less than 15.36 k.

Therefore, $Q = 4.35$ k will not serve the load requirement, and hence providing $Q = 7.22$ k is necessary for a load of 15.36 k to be attained. Then use

$$Q = 7.22 \text{ k and } F = 0$$

These are not final design values of Q and F. Rather, they are minimum required values. The next step in the analysis is to specify a suitable diaphragm obtainable from Diaphragm Catalogues or from previous cantilever shear diaphragm tests. Before such a step is considered, the use of approximate formulas as well as design by the aid of charts will be demonstrated.

ii) Design by the use of the Approximate Formulas (Chapter 4)

The shear rigidity $Q = 7.22$ k, as obtained in case (i), is required for a flexural buckling load of 15.36 k to occur.

The possibility of the torsional-flexural buckling mode will be investigated herein by using the approximate formula

(141). The procedure is different than that considered in the use of the governing equations case (i). Herein using Q = 7.22 k and F = 0 in Eq. (141) the buckling load will be computed. If such a load is less than that load (15.36 k) governing the flexural behavior, then torsional-flexural buckling governs; otherwise flexural buckling controls.

From Eq. (141b), with Q = 7.22 k and F = 0,

$$P'_\phi = 18.73 \text{ k.}$$

From Eq. (141) $\quad P = 16.5 \text{ k} \quad 15.36$

Then flexural buckling (Eq. 2) giverns, and as before, Q = 7.22 k and F = 0 are the minimum required values of Q and F.

iii) Design by Use of Chart Fig. 14 (Chapter 4)

With Q = 7.22 and F = 0, then

From Eq. (146) $\quad P'_\phi = 18.73$ k

From Eq. (147) $\quad R = 0.325$

From Fig. 14 with $\quad b/a = 0.4$

$c/a = 0.14$

and $\quad R = 0.325$

then $\quad = 0.29$.

Therefore $P_{cr} = \alpha P_x = 16.8 \text{ k} > 15.36$. Hence flexural buckling governs and as before, Q = 7.22 k and F = 0 are minimum required values.

Choosing the diaphragm and checking its adequacy

The procedure is outlined in Section 6.3 (part B, sections braced on both sides, provisions 3b and 3c).

From Diaphragm Catalogues choose a diaphragm for which Q and F are larger than their respective values obtained in the previous step of analysis (i.e. Q = 7.22 and F = 0).

Try 1/2" Impregnated Celotex boards with fasteners every 7".

From diaphragm test results, the following has been obtained:

$$G' = 0.66 \text{ k/in}$$
$$\gamma_d = 0.0096 \text{ in/in}$$
$$F' = 0.01 \text{ k.in/in.rad.}$$
$$\phi_d = 0.23 \text{ rad.}$$

Therefore $Q_r = \frac{2}{3} \times 0.66 \times 24 \times 2 = 21.3 > 7.22$ O.K.

$$F_r = \frac{2}{3} \times 0.01 \times 2 = 0.014 > 0 \qquad \text{O.K.}$$

However, this is not sufficient; it is still necessary to check the diaphragm adequacy, that is, to verify that at a load P_r = 15.36 k, the resulting γ_{max} and ϕ_{max} are smaller than γ_d and ϕ_d of the chosen diaphragm.

Check diaphragm adequacy

From Eqs. (11), the initial imperfections are

$$C_o = 0.411 \text{ in.}$$
$$D_o = 0.206 \text{ in.}$$
$$E_o = 0.004 \text{ rad.}$$

From Eq. (14a) with P_r = 15.36 k,

$$C_o = 0.44 \text{ in.}$$

From Eq. (14b) with $P_r = 15.36$ k,

$$E_1 = 0.05 \text{ rad.}$$

From Eqs. (12) and (13)

$$\gamma_{max} = 0.014 > 0.0096 \qquad \text{N.G.}$$
$$\phi_{max} = 0.05 < 0.23 \qquad \text{O.K.}$$

Therefore this diaphragm is <u>not</u> adequate and hence choose another trial diaphragm with larger values of Q and F.

<u>Try 1/2" Homosote Boards</u> with fasteners @ 12".

$$G' = 0.845 \text{ k.in}$$
$$F' = 0.012 \text{ k.in/in.rad.}$$
$$\gamma_d = 0.012 \text{ in/in.}$$
$$\phi_d = 0.175 \text{ rad.}$$

Therefore $\quad Q_r = \frac{2}{3} \times 0.845 \times 24 \times 2 = 27.0$ k

$$F_r = \frac{2}{3} \times 0.012 \times 2 = 0.016 \text{ k.in/in.rad.}$$

With the previous values of C_o, D_o and E_o as well as $P_r = 15.36$ k, the following is computed:

From Eq. (14a) $\quad C_1 = 0.320$

(14b) $\quad E_1 = 0.04$

Then from Eqs. (12) and (13),

$$\gamma_{max} = 0.010 < 0.012 \qquad \text{O.K.}$$
$$\phi_{max} = 0.04 < 0.175 \qquad \text{O.K.}$$

<u>Therefore 1/2" Homosote boards with fasteners @ 12" is adequate</u>

for bracing. The next and final step is to check the possibility of buckling between the fasteners.

Check the possibility of buckling between the fasteners

Following the procedure of analysis outlined in Example 1 for the channel section, the following has been computed:

a) Flexural buckling about the y-axis

$$P_{crf} = 1172.69 \text{ k.}$$

b) Torsional buckling

$$P_{crf} = 812.2 < 1172.69 \text{ k} \quad \text{controls}$$

Therefore from Eq. (24), the inelastic buckling load

$$P_{crf} = 44.35 \text{ k} >> P_r = 15.36 \text{ k}$$

Therefore, buckling between the fasteners does not govern and 12" fastener spacing is acceptable.

It follows that 1/2" Homosote boards with fasteners @ 12" satisfies all the design requirements.

Computer Output

Program B1 has been used to solve the present example. The output, shown in Appendix 4, includes a list of different values of Q and F as well as their respective values of γ_{max}, ϕ_{max}, C_1 and E_1. Each value of Q and its respective F represents a diaphragm adequate for bracing the stud so that P_r = 15.36 k can be sustained safety. The designer may use such a list to specify a suitable diaphragm material by the aid of Diaphragm Catalogues or previous diaphragm test results.

For example, one may choose from the list

$Q = 22.216 \qquad F = 0.015 \qquad \gamma_{max} = 0.012 \qquad \phi_{max} = 0.047$

Hence, find from Diaphragm Catalogues a certain diaphragm for which Q and F are equal to or larger than those chosen from the list. In addition, check from the Diaphragm Catalogues that γ_d and ϕ_d of the chosen diaphragm are larger than the listed γ_{max} and ϕ_{max}. In the present example, 1/2" Homosote boards with fasteners every 12" satisfy these requirements. On the other hand, one may notice from the list that 1/2" Impregnated Celotex with fasteners every 7" do not satisfy γ_{max} and ϕ_{max} requirements; therefore, such a diaphragm is not adequate for the present design case.

SOLUTION CASE (II). $P_{all} = 16.0$ k

For a F.S. = 1.92 the load capacity P_o is

$$P_o = 1.92 \times 16.0 = 30.72 \text{ k}$$

As in case (I), for an unbraced stud,

$$P_{cr,UB} = 8.14 < 30.72 \qquad \text{O.K.}$$

$$P_{cr,x} = 56.55 > 30.72 \qquad \text{O.K.}$$

$$P_{yield} = 47.0 > 30.72 \qquad \text{O.K.}$$

Thus the stud cross-section is satisfactory; it is left to specify a suitable diaphragm and check the possibility of buckling between the fasteners.

Check inelastic behavior

The stress at $P_o = 30.72$ k is

$$\frac{30.72}{1.048} = 29.4 \text{ ksi} > 0.5 \times 0.907 \times 50$$

Then according to Eq. (23), inelastic buckling governs.

The next step is to find from the governing equations the values of Q and F that satisfy the requirement, i.e. $P_o = 30.72$

k (inelastic).

(1) Design by the Use of the Governing Equations

The design procedure outlined in Section 6.3 (part B, provisions 4a, 4b and 4c) suggests the use of a trial and error method to obtain values of Q and F. Herein an alternative e quivalent to such an approach will be used.

Equation (24) gives the value of the inelastic buckling load P_a for a known value of stress σ_{cr} corresponding to P_{cr} computed on an elastic bases. Therefore, in our case, knowing P_a, then σ_{cr} can be calculated. Hence the corresponding critical elastic load ($P_{cr} = \sigma_{cr} \times$ Area) can be obtained. Knowing such a load will allow the direct use of the governing equations, based on elastic behavior, to obtain Q and F.

Therefore, substituting $P_o = 20.72$ for P_a in Eq. (24) gives:

$$\sigma_{cr} = 32.0 \text{ ksi}$$

Then the inelastic buckling load = 32.0 × 1.05 = 33.60 k where the area of the cross-section A = 1.05 in^2.

From Eq. (2) (flexural buckling),

$$Q = 33.6 - 8.14 = 25.46 \text{ k} \quad \text{(see Fig. 17)}$$

From Eq. (3) (torsional-flexural buckling) with F = 0,

$$Q = 40.0 \text{ k} > 25.46$$

Torsional-flexural buckling governs (see explanation in previous design case (I)). Therefore Q = 40 and F = 0 are the minimum required values of the expected diaphragm. This gives an idea from where to start assuming values of Q and F.

(ii), (iii) Design by Approximate Formulas and Design Charts

The approximate formulas as well as the charts lose their simplicity in the present design situation. Therefore their use is not recommended. The computer program B1 of Appendix 4 may be utilized as a design aid.

Choosing the diaphragm and checking its adequacy

Try 3/8" GYPSUM BOARDS with fasteners @ 12".

$$G' = 1.6 \text{ k/in.} \qquad \gamma_d = 0.013 \text{ in/in}$$

$$F' = 0.0355 \text{ k.in/in.rad.} \qquad \phi_d = 0.12 \text{ rad.}$$

then
$$Q = \frac{2}{3} \quad 1.6 \times 24 \times 2 = 51.4 \text{ k}$$

and
$$F = \frac{2}{3} \times 0.0355 \times 2 = 0.048 \text{ k.in/in.rad.}$$

Since inelastic behavior governs, then:

From Eq. (25) with σ = 29.4 ksi

$$E^* = 26967.5 \text{ ksi}$$

From Eq. (27)

$$G^* = 10329.9 \text{ ksi}$$

Therefore From Eqs. (1) with n = 1 and the above-computed values of E* and G*,

$$P_x = 51.699 \text{ k}$$

$$P_y = 7.445 \text{ k}$$

$$P_\phi = 10.991 \text{ k}$$

From Eqs. (11)

$$C_o = 0.411''$$
$$D_o = 0.206''$$
$$E_o = 0.004 \text{ rad.}$$

From Eq. (14a) and $P_r = 30.72$ $\quad C_1 = 0.531$

(14b) $\quad E_1 = 0.150$

(12) $\quad \gamma_{max} = 0.02 > 0.013$ N.G.

(13) $\quad \phi_{max} = 0.15 > 0.12$ N.G.

Try 3/8" GYPSUM BOARDS with fasteners @ 9".

$$G' = 2.050 \text{ k/in}$$
$$F' = 0.055 \text{ k.in/in.rad.}$$
$$\gamma_d = 0.014 \text{ in/in}$$
$$\phi_d = 0.15 \text{ rad.}$$

Then $\quad Q_r = Q = \frac{2}{3} \times 2.05 \times 24 \times 2 = 66.0$ k.

and $\quad F_r = F = \frac{2}{3} \times 0.055 \times 2 = 0.073$ k.in/in.rad

Using the parameters P_x, P_y, P_ϕ, C_o, D_o and E_o computed in the previous trial case, the following is obtained:

From Eq. (14a) $\quad C_1 = 0.29$

(14b) $\quad E_1 = 0.072$

(12) $\quad \gamma_{max} = 0.01 < 0.014$ O.K.

(13) $\quad \phi_{max} = 0.072 < 0.15$ O.K.

Therefore, the diaphragm is adequate and as a final step in the analysis, check the possibility of buckling between the fasteners.

Buckling between fasteners

Following the procedure of analysis outlined in Example 1 for channel sections, the following has been computed by considering the distance between the fasteners (s = 9.0"):

a) Flexural buckling about y-axis

$$P_{crf} = 2100.0 \text{ k}$$

b) Torsional buckling

$$P_{crf} = 1440.0 \text{ k} < 2100 \text{ k} \qquad \text{(governs)}$$

Since such behavior is in the inelastic range then from Eq. (24), the inelastic load is

$$P_{crf} = 44.80 \text{ k} > P_o = 30.72 \text{ k}$$

Then buckling between fasteners does not govern; hence 9" fastener spacing is acceptable.

Therefore the 3/8" GYPSUM BOARD with fasteners @ 9" satisfies all diaphragm requirements.

For a wide variety of Q and F-values see the sample output of Computer Program B1 in Appendix 4.

EXAMPLE 3 (bracing on one side)

Case (a): Calculate the ultimate and allowable loads of a 12'-0" wall stud of a 4" × $1\frac{1}{8}$" channel section-12 gage, with both ends hinged. The studs are spaced every 2-'0" and are attached on one side only to 3/8" GYPSUM WALLBOARD with fasteners every 12". Consider the following properties of the diaphragm, obtained from diaphragm test results:

$$G' = 1.88 \text{ k/in.}$$

$$\gamma_d = 0.014 \text{ in/in.}$$

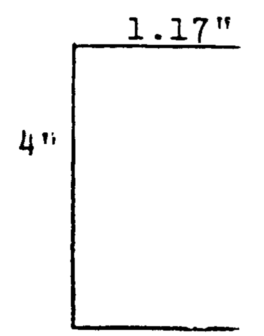
$$F' = 0.06 \text{ k.in/in.rad.}$$

$$\phi_d = 0.15 \text{ rad.}$$

The yield stress of steel used in the studs is 50 ksi.

Case (b): Specify a suitable wallboard material so that the same stud can safely carry an allowable load of 4.40 k.

Note: The computer programs A2 and B2 given in Appendix 4 are used to obtain the solution of case (a) and case (b), respectively. In such a design case, the computer program provides a convenient design tool.



$t = 0.105"$
$Q_A = 0.953$

SOLUTION CASE (a)

For the diaphragm:

$$Q_r = \frac{2}{3} \times 1.88 \times 24" = 30 \text{ k.}$$

$$\gamma_d = 0.014 \text{ in/in.}$$

$$F_r = \frac{2}{3} \times 0.06 = 0.04 \text{ k.in/in.rad.}$$

$$\phi_d = 0.15 \text{ rad.}$$

The output of the computer program A2, shown in Appendix 4, gives the elastic critical buckling load P_{cr}, computer by Eq. (7),

$$P_{cr} = 10.234 \text{ k. and } n = 2$$

Checking the higher buckling modes has been considered in the analysis by taking n = 1,2,3,...10, where n is the number of half-sine waves into which the stud may buckle. The following is obtained from the detailed output of program A2. These details are not shown in the output given in Appendix 4; however, these are obtainable only if PRINT = 1 is used in the program.

$$\begin{aligned} P_{cr} &= 12.474 \quad \text{for} \quad n = 1 \\ &= 10.234 \qquad\qquad 2 \\ & 13.966 \qquad\qquad 3 \\ & \quad \cdot \qquad\qquad\quad \cdot \\ & \quad \cdot \qquad\qquad\quad \cdot \\ & \quad \cdot \qquad\qquad\quad \cdot \\ &= 105.048 \quad \text{for } n = 10 \end{aligned}$$

From the values of P_{cr} given above, the following may be concluded:

i) Choosing n = 1,2,3,...10 for checking the possibility of higher buckling loads is more than sufficient in the present case.

ii) P_{cr}-values for $n > 2$ are increasing.

Check inelastic behavior

$$0.5Q_A\sigma_y = .05 \times 0.953 \times 50 = 23.83 \text{ ksi.}$$

$$\frac{P_{cr}}{A} = \frac{10.234}{0.644} = 15.897 < 23.83$$

Therefore according to Eq. (22), elastic behavior governs.

Check the possibility of buckling between the fasteners

Following the same procedure used in Example 1 (channel section), the following loads are obtained:

a) buckling about y-axis

$$P_{crf} = 143.681 \text{ k.}$$

b) torsional buckling

$$P_{crf} = 161.425 \text{ k.}$$

Both loads are larger than $P_{cr} = 10.234$; hence buckling between the fasteners does not govern. This can also be shown from the computer output and is given as

Elastic critical buckling load, considering buckling between the fasteners = 10.234 k.

Now it is left to satisfy the requirements that the resulting shear deformations and rotation of the stud are less than γ_d and ϕ_d (of the diaphragm), respectively, that is,

$$\gamma_{max} < 0.014 \text{ in/in.}$$

and $$\phi_{max} < 0.15 \text{ rad.}$$

Consider initial imperfections:

$$C_o = 0.411 \text{ in.}$$

$$D_o = 0.206 \text{ in.}$$

$$E_o = 0.004 \text{ rad.}$$

For $n = 2$ $P_x = 79.051$ k

$P_y = 3.99$ k

$P_\phi = 14.451$ k

Consider a trial load $P_r = \lambda P_{cr}$, where $\lambda < 1.0$. Hence compute C_1, E_1, and γ_{max} and ϕ_{max} from Eqs. (19a), (19b), (17) and (18), respectively. Therefore

Trial 1

$\lambda = 0.94$

$P_r = 10.234 \quad 0.94 = 9.62$ k.

$C_1 = 2.008$

$E_1 = 0.780$

$\gamma_{max} = 0.019 > 0.014$ N.G.

$\phi_{max} = 0.783 > 0.15$ N.G.

Trial 2

$\lambda = 0.84$

$P_r = 10.234 \times 0.84 = 8.59$ k.

$C_1 = 0.680$

$E_1 = 0.258$

$\gamma_{max} = 0.007 < 0.014$ O.K.

$\phi_{max} = 0.239 > 0.15$ N.G.

Trial 3

$\lambda = 0.75$

$P_r = 10.234 \quad 0.75 = 7.675$ k.

$C_1 = 0.392$

$E_1 = 0.146$

$\gamma_{max} = 0.004 < 0.014$ O.K.

$\phi_{max} = 0.146 < 0.15$ O.K.

Therefore the load capacity of the stud $P_r = 7.675$ k.

$$\text{Allowable load } P_{all} = \frac{7.675}{1.92} = \underline{3.998 \text{ k}}$$

(see computer output of Program A2, given in Appendix 4.)

Notes:

1) It has been conclusively assumed that the number of half-sine waves, n, into which the column may buckle is the same for both perfect and imperfect columns. That is, n = 2, obtained for P_{cr} = 10.234 k, has been used in the calculations of C_1, E_1 and hence P_r and P_{all}. This assumption has been disregarded in the Computer Program A2 and hence the solution routine includes calculating $P_r = \lambda P_{cr}$ for n = 1,2,3,...10, and then choosing the smallest P_r and the corresponding n. The results of the computations of this example and other examples substantiate the considered assumption. In the output of Program A2 of Appendix 4, the following is printed.

Critical buckling load P_{cr} = 10.234 and n = 2

Load capacity P_r = 7.675 and n = 2

Details of the above computations are obtainable from the computer output of Program A2 with PRINT = 1.

2) It is of interest to note that the critical buckling load of the unbraced stud is equal to 0.998 k. Therefore, by bracing the stud on one side, the load capacity increased to 7.675 k (i.e. about 7.5 times the unbraced buckling load).

Case (b)

$$\text{Load capacity } P_o = 4.4 \times 1.92 = 8.445 \text{ k}$$

Check that $P_o > P_{cr,UB}$

$P_o < P_{cr,x}$

$$P_o < P_{yield}$$

where $P_{cr,UB}$ = critical buckling load of the unbraced stud

$P_{cr,x}$ = strong axis buckling

P_{yield} = yield load of the stud

For the given stud,

$$P_{cr,UB} = 0.998 < 8.445 \text{ k} \qquad \text{O.K.}$$

$$P_{cr,x} = 19.763 > 8.445 \text{ k} \qquad \text{O.K.}$$

$$P_{yield} = 30.50 > 8.445 \text{ k} \qquad \text{O.K.}$$

Therefore, the stud cross-section is satisfactory. The next step is to specify a suitable diaphragm and hence check the possibility of buckling between the fasteners.

Check inelastic behavior

$$0.5Q_A\sigma_y = 23.83 \text{ ksi}$$

$$\frac{P_o}{A} = \frac{8.445}{0.644} = 13.2 < 23.83$$

Hence elastic behavior governs.

Diaphragm bracing

1) Min. Q and F-values

Consider n = 1,2,3,...10.

Assume trial values of Q and F. Then from Eq. (7), find P_{cr} and the corresponding n.

If $P_{cr} < P_o$, then increase Q and F and repeat the analysis until a value of $P_{cr} > P_o$ is obtained. Such values are termed as the minimum Q and F-values.

From the computer output of Program B2, these two values are:

$$Q = 25.0 \text{ k.}$$

and $$F = 0.04 \text{ k.in/in.rad.}$$

By using these values in Eq. (7) and different values of n, the critical buckling load is

$$P_{cr} = 10.055 > 8.45 \quad \text{and} \quad n = 2$$

ii) <u>Trial of available diaphragms</u>

<u>Trial 1</u>

From diaphragm test results of 3/8" GYP. and fasteners every 9",

$$G' = 2.05 \text{ k/in.}$$

$$F' = 0.060 \text{ k.in/in.rad.}$$

$$\gamma_d = 0.014$$

$$\phi_d = 0.15$$

Then $$Q = Q_r = \frac{2}{3} \times 2.05 \times 24 = 33 \qquad \text{O.K.}$$

$$F = F_r = \frac{2}{3} \times 0.060 \approx 0.040 \text{ k.in/in.rad}$$

With Q = 33.0 and F = 0.037 (n = 1,2,3,...10), P_{cr} is calculated from Eq. (7) and its value is equal to

$$P_{cr} = 10.30 > 8.45 \text{ k} \qquad \text{O.K.}$$

and the corresponding n = 2.

Consider initial imperfections:

$$C_o = 0.411$$

$$D_o = 0.206$$

$$E_o = 0.004$$

Therefore, from Eqs. (19a), (19b), (17) and (18) with P_r = 8.45 and n = 2, and above values of the initial imperfections, then

$$C_1 = 0.58 \text{ in.}$$
$$E_1 = 0.226 \text{ rad.}$$
$$\gamma_{max} = 0.006 < 0.014 \qquad \text{O.K.}$$
$$\phi_{max} = 0.226 > 0.15 \qquad \text{N.G.}$$

Thus the diaphragm is not adequate for bracing.

Trial 2

Try 1/2" Homosote boards and fasteners every 6", and consider the following:

$$G' = 2.80 \text{ k/in.}$$
$$F' = 0.07 \text{ k.in/in.rad}$$
$$\gamma_d = 0.012 \text{ in/in}$$
$$\phi_d = 0.175 \text{ rad.}$$

Then $$Q = Q_r = \frac{2}{3} \times 2.80 \times 24 = 45.0 \text{ k}$$

$$F = F_r = \frac{2}{3} \times 0.07 = 0.045 \text{ k.in/in.rad.}$$

Following the same steps of analysis considered in the previous trial,

$$P_{cr} = 10.8 \text{ k} > 8.45 \qquad \text{O.K.}$$

and n = 2. Hence,

$$C_1 = 0.431$$
$$E_1 = 0.175$$

$$\gamma_{max} = 0.004 < 0.012 \qquad \text{O.K.}$$

$$\phi_{max} = 0.1748 < 0.175 \qquad \text{O.K.}$$

Thus the diaphragm is adequate for bracing.

Check possibility of buckling between the fasteners

Distance between fasteners s = 6"

a) Buckling about y-axis

$$P_{crf} = 582.0 \text{ k.}$$

b) Torsional buckling

$$P_{crf} = 650.0 \text{ k.}$$

Both loads are much larger than P_o = 8.45 k; therefore buckling between the fasteners does not govern.

Thus 1/2" Homosote boards with fasteners every 6" satisfies all the design requirements.

Note: The computer output of Program B2 shown in Appendix 4 includes a list of Q, F, γ_{max} and ϕ_{max}. With the aid of such a list, the suitable diaphragm may be chosen from Diaphragm Catalogues or from diaphragm test results, provided that Q, F, γ_d, ϕ_d of the diaphragm are at least equal to one of the values of Q, F, γ_{max} and ϕ_{max} listed in the output.

Appendix 2

COMMENTS ON THE METHODS USED TO SIMPLIFY THE GOVERNING EQUATIONS

Methods 1 and 2 are iterative procedures commonly used to give the roots of a polynomial when the coefficients of the variables have numerical values; then the iterative procedure is possible[39,40,41]. However, in the cases considered herein the coefficients are parameters forming complicated algebraic expressions as in Eqs. (39) and (42); therefore the intent is to find the smallest root only of these equations expressed in a linear form in terms of the known parameters. Therefore iteration for more than two cycles at the most is not possible. This disadvantage has been overcome by choosing the first trial root as close as possible to the real root, so that fast convergence would be possible. Method 3 is simpler to use but poor in accuracy unless at least the first three terms of the expansion are considered. The abovementioned three methods are used to obtain an expression of the smallest root of the governing equations.

Approximation by piecewise linear functions, Method 4, involves reducing the nonlinear equations to a set of linear functions (Fig. 13). This is done by selecting points lying on f(p) as break points at which the slope changes. The points should be chosen sc that the equations of the linear segments would approximate as accurately as possible the original function and most important, that the equations of the segments are expressed in simple terms. Application of the method to zee-sections braced on both sides is illustrated in Fig. 13. The resulting equations are not simple and the approximation is not satisfactory in the ragion of small values of Q.

Method 5[41] is one of the strategies to solve nonlinear algebraic equations by treating them as linear equations, delegating the higher powers to an unimportant place on the right hand side of the equation. For example, applying the method to Eq. (59), then

$$P_{i+1} = \frac{P_x P_y + P_x Q - P_{xy}^2 + P_i^2}{-(P_x + P_y + Q)} \qquad \text{where } i = 0,1,2,\ldots$$

For the first approximation set $P_o = 0$ and get the first approximate root. Repeat the steps until convergence is obtained. The method is not as effective as Newton's method since it is not possible to perform more than one iteration.

The method of split rigidity[42] was developed by Bijlaard in 1932 to calculate the buckling loads of structures that buckle in the composite mode. However, the method had been known and used by F. Buckens, 1943, without any reference to Bijlaard. Buckens used the method to overcome the difficulties which are inhibited in certain relations of stability problems. The method consists of splitting the buckling deflections into two or more component modes and expressing the buckling stress in terms of the critical loads for these component modes. Simple answers are obtained for sandwich plates for which the basic assumption that the split deflections have the same shape is fulfilled. However, when the deflection has components in more than one direction (for example u, v, ϕ) the solution becomes as complicated as the solutions obtained in the present investigation. This has been shown by Bijlaard in a paper dealing with torsional-flexural buckling of open sections[43]. The question of whether the method yields conservative answers or not has been discussed by Plantema[44]. Siede[45] found that in some cases of buckling of flat plates, the method gives unconservative answers. It became clear after a few attempts to solve the simplest case in the present investigation, the method will not yield a simple expression of the buckling load. This is mainly due to the involvement of more than one component of the deflection in the buckling mode.

Neglecting the term thought to be of minor influence, method 6, did not lend itself to any logical answer. After many trials it has been realized that the equations of stability are very sensitive to inconsistent changes in the quantities forming the coefficients of the variables.

The governing equations of sections braced on both sides are much simpler than those for one sided bracing. Method 8 has been suggested to investigate the possibility of obtaining a simple expression of the buckling load in the case of one sided bracing in terms of the solution of two sided bracing. Comparison of the exact numerical results of channel sections braced on both sides and channel sections braced on one side revealed that a certain reduction factor can be introduced to the diaphragm shear rigidity Q (Eq. 57) so that the modified equation can handle the case of one sided bracing. However, it has been realized after examining numerically different cases that the method lacks generality.

Appendix 3

SAMPLE DERIVATION OF LOAD-DEFLECTION RELATIONSHIP OF AN IMPERFECT COLUMN
(general-shaped section braced on both sides)

End Condition $\quad u = v = \phi = 0 \quad$ at $z = 0,L$

$u'' = v'' = \phi'' = 0 \quad$ at $z = 0,L$

The following are the details of deriving Eqs. (80) of Section 2.7B.1.

From Eq. (53) the following functions are chosen since they satisfy the above end conditions.

$$u = C_1\sin\frac{\pi z}{L} \qquad u_o = C_o\sin\frac{\pi z}{L}$$

$$v = D_1\sin\frac{\pi z}{L} \qquad v_o = D_o\sin\frac{\pi z}{L}$$

$$\phi = E_1\sin\frac{\pi z}{L} \qquad \phi_o = E_o\sin\frac{\pi z}{L}$$

Considering that $u_t = u + u_o$, $v_t = v + v_o$, $\phi_t = \phi + \phi_o$, and substituting with the above listed displacement functions into Eq. (79), the following equation is obtained:

$$\Pi = \frac{1}{2}\int_o^L \{EI_yC_1^2(\tfrac{\pi}{L})^4\sin^2\tfrac{\pi z}{L}+2EI_{xy}C_1D_1(\tfrac{\pi}{L})^4\sin^2\tfrac{\pi z}{L}+EC_wE_1^2(\tfrac{\pi}{L})^4\sin^2\tfrac{\pi z}{L}+GJE_1^2(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L}$$

$$-P[C_1^2(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L}+2C_1C_o(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L}+D_1^2(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L}+2D_1D_o(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L}$$

$$+r_o^2(E_1^2(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L}+2E_1E_o(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L})-2x_o(D_1E_1(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L}$$

$$+D_1E_o(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L}+D_oE_1(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L})+2y_o(C_1E_1(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L}$$

$$+C_1E_o(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L}+C_oE_1(\tfrac{\pi}{L})^2\cos^2\tfrac{\pi z}{L})] \qquad \text{(contd.)}$$

$$+Q[C_1^2(\frac{\pi}{L})^2\cos^2\frac{\pi z}{L}+E_1^2(\frac{\pi}{L})^2\cos^2\frac{\pi z}{L}(\frac{d_1^2+d_2^2}{2})+C_1E_1(\frac{\pi}{L})^2\cos^2\frac{\pi z}{L}(d_1-d_2)]+F\cdot E_1^2\sin^2\frac{\pi z}{L}\}dz$$

Then $$\Pi = \frac{1}{2}(\frac{\pi}{L})^2\frac{L}{2}\{P_yC_1^2+2P_{xy}C_1D_1+EC_w(\frac{\pi}{L})^2E_1^2+GJE_1^2+P_xD_1^2$$

$$-P[C_1^2+2C_1C_o+D_1^2+2D_1D_o+r_o^2(E_1^2+2E_1E_o)$$

$$-2x_o(D_1E_1+D_1E_o+D_oE_1)+2y_o(C_1E_1+C_1E_o+C_oE_1)]$$

$$+Q[C_1^2+E_1^2(\frac{d_1^2+d_2^2}{2})+C_1E_1(d_1-d_2)]+E_1^2F\frac{L^2}{\pi^2}\}$$

Using the Rayleigh-Ritz method to minimize the above energy expression with respect to C_1, D_1 and E_1, hence the following 3 equations are obtained:

$$\frac{\partial\Pi}{\partial C_1} = 0$$

$$C_1(P_x-P+Q)+D_1(P_{xy})+E_1(Q(\frac{d_1-d_2}{2})-Py_o) = P(C_o+y_oE_o)$$

$$\frac{\partial\Pi}{\partial D_1} = 0$$

$$C_1(P_{xy})+D_1(P_x-P)+E_1(Px_o) = P(D_o-x_oE_o)$$

$$\frac{\partial\Pi}{\partial E_1} = 0$$

$$C_1(Q(\frac{d_1-d_2}{2})-Py_o)+D_1(Px_o)+E_1(r_o^2(P_\phi-P)+Q(\frac{d_1^2+d_2^2}{2})+F\frac{L^2}{\pi^2})$$

$$= P(r_o^2E_o-x_oD_o+y_oC_o)$$

Rearranging these equations in matrix form leads to Eq. (80).

Appendix 4

WALL STUDS DESIGN PROGRAMS

(Documented Listings, Flow Charts and Sample Outputs)

4A. General

Four programs, written in Basic FORTRAN IV Language for the IBM 360/65 are included herein.

The input data and its format are described in the beginning of each program listing.

Three cards within the program may need to be cahnged for a given compiler and application. These are:

1 and 2) LOGICAL RECORD UNITS of READ and WRITE statements are replaced by J and K, respectively, provided that J and K units, required by a certain compiler, are declared before any READ or WRITE statements. Herein the units of J and K used in the program are

$$J = 5$$

$$\text{and} \quad K = 6$$

3) The card containing the CONTROL VARIABLE (PRINT) is to transfer control of the WRITE statements. If details of the computations as well as the final answers are needed in the output, then let PRINT = 1; if only the final answers are needed, then let PRINT = 0.

In this Appendix, the flow charts, samples of computer outputs and the documented listing of the programs follow, in order, Section 4D (Definitions of Variables). Sample outputs are the solutions, without details (i.e. PRINT = 0), of the design examples of Appendix 1.

4B. Sources of Equations

The solution routines are based on the suggested design procedure outlined in Chapter 6, Section 6.3 and have the same limitations specified in this report. The design equations coded in Section 6.4 are utilized throughout the programs, while formulas for computing section properties are obtained from Refs. 49 and 53.

4C. Limitation of the Programs

1) The programs have been prepared to serve as design aids for the analysis of wall studs made of I, channel and zee-sections.

2) The studs are braced with diaphragms whose properties are within the practical range of wall stud applications.

3) Units for each design parameter are given in the beginning of each program, as well as in Section 4D of this appendix (Definitions of Variables).

4) The programs provide for the design of diaphragm-braced wall studs of I, channel and zee-sections for the following cases:

- Sections braced on both sides
    - Find P_{all} for given S and F-values (Prog. A1)
    - Find S and F-values for given P_{all} (Prog. B1)
- Sections braced on one side only
    - Find P_{all} for given S and F-values (Prog. A2)
    - Find S and F-values for given P_{all} (Prog. B2)

4D. Definitions of Variables

Some of the important variables in the program will now be defined. Some of these variables appear in the program output. Variables which appear in the READ statements are defined in the beginning of each program. Units only for those variables appearing in the output are given below.

AN = n^2, where n is the number of half-sine waves into which the stud may buckle

AREA = stud cross-sectional area

C1 = deflection in the direction of the wallboards, in.

CW = warping constant, in^6

E1 = E_1 = computed rotation, rad.

F.S. = factor of safety

FEMAX = computed rotation, rad.

GAMAX = computed shear strain in the diaphragm, in/in.

GI = inelastic shear modulus, ksi

NU = number of half-sine waves to be examined

NWAVE = number of waves corresponding to the buckling load under consideration

P_{all} = allowable load, k

PO = given allowable load, k

PA = inelastic buckling load, k

PC = elastic buckling load computed for each value NU, k

PCF = buckling load of the unbraced stud with buckling length equal to the distance between the fasteners, k

PCR = elastic critical buckling load, k

PCUNB = buckling load of unbraced stud after investigating inelastic behavior, k

PE = elastic buckling load obtained from Eq. (24) of Section 6.4 for a known value of inelastic buckling load PA, k

PFE = torsional buckling load of the braced stud, k

PFEF = torsional buckling load of the unbraced stud with buckling length equal to the distance between the fasteners, k

PI = polar moment of inertia about the shear center, in^4

PR = load capacity of the stud, k

PUNB = buckling load of the unbraced stud, k

PX1 = Euler buckling load of the unbraced stud about the minor axis of inertia, k

PX2 = Euler buckling load of the unbraced stud about the major axis of inertia, k

PXX = Euler buckling load of the unbraced stud about the x-axis, k

PXXF = Euler buckling load of the unbraced stud about the x-axis, used in checking the possibility of buckling between the fasteners of a channel section (= PXX), k

PXY = defined by Eq. (1c)

PYIELD = yield load of the stud (= Area × σ_y), k

PYY = Euler buckling load of the unbraced stud about the y-axis, k

PYYF = buckling load of the unbraced stud about the y-axis, with the buckling length equal to the distance between the fasteners, k

R2 = IP/AREA, where IP is the polar moment of inertia about the shear center (equivalent to r_o^2 used in the governing equations, in^2

TMOD = inelastic modulus defined by Eq. (25), ksi

XI1 = moment of inertia about minor axis, in^4

XI2 = moment of inertia about major axis, in^4

XL = stud length

XL1 = distance between the fasteners

XLAM = a factor less than 1.0 (equivalent to λ used in the design procedure

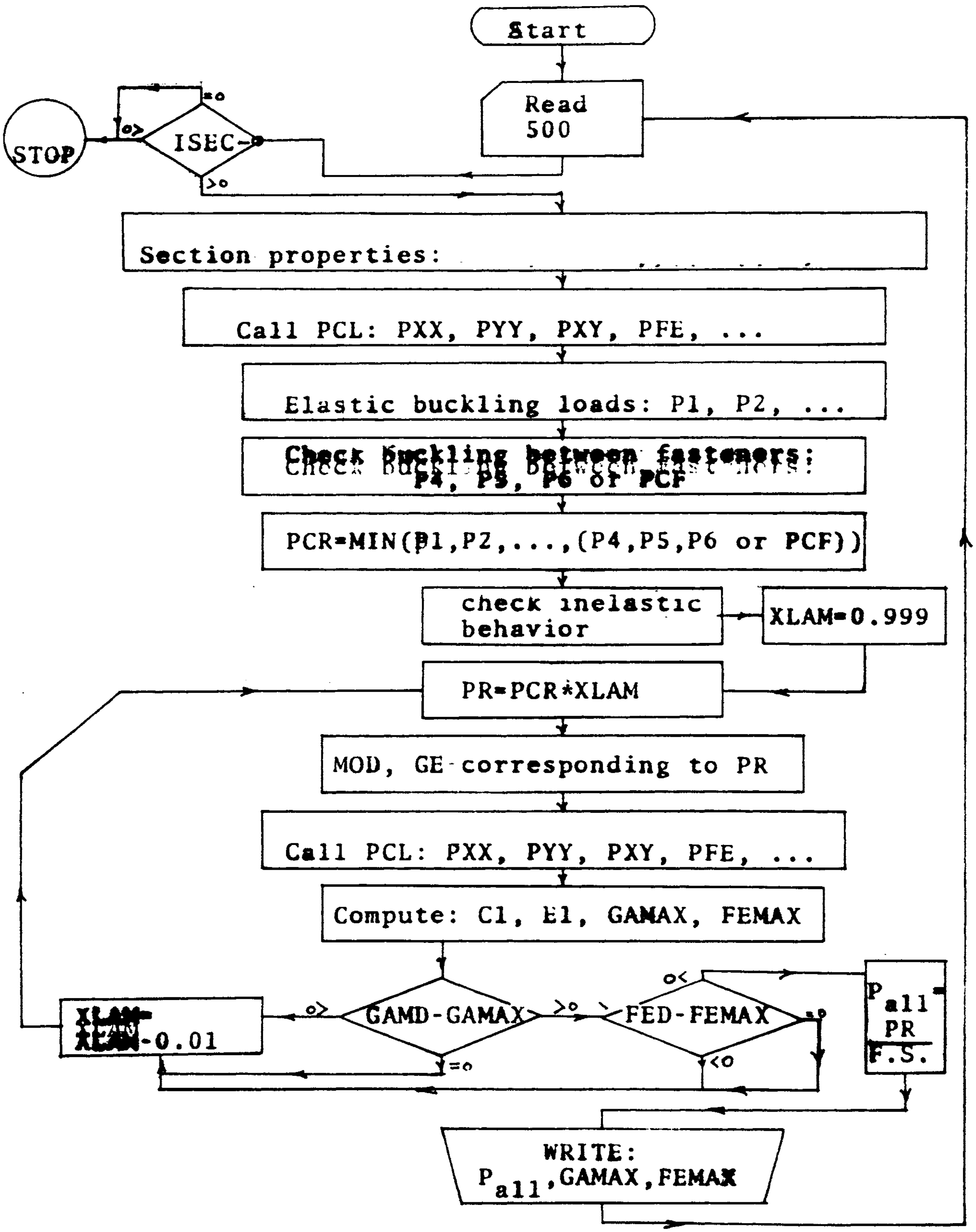
XJ = St. Venant torsion constant, in^4

XO = distance between centroid and shear center of the section, in.

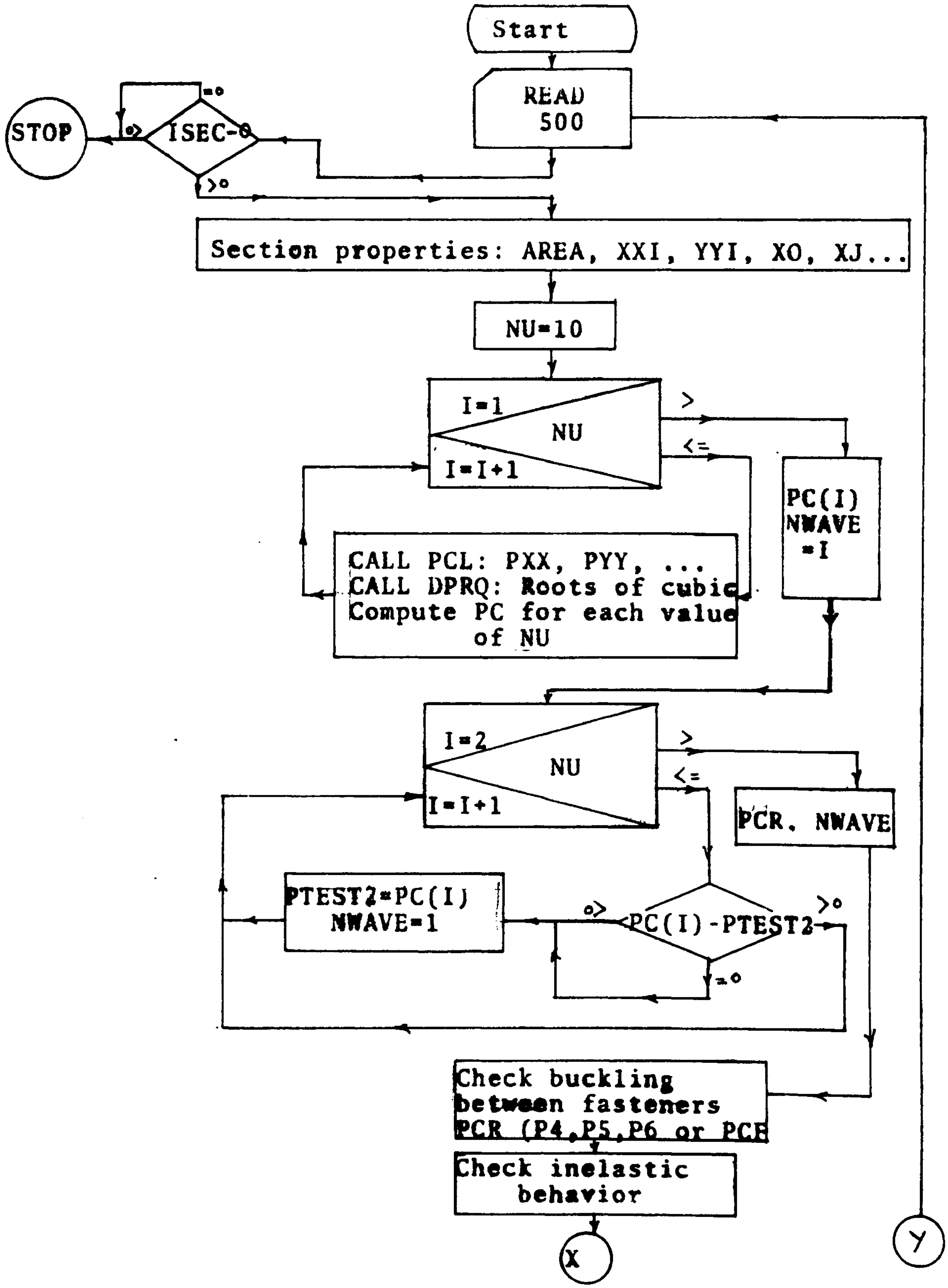
XXI = moment of inertia about x-axis, in^4

XYI = product of inertia with respect to x- and y-axes, in^4

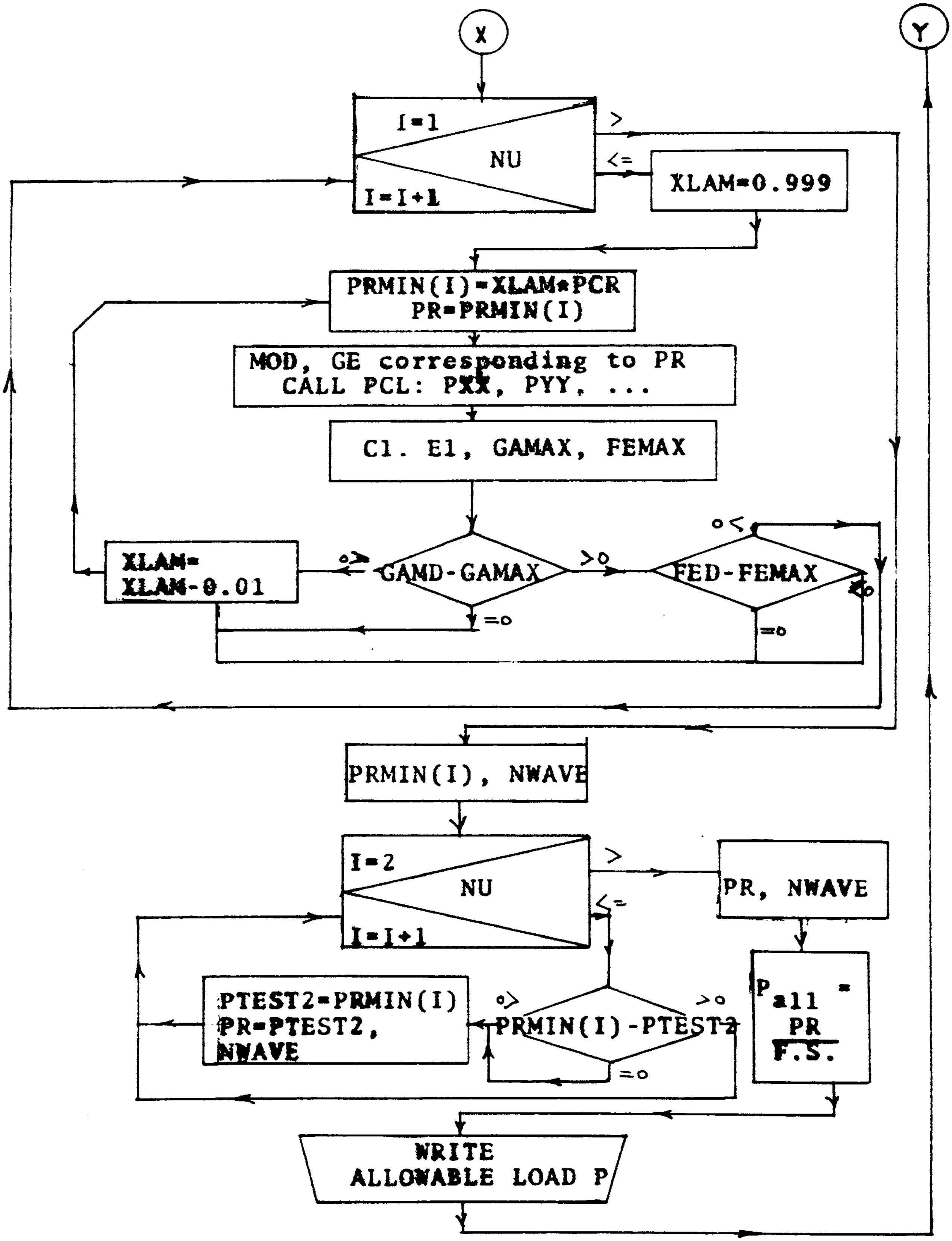
YYI = moment of inertia with respect to y-axis



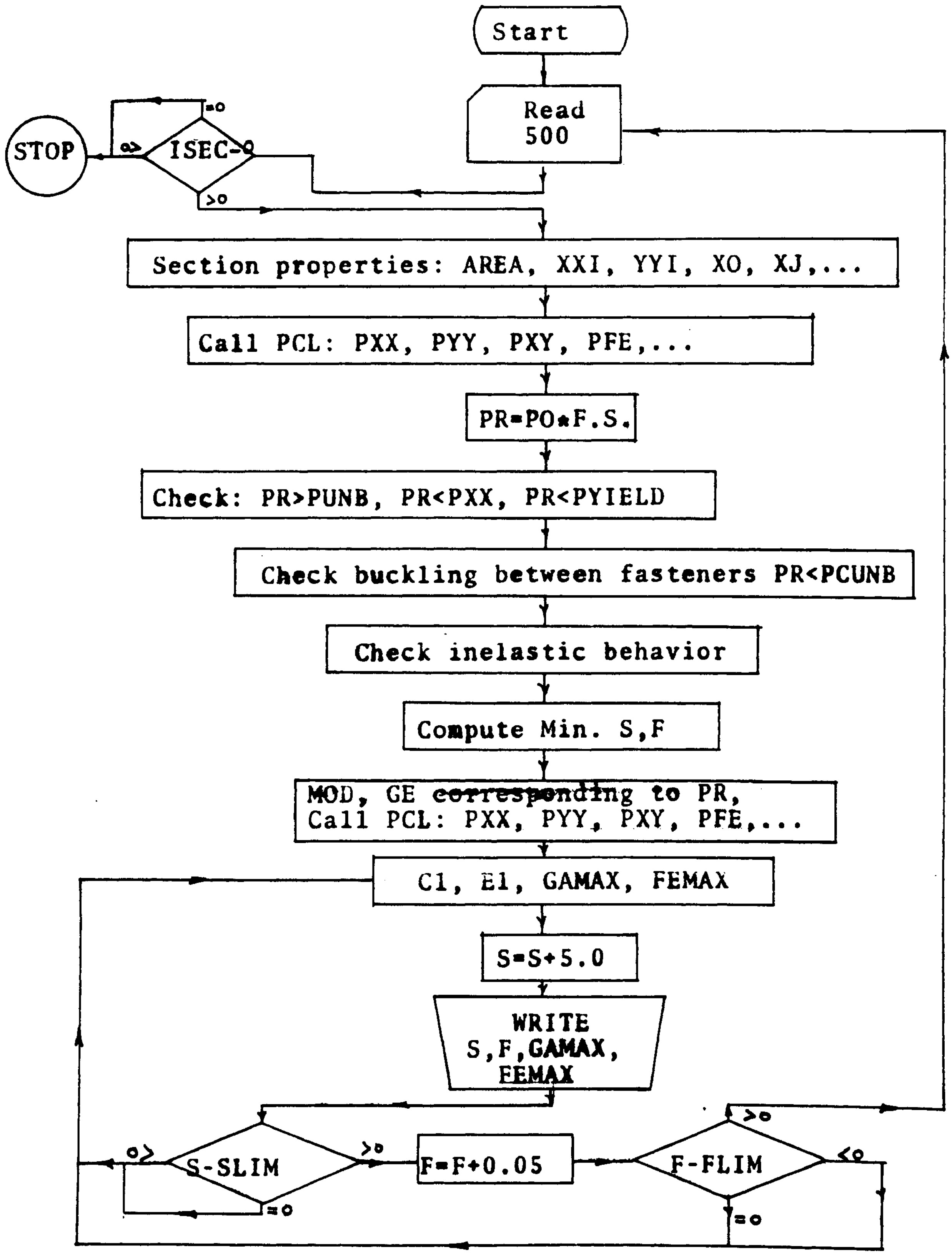
FLOW CHART FOR PROGRAM (A1)



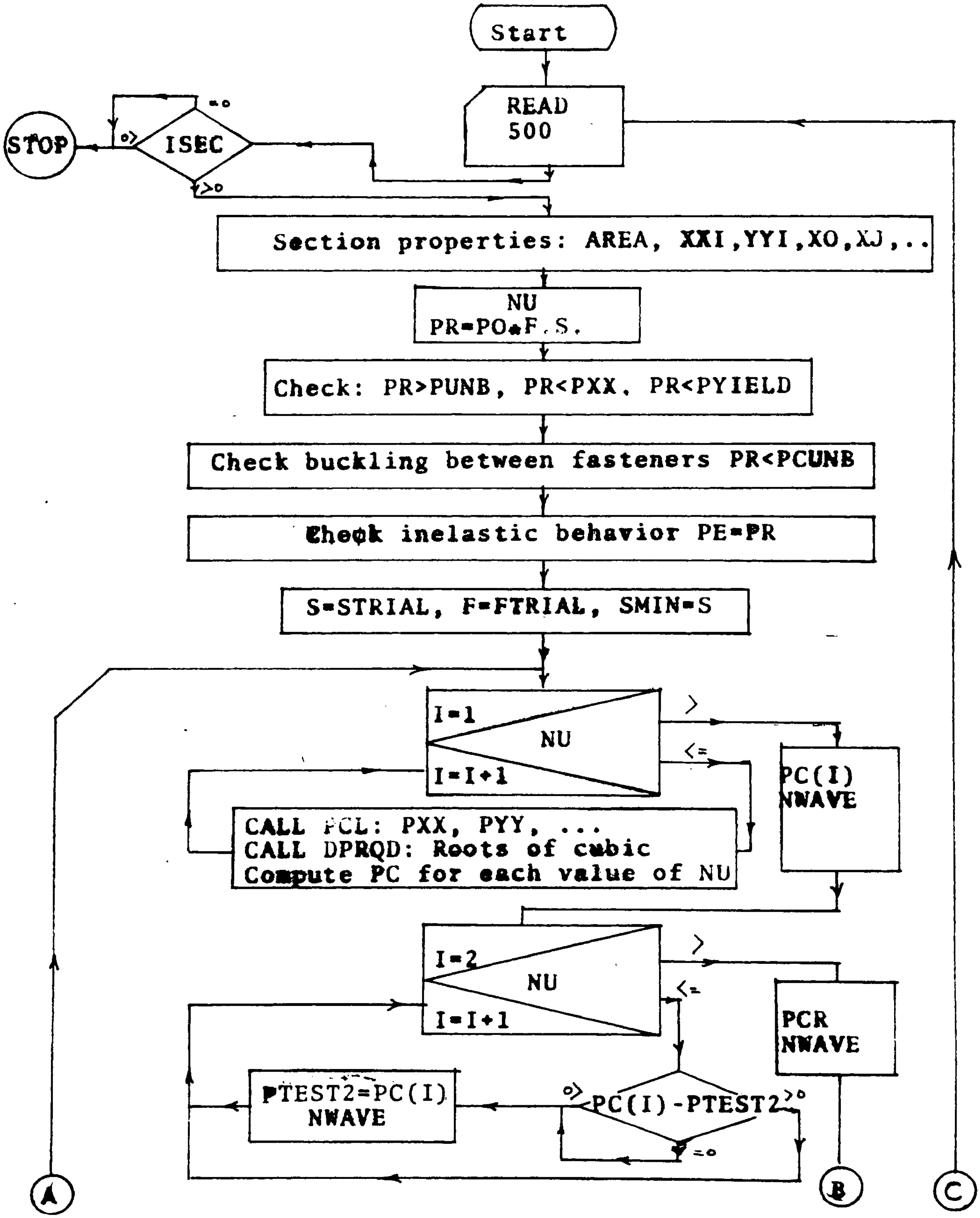
FLOW CHART FOR PROGRAM (A2)



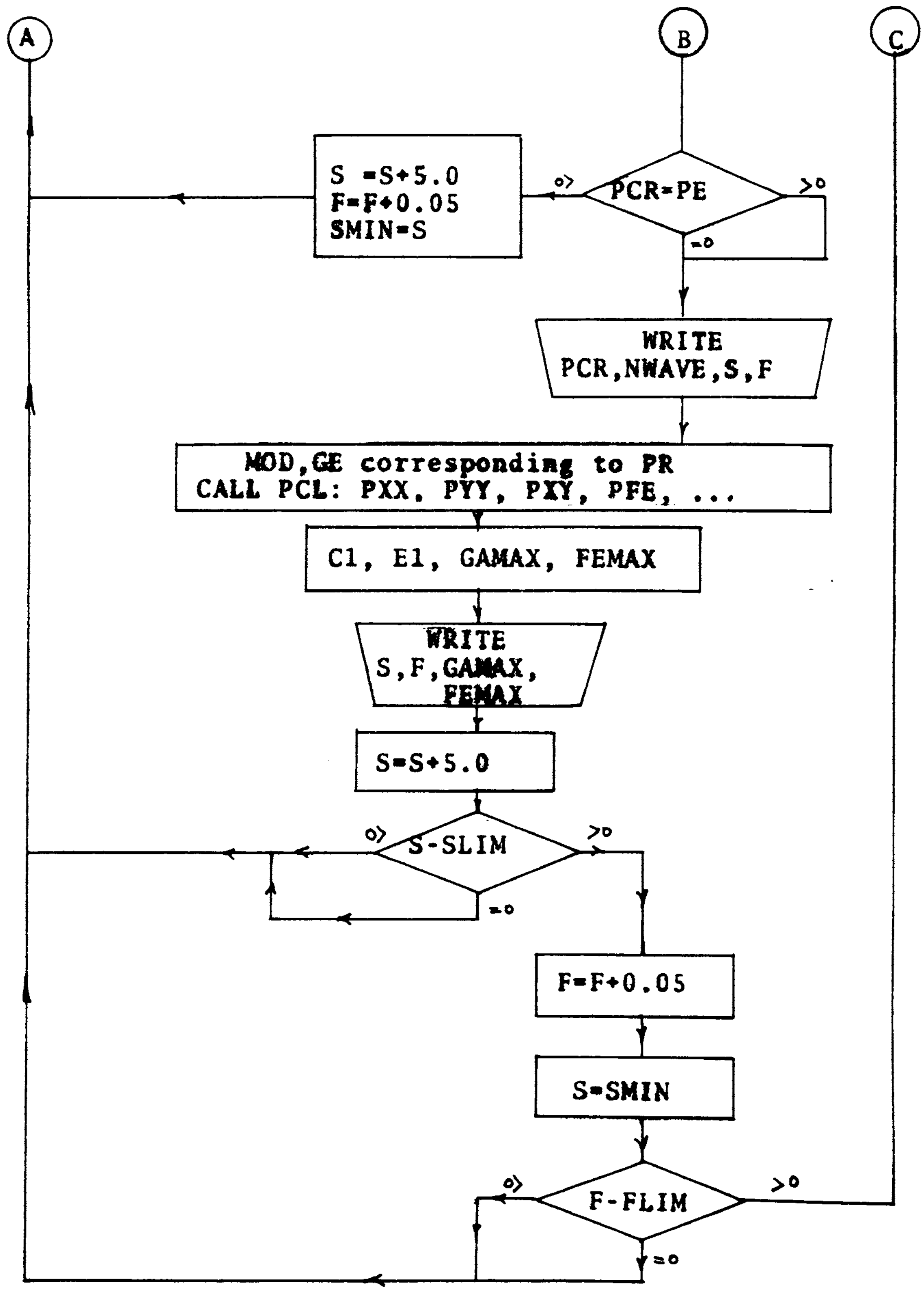
FLOW CHART FOR PROGRAM (A2) (contd.)



FLOW CHART FOR PROGRAM (B1)



FLOW CHART FOR PROGRAM (B2)



FLOW CHART FOR PROGRAM (B2) (contd.)

SAMPLE OUTPUT OF PROGRAM A1

Solution of DESIGN EXAMPLE 1 (case I)

```
ALLOWABLE  LOAD OF STUD BRACED CN BCTH SIDES        (PROG. A1)
=============================================

 ZFE - SECTION STUD                  LENGTH= 96.CO
 ----------------

 SECTICN DIMENSIONS
 DEPTH= 3.500  H= 3.440  B= 1.940  D= C.470  T= 0.060
 CA= 0.861

 DIAPHRAGM PROPERTIES
 S=  50.000     F= 0.060    GAMD= 0.01100    FED= 0.15000

 YIELD STRESS FY=50.000
 INITIAL IMPERFECTIONS CO=0.274  DO=0.137  EC=0.002

 SECTICN PROPERTIES
 AREA= 0.496  IXX= 1.018  IYY= 0.504  IXY= 0.470  XC= 0.000
 R2= 3.071  J= 0.001  CW= 0.947  IX1= C.226  IX2= 1.296

 MOD= 29500.0  GE= 11300.0  PXX= 32.15C  PYY= 15.933
 PFE= 11.929  PXY= 14.837  PX1=  7.133  PX2= 40.950

 ELASTIC CRITAL B. LOAD=  26.559

          ALLOWABLE   LOAD   P=   8.88C
         ================================
```

SAMPLE OUTPUT OF PROGRAM A1

Solution of DESIGN EXAMPLE 1 (case II)

```
ALLOWABLE  LOAD OF STUD BRACED ON BOTH SIDES         (PROG. A1)
===========================================

CHANNEL  SECTION  STUD          LENGTH= 96.CC
-----------------

SECTICN DIMENSICNS
DEPTH= 3.500  H= 3.440  B= 1.940  D= 0.470  T= 0.060
CA= 0.861

DIAPHRAGM PROPERTIES
S=  5C.000    F= 0.060    GAMD= 0.01100    FED= 0.15000

YIELD STRESS FY=50.000
INITIAL IMPERFECTIONS C0=0.274  D0=0.137  E0=0.002

SECTICN PROPERTIES
AREA= 0.496  IXX= 1.018  IYY= 0.278  IXY= C.000
XC= 1.643  R2= 5.313  J= 0.001  CW= 0.649

MOD= 29500.0  GE= 11300.0
PXX= 32.150  PYY=  8.769  PFE=  5.125  PXY=  0.000

ELASTIC CRITAL B. LOAD=  21.680

          ALLOWABLE   LOAD   P=   8.211
          ==============================
```

SAMPLE OUTPUT OF PROGRAM B1

Solution of DESIGN EXAMPLE 2 (Case I)

```
   ALLOWABLE  LOAD OF STUD BRACED ON BOTH SIDES (PROG. B1)
   ==============================================
CHANNEL  SECTION  STUD          LENGTH=144.00
------------------


GIVEN ALL.  LOAD   (PO)  =   8.000

SECTION DIMENSIONS
DEPTH= 5.000  H= 4.895  B= 1.895  D= 0.647  T= 0.105
QA= 0.907

DIAPHRAGM PROPERTIES
SLIM=  35.000    FLIM=   0.020    XL1= 12.00

YIELD STRESS FY=50.000
INITIAL IMPERFECTIONS CO=0.411  DO=0.206  EO=0.004

SECTION PROPERTIES
AREA= 1.048  IXX= 4.028  IYY= 0.580  IXY= 0.000
XO= 1.540  R2= 6.770  J= 0.004  CW= 2.698

MOD= 29500.0  GE= 11300.0
PXX= 56.554  PYY=  8.144  PFE= 12.023  PXY=  0.000
```

S	F	GAMAX	FEMAX	C1	E1
12.216	0.000	0.036	0.156	1.264	0.156
17.216	0.000	0.019	0.088	0.632	0.088
.	.				
.	.				
.	.				
.	.				
27.216	0.010	0.009	0.040	0.316	0.040
32.216	0.010	0.007	0.034	0.253	0.034
12.216	0.015	0.032	0.087	1.264	0.087
17.216	0.015	0.017	0.061	0.632	0.061
22.216	0.015	0.012	0.047	0.421	0.047
27.216	0.015	0.009	0.038	0.316	0.038
32.216	0.015	0.007	0.032	0.253	0.032
12.216	0.020	0.032	0.076	1.264	0.076
17.216	0.020	0.017	0.055	0.632	0.055
22.216	0.020	0.012	0.043	0.421	0.043
27.216	0.020	0.009	0.036	0.316	0.036
32.216	0.020	0.007	0.030	0.253	0.030

SAMPLE OUTPUT OF PROGRAM B1

Solution of DESIGN EXAMPLE 2 (case II)

```
ALLOWABLE  LOAD OF STUD BRACED ON BOTH SIDES        (PROG. B1)
==========================================

 CHANNEL  SECTION  STUD          LENGTH=144.00
 -----------------

 GIVEN ALL.  LOAD   (PO)  =  16.000

 SECTION DIMENSIONS
 DEPTH= 5.000  H= 4.895  B= 1.895  D= 0.647  T= 0.105
 QA= 0.907

 DIAPHRAGM PROPERTIES
 SLIM=  80.000    FLIM=   0.070    XL1= 12.00

 YIELD STRESS FY=50.000
 INITIAL IMPERFECTIONS C0=0.411  D0=0.206  E0=0.004

 SECTION PROPERTIES
 AREA= 1.048  IXX= 4.028  IYY= 0.580  IXY= 0.000
 XO= 1.540  R2= 6.770  J= 0.004  CW= 2.698

 MOD= 29500.00  GE= 11300.00
  PE= 33.605  PR= 30.720


 MOD= 26967.5  GE= 10329.9
 PXX= 51.699  PYY=  7.445  PFE= 10.991  PXY=  0.000
```

S	F	GAMAX	FEMAX	C1	E1
42.060	0.000	0.070	1.013	0.673	1.013
47.060	0.000	0.035	0.425	0.531	0.425
52.060	0.000	0.024	0.269	0.439	0.269
57.060	0.000	0.019	0.196	0.374	0.196
62.060	0.000	0.016	0.155	0.326	0.155
67.060	0.000	0.013	0.128	0.289	0.128
72.060	0.000	0.012	0.109	0.259	0.109
77.060	0.000	0.010	0.095	0.235	0.095
42.060	0.005	0.052	0.691	0.673	0.691
47.060	0.005	0.031	0.355	0.531	0.355
52.060	0.005	0.023	0.239	0.439	0.239

contd. on next page

62.060	0.040	0.012	0.099	0.326	0.099
67.060	0.040	0.011	0.087	0.289	0.087
72.060	0.040	0.010	0.C78	0.259	0.078
77.060	0.040	0.009	0.070	0.235	0.070
42.060	0.045	0.025	0.195	0.673	0.195
47.060	0.045	0.020	0.154	C.531	0.154
52.060	0.045	0.017	0.127	0.439	0.127
57.060	0.045	0.014	0.108	0.374	0.108
62.060	0.045	0.012	0.094	0.326	0.094
67.060	0.045	0.011	0.084	0.289	0.084
72.060	0.045	0.010	0.075	0.259	0.075
77.060	0.045	0.009	0.068	0.235	0.068
42.060	0.050	0.024	0.179	0.673	0.179
47.060	0.050	0.019	0.144	0.531	0.144
52.060	0.050	0.016	0.120	0.439	0.120
57.060	0.050	0.014	0.103	0.374	0.103
62.060	0.050	0.012	0.090	0.326	0.090
67.060	0.050	0.011	0.081	0.289	0.081
72.060	0.050	0.010	0.073	0.259	0.073
77.060	0.050	0.009	0.066	0.235	0.066
42.060	0.055	0.024	0.165	0.673	0.165
47.060	0.055	0.019	0.135	C.531	0.135
52.060	0.055	0.016	0.114	0.439	0.114
57.060	0.055	0.014	0.099	0.374	0.099
62.060	0.055	0.012	0.087	0.326	0.087
67.060	0.055	0.011	0.078	0.289	0.078
72.060	0.055	0.009	0.070	0.259	0.070
77.060	0.055	0.009	0.064	0.235	0.064
42.060	0.060	0.023	0.154	0.673	0.154
47.060	0.060	0.019	0.127	0.531	0.127
52.060	0.060	0.015	0.108	0.439	0.108
57.060	0.060	0.013	0.094	0.374	0.094
62.060	0.060	0.012	0.084	0.326	0.084
67.060	0.060	0.010	0.075	0.289	0.075
72.060	0.060	0.009	0.068	0.259	0.068
77.060	0.060	0.009	0.062	0.235	0.062
42.060	0.065	0.023	0.144	0.673	0.144
47.060	0.065	0.018	0.120	C.531	0.120
52.060	0.065	0.015	0.103	0.439	0.103
57.060	0.065	0.013	0.090	0.374	0.090
62.060	0.065	0.011	0.C80	0.326	0.080
67.060	0.065	0.010	0.072	0.289	0.072
72.060	0.065	0.009	0.066	0.259	0.066
77.060	0.065	0.008	0.060	0.235	0.060
42.060	0.070	0.022	0.135	0.673	0.135
47.060	0.070	0.018	0.114	0.531	0.114
52.060	0.070	0.015	0.098	0.439	0.098
57.060	0.070	0.013	0.C87	0.374	0.087
62.060	0.070	0.011	0.078	0.326	0.078
67.060	0.070	0.010	0.070	0.289	0.070
72.060	0.070	0.009	0.064	0.259	0.064
77.060	0.070	0.008	0.059	0.235	0.059

SAMPLE OUTPUT OF PROGRAM A2

Solution of DESIGN EXAMPLE 3 (case a)

```
ALLOWABLE  LOAD OF STUD BRACED ON ONE SIDE ONLY     (PROG. A2)
==========================================


CHANNEL  SECTION  STUD          LENGTH=144.00
-----------------

SECTION DIMENSIONS
DEPTH= 4.000  H= 3.895  B= 1.118  D= 0.000  T= 0.105
CA= 0.953

DIAPHRAGM PROPERTIES
S=  30.000    F= 0.040    GAMD= 0.01400    FED= 0.15000  XL1=
 12.0
YIELD STRESS FY=50.000
INITIAL IMPERFECTIONS CO=0.411  DO=0.206  EO=0.004

SECTION PROPERTIES
AREA= 0.644  IXX= 1.408  IYY= 0.071  IXY= 0.000
XO= 0.558  R2= 2.608  J= 0.002  CW= 0.195

ELASTIC CRITICAL B. LOAD PCR=  10.234    NWAVE= 2

 CRITICAL B. LOAD ,CONSIDER. B. BETWEEN FASTENERS,=    10.234

 LOAD CAPACITY   PR=      7.675      NWAVE= 2


   ALLOWABLE DESIGN LOAD  (( PALL )) =     3.998

   ==========================================
```

SAMPLE OUTPUT OF PROGRAM B2

Solution of DESIGN EXAMPLE 3 (case b)

```
ALLOWABLE  LOAD OF STUD BRACED ON ONE SIDE ONLY     (PROG. B2)
=========================================================


CHANNEL  SECTION  STUD           LENGTH=144.00
-----------------


GIVEN ALL.  LOAD   (PO)  =   4.400

SECTION DIMENSIONS
DEPTH= 4.000  H= 3.895  B= 1.118  D= 0.000  T= 0.105
QA= 0.953

DIAPHRAGM PROPERTIES
SLIM=  50.000     FLIM=   0.050    XL1= 12.00
STRIAL= 25.000     FTRIAL= 0.0400

YIELD STRESS FY=50.000
INITIAL IMPERFECTIONS CO=0.411  DO=0.206  EO=0.004 NU=10

SECTION PROPERTIES
AREA= 0.644  IXX= 1.408  IYY= 0.071  IXY= 0.000
XO= 0.558  R2= 2.608  J= 0.002  CW= 0.195
```

S	F	GAMAX	FEMAX	C1	E1
25.000	0.040	0.009	0.254	0.704	0.254
30.000	0.040	0.007	0.232	0.614	0.232
35.000	0.040	0.005	0.219	0.558	0.219
40.000	0.040	0.004	0.209	0.520	0.209
45.000	0.040	0.004	0.203	0.493	0.203
50.000	0.040	0.003	0.198	0.472	0.198
25.000	0.045	0.008	0.216	0.610	0.216
30.000	0.045	0.006	0.198	0.534	0.198
35.000	0.045	0.005	0.188	0.487	0.188
40.000	0.045	0.004	0.180	0.455	0.180
45.000	0.045	0.004	0.175	0.431	0.175
50.000	0.045	0.003	0.171	0.413	0.171
25.000	0.050	0.007	0.187	0.540	0.187
30.000	0.050	0.006	0.173	0.475	0.173
35.000	0.050	0.005	0.164	0.434	0.164
40.000	0.050	0.004	0.158	0.405	0.158
45.000	0.050	0.003	0.154	0.384	0.154
50.000	0.050	0.003	0.150	0.368	0.150

```
C    ************************************************
C    *                                              *
C    *              PROGRAM  'A1'                   *
C    *                                              *
C    *          STUD BRACED ON BOTH SIDES           *
C    *   FIND ALL. LOAD 'P' FOR GIVEN 'S' & 'F' VALUES  *
C    *                                              *
C    ************************************************
C
C
C
C       THE *INPUT DATA* CONSISTS OF THE FOLLOWING PER CASE :
C       (ISEC,XL,HH,H,B,D,T,QA,S,F,GAMAD,FED,FY,XL1)
C    THESE PARAMETERS ARE PUNCHED IN  2 CARDS ACCORDING TO THE
C    FORMAT STATEMENT NUMBER 500   FORMAT(I10,7F10.3,/,6F10.5)
C    THE ABOVE  MAY  BE  REPEATED FOR EACH  CASE INVOLVING
C    NEW VALUES OF THE ABOVE PARAMETERS.
C
C    TWO BLANK CARDS  'WITH  ISEC=0 '   MUST BE PROVIDED AFTER
C    THE DATA CARDS TO SIGNIFY  THE LOGICAL TERMINATION OF THE
C    PROGRAM
C
C
C                   -----------*----------
C
C    THE FOLLOWING DEFINES THE INPUT DATA AS WELL AS IMPORTANT
C    PARAMETERS USED IN THE PROGRAM.     DEFINITIONS OF OTHER
C    PARAMETERS ARE GIVEN IN THE NOMENCLATURE OF  APPENDIX # 4
C    OF THE MAIN REPORT.
C
C
C                   FOR  I-SECTION        ISEC=1
C                        CHANNEL-SEC.     ISEC=2
C                        ZEE-SECTION      ISEC=3
C                        STOP PROGRAM     ISEC=0
C
C
C    ALL DIMENSIONS , LOADS & STRESSES ARE IN THE FOLLOWING
C    UNITS EXCEPT OTHERWISE NOTED :
C    DIMENSIONS        IN    INCHES
C    LOADS             IN    KIPS
C    STRESSES          IN    KSI
C
C
C                   SECTION DIMENSIONS:
C                   XL=      STUD LENGTH
C                   QA=      SHAPE FACTOR
C                   HH=      TOTAL DEPTH OF SECTION
C                   T =      THICKNESS OF SECTION
C                   H,B,D  ARE CENTER LINE DIMENSIONS OF WEB,
C                   FLANGE & LIP
C
C
C                   DIAPHRAGM  PROPERTIES:
```

```
C                    S= RELIABLE SHEAR RIGIDITY  K
C                    F= RELIABLE ROT. RESTRAINT  K.IN/IN.RAD
C                    GAMAD & FED ARE DESIGN SHEAR STRAIN AND
C                    ROTATIONAL CAPACITY   IN RAD.
C                    XL1= DISTANCE BETWEEN FASTENERS
C
C
C                    INITIAL IMPERFECTIONS:
C                    CO= STUD LENGTH /700.
C                    DO= STUD LENGTH /700.
C                    EO= 0.0006 RAD. PER FOOT LENGTH OF STUD
C
C
C                    MATERIAL PROPERTIES OF STUD:
C                    FY = YIELD STRESS OF STEEL
C                    FLT= PROPORTIONAL LIMIT       (FLT= 0.5FY)
C                    MOD= MODULUS OF ELASTICITY  (29500. KSI)
C                    GE = SHEAR MODULUS          (11300. KSI)
C
C
C
C  J & K ARE LOGICAL RECORD UNITS OF READ & WRITE STATEMENTS
      REAL MOD
      J=5
      K=6
C
C
 800  READ(J,500)ISEC,XL,HH,H,B,D,T,QA,S,F,GAMD,FED,FY,XL1
      MOD=29500.0
      GE=11300.0
      PIE=3.14159
      WRITE(K,999)
C
C
C  INITIAL  IMPERECTIONS
C
      CO=XL/700.
      DO=XL/700.
      EO=0.0006*(XL/2.)/12.
C
C  FOR INITIAL IMPER. AND ACCEDENTAL LOAD ECCENTRICITY
C
      CO=2.*CO
C
C
C  LET PRINT=1 IF DETAILS OF COMPUTATIONS ARE NEEDED
C  LET PRINT=0 IF DETAILS OF COMPUTATIONS ARE NOT NEEDED
C
      PRINT=0
C
      IF(ISEC-0) 802,802,801
 801  GO TO(771,772,773),ISEC
 771  WRITE(K,774)XL
      WRITE(K,764)
```

```
      GO TO 807
 772  WRITE(K,775)XL
      WRITE(K,765)
      GO TO 807
 773  WRITE(K,776)XL
      WRITE(K,766)
 807  WRITE(K,502)HH,H,B,D,T,QA
      WRITE(K,503)S,F,GAMD,FED
      WRITE(K,504)FY,CO,DO,EO
      GO TO(101,201,301),ISEC
C
C
C  CALCULATION OF SECTION PROPERTIES     I-SECTION
C
 101  AREA=2.*T*(H+B+2.*D)
      XXI=T*(H**3+3.*B*H**2+6.*D*(H-D)**2+2.*D**3)/6.
      YYI=B**2*T*(B+6.*D)/6.
      XYI=0.0
      XO=0.0
      XJ=2.*T**3*(B+H+2.*D)/3.
      CW=B**2*T*(B*H**2+6.*D*H**2+12.*H*D**2+8.*D**3)/24.
      PI=XXI+YYI
      R2=PI/AREA
      XI1=YYI
      XI2=XXI
      WRITE(K,600)AREA,XXI,YYI,XYI,XO,R2,XJ,CW
C
C  CALCULATION OF ELASTIC BUCKLING LOADS   I-SECTION
C
      CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,PXX,PY
     1Y,PFE,PXY,PX1,PX2)
      WRITE(K,601)MOD,GE,PXX,PYY,PFE,PXY
      P1=PYY+S
      P2=PXX
      IF(P1-P2) 111,111,112
 111  PCR=P1
      GO TO 42
 112  PCR=P2
 42   CONTINUE
C
C  CHECK POSSIBILITY OF BUCKLING BETWEEN FASTENERS
C  DISTANCE BETWEEN FASTENERS  = XL1
C
      PCF=(PIE**2)*MOD*YYI/(XL1**2)
      IF(PRINT-1) 950,951,951
 951  WRITE(K,602)P1,P2,PCF
  950 CONTINUE
      IF(PCR-PCF) 211,211,212
 212  PCR=PCF
 211  WRITE(K,603)PCR
      GO TO 44
C
C
```

```
C   CALCULATION OF SECTION PROPERTIES     CHANNEL SECTION
C
 201   AREA=T*(H+2.0*B+2.Q*D)
       XBAR=T*(B**2+2.0*D*B)/AREA
       XXI=T*(H**3+6.0*B*H**2+6.0*D*(H-D)**2+2.0*D**3)/12.0
       YYI=T*B**2*(2.0*H*B+B**2+2.0*D*(2.0*B+3.0*H))/(3.0*(H+
      C  2.0*B+2.0*D)
      1)
       XYI=0.0
       XMBAR=(B*H)**2*T*(1.0+2.0*D/B-8.0*D**3/(3.0*B*H**2))/(
      C  4.0*XXI)
       XO=XMBAR+XBAR
       PI=XXI+YYI+AREA*XO**2
       R2=PI/AREA
       XJ=T**3*(H+2.0*B+2.0*D)/3.0
       CW=(B*H*T)**2*(2.0*B*H**3+3.0*(B*H)**2+6.0*D*(H+2.0*B)
      C  *H**2+12.0*D
      1**2*(H+4.0*B)+8.0*D**3*(H+14.0*B)+48.0*D**4)/(144.0*XX
      C  I)
       XI1=YYI
       XI2=XXI
       WRITE(K,600)AREA,XXI,YYI,XYI,XO,R2,XJ,CW
C
C   CALCULATION OF ELASTIC BUCKLING LOAD    CHANNEL SECTION
C
       CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
      C  1,XI2,PXX,PY
      1Y,PFE,PXY,PX1,PX2)
       WRITE(K,601)MOD,GE,PXX,PYY,PFE,PXY
       A1=R2-XO**2
       A2=-(R2*(PFE+PXX)+S*(HH/2.)**2+F*(XL/PIE)**2)
       A3=PXX*(R2*PFE+S*(HH/2.)**2+F*(XL/PIE)**2)
       P1=(-A2+SQRT(A2**2-4.*A1*A3))/(2.*A1)
       P2=(-A2-SQRT(A2**2-4.*A1*A3))/(2.*A1)
       P3=PYY+S
C
C   CHECK POSSIBILITY OF BUCKLING BETWEEN FASTENERS
C   DISTANCE BETWEEN FASTENERS  = XL1
C
       PYYF=(PIE**2)*MOD*YYI/(XL1**2)
       PFEF=(GE*XJ+(PIE**2)*MOD*CW/XL1**2)/R2
       PXXF=(PIE**2)*MOD*XXI/(XL1**2)
       P4=PYYF
       P5=PFEF
       P6=PXXF
       IF(PRINT-1) 952,953,953
 953   WRITE(K,604)P1,P2,P3,P4,P5,P6
  952 CONTINUE
       PCR=AMIN1(P1,P2,P3,P4,P5,P6)
       WRITE(K,603)PCR
       GO TO 44
C
C
C   CALCULATION OF SECTION PROPERTIES     Z-SECTION
```

```
C
 301   AREA=T*(H+2.0*B+2.0*D)
       XXI=T*(H**3+6.0*B*H**2+6.0*D*(H-D)**2+2.0*D**3)/12.0
       YYI=2.0*B**2*T*(B+3.0*D)/3.0
       XYI=B*T*(B*H+D*(H-D))/2.0
       XO=0.0
       XJ=T**3*(2.0*B+H+2.0*D)/3.0
       CW=(B*T)**2*(2.0*H**3*B+(H*B)**2+2.0*D*H**2*(3.0*H+2.0
      C  *B)+12.0*D**
      12*H*(H+B)+8.0*D**3*(H+2.0*B)+D**4)/(12.0*AREA)
       PI=XXI+YYI
       R2=PI/AREA
       XI1=((XXI+YYI)/2.)-SQRT(((XXI-YYI)/2.)**2+XYI**2)
       XI2=((XXI+YYI)/2.)+SQRT(((XXI-YYI)/2.)**2+XYI**2)
       WRITE(K,666)AREA,XXI,YYI,XYI,XO,R2,XJ,CW,XI1,XI2
C
C  CALCULATION OF ELASTIC BUCKLING LOADS   Z-SECTION
C
       CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
      C  1,XI2,PXX,PY
      1Y,PFE,PXY,PX1,PX2)
       WRITE(K,606)MOD,GE,PXX,PYY,PFE,PXY,PX1,PX2
       G1=-(PXX+PYY+S)
       G2=-PXY**2+PXX*PYY+PXX*S
       P1=(-G1+SQRT(G1**2-4.*G2))/2.
       P2=(-G1-SQRT(G1**2-4.*G2))/2.
       IF(P1-P2) 113,113,114
 113   PCR=P1
       GO TO 43
 114   PCR=P2
 43    CONTINUE
C
C  CHECK POSSIBILITY OF BUCKLING BETWEEN FASTENERS
C  DISTANCE BETWEEN FASTENERS  = XL1
C
       PCF=(PIE**2)*MOD*XI1/(XL1**2)
       IF(PRINT-1) 954,955,955
 955   WRITE(K,602)P1,P2,PCF
  954 CONTINUE
       IF(PCR-PCF)311,311,312
 312   PCR=PCF
 311   WRITE(K,603)PCR
C
C
C  CALCULATICN OF INELASTIC BUCKLING LOAD
C
 44    FCR=PCR/AREA
       FY=FY*QA
       FLT=.5*FY
       IF(PRINT-1) 956,957,957
 957   WRITE(K,61)FCR,FLT
  956 CONTINUE
       IF(FCR-FLT) 20,20,40
 40    PA=AREA*(FY-FY**2/(4.*FCR))
```

```
      IF(PRINT-1) 958,959,959
 959  WRITE(6,62)PA
  958 CONTINUE
      PCR=PA
C
C
C  LOAD CAPACITY OF STUD
C
 20   XLAM=0.99999
 5    PR=XLAM*PCR
      FR=PR/AREA
      IF(PRINT-1) 960,961,961
 961  WRITE(K,63)XLAM,PR,FR
  960 CONTINUE
      IF(FR-FLT) 90,90,91
 90   MOD=29500.0
      GE=11300.0
      GO TO 92
 91   TMOD=29500.*(FR*(FY-FR)/(FLT*(FY-FLT)))
      MOD=TMOD
      GI=11300.*TMOD/29500.0
      GE=GI
      IF(PRINT-1) 962,963,963
 963  WRITE(K,64)TMOD,GI
  962 CONTINUE
 92   CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,PXX,PY
     1Y,PFE,PXY,PX1,PX2)
      IF(PRINT-1) 964,965,965
 965  WRITE(K,601)MOD,GE,PXX,PYY,PFE,PXY
  964 CONTINUE
C
C
C  CHECKING THE DIAPHRAGM ADEQUACY
C
      GO TO(103,203,303),ISEC
C
C
C  CHECK   'GAMAD'  &  'FED'  REQUIREMENTS            I-SEC
C
 103  C1=CO*PR/(PYY+S-PR)
      GAMAX=C1*PIE/XL
      FEMAX=.0
      IF(PRINT-1) 966,967,967
 967  WRITE(K,66)C1,GAMAX,GAMD,FEMAX,FED
  966 CONTINUE
      GO TO 22
C
C
C  CHECK   'GAMAD'  &  'FED'  REQUIREMENTS        CHANNEL-SEC
C
 203  A4=PYY-PR+S
      A5=PXX-PR
      A6=PR*XO
```

```
      A7=R2*(PFE-PR)+S*(HH/2.)**2+F*(XL/PIE)**2
      DET=A4*(A5*A7-A6**2)
      C1=PR*(CO*(A5*A7-A6**2))/DET
      E1=PR*(-A4*A6*(DO-XO*EO)+A4*A5*(R2*EO-XO*DO))/DET
      E1=ABS(E1)
      GAMAX=PIE*(C1+E1*HH/2.)/XL
      FEMAX=E1
      IF(PRINT-1) 968,969,969
 969  WRITE(K,67)C1,E1,GAMAX,GAMD,FEMAX,FED
  968 CONTINUE
      GO TO 22
C
C
C  CHECK   'GAMAD'  &  'FED'  REQUIREMENTS                  Z-SEC
C
 303  G3=PYY-PR+S
      G4=PXY
      G5=PXX-PR
      G6=(PFE-PR)*R2+S*(HH/2.)**2+F*(XL/PIE)**2
      DET=G3*G5*G6-G4**2*G6
      C1=PR*(CO*G5*G6-DO*G4*G6)/DET
      C1=ABS(C1)
      E1=PR*(EO*R2*(G3*G5-G4**2))/DET
      E1=ABS(E1)
      GAMAX=PIE*(C1+E1*HH/2.)/XL
      FEMAX=E1
      IF(PRINT-1) 978,979,979
 979  WRITE(K,67)C1,E1,GAMAX,GAMD,FEMAX,FED
 978  CONTINUE
 22   IF(GAMD-GAMAX) 10,11,11
 10   XLAM=XLAM-.01
      GO TO 5
 11   IF(FED-FEMAX) 12,14,14
 12   GO TO 10
C
C
C    ALLOWABLE   LOAD  OF  STUD
C    ---------------------------
C
 14   P=PR/1.92
      WRITE(K,80)P
C
C
C
 999  FORMAT('1',4X,'ALLOWABLE  LOAD OF STUD BRACED ON BOTH
     C  SIDES
     1(PROG. A1)',/,5X,44('='),////)
 500  FORMAT(I10,7F10.3,/,6F10.5)
 502  FORMAT(' ',1X,'SECTION DIMENSIONS',/,2X,'DEPTH=',F6.3,
     C  2X,'H=',F6.3
     1,2X,'B=',F6.3,2X,'D=',F6.3,2X,'T=',F6.3,/,2X,'QA=',F6.
     C  3,/)
 503  FORMAT(' ',1X,'DIAPHRAGM PROPERTIES',/,2X,'S=',F8.3,4X
     C  ,'F=',F6.3,4
```

```
     1X,'GAMD=',F8.5,4X,'FED=',F8.5,/)
504   FORMAT(' ',1X,'YIELD STRESS FY=',F6.3,/,2X,'INITIAL IM
     C  PERFECTIONS
     1CO=',F5.3,2X,'DO=',F5.3,2X,'EQ=',F5.3,/)
61    FORMAT(' ',1X,'FCR=',F8.3,5X,'FLT=',F8.3)
62    FORMAT(' ',1X,'PA=',F8.3,//)
63    FORMAT(' ',1X,'XLAM=',F5.3,5X,'PR=',F8.3,5X,'FR=',F10.
     C  3)
64    FORMAT(' ',1X,'TMOD=',F12.3,8X,'GI=',F12.3,/)
66    FORMAT(' ',1X,'C1=',F10.5,5X,'GAMAX=',F10.5,5X,'GAMD='
     C  ,F10.5,5X,'F
     1EMAX=',F10.5,5X,'GAMD=',F10.5)
67    FORMAT(' ',1X,'C1=',F10.5,5X,'E1=',F8.5,5X,'GAMAX=',F1
     C  0.5,5X,'GAMD
     1=',F10.5,5X,'FEMAX=',F10.5,5X,'FED =',F10.5,//)
600   FORMAT(' ',1X,'SECTION PROPERTIES',/,2X,'AREA=',F6.3,2
     C  X,'IXX=',F6.
     13,2X,'IYY=',F6.3,2X,'IXY=',F6.3,/,2X,'XO=',F6.3,2X,'R2
     C  =',F6.3,2X,'
     2J=',F6.3,2X,'CW=',F6.3,/)
601   FORMAT(' ',1X,'MOD=',F8.1,2X,'GE=',F8.1,/,2X,'PXX=',F7
     C  .3,2X,'PYY='
     1,F7.3,2X,'PFE=',F7.3,2X,'PXY=',F7.3,//)
602   FORMAT(' ',1X,'P1=',F8.3,2X,'P2=',F8.3,2X,'PCF=',F8.3)
603   FORMAT(' ',1X,'ELASTIC CRITAL B. LOAD=',F8.3,//)
604   FORMAT(' ',1X,'P1=',F8.3,2X,'P2=',F8.3,2X,'P3=',F8.3,2
     C  X,'P4=',F8.3
     1,2X,'P5=',F8.3,2X,'P6=',F8.3)
606   FORMAT(' ',1X,'MOD=',F8.1,2X,'GE=',F8.1,2X,'PXX=',F7.3
     C  ,2X,'PYY=',F
     17.3,/,2X,'PFE=',F7.3,2X,'PXY=',F7.3,2X,'PX1=',F7.3,2X,
     C  'PX2=',F7.3,
     2//)
666   FORMAT(' ',1X,'SECTION PROPERTIES',/,2X,'AREA=',F6.3,2
     C  X,'IXX=',F6.
     13,2X,'IYY=',F6.3,2X,'IXY=',F6.3,2X,'XO=',F6.3,/,2X,'R2
     C  =',F6.3,2X,'
     2J=',F6.3,2X,'CW=',F6.3,2X,'IX1=',F6.3,2X,'IX2=',F6.3,/
     C  )
774   FORMAT(' ',1X,'I - SECTION  STUD                LENGTH=',F
     C  6.2)
764   FORMAT(' ',1X,24('_'),//)
775   FORMAT(' ',1X,'CHANNEL  SECTION  STUD            LENGTH='
     C  ,F6.2)
765   FORMAT(' ',1X,18('_'),//)
776   FORMAT(' ',1X,'ZEE - SECTION STUD                LENGTH='
     C  ,F6.2)
766   FORMAT(' ',1X,17('_'),//)
80    FORMAT(' ',12X,'ALLOWABLE   LOAD   P=',F8.3,/,12X,31('
     C  ='),/)
      GO TO 800
802   STOP
      END
      SUBROUTINE PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE
```

```
C    ,XL,XI1,XI2,
     1PXX,PYY,PFE,PXY,PX1,PX2)
      REAL MOD
      PXX=(PIE**2)*MOD*XXI/(XL**2)
      PYY=(PIE**2)*MOD*YYI/(XL**2)
      PFE=(GE*XJ+(PIE**2)*MOD*CW/XL**2)/R2
      PXY=(PIE**2)*MOD*XYI/(XL**2)
      PX1=(PIE**2)*MOD*XI1/(XL**2)
      PX2=(PIE**2)*MOD*XI2/(XL**2)
      RETURN
      END
*DATA
```

```
C    ************************************************
C    *                                              *
C    *               PROGRAM  'A2'                  *
C    *                                              *
C    *        STUD BRACED ON  ONE  SIDE ONLY        *
C    *  FIND ALL. LOAD 'P' FOR GIVEN 'S' & 'F' VALUES  *
C    *                                              *
C    ************************************************
C
C
C
C       THE *INPUT DATA* CONSISTS OF THE FOLLOWING PER CASE :
C       (ISEC,XL,HH,H,B,D,T,QA,S,F,GAMAD,FED,FY,XL1)
C    THESE PARAMETERS ARE PUNCHED IN  2 CARDS ACCORDING TO THE
C    FORMAT STATEMENT NUMBER 500   FORMAT(I10,7F10.3,/,6F10.5)
C    THE ABOVE  MAY  BE  REPEATED FOR EACH  CASE INVOLVING
C    NEW VALUES OF THE ABOVE PARAMETERS.
C
C    TWO BLANK CARDS  'WITH  ISEC=0 '   MUST BE PROVIDED AFTER
C    THE DATA CARDS TO SIGNIFY  THE LOGICAL TERMINATION OF THE
C    PROGRAM
C
C
C                    ----------*----------
C
C    THE FOLLOWING DEFINES THE INPUT DATA AS WELL AS IMPORTANT
C    PARAMETERS USED IN THE PROGRAM.    DEFINITIONS OF OTHER
C    PARAMETERS ARE GIVEN IN THE NOMENCLATURE OF  APPENDIX # 4
C    OF THE MAIN REPORT.
C
C
C                    FOR  I-SECTION       ISEC=1
C                         CHANNEL-SEC.    ISEC=2
C                         ZEE-SECTION     ISEC=3
C                         STOP PROGRAM    ISEC=0
C
C
C    ALL DIMENSIONS , LOADS & STRESSES ARE IN THE FOLLOWING
C    UNITS EXCEPT OTHERWISE NOTED :
C    DIMENSIONS         IN    INCHES
C    LOADS              IN    KIPS
C    STRESSES           IN    KSI
C
C
C                    SECTION DIMENSIONS:
C                    XL=      STUD LENGTH
C                    QA=      SHAPE FACTOR
C                    HH=      TOTAL DEPTH OF SECTION
C                    T =      THICKNESS OF SECTION
C                    H,B,D  ARE CENTER LINE DIMENSIONS OF WEB,
C                    FLANGE & LIP
C
C
C                    DIAPHRAGM  PROPERTIES:
```

```
C                    S= RELIABLE SHEAR RIGIDITY  K
C                    F= RELIABLE ROT. RESTRAINT  K.IN/INHRAD
C                    GAMAD & FED ARE DESIGN SHEAR STRAIN AND
C                    ROTATIONAL CAPACITY   IN RAD.
C                    XL1= DISTANCE BETWEEN FASTENERS
C
C
C                    INITIAL IMPERFECTIONS:
C                    CO= STUD LENGTH /700.
C                    DO= STUD LENGTH /700.
C                    EO= 0.0006 RAD. PER FOOT LENGTH OF STUD
C
C
C                    MATERIAL PROPERTIES OF STUD:
C                    FY = YIELD STRESS OF STEEL
C                    FLT= PROPORTIONAL LIMIT      (FLT= 0.5FY)
C                    MOD= MODULUS OF ELASTICITY  (29500. KSI)
C                    GE = SHEAR MODULUS           (11300. KSI)
C
C
C             HIGHER BUCKLING MODES ARE EXAMINED BY CONSIDERING
C  SUFFICIENT NUMBERS OF 'NU'.IN THIS PROGRAM NU=1,2,.....,10
C  IF MORE VALUES ARE DESIRED,THEN CHANGE PRESENT 'NU' VALUE
C
C
C  J & K ARE LOGICAL RECORD UNITS OF READ & WRITE STATEMENTS
C
C
      REAL*8 C,Q,E,POL
      DIMENSION P(3)
      DIMENSION C(4),Q(4),E(4),POL(4)
      DIMENSION PC(40)
      DIMENSICN PRMIN(10)
      REAL MOD
      J=5
      K=6
C
C
 800  READ(J,500)ISEC,XL,HH,H,B,D,T,QA,S,F,GAMD,FED,FY,XL1
      MOD=29500.0
      GE=11300.0
      PIE=3.14159
C
C   N U M B E R   OF  BUCKLING  MODES  TO  BE  EXAMINED 'NU'
C
      NU=10
C
C
      WRITE(K,999)
C
C
C  INITIAL  IMPERECTIONS
C
      CO=XL/700.
```

```
      DO=XL/700.
      EO=0.0006*(XL/2.)/12.
C
C  FOR INITIAL IMPER. AND ACCEDENTAL LOAD ECCENTRICITY
C
      CO=2.*CO
C
C
C  LET PRINT=1 IF DETAILS OF COMPUTATIONS ARE NEEDED
C  LET PRINT=0 IF DETAILS OF COMPUTATIONS ARE NOT NEEDED
C
      PRINT=1
C
      IF(ISEC-0) 802,802,801
 801  GO TO(771,772,773),ISEC
 771  WRITE(K,774)XL
      WRITE(K,764)
      GO TO 807
 772  WRITE(K,775)XL
      WRITE(K,765)
      GO TO 807
 773  WRITE(K,776)XL
      WRITE(K,766)
 807  WRITE(K,502)HH,H,B,D,T,QA
      WRITE(K,503)S,F,GAMD,FED,XL1
      WRITE(K,504)FY,CO,DO,EO
      GO TO(101,201,301),ISEC
C
C
C  CALCULATION OF SECTION PROPERTIES      I-SECTION
C
 101  AREA=2.*T*(H+B+2.*D)
      XXI=T*(H**3+3.*B*H**2+6.*D*(H-D)**2+2.*D**3)/6.
      YYI=B**2*T*(B+6.*D)/6.
      XYI=0.0
      XO=0.0
      XJ=2.*T**3*(B+H+2.*D)/3.
      CW=B**2*T*(B*H**2+6.*D*H**2+12.*H*D**2+8.*D**3)/24.
      PI=XXI+YYI
      R2=PI/AREA
      XI1=YYI
      XI2=XXI
      WRITE(K,600)AREA,XXI,YYI,XYI,XO,R2,XJ,CW
C
C  CALCULATION OF ELASTIC BUCKLING LOAD     I-SECTION
C
      IF(PRINT-1) 420,421,421
421   WRITE(K,652)
      WRITE(K,653)
 420  CONTINUE
      DO 50 I=1,NU,1
      AN=(I*1.)**2
      CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,AN,PXX
```

```
      1,PYY,PFE,PXY,PX1,PX2)
       G1=-(PFE+PYY+S+(S*(HH/2.)**2+F*(1./AN)*(XL/PIE)**2)/R)
       G2=(PYY+S)*(PFE+(S*(HH/2.)**2+F*(1./AN)*(XL/PIE)**2)/R
      C  2)-(S*(HH/2.
      1)**2)/R2
       P1=(-G1+SQRT(G1**2-4.*G2))/2.
       P2=(-G1-SQRT(G1**2-4.*G2))/2.
       P3=(PIE**2)*MOD*XXI/(XL**2)
       PC(I)=AMIN1(P1,P2,P3)
       IF(PRINT-1) 422,423,423
423    WRITE(K,601)MOD,GE,PXX,PYY,PFE,PXY,P1,P2,P3,PC(I),I
 422   CONTINUE
 50    CONTINUE
C
C
C              TESTING FO THE CRITICAL BUCKLING MODE AMONG THE
C   'NU' MODES CONSIDERED  AND THE CORRESPONDING HALF-SINE
C   WAVE  (VALUE OF NU)
C
       PTEST2=PC(1)
       AN=1.0
       NWAVE=1
       PCR=PTEST2
       DO 51 I=2,NU
       IF(PC(I)-PTEST2) 52,52,51
 52    PTEST2=PC(I)
       AN=(I*1.)**2
       NWAVE=I
       PCR=PTEST2
 51    CONTINUE
       WRITE(K,603)PCR,NWAVE
C
C   CHECK POSSIBILITY OF BUCKLING BETWEEN FASTENERS
C   DISTANCE BETWEEN FASTENERS   = XL1
C
       PCF=(PIE**2)*MOD*YYI/(XL1**2)
       IF(PRINT-1) 950,951,951
 951   WRITE(K,808)PCR,PCF
  950  CONTINUE
       IF(PCR-PCF) 211,211,212
 212   PCR=PCF
 211   WRITE(K,803)PCR
C
       GO TO 44
C
C
C   CALCULATION OF SECTION PROPERTIES     CHANNEL SECTION
C
 201   AREA=T*(H+2.0*B+2.0*D)
       XBAR=T*(B**2+2.0*D*B)/AREA
       XXI=T*(H**3+6.0*B*H**2+6.0*D*(H-D)**2+2.0*D**3)/12.0
       YYI=T*B**2*(2.0*H*B+B**2+2.0*D*(2.0*B+3.0*H))/(3.0*(H+
      C  2.0*B+2.0*D)
      1)
```

```
      XYI=0.0
      XMBAR=(B*H)**2*T*(1.0+2.0*D/B-8.0*D**3/(3.0*B*H**2))/(
     C  4.0*XXI)
      XO=XMBAR+XBAR
      PI=XXI+YYI+AREA*XO**2
      R2=PI/AREA
      XJ=T**3*(H+2.0*B+2.0*D)/3.0
      CW=(B*H*T)**2*(2.0*B*H**3+3.0*(B*H)**2+6.0*D*(H+2.0*B)
     C  *H**2+12.0*D
     1**2*(H+4.0*B)+8.0*D**3*(H+14.0*B)+48.0*D**4)/(144.0*XX
     C  I)
      XI1=YYI
      XI2=XXI
      WRITE(K,600)AREA,XXI,YYI,XYI,XO,R2,XJ,CW
C
C  CALCULATION OF ELASTIC BUCKLING LOAD      CHANNEL-SECTION
C
      IF(PRINT-1) 424,425,425
 425  WRITE(K,652)
      WRITE(K,653)
 424  CONTINUE
      DO 71 I=1,NU,1
      AN=(I*1.)**2
      CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,AN,PXX
     1,PYY,PFE,PXY,PX1,PX2)
      FO=-(PYY+S)*(PXX*(R2*PFE+S*(HH/2.)**2+F*(1./AN)*(XL/PI
     C  E)**2))+PXX*
     1(S*HH/2)**2
      F1=(PXX*(R2*PFE+S*(HH/2.)**2+F*(1./AN)*(XL/PIE)**2)+(P
     C  YY+S)*(R2*(P
     1FE+PXX)+S*(HH/2.)**2+F*(1./AN)*(XL/PIE)**2)-(S*HH/2.)*
     C  *2)
      F2=-(R2*(PFE+PXX)+S*(HH/2.)**2+F*(1./AN)*(XL/PIE)**2+(
     C  R2-XO**2)*(P
     1YY+S))
      F3=R2-XO**2
      C(1)=FO
      C(2)=F1
      C(3)=F2
      C(4)=F3
      IC=4
      IR=3
      CALL DPRQD(C,IC,Q,E,POL,IR,IER)
      V1=Q(1)
      V2=Q(2)
      V3=Q(3)
      W1=E(1)
      W2=E(2)
      W3=E(3)
      IF(W1-0.0) 6,5,6
 6    V1=0.0
 5    P1=V1
      IF(W2-0.0) 10,11,10
```

```
 10    V2=0.0
 11    P2=V2
       IF(W3-0.0) 14,12,14
 14    V3=0.0
 12    P3=V3
       P(1)=P1
       P(2)=P2
       P(3)=P3
       PTEST1=10000000.0
       DO 25 N=1,3
       IF(P(N)-0.0) 25,25,24
 24    IF(P(N)-PTEST1) 23,23,25
 23    PTEST1=P(N)
 25    CONTINUE
       PC(I)=PTEST1
       IF(PRINT-1) 426,427,427
427    WRITE(K,601)MOD,GE,PXX,PYY,PFE,PXY,P1,P2,P3,PC(I),I
 426   CONTINUE
 71    CONTINUE
C
C              TESTING FO THE CRITICAL BUCKLING MODE AMONG THE
C   'NU' MODES CONSIDERED  AND THE CORRESPONDING HALF-SINE
C   WAVE  (VALUE OF NU)
C
       PTEST2=PC(1)
       AN=1.0
       NWAVE=1
       PCR=PTEST2
       DO 72 I=2,NU
       IF(PC(I)-PTEST2) 73,73,72
 73    PTEST2=PC(I)
       AN=(I*1.)**2
       NWAVE=I
       PCR=PTEST2
 72    CONTINUE
       WRITE(K,603)PCR,NWAVE
C
C  CHECK POSSIBILITY OF BUCKLING BETWEEN FASTENERS
C  DISTANCE BETWEEN FASTENERS  = XL1
C
       PYYF=(PIE**2)*MOD*YYI/(XL1**2)
       PFEF=(GE*XJ+(PIE**2)*MOD*CW/XL1**2)/R2
       PXXF=(PIE**2)*MOD*XXI/(XL**2)
       P4=PYYF
       P5=PFEF
       P6=PXXF
       IF(PRINT-1) 952,953,953
 953   WRITE(K,804)PCR,P4,P5,P6
  952 CONTINUE
       PCR=AMIN1(PCR,P4,P5,P6)
       WRITE(K,803)PCR
       GO TO 44
C
C  CALCULATION OF SECTION PROPERTIES      Z-SECTION
```

```
C
 301   AREA=T*(H+2.0*B+2.0*D)
       XXI=T*(H**3+6.0*B*H**2+6.0*D*(H-D)**2+2.0*D**3)/12.0
       YYI=2.0*B**2*T*(B+3.0*D)/3.0
       XYI=B*T*(B*H+D*(H-D))/2.0
       XO=0.0
       XJ=T**3*(2.0*B+H+2.0*D)/3.0
       CW=(B*T)**2*(2.0*H**3*B+(H*B)**2+2.0*D*H**2*(3.0*H+2.0
     C   *B)+12.0*D**
     12*H*(H+B)+8.0*D**3*(H+2.0*B)+D**4)/(12.0*AREA)
       PI=XXI+YYI
       R2=PI/AREA
       XI1=((XXI+YYI)/2.)-SQRT(((XXI-YYI)/2.)**2+XYI**2)
       XI2=((XXI+YYI)/2.)+SQRT(((XXI-YYI)/2.)**2+XYI**2)
       WRITE(K,666)AREA,XXI,YYI,XYI,XO,R2,XJ,CW,XI1,XI2
C
C   CALCULATION OF ELASTIC  BUCKLING LOAD    Z-SECTION
C
       IF(PRINT-1) 428,429,429
429    WRITE(K,652)
       WRITE(K,653)
 428   CONTINUE
       DO 74 I=1,NU,1
       AN=(I*1.)**2
       CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C   1,XI2,AN,PXX
     1,PYY,PFE,PXY,PX1,PX2)
       B1=PFE+PYY+PXX+S
       B2=(((HH/2.)**2)*S+F*(1./AN)*(XL/PIE)**2)/R2
       B3=(PYY+S)*PXX-PXY**2
       B4=(PYY+PXX+S)*PFE
       B5=PYY+PXX+S
       B6=((S*HH/2.)**2)/R2
       FO=-B3*PFE-B3*B2+B6*PXX
       F1=B3+B4+B5*B2-B6
       F2=-B1-B2
       F3=1.0
       C(1)=FO
       C(2)=F1
       C(3)=F2
       C(4)=F3
       IC=4
       IR=3
       CALL DPRQD(C,IC,Q,E,POL,IR,IER)
       V1=Q(1)
       V2=Q(2)
       V3=Q(3)
       W1=E(1)
       W2=E(2)
       W3=E(3)
       IF(W1-0.0) 8,7,8
 8     V1=0.0
 7     P1=V1
       IF(W2-0.0) 15,16,15
```

```
 15    V2=0.0
 16    P2=V2
       IF(W3-0.0) 18,17,18
 18    V3=0.0
 17    P3=V3
       P(1)=P1
       P(2)=P2
       P(3)=P3
       PTEST1=10000000.0
       DO 75 N=1,3
       IF(P(N)-0.0) 75,75,76
 76    IF(P(N)-PTEST1) 77,77,75
 77    PTEST1=P(N)
 75    CONTINUE
       PC(I)=PTEST1
       IF(PRINT-1) 430,431,431
431    WRITE(K,601)MOD,GE,PXX,PYY,PFE,PXY,P1,P2,P3,PC(I),I
 430   CONTINUE
 74    CONTINUE
C
C              TESTING FO THE CRITICAL BUCKLING MODE AMONG THE
C   'NU' MODES CONSIDERED  AND THE CORRESPONDING HALF-SINE
C   WAVE  (VALUE OF NU)
C
       PTEST2=PC(1)
       AN=1.0
       NWAVE=1
       PCR=PTEST2
       DO 78 I=2,NU
       IF(PC(I)-PTEST2) 79,79,78
 79    PTEST2=PC(I)
       AN=(I*1.)**2
       NWAVE=I
       PCR=PTEST2
 78    CONTINUE
       WRITE(K,603)PCR,NWAVE
C
C  CHECK POSSIBILITY OF BUCKLING BETWEEN FASTENERS
C  DISTANCE BETWEEN FASTENERS  = XL1
C
       PCF=(PIE**2)*MOD*XI1/(XL1**2)
       IF(PRINT-1) 954,955,955
 955   WRITE(K,808)PCR,PCF
  954  CONTINUE
       IF(PCR-PCF)311,311,312
 312   PCR=PCF
 311   WRITE(K,803)PCR
       GO TO 44
C
C  CALCULATION OF INELASTIC BUCKLING LOAD
C
 44    FCR=PCR/AREA
       FY=FY*QA
       FLT=.5*FY
```

```
      IF(PRINT-1) 432,433,433
 433  WRITE(K,61)FCR,FLT
 432  CONTINUE
      IF(FCR-FLT) 20,20,40
 40   PA=AREA*(FY-FY**2/(4.*FCR))
      IF(PRINT-1) 434,435,435
 435  WRITE(6,62)PA
 434  CONTINUE
      PCR=PA
C
C  LOAD CAPACITY OF STUD
C
 20   DO 111 I=1,NU
      AN=(I*1.)**2
      NWAVE=I
      XLAM=0.99999
 42   PRMIN(I)=XLAM*PCR
      PR=PRMIN(I)
      FR=PR/AREA
      IF(PRINT-1) 436,437,437
 437  WRITE(K,63)XLAM,PR,FR
 436  CONTINUE
      IF(FR-FLT) 90,90,91
 90   MOD=29500.0
      GE=11300.0
      GO TO 92
 91   TMOD=29500.*(FR*(FY-FR)/(FLT*(FY-FLT)))
      MOD=TMOD
      GI=11300.*TMOD/29500.0
      GE=GI
 92   CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,AN,PXX
     1,PYY,PFE,PXY,PX1,PX2)
      IF(PRINT-1) 438,439,439
 439  WRITE(K,602)MOD,GE,PXX,PYY,PFE,PXY
 438  CONTINUE
C
C
C  CHECKING THE DIAPHRAGM ADEQUACY
C
      W=NWAVE*1.0
      GO TO(103,203,303),ISEC
C
C  CHECK   'GAMAD'  &  'FED'  REQUIREMENTS             I-SEC
C
 103  A1=PYY-PR+S
      A2=-S*HH/2.
      A5=R2*(PFE-PR)+S*(HH/2.)**2+(1./AN)*F*(XL/PIE)**2
      C1=PR*(A5*CO/W-R2*(EO/W)*A2)/(A1*A5-A2**2)
      E1=PR*(A2*CO/W-R2*(EO/W)*A1)/(A2**2-A1*A5)
      GAMAX=PIE*W*(C1-E1*HH/2.)/XL
      GAMAX=ABS(GAMAX)
      FEMAX=ABS(E1)
      IF(PRINT-1) 440,441,441
```

```
 441   WRITE(K,67)C1,E1,GAMAX,GAMD,FEMAX,FED,NWAVE
 440   CONTINUE
       GO TO 22
C
C  CHECK   'GAMAD'  &  'FED'  REQUIREMENTS        CHANNEL-SEC
C
 203   F4=PYY-PR+S
       F5=-S*HH/2.
       F6=PXX-PR
       F7=PR*XO
       F8=R2*(PFE-PR)+S*(HH/2.)**2+(1./AN)*F*(XL/PIE)**2
       DET=F4*(F6*F8-F7**2)-F6*F5**2
       C1=PR*(CO/W*(F6*F8-F7**2)+F7*F5*(DO/W-XO*EO/W)-F6*F5*(
      C  R2*EO/W-XO*D
      1O/W))/DET
       E1=PR*(-CO/W*F5*F6-F4*F7*(DO/W-XO*EO/W)+F4*F6*(R2*EO/W
      C  -XO*DO/W))/D
      1ET
       GAMAX=PIE*W*(C1-E1*HH/2.)/XL
       GAMAX=ABS(GAMAX)
       FEMAX=ABS(E1)
       IF(PRINT-1) 442,443,443
 443   WRITE(K,67)C1,E1,GAMAX,GAMD,FEMAX,FED,NWAVE
 442   CONTINUE
       GO TO 22
C
C  CHECK   'GAMAD'  &  'FED'  REQUIREMENTS              Z-SEC
C
 303   F1=PYY-PR+S
       F2=PXY
       F3=-S*HH/2.
       F4=PXX-PR
       F5=(PFE-PR)*R2+S*(HH**2)/4.+(1./AN)*F*(XL/PIE)**2
       DET=F1*F4*F5-(F2**2)*F5-(F3**2)*F4
       C1=PR*(CO*F4*F5/W-DO*F2*F5/W-EO*F3*F4*R2/W)/DET
       E1=PR*(-CO*F3*F4/W+DO*F3*F2/W+EO*R2*(F1*F4-F2**2)/W)/D
      C  ET
       GAMAX=PIE*W*(C1-E1*HH/2.)/XL
       GAMAX=ABS(GAMAX)
       FEMAX=ABS(E1)
       IF(PRINT-1) 444,445,445
 445   WRITE(K,67)C1,E1,GAMAX,GAMD,FEMAX,FED,NWAVE
 444   CONTINUE
 22    IF(GAMD-GAMAX) 46,47,47
 46    XLAM=XLAM-.01
       GO TO 42
 47    IF(FED-FEMAX) 48,49,49
 48    GO TO 46
 49    PALL=PR/1.92
       IF(PRINT-1) 810,811,811
 811   WRITE(K,80)PALL,PR,NWAVE
 810   CONTINUE
 111   CONTINUE
C  ALLOWABLE  LOAD
```

```
      PTEST2=PRMIN(1)
      NWAVE=1
      PR=PTEST2
      DO 112 I=2,NU
      IF(PRMIN(I)-PTEST2) 113,113,112
 113  PTEST2=PRMIN(I)
      NWAVE=I
      PR=PTEST2
 112  CONTINUE
      WRITE(K,820)PR,NWAVE
C
C
C
C     ALLOWABLE    LOAD  OF  STUD
C     ---------------------------
C
      PALL=PR/1.92
      WRITE(K,577)PALL
      WRITE(K,578)
C
C
 999  FORMAT('1',4X,'ALLOWABLE  LOAD OF STUD BRACED ON ONE S
     C  IDE ONLY
     1(PROG. A2)',/,5X,44('='),////)
 500  FORMAT(I10,7F10.3,/,6F10.5)
 774  FORMAT(' ',1X,'I - SECTION  STUD                LENGTH=',F
     C  6.2)
 764  FORMAT(' ',1X,24('_'),//)
 775  FORMAT(' ',1X,'CHANNEL  SECTION  STUD            LENGTH='
     C   ,F6.2)
 765  FORMAT(' ',1X,18('_'),//)
 776  FORMAT(' ',1X,'ZEE - SECTION STUD                LENGTH='
     C   ,F6.2)
 766  FORMAT(' ',1X,17('_'),//)
 502  FORMAT(' ',1X,'SECTION DIMENSIONS',/,2X,'DEPTH=',F6.3,
     C  2X,'H=',F6.3
     1,2X,'B=',F6.3,2X,'D=',F6.3,2X,'T=',F6.3,/,2X,'QA=',F6.
     C  3,/)
 503  FORMAT(' ',1X,'DIAPHRAGM PROPERTIES',/,2X,'S=',F8.3,4X
     C   ,'F=',F6.3,4
     1X,'GAMD=',F8.5,4X,'FED=',F8,5,2X,'XL1=',F5.1,/)
 504  FORMAT(' ',1X,'YIELD STRESS FY=',F6.3,/,2X,'INITIAL IM
     C  PERFECTIONS
     1CO=',F5,3,2X,'DO=',F5.3,2X,'EO=',F5.3,/)
 600  FORMAT(' ',1X,'SECTICN PROPERTIES',/,2X,'AREA=',F6.3,2
     C   X,'IXX=',F6.
     13,2X,'IYY=',F6.3,2X,'IXY=',F6.3,/,2X,'XO=',F6.3,2X,'R2
     C   =',F6.3,2X,'
     2J=',F6.3,2X,'CW=',F6.3,/)
 666  FORMAT(' ',1X,'SECTICN PROPERTIES',/,2X,'AREA=',F6.3,2
     C   X,'IXX=',F6.
     13,2X,'IYY=',F6.3,2X,'IXY=',F6.3,2X,'XO=',F6.3,/,2X,'R2
     C   =',F6.3,2X,'
     2J=',F6.3,2X,'CW=',F6.3,2X,'IX1=',F6.3,2X,'IX2=',F6.3,/
```

```
    C   )
601  FORMAT(' ',1X,2F12.2,2X,8F10.3,2X,I2)
652  FORMAT(' ',9X,'MOD          GE              PXX         PYY
    C         PFE
    1     PXY          P1          P2          P3          PC  NWAVE'
    C   )
653  FORMAT(' ',2X,116('='),/)
602  FORMAT(' ',1X,'MOD=',F8.1,4X,'GE=',F8.1,4X,'PXX=',F8.3
    C   ,4X,'PYY=',F
    18.3,4X,'PFE=',F8.3,4X,'PXY=',F8.3)
603  FORMAT(' ',1X,'ELASTIC CRITICAL B. LOAD PCR=',F8.3,4X,
    C   'NWAVE=',I2,
    1/)
808  FORMAT(' ',2X,'ELASTIC B. LOAD=',F10.3,2X,'PCF=',F10.3
    C   ,/)
803  FORMAT(' ',2X,'CRITICAL B. LOAD ,CONSIDER. B. BETWEEN
    C   FASTENERS,='
    1,F10.3,/)
804  FORMAT(' ',2X,'ELAST. B. LOAD=',F10.3,2X,': BUCKLING B
    C   ET. FAST.: P
    14,P5,P6  =' 3F10.3,/)
61   FORMAT(' ',1X,'FCR=',F8.3,5X,'FLT=',F8.3)
62   FORMAT(' ',1X,'PA=',F8.3,//)
63   FORMAT(' ',1X,'XLAM=',F5.3,5X,'PR=',F8.3,5X,'FR=',F10.
    C   3)
67   FORMAT(' ',1X,'C1=',F10.5,5X,'E1=',F8.5,5X,'GAMAX=',F1
    C   0.5,5X,'GAMD
    1=',F10.5,5X,'FEMAX=',F10.5,5X,'FED =',F10.5,2X,'NWAVE=
    C   ',I2,/)
577  FORMAT(' ',4X,'ALLOWABLE DESIGN LOAD  (( PALL )) =',F1
    C   0.3,/)
578  FORMAT(' ',4X,44('='),/)
820  FORMAT(' ',2X,'LOAD CAPACITY   PR=',F10.3,4X,'  NWAVE=
    C   ',I2,///)
80   FORMAT(' ',4X,'ALL.LOAD =',F10.3,2X,'PR=',F10.3,4X,'NW
    C   AVE=',I2,//)
     GO TO 800
802  STOP
     END
     SUBROUTINE PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE
    C   ,XL,XI1,XI2,
    1AN,PXX,PYY,PFE,PXY,PX1,PX2)
     REAL MOD
     PXX=AN*(PIE**2)*MOD*XXI/(XL**2)
     PYY=AN*(PIE**2)*MOD*YYI/(XL**2)
     PFE=(GE*XJ+AN*(PIE**2)*MOD*CW/XL**2)/R2
     PXY=AN*(PIE**2)*MOD*XYI/(XL**2)
     PX1=AN*(PIE**2)*MOD*XI1/(XL**2)
     PX2=AN*(PIE**2)*MOD*XI2/(XL**2)
     RETURN
     END

     SUBROUTINE DPRQD(C,IC,Q,E,POL,IR,IER)
     DIMENSION C(4),Q(4),E(4),POL(4)
```

```
      DOUBLE PRECISION Q,E,O,P,T,EXPT,ESAV,U,V,W,C,POL,EPS,D
     C   ABS,DSQRT
      IR=IC
      IER=0
      EPS=1.D-16
      TOL=1.E-6
      LIMIT=10*IC
      KOUNT=0
    1 IF(IR-1)79,79,2
    2 IF(C(IR))4,3,4
    3 IR=IR-1
      GOTO 1
    4 O=1.0D0/C(IR)
      IEND=IR-1
      ISTA=1
      NSAV=IR+1
      JBEG=1
      DO 9 I=1,IR
      J=NSAV-I
      IF(C(I))7,5,7
    5 GOTO(6,8),JBEG
    6 NSAV=NSAV+1
      Q(ISTA)=0.D0
      E(ISTA)=0.D0
      ISTA=ISTA+1
      GOTO 9
    7 JBEG=2
    8 Q(J)=C(I)*O
      C(I)=Q(J)
    9 CONTINUE
      ESAV=0.D0
      Q(ISTA)=0.D0
   10 NSAV=IR
      EXPT=IR-ISTA
      E(ISTA)=EXPT
      DO 11 I=ISTA,IEND
      EXPT=EXPT-1.0D0
      POL(I+1)=EPS*DABS(Q(I+1))+EPS
   11 E(I+1)=Q(I+1)*EXPT
      IF(ISTA-IEND)12,20,60
   12 JEND=IEND-1
      DO 19 I=ISTA,JEND
      IF(I-ISTA)13,16,13
   13 IF(DABS(E(I))-POL(I+1))14,14,16
   14 NSAV=I
      DO 15 K=I,JEND
      IF(DABS(E(K))-POL(K+1))15,15,80
   15 CONTINUE
      GOTO 21
   16 DO 19 K=I,IEND
      E(K+1)=E(K+1)/E(I)
      Q(K+1)=E(K+1)-Q(K+1)
      IF(K-I)18,17,18
   17 IF(DABS(Q(I+1))-POL(I+1))80,80,19
```

```
 18 Q(K+1)=Q(K+1)/Q(I+1)
    POL(K+1)=POL(K+1)/DABS(Q(I+1))
    E(K)=Q(K+1)-E(K)
 19 CONTINUE
 20 Q(IR)=-Q(IR)
 21 E(ISTA)=0.D0
    NRAN=NSAV-1
 22 E(NRAN+1)=0.D0
    IF(NRAN-ISTA)24,23,31
 23 Q(ISTA+1)=Q(ISTA+1)+EXPT
    E(ISTA+1)=0.D0
 24 E(ISTA)=ESAV
    IF(IR-NSAV)60,60,25
 25 ISTA=NSAV
    ESAV=E(ISTA)
    GOTO 10
 26 P=P+EXPT
    IF(O)27,28,28
 27 Q(NRAN)=P
    Q(NRAN+1)=P
    E(NRAN)=T
    E(NRAN+1)=-T
    GOTO 29
 28 Q(NRAN)=P-T
    Q(NRAN+1)=P+T
    E(NRAN)=0.D0
 29 NRAN=NRAN-2
    GOTO 22
 30 Q(NRAN+1)=EXPT+P
    NRAN=NRAN-1
    GOTO 22
 31 JBEG=ISTA+1
    JEND=NRAN-1
    TEPS=EPS
    TDELT=1.E-2
 32 KOUNT=KOUNT+1
    P=Q(NRAN+1)
    R=ABS(SNGL(E(NRAN)))
    IF(R-TEPS)30,30,33
 33 S=ABS(SNGL(E(JEND)))
    IF(S-R)38,38,34
 34 IF(R-TDELT)36,35,35
35  P=0.D0
 36 O=P
    DO 37 J=JBEG,NRAN
    Q(J)=Q(J)+E(J)-E(J-1)-O
    IF(DABS(Q(J))-POL(J))81,81,37
 37 E(J)=Q(J+1)*E(J)/Q(J)
    Q(NRAN+1)=-E(NRAN)+Q(NRAN+1)-O
    GOTO 54
 38 P=0.5D0*(Q(NRAN)+E(NRAN)+Q(NRAN+1))
    O=P*P-Q(NRAN)*Q(NRAN+1)
    T=DSQRT(DABS(O))
    IF(S-TEPS)26,26,39
```

```
39 IF(O)43,40,40
40 IF(P)42,41,41
41 T=-T
42 P=P+T
   R=S
   GOTO 34
43 IF(S-TDELT)44,35,35
44 O=Q(JBEG)+E(JBEG)-P
   IF(DABS(O)-POL(JBEG))81,81,45
45 T=(T/O)**2
   U=E(JBEG)*Q(JBEG+1)/(O*(1.0D0+T))
   V=O+U
   KOUNT=KOUNT+2
   DO 53 J=JBEG,NRAN
   O=Q(J+1)+E(J+1)-U-P
   IF(DABS(V)-POL(J))46,46,49
46 IF(J-NRAN)81,47,81
47 EXPT=EXPT+P
   IF(ABS(SNGL(E(JEND)))-TOL)48,48,81
48 P=0.5D0*(V+O-E(JEND))
   O=P*P-(V-U)*(O-U*T-O*W*(1.D0+T)/Q(JEND))
   T=DSQRT(DABS(O))
   GOTO 26
49 IF(DABS(O)-POL(J+1))46,46,50
50 W=U*O/V
   T=T*(V/O)**2
   Q(J)=V+W-E(J-1)
   U=0.D0
   IF(J-NRAN)51,52,52
51 U=Q(J+2)*E(J+1)/(O*(1.D0+T))
52 V=O+U-W
   IF(DABS(Q(J))-POL(J))81,81,53
53 E(J)=W*V*(1.0D0+T)/Q(J)
   Q(NRAN+1)=V-E(NRAN)
54 EXPT=EXPT+P
   TEPS=TEPS*1.1
   TDELT=TDELT*1.1
   IF(KOUNT-LIMIT)32,55,55
55 IER=1
56 IEND=NSAV-NRAN-1
   E(ISTA)=ESAV
   IF(IEND)59,59,57
57 DO 58 I=1,IEND
   J=ISTA+I
   K=NRAN+1+I
   E(J)=E(K)
58 Q(J)=Q(K)
59 IR=ISTA+IEND
60 IR=IR-1
   IF(IR)78,78,61
61 DO 62 I=1,IR
   Q(I)=Q(I+1)
62 E(I)=E(I+1)
   POL(IR+1)=1.D0
```

```
      IEND=IR-1
      JBEG=1
      DO 69 J=1,IR
      ISTA=IR+1-J
      O=0.DO
      P=Q(ISTA)
      T=E(ISTA)
      IF(T)65,63,65
   63 DO 64 I=ISTA,IR
      POL(I)=O-P*POL(I+1)
   64 O=POL(I+1)
      GOTO 69
   65 GOTO(66,67),JBEG
   66 JBEG=2
      POL(ISTA)=0.DO
      GOTO 69
   67 JBEG=1
      U=P*P+T*T
      P=P+P
      DO 68 I=ISTA,IEND
      POL(I)=O-P*POL(I+1)+U*POL(I+2)
   68 O=POL(I+1)
      POL(IR)=O-P
   69 CONTINUE
      IF(IER)78,70,78
   70 P=0.DO
      DO 75 I=1,IR
      IF(C(I))72,71,72
   71 O=DABS(POL(I))
      GOTO 73
   72 O=DABS((POL(I)-C(I))/C(I))
   73 IF(P-O)74,75,75
   74 P=O
   75 CONTINUE
      IF(SNGL(P)-TOL)77,76,76
   76 IER=-1
   77 Q(IR+1)=P
      E(IR+1)=0.DO
   78 RETURN
   79 IER=2
      IR=0
      RETURN
   80 IER=4
      IR=ISTA
      GOTO 60
   81 IER=3
      GOTO 56
      END
*DATA
```

```
C     **************************************************
C     *                                                *
C     *                PROGRAM  'B1'                   *
C     *                                                *
C     *           STUD BRACED ON BOTH SIDES            *
C     *  FIND 'S' & 'F' VALUES FOR GIVEN ALL. LOAD  PO *
C     *                                                *
C     **************************************************
C
C
C
C       THE *INPUT DATA* CONSISTS OF THE FOLLOWING PER CASE :
C       (ISEC,XL,HH,H,B,D,T,QA,FYIELD,SLIM,FLIM,PO,XL1)
C   THESE PARAMETERS ARE PUNCHED IN  2 CARDS ACCORDING TO THE
C   FORMAT STATEMENT NUMBER 500   FORMAT(I10,7F10.3,/,5F10.5)
C   THE ABOVE  MAY  BE  REPEATED FOR EACH  CASE INVOLVING
C   NEW VALUES OF THE ABOVE PARAMETERS.
C
C   TWO BLANK CARDS  'WITH  ISEC=0 '   MUST BE PROVIDED AFTER
C   THE DATA CARDS TO SIGNIFY  THE LOGICAL TERMINATION OF THE
C   PROGRAM
C
C
C                   ----------*---------
C
C   THE FOLLOWING DEFINES THE INPUT DATA AS WELL AS IMPORTANT
C   PARAMETERS USED IN THE PROGRAM.     DEFINITIONS OF OTHER
C   PARAMETERS ARE GIVEN IN THE NOMENCLATURE OF  APPENDIX # 4
C   OF THE MAIN REPORT.
C
C
C                   FOR  I-SECTION       ISEC=1
C                        CHANNEL-SEC.    ISEC=2
C                        ZEE-SECTION     ISEC=3
C                        STOP PROGRAM    ISEC=0
C
C
C   ALL DIMENSIONS , LOADS & STRESSES ARE IN THE FOLLOWING
C   UNITS EXCEPT OTHERWISE NOTED :
C   DIMENSIONS          IN    INCHES
C   LOADS               IN    KIPS
C   STRESSES            IN    KSI
C
C
C                   SECTION DIMENSIONS:
C                   XL=      STUD LENGTH
C                   QA=      SHAPE FACTOR
C                   HH=      TOTAL DEPTH OF SECTION
C                   T =      THICKNESS OF SECTION
C                   H,B,D  ARE CENTER LINE DIMENSIONS OF WEB,
C                   FLANGE & LIP
C
C
C                   DIAPHRAGM  PROPERTIES:
```

```
C                    S= RELIABLE SHEAR RIGIDITY  K
C                    F= RELIABLE ROT. RESTRAINT  K.IN/IN.RAD
C                    GAMAD & FED ARE DESIGN SHEAR STRAIN AND
C                    ROTATIONAL CAPACITY   IN RAD.
C
C
C                    PARAMETERS INTUITIVELY ESTIMATED WITHIN
C                    PRACTICAL RANGE OF WALL STUDS APPLICATION
C                    XL1 =  DISTANCE BETWEEN FASTENERS
C                    SLIM=  MAX. AVAILABLE VALUE OF  'S'
C                    FLIM=  MAX. AVAILABLE VALUE OF  'F'
C
C
C                    INITIAL IMPERFECTIONS:
C                    CO= STUD LENGTH /700.
C                    DO= STUD LENGTH /700.
C                    EO= 0.0006 RAD. PER FOOT LENGTH OF STUD
C
C
C                    MATERIAL PROPERTIES OF STUD:
C                    FYIELD= YIELD STRESS OF STEEL
C                    FLT= PROPORTIONAL LIMIT      (FLT= 0.5FY)
C                    MOD= MODULUS OF ELASTICITY  (29500. KSI)
C                    GE = SHEAR MODULUS          (11300. KSI)
C
C
C
C
C   J & K ARE LOGICAL RECORD UNITS OF READ & WRITE STATEMENTS
C
      REAL MOD
      J=5
      K=6
C
C
 800  READ(J,500)ISEC,XL,HH,H,B,D,T,QA,FYIELD,SLIM,FLIM,PO,X
     C  L1
      MOD=29500.0
      GE=11300.0
      PIE=3.14159
      WRITE(K,999)
C
C
C   INITIAL  IMPERECTIONS
      CO=XL/700.
      DO=XL/700.
      EO=0.0006*(0.5*XL/12.)
C
C   FOR INITIAL IMPER. AND ACCEDENTAL LOAD ECCENTRICITY
C
      CO=2.*CO
C
C
C   LET PRINT=1 IF DETAILS OF COMPUTATIONS ARE NEEDED
```

```
C  LET PRINT=0 IF DETAILS OF COMPUTATIONS ARE NOT NEEDED
C
      PRINT=0
C
      IF(ISEC-0) 802,802,801
 801  GO TO(771,772,773),ISEC
 771  WRITE(K,774)XL
      WRITE(K,764)
      GO TO 807
 772  WRITE(K,775)XL
      WRITE(K,765)
      GO TO 807
 773  WRITE(K,776)XL
      WRITE(K,766)
 807  WRITE(K,522)PO
      WRITE(K,502)HH,H,B,D,T,QA
      WRITE(K,503)SLIM,FLIM,XL1
      WRITE(K,504)FYIELD,CO,DO,EO
      GO TO(101,201,301),ISEC
C
C
C  CALCULATION OF SECTION PROPERTIES     I-SECTION
C
 101  AREA=2.*T*(H+B+2.*D)
      XXI=T*(H**3+3.*B*H**2+6.*D*(H-D)**2+2.*D**3)/6.
      YYI=B**2*T*(B+6.*D)/6.
      XYI=0.0
      XO=0.0
      XJ=2.*T**3*(B+H+2.*D)/3.
      CW=B**2*T*(B*H**2+6.*D*H**2+12.*H*D**2+8.*D**3)/24.
      PI=XXI+YYI
      R2=PI/AREA
      XI2=XXI
      XI1=YYI
      WRITE(K,600)AREA,XXI,YYI,XYI,XO,R2,XJ,CW
      GO TO 44
C
C
C  CALCULATION OF SECTION PROPERTIES    CHANNEL-SEC
C
 201  AREA=T*(H+2.0*B+2.0*D)
      XBAR=T*(B**2+2.0*D*B)/AREA
      XXI=T*(H**3+6.0*B*H**2+6.0*D*(H-D)**2+2.0*D**3)/12.0
      YYI=T*B**2*(2.0*H*B+B**2+2.0*D*(2.0*B+3.0*H))/(3.0*(H+
     C  2.0*B+2.0*D)
     1)
      XYI=0.0
      XMBAR=(B*H)**2*T*(1.0+2.0*D/B-8.0*D**3/(3.0*B*H**2))/(
     C  4.0*XXI)
      XO=XMBAR+XBAR
      PI=XXI+YYI+AREA*XO**2
      R2=PI/AREA
      XJ=T**3*(H+2.0*B+2.0*D)/3.0
      CW=(B*H*T)**2*(2.0*B*H**3+3.0*(B*H)**2+6.0*D*(H+2.0*B)
```

```
     C   *H**2+12.0*D
     1**2*(H+4.0*B)+8.0*D**3*(H+14.0*B)+48.0*D**4)/(144.0*XX
     C   I)
      XI1=YYI
      XI2=XXI
      WRITE(K,600)AREA,XXI,YYI,XYI,XO,R2,XJ,CW
      GO TO 44
C
C
C  CALCULATION OF SECTION PROPERTIES      Z-SECTION
C
 301  AREA=T*(H+2.0*B+2.0*D)
      XXI=T*(H**3+6.0*B*H**2+6.0*D*(H-D)**2+2.0*D**3)/12.0
      YYI=2.0*B**2*T*(B+3.0*D)/3.0
      XYI=B*T*(B*H+D*(H-D))/2.0
      XO=0.0
      XJ=T**3*(2.0*B+H+2.0*D)/3.0
      CW=(B*T)**2*(2.0*H**3*B+(H*B)**2+2.0*D*H**2*(3.0*H+2.0
     C   *B)+12.0*D**
     12*H*(H+B)+8.0*D**3*(H+2.0*B)+D**4)/(12.0*AREA)
      PI=XXI+YYI
      R2=PI/AREA
      XI1=((XXI+YYI)/2.)-SQRT(((XXI-YYI)/2.)**2+XYI**2)
      XI2=((XXI+YYI)/2.)+SQRT(((XXI-YYI)/2.)**2+XYI**2)
      WRITE(K,666)AREA,XXI,YYI,XYI,XO,R2,XJ,CW,XI1,XI2
 44   PR=PO*1.92
      IF(PRINT-1) 690,691,691
 691  WRITE(K,680)PR
 690  CONTINUE
C
C
C  CHECK IF PR (GIVEN LOAD X F.S.) SATISFIES THE CONDITIONS:
C         PR > UNBRACED BUCKLING LOAD  (PUNB)
C         PR < THE CRITICAL LOAD OF BUCKLING PERPENDICULAR TO
C         THE WALL
C         PR < YIELDING OF SECTION
C
C
      CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,PXX,PY
     1Y,PFE,PXY,PX1,PX2)
      IF(PRINT-1) 692,693,693
 693  WRITE(K,681)PXX,PYY,PFE,PXY,PX1
 692  CONTINUE
      GO TO(102,202,302),ISEC
 102  PUNB=PYY
      CALL PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
      IF(PR-PCUNB) 113,113,114
 113  WRITE(K,115)
      GO TO 799
 114  PUNB=PXX
      CALL PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
      IF(PR-PCUNB) 116,116,117
 117  WRITE(K,118)
```

```
      GO TO 799
 116  PYIELD=FYIELD*QA*AREA
      IF(PR-PYIELD) 119,120,120
 120  WRITE(K,121)
      GO TO 799
 202  A1=R2-XO**2
      A2=-R2*(PFE+PXX)
      A3=PXX*PFE*R2
      P1=(-A2+SQRT(A2**2-4.*A1*A3))/(2.*A1)
      P2=(-A2-SQRT(A2**2-4.*A1*A3))/(2.*A1)
      P3=PYY
      PUNB=AMIN1(P1,P2,P3)
      GO TO 135
 302  PUNB=PX1
 135  CALL PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
      IF(PR-PCUNB) 123,123,124
 123  WRITE(K,125)
      GO TO 799
 124  PUNB=PXX
      CALL PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
      IF(PR-PCUNB) 126,127,127
 127  WRITE(K,128)
      GO TO 799
 126  PYIELD=FYIELD*QA*AREA
      IF(PR-PYIELD) 119,130,130
 130  WRITE(K,131)
      GO TO 799
 119  CONTINUE
C
C
C  CHECK POSSIBILITY OF BUCKLING BETWEEN FASTENERS
C  DISTANCE BETWEEN FASTENERS  = XL1
C
      GO TO(105,205,305),ISEC
C
C  CHECKING BUCKLING BETWEEN FASTENERS     I-SEC
C
 105  PCF=(PIE**2)*MOD*YYI/(XL1**2)
      IF(PRINT-1) 694,695,695
 695  WRITE(K,682)PCF
 694  CONTINUE
      GO TO 899
C
C  CHECKING BUCKLING BETWEEN FASTENERS     CHANNEL -SEC
C
 205  PYYF=(PIE**2)*MOD*YYI/(XL1**2)
      PFEF=(GE*XJ+(PIE**2)*MOD*CW/XL1**2)/R2
      PXXF=(PIE**2)*MOD*XXI/(XL**2)
      P4=PYYF
      P5=PFEF
      P6=PXXF
      PCF=AMIN1(P4,P5,P6)
      IF(PRINT-1) 696,697,697
 697  WRITE(K,683)P4,P5,P6,PCF
```

```
 696   CONTINUE
       GO TO 899
C
C  CHECKING BUCKLING BETWEEN FASTENERS     Z-SEC
C
 305   PCF=(PIE**2)*MOD*XI1/(XL1**2)
 899   PUNB=PCF
       CALL PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
       IF(PRINT-1) 698,699,699
 699   WRITE(K,684)PR,PCUNB
 698   CONTINUE
       IF(PR-PCUNB) 219,219,889
 889   WRITE(K,888)
       GO TO 799
C
C
C  CHECK IF PR (GIVEN LOAD X F.S.) IS IN THE INELASTIC RANGE
C  IF SO , THEN FIND THE EQUIVALENT ELASTIC LOAD  (PE)
C  CORRESPONDING TO  (PR)
C
C
 219   FY=FYIELD*QA
       FLT=.5*FY
       FPR=PR/AREA
       IF(FPR-FLT) 90,90,91
 90    PE=PR
       GO TO 94
C
C  EQUIVALENT ELASTIC LOAD 'PE' CORRESPONDING TO  'PR'
C
 91    PE=(AREA*FY)**2/(4.*(AREA*FY-PR))
       WRITE(K,679)MOD,GE,PE,PR
 94    CONTINUE
C
C
C  COMPUTATIONS OF A LIST OF 'S'&'F' ,ALSO THE CORRESPONDING
C  'GAMAX' & 'FEMAX' , SO THAT A SUITABLE DIAPHRAGM CAN  BE
C     CHOOSEN
C
       GO TO(103,203,303),ISEC
C
C      D I A P H R A G M          FOR  I-SECTION
C
 103   S=PE-PYY
C
C  ROTATIONAL RESTAINT OF DIAPHRAGM IS NOT NEEDED
C
       F=0.0
       CALL CONST(PR,FYIELD,QA,AREA,MOD,GE)
       CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,PXX,PY
     1Y,PFE,PXY,PX1,PX2)
       WRITE(K,669)MOD,GE,PXX,PYY,PFE,PXY
       WRITE(K,668)
```

```
 80    C1=CO*PR/(PYY+S-PR+.001)
       E1=0.0
       GAMAX=C1*PIE/XL
       GAMAX=ABS(GAMAX)
       FEMAX=.0
       IF(GAMAX-1.0) 60,61,61
 60    WRITE(K,667)S,F,GAMAX,FEMAX,C1,E1
 61    S=S+5.
       IF(S-SLIM) 80,80,81
 81    GO TO 799
C
C      D I A P H R A G M            FOR  CHANNEL-SEC
C
 203   S1=PE-PYY
       F=0.0
       S2=((PE*XO)**2-(PXX-PE)*(R2*(PFE-PE)+F*(XL/PIE)**2))/(
      C  (PXX-PE)*(HH
      1/2.)**2)
       IF(S2-S1) 82,82,83
 82    SREQ=S1
       GO TO 84
 83    SREQ=S2
 84    S=SREQ
       CALL CONST(PR,FYIELD,QA,AREA,MOD,GE)
       CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
      C  1,XI2,PXX,PY
      1Y,PFE,PXY,PX1,PX2)
       WRITE(K,669)MOD,GE,PXX,PYY,PFE,PXY
       WRITE(K,668)
 85    A4=PYY-PR+S
       A5=PXX-PR
       A6=PR*XO
       A7=R2*(PFE-PR)+S*(HH/2.)**2+F*(XL/PIE)**2
       DET=A4*(A5*A7-A6**2)+.001
       C1=PR*(CO*(A5*A7-A6**2))/DET
       E1=PR*(-A4*A6*(DO-XO*EO)+A4*A5*(R2*EO-XO*DO))/DET
       E1=ABS(E1)
       GAMAX=PIE*(C1+E1*HH/2.)/XL
       FEMAX=E1
       IF(GAMAX-1.0) 62,63,63
 62    WRITE(K,667)S,F,GAMAX,FEMAX,C1,E1
 63    S=S+5.
       IF(S-SLIM) 85,85,86
 86    F=F+0.005
       S=SREQ
       IF(F-FLIM) 85,85,87
 87    GO TO 799
C
C      D I A P H R A G M            FOR  Z-SECTION
C
 303   S1=((PE-PYY)*(PXX-PE)+PXY**2)/(PXX-PE)
       SREQ=S1
       S=SREQ
       F=0.0
```

```
CALL CONST(PR,FYIELD,QA,AREA,MOD,GE)
CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
C  1,XI2,PXX,PY
1Y,PFE,PXY,PX1,PX2)
WRITE(K,669)MOD,GE,PXX,PYY,PFE,PXY
WRITE(K,668)
185  G3=PYY-PR+S
G4=PXY
G5=PXX-PR
G6=(PFE-PR)*R2HS*(HH/2.)**2+F*(XL/PIE)**2
DET=G3*G5*G6-G4**2*G6+.001
C1=PR*(CO*G5*G6-DO*G4*G6)/DET
C1=ABS(C1)
E1=PR*(EO*R2*(G3*G5-G4**2))/DET
E1=ABS(E1)
GAMAX=PIE*(C1+E1*HH/2.)/XL
FEMAX=E1
IF(GAMAX-1.0) 64,65,65
64   WRITE(K,667)S,F,GAMAX,FEMAX,C1,E1
65   S=S+5.
IF (S-SLIM) 185,185,186
186  F=F+0.005
S=SREQ
IF(F-FLIM) 185,185,799
799  GO TO 800
888  FORMAT(' ',1X,'BUCKLING BETWEEN FASTENERS GOVERNS , DE
C  CREASE
1DISTANCE BETWEEN FASTENERS  , OR USE STRONGER STUD')
669  FORMAT(' ',1X,'MOD=',F8.1,2X,'GE=',F8.1,/,2X,'PXX=',F7
C  .3,2X,'PYY='
1,F7.3,2X,'PFE=',F7.3,2X,'PXY=',F7.3,//)
679  FORMAT(' ',1X,'MOD=',F9.2,2X,'GE=',F9.2,/,2X,' PE=',F7
C  .3,2X,'PR=',
1F7.3,//)
500  FORMAT(I10,7F10.3,/,5F10.3)
600  FORMAT(' ',1X,'SECTION PROPERTIES',/,2X,'AREA=',F6.3,2
C  X,'IXX=',F6.
13,2X,'IYY=',F6.3,2X,'IXY=',F6.3,/,2X,'XO=',F6.3,2X,'R2
C  =',F6.3,2X,'
2J=',F6.3,2X,'CW=',F6.3,/)
666  FORMAT(' ',1X,'SECTION PROPERTIES',/,2X,'AREA=',F6.3,2
C  X,'IXX=',F6.
13,2X,'IYY=',F6.3,2X,'IXY=',F6.3,2X,'XO=',F6.3,/,2X,'R2
C  =',F6.3,2X,'
2J=',F6.3,2X,'CW=',F6.3,2X,'IX1=',F6.3,2X,'IX2=',F6.3,/
C  )
667  FORMAT(' ',2X,6F9.3)
668  FORMAT(' ','       S         F        GAMAX     FEMAX
C  C1          E1
1',/,5X,53('-'),/)
999  FORMAT('1',4X,'ALLOWABLE  LOAD OF STUD BRACED ON BOTH
C  SIDES
1(PROG. B1)',/,5X,44('='),////)
774  FORMAT(' ',1X,'I - SECTION  STUD                LENGTH=',F
```

```
    C  6.2)
764  FORMAT(' ',1X,24('_'),//)
775  FORMAT(' ',1X,'CHANNEL  SECTION  STUD          LENGTH='
    C  ,F6.2)
765  FORMAT(' ',1X,18('_'),//)
776  FORMAT(' ',1X,'ZEE - SECTION STUD               LENGTH='
    C  ,F6.2)
766  FORMAT(' ',1X,17('_'),//)
502  FORMAT(' ',1X,'SECTION DIMENSIONS',/,2X,'DEPTH=',F6.3,
    C  2X,'H=',F6.3
    1,2X,'B=',F6.3,2X,'D=',F6.3,2X,'T=',F6.3,/,2X,'QA=',F6.
    C  3,/)
503  FORMAT(' ',1X,'DIAPHRAGM PROPERTIES',/,2X,'SLIM=',F8.3
    C  ,4X,'FLIM=',
    1F8.3,4X,'XL1=',F6.2,/)
684  FORMAT(' ',2X,'PR=',F10.3,2X,'PCUNBRACED=',F10.3,/)
683  FORMAT(' ',2X,'P4=',F10.3,2X,'P5=',F10.3,2X,'P6=',F10.
    C  3,2X,'PCF=',
    1F10.3,/)
682  FORMAT(' ',2X,'PCF=',F10.3,/)
681  FORMAT(' ',2X,'PXX=',F7.3,2X,'PYY=',F7.3,2X,'PFE=',F7.
    C  3,2X,'PXY=',
    1F7.3,2X,'PX1=',F7.3,/)
680  FORMAT(' ',2X,'PR=',F7.3,/)
115  FORMAT(' ',4X,'UNBRACED STUD CAN CARRY THE LOAD ,DIAPH
    C  RAGM ACTION
    1IS NOT NEEDED , FOR ECCONOMICAL DESIGN TRY SMALLER SEC
    C  TION',//)
118  FORMAT(' ',4X,'DESIGN LOAD CAN NOT BE REACHED SINCE BU
    C  CKLING PERPE
    1NDICULAR TO WALL IS SMALLER , USE STUD OF STRONGER SEC
    C  TION',//)
121  FORMAT(' ',4X,'IT IS NOT ECCONOMICAL TO DESIGN SUCH ST
    C  UD,SINCE LAR
    1GE VALUES OF S&F WOULD BE REQUIRED,TRY STUD OF STRONGE
    C  R SECTION',/
    2/)
125  FORMAT(' ',4X,'UNBRACED STUD CAN CARRY THE LOAD ,DIAPH
    C  RAGM ACTION
    1IS NOT NEEDED , FOR ECCONOMICAL DESIGN TRY SMALLER SEC
    C  TION',//)
128  FORMAT(' ',4X,'DESIGN LOAD CAN NOT BE REACHED SINCE BU
    C  CKLING PERPE
    1NDICULAR TO WALL IS SMALLER , USE STUD OF STRONGER SEC
    C  TION',//)
131  FORMAT(' ',4X,'IT IS NOT ECCONOMICAL TO DESIGN SUCH ST
    C  UD,SINCE LAR
    1GE VALUES OF S&F WOULD BE REQUIRED,TRY STUD OF STRONGE
    C  R SECTION',/
    2/)
504  FORMAT(' ',1X,'YIELD STRESS FY=',F6.3,/,2X,'INITIAL IM
    C  PERFECTIONS
    1CO=',F5.3,2X,'DO=',F5.3,2X,'EO=',F5.3,/)
522  FORMAT(' ',1X,'GIVEN ALL.  LOAD   (PO)  =',F8.3,/)
```

```
 802  STOP
      END
      SUBROUTINE CONST(PR,FYIELD,QA,AREA,MOD,GE)
      REAL MOD
      FY=FYIELD
      FY=FY*QA
      FLT=.5*FY
      FR=PR/AREA
      IF(FR-FLT) 90,90,91
 90   MOD=29500.0
      GE=11300.0
      GO TO 92
 91   TMOD=29500.*(FR*(FY-FR)/(FLT*(FY-FLT)))
      MOD=TMOD
      GI=11300.*TMOD/29500.0
      GE=GI
 92   RETURN
      END
      SUBROUTINE PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE
     C  ,XL,XI1,XI2,
     1PXX,PYY,PFE,PXY,PX1,PX2)
      REAL MOD
      PXX=(PIE**2)*MOD*XXI/(XL**2)
      PYY=(PIE**2)*MOD*YYI/(XL**2)
      PFE=(GE*XJ+(PIE**2)*MOD*CW/XL**2)/R2
      PXY=(PIE**2)*MOD*XYI/(XL**2)
      PX1=(PIE**2)*MOD*XI1/(XL**2)
      PX2=(PIE**2)*MOD*XI2/(XL**2)
      RETURN
      END
      SUBROUTINE PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
      FCR=PUNB/AREA
      FY=FYIELD*QA
      FLT=.5*FY
      IF(FCR-FLT) 20,20,40
 40   PA=AREA*(FY-FY**2/(4.*FCR))
      PCUNB=PA
      GO TO 21
 20   PCUNB=PUNB
 21   RETURN
      END
*DATA
```

```
C     ************************************************
C     *                                              *
C     *              PROGRAM   'B2'                  *
C     *                                              *
C     *         STUD BRACED ON ONE SIDE ONLY         *
C     *  FIND 'S' & 'F' VALUES FOR GIVEN ALL. LOAD  PO *
C     *                                              *
C     ************************************************
C
C
C
C      THE *INPUT DATA* CONSISTS OF THE FOLLOWING PER CASE :
C                              (ISEC,XL,HH,H,B,D,T,QA,FYIELD,
C                             SLIM,FLIM,PO,XL1,STRIAL,FTRIAL)
C   THESE PARAMETERS ARE PUNCHED IN  2 CARDS ACCORDING TO THE
C   FORMAT STATEMENT NUMBER 500   FORMAT(I10,7F10.3,/,7F10.5)
C   THE ABOVE  MAY  BE  REPEATED FOR EACH  CASE INVOLVING
C   NEW VALUES OF THE ABOVE PARAMETERS.
C
C   TWO BLANK CARDS  'WITH  ISEC=0 '   MUST BE PROVIDED AFTER
C   THE DATA CARDS TO SIGNIFY  THE LOGICAL TERMINATION OF THE
C   PROGRAM
C
C
C                  ----------*----------
C
C   THE FOLLOWING DEFINES THE INPUT DATA AS WELL AS IMPORTANT
C   PARAMETERS USED IN THE PROGRAM.     DEFINITIONS OF OTHER
C   PARAMETERS ARE GIVEN IN THE NOMENCLATURE OF  APPENDIX # 4
C   OF THE MAIN REPORT.
C
C
C                  FOR  I-SECTION       ISEC=1
C                       CHANNEL-SEC.    ISEC=2
C                       ZEE-SECTION     ISEC=3
C                       STOP PROGRAM    ISEC=0
C
C
C   ALL DIMENSIONS , LOADS & STRESSES ARE IN THE FOLLOWING
C   UNITS EXCEPT OTHERWISE NOTED :
C   DIMENSIONS        IN    INCHES
C   LOADS             IN    KIPS
C   STRESSES          IN    KSI
C
C
C                  SECTION DIMENSIONS:
C                  XL=      STUD LENGTH
C                  QA=      SHAPE FACTOR
C                  HH=      TOTAL DEPTH OF SECTION
C                  T =      THICKNESS OF SECTION
C                  H,B,D  ARE CENTER LINE DIMENSIONS OF WEB,
C                  FLANGE & LIP
C
C
```

```
C                    DIAPHRAGM  PROPERTIES:
C                    S= RELIABLE SHEAR RIGIDITY  K
C                    F= RELIABLE ROT. RESTRAINT  K.IN/IN.RAD
C                    GAMAD & FED ARE DESIGN SHEAR STRAIN AND
C                    ROTATIONAL CAPACITY   IN RAD.
C
C
C                    PARAMETERS INTUITIVELY ESTIMATED WITHIN
C                    PRACTICAL RANGE OF WALL STUDS APPLICATION
C                    XL1 =  DISTANCE BETWEEN FASTENERS
C                    SLIM=  MAX. AVAILABLE VALUE OF  'S'
C                    FLIM=  MAX. AVAILABLE VALUE OF  'F'
C                    STRIAL= FIRST TRIAL VALUE OF 'S'
C                    FTRIAL= FIRST TRIAL VALUE OF 'F'
C
C
C                    INITIAL IMPERFECTIONS:
C                    CO= STUD LENGTH /700.
C                    DO= STUD LENGTH /700.
C                    EO= 0.0006 RAD. PER FOOT LENGTH OF STUD
C                    MATERIAL PROPERTIES OF STUD:
C                    FYIELD= YIELD STRESS OF STEEL
C                    FLT= PROPORTIONAL LIMIT      (FLT= 0.5FY)
C                    MOD= MODULUS OF ELASTICITY  (29500. KSI)
C                    GE = SHEAR MODULUS          (11300. KSI)
C
C
C            HIGHER BUCKLING MODES ARE EXAMINED BY CONSIDERING
C   SUFFICIENT NUMBERS OF 'NU'.IN THIS PROGRAM NU=1,2,....,10
C   IF MORE VALUES ARE DESIRED,THEN CHANGE PRESENT 'NU' VALUE
C
C
      REAL*8 C,Q,E,POL
      DIMENSION P(3)
      DIMENSION C(4),Q(4),E(4),POL(4)
      DIMENSION PC(40)
      REAL MOD
C
C
C   J & K ARE LOGICAL RECORD UNITS OF READ & WRITE STATEMENTS
C
      J=5
      K=6
C
 800  READ(J,500)ISEC,XL,HH,H,B,D,T,QA,FYIELD,SLIM,FLIM,PO,X
     C  L1,STRIAL,FT
     1RIAL
      MOD=29500.0
      GE=11300.0
      PIE=3.14159
C
C   N U M B E R   OF  BUCKLING  MODES  TO  BE  EXAMINED 'NU'
C
      NU=10
```

```
      WRITE(K,999)
C
C
C  INITIAL  IMPERECTIONS
      CO=XL/700.
      DO=XL/700.
      EO=0.0006*(0.5*XL/12.)
C
C  FOR INITIAL IMPER. AND ACCEDENTAL LOAD ECCENTRICITY
C
      CO=2.*CO
C
C
C  LET PRINT=1 IF DETAILS OF COMPUTATIONS ARE NEEDED
C  LET PRINT=0 IF DETAILS OF COMPUTATIONS ARE NOT NEEDED
C
      PRINT=0
      IF(ISEC-0) 802,802,801
 801  GO TO(771,772,773),ISEC
 771  WRITE(K,774)XL
      WRITE(K,764)
      GO TO 807
 772  WRITE(K,775)XL
      WRITE(K,765)
      GO TO 807
 773  WRITE(K,776)XL
      WRITE(K,766)
 807  WRITE(K,522)PO
      WRITE(K,502)HH,H,B,D,T,QA
      WRITE(K,503)SLIM,FLIM,XL1,STRIAL,FTRIAL
      WRITE(K,504)FYIELD,CO,DO,EO,NU
      GO TO(101,201,301),ISEC
C
C
C  CALCULATION OF SECTION PROPERTIES     I-SECTION
C
 101  AREA=2.*T*(H+B+2.*D)
      XXI=T*(H**3+3.*B*H**2+6.*D*(H-D)**2+2.*D**3)/6.
      YYI=B**2*T*(B+6.*D)/6.
      XYI=0.0
      XO=0.0
      XJ=2.*T**3*(B+H+2.*D)/3.
      CW=B**2*T*(B*H**2+6.*D*H**2+12.*H*D**2+8.*D**3)/24.
      PI=XXI+YYI
      R2=PI/AREA
      XI1=YYI
      XI2=XXI
      WRITE(K,600)AREA,XXI,YYI,XYI,XO,R2,XJ,CW
      GO TO 44
C
C
C  CALCULATION OF SECTION PROPERTIES     CHANNEL SECTION
C
 201  AREA=T*(H+2.0*B+2.0*D)
```

```
      XBAR=T*(B**2+2.0*D*B)/AREA
      XXI=T*(H**3+6.0*B*H**2+6.0*D*(H-D)**2+2.0*D**3)/12.0
      YYI=T*B**2*(2.0*H*B+B**2+2.0*D*(2.0*B+3.0*H))/(3.0*(H+
     C  2.0*B+2.0*D)
     1)
      XYI=0.0
      XMBAR=(B*H)**2*T*(1.0+2.0*D/B-8.0*D**3/(3.0*B*H**2))/(
     C  4.0*XXI)
      XO=XMBAR+XBAR
      PI=XXI+YYI+AREA*XO**2
      R2=PI/AREA
      XJ=T**3*(H+2.0*B+2.0*D)/3.0
      CW=(B*H*T)**2*(2.0*B*H**3+3.0*(B*H)**2+6.0*D*(H+2.0*B)
     C  *H**2+12,0*D
     1**2*(H+4.0*B)+8.0*D**3*(H+14.0*B)+48.0*D**4)/(144.0*XX
     C  I)
      XI1=YYI
      XI2=XXI
      WRITE(K,600)AREA,XXI,YYI,XYI,XO,R2,XJ,CW
      GO TO 44
C
C
C  CALCULATION OF SECTION PROPERTIES     Z-SECTION
C
 301  AREA=T*(H+2.0*B+2.0*D)
      XXI=T*(H**3+6.0*B*H**2+6.0*D*(H-D)**2+2.0*D**3)/12.0
      YYI=2.0*B**2*T*(B+3.0*D)/3.0
      XYI=B*T*(B*H+D*(H-D))/2.0
      XO=0.0
      XJ=T**3*(2.0*B+H+2.0*D)/3.0
      CW=(B*T)**2*(2.0*H**3*B+(H*B)**2+2.0*D*H**2*(3.0*H+2.0
     C  *B)+12.0*D**
     12*H*(H+B)+8.0*D**3*(H+2.0*B)+D**4)/(12.0*AREA)
      PI=XXI+YYI
      R2=PI/AREA
      XI1=((XXI+YYI)/2.)-SQRT(((XXI-YYI)/2.)**2+XYI**2)
      XI2=((XXI+YYI)/2.)+SQRT(((XXI-YYI)/2.)**2+XYI**2)
      WRITE(K,666)AREA,XXI,YYI,XYI,XO,R2,XJ,CW,XI1,XI2
C
C
C  L O A D    C A P A C I T Y        OF  STUD
C
 44   PR=PO*1.92
      IF(PRINT-1) 690,691,691
 691  WRITE(K,680)PR
 690  CONTINUE
C
C
C  CHECK IF PR (GIVEN LOAD X F.S.) SATISFIES THE CONDITIONS:
C         PR > UNBRACED BUCKLING LOAD  (PUNB)
C         PR < THE CRITICAL LOAD OF BUCKLING PERPENDICULAR TO
C         THE WALL
C         PR < YIELDING OF SECTION
C
```

```
C
      AN=1.0
      CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,AN,PXX
     1,PYY,PFE,PXY,PX1,PX2)
      IF(PRINT-1) 692,693,693
 693  WRITE(K,681)PXX,PYY,PFE,PXY,PX1
 692  CONTINUE
      GO TO(102,202,302),ISEC
 102  PUNB=PYY
      CALL PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
      IF(PR-PCUNB) 113,113,114
 113  WRITE(K,115)
      GO TO 799
 114  PUNB=PXX
      CALL PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
      IF(PR-PCUNB) 116,116,117
 117  WRITE(K,118)
      GO TO 799
 116  PYIELD=FYIELD*QA*AREA
      IF(PR-PYIELD) 119,120,120
 120  WRITE(K,121)
      GO TO 799
 202  A1=R2-XO**2
      A2=-R2*(PFE+PXX)
      A3=PXX*PFE*R2
      P1=(-A2+SQRT(A2**2-4.*A1*A3))/(2.*A1)
      P2=(-A2-SQRT(A2**2-4.*A1*A3))/(2.*A1)
      P3=PYY
      PUNB=AMIN1(P1,P2,P3)
      GO TO 135
 302  PUNB=PX1
 135  CALL PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
      IF(PR-PCUNB) 123,123,124
 123  WRITE(K,125)
      GO TO 799
 124  PUNB=PXX
      CALL PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
      IF(PR-PCUNB) 126,127,127
 127  WRITE(K,128)
      GO TO 799
 126  PYIELD=FYIELD*QA*AREA
      IF(PR-PYIELD) 119,130,130
 130  WRITE(K,131)
      GO TO 799
 119  CONTINUE
C
C
C  CHECK POSSIBILITY OF BUCKLING BETWEEN FASTENERS
C  DISTANCE BETWEEN FASTENERS  = XL1
C
      GO TO(105,205,305),ISEC
C
C  CHECKING BUCKLING BETWEEN FASTENERS     I-SEC
```

```
C
 105  PCF=(PIE**2)*MOD*YYI/(XL1**2)
      IF(PRINT-1) 694,695,695
 695  WRITE(K,682)PCF
 694  CONTINUE
      GO TO 899
C
C  CHECKING BUCKLING BETWEEN FASTENERS      CHANNEL -SEC
C
 205  PYYF=(PIE**2)*MOD*YYI/(XL1**2)
      PFEF=(GE*XJ+(PIE**2)*MOD*CW/XL1**2)/R2
      PXXF=(PIE**2)*MOD*XXI/(XL**2)
      P4=PYYF
      P5=PFEF
      P6=PXXF
      PCF=AMIN1(P4,P5,P6)
      IF(PRINT-1) 696,697,697
 697  WRITE(K,683)P4,P5,P6,PCF
 696  CONTINUE
      GO TO 899
C
C  CHECKING BUCKLING BETWEEN FASTENERS      Z-SEC
C
 305  PCF=(PIE**2)*MOD*XI1/(XL1**2)
 899  PUNB=PCF
      CALL PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
      IF(PRINT-1) 698,699,699
 699  WRITE(K,684)PR,PCUNB
 698  CONTINUE
      IF(PR-PCUNB) 219,219,889
 889  WRITE(K,888)
      GO TO 799
C
C
C  CHECK IF PR (GIVEN LOAD X F.S.) IS IN THE INELASTIC RANGE
C  IF SO , THEN FIND THE EQUIVALENT ELASTIC LOAD  (PE)
C  CORRESPONDING TO  (PR)
 219  FY=FYIELD*QA
      FLT=.5*FY
      FPR=PR/AREA
      IF(FPR-FLT) 90,90,91
 90   PE=PR
      GO TO 94
C
C  EQUIVALENT ELASTIC LOAD 'PE' CORRESPONDING TO  'PR'
C
 91   PE=(AREA*FY)**2/(4.*(AREA*FY-PR))
      WRITE(K,679)MOD,GE,PE,PR
 94   CONTINUE
C
C
C  COMPUTATIONS OF A LIST OF 'S'&'F' ,ALSO THE CORRESPONDING
C  'GAMAX' & 'FEMAX' , SO THAT A SUITABLE DIAPHRAGM CAN  BE
C       CHOOSEN
```

```
C
      S=STRIAL
      F=FTRIAL
      SMIN=S
      WRITE(K,668)
      GO TO (103,203,303),ISEC
C
C
C     D I A P H R A G M          FOR  I-SECTION
C
 103  DO 50 I=1,NU,1
      AN=(I*1.)**2
C
C  CALCULATION OF ELASTIC BUCKLING LOAD           I-SECTION
C
      CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,AN,PXX
     1,PYY,PFE,PXY,PX1,PX2)
      G1=-(PFE+PYY+S+(S*(HH/2.)**2+F*(1./AN)*(XL/PIE)**2)/R)
      G2=(PYY+S)*(PFE+(S*(HH/2.)**2+F*(1./AN)*(XL/PIE)**2)/R
     C  2)-(S*(HH/2.
     1)**2)/R2
      P1=(-G1+SQRT(G1**2-4.*G2))/2.
      P2=(-G1-SQRT(G1**2-4.*G2))/2.
      P3=(PIE**2)*MOD*XXI/(XL**2)
      PC(I)=AMIN1(P1,P2,P3)
 50   CONTINUE
      PTEST2=PC(1)
      AN=1.0
      NWAVE=1
      PCR=PTEST2
      DO 57 I=2,NU
      IF(PC(I)-PTEST2) 56,56,57
 56   PTEST2=PC(I)
      AN=(I*1.)**2
      NWAVE=I
      PCR=PTEST2
 57   CONTINUE
      IF(PRINT-1) 260,261,261
 261  WRITE(K,603)PCR,NWAVE,S,F
 260  CONTINUE
      IF(PCR-PE) 195,196,196
 195  S=S+5.0
      F=F+0.005
      SMIN=S
      GO TO 103
 196  CONTINUE
C
C  CHECK  'GAMAD'  &  'FED'   REQUIREMENTS
C
      W=NWAVE*1.0
      CALL CONST(PR,FYIELD,QA,AREA,MOD,GE)
      CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,AN,PXX
```

```
     1,PYY,PFE,PXY,PX1,PX2)
      A1=PYY-PR+S
      A2=-S*HH/2.
      A5=R2*(PFE-PR)+S*(HH/2.)**2+(1./AN)*F*(XL/PIE)**2
      C1=PR*(A5*CO/W-R2*(EO/W)*A2)/(A1*A5-A2**2)
      E1=PR*(A2*CO/W-R2*(EO/W)*A1)/(A2**2-A1*A5)
      GAMAX=PIE*W*(C1-E1*HH/2.)/XL
      GAMAX=ABS(GAMAX)
      FEMAX=ABS(E1)
      IF(PRINT-1) 266,267,267
 267  WRITE(K,818)PCR,PR,PE,MOD,GE,NWAVE
 266  CONTINUE
      WRITE(K,667)S,F,GAMAX,FEMAX,C1,E1
      S=S+5.
      IF(S-SLIM) 103,103,61
 61   F=F+.005
      S=SMIN
      IF(F-FLIM) 103,103,62
 62   GO TO 799
C
C
C     D I A P H R A G M          FOR  CHANNEL-SEC
C
 203  DO 71 I=1,NU,1
      AN=(I*1.)**2
C
C  CALCULATION OF ELASTIC BUCKLING LOAD          CHANNEL-SEC
C
      CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,AN,PXX
     1,PYY,PFE,PXY,PX1,PX2)
      FO=-(PYY+S)*(PXX*(R2*PFE+S*(HH/2.)**2+F*(1./AN)*(XL/PI
     C  E)**2))+PXX*
     1(S*HH/2)**2
      F1=(PXX*(R2*PFE+S*(HH/2.)**2+F*(1./AN)*(XL/PIE)**2)+(P
     C  YY+S)*(R2*(P
     1FE+PXX)+S*(HH/2.)**2+F*(1./AN)*(XL/PIE)**2)-(S*HH/2.)*
     C  *2)
      F2=-(R2*(PFE+PXX)+S*(HH/2.)**2+F*(1./AN)*(XL/PIE)**2+(
     C  R2-XO**2)*(P
     1YY+S))
      F3=R2-XO**2
      C(1)=FO
      C(2)=F1
      C(3)=F2
      C(4)=F3
      IC=4
      IR=3
      CALL DPRQD(C,IC,Q,E,POL,IR,IER)
      V1=Q(1)
      V2=Q(2)
      V3=Q(3)
      W1=E(1)
      W2=E(2)
```

```
      W3=E(3)
      IF(W1-0.0) 6,5,6
 6    V1=0.0
 5    P1=V1
      IF(W2-0.0) 10,11,10
 10   V2=0.0
 11   P2=V2
      IF(W3-0.0) 14,12,14
 14   V3=0.0
 12   P3=V3
      P(1)=P1
      P(2)=P2
      P(3)=P3
      PTEST1=10000000.0
      DO 25 N=1,3
      IF(P(N)-0.0) 25,25,24
 24   IF(P(N)-PTEST1) 23,23,25
 23   PTEST1=P(N)
 25   CONTINUE
      PC(I)=PTEST1
 71   CONTINUE
      PTEST2=PC(1)
      AN=1.0
      NWAVE=1
      PCR=PTEST2
      DO 72 I=2,NU
      IF(PC(I)-PTEST2) 73,73,72
 73   PTEST2=PC(I)
      AN=(I*1.)**2
      NWAVE=I
      PCR=PTEST2
 72   CONTINUE
      IF(PRINT-1) 262,263,263
 263  WRITE(K,603)PCR,NWAVE,S,F
 262  CONTINUE
      IF(PCR-PE) 95,96,96
 95   S=S+5.0
      F=F+0.005
      SMIN=S
      GO TO 203
 96   CONTINUE
C
C  CHECK 'GAMAD'  &  'FED'   REQUIREMENTS
C
      W=NWAVE*1.0
      CALL CONST(PR,FYIELD,QA,AREA,MOD,GE)
      CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,AN,PXX
     1,PYY,PFE,PXY,PX1,PX2)
      F4=PYY-PR+S
      F5=-S*HH/2.
      F6=PXX-PR
      F7=PR*XO
      F8=R2*(PFE-PR)+S*(HH/2.)**2+(1./AN)*F*(XL/PIE)**2
```

```
      DET=F4*(F6*F8-F7**2)-F6*F5**2
      C1=PR*(CO/W*(F6*F8-F7**2)+F7*F5*(DO/W-XO*EO/W)-F6*F5*(
     C  R2*EO/W-XO*D
     1O/W))/DET
      E1=PR*(-CO/W*F5*F6-F4*F7*(DO/W-XO*EO/W)+F4*F6*(R2*EO/W
     C  -XO*DO/W))/D
     1ET
      GAMAX=PIE*W*(C1-E1*HH/2.)/XL
      GAMAX=ABS(GAMAX)
      FEMAX=ABS(E1)
      IF(PRINT-1) 268,269,269
 269  WRITE(K,818)PCR,PR,PE,MOD,GE,NWAVE
 268  CONTINUE
      WRITE(K,667)S,F,GAMAX,FEMAX,C1,E1
      S=S+5.
      IF(S-SLIM) 203,203,63
 63   F=F+.005
      S=SMIN
      IF(F-FLIM) 203,203,64
 64   GO TO 799
C
C
C     D I A P H R A G M          FOR  Z-SECTION
C
 303  DO 74 I=1,NU,1
      AN=(I*1.)**2
C
C  CALCULATION OF ELASTIC BUCKLING LOAD             Z-SECTION
C
      CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,AN,PXX
     1,PYY,PFE,PXY,PX1,PX2)
      B1=PFE+PYY+PXX+S
      B2=(((HH/2.)**2)*S+F*(1./AN)*(XL/PIE)**2)/R2
      B3=(PYY+S)*PXX-PXY**2
      B4=(PYY+PXX+S)*PFE
      B5=PYY+PXX+S
      B6=((S*HH/2.)**2)/R2
      FO=-B3*PFE-B3*B2+B6*PXX
      F1=B3+B4+B5*B2-B6
      F2=-B1-B2
      F3=1.0
      C(1)=FO
      C(2)=F1
      C(3)=F2
      C(4)=F3
      IC=4
      IR=3
      CALL DPRQD(C,IC,Q,E,POL,IR,IER)
      V1=Q(1)
      V2=Q(2)
      V3=Q(3)
      W1=E(1)
      W2=E(2)
```

```
      W3=E(3)
      IF(W1-0.0) 8,7,8
 8    V1=0.0
 7    P1=V1
      IF(W2-0.0) 15,16,15
 15   V2=0.0
 16   P2=V2
      IF(W3-0.0) 18,17,18
 18   V3=0.0
 17   P3=V3
      P(1)=P1
      P(2)=P2
      P(3)=P3
      PTEST1=10000000.0
      DO 75 N=1,3
      IF(P(N)-0.0) 75,75,76
 76   IF(P(N)-PTEST1) 77,77,75
 77   PTEST1=P(N)
 75   CONTINUE
      PC(I)=PTEST1
 74   CONTINUE
      PTEST2=PC(1)
      AN=1.0
      NWAVE=1
      PCR=PTEST2
      DO 78 I=2,NU
      IF(PC(I)-PTEST2) 79,79,78
 79   PTEST2=PC(I)
      AN=(I*1.)**2
      NWAVE=I
      PCR=PTEST2
 78   CONTINUE
      IF(PRINT-1) 264,265,265
 265  WRITE(K,603)PCR,NWAVE,S,F
 264  CONTINUE
      IF(PCR-PE) 93,34,34
 93   S=S+5.0
      F=F+0.005
      SMIN=S
      GO TO 303
 34   CONTINUE
C
C  CHECK  'GAMAD'  &  'FED'   REQUIREMENTS
C
      W=NWAVE*1.0
      CALL CONST(PR,FYIELD,QA,AREA,MOD,GE)
      CALL PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE,XL,XI
     C  1,XI2,AN,PXX
     1,PYY,PFE,PXY,PX1,PX2)
      F1=PYY-PR+S
      F2=PXY
      F3=-S*HH/2.
      F4=PXX-PR
      F5=(PFE-PR)*R2+S*(HH**2)/4.+(1./AN)*F*(XL/PIE)**2
```

```
      DET=F1*F4*F5-(F2**2)*F5-(F3**2)*F4
      C1=PR*(CO*F4*F5/W-DO*F2*F5/W-EO*F3*F4*R2/W)/DET
      E1=PR*(-CO*F3*F4/W+DO*F3*F2/W+EO*R2*(F1*F4-F2**2)/W)/D
     C  ET
      GAMAX=PIE*W*(C1-E1*HH/2.)/XL
      GAMAX=ABS(GAMAX)
      FEMAX=ABS(E1)
      IF(PRINT-1) 270,271,271
271   WRITE(K,818)PCR,PR,PE,MOD,GE,NWAVE
270   CONTINUE
      WRITE(K,667)S,F,GAMAX,FEMAX,C1,E1
      S=S+5.
      IF(S-SLIM) 303,303,66
66    F=F+.005
      S=SMIN
      IF(F-FLIM) 303,303,799
799   GO TO 800
500   FORMAT(I10,7F10.3,/,7F10.5)
818   FORMAT(' ',2X,'PCR=',F8.3,4X,'PR=',F8.3,4X,'PE=',F8.3,
     C  4X,'MOD=',F1
     12.3,4X,'GE=',F12.3,4X,'NWAVE=',I2)
600   FORMAT(' ',1X,'SECTION PROPERTIES',/,2X,'AREA=',F6.3,2
     C  X,'IXX=',F6.
     13,2X,'IYY=',F6.3,2X,'IXY=',F6.3,/,2X,'XO=',F6.3,2X,'R2
     C  =',F6.3,2X,'
     2J=',F6.3,2X,'CW=',F6.3,/)
666   FORMAT(' ',1X,'SECTION PROPERTIES',/,2X,'AREA=',F6.3,2
     C  X,'IXX=',F6.
     13,2X,'IYY=',F6.3,2X,'IXY=',F6.3,2X,'XO=',F6.3,/,2X,'R2
     C  =',F6.3,2X,'
     2J=',F6.3,2X,'CW=',F6.3,2X,'IX1=',F6.3,2X,'IX2=',F6.3,/
     C  )
680   FORMAT(' ',2X,'PR=',F7.3,/)
681   FORMAT(' ',2X,'PXX=',F7.3,2X,'PYY=',F7.3,2X,'PFE=',F7.
     C  3,2X,'PXY=',
     1F7.3,2X,'PX1=',F7.3,/)
115   FORMAT(' ',4X,'UNBRACED STUD CAN CARRY THE LOAD ,DIAPH
     C  RAGM ACTION
     1IS NOT NEEDED , FOR ECCONOMICAL DESIGN TRY SMALLER SEC
     C  TION',//)
118   FORMAT(' ',4X,'DESIGN LOAD CAN NOT BE REACHED SINCE BU
     C  CKLING PERPE
     1NDICULAR TO WALL IS SMALLER , USE STUD OF STRONGER SEC
     C  TION',//)
121   FORMAT(' ',4X,'IT IS NOT ECCONOMICAL TO DESIGN SUCH ST
     C  UD,SINCE LAR
     1GE VALUES OF S&F WOULD BE REQUIRED,TRY STUD OF STRONGE
     C  R SECTION',/
     2/)
125   FORMAT(' ',4X,'UNBRACED STUD CAN CARRY THE LOAD ,DIAPH
     C  RAGM ACTION
     1IS NOT NEEDED , FOR ECCONOMICAL DESIGN TRY SMALLER SEC
     C  TION',//)
128   FORMAT(' ',4X,'DESIGN LOAD CAN NOT BE REACHED SINCE BU
```

```
      C  CKLING PERPE
      1NDICULAR TO WALL IS SMALLER , USE STUD OF STRONGER SEC
      C  TICN',//)
131    FORMAT(' ',4X,'IT IS NOT ECCONOMICAL TO DESIGN SUCH ST
      C  UD,SINCE LAR
      1GE VALUES OF S&F WOULD BE REQUIRED,TRY STUD OF STRONGE
      C  R SECTION',/
      2/)
682    FORMAT(' ',2X,'PCF=',F10.3,/)
683    FORMAT(' ',2X,'P4=',F10.3,2X,'P5=',F10.3,2X,'P6=',F10.
      C  3,2X,'PCF=',
      1F10.3,/)
684    FORMAT(' ',2X,'PR=',F10.3,2X,'PCUNBRACED=',F10.3,/)
888    FORMAT(' ',1X,'BUCKLING BETWEEN FASTENERS GOVERNS , DE
      C  CREASE
      1DISTANCE BETWEEN FASTENERS  , OR USE STRONGER STUD')
774    FORMAT(' ',1X,'I - SECTION  STUD                LENGTH=',F
      C  6.2)
764    FORMAT(' ',1X,24('_'),//)
775    FORMAT(' ',1X,'CHANNEL  SECTION  STUD           LENGTH='
      C  ,F6.2)
765    FORMAT(' ',1X,18('_'),//)
776    FORMAT(' ',1X,'ZEE - SECTION STUD               LENGTH='
      C  ,F6.2)
766    FORMAT(' ',1X,17('_'),//)
999    FORMAT('1',4X,'ALLOWABLE  LOAD OF STUD BRACED ON ONE S
      C  IDE ONLY
      1(PROG. B2)',/,5X,44('='),////)
502    FORMAT(' ',1X,'SECTION DIMENSIONS',/,2X,'DEPTH=',F6.3,
      C  2X,'H=',F6.3
      1,2X,'B=',F6.3,2X,'D=',F6.3,2X,'T=',F6.3,/,2X,'QA=',F6.
      C  3,/)
504    FORMAT(' ',1X,'YIELD STRESS FY=',F6.3,/,2X,'INITIAL IM
      C  PERFECTIONS
      1CO=',F5.3,2X,'DO=',F5.3,2X,'EO=',F5.3,1X,'NU=',I2,/)
503    FORMAT(' ',1X,'DIAPHRAGM PROPERTIES',/,2X,'SLIM=',F8.3
      C  ,4X,'FLIM=',
      1F8.3,4X,'XL1=',F6.2,/,2X,'STRIAL=',F7.3,4X,'FTRIAL=',F
      C  7.4,/)
522    FORMAT(' ',1X,'GIVEN ALL.  LOAD   (PO)  =',F8.3,/)
 667   FORMAT(' ',2X,6F9.3)
668    FORMAT(' ','       S         F        GAMAX    FEMAX
      C    C1          E1
      1',/,5X,53('-'),/)
679    FORMAT(' ',1X,'MOD=',F9.2,2X,'GE=',F9.2,/,2X,' PE=',F7
      C  .3,2X,'PR=',
      1F7.3,//)
603    FORMAT(' ',1X,'ELASTIC CRITICAL B. LOAD PCR=',F8.3,4X,
      C  'NWAVE=',I2,
      12F10.5)
802    STOP
       END
       SUBROUTINE CONST(PR,FYIELD,QA,AREA,MOD,GE)
       REAL MOD
```

```
      FY=FYIELD
      FY=FY*QA
      FLT=.5*FY
      FR=PR/AREA
      IF(FR-FLT) 90,90,91
90    MOD=29500.0
      GE=11300.0
      GO TO 92
91    TMOD=29500.*(FR*(FY-FR)/(FLT*(FY-FLT)))
      MOD=TMOD
      GI=11300.*TMOD/29500.0
      GE=GI
92    RETURN
      END
      SUBROUTINE PCL(ISEC,XXI,YYI,XYI,XO,R2,XJ,CW,MOD,GE,PIE
     C   ,XL,XI1,XI2,
     1AN,PXX,PYY,PFE,PXY,PX1,PX2)
      REAL MOD
      PXX=AN*(PIE**2)*MOD*XXI/(XL**2)
      PYY=AN*(PIE**2)*MOD*YYI/(XL**2)
      PFE=(GE*XJ+AN*(PIE**2)*MOD*CW/XL**2)/R2
      PXY=AN*(PIE**2)*MOD*XYI/(XL**2)
      PX1=AN*(PIE**2)*MOD*XI1/(XL**2)
      PX2=AN*(PIE**2)*MOD*XI2/(XL**2)
      RETURN
      END
      SUBROUTINE PCUNBR(PUNB,AREA,QA,FYIELD,PCUNB)
      FCR=PUNB/AREA
      FY=FYIELD*QA
      FLT=.5*FY
      IF(FCR-FLT) 20,20,40
40    PA=AREA*(FY-FY**2/(4.*FCR))
      PCUNB=PA
      GO TO 21
20    PCUNB=PUNB
21    RETURN
      END
```

```
      SUBROUTINE DPRQD(C,IC,Q,E,POL,IR,IER)
      DIMENSION C(4),Q(4),E(4),POL(4)
      DOUBLE PRECISION Q,E,O,P,T,EXPT,ESAV,U,V,W,C,POL,EPS,D
     C  ABS,DSQRT
      IR=IC
      IER=0
      EPS=1.D-16
      TOL=1.E-6
      LIMIT=10*IC
      KOUNT=0
    1 IF(IR-1)79,79,2
    2 IF(C(IR))4,3,4
    3 IR=IR-1
      GOTO 1
    4 O=1.0DO/C(IR)
      IEND=IR-1
      ISTA=1
      NSAV=IR+1
      JBEG=1
      DO 9 I=1,IR
      J=NSAV-I
      IF(C(I))7,5,7
    5 GOTO(6,8),JBEG
    6 NSAV=NSAV+1
      Q(ISTA)=0.DO
      E(ISTA)=0.DO
      ISTA=ISTA+1
      GOTO 9
    7 JBEG=2
    8 Q(J)=C(I)*O
      C(I)=Q(J)
    9 CONTINUE
      ESAV=0.DO
      Q(ISTA)=0.DO
   10 NSAV=IR
      EXPT=IR-ISTA
      E(ISTA)=EXPT
      DO 11 I=ISTA,IEND
      EXPT=EXPT-1.0DO
      POL(I+1)=EPS*DABS(Q(I+1))+EPS
   11 E(I+1)=Q(I+1)*EXPT
      IF(ISTA-IEND)12,20,60
   12 JEND=IEND-1
      DO 19 I=ISTA,JEND
      IF(I-ISTA)13,16,13
   13 IF(DABS(E(I))-POL(I+1))14,14,16
   14 NSAV=I
      DO 15 K=I,JEND
      IF(DABS(E(K))-POL(K+1))15,15,80
   15 CONTINUE
      GOTO 21
   16 DO 19 K=I,IEND
      E(K+1)=E(K+1)/E(I)
      Q(K+1)=E(K+1)-Q(K+1)
```

```
      IF(K-I)18,17,18
   17 IF(DABS(Q(I+1))-POL(I+1))80,80,19
   18 Q(K+1)=Q(K+1)/Q(I+1)
      POL(K+1)=POL(K+1)/DABS(Q(I+1))
      E(K)=Q(K+1)-E(K)
   19 CONTINUE
   20 Q(IR)=-Q(IR)
   21 E(ISTA)=0.D0
      NRAN=NSAV-1
   22 E(NRAN+1)=0.D0
      IF(NRAN-ISTA)24,23,31
   23 Q(ISTA+1)=Q(ISTA+1)+EXPT
      E(ISTA+1)=0.D0
   24 E(ISTA)=ESAV
      IF(IR-NSAV)60,60,25
   25 ISTA=NSAV
      ESAV=E(ISTA)
      GOTO 10
   26 P=P+EXPT
      IF(O)27,28,28
   27 Q(NRAN)=P
      Q(NRAN+1)=P
      E(NRAN)=T
      E(NRAN+1)=-T
      GOTO 29
   28 Q(NRAN)=P-T
      Q(NRAN+1)=P+T
      E(NRAN)=0.D0
   29 NRAN=NRAN-2
      GOTO 22
   30 Q(NRAN+1)=EXPT+P
      NRAN=NRAN-1
      GOTO 22
   31 JBEG=ISTA+1
      JEND=NRAN-1
      TEPS=EPS
      TDELT=1.E-2
   32 KOUNT=KOUNT+1
      P=Q(NRAN+1)
      R=ABS(SNGL(E(NRAN)))
      IF(R-TEPS)30,30,33
   33 S=ABS(SNGL(E(JEND)))
      IF(S-R)38,38,34
   34 IF(R-TDELT)36,35,35
   35 P=0.D0
   36 O=P
      DO 37 J=JBEG,NRAN
      Q(J)=Q(J)+E(J)-E(J-1)-O
      IF(DABS(Q(J))-POL(J))81,81,37
   37 E(J)=Q(J+1)*E(J)/Q(J)
      Q(NRAN+1)=-E(NRAN)+Q(NRAN+1)-O
      GOTO 54
   38 P=0.5D0*(Q(NRAN)+E(NRAN)+Q(NRAN+1))
      O=P*P-Q(NRAN)*Q(NRAN+1)
```

```
      T=DSQRT(DABS(O))
      IF(S-TEPS)26,26,39
   39 IF(O)43,40,40
   40 IF(P)42,41,41
   41 T=-T
   42 P=P+T
      R=S
      GOTO 34
   43 IF(S-TDELT)44,35,35
   44 O=Q(JBEG)+E(JBEG)-P
      IF(DABS(O)-POL(JBEG))81,81,45
   45 T=(T/O)**2
      U=E(JBEG)*Q(JBEG+1)/(O*(1.0D0+T))
      V=O+U
      KOUNT=KOUNT+2
      DO 53 J=JBEG,NRAN
      O=Q(J+1)+E(J+1)-U-P
      IF(DABS(V)-POL(J))46,46,49
   46 IF(J-NRAN)81,47,81
   47 EXPT=EXPT+P
      IF(ABS(SNGL(E(JEND)))-TOL)48,48,81
   48 P=0.5D0*(V+O-E(JEND))
      O=P*P-(V-U)*(O-U*T-O*W*(1.D0+T)/Q(JEND))
      T=DSQRT(DABS(O))
      GOTO 26
   49 IF(DABS(O)-POL(J+1))46,46,50
   50 W=U*O/V
      T=T*(V/O)**2
      Q(J)=V+W-E(J-1)
      U=0.D0
      IF(J-NRAN)51,52,52
   51 U=Q(J+2)*E(J+1)/(O*(1.D0+T))
   52 V=O+U-W
      IF(DABS(Q(J))-POL(J))81,81,53
   53 E(J)=W*V*(1.0D0+T)/Q(J)
      Q(NRAN+1)=V-E(NRAN)
   54 EXPT=EXPT+P
      TEPS=TEPS*1.1
      TDELT=TDELT*1.1
      IF(KOUNT-LIMIT)32,55,55
   55 IER=1
   56 IEND=NSAV-NRAN-1
      E(ISTA)=ESAV
      IF(IEND)59,59,57
   57 DO 58 I=1,IEND
      J=ISTA+I
      K=NRAN+1+I
      E(J)=E(K)
   58 Q(J)=Q(K)
   59 IR=ISTA+IEND
   60 IR=IR-1
      IF(IR)78,78,61
   61 DO 62 I=1,IR
      Q(I)=Q(I+1)
```

```
 62 E(I)=E(I+1)
    POL(IR+1)=1.D0
    IEND=IR-1
    JBEG=1
    DO 69 J=1,IR
    ISTA=IR+1-J
    O=0.D0
    P=Q(ISTA)
    T=E(ISTA)
    IF(T)65,63,65
 63 DO 64 I=ISTA,IR
    POL(I)=O-P*POL(I+1)
 64 O=POL(I+1)
    GOTO 69
 65 GOTO(66,67),JBEG
 66 JBEG=2
    POL(ISTA)=0.D0
    GOTO 69
 67 JBEG=1
    U=P*P+T*T
    P=P+P
    DO 68 I=ISTA,IEND
    POL(I)=O-P*POL(I+1)+U*POL(I+2)
 68 O=POL(I+1)
    POL(IR)=O-P
 69 CONTINUE
    IF(IER)78,70,78
 70 P=0.D0
    DO 75 I=1,IR
    IF(C(I))72,71,72
 71 O=DABS(POL(I))
    GOTO 73
 72 O=DABS((POL(I)-C(I))/C(I))
 73 IF(P-O)74,75,75
 74 P=O
 75 CONTINUE
    IF(SNGL(P)-TOL)77,76,76
 76 IER=-1
 77 Q(IR+1)=P
    E(IR+1)=0.D0
 78 RETURN
 79 IER=2
    IR=0
    RETURN
 80 IER=4
    IR=ISTA
    GOTO 60
 81 IER=3
    GOTO 56
    END
*DATA
```

Appendix 5

BUCKLING LOAD CORRESPONDING TO ASSUMED DISPLACEMENT FUNCTIONS OF DIFFERENT SHAPES

The governing equations derived in Chapter 2 are based on assumed displacement functions of similar shapes of the displacements u, v and the rotation ϕ. These functions, represented by the infinite series equations (18) and (20) for a column with hinged and fixed ends, respectively, satisfy such an assumption, since the number of half-sine or cosine waves (n = 1,2,3,...) appears simultaneously in each of these series. On the other hand, if the number of half-since or cosine waves take different values in each of these series, different shapes of displacement functions ensue.

It is of interest to note that there is a possibility that the buckling load obtained by assuming different shapes of displacement functions of u, v and ϕ is lower than the buckling load obtained by assuming displacement functions of similar shapes (Chapter 2). For a column with hinged ends, displacement functions of different shapes may be represented by the following infinite series:

$$u = \sum_i C_i \sin\frac{i\pi Z}{L} \tag{5.18a}$$

$$v = \sum_j D_j \sin\frac{j\pi Z}{L} \tag{5.18b}$$

$$\phi = \sum_m E_m \sin\frac{m\pi Z}{L} \tag{5.18c}$$

where i, j, m are the number of half-sine waves chosen so that different shapes of displacement functions result.

For a column with fixed ends, the displacements are given in the form of the following series:

$$u = \sum_i C_i\left(1 - \cos\frac{2i\pi Z}{L}\right) \tag{5.20a}$$

$$v = \sum_j D_j\left(1 - \cos\frac{2j\pi Z}{L}\right) \tag{5.20b}$$

$$\phi = \sum_m E_m\left(1 - \cos\frac{2m\pi Z}{L}\right) \tag{5.20c}$$

where (i,j,m) are as defined in Eqs. (5.18).

Following the analytical procedure presented in Section 2.4, it has been found that for hinged ends columns an equation similar to Eq. (23) results, from which it is concluded that Eqs. (5.18) may be replaced by the following simpler functions of displacement, without any effect on the final results:

$$u = C_i \sin\frac{i\pi Z}{L} \tag{5.29a}$$

$$v = D_j \sin\frac{j\pi Z}{L} \tag{5.29b}$$

$$\phi = E_m \sin\frac{m\pi Z}{L} \tag{5.29c}$$

where (i,j,m) are as defined in Eqs. (5.18).

This conclusion is due to the fact that uncoupled modes of buckling corresponding to each combination of the values of (i, j,m) exist. However, for a column with fixed ends or other types of end conditions listed in Table 1, this conclusion is not valid, since in such cases the buckling mode resulting from using Eqs. (5.20) is all coupled. Therefore, only the case of hinged ends columns will be considered in detail herein, since such a case is of particular interest to the suggested design approach given in Chapter 6 and also to avoid presenting lengthy and complicated equations of the cases with end conditions other than hinged. Nevertheless, the analytical procedure given in Chapter 2 lends itself easily to applications of columns with these different end conditions.

1. General Equations of Stability of Two Sides Braced Column (Hinged Ends)

Following the same procedure of deriving Eq. (35) (with similar displacement functions) equations of stability are derived for the following cases by considering different shapes of displacement functions, Eqs. (5)-(29).

a) $i = j \neq m$

b) $i = m \neq j$

c) $j = m \neq i$

d) $i \neq j \neq m$

where i, j and m take certain chosen values to satisfy the abovementioned four cases; for example, in case (a) possible values of i, j and m would be $i = j = 1,3,5,\ldots$ and $m = 2,4,6, \ldots$. The parameters P_x, P_y and P_{xy} appearing in the following equations are given by Eqs. (25).

Case (a) $i = j \neq m$

$$\begin{bmatrix} i^2P_y-P+Q & i^2P_{xy} & 0 \\ i^2P_{xy} & i^2P_x-P & 0 \\ 0 & 0 & r_o^2(P_\phi-P)+Q(\frac{d_1^2+d_2^2}{2})+\frac{F}{m^2}\cdot\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_i \\ D_i \\ E_m \end{Bmatrix} = 0 \tag{5.35a}$$

where $P_\phi = \frac{1}{r_o^2}(m^2 EC_w \frac{\pi^2}{L^2} + GJ)$

<u>Case (b) $i = m \neq j$</u>

$$\begin{bmatrix} i^2P_y-P+Q & 0 & -Py_o+Q(d_1-d_2) \\ 0 & j^2P_x-P & 0 \\ -Py_o+Q(d_1-d_2) & 0 & r_o^2(P_\phi-P)+Q(\frac{d_1^2+d_2^2}{2})+\frac{F}{i^2}\cdot\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_i \\ D_j \\ E_i \end{Bmatrix} = 0 \tag{5.35b}$$

where $P_\phi = \frac{1}{r_o^2}(i^2 EC_w \frac{\pi^2}{L^2} + GJ)$

<u>Case (c) $j = m \neq i$</u>

$$\begin{bmatrix} i^2P_y-P+Q & 0 & 0 \\ 0 & j^2P_x-P & Px_o \\ 0 & Px_o & r_o^2(P_\phi-P)+Q(\frac{d_1^2+d_2^2}{2})+\frac{F}{j^2}\cdot\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_i \\ D_j \\ E_j \end{Bmatrix} = 0 \tag{5.35c}$$

where $P_\phi = \frac{1}{r_o^2}(j^2 EC_w \frac{\pi^2}{L^2} + GJ)$

<u>Case (d) $i \neq j \neq m$</u>

$$\begin{bmatrix} i^2P_y-P+Q & 0 & 0 \\ 0 & j^2P_x-P & 0 \\ 0 & 0 & r_o^2(P_\phi-P)+Q(\frac{d_1^2+d_2^2}{2})+\frac{F}{m^2}\cdot\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_i \\ D_j \\ E_m \end{Bmatrix} = 0 \tag{5.35d}$$

where $P_\phi = \frac{1}{r_o^2}(m^2 EC_w \frac{\pi^2}{L^2} + GJ)$

2. General Equations of Stability of One Side Braced Column (Hinged Ends)

Using Eqs. (5.29) and following the same procedure of deriving Eq. (38) (similar displacement functions), the following stability equations are obtained (i, j and m are defined in the previous section):

Case (a) $i = j \neq m$

$$\begin{bmatrix} i^2P_y - P + Q & i^2P_{xy} & 0 \\ i^2P_{xy} & i^2P_x - P & 0 \\ 0 & 0 & r_o^2(P_\phi - P) + Qd_2^2 + \frac{F}{m^2}\cdot\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_i \\ D_i \\ E_m \end{Bmatrix} = 0 \qquad (5.38a)$$

where $P_\phi = \frac{1}{r_o^2}(m^2 EC_w \frac{\pi^2}{L^2} + GJ)$

Case (b) $i = m \neq j$

$$\begin{bmatrix} i^2P_y - P + Q & 0 & -Py_o - Qd_2 \\ 0 & j^2P_x - P & 0 \\ -Py_o - Qd_2 & 0 & r_o^2(P_\phi - P) + Qd_2^2 + \frac{F}{i^2}\cdot\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_i \\ D_j \\ E_i \end{Bmatrix} = 0 \qquad (5.38b)$$

where $P_\phi = \frac{1}{r_o^2}(i^2 EC_w \frac{\pi^2}{L^2} + GJ)$

Case (c) $j = m \neq i$

$$\begin{bmatrix} i^2P_y-P+Q & 0 & 0 \\ 0 & j^2P_x-P & 0 \\ 0 & Px_o & r_o^2(P_\phi-P)+Qd_2^2+\frac{F}{j^2}\cdot\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_i \\ D_j \\ E_j \end{Bmatrix} = 0 \qquad (5.38c)$$

where $P_\phi = \frac{1}{r_o^2}(j^2EC_w \frac{\pi^2}{L^2} + GJ)$

Case (d) $i \neq j \neq m$

$$\begin{bmatrix} i^2P_y-P+Q & 0 & 0 \\ 0 & j^2P_x-P & 0 \\ 0 & 0 & r_o^2(P_\phi-P)+Qd_2^2+\frac{F}{m^2}\cdot\frac{L^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} C_i \\ D_j \\ E_m \end{Bmatrix} = 0 \qquad (5.38d)$$

where $P_\phi = \frac{1}{r_o^2}(m^2EC_w \frac{\pi^2}{L^2} + GJ)$

3. P_{cr} of Particular Column Sections (Hinged Ends)

Equations (5.35) and (5.38) can be used to derive the governing equations of the cases of channel, zee- and I-section columns braced either on both sides or on one side.

For a particular cross-section the governing equations of the buckling loads can be derived - for each of the given cases of i, j, m combinations - by substituting for the geometric terms appearing in the stability equations, those of the particular cross-section under consideration. Such a procedure is outlined in detail in Section 2.4.3. In the present section only samples of these derivations will be given; other governing equations can be similarly obtained.

Channel Section Braced on Both Sides (Hinged Ends)

For channel sections $y_o = 0$

$$d_1 = d_2 = d/2$$

$$I_{xy} = 0 \quad \text{hence } P_{xy} = 0$$

Substituting the above parameters into Eqs. (5.35) yields the following:

Case (a) $i = j \neq m$

From Eq. (5.35a) the critical buckling load P_{cr} is given by the smallest value of P obtained from the following equations:

$$P = i^2 P_x \tag{a1}$$

$$P = i^2 P_y + Q \tag{a2}$$

$$P = P_\phi + \frac{1}{r_o^2}\left(Q\frac{d^2}{4} + \frac{F}{m^2}\cdot\frac{L^2}{\pi^2}\right) \tag{a3}$$

where $P_\phi = \frac{1}{r_o^2}(m^2 EC_w \frac{\pi^2}{L^2} + GJ)$ and $i \neq m$

To obtain the smallest value of P given by the above three equations, let us start with Eq. (a3). Then, to find the value of m that minimizes the expression (a3) differentiate Eq. (a3) with respect to m and equate the results to zero; it follows that

$$m = \frac{L}{\pi}\sqrt[4]{\frac{F}{EC_w}}$$

If L is in inches, F in units of k.in/in. rad, C_w in units of

in^6, and $E = 29.5 \times 10^3$ ksi, then

$$m = \frac{L}{41} \sqrt[4]{\frac{F}{C_w}} \qquad (a4)$$

Equation (a4) gives the value of m in terms of known parameters L, F and C_w which makes P minimum. It should be noted that m must be an integer; however, m given by Eq. (a4) will be in general a rational number. Therefore, in such a case m should be rounded off to the nearest smaller and larger integer number. Hence for these two values of m, the smallest value of P obtained from Eq. (a3) and its corresponding value of m will be compared with the smallest value of P obtained from Eqs. (a1) and (a2) and the corresponding value of i, respectively, as will be illustrated in the following step.

It is obvious that the smallest buckling load given by Eqs. (a1) and (a2) corresponds to $i = 1.0$. Then one of the following two cases may result:

<u>I. If $i = 1.0 < m$</u>

In such a case the critical buckling load P_{cr} is the smallest value of P obtained from Eqs. (a1) and (a2) with $i = 1.0$ and P obtained from Eq. (a3) as outlined above.

<u>II. If $m = 1.0$ (as obtained by Eq. a4)</u>

In this case i must be equal to 2.0 (i.e. $i = 2.0$) since by definition $i \neq m$. Hence the critical buckling load P_{cr} is given by the smallest value of P obtained from Eqs. (a1) and (a2) with $i = 2.0$ and P from Eq. (a3) with $m = 1.0$.

<u>Case (b) $i = m \neq j$</u>

From Eq. (5.35b) the critical buckling load P_{cr} is given by the smallest value of P obtained from the following equations:

$$P = j^2 P_x \tag{b1}$$

$$P = i^2 P_y + Q \tag{b2}$$

$$P = P_\phi + \frac{1}{r_o^2}\left(Q\,\frac{d^2}{4} + \frac{F}{i^2}\cdot\frac{L^2}{\pi^2}\right) \tag{b3}$$

where $P_\phi = \frac{1}{r_o^2}\left(i^2 EC_w \frac{\pi^2}{L^2} + GJ\right)$

Following the procedure outlined in the previous case (a) ($i = j \neq m$) to determine P_{cr}, the present and the following cases can be accordingly treated.

Case (c) $j = m \neq i$

From Eq. (5.35c) the critical buckling load P_{cr} is given by the smallest value of P obtained from the following equations:

$$P = i^2 P_y + Q \tag{c1}$$

$$P^2\left(1-\frac{x_o^2}{r_o^2}\right)-P\left[j^2P_x+P_\phi+\frac{1}{r_o^2}\left(Q\frac{d^2}{4}+\frac{F}{j^2}\cdot\frac{L^2}{\pi^2}\right)\right]+j^2P_x\left[P_\phi+\frac{1}{r_o^2}\left(Q\frac{d^2}{4}+\frac{F}{j^2}\cdot\frac{L^2}{\pi^2}\right)\right]=0 \tag{c2}$$

where $P_\phi = \frac{1}{r_o^2}\left(j^2 EC_w \frac{\pi^2}{L^2} + GJ\right)$

In this case an expression for j which makes P minimum cannot be obtained in a simple form as in Case (a) (see Eq. a4). Hence sufficient values of j where j = 1,2,3... must be considered so that the smallest root of Eq. (c2) is a minimum, then proceeding as outlined in Case (a)

Case (d) $i \neq j \neq m$

From Eq. (5.35d), the critical buckling load P_{cr} is given by the smallest value of P obtained from the following equations:

$$P = j^2 P_x \tag{d1}$$

$$P = i^2 P_y + Q \tag{d2}$$

$$P = P_\phi + \frac{1}{r_o^2}\left(Q\,\frac{d^2}{4} + \frac{F}{m^2}\cdot\frac{L^2}{\pi^2}\right) \tag{d3}$$

where $P_\phi = \frac{1}{r_o^2}\left(m^2 EC_w \frac{\pi^2}{L^2} + GJ\right)$

Conclusion

Considering higher buckling modes is in fact a step towards the refinement of the assumed displacement functions in which only the first mode is considered (i.e. n = 1) and therefore a better approximation of the exact buckling load can be achieved.

Higher buckling modes have been considered in this investigation in two stages: first by assuming displacement functions with similar shapes (given in Chapter 2), and second by an attempt to improve the analysis by assuming displacement functions of different shapes, as illustrated in the present appendix. Both stages have introduced complication to that method of analysis which considers only the first term of displacement functions (see Section 2.6). However, the complication introduced by assuming different shapes of displacement functions is relatively greater than that resulting from assuming similar shapes. This is so since the latter includes only one varying parameter, namely n, while the former includes three parameters (i,j,m), which requires that four cases (a, b, c and d) of different combinations of i, j amd m must be investigated.

Fortunately, higher buckling modes resulting from assuming functions of different shapes do not govern the buckling behavior of all the cases considered except the case of I-sections braced on one side only. This is so because in these cases the resulting buckling modes are uncoupled and in principle, such behavior is similar to imposing certain constraints on the freedom of the section to undergo one or more of the displacements of u, v and ϕ. This is analogous to the cases of enforced axis of rotation or prescribed plane of deflection. Such cases are known to give higher buckling loads than if the section is free (if its geometry allows) to displace and rotate, i.e. in a coupled buckling mode. Therefore, it has been concluded that higher buckling modes resulting from assuming displacement functions of similar shapes (n only) would give lower buckling loads than if functions of different shapes (i, j,m) are assumed. This conclusion is valid for the following cases:

- channel section braced on one or both sides
- zee-section braced on one of both sides
- I-section braced on <u>both sides</u>.

Contrary to these cases is the I-section braced on one side only. Equation (5.38b), which is based on displacement functions of different shapes, gives the following two possible solutions of the critical buckling load. Note that $i \neq j$.

$$j^2P_x - P = 0 \tag{5.1}$$

$$(i^2P_y - P + Q)[r_o^2(P_\phi - P) + Q\frac{d^2}{4} + \frac{F}{i^2}\cdot\frac{L^2}{\pi^2}]-(Q\frac{d}{2})^2=0 \tag{5.2}$$

On the other hand, Eq. (38) (Chapter 2) which is based on displacement functions of similar shapes gives the following two possible solutions of the critical buckling load.

$$n^2P_x - P = 0 \tag{2.1}$$

$$(n^2P_y - P + Q)[r_o^2(P_\phi - P) + Q\frac{d^2}{4} + \frac{F}{n^2}\cdot\frac{L^2}{\pi^2}] - Q^2\frac{d^2}{4} = 0 \tag{2.2}$$

It is easily seen that Eqs. (5.1) and (5.2) for $i = j$ reduce to Eqs. (2.1) and (2.2), respectively. Therefore, if one solves Eqs. (5.1) and (5.2) for all integer values of i and j the lowest buckling load can be obtained.

Evidently, the smallest P given by Eq. (5.2) is for $j = 1$. Therefore investigating the possibility of higher buckling modes applies only to Eq. (5.2) and $j = 1,2,3,...$ must be considered only when this equation is tuilized.

Appendix 6

NOTES ON THE DESIGN CRITERIA

This appendix provides a record of the reasoning behind and justification for the different sections of the design criteria outlined in Chapter 6. Herein each section is given the same number as the corresponding section of Chapter 6 (except that they are preceded by the letter X for cross-referencing.

X.6.1 Introduction

The design procedure suggested in Chapter 6 is based on the ultimate capacity of the column, utilizing a conservative estimate of the shear rigidity Q and rotational restraint F of the wallboards acting as bracing diaphragms. A factor of safety (F.S.)[52] of 1.92 on the ultimate loads is incorporated in the method of analysis.

Tests of 11 diaphragm braced assemblies as reported in Chapter 5, Experimental Verification of the Theory, substantiate the theoretical findings of the present investigation on which the design procedure is based.

In order to achieve better approximation of the exact buckling load, higher buckling modes based on assumed displacement functions of similar as well as of different shapes have been investigated in Chapter 2 and Appendix 5, respectively.

X.6.3 Method of Analysis

Comments regarding inelastic analysis and the initial imperfections are given below.

• Load Capacity P_r. Computation of the amplitudes of deflections C_1 as well as the rotation E_1, and then the maximum shear strain γ_{max} and maximum rotation ϕ_{max}, are essential for checking the diaphragm adequacy. However, since deflections and rotation become indefinitely large as P_{cr} is reached, values of these parameters are computed at load levels equal to λP_{cr} where λ is a trial reduction factor less than 1.0, for example, $\lambda = 0.98, 0.96, 0.94, \ldots$. The factor λ is so chosen that the computed γ_{max} and ϕ_{max} do not exceed those available by the bracing diaphragm (for additional explanation regarding λ, see Section 2.7 of Chapter 2).

· Possibility of Higher Buckling Modes. Higher buckling modes are conventionally associated with buckling in more than one half-wave, i.e. $n > 1$. In some cases, depending on the relative magnitude of the diaphragm characteristics and the column stiffness, higher buckling modes govern the behavior of the stud. Section 4.2 of Chapter 4 includes a numerical investigation conducted to examine the possibility of higher buckling modes. In the numerical investigation the variation of the diaphragm shear rigidity Q and the rotational restraint F, as well as the column's flexural and torsional rigidities, were chosen to be within the practical range of wall stud construction. The results indicate that higher buckling modes do not govern the behavior of studs of channel, zee and I-sections braced on both sides. Therefore, for these cases governing equations based on $n = 1$ are listed in Section 6.4.1.1. However, if a diaphragm of unusual characteristics is utilized it is recommended that the possibility of higher buckling mode be checked. For this purpose Eqs. (44) and (45) for channel sections and Eq. (48) for zee-sections can be used (see Sections 2.4.3.1, 2.4.3.2 of Chapter 2). It should be noted that higher buckling modes do not govern the behavior of I-sections braced on both sides, regardless of the relative stiffness of the stud and the diaphragm (see Section 2.4). However, for sections braced on one side only, higher buckling modes are possible in some cases and such a possibility should always be considered. Hence for these cases, governing equations based on n = 1,2,3, ... are listed in Section 6.4.1.2. Higher buckling modes based on displacement functions of different shapes influence only the I-section braced on one side. This has been indicated in the conclusion and the end of Appendix 5. For this purpose Eq. (9) of Section 6.4.1.2 gives a flexural buckling load $P = P_x$ based on n = 1, while Eq. (10) (torsional-flexural buckling) requires that the possibility of higher buckling modes be investigated (i.e. n = 1,2,3,...).

• Values of n. In the design procedure (Section 6.3) it has been suggested to use n = 1,2,3,...6. Such a suggested number of n's is not mandatory; it can be increased or decreased depending on the case under consideration. However, in all the cases considered in the numerical investigation, higher buckling modes have never occurred beyond n = 4. Yet, consideration of any value of n is a simple task if computer subroutines are utilized in the analysis.

• Required Q and F if P_{all} is known (channel braced on both sides). Figure 16 illustrates the two buckling modes of channel sections braced on both sides, namely flexural and torsional-flexural. These two modes are given by Eqs. (2) and (3), respectively. If at a given load P_1 (see Fig. 16), Q_I obtained from Eq. (2) is larger than Q_{II} obtained from Eq. (3) with F = 0, i.e. $Q_I > Q_{II}$, flexural buckling governs and Q_I obtained from Eq. (2) controls, since with Q_{II}, flexural buckling will occur at a load smaller than that given by P_1. On the other hand, if at a load P_2, $Q_{II} > Q_I$, then otrsional-flexural buckling governs; therefore Q_{II} controls and in sich a case, including F in the analysis will result in economical design. Otherwise larger values of Q would be required.

• <u>Value of n Associated with P_a.</u> It has been stated in different parts of the design procedure that when inelastic behavior governs (i.e. $P_{cr}/A > 0.5Q_A\sigma_y$), then knowing P_{cr} (elastic), the inelastic buckling load P_a may be determined by Eq. (24) (AISI formula). Accordingly, in computing γ_{max} and ϕ_{max} from equations of Section 6.4.2 it has been conclusively assumed that the value of n used in these equations is the value of n corresponding to P_{cr}. Such a consideration has been examined numerically and it has been found that the lowest value of the load capacity (i.e. λP_a corresponding to $\gamma_{max} < \gamma_d$ and $\phi_{max} < \phi_d$) is always associated with that particular value of n corresponding to P_{cr}. However, in case that such an assumption is to be verified, the procedure can be summarized in the following. Having obtained P_{cr} and the corresponding n, determine P_a from Eq. (24). Consider n = 1,2,3,... and for each value of n, check the diaphragm adequacy by using equations of Section 6.4.2, calculating γ_{max} and ϕ_{max} and hence determining the trial load $P_r = \lambda P_a$, at which γ_{max} and ϕ_{max} are smaller than γ_d and ϕ_d of the diaphragm, respectively. The output of this algorithm is a set of load capacities P_r, each corresponding to a certain value of n. The lowest value of these loads determines the load capacity of the stud. Consequently, check wehther or not it is associated with the same value of n corresponding to P_{cr}. Such a procedure can be executed by the computer programs of Appendix 4.

X.6.4.2 Initial Imperfections (Eqs. 11)

The initial imperfections are the primary cause of deflections and rotations prior to the state of instability of the column. The required strength of the bracing is a function of these initial imperfections. In order to obtain a method of analysis for practical design it is necessary to investigate real rather than ideally straight columns. This is so because the rigidity and restraint calculated for bracing an ideal column are not sufficient to achieve the required bracing of an imperfect column(9). Hence it is essential that the suggested design criteria, which will be explained in detail in the next sections, should provide a check to insure that the shear strength and the rotational capacity of the diaphragm are not exceeded before the design load is reached. Such a check will be made by calculating the additional deflections and rotation corresponding to the design load. Then calculate the maximum shear slope γ_{max} and rotation ϕ_{max} of the diaphragm, and compare these values with the available diaphragm shear strength and rotational capacity.

The amplitude of the initial imperfections may be taken from about 1/500 to 1/1000 of the column length. However, initial imperfections accounting for initial sweep plus accidental load eccentricities may be considered according to the following tentatively suggested formulas:

$$C_o = 2(L/700)$$

$$\text{and} \qquad D_o = L/700$$

Based on limited information available, the amplitude of the initial twist is arbitrarily taken equal to 0.0006 radians per foor of length[17].

X.6.4.3 Inelastic and Local Buckling Behavior

Depending on the values of Q and F, and the slenderness of the stud, the compressive stress may exceed the proportional limit σ_p of the stud material. As a result buckling will occur at a stress lower than that predicted by the elastic governing equations. To modify the elastic design equations, Section 6.4, so as to account for the inelastic range[12,15,17,38], E will be replaced by E* and G by G* = G(E*/E), where E* is the inelastic modulus corresponding to the average stress level (σ) and is given by:

$$E^* = E\left[\frac{\sigma(\sigma_y - \sigma)}{\sigma_p(\sigma_y - \sigma_p)}\right]$$

In addition, it is assumed that the behavior of the diaphragm remains elastic until failure.

If inelastic buckling governs the behavior of the stud, then two methods are available to compute the inelastic buckling load. Both methods are recorded herein, even though the second method is recommended for the design procedure.

a) Iterative Approach

1) From Eqs. (25) and (27) find the value of E* corresponding to P_{cr}.

2) Substitute E* for E in the elastic governing equation and compute the new value of E*.

3) Find the corresponding value of P_{cr} and compare with the previous value. Repeat the procedure until the loads converge to the desired accuracy.

Such an iterative procedure is not desirable for design use, though it is accurate.

b) AISI Formula[1]

In a previous design recommendation[17] the AISI formula of Section 3.6.12 has been used for the design of diaphragm braced columns.

In deriving these formulas, the general form of the inelastic buckling stress vs. the slenderness ratio is assumed, obviating the necessity of obtaining the inelastic buckling stress by iteration, as may be required when the buckling stress relation is obtained from an assumed analytical stress-strain relation.

The formula gives a limit to the buckling load of the stud in the inelastic range by the following value (without a factor of safety):

$$P_a = A(\sigma_y - \frac{\sigma_y^2}{4\sigma_{cr}})$$

where P_a = inelastic buckling load.

The effects which local buckling of thin-walled compression members can have in reducing the column strength is presented in Section 3.6.1 of the current AISI Specification by a form factor Q, here designated as Q_A. If this form factor is less than 1.0, then replacing σ_y by $Q_A\sigma_y$ in all equations involving σ_y will furnish design formulas which provide adequate safety against local buckling and accounts for cases in which combinations of overall and local buckling occur.

6.4.4 Diaphragm Characteristics

In order to predict the behavior of the braced stud it is necessary to know the nature and magnitude of the restraint provided by the wallboards.

The two important parameters which characterize the bracing diaphragm are its shear rigidity Q and its rotational restraint F. These parameters are determined experimentally. Methods of testing as well as values of Q and F of different wall materials are presented in Chapter 5. The specific values obtained in the test program are only indicative and design values should be obtained from tests representing the actual structure.

In a previous "Design Recommendations for Diaphragm-Braced Beams, Column and Wall Studs,"(17) a recommended value of reliable shear rigidity Q_r was given by,

$$Q_r = G_r' \cdot w$$

or $$Q_r = \frac{2}{3}G' \cdot w$$

where G' is the shear stiffness obtained from a cantilever shear test at 0.8 of the ultimate load of the diaphragm and w is the width of the diaphragm contributing to the bracing of one member.

Similarly, the design value of the shear strain of the diaphragm γ_d is determined from the same cantilever test and is given by

$$\gamma_d = \frac{\Delta_d}{a}$$

where Δ_d is the shear deflection at $0.8P_{ult}$ and a is the dimension of the shear diaphragm perpendicular to the test load direction.

Since F is as important as Q in providing for the stability of studs subjected to torsional-flexural buckling, it would be reasonable to adopt the same reduction factors of Q and γ for the rotational restraint F and the rotational capacity ϕ of the diaphragm. The details of the test set-up to evaluate F and ϕ for a certain diaphragm are included in Chapter 5. Hence a reliable value of the rotational restraint F is given by

$$F_r = \frac{2}{3}F$$

where F is the rotational restraint coefficient at $0.8P_{ult}$. Similarly, the design rotational capacity of the diaphragm $_d$ is obtained at $0.8P_{ult}$ and it represents the amount of rotation in radians that the diaphragm can undergo at $0.8P_{ult}$ (see Figs. 19, 20 and 21).

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TABLE 1

COEFFICIENTS K FOR VARIOUS END CONDITIONS (n=1)

COEFFICIENTS END CONDITIONS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}	K_{11}	K_{12}
$u''=v''=\phi''=0$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$u''=v''=\phi'=0$	1.0	1.0	1.0	4.0	0.849	0.849	0.849	0.75	0.849	0.849	0.849	0.849
$u''=v'=\phi'=0$	4.0	1.0	0.849	4.0	0.849	0.849	1.0	0.75	1.0	1.0	0.849	0.849
$u''=v'=\phi''=0$	4.0	1.0	0.849	1.0	1.0	1.0	0.849	1.0	0.849	0.849	0.849	0.849
$u'=v'=\phi''=0$	4.0	4.0	4.0	1.0	0.849	0.849	0.849	1.0	0.849	0.849	0.849	0.849
$u'=v''=\phi''=0$	1.0	4.0	0.849	1.0	0.849	0.849	1.0	1.0	1.0	1.0	0.849	0.849
$u'=v''=\phi'=0$	1.0	4.0	0.849	4.0	1.0	1.0	0.849	0.75	0.849	0.849	1.0	1.0
$v'=v'=\phi'=0$	4.0	4.0	4.0	4.0	1.0	1.0	1.0	0.75	1.0	1.0	1.0	1.0

Notes

$u = v = \phi = 0$ at $Z = 0,L$ for all cases

All end conditions shown are for $Z = 0,L$

TABLE 2

DIAPHRAGM PROPERTIES USED IN THE TESTS

TYPE OF DIAPHRAGM	FASTENER SPACING	G'	γ_d in/in	F' k.in/in.rad.	ϕ_d rad.
$\frac{5}{8}$" GYPSUM	6"	2300	0.0041	NOT TESTED	
	9"	2700*	0.0132	NOT TESTED	
$\frac{3}{8}$" GYPSUM	9"	2050	0.014	0.055	0.15
	11"	1600	0.013	0.0355	0.15
$\frac{1}{2}$" HOMOSOTE	11"	845	0.012	0.024	0.175
$\frac{1}{2}$" CELOTEX	7"	620	0.0083	0.0135	0.21
	11"	490	0.0078	0.0094	0.21
$\frac{1}{2}$" IMPREGNATED CELOTEX	7"	660	0.0096	0.021	0.23
	11"	530	0.0086	0.014	0.23
$\frac{1}{2}$" HEAVY IMPREG. CELOTEX	11"	940	0.0106	0.018	0.18

G' = Diaphragm shear stiffness at $0.8P_{ult}$

γ_d = shear strain at $0.8P_{ult}$

F' = rotational restraint coefficient at $0.8P_{ult}$

ϕ_d = rotational capacity of the diaphragm at $0.8P_{ult}$

* Fastened along 4 sides

TABLE 3

SUMMARY OF TEST RESULTS

TYPE OF TEST	WALL MATERIAL AND FASTENER SPACING	Q	F	LOADS THEORY	LOADS TEST	P_{TEST}/P_{THEORY}	TYPE OF FAILURE
5B	$\frac{3}{8}$" GYP. @ $11\frac{1}{2}$"	27.2	0.071	24.2	23.4	.97	TOR. FLEX.
6C	$\frac{1}{2}$" CELOTEX @ $11\frac{1}{2}$"	11.8	0.019	16.5	15.5	.94	FLEX.
7C	$\frac{1}{2}$" CELOTEX @ 7" IMPREG.	19.8	0.042	24.0	23.7	.99	FLEX.
8D	$\frac{3}{8}$" GYP. @ $11\frac{1}{2}$"	27.2	0.071	28.8	26.5	.92	FLEX.
9D	$\frac{1}{2}$" CELOTEX @ $11\frac{1}{2}$" IMPREG.	22.4	0.06	27.4	26.9	.98	FLEX.
10C	$\frac{3}{8}$" GYP. @ $11\frac{1}{2}$"	13.6	0.036	14.7	14.5	.985	T.F.
11D	$\frac{3}{8}$" GYP. @ $11\frac{1}{2}$"	13.6	0.036	19.26	18.6	.97	T.F.
1A	$\frac{5}{8}$" GYP. @ 9"	41.5	NOT TESTED	11.3	11.5	1.02	*
2A	$\frac{1}{2}$" HOMOSOTE @ $11\frac{1}{2}$"	21.0	0.03	11.3	10.6	0.94	*
3A	$\frac{1}{2}$" HOMOSOTE @ $11\frac{1}{2}$"	10.5	0.015	5.93	6.0	1.01	T.F.
4A	$\frac{3}{8}$" GYP. @ $11\frac{1}{2}$"	15.5	0.029	6.44	5.0	0.78	T.F.

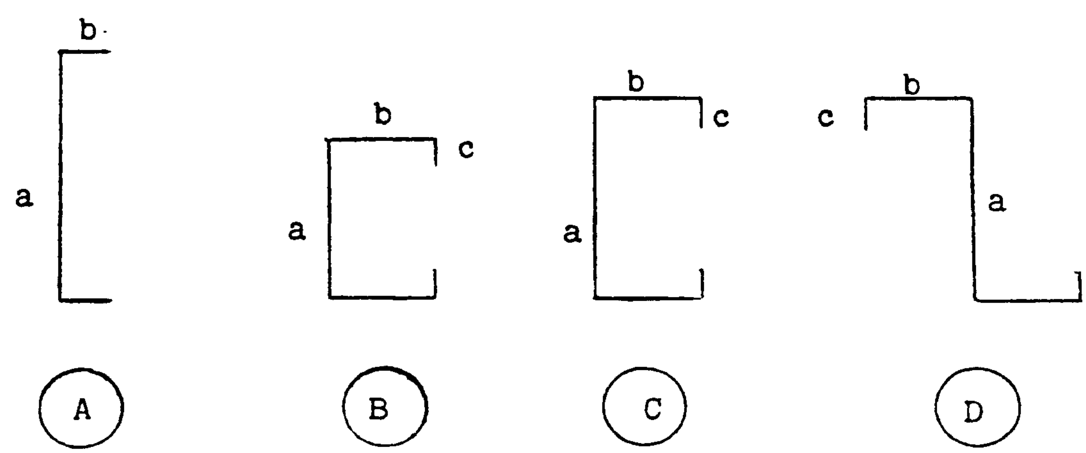
* Sudden local buckling

Table 4

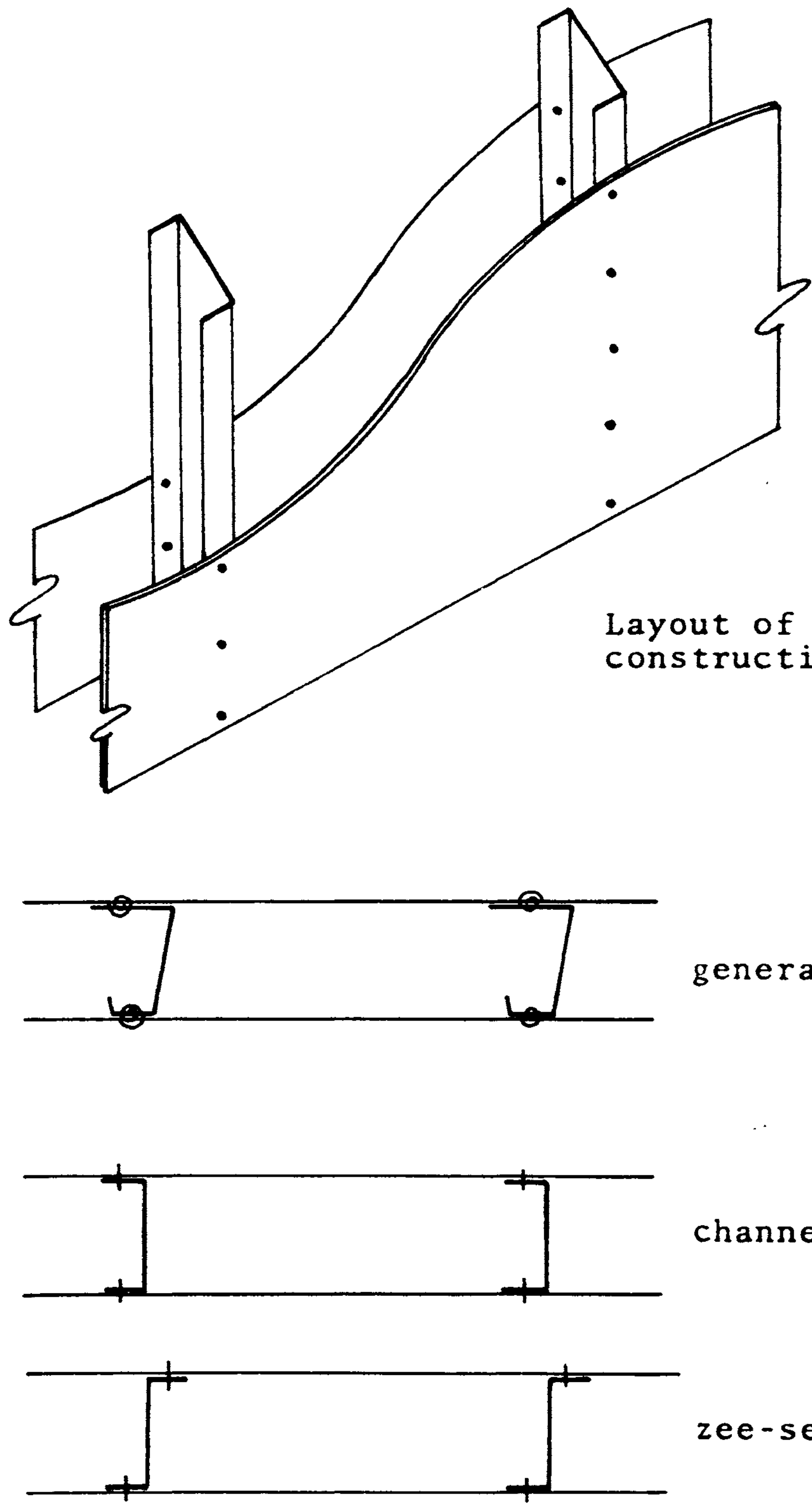
SECTIONS USED IN THE EXPERIMENTAL PROGRAM

Test No.	Type of Sec.	t	a	b	c
1A	A	0.061	3.628	0.88	0.0
2A	A	0.062	3.628	0.88	0.0
3A	A	0.062	3.625	0.88	0.0
4A	A	0.061	3.630	0.88	0.0
5B	B	0.106	2.506	1.75	0.62
6C	C	0.106	3.07	1.76	0.67
7C	C	0.106	3.07	1.75	0.66
8D	D	0.105	3.07	1.76	0.66
9D	D	0.105	3.07	1.76	0.66
10C	C	0.106	3.07	1.75	0.66

Dimensions shown are the average along the column length.

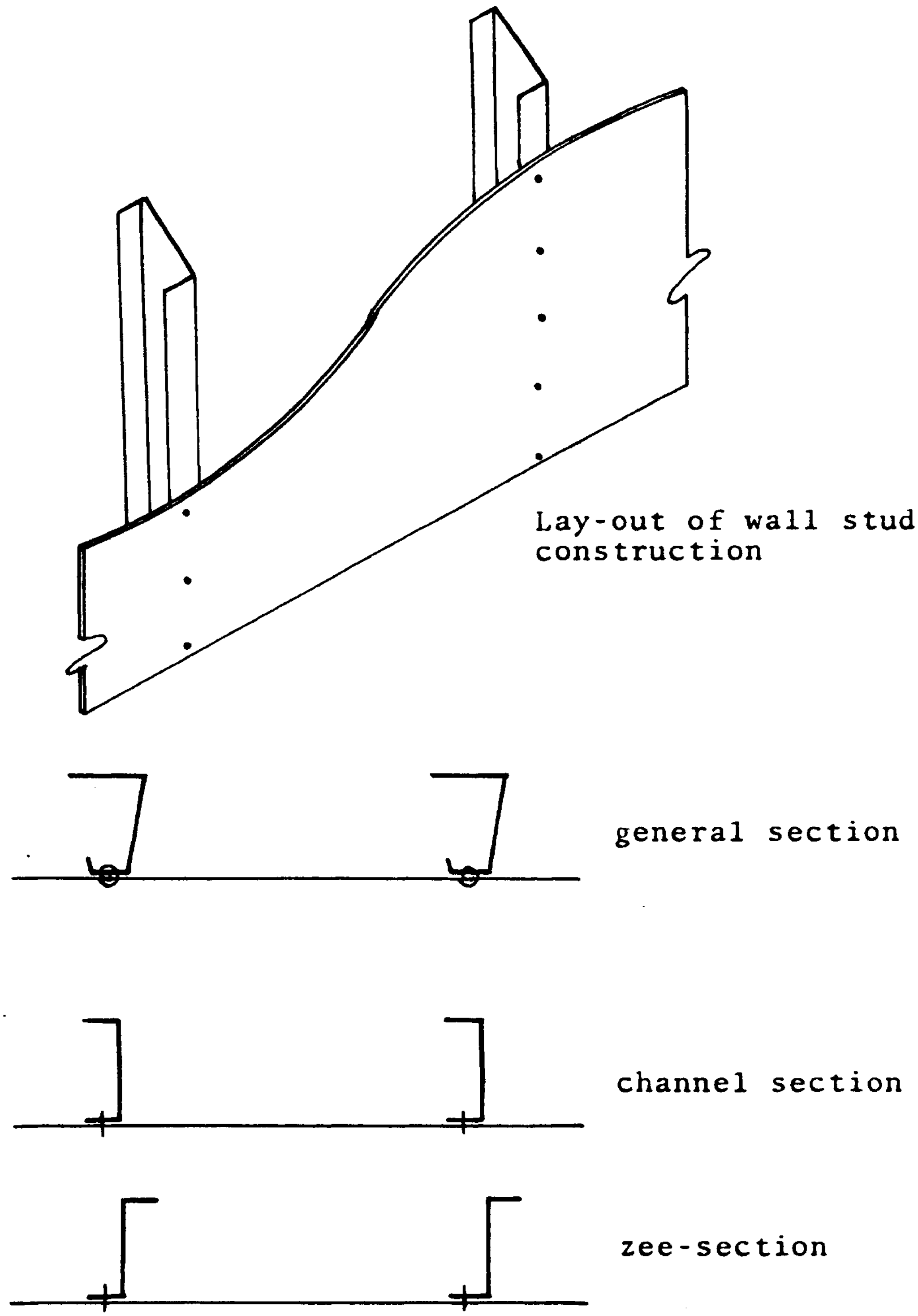


16 gage 12 gage 12 gage 12 gage



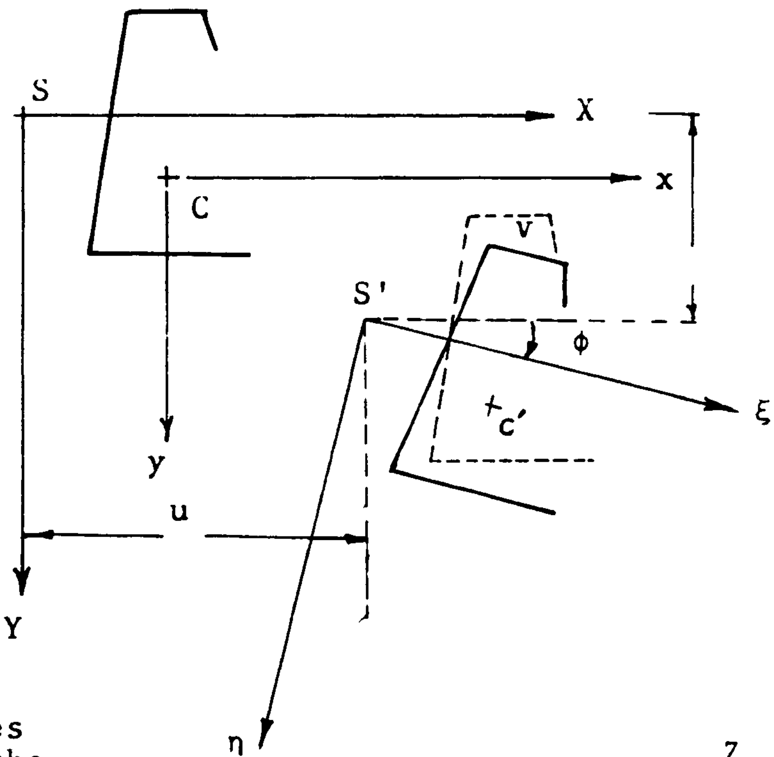
general and specific sections considered in the analysis

Fig. 1) Columns braced with diaphragms on both sides.



general and specific sections considered in the analysis

Fig. 2) Columns braced with diaphragms on one side.



X,Y are arbitrary axes passing through the shear center (S)

x,y axes passing through centroid C, parallel to and perpendicular to the diaphragm

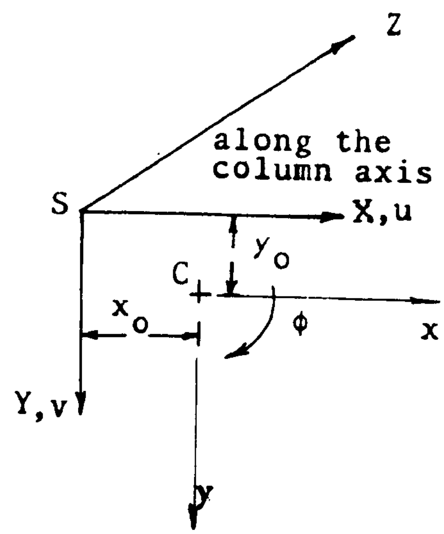


Fig. 3) Sign convention and displaced position of the column cross-section. (Ref. 5)

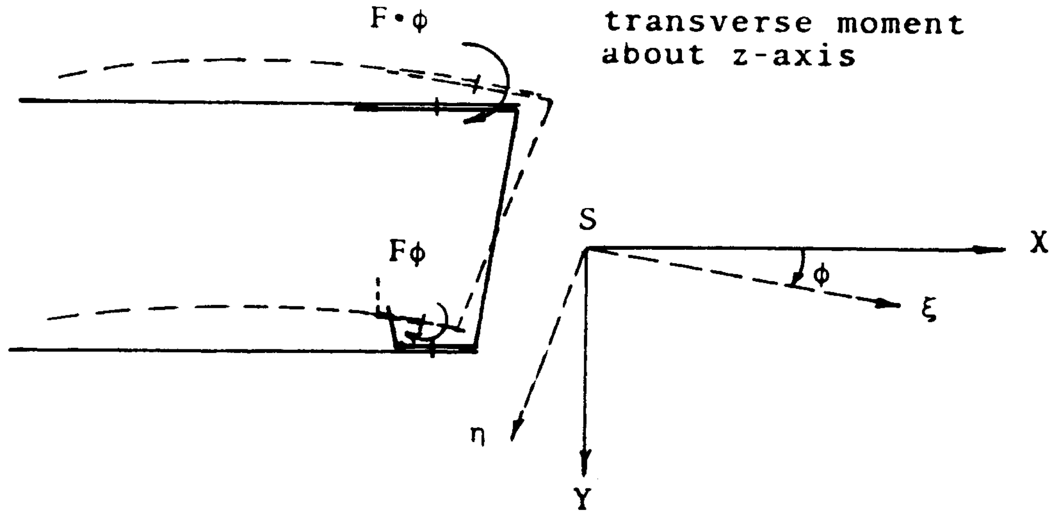
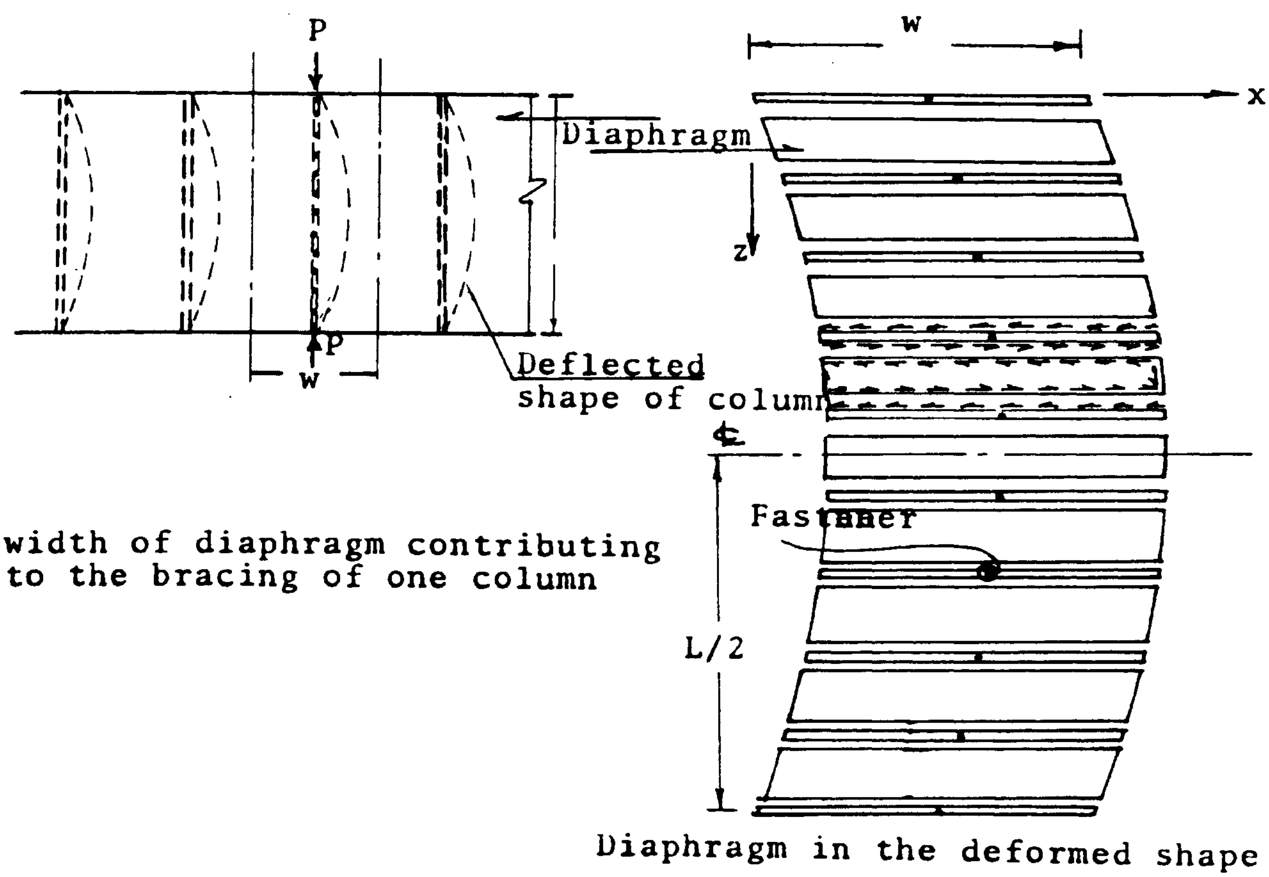
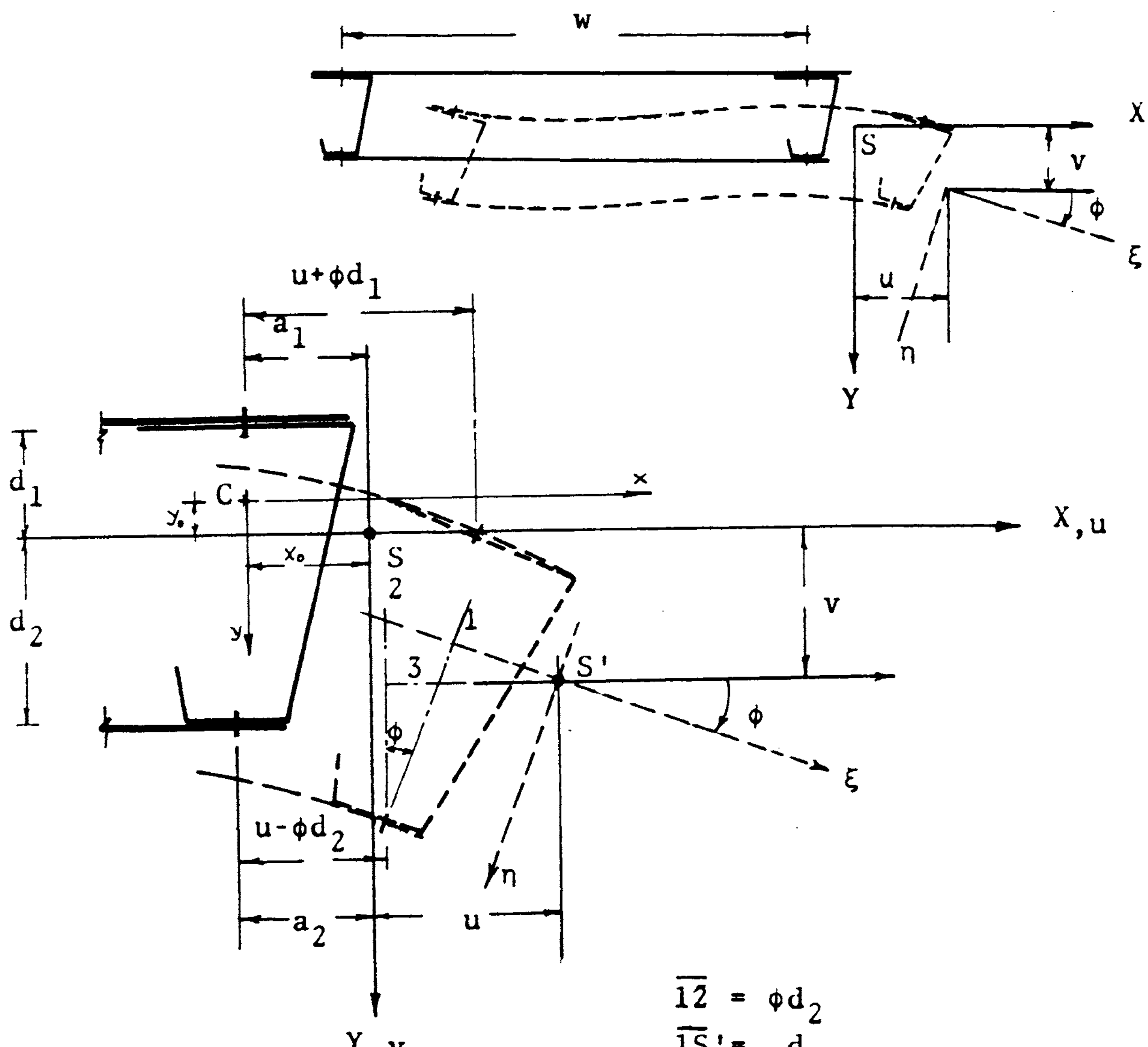


Fig. 4) Transverse rotation of the diaphragm.



w = width of diaphragm contributing to the bracing of one column

Fig. 5) In-plane shear deformation of the diaphragm.



$\overline{12} = \phi d_2$

$\overline{1S}' = d_2$

$\overline{2S}' = \phi d_2 + d_2$

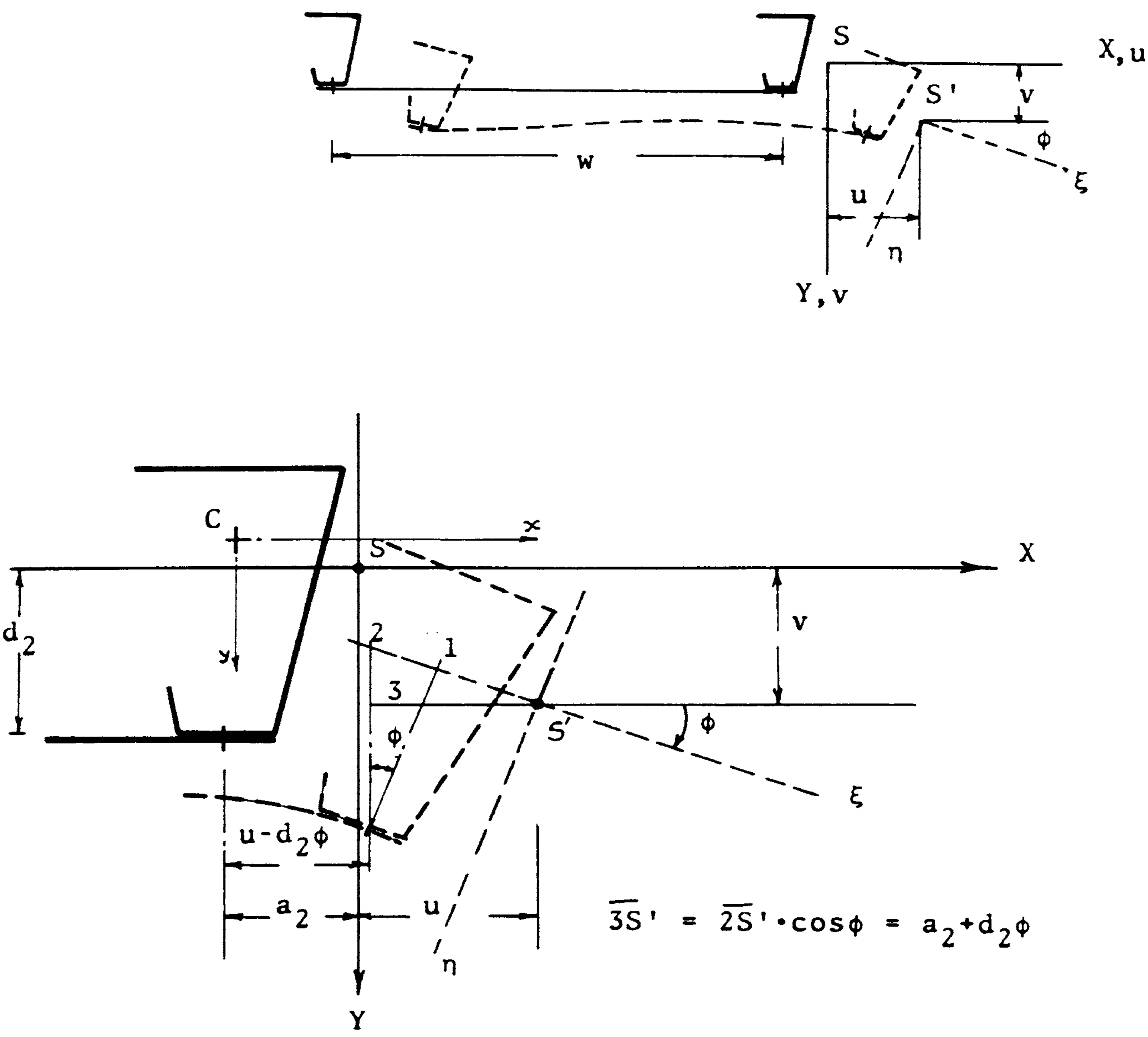
$\overline{3S}' = \overline{2S}'\cos\phi = a_2 + \phi d_2$

lateral displacement in the plane of the diaphragm:

bottom diaphragm $= a_2 + u - 3S' = u - \phi d_2$

top diaphragm $= u + \phi d_2$

Fig. 6) Generalized displaced position of column braced on both sides.



lateral displacement in the plane of diaphragm $u_D = a + u - 3S'$
$= u - d_2\phi$

Fig. 7) Generalized displaced position of column braced on one side only.

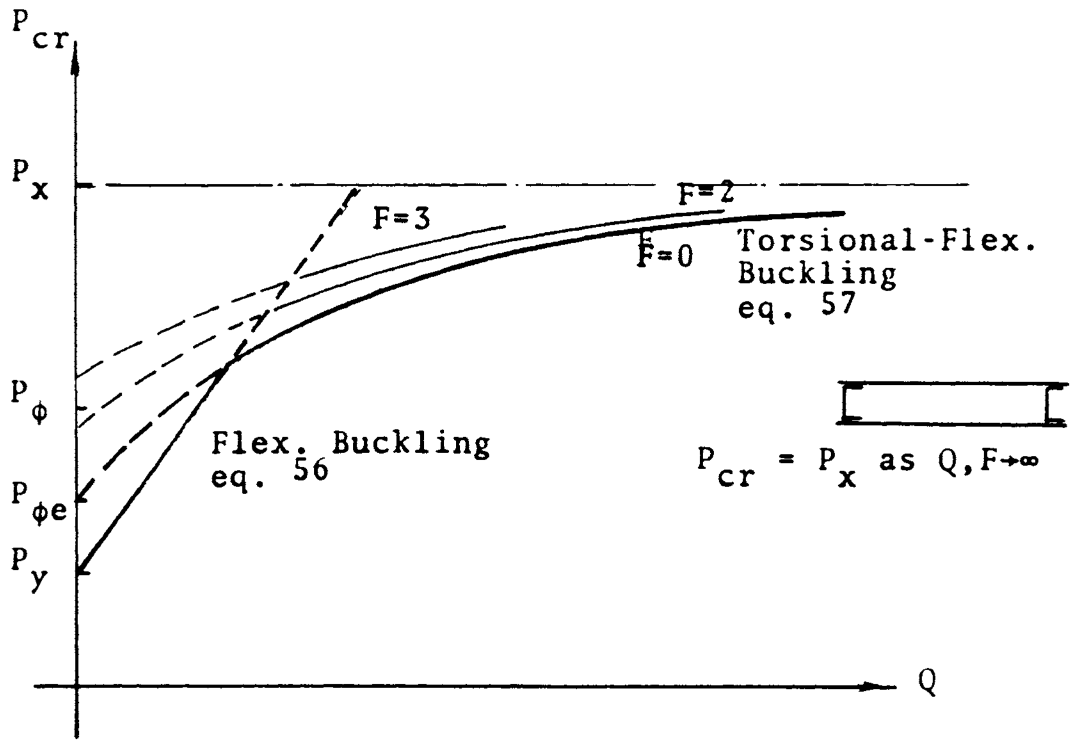


Fig. 8) P_{cr}, (Q,F) relationship for channel section braced on both sides.

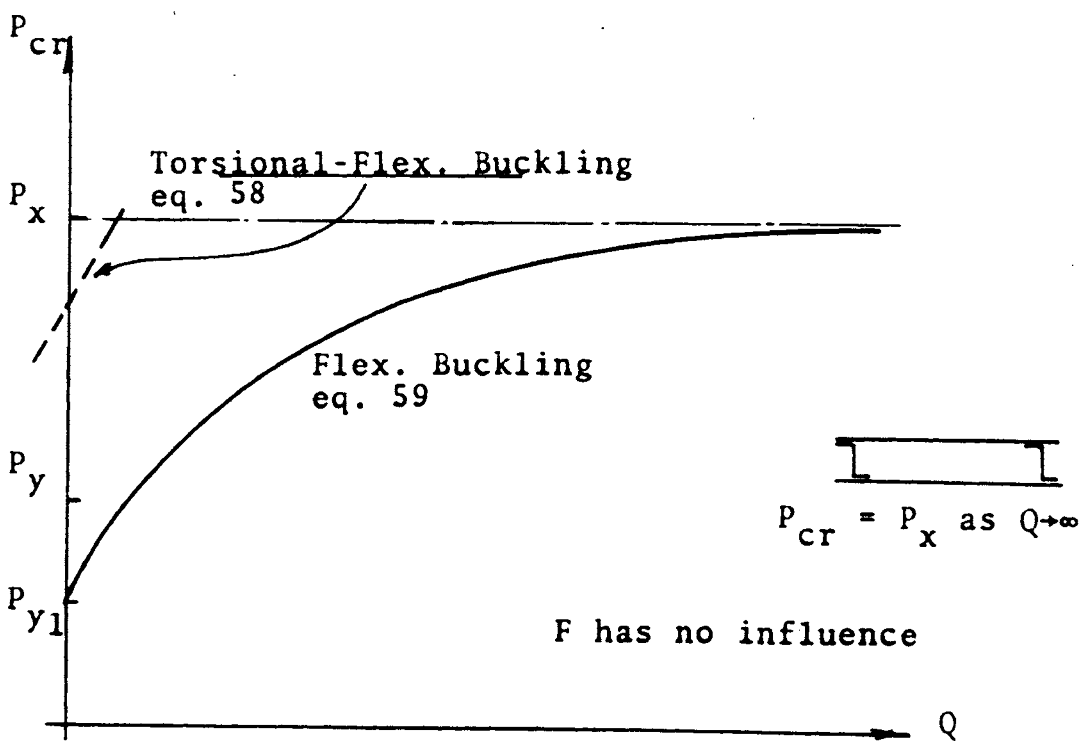


Fig. 9) P_{cr},Q relationship for zee-section braced on both sides.

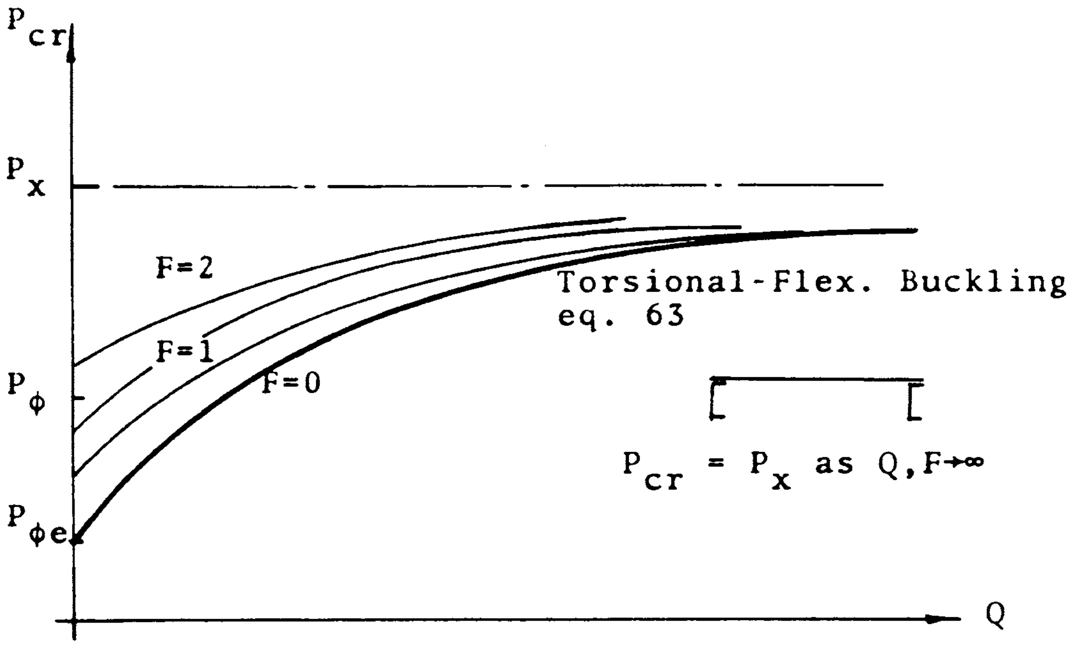


Fig. 10) P_{cr}, (Q,F) relationship for channel section braced on one side.

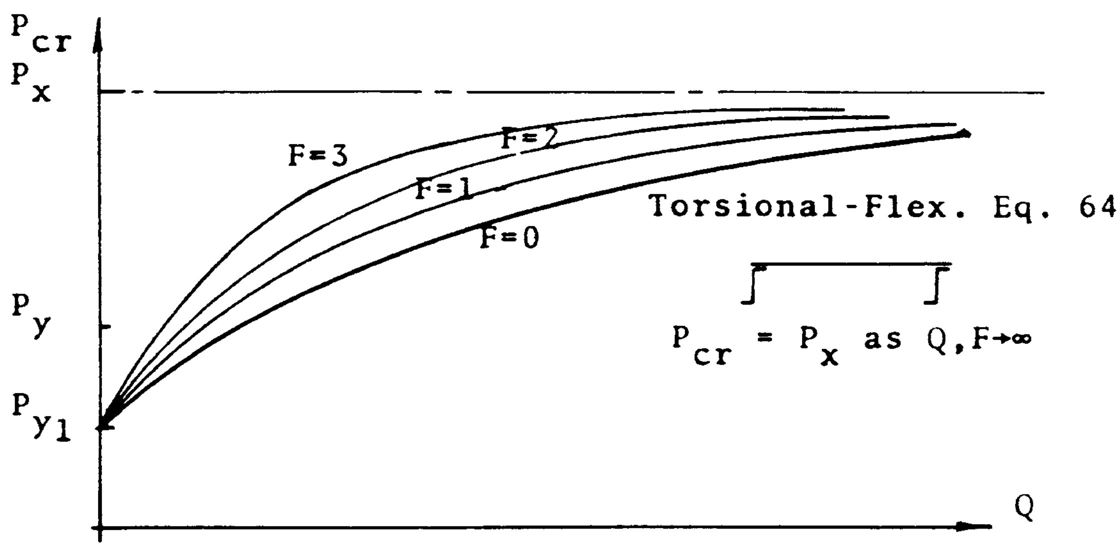


Fig. 11) P_{cr},Q relationship for zee-section braced on one side

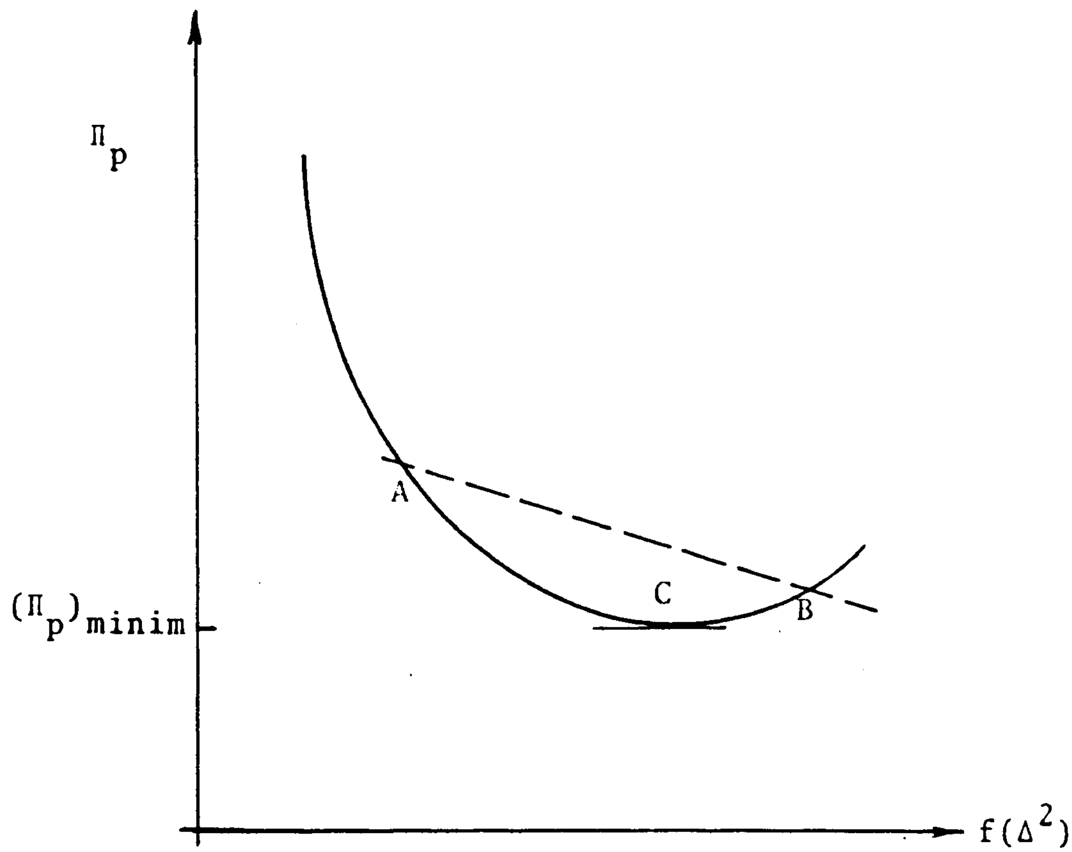
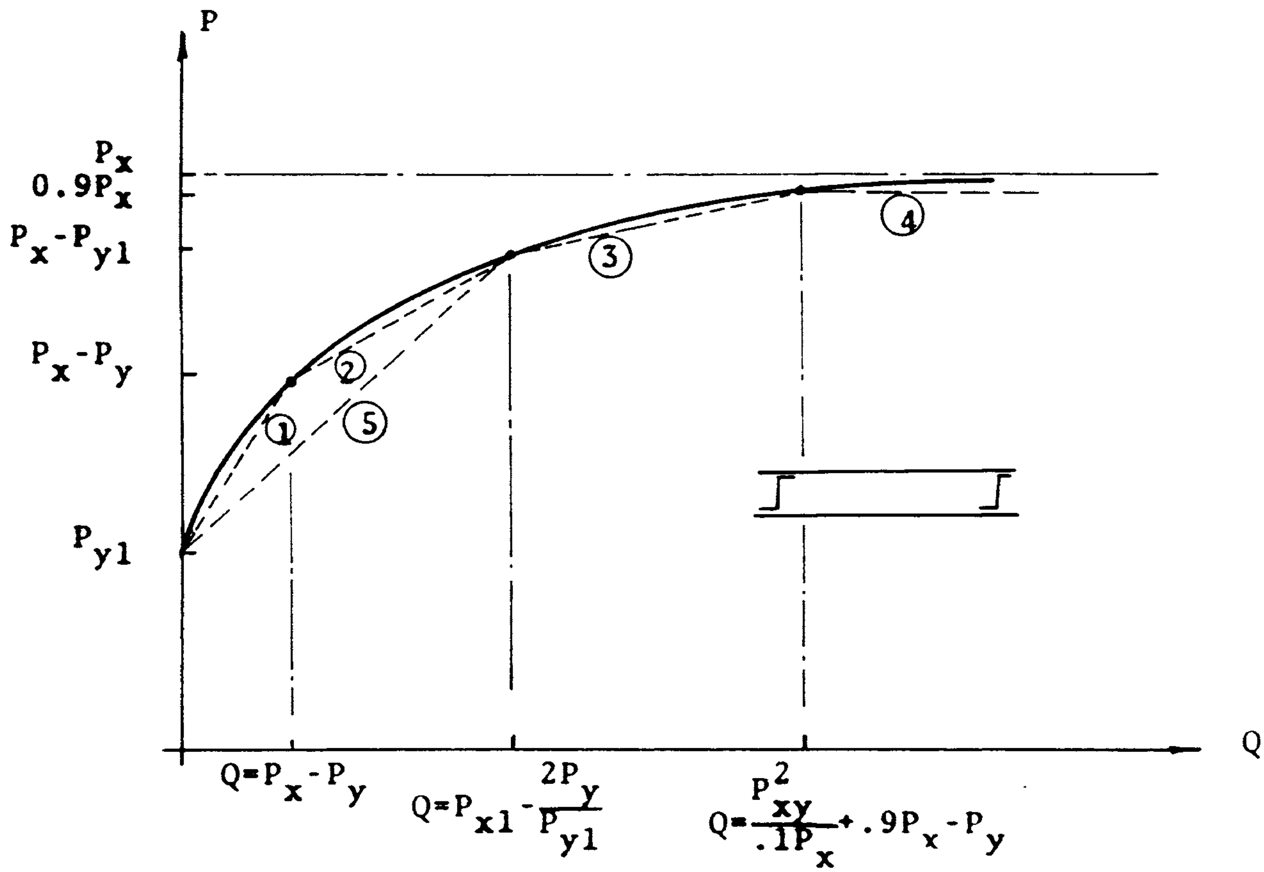


Fig. 12) Effect of constraint on the potential energy function Π_p (Ref. 35).



Equation of Line 1 $\quad P = P_{y1}+Q(P_x-P_{y1}-P_{xy})/(P_x-P_y)$

2 $\quad P = P_x-P_{xy}+(Q-P_x+P_y)\ \dfrac{P_{xy}-P_{y1}}{2P_y-P_{y1}-\dfrac{2P_y}{P_{y1}}}$

3 $\quad P = P_x-P_{y1}+(Q-P_{x1}+\dfrac{2P_y}{P_{y1}})\ \dfrac{.9P_x-P_x+P_{y1}}{\dfrac{P_{xy}^2}{.1P_x}+.9P_x-P_y}$

4 $\quad P = 0.9P_x$

Fig. 13) Approximation by Piecewise Linear Function

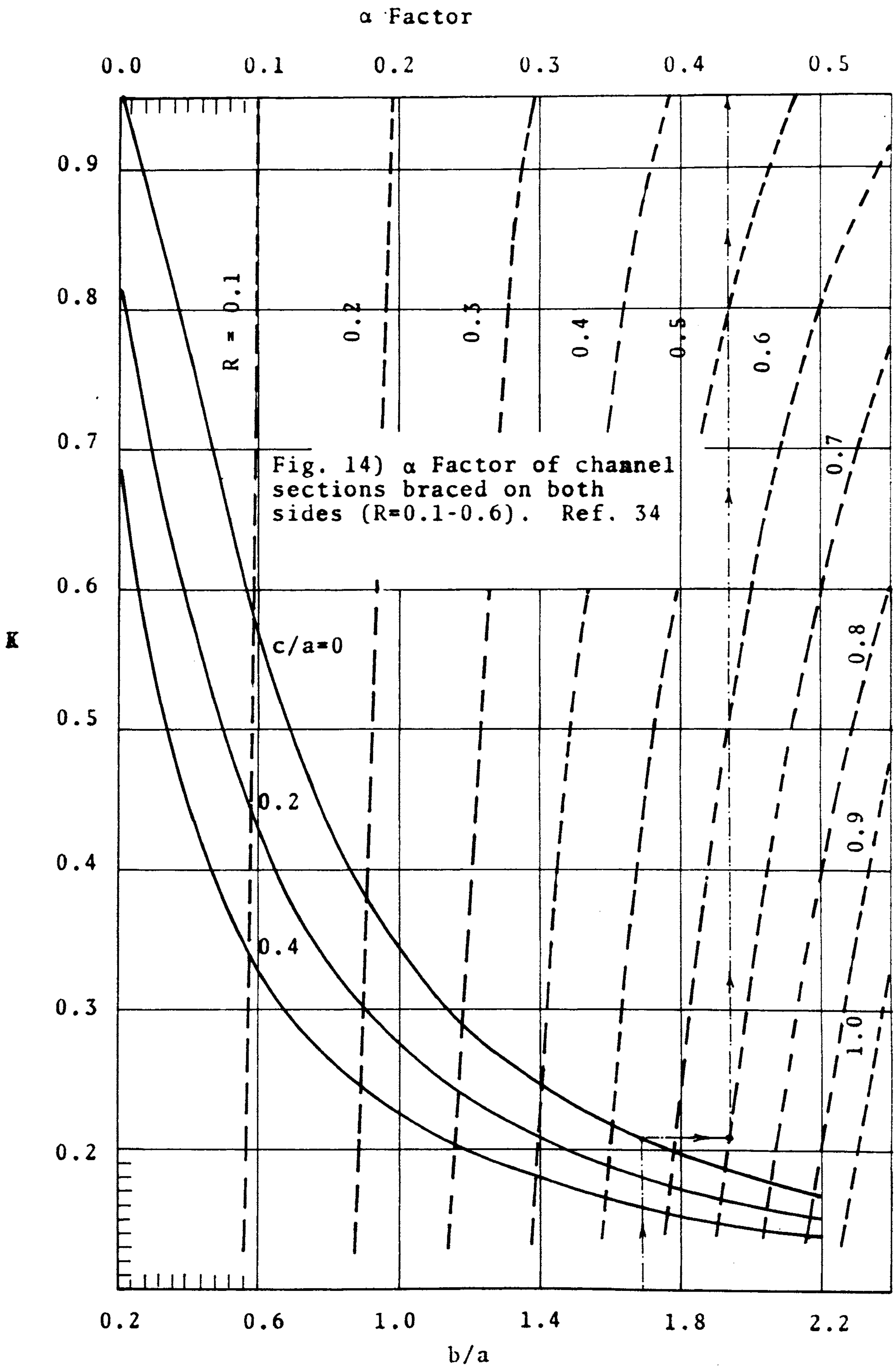


Fig. 14) α Factor of channel sections braced on both sides (R=0.1-0.6). Ref. 34

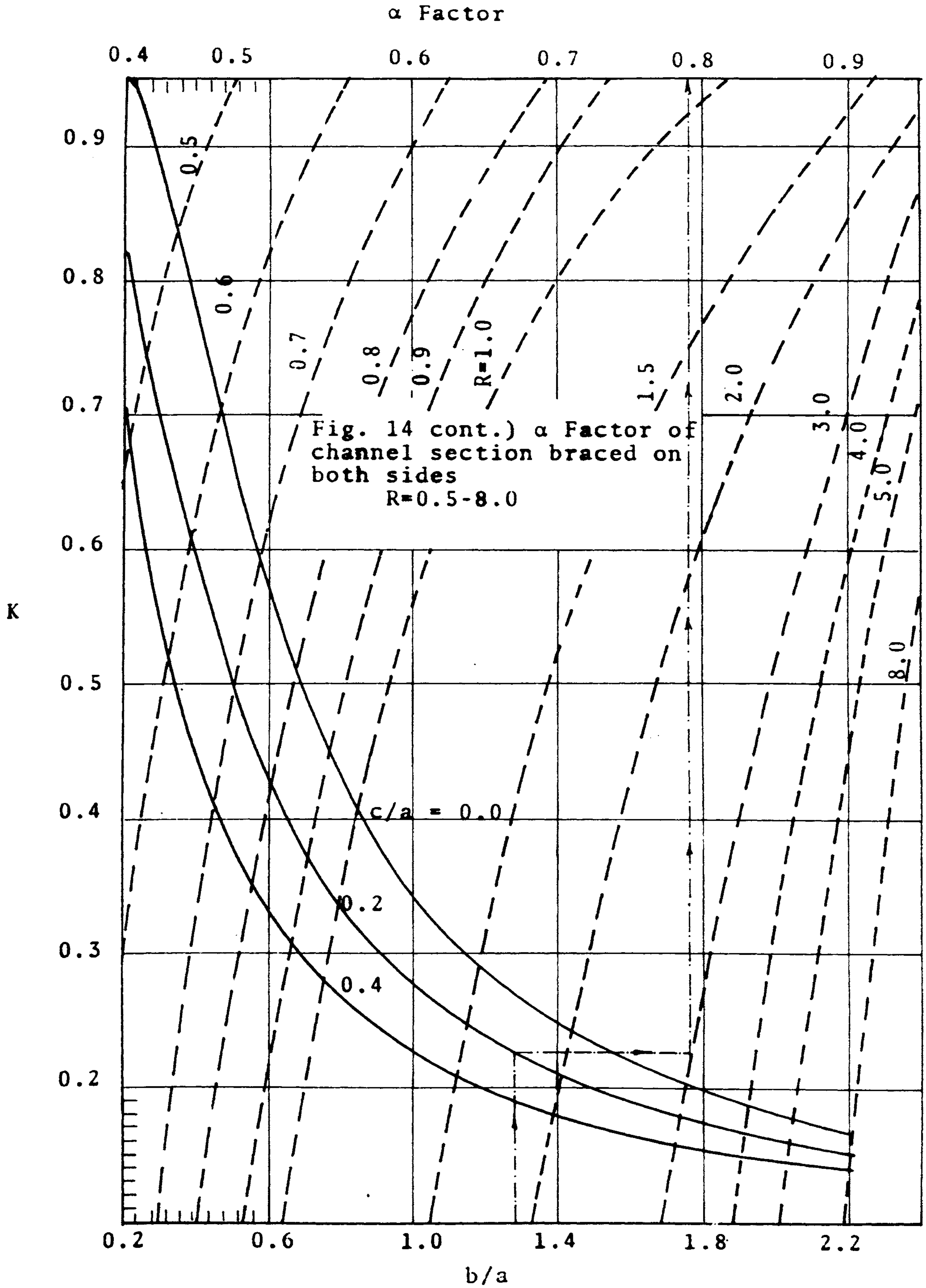


Fig. 14 cont.) α Factor of channel section braced on both sides
R=0.5-8.0

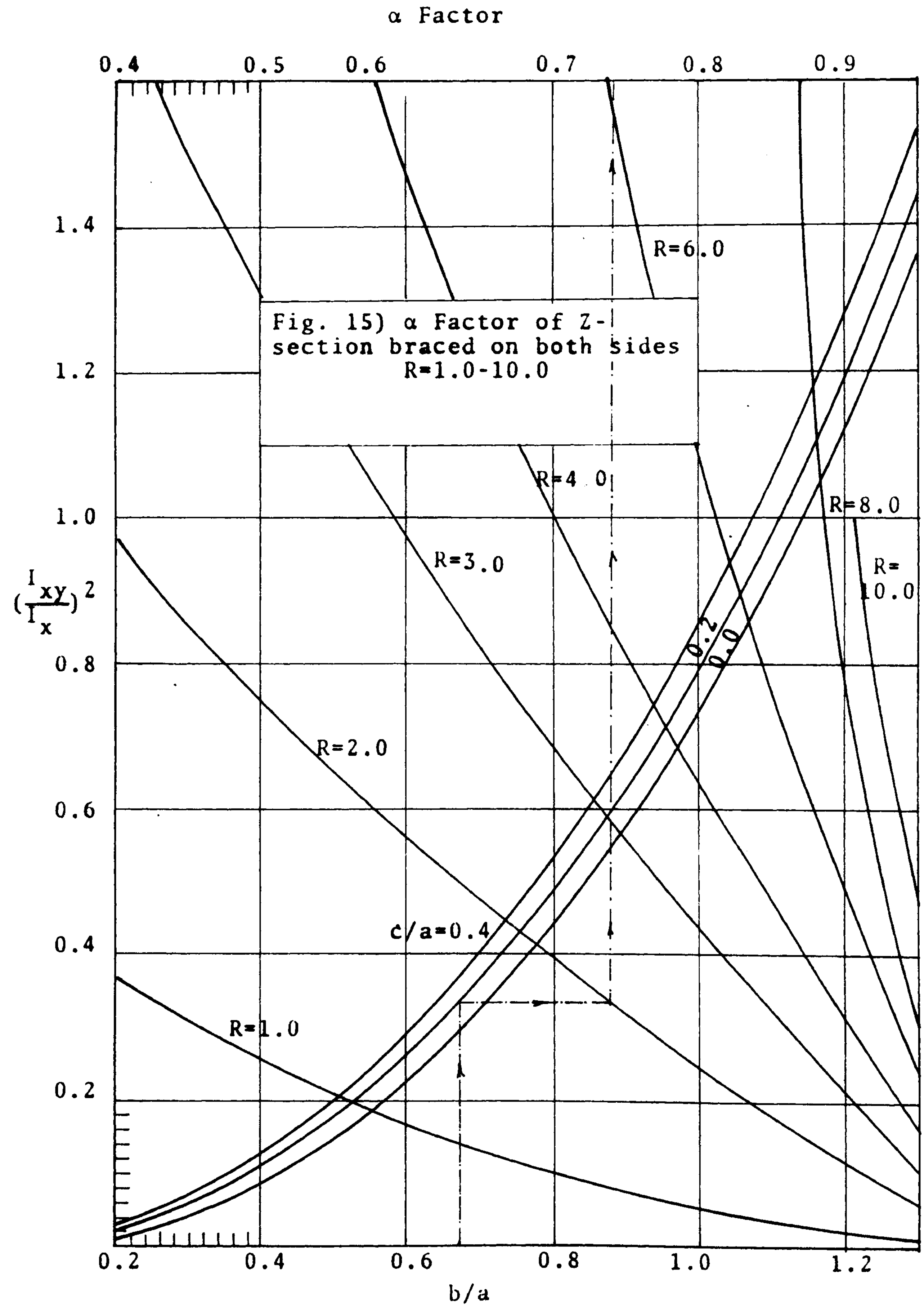


Fig. 15) α Factor of Z-section braced on both sides R=1.0-10.0

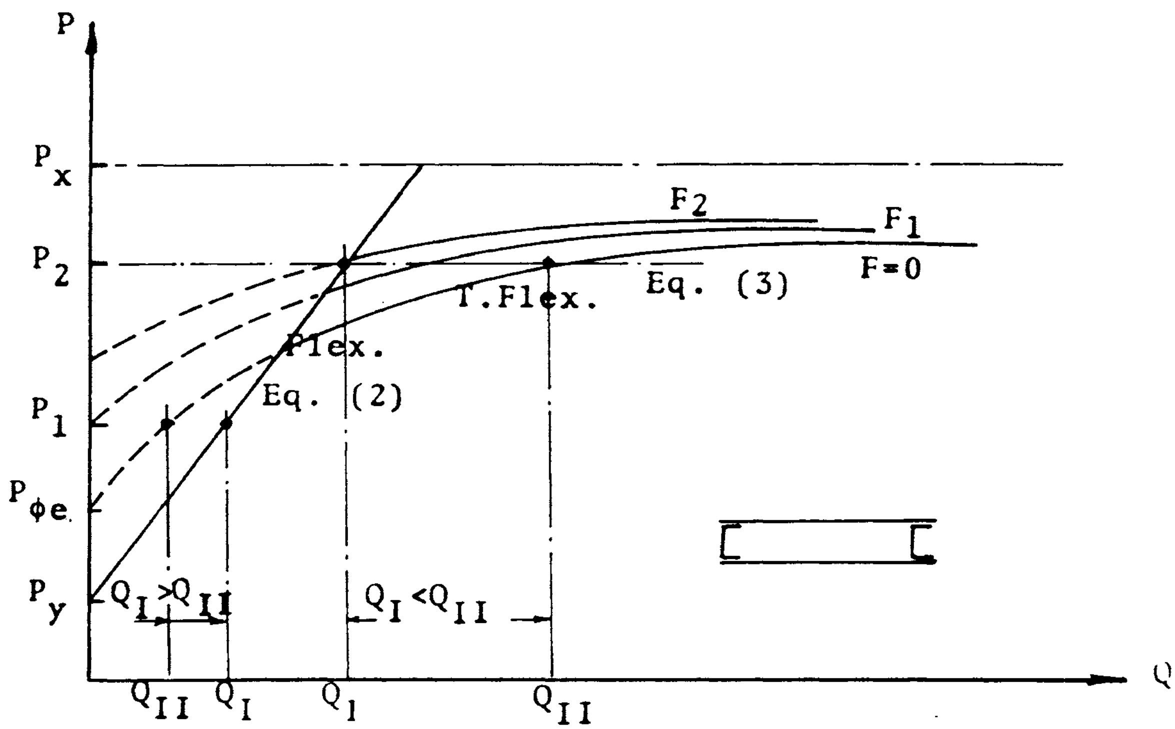


Fig. 16) Q and F required for a certain load P.
DESIGN PROCEDURE

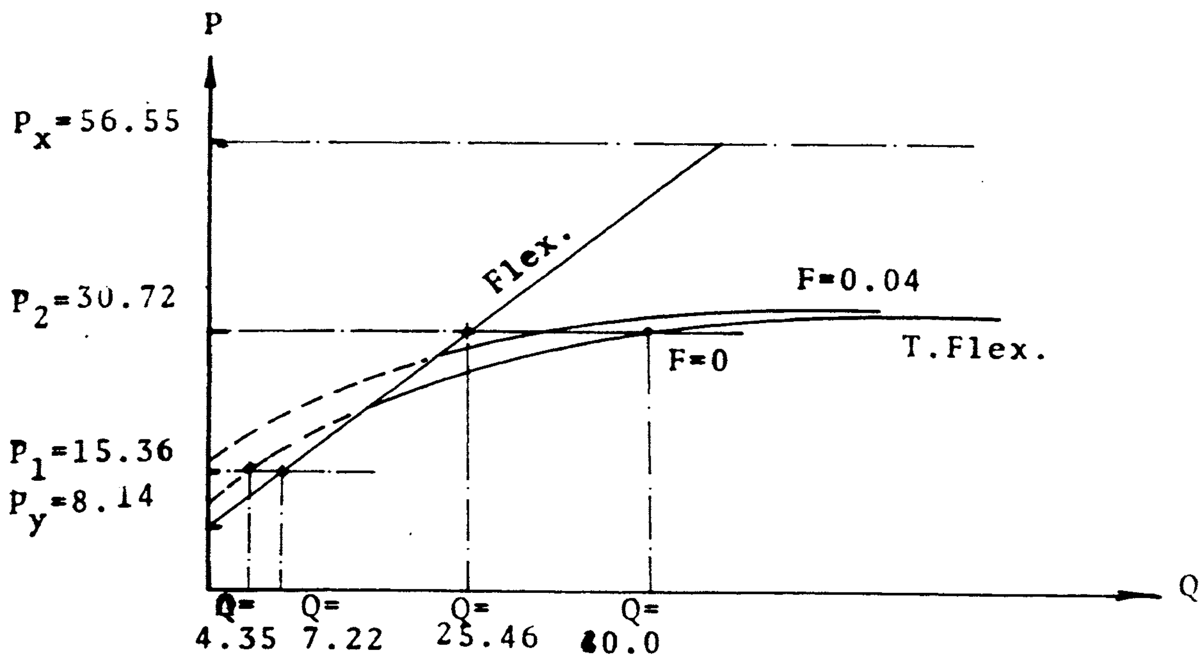


Fig. 17) Q and F required for a certain load P.
DESIGN EXAMPLE 2

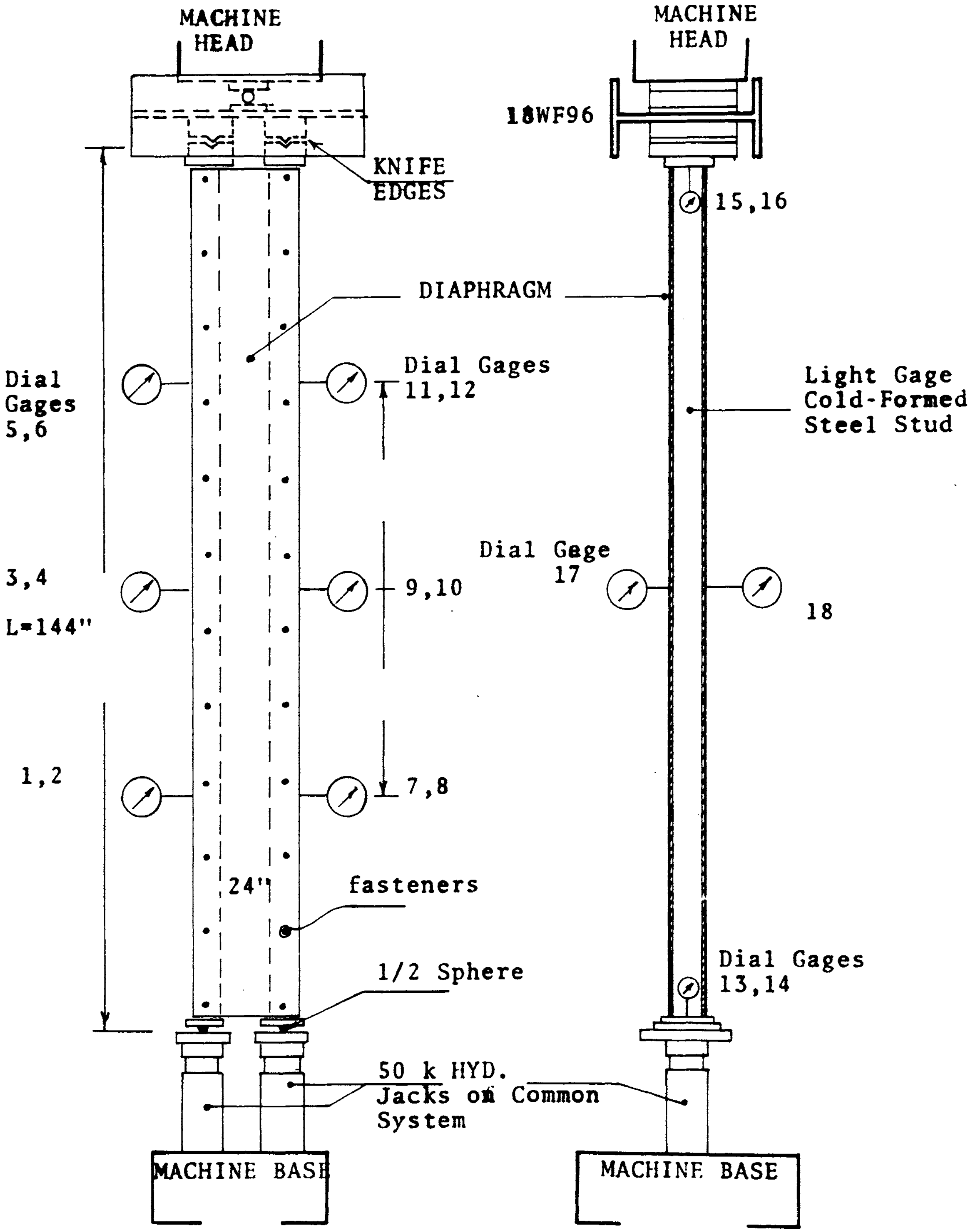
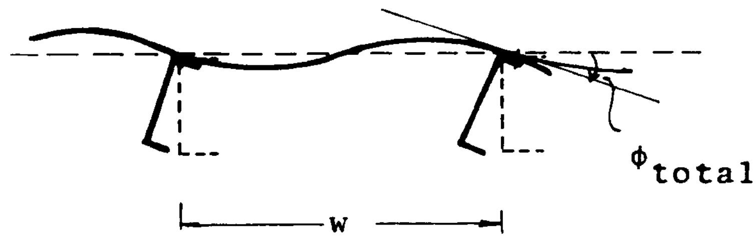
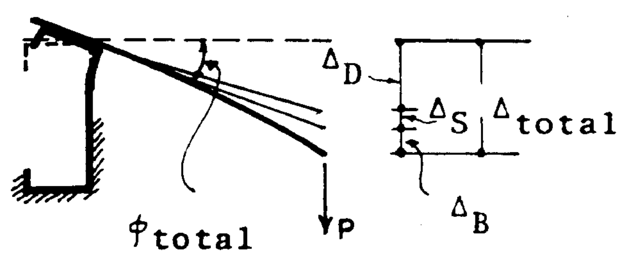


Fig. 18) Double-column assembly test set-up.



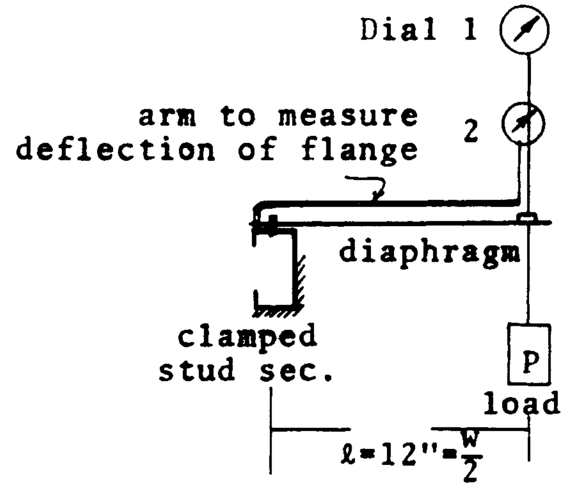
$$\phi_{total} = \phi_D + \phi_B + \phi_S$$

Fig. 19a) Rotation of diaphragm and column assembly.



$$\Delta_{total} = \Delta_D + \Delta_S + \Delta_B$$

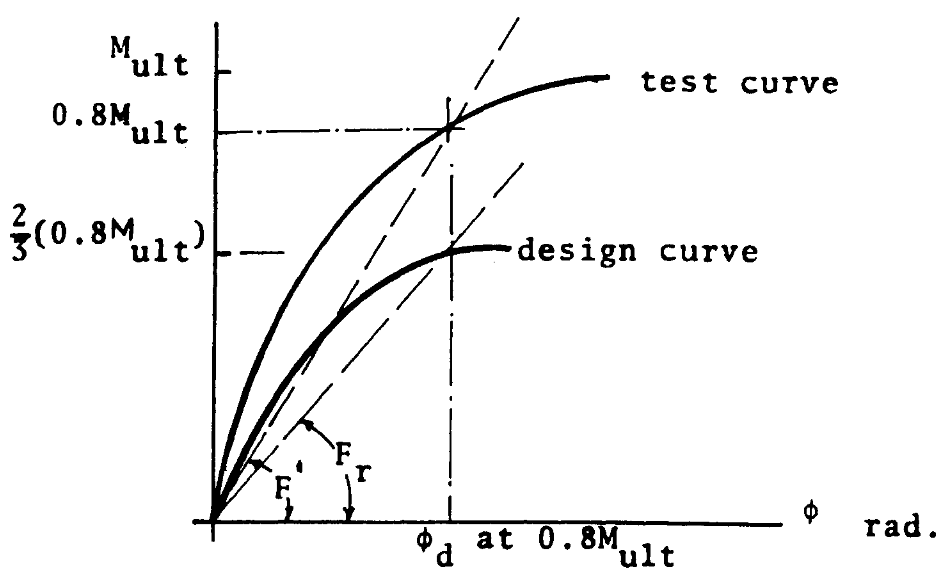
Fig. 19b) Deflection at the free end
(in the test set-up).



Dial 1 to measure Δ_{total}

2 to measure deflection Δ_S due to flange deformation

Test set-up



Moment-rotation curves

Fig. 20) Test set-up and moment-rotation curves for ~~determination of~~ F.

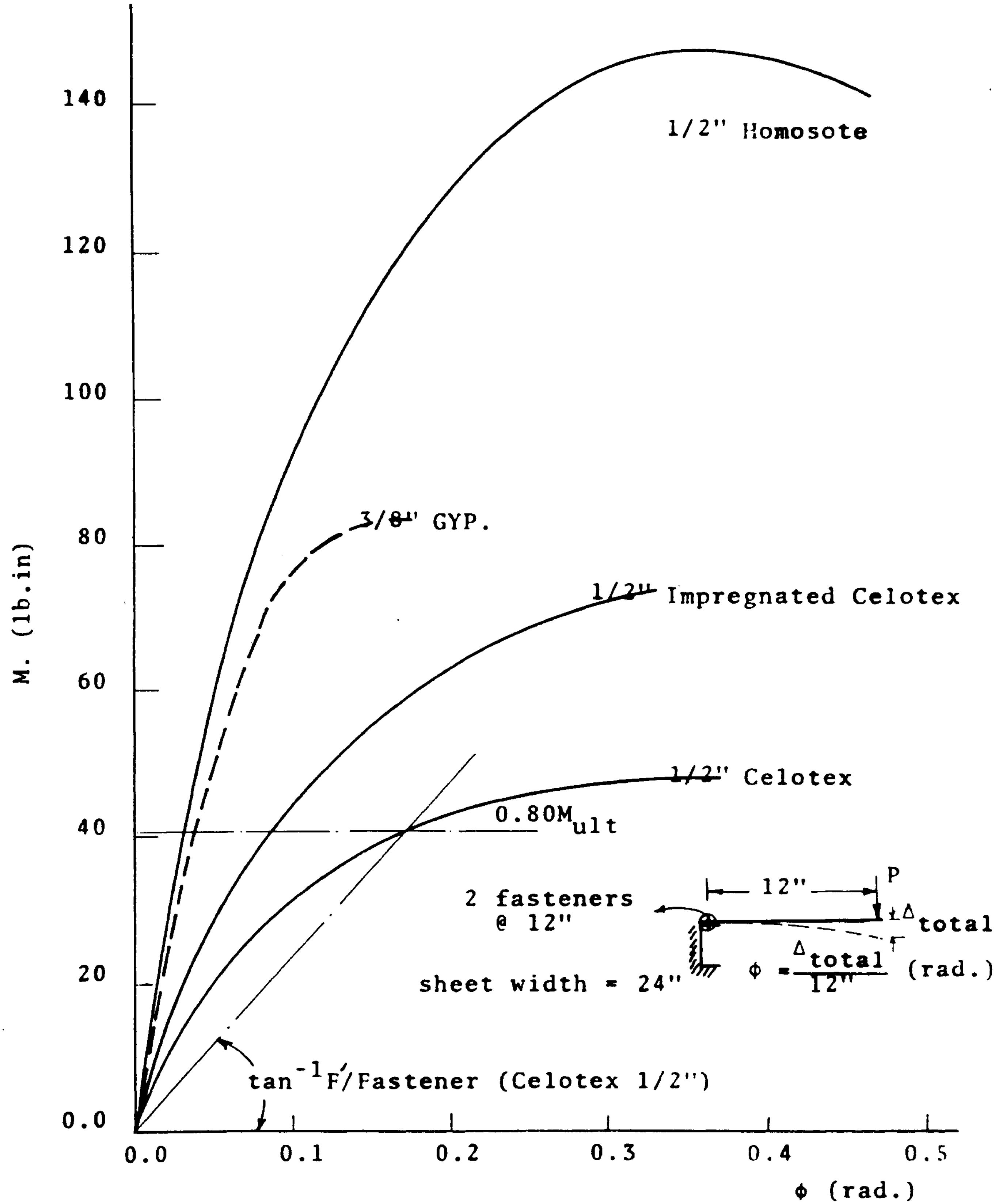
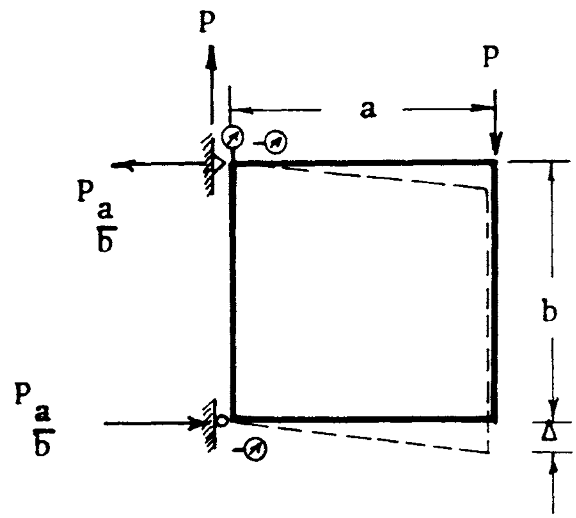
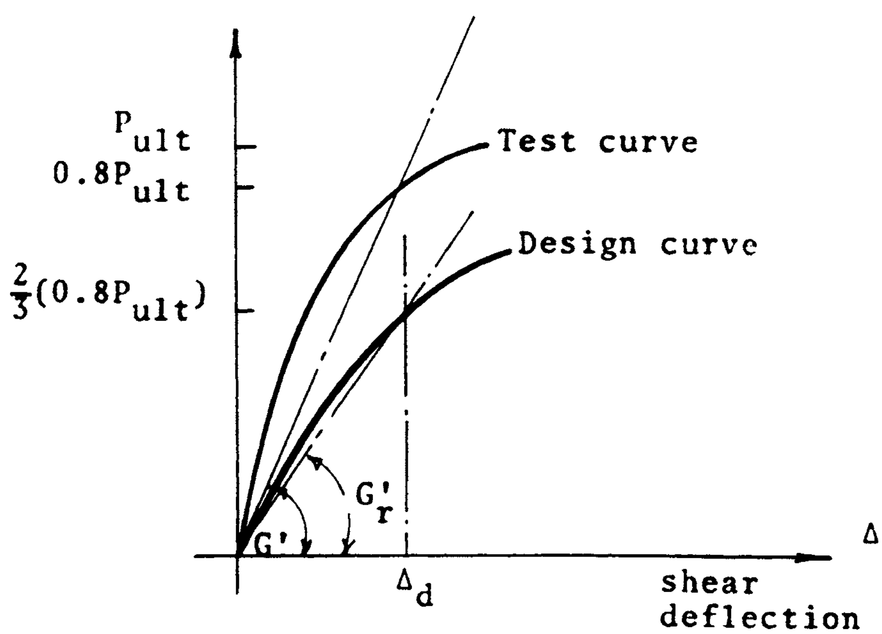


Fig 21) Rotational restraint (F) of different wall materials.



Test layout



Load-delfection curves

Fig. 22) Cantilever shear diaphragm test arrangement.

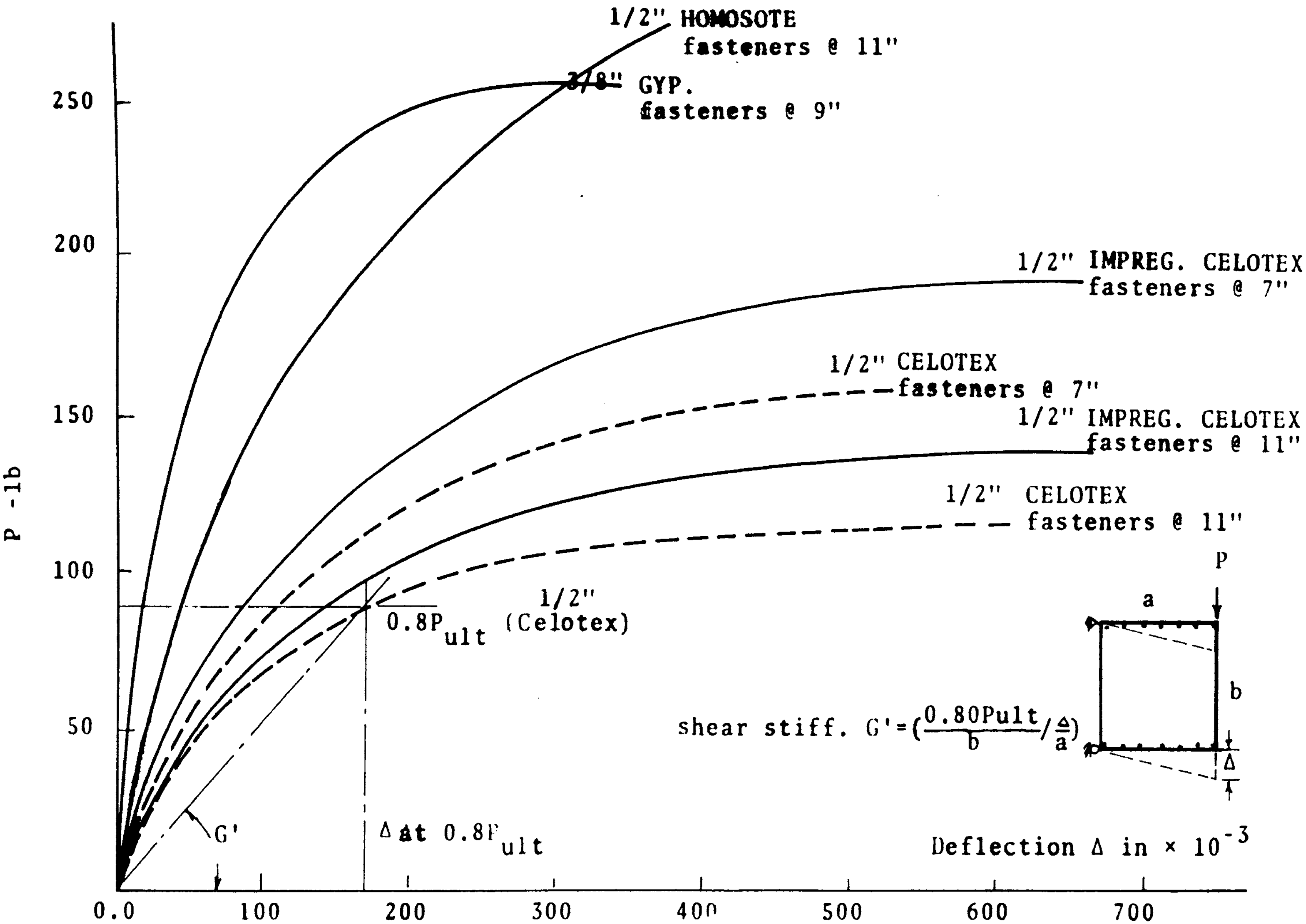


Fig. 23) Load-deflection curves of different wall materials - Cantilever test.

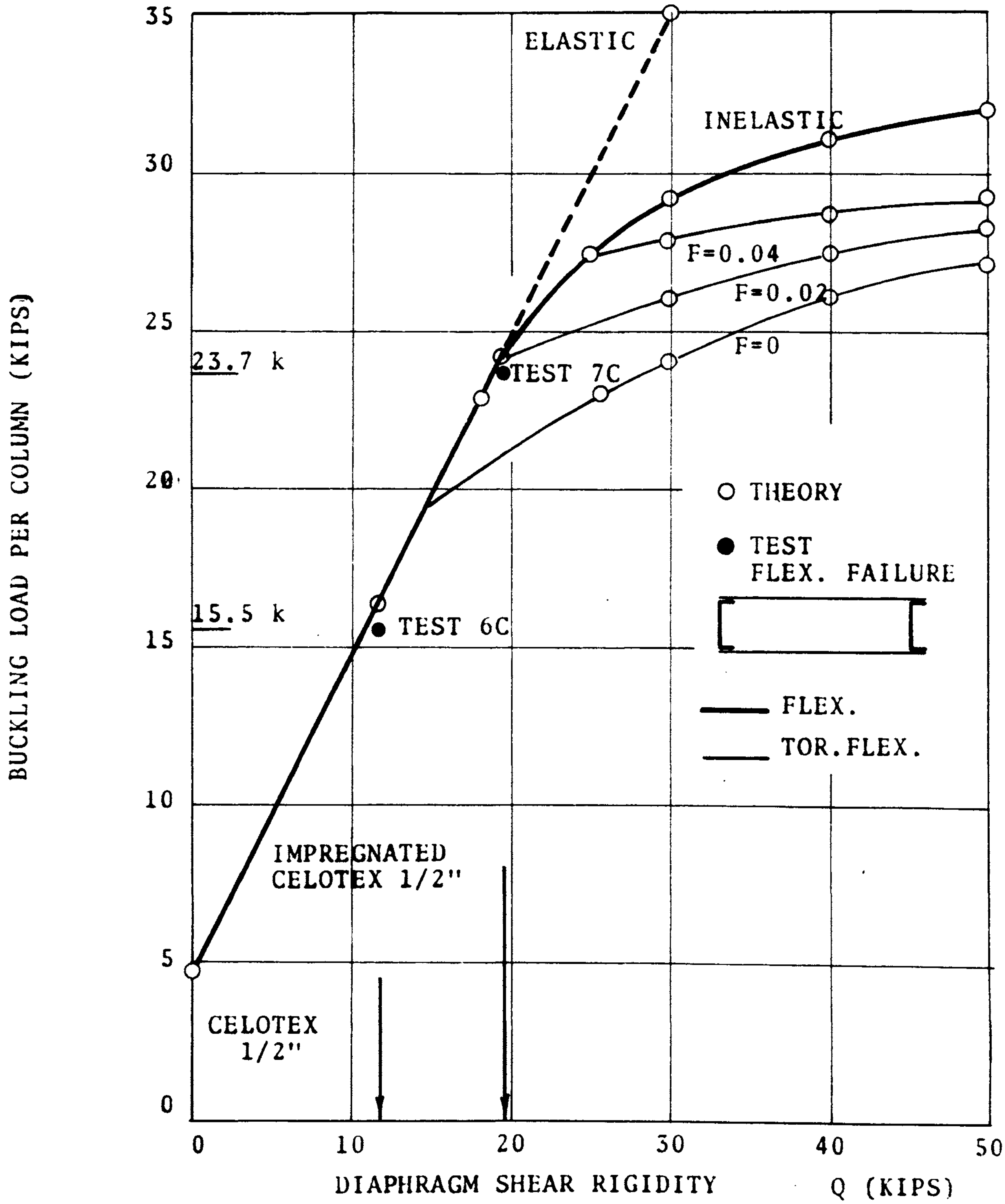


Fig. 24) Channel section type C - Braced on two sides.

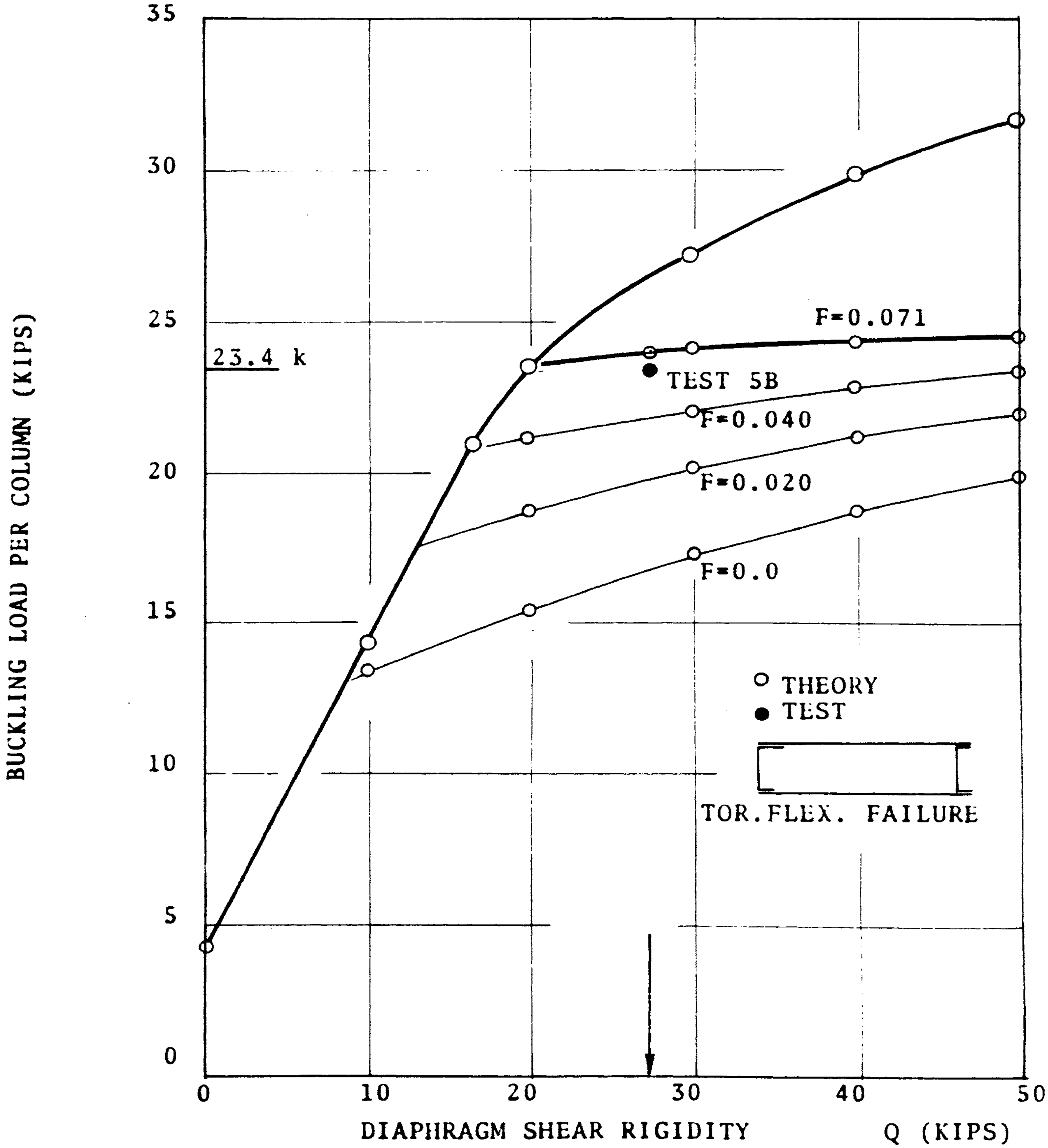


Fig. 25) Channel section type B - Braced on both sides.

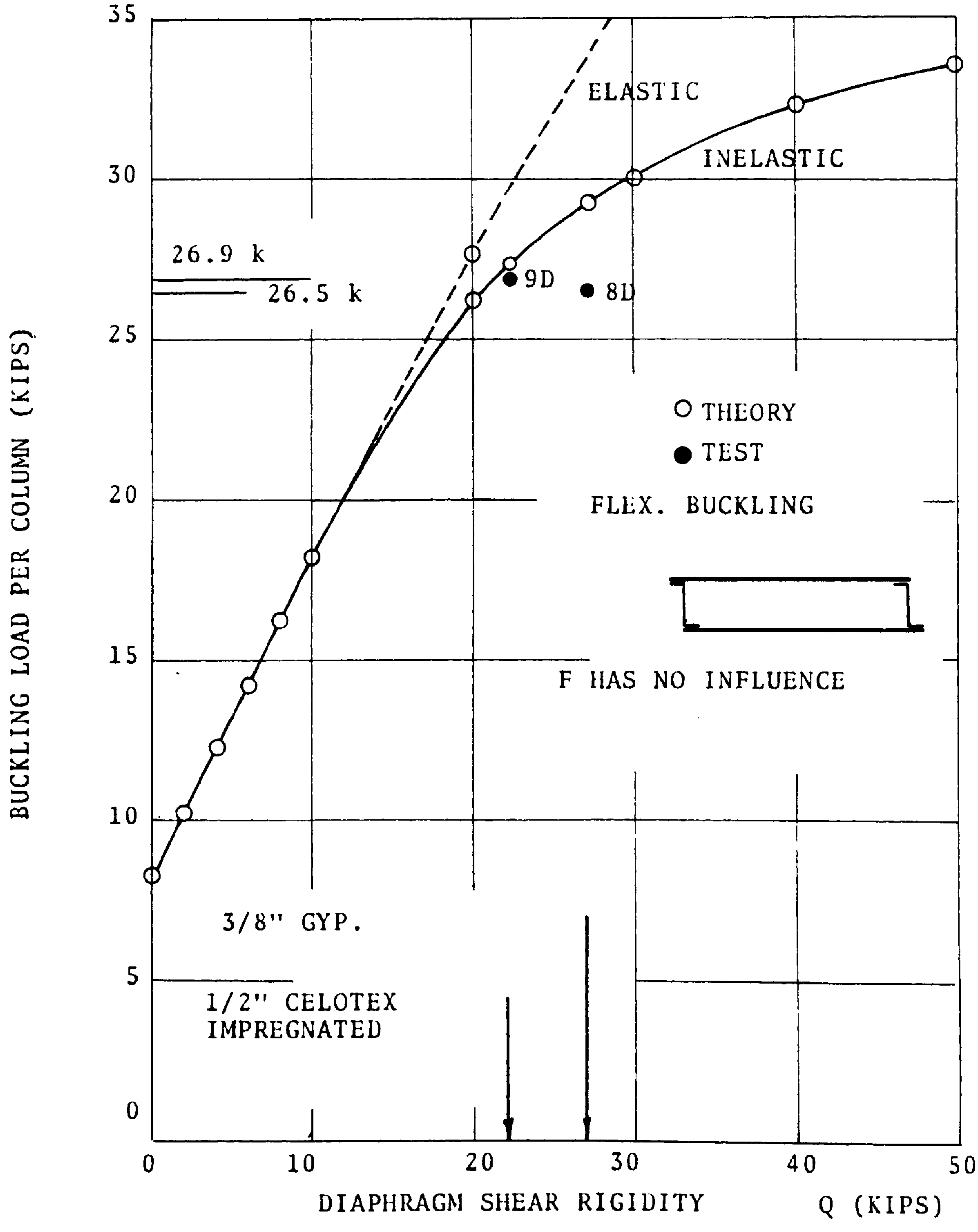


Fig. 26) Z-section type D braced on two sides.

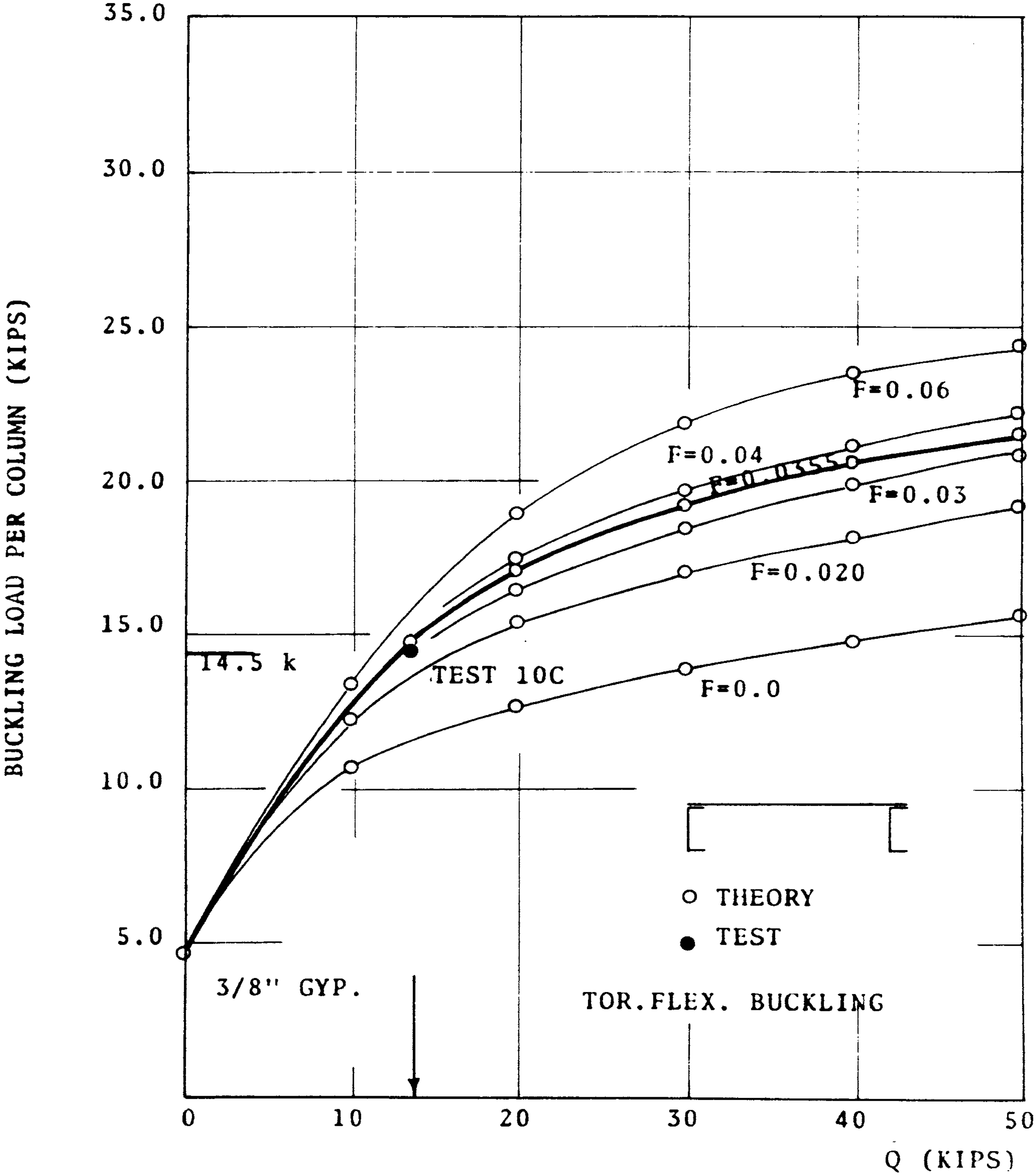


Fig. 27) Channel section type C - Braced on one side.

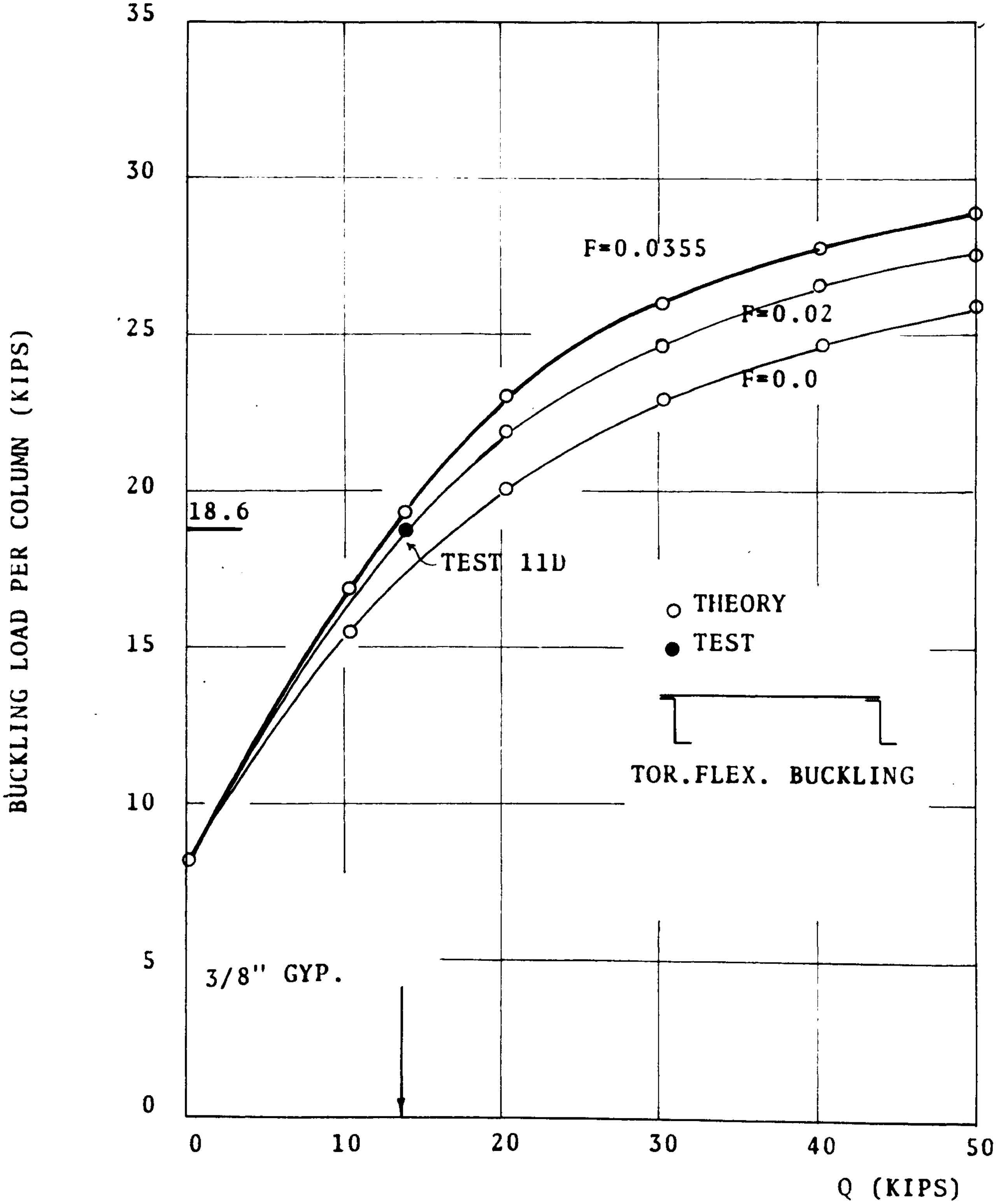


Fig. 28) Z-section type D - Braced on one side.

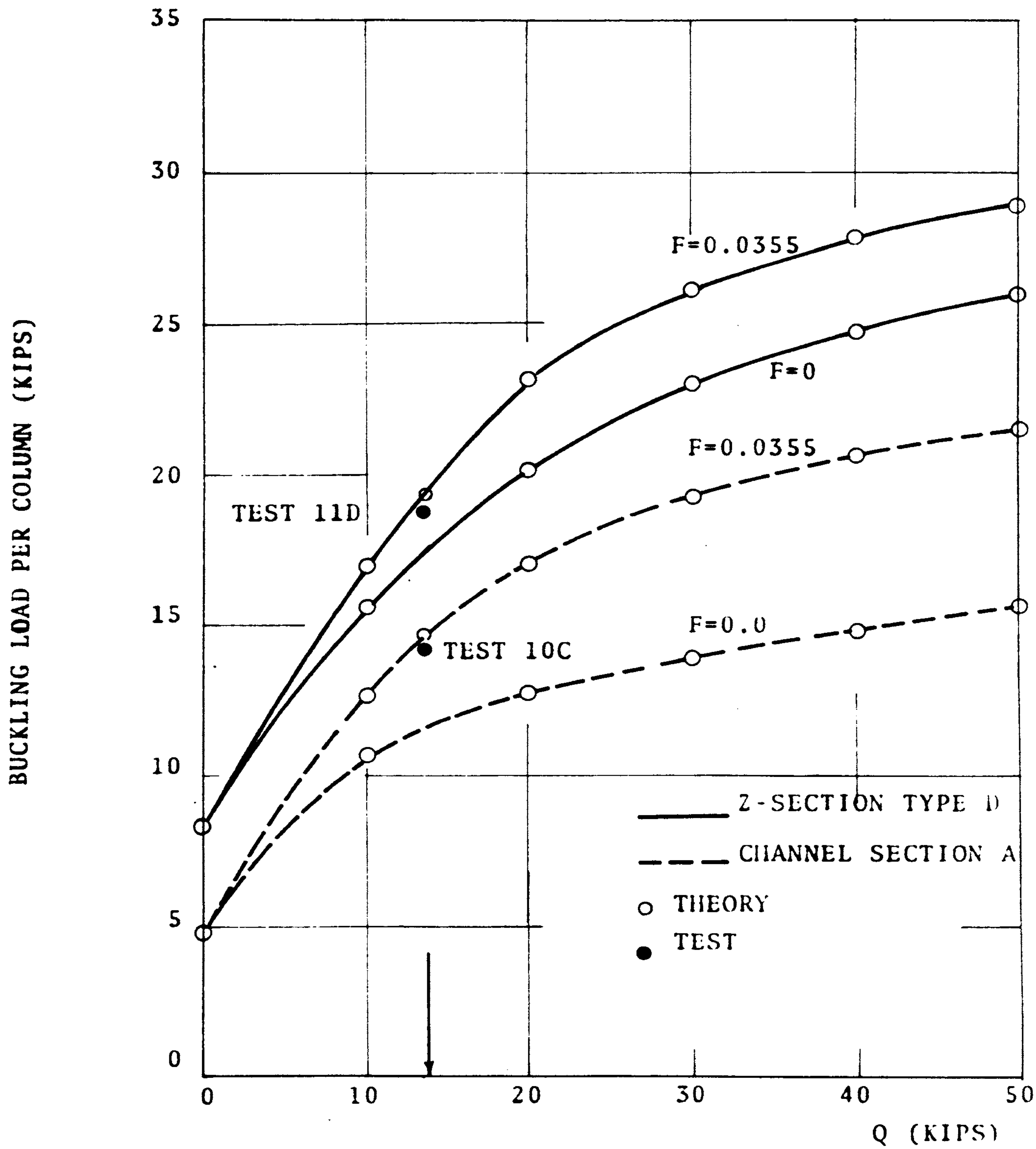


Fig. 29) Comparison of buckling load of Z and channel sections braced on one side.

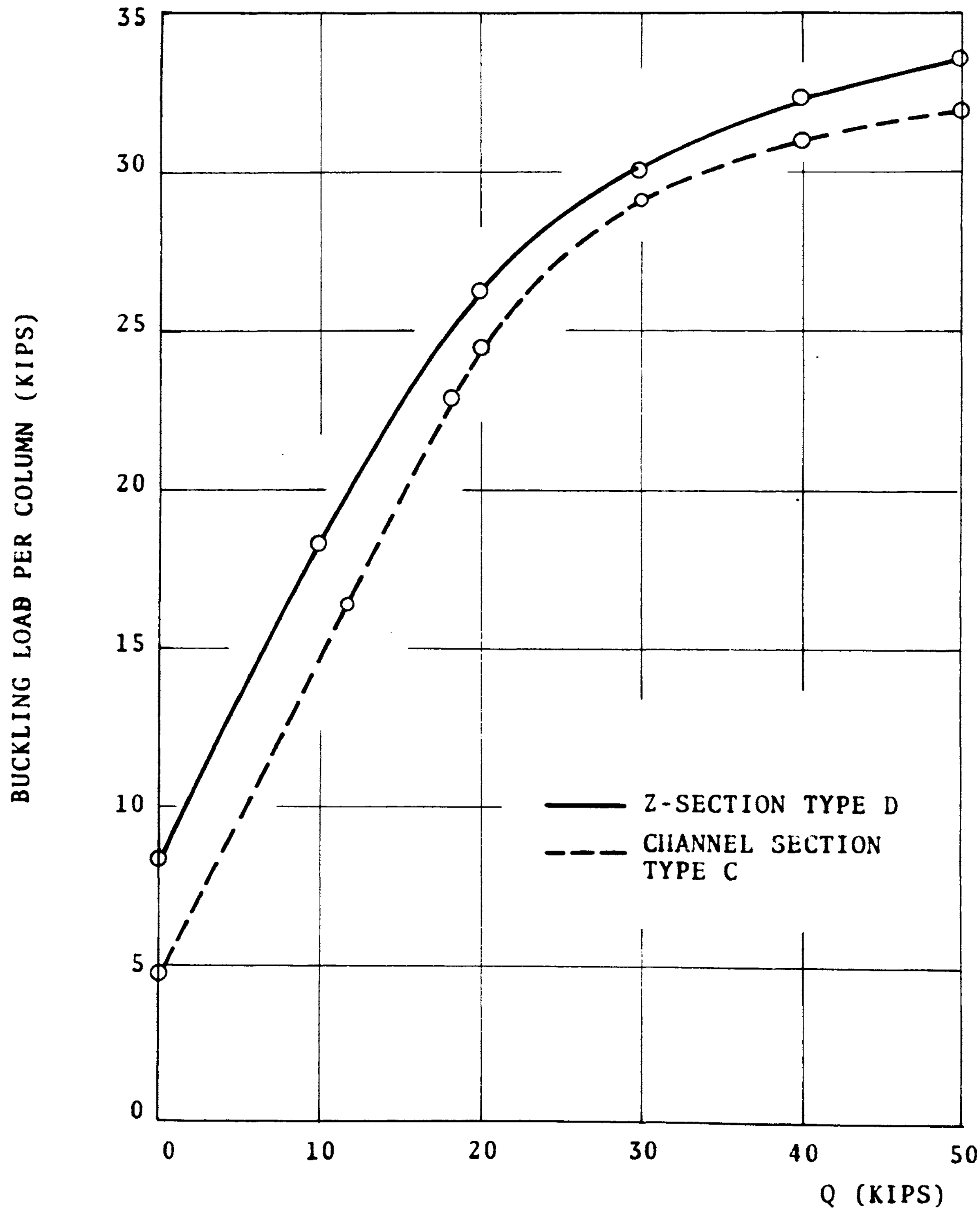
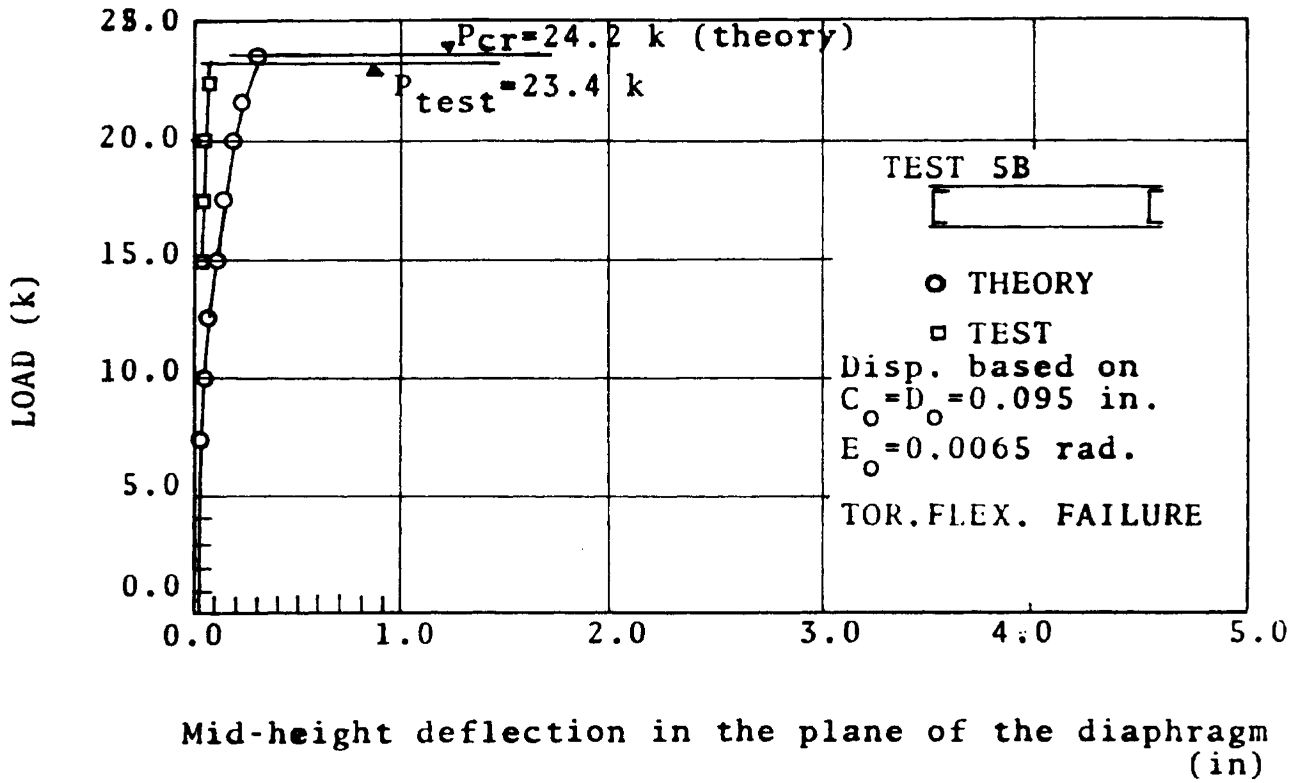


Fig. 30) Comparison of buckling load of Z and Channel sections braced on two sides.



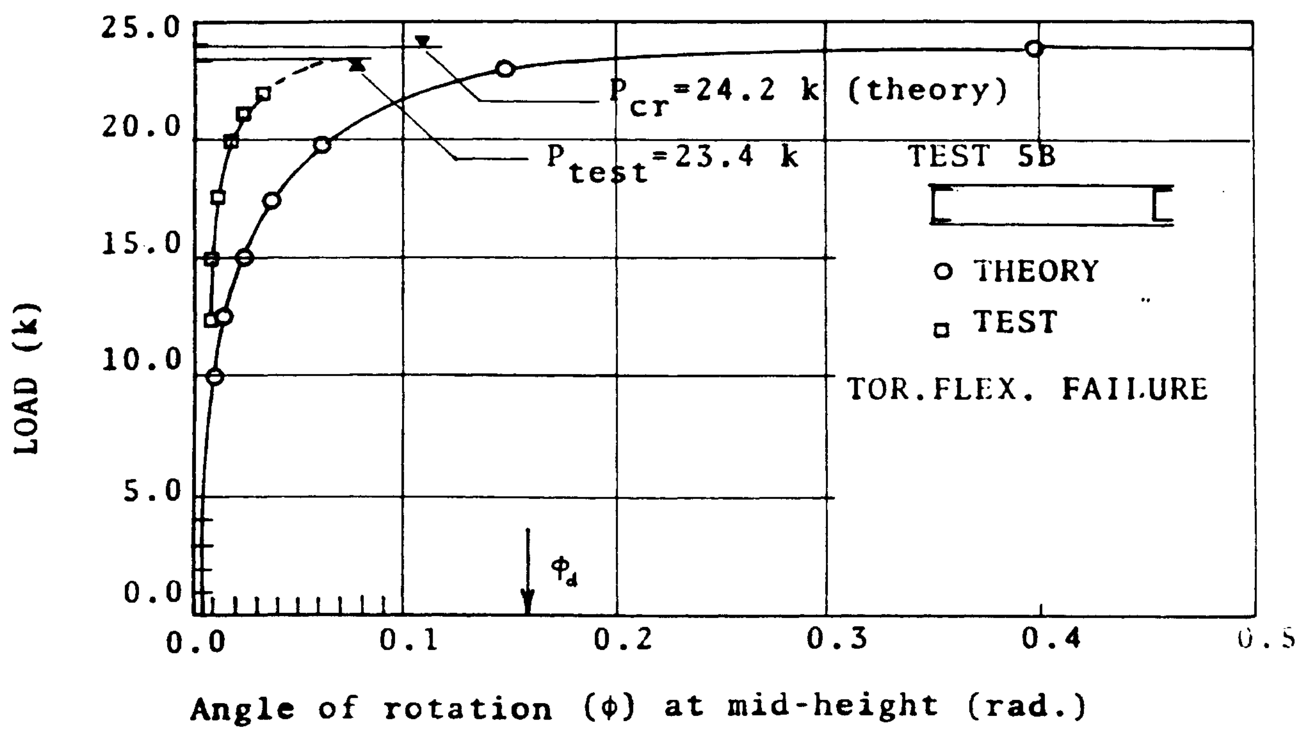
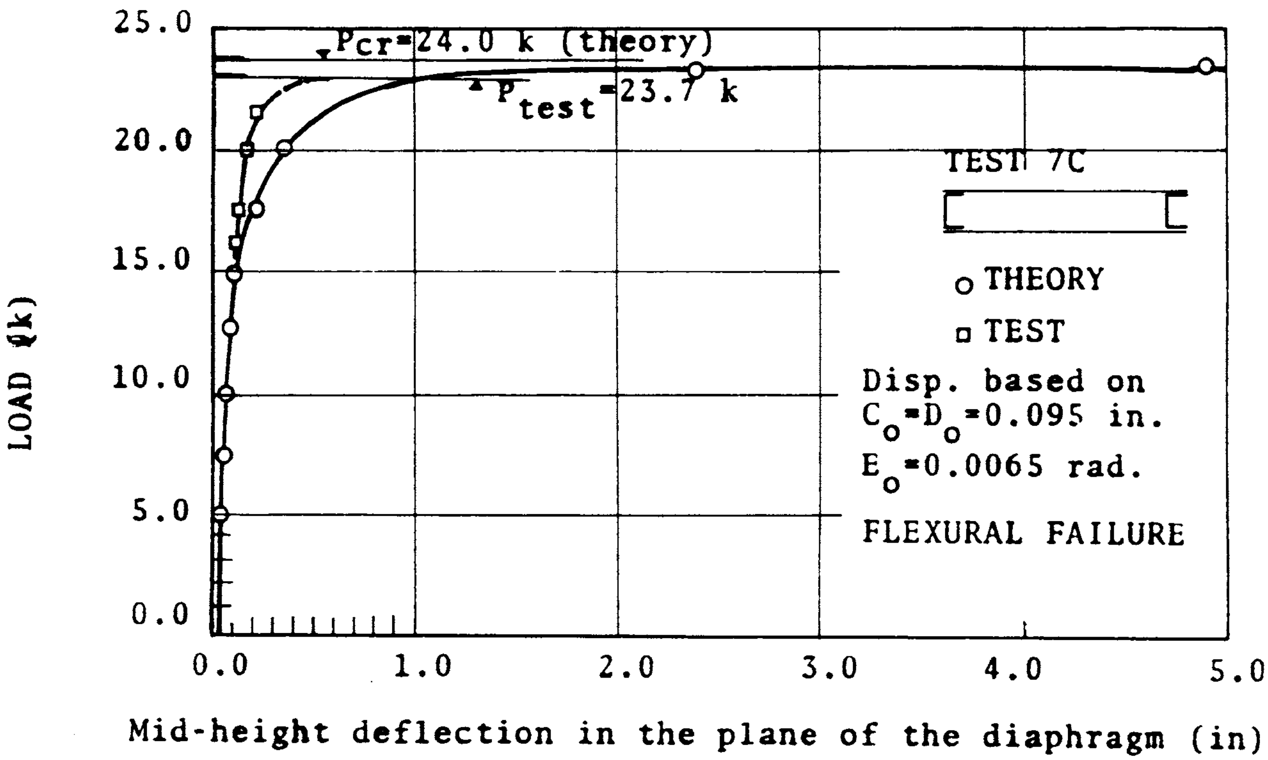


Fig. 31) Deflection and rotation of assembly 5B.



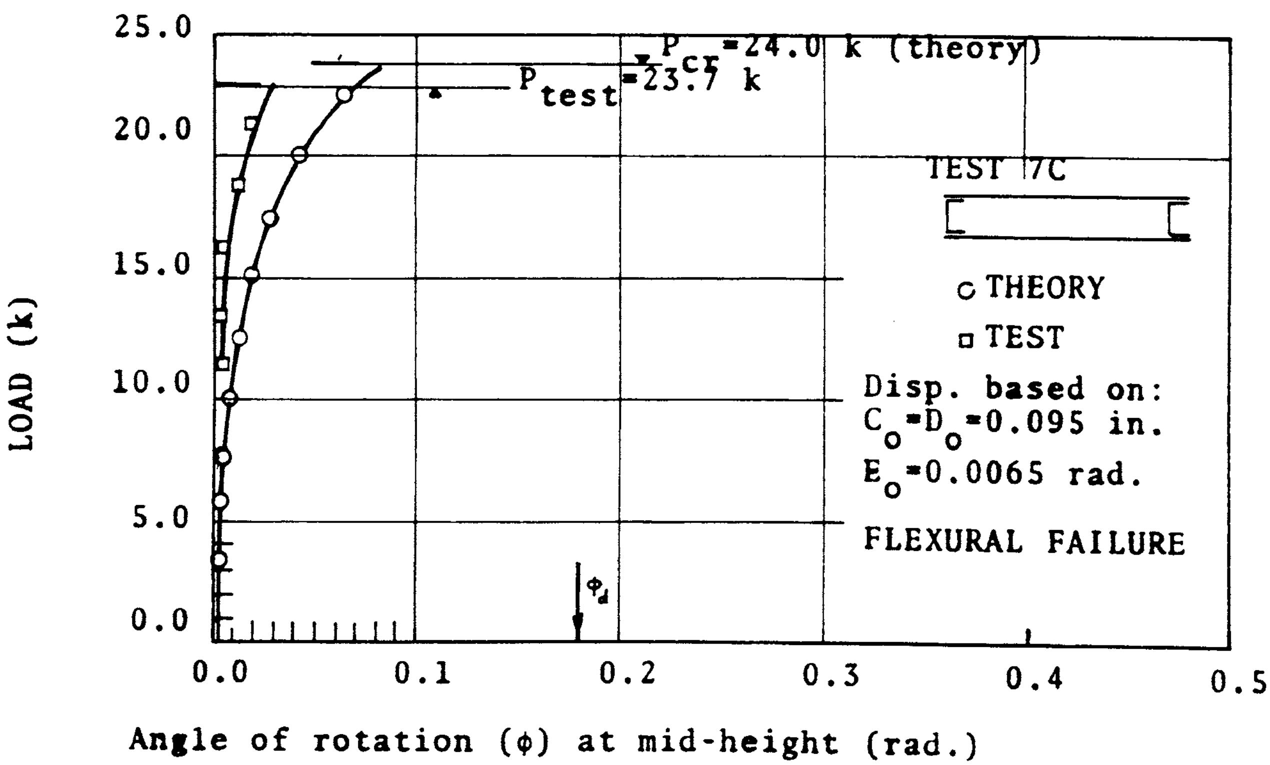


Fig. 32) Deflection and rotation of assembly 7C.

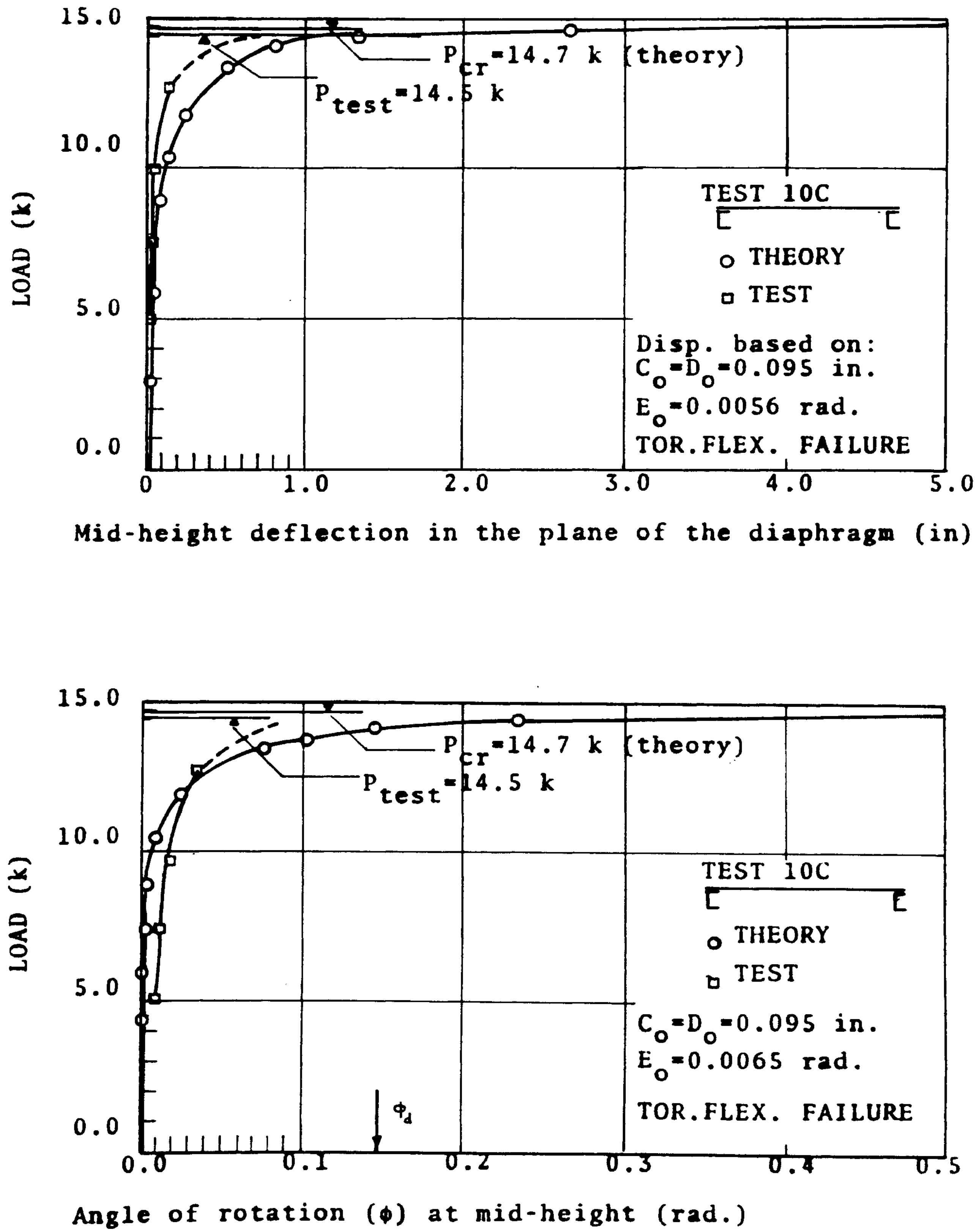
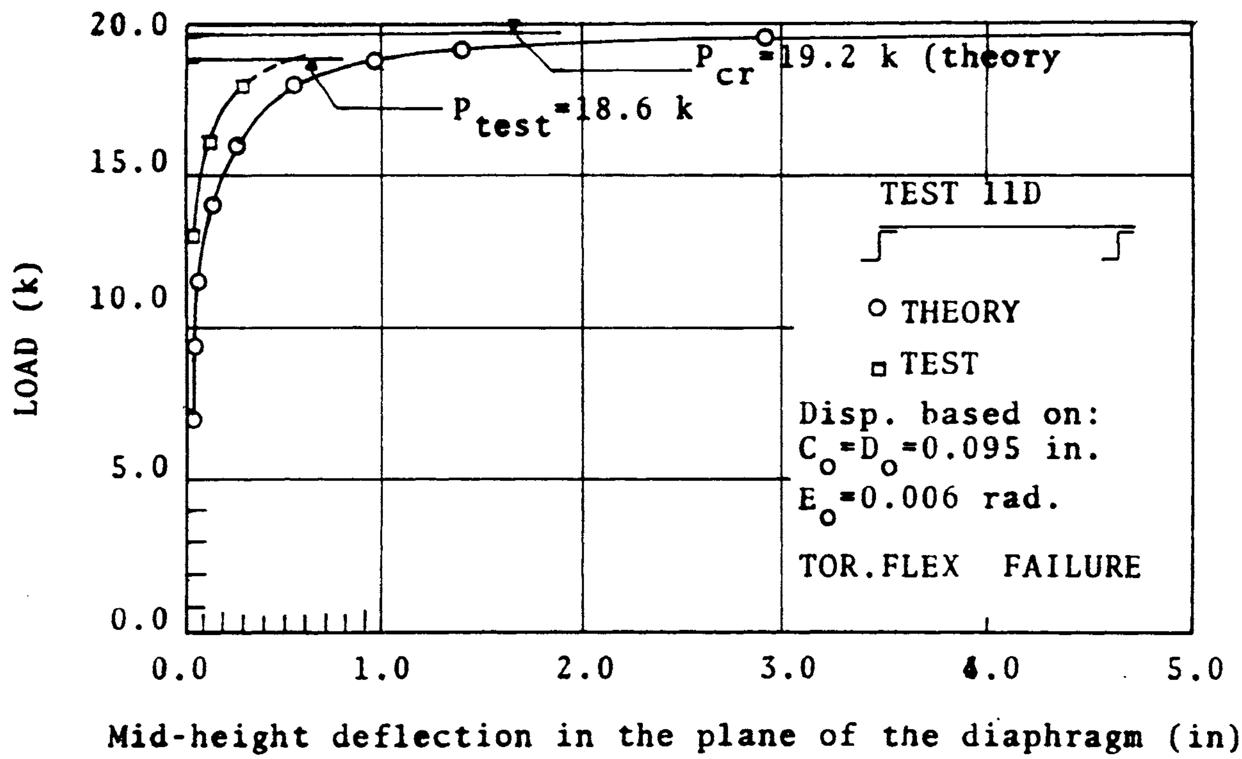


Fig. 33) Deflection and rotation of assembly 10C.



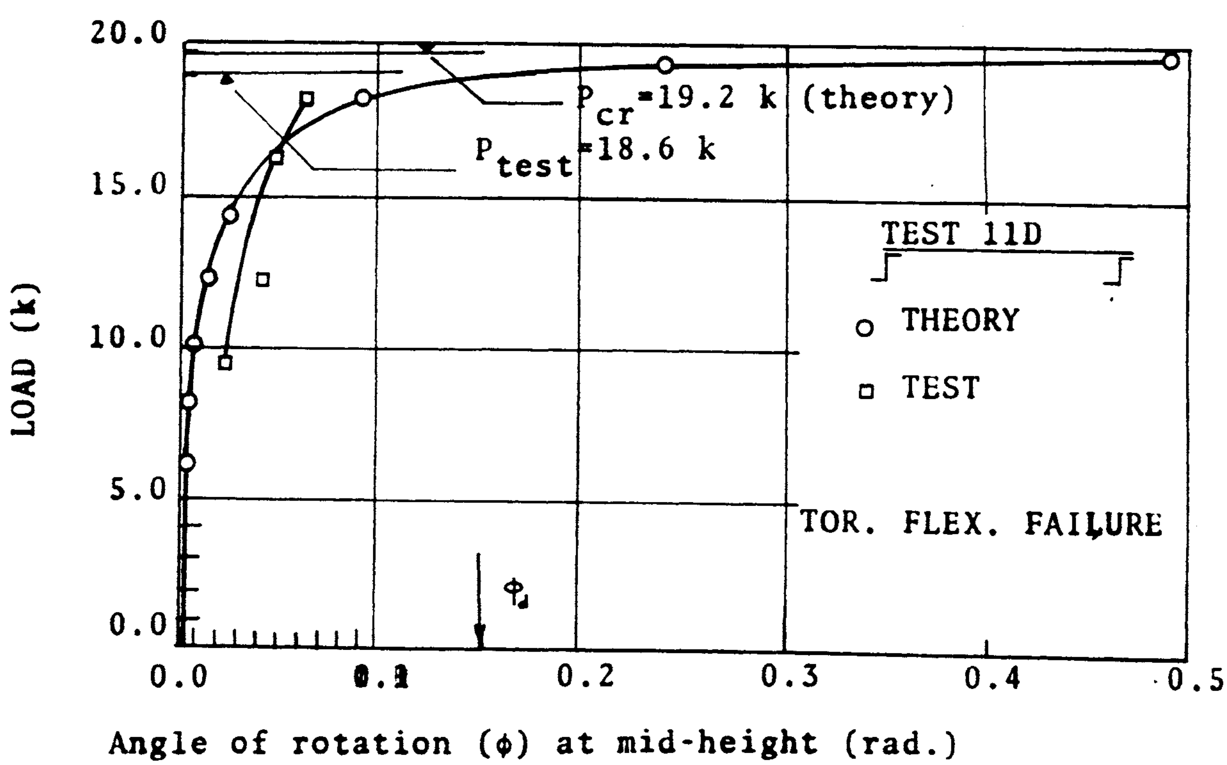
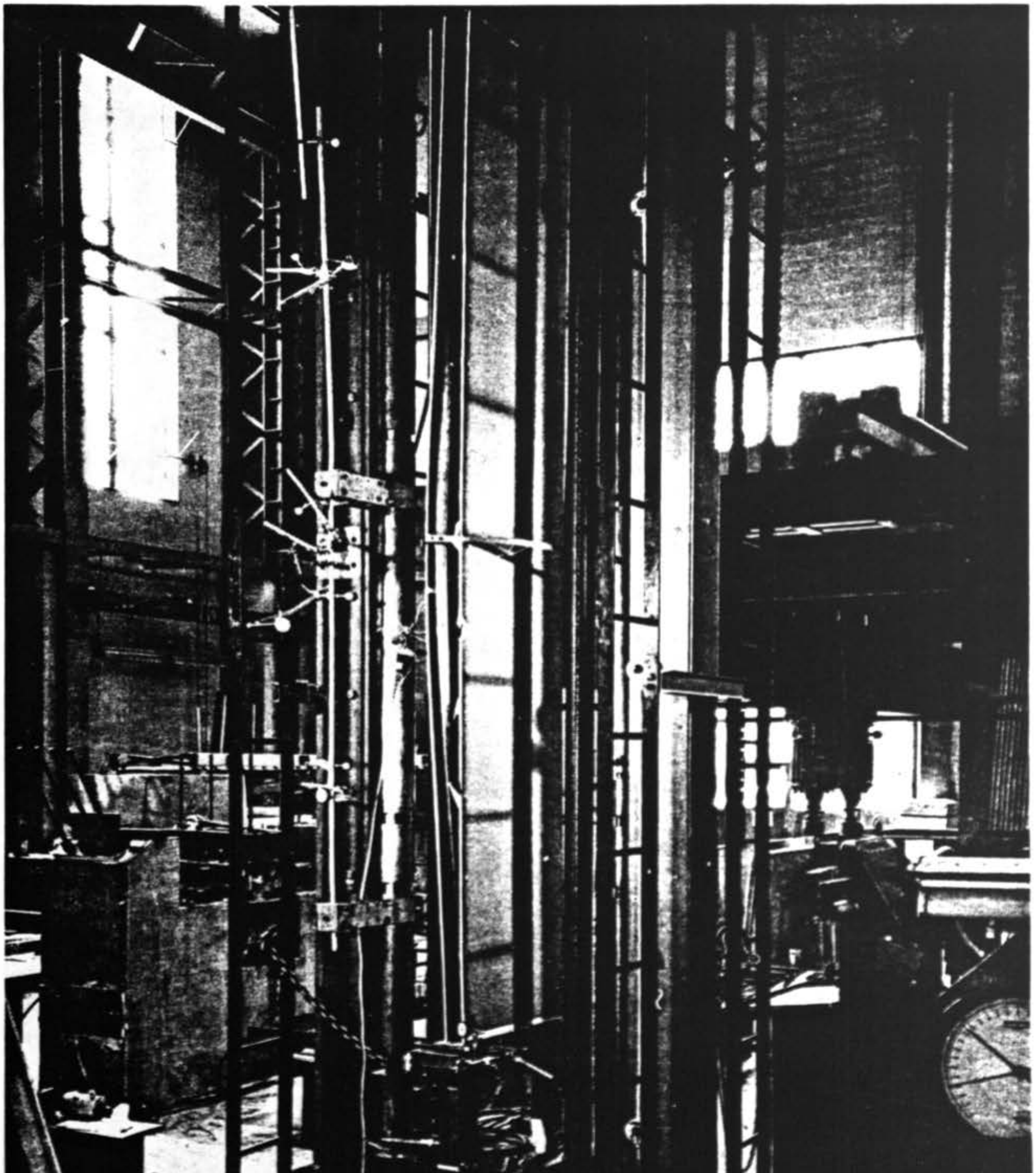
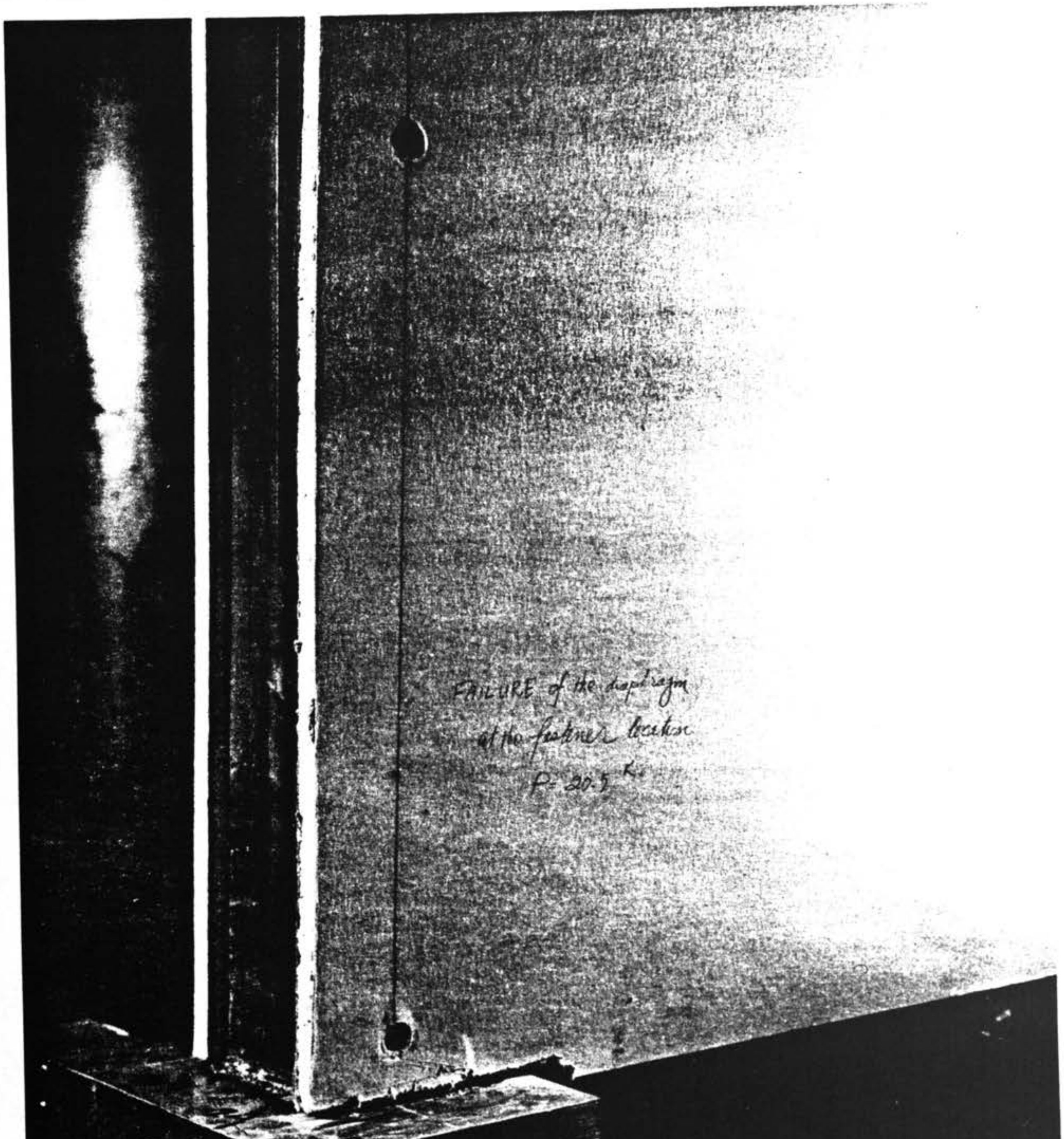


Fig. 34) Deflection and rotation of assembly 11D.



PHOTOGRAPH 1

COLUMN ASSEMBLY AFTER FAILURE



PHOTOGRAPH 2

FAILURE OF DIAPHRAGM AT FASTENER LOCATION