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Stresses in Suspension Bridges.

T.15 9.

W. M. CLAYPOOL.

The second se MSM HISTORICAL COLLECTION Stresses Auskusion Bridger. 7674

Stresses --in --Suspension Bridges

Ausfamin Bridger a surficien bridge is the which the roadway over the stream on spon to be cround in suspinded from charment win ropes. a suspinion bridge consists of the town or fine our which the main chains on caller paris; the anchorages to which the ends of the cables are attached; the main chains on cables from which the roadway is ruspended; the suspending rode on chaine which connet the roadway with the cable and the roadway. The pub-structure consists of the boundations, piere +c. The super structure consists of the roadway and the channe on cables.

Jowner -The towns, frequently timed fierd, an generally moder of maronry, autough iron has sometime bear

Suspension Bridges

A suspension bridge is one in which the roadway over the stream or span to be crossed is suspended from chains or wire ropes.

A suspension bridge consists of the towers or piers over which the main chains or cables pass; the anchorages to which the ends of the cables are attached; the main chains or cables from which the roadway is suspended; the suspending rods or chains which connect the roadway with the cable and the roadway.

The sub-structure consists of the foundations, piers &c. The super-structure consists of the roadway and the chains or cables.

Towers-

The towers, frequently termed piers, are generally made of masonry, although iron has sometimes been

used. The particular triad form of towns will depend to some sextent upon the locality and character of surroundings. This diminious will depaired upon Their hight and the amount of strain they will have to mist. Where the caller pass our the former an saddles. Our construction of saddles in which the cable passes over friction melen rigidly attached to the top of the pier, areams the caller to slip backworde and forwards over it with comparationly little priction, so that the stress on the cable may be taken as equal on both sider of the saddle. In another construction whe chain is second to the saddle, which, however, in four to moor horizontally on the Top of the firm. he the first form of radder the usuelant presence on the fin will not be vertical unless the chain

used. The particular kind or form of towers will depend to some extent upon the locality and character of surroundings. Their dimensions will depend upon their height and the amount of strains they will haves to resist. Where the cables pass over the towers are saddles. Our construction of saddles in which the cable passed our friction rollers rigidly attached to the top of the pier, allows the cable to slip backwards and forwards over it with comparatively little friction, so that the stress on the cable may be taken as equal on both sides of the saddle. In another construction the chain is secured to its saddle, which, however, is free to move horizontally on the top of the piers. In the first form of saddles the resultant pressure on the pier will

not be vertical unless the chain

2

leaving the firm at un equal meterration on each side, and even when the bridge is designed with an equal slafe of claim on both sides of the pur, a change in the distribution of wight due to any paring load, wie cause some departure from the equal slope of the chaine, and therefore from the truly vistical pressure on the pine. This departure is easily allowed for im the design of the bridge firm. The friction on the saddle runder the assumption of equal stresses on doch side slightly incomet, and with This lype of raddle, care must be taken to provide against the war produced by the motion of the chain. In the second type, the use of rolling under the solid radde leave the motion of the saddle very fru; the resultant pressure on the town is always vertical, and the chains may

leaves the pier at an equal inclination on each side, and even when the bridge is designed with an equal slope of chain on both sides of the pier, a change in the distribution of weight due to any passing load, will cause some departure from the equal slope of the chains, and therefore from the truly vertical pressure of the piers. This departure is easily allowed for in the design of the bridge piers. The friction on the saddle renders the assumption of equal stresses on each side slightly incorrect, and with this type of saddle, care must be taken to provide against the wear produced by the motion of the chain. In the second type, the use of rollers under the solid saddle leaves the motion of the saddle very free; its resultant pressure on the tower is always vertical, and the chains may

leave the town at any angle, equal or mequal. The chain must in no case be rigidly attached to the firm, muless the firm, on racher support in this instance, is for to rock on its have, as for example, when the place of the pier is taken by in struts working on a lorizontal axis. andwage - If the show or bank be of wek, a virtical passage should be excounted and a strong iron plate placed in the hottom and finnely imbadded in the side of the passage. Though this feate the bude of the caller an passed and second on the under side. after the cubies an put in place, the parrage planed be filled with concrede and masonny, If the bank is not suitable for the

leave the tower at any angle, equal or unequal.

The chain must in no case be rigidly attached to the pier, unless the pier, or rather support in this instance, is free to rock on its base, or for example, when the place of the pier is taken by iron struts working on a horizontal axis.

<u>Anchorage</u>

If the shore or bank be of rock, a vertical passage should be excavated and a strong iron plate placed in the bottom and firmly imbedded in the side of the passage. Through this plate the ends of the cables are passed and secured on the under side.

After the cables are put in place, the passage should be filled with concrete and masonry.

If the bank is not suitable for the

anchorage, a heavy man of marany should be built of larger blocks of cut stone well louded together for the purpose. In this can it is well to construct a parsage way so that the chains and facturings may at any time be examined. The mass of masonry or the natural rock to which the ende of the cables are fastered is prequently called the abutunt. Str stability must be greater stan The tension of the cobler. It wight and thekness must be sufficient to prevent its bring over turned, and its cuter of resistance must be in rafe himile, The coleneations in regard to the anchorage, when it is artificial, properly belong with the puspinion bidge, but I have left them out, since they can be very appropriately included in marony.

anchorage, a heavy mass of masonry should be built of large blocks of cutstone well bonded together for this purpose. In this case it is well to construct a passage way so that the chains and fastenings may at any time be examined. The mass of masonry or the natural rock to which the ends of the cables are fastened is frequently called the abutment. Its stability must be greater than the tension of the cables. Its weight and thickness must be sufficient to prevent its being over turned, and its center of resistance must be in safe limits. The calculations in regard to the anchorage, when it is artificial, properly belong with the suspension bridge, but I have left them out, since they can be very appropriately included in masonry.

Cables - These may be made of iron born connected by eye bar and fin juinte, of iron links se, but the custom now is to use wine roper on cables. The preacent muchor of calilies is two, one to support each side of the roadway. Sunnakly more than two are used, since, boi the same amount of material, they offer at least the parme vistance, are more accurately manufactured, an liable to her dauge of accident, and can be more early put in place and replaced than a single cable of an equal amount of material. Great care is taken to give each wine the som degree of tension. To ensure this it used to be alonght necessary to strain each win reparately our the actual piers, on piers similarly placed, and bind this Together when hanging, strained by their own weight with the dip proposed for the bridge. It was also shonght essential that each roke

<u>Cables</u>

These may be made of iron bars connected by eye bar and pin joints, of iron links &c, but the custom now is to use wire ropes or cables. The smallest number of cables is two, one to support each side of the roadway. Generally more than two are used, since, for the same amount of material, they offer at least the same resistance, are more accurately manufactured, are liable to less danger of accident, and can be more easily put in place and replaced than a single cable of an equal amount of material. Great care is taken to give each wire the same degree of tension. To ensure this, it used to be thought necessary to strain each wire separately over the actual piers, or piers similarly placed, and bind them together when hanging, strained by their own weight with the dip proposed for the bridge.

It was also thought essential that each rope

should be an aggregate of paraelel wires, not spin, as in a rope. Experiment. howwww, has shown that wire ropes spine with a machine which dors not put a worst in each wine, but lays it helically and untwisted, and with no straight central win, are as strong as win roken of equal wight made with straight wires. It is the custon now to make the cable of win 16 to 15' in diameter, and bring them to a cylindrical slape by a spiral wrapping of une. The winer are coated with varnish bafor bring bound rep, and the cable itself is mitably protected from atmospheric influences.

Surpineion Rada -

when the cable is compared of links or home, then are attached directly to item. If of rope, the purposition rod is attached to a colear of iron of suitable plaps but around the cable, or to a saddle piece return on it. When there are two ealer,

should be an aggregate of parallel wires, not spun, as in a rope. Experiment, however, has shown that wire ropes spun with a machine which does not put a twist in each wire, but lays it helically and untwisted, and with no straight central wire, are as strong as wire ropes of equal weight made with straight wires. It is the custom now to make the cable of wire $\frac{1}{6}$ " to $\frac{1}{5}$ " in diameter, and bring them to a cylindrical shape by a spiral wrapping of wire. The wires are coated with varnish before being bound up, and the cable itself is suitably protected from atmospheric influences.

Suspension Rods

When the cable is composed of links or bars, they are attached directly to them. If of rope, the suspension rod is attached to a collar of iron of suitable shape bent around the cable, or to a saddle piece resting on it. When there are two cables,

can must be taken to distribute the load upon the cables according to this degree of struget. Nuadwary -The roadway brarers an supported by the suspension rode. On the bearers are laid longitudinal juste, and on them the planking, in the planking is laid directly on the roadway branese. The latter an stiffered by diagonal ties of iron placed larizontally between each pain of roadway bearers. Sund finisplus - The great ment of a suspinion bridge is its cheapmen, arising from the composatively small quantity of material required to carry a given passing load across a given span. This cheapment may be seen more clearly by considering an example. a man might cross a chasm of 100' by hanging to a stul won 21" in diameter, dipping 10

care must be taken to distribute the load upon the cables according to their degree of strength.

<u>Roadway</u>

The roadway bearers are supported by the suspension rods. On the bearers are laid longitudinal joists, and on them the planking, or the planking is laid directly on the roadway bearers. The latter are stiffened by diagonal ties of iron placed horizontally between each pair of roadway bearers.

General Principles

The great merit of a suspension bridge is its cheapness, arising from the comparatively small quantity of material required to carry a given passing load across a given span. This cheapness may be seen more clearly by considering an example. A man might cross a chasm of 100' by hanging to a steel wire .21" in diameter, dipping 10';

the weight of the iron would be 12.75 ebs. a wright iron bran of rectanquelar cross section then times as deep as it is hroad, would have to be about 27" deep and q" broad to carry him and its own weight. It would be wiegh 87,500 lbr. an iron I beam of best construction 10' duk would whigh about 120 lbs. In each case 4' love been allowed for branings at the ende of the spone. The enormous difference would not exist if the bran and wire had only to carry the man, wen then there would be a great difference in favor of the main difference arise from the fact that the bridge harts carry its own wight. The chip-merit of a suspinsion bridge done not therefore, come into play, until the weight of the rope or bram 'is considerable when camponed with the platform and roleing load; for attrangt

the weight of the iron would be 12.75 lbs. A wrought iron beam of rectangular cross-section three times as deep as it is broad, would have to be about 27" deep and 9" broad to carry him and its own weight. It would be weigh 87,500 lbs. An iron I beam of best construction 10' deep would weigh about 120 lbs. In each case 4' have been allowed for bearings at the end of the spans. The enormous differences would not exist if the beam and wire had only to carry the man, even then there would be a great difference in favor of the wire. The main difference arises from the fact that the bridge has to carry its own weight. The chief merit of a suspension bridge does not, therefore, come into play, until the weight of the rope or beam is considerable when compared with the platform and rolling load; for although

the chain will for any given load be

lighter than a beau, the paving in this respect will, for male span, 'be non than compensated by the expense of the anchorages. The disadvantages of the suspension bidge are minimum. A change in the distribution of the load courses a very smiller deformation of the structure, for the cuble of the suspinision bridge must adapt its form to the new position of The boad, whereas in the brain the deformation is hardly surrible, equilibrium bring attained by a new distribution of the stresser through the material. This plexiclify of the suspinion budge under it minitable for the passage of a railway train at any considerable speed. The platform rises up an a wave in brout of any rapidly advancing load, and the masser in motion produce stresses much greater Than their which would reall from

lighter than a beam, the saving in this respect will, for small spans, be more than compensated by the expense of the anchorages.

The disadvantages of the suspension bridge are numerous. A change in the distribution of the load causes a very sensible deformation of the structure, for the cable of the suspension bridge must adapt is form to the new position of the load, whereas in the beam the deformation is hardly sensible, equilibrium being attained by a new distribution of the stresses through the material. This flexibility of the suspension bridge renders it unsuitable for the passage of a railway train at any considerable speed. The platform rises up as a wave in front of any rapidly advancing load, and the masses in motion produce stresses much greater than those which would result from

the same wights when at rest. The kinute effect of the ascittations produced by linding of men marching, on by impulse due to wind, jur nieto straim which counted by forseen.

the same weights when at rest. The kinetic effect of the oscillations produced by bodies of men marching, or by impulses due to wind, give rise to strains which cannot be foreseen.



Lit Etf'C be cable of a puspension bridge carrying a load which extends own the whole ppour. In practice the load carried by a suspension bridge cable is uniform in intensity in reference to a harizontal line. I heardically this assumption waved not do, but the load in so marly migone for foot of span abot it is taken to be exactly so. Lit ED + BC = l = plan BH = h, = hight of highest lower D'f'= h_2 = " " low low. w = load for haringantal foot x = distance measured harizontally from If, the lowest point in the cable.

Fig. 1

Let EH'C be cable of a suspension bridge carrying a load which extends over the whole span. In practice the load carried by a suspension bridge cable is uniform in intensity in reference to a horizontal line. Theoretically this assumption would not do, but the load is so nearly uniform per foot of span that it is taken to be exactly so.

- Let ED + BC = I = span
 - $BH' = L_1 = height of highest tower$ $DH' = L_2 =$ " lower tower
 - DH = L2 = 10 Novel to ve
 - w = load for horizontal foot
 - x = distance measured horizontally from
 - H', the lowest point in the cable.

The ordinate of any paint & is x, Than The load on H'M in W = wx, since the total load is equal to the number of while of length into the load on one with of fingth. Prow PK tangent to the error of P, them, since the resultant of the load between I and If acts through the point of intersection of the tangents at P and If', and the load and turione on the clain at Paul A' are aspectively proportional to the side of a triangle paraelel to this directions, the cable Territor at P and If' and the direction of W must interset in one point. Since w is miniform along X, the resultant direction of W passis strongh IV, half way between H' and M. Theyon, FH' = H'K, or, since H'K in the substangent, the absense, FH', of the curve is equal to the sub-tangent, have, the enver in the

The ordinate of any point P is x, then the load on H'M is

W = wx, since the total load is equal to the number of units of length into the load on one unit of length. Draw PK tangent to the curve at P, then, since the resultant of the load between P and H' acts through the point of intersection of the tangents at P and H', and the load and tensions on the chain at P and H' are respectively proportional to the sides of a triangle parallel to their directions, the cable tension at P and H' and the direction of W must intersect in one point. Since w is uniform along x, the resultant direction of W passes through N, half way between H' and M. Therefore FH' = H'K, or, since H'K is the sub-tangent, the abscissa, FH', of the curve is equal to the sub-tangent, hence, the curve is the

ordinary parabola. also -It is known that the horizontal component of the tension of a cable will be a constant quantity if the loading, as is assumed in this case, be vertical; let that component be denoted by A. Lit the right triangh ShP be taken for the triangle of forces at P, in which NP represents the cable timeion at P, In the load W = wx, and Is I the constant harizontal component of. PA bring normal to the curver at P, The is APF and GNP will be similar, and we have the proportion - $\frac{\mathcal{H}\mathcal{F}}{\mathcal{H}\mathcal{P}} = \frac{\mathcal{F}\mathcal{P}}{\mathcal{H}\mathcal{N}} = \frac{\mathcal{X}}{\mathcal{W}\mathcal{X}} = \frac{1}{\mathcal{W}} = \alpha \text{ constant.}$ IF is the sub-normal of the curver of the cable, and since it is constant, the surver must be the common parabola. If the load placed on a cable 'be a

ordinary parabola. Also –

It is known that the horizontal component of the tension of a cable wire be a constant quantity if--the loading, as is assumed in this case, be vertical; let that component be denoted by H.

Let the right triangle GNP be taken for the triangle of forces at P, in which NP represents the cable tension at P, GN the load W = wx, and GP the constant horizontal component H. PH being normal to the curve at P, the Δ 's HPF and GNP will be similar, and we have the proportion: $\frac{HF}{GP} = \frac{FP}{GN} = \frac{x}{wx} = \frac{1}{w} = a \ constant$ HF is the sub-normal of the curve of the cable, and since it is constant, the

curve must be the common parabola. If the load placed on a cable be a

direct function of its length, The enve arsund by the mean fiber of the cable will be a caturary. If it be a direct punction of els span it will be a parabola. But the weight resting on the main chains is metter a direct function of the length of the cable nor of the span, but a function of both. The curve in therefore, mather a calmary nor a parabola. But since the roadway, which forme the principal part of the load, in distributed very nearly uniformly over the span, the curve approaches now the parabola, and in practice, is usually regarded ai such a curver, now if any two points, P and Q, ba considered fixed, and the portion P& of the cable carries the same intensity of load as before, we have - a cable conjing a load whose intensity along a shringet him and direction on miniform. Hance - if- a perfectly fligible cable

direct function of its length, the curve assumed by the mean fibre of the cable will be a catenary. If it be a direct function of its span it will be a parabola. But the weight resting on the main chains is neither a direct function of the length of the cable nor of the span, but a function of both. The curve is, therefore, neither a catenary nor a parabola. But since the roadway, which forms the principal part of the load, is distributed very nearly uniformly over the span, the curve approaches nearer the parabola, and in practice, is usually regarded as such a curve. Now if any two points, P and Q, be considered fixed, and the portion PQ of the cable carries the same intensity of load as before, we have - a cable carrying a load whose intensity along a straight line and direction are uniform. Hence – if - a perfectly flexible cable

carry a load mifarm in direction and intuily in reference to a straight line, the cable will assume the form of an ordinary parabola where ages will In parallel to the funding. Jarameter of Curva have the equation of the curve -X2 = 2 k y/11 in which 2 k in the parameter. Let $BC = X_1$, $ED = X_2$, then $\chi_{i}^{2} = 2 ph_{i} (h_{i} = H'B), \quad \chi = V2ph_{i} (2)$ $\chi_{2}^{2} = 2 \mu h_{2} (h_{2} = D H), \quad (\lambda_{2} = \sqrt{2 \mu h_{2}} (3)$ Then, multiplying together equations (2) and (3), X, X2 = 2/2/h, h2, and 2X, X2 = 4/2 (h, h2. (4) Hance- $(x_1 + x_2)^{T} = x_1^{T} + 2x_1x_2 + x_2^{T} = l^2 = 2p(\sqrt{h_1} + \sqrt{h_2})^2$ $= 2p(h_1 + 2h_1, h_2 + h_2)(5)$. $l^{2} = 2p(h_{1} + 2\ln_{1}h_{2} + h_{2}) (b)$ $i \cdot p = \frac{l^{2}}{2(\ln_{1} + \ln_{2})^{2}} = \frac{l^{2}}{2(h_{1} + 2\ln_{1}h_{2} + h_{2})} (b)$

carry a load uniform in direction and intensity in reference to a straight line, the cable will assume the form of an ordinary parabola whose axis will be parallel to the loading.

Parameter of CurveFrom Fg. 1 wehave the equation of the curve $x^2 = 2py$, (1) in which 2p isthe parameter.

Let BC = x1, ED = X2, then $x_1^2 = 2ph_1$ (h1=H'B), therefore $x_1 = \sqrt{2ph_1}$ (2) $x_2^2 = 2ph_2$ (h2=DH'), therefore $x_2 = \sqrt{2ph_2}$ (3) Then, multiplying together equations (2) and (3), $x_2 x_1 = 2p\sqrt{h_1h_2}$ = and $2x_2 x_1 = 4p(\sqrt{h_1} + \sqrt{h_2})^2$ (4) Hence $(x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2 = l^2 = 2p(\sqrt{h_1} + \sqrt{h_2})^2$ (5) $l^2 = 2p(h_1 + 2\sqrt{h_1h_2} + h_2$ (6)

$$p = \frac{l^2}{2(\sqrt{h_1} + \sqrt{h_2})^2} = \frac{l^2}{2(h_1 + 2\sqrt{h_1h_2} + h_2)}$$
(6)

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If the town on of the same hight,
then
$$k_1 = k_2 = k_1$$
, and equation (b)
become $-\frac{k_1}{p} = \frac{k_2}{8k_1} - -(7)$
If originated distances from lowest faint of
even to paints of suffort.
distance from the lowest faint of the caller
to the highest town in , Jiq 1, BC = X,
walso ED, the lower town , = X_2
 $X_1 = 12ph_1 - -(2)$
But $p = 2(h_1 + h_2) - (b)$. Indictuding there
is also a primitar, we have from (3) -
 $X_2 = \frac{k_1 + k_2}{k_1 + k_2} - -(9)$

If the towers* of the same height, the $h_1 = h_2 = h$, and equation (6) becomes:

$$p = \frac{l^2}{8h} (7)$$

Horizontal distances from lowest point of curve to points of support.

The horizontal distance from the lowest point of the cable to the highest tower is, Fig. 1, BC = x_1 so also ED, the lower tower, = x_2 $x_1 = \sqrt{2ph_1}$ ---(2) But $p = \frac{l^2}{2(\sqrt{h_1} + \sqrt{h_2})^2}$ (6). Substituting this value of p in (2) above-- $x_1 = \sqrt{2ph_1} = \sqrt{\frac{2h_1l^2}{2(h_1 + h_2)^2}} = \sqrt{\frac{l^2h_1}{(h_1 + h_2)^2}}$

$$=\frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}$$
----(8)

In a similar, we have from (3):

$$x_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}} - - - (9)$$

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lf

If
$$h_1 = h_2$$
; $x_1 = x_2 = \frac{l}{2}$ ----(10)

Inclination of cable at any point Since KH' = H'F = y, if i is the inclination to a horizontal line of the curve at any point P, then we have from the Δ FPK, (FK = 2y), x tan(i)=2y,

$$tan(i) = \frac{2y}{x}$$
 therefore $sec(i) = \sqrt{1 + \frac{4y^2}{x^2}}$ ----(11)

At the tops of the towers

$$tan(i_1) = \frac{2h_1}{x_1} - (12). \quad tan(i_2) = \frac{2h_2}{x_2} - (13)$$
$$h_1 = h_2, tan(i_1) = tan(i_2) = \frac{4h}{l} - (14)$$

Resultant tension at any point of cable It has

been shown that if the loading on a cable is uniform in direction, the component of cable tension normal to that direction will be constant at all points of the cable. Let the resultant tension at the lowest point of the cable be this constant component, denoted by H.

We have seen that
$$f = wHF$$
. But HF in
the sub-normal of the curve, and we
know from General grownery, that the
subnormal to all farabola is equal to
one last the parameter 210, or equal to p.
Hence, $H = wp - - (15)$
Aubstituting in the above the value of
 p as found in equation (6), we have -
 $H = \frac{wl^2}{2(K_1 + 2K_1, K_1 + K_2)} - - (16)$
And R informed the number beneries
of any faint, the four the triangle of
forces, Is N P,
 $R = H$ see $i = H \prod \frac{WL^2}{X^2} - - (17)$ by
substituting the value of see i in (11),
 $R = H \left[1 + \frac{WL^2}{X^2} - - (18) \right]$
 $R_2 = H \sqrt{1 + \frac{WL^2}{X^2}} - - (19)$

We have seen that H = wAF. But AF is the sub-normal of the curve, and we know from General geometry, that the subnormal to the parabola is equal to one half the parmeter 2p, or equalt to p. Hence, H=wp----(15)

Substituting in the above the value of p as found in equation (6), we have:

$$H = \frac{wl^2}{2(h_1 + h_2)^2} = \frac{wl^2}{2(h_1 + 2\sqrt{h_1 h_2} + h_2)} - ---(16)$$

Let R represent the resultant tension at any point, then from the triangle of forces, GNP,

PN=GPsec(i), or

$$R = Hsec(i) = H\sqrt{1 + \frac{4y^2}{x^2}}$$
----(17) but

substituting the value of sec(i) in (11). At the tops of the towers the tensions

are:

$$R_{1} = H \sqrt{1 + \frac{4h_{1}^{2}}{x_{1}^{2}}} - ----(18)$$

$$R_{2} = H \sqrt{1 + \frac{4h_{2}^{2}}{x_{2}^{2}}} - ----(19)$$

If h, = h2, then x, = x2 = 2, and from
Equation (16) -
$$H = \frac{WL^2}{8h^2} - --(20)$$

Also $N_1 = R_2 = H \sqrt{1 + \frac{10h^2}{22}} - --(21)$
Lungth of caller between a human friend
and the vertex, on between vertex and
a faint of which the inclination to a
henzynthe line in 2.
From the
calculus we have the formula for
the reclification of plane curves
 $H_2 = Vdx + dy^2$, in which 2 referents
the hugth of curve, and x and y the
guid coin directer.
 $X^2 = 2 fry, we have
 $dy^2 = \frac{y^2 dy^2}{p^2}, and$
Lo integral this expression, affly
formule of reduction -$

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If $L_1 = L_2$, then $x_1 = x_2 = \frac{l}{2}$, and from Equation (16) $H = \frac{wl^2}{8h}$ ----(20) Also, $R_1 = R_2 = H\sqrt{1 + [illegible]}$

Length of cables between a known point and the vertex, or between vertex and a point at which the inclination to a horizontal line in "i". From the calculus we have the formula for

the rectification of plane curves

 $dz = \sqrt{dx^2 + dy^2}$, in which z represents

the length of the curve, and x and y the general coordinates.

From the equation of the curve

$$x^{2} = 2py$$
, we have
 $dy^{2} = \frac{x^{2}dx^{2}}{pz}$, and
 $dz = \frac{1}{p}\sqrt{p^{2} + x^{2}}dx$

To integrate this expression, apply formula C of reduction.

$$y = \int \chi^{m} (\alpha + b\chi^{m})^{p} d\chi = \frac{\chi^{m+1}(\alpha + b\chi^{m})^{p+1} d\chi}{mp + m + 1} \qquad (c)$$

$$Z = \frac{\chi \sqrt{\chi^{2} + p^{-}}}{2 p} + \frac{p}{2} \int \frac{dy}{\sqrt{p^{2} + \chi^{2}}} - (22)$$

$$Z = \frac{\chi \sqrt{\chi^{2} + p^{-}}}{2 p} + \frac{p}{2} \int \frac{dy}{\sqrt{p^{2} + \chi^{2}}} - (22)$$

$$J_{0} \text{ integrate } \frac{d\chi}{\sqrt{p^{2} + \chi^{2}}}, \quad p_{0}t = \chi + \sqrt{p^{2} + \chi^{2}} - (23)$$

$$Then d z = d\chi + \frac{\chi d\chi}{\sqrt{p^{2} + \chi^{2}}} = d\chi (1 + \frac{\chi}{\sqrt{p^{2} + \chi^{2}}})$$

$$dz = \frac{\chi + \sqrt{p^{2} + \chi^{2}}}{\sqrt{p^{2} + \chi^{2}}} d\chi$$

$$Mow \quad curv \quad hove = \frac{\chi + \sqrt{p^{2} + \chi^{2}}}{\chi + \sqrt{p^{2} + \chi^{2}}} = \sqrt{\chi}$$

$$\int \frac{dz}{z} = \frac{\chi + \sqrt{p^{2} + \chi^{2}}}{\chi + \sqrt{p^{2} + \chi^{2}}} d\chi$$

$$\int \frac{dz}{z} = \int \frac{d\chi}{\sqrt{p^{2} + \chi^{2}}} = \log 2 \quad \text{Reilning the value of } 2 - \int \frac{d\chi}{\sqrt{p^{2} + \chi^{2}}} = \log (\chi + \sqrt{p^{2} + \chi^{2}}) - (23)$$

Thursdaw -
$$Z = \frac{\chi \sqrt{p^2 + \chi^2}}{2p} + \frac{p}{2} \log(\chi + \sqrt{p^2 + \chi^2}) + C$$

Estimating the arc from the viter, it
bring the origin,
$$C = -\frac{p}{2}\log p$$

$$y = \int x^m (a + bx^n)^p dx = \frac{x^{m+1}(a+bx^n)^p + anp \int x^m (a+bx^n)^{p-1} dx}{np+m+1}$$
 (C)
and we have

$$z = \frac{x\sqrt{x^2 + p^2}}{2p} + \frac{p}{2}\int \frac{dx}{\sqrt{p^2 + x^2}}$$
-----(22)

To integrate
$$\frac{dx}{\sqrt{p^2+x^2}}$$
, put $z = x + \sqrt{p^2 + x^2}$ ----(23)
then $dz = \frac{x+\sqrt{p^2+x^2}}{\sqrt{p^2+x^2}} dx$

Now we have

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$$\frac{dz}{z} = \frac{\frac{x + \sqrt{p^2 + x^2}}{\sqrt{p^2 + x^2}} dx}{x + \sqrt{p^2 + x^2}} = \frac{dx}{\sqrt{p^2 + x^2}} - --(24)$$

$$\int \frac{dz}{z} = \int \frac{dx}{\sqrt{p^2 + x^2}} = \log(z) \text{ Restoring the value}$$
of z:
$$\int \frac{dx}{\sqrt{p^2 + x^2}} = \log(x + \sqrt{p^2 + x^2}) - --(25)$$

Therefore

$$z = \frac{x\sqrt{p^2 + x^2}}{2p} + \frac{p}{2}\log(x + \sqrt{p^2 + x^2} + C)$$

Estimating the arc from the vertex, it
being the origin $C = -\frac{p}{2}\log p$

Thus the corrected integral is -

$$Z = \frac{x\sqrt{p^2 + x^2}}{2p} + \frac{p}{2}\log\left[\frac{x+\sqrt{p^2 + x^2}}{p}\right] - -(26)$$
Now by substituting in the above for pite
ralm $\frac{1}{2\sqrt{q}}$, the equation can be part in the porm-

$$Z = \frac{x^2}{4\sqrt{q}}\left[\frac{24}{x}\sqrt{1+\frac{44}{x^2}} + \log\left(\frac{24}{x} + \sqrt{1+\frac{44}{x^2}}\right)\right] - -(27)$$
Now as how piece stat $\frac{x^2}{x} = \frac{p}{2}$; $\frac{24}{x} = \tan i$;
 $1+\frac{44}{x} = peei, and by substituting then
using, eq. (27) becomes -
$$Z = \frac{1}{2}\left[\tan i \operatorname{aci} + \log(\tan i + \operatorname{aci})\right] - -(28)$$
In the above formulas the modules is 1.
Direct the above formulas in the vertex to any
for the distances from the vertex to any
for x and y in (27), $m - i_2$ for is in (27)
and odding the results. For $2\sqrt{1-1}$$

22

Thus the corrected integral is $z = \frac{x\sqrt{p^2 + x^2}}{2p} + \frac{p}{2}\log(\frac{x + \sqrt{p^2 + x^2}}{p})$ -----(26) Now by substituting in the above for p its value $\frac{x^2}{2y}$, the equation can be put in the form-- $z = \frac{x^2}{4y} \left(\frac{2y}{x} \sqrt{1 + \frac{4y^2}{x^2}} + \log\left[\frac{2y}{x} + \sqrt{1 + \frac{4y^2}{x^2}}\right]\right) - \dots - (27)$ Now we have seen that $\frac{x^2}{4y} = \frac{p}{2}$; $\frac{2y}{x} = \tan(i)$; $\sqrt{1 + \frac{4y^2}{x^2}} = sec(i)$, and by substituting these values, eq. (27) becomes $z = \frac{p}{2} [tan(i)sec(i) + log(tan(i) + sec(i))] ----(28)$ In the above formulas, the Naperian logarith is used, since the modulus is 1. Since the above formulae were deduced for the distance from the vertex to any particular point, the total length of cable will be found by substituting for $y_1 h_1$; and for $x_1 x_1$ in equation (27); or i_1 for i in Eq. (28); then x_2 and h_2

for x and y in (27), or i_2 for i in (28) and adding the results. Denoting these

usuets by l, and lz, the total lingth
with be-
l, + lz --- (29)
a formula, which is class enough for
practical femposes, and which is frequently
used, is deduced as foreaus.
In fig 1. suppose H'P is an are of a
circle where radius is R. The
conditions x and y as the pare as before
The expression for a circular are in the
integral calculus is

$$\int_{(1-\frac{1}{R})^{R_2}} = \int_{1-\frac{1}{2}R_2}^{\frac{1}{2}}$$
, approximately,

results by l_1 and l_2 , then total length will be:

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 $l_1 + l_2$ ----(29) A formula, which is close enough for practical purposes, and which is frequently used, is deduced as follows. In fig. 1 suppose H'P is an arc of a circle whose radius is R. The coordinates x and y are the same as before. The expression for a circular arc in the integral calculus is:

$$\frac{dx}{\sqrt{1-\frac{x^2}{R^2}}} = \int \frac{dx}{1-\frac{x^2}{2R^2}}, \text{ approximately,}$$

considering R very large as compared with x.

Suspinion rods -It is usually assumed in the calculations relating to puspinsion rode, that the cable lies in a vertical from, and that the englission rods on virtual. Since in all cases the suspension rode on parallel to each other, the above person plin door not affect the generality of the results. If the rode are inclined, it true lengths can be found by multiplying the value obtained by the search of the inclination of the rode to a vortical line. a flat parabala' nearly coincides with a circle and we may suppose the camber to In formed by a parabolic arc.

Suspension Rods

It is usually assumed in the calculations relating to suspension rods, that the cable lies in a vertical beam, and that the suspension rods are vertical. Since in all cases the suspension rods are parallel to each other, the above assumption does not affect the generality of the results. If the rods are inclined, the true lengths can be found by multiplying the values obtained by the secant of the inclination of the rods to a vertical line.

A flat parabola nearly coincides with a circle and we may suppose the camber to be found by a parabolic arc.





Figure 2.

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Let the ordinate AD=x be measured from A towards B, and DC=y perpendicular to it. Let $AB=x_1$ = the half span. The curve of the cables is the parabola as before.

$$x^{2} = 2py,$$

$$y = \frac{x^{2}}{2p}$$

From Eq. (1) $p = \frac{l^{2}}{8h}$ substituting:

$$y = \frac{4hx^{2}}{l^{2}}$$

 $AB = x_{1} = \frac{1}{2}l, \therefore l=2x, \text{ and we have}$

$$y = \frac{4hx^{2}}{l^{2}} = \frac{4hx^{2}}{4x_{1}^{2}} = L\frac{x^{2}}{x_{1}^{2}}$$

Now L=y, in Fig II, or calling y, the ordinate and x the abscissa of any point in the curve above AB, we have the general equation:

$$y' = y_1 \frac{x^2}{x_1^2} - \dots - (33)$$

In the same manner for the lower curver or the camber $y'' = z \frac{x'^2}{x^2} - -(34)$, in which I'm is taken as the axis of abscissas and z is the ordinate Representing the kingth of any suspender as 'ce' by k, we have L = CC' = CD + DD' + DC' - - (35)Now DD'= ho= c, and taking the value of CD and D'C' as given in Eqs. (33) and (34), $h = \gamma' + \gamma'' + e - - (36)$ From this we are that each puspinder is composed of the constant length c and the two voriable ones y and y". adding equatione (33) and (34), and representing the sum of the variable lugthe hy y $y = (y' + y'') = (y_1 + 2) \frac{x_1}{x_2} - (37)$ now let ate suspenders are be the same distance abort, and represent this constant distance by d, them.

In the same manner for the lower curve or the camber

 $y'' = z \frac{x^2}{x_1^2}$ ----(34) in which

A'B' is taken as the axis of abscissae and z is the ordinate Representing the length of any suspender as CC' by L, we have -

L=CC' = C'D+DD'+D'C'----(35) Now DD' = h_0 = C, and taking the value of CD and D'C' as given in Eqs. (33) and (34), h = y'+y''+C ----(36) From this we see that each suspender is composed of the constant length c and the two variable ones y' and y''. Adding equations (33) and (34), and representing the sum of the variable lengths by y --

$$y = (y' + y'') = (y_1 + 2)\frac{x^2}{x_1^2}$$
----(37)

Now let the suspenders all be the same distance apart, and represent this constant distance by d, then

$$h_{2} = e + \frac{4d^{2}}{x_{1}^{2}}(y_{1}+z)$$

$$h_{3} = e + \frac{9d^{2}}{x_{1}^{2}}(y_{1}+z)$$

$$h_{n-1} = e + \frac{(y_{1}-1)^{2}}{x_{1}^{2}}(y_{1}+z)$$

$$h_{n-1} = e + \frac{(y_{1}-1)^{2}}{x_{1}^{2}}(y_{1}+z)$$

$$h_{n} = e + \frac{(y_{1}^{2}d^{2}}{x_{1}^{2}} = 1)(y_{1}+z) = e + y_{1} + z_{2} - -(38)$$

Since h, was assumed equal to h2 in the above calculations, on the towns of the same hight and equal to h, the lengths of the surfunders on each side of the lowest paint in the cable will be equal, and having computed one side we use this values for the other. The vertical load which any rod conine mostiplied by the second of its inclination to a vertical line gives the phase on such rod, 27

$$h_{2} = c + \frac{4d^{2}}{x_{1}^{2}}(y_{1} + z)$$

$$h_{3} = c + \frac{9d^{2}}{x_{1}^{2}}(y_{1} + z)$$

$$h_{n-1} = c + \frac{(n-1)^{2}}{x_{1}^{2}}(y_{1} + z)$$

$$h_{n} = c + (\frac{n^{2}d^{2}}{x_{1}^{2}})(y_{1} + z) = c + y_{1} + z \text{----}(38)$$

Since h_1 was assumed equal to h_2 in the above calculations, or the towers of the same height and equal to h, the lengths of the suspenders on each side of the lowest point in the cable will be equal, and having computed one side we use these values for the other. The vertical load which any rod carries multiplied by the secant of its inclination to a vertical line gives the stress on such rod.

Deflection of calle for change in high,
the chair numaining the cause.
X.(1+24), aubititute X, for X and b, for
Y, and we have -
X.(1+3X), cubititute X, for X and b, for
Y, and we have -
X.(1+
$$\frac{h_1}{3X_1^2}$$
) --- (39)
also subitituding X₂ and b₂ for X and b
in the same equation, it becomes
X₂(1+ $\frac{2h_1}{3X_2^2}$) -- (40)
alding the above equations and denoting
the Two sequences of the parabola lay e,
and e₂, we have the total langth of
caller -
C₁+e₂ = X₁+X₂+ $\frac{2}{3}$ ($\frac{h_1^2}{X_1}$ + $\frac{h_2}{X_2}$) -- (41)
Differentiating -
d(e₁+e₂) = $\frac{4}{3}$ ($\frac{h_1}{X_1}$ + $\frac{h_2}{X_2}$) dX -- (42)
Now $h_1 - h_2$ being equal to a constant,
dh₁ = dh₂ = dh
 $\frac{30(e_1 + e_2)}{H(\frac{h_1}{X_1} + \frac{h_2}{X_2})}$

Deflection of a cable for change in length, the span remaining the same In Eq. (3) $x(1 + \frac{2y^2}{3x^{-2}})$, substitute x_1 for x and h_1 for y, and we have $x_1(1 + \frac{h_1^2}{3x_1^2})$ -----(39)

Also substituting x_2 and h_2 for x and h in the same equation, it becomes

$$x_2(1+\frac{h_2^2}{3x_2^2})$$
-----(40)

Adding the above equations and denoting the two segments of the parabola by c_1 and c_2 , we have the total length of cable --

$$c_1 + c_2 = x_1 + x_2 + \frac{2}{3}\left(\frac{h_1^2}{x_1} + \frac{h_2^2}{x_2}\right)$$
-----(41)

Differentiating:

28

$$d(c_1 + c_2) = \frac{4}{3} \left(\frac{h_1}{x_1} + \frac{h_2}{x_2}\right) dx - \dots - (42)$$

Now $h_1 - h_2$ being equal to a constant,

$$\therefore dh = \frac{3d(c_1 + c_2)}{4(\frac{h_1}{x_1} + \frac{h_2}{x_2})}$$
(43)

 $dh_1 = dh_2 = dh$

From whatever cause the cable may vary in length, this variation in to be put for d(c,+c2) in equations (42) and (43), and the de will be The companding deflection of the lawest paint of the cable. If the town on of the purse hight - $C_1 = C_2$, $W_1 = W_2$, $X_1 = X_2 = \frac{l}{2}$ Thu we have from (42) -2 de, = 16 h, --- (44) $dh = \frac{3h_1}{11} 2dc_1 - -- (45)$ In equation (42) and (43) the assumption, though not strictly true, is that the lowest paint of the cable remaine at the same horizontal distance from the towers.

From whatever cause the cable may vary in length, this variation is to be put for $d(c_1 + c_2)$ in equations (42) and (43), and then *dh* will be the corresponding deflection of the lowest point of the cable. If the towers are of the same height-

$$c_1 = c_2, h_1 = h_2, x_1 = x_2 = \frac{l}{2}$$

Then we have from (42)-

29

$$2dc_1 = \frac{16}{3} \frac{h_1}{l} - \dots - (44)$$
$$dh = \frac{3h_1}{16l} 2dc_1 - \dots - (45)$$

In equations (42) and (43) the assumption, though not strictly true, is that the lowest point of the cable remains at the same horizontal distance from the towers.

To ablain the true length of the curr since the above relations were deduced from the approximate formula (), we much take the true equation for the length of the curve. as before, let (c, +c2) by the known ligh of envi before variation taken place ; let b, and b2, X, and X2 be the original highly of towns also agreente of span also known. Lit y, and yz be the height of the lowing about the hewest point in the cable, after variation in its length hers taken place. X, and Xz are she constante. Lit the variation in length of the cable be represented by V Thue $v = -(c_1 + c_2) + (c_1 + c_2 + v) - - - (46)$ $V = \frac{\chi_{i}}{4 \chi_{i}} \left(\frac{2 \chi_{i}}{\chi_{i}} \sqrt{1 + \frac{4 \chi_{i}^{2}}{\chi_{i}^{2}}} + \log \left(\frac{2 \chi_{i}}{\chi_{i}} + \sqrt{1 + \frac{4 \chi_{i}^{2}}{\chi_{i}^{2}}} \right) \right)$ $+\frac{x_{2}}{4y_{2}^{2}}\left[\frac{2y_{2}}{x_{2}}\sqrt{1+\frac{4y_{2}^{2}}{x_{2}^{2}}}+\log\left(\frac{2y_{2}}{x_{2}}+\sqrt{1+\frac{4y_{2}^{2}}{x_{2}^{2}}}\right)\right]-\left(c_{1}+c_{2}\right)-$

To obtain the true length of the curve since the above relations were deduced from the approximate formula (), we must take the true equation for the length of the curve. As before, let $(c_1 + c_2)$ be the known length of curve before variation takes places; let h_1 and h_2 , x_1 and x_2 be the original heights of the towers also segments of span also known. Let y_1 and y_2 be the heights of the towers above the lowest point in the cable, after variation in its length has taken place. x_1 and x_2 are still constants. Let the variation in length of the cable be represented by v.

Thus
$$v = -(c_1 + c_2) + (c_1 + c_2 + v)$$
----(46)
 $v = \frac{x_1^2}{4y_1} \left[\frac{2y_1}{x_1} \sqrt{1 + \frac{4y_1^2}{x_1^2}} + \log\left(\frac{2y_1}{x_1} + \sqrt{1 + \frac{4y_1^2}{x_1^2}}\right) \right]$
 $+ \frac{x_2^2}{4y_2^2} \left[\frac{2y_2}{x_2} \sqrt{1 + \frac{4y_2^2}{x_2^2}} + \log\left(\frac{2y_2}{x_2} + \sqrt{1 + \frac{4y_2^2}{x_2^2}}\right) \right] - (c_1 + c_2)$ ----(47)

But since y, -y_ = h, -h_2 = a constant, we can take the value of y, or yz and substitute in Equation (47), and we win the have only one unknown quantity in the right member, and this unknown is determined by trial, 14, or y2, h, m h2 may be taken as the) The first value of y, on y2 taken may b h, wh z menared or decreased, as the care may by by dh. Kl is taken from Eq. (45) y, - h, = y_2 - h 2 is the deflection sought, The wariation of length can be abtained at one from equation (47) when the new hight y, and yz an given. When the town an of the parm height , X, = X2 = 2, C, = C2, and y, = Y2=h Making substitution of the value in equation (47), then results, after adding-

But since $y_1 - y_2 = h_1 - h_2 = a \ constant$, we can take the value of y_1 or y_2 and substitute in equation (47), and we will then have only one unknown quantity in the right member, and this unknown is determined by trial $(y_1 \text{ or } y_2, h_1 \text{ or } h_2 \text{ may be taken as the})$. The first value of y_1 or y_2 taken may be h_1 or h_2 increased or decreased, as the case* may be, by *dh*. *dh* is taken from Eq. (45).

 $y_1 - h_1 = y_2 - h_2$ is the deflection sought.

The variation of length can be obtained at once from equation (47) where the new heights y_1 and y_2 are given. When the towers are of the same height,

 $x_1 = x_2 = \frac{l}{2}, c_1 = c_2, y_1 = y_2 = h$ Making substitutions of the values in equation (47), then results, after adding,

V = 2 2 4 4 VI+ 16 h + log (4h + VI+ 16h2) -28 (48) If h is Known, we can find a from the above equation. If v is Known, h is found by trial, and a then resulte. h-h, = deflection of the middle paint of the trues.

$$v = \frac{2l^2}{16h} \left[\frac{4h}{l} \sqrt{1 + \frac{16h^2}{l^2}} + log\left(\frac{4h}{l} + \sqrt{1 + \frac{16h^2}{l^2}}\right) \right] - 2c - \dots - (48)$$

If *h* is known, we can find *v* from the above equation. If *v* is known, *h* is found by trial, and *v* then results. h-h = deflection of the middle

 $h-h_1$ = deflection of the middle point of the truss.



Pressure on Tower

Fig. 3

Let	P∟ = th	e vertical cor	npone	nt of pre	ssure	on	tower	head
"	Ph = "	horizontal	"	"	"	"	u	"
"	R = "	resultant	u	"	"	"	u	"
"	T_p and	T'p = tensior	n of cal	ole on di	fferent	* si	des"	"
"	x, x' ar	nd θ represer	nt inclir	nations to	o a ve	rtica	al as s	hown.
When we consider friction on the saddle								
$P_L = T_P cos(\alpha) + T'_p cos(\alpha'); $ (49)								
$P_h = T_p sin(\alpha) - T'_p sin(\alpha'); $ (50)								
	$R = \sqrt{R}$	$P_L^2 + P_h^2$; (51)		cos(l	9) = -	$\frac{P_L}{R};$	(52)	
When friction is not considered								
$T_p = T'_p \therefore P_L = T_p(\cos(\alpha) + \cos(\alpha'))$								
$P_h = T_p(sin(\alpha) - sin(\alpha'))$								
$R = \sqrt{P_L^2 + P_h^2}$								
$cos(\theta) = \frac{P_L}{R}$								
If $\alpha = \alpha'$, $P_L = 2W$, $P_h = 0$, $R = 2W = P_L$,								
$\theta = 0$, then W represents one half the weight of								
the load and structure.								

Bracing to risist heavy traveling load -Variance methods have been proposed, and some of stem tried, to enable a suspinion bridge to resist the setion of a heavily traveling had as as to undergo no more diefigurentet atom a girder. I a men this in a bridge of several bags, the price must be made very strong, and the chains security factured to them. The bit way of bracing in dry means of anxilary girders, a adain of straight girden of any convenient form hing from the cobler by suspending rode and supporting the cross justs of the platform. This girder should be supported of each end, and also fastund down, as iten an cirtain paintions of the rolling load which would tend to rain and end of the truck. The ends should be for to move horizontally, howward,

Bracing to resist heavy traveling load Various

methods have been proposed, and some of them tried, to enable a suspension bridge to resists the action of a heavily traveling load so as to undergo no more disfigurement than a girder. To ensure this in a bridge of several bays, the piers must be made very strong, and the chains securely fastened to them.

The best way of bracing is by means of auxiliary girders, or a pair of straight girders of any convenient form hung from the cable by suspending rods and supporting the cross joists of the platform. These girders should be supported at each end, and also fastened down, as there are certain positions of the rolling load which would tend to raise one end of the truss. The ends should be free to move horizontally, however.

By privating a change in the form of the cable, which is accomplished by the sliffening trues, we not only prevent very injurious undulations, but also bessen the work of computing stresses, which would be very difficult if the cable did not retain the form. The calle will assume the same parabolic curve only when the is a uniform full on the suspension rode from und to und. Let I be the uniform fired on any suspinion rod, and t its intuity for mit of span, how if - & represents one pavel light in the truce, I= pt Let ur bi the fixed load per mit of span sustained by the cables, and with moving load sustained by them; Let I be the span; R the reaction of B (fig.); R' the reaction A. Suppose the moving load to pass on from 13. It x, bit the distance from B to the front of the moving food. The load is supposed continuous.

35

35

By preventing a change in the form of the cable, which is accomplished by the stiffening truss, we not only prevent very injurious undulations, but also lessen the work of computing stresses, which would be very difficult if the cable did not retain the same form. The cable will assume the same parabolic curve only when there is a uniform pull on the suspension rods from end to end. Let *T* be the uniform pull on any suspension rod, and *t* its intensity per unit of span. Now if p represents one panel length in the russ, T=ptLet *w* be the fixed load per unit of span sustained by the cables, and w' the moving load sustained by them; Let ℓ be the span; R the reaction at B (fig.); *R*' the reaction A. Suppose the moving load to pass on from B. Let x_1 be the distance from B to the front of the moving load. The load is supposed continuous.



Fig. 4

All the forces acting on the truss are vertical in their direction, and we have two equations of equilibrium, indicating that the sum of the external vertical forces and that the sum of these moments about any point must each equal zero.

 $wl + w'x_1 - tl - R - R' = 0$ ----(53) Thus by taking the moments about the head of the moving load-

 $(w+w')\frac{x_1^2}{2} - t\frac{x_1^2}{2} - Rx_1 + (t-w)\frac{(l-x_1)^2}{2} + R'(l-x_1) = 0--(54)$

In each of the above equations are three unknown quantities, *t*, *R*, and *R'*. If any one of these is known, the others may found from the two conditional equations. From this we see that unless at least one condition is imposed, it

In upossible to ascertain how much The trush will carry, wither in connection with the cable, or above. assuming a value for 2, R, ~ R', makes the stiffering truck all allegether in connection with the cable, and carry no load as an ordinary buse. From the above it is seen that the sum of see the loads w, wire, must be equa to the own of all the uniform upward forces, T=pt. The resultants of the two forces act in different fines, and the trun is the subjected to the action of a careful, which much a constracted abother couple of equalmonut but appoint live of action. These couples must set at the extructive I and B - They are the reaction R and R. Therefore -R = -RiSubstituting this value in Equation (53),

is impossible to ascertain how much the truss will carry, either in connection with the cable, or alone. Assuming a value for *t*, *R*, or *R*', makes the stiffening truss act altogether in connection with the cable, and carry no load as an ordinary truss. From the above it is seen that the sum of all the loads w, w', and c*, must be equal to the sum of all the uniform upward forces, T=pt. The resultant of the two forces act in different lines, and the truss is then subjected to the action of a couple, which must be counteracted another couple of equal moment but opposite lines of action. These couples must act at the extremities A and B. They are the reactions R and R'. Therefore- $R = -R^{\prime\prime}$ Substituting this value in Equation (53),

and solving for z, $z = w + w' \frac{x_1}{l}$ ----(55) From substituting the same in (54), $R = -R' = \frac{w'x_1}{2} (1 - \frac{x_1}{l})$ ----(56) In (56), if $x_1 = l$, or w' = 0, both reactions become zero, R = -R' = 0It is also seen that R and R' are numerically equal, but have opposite directions. R' is a downward reaction and will

show the amount of anchorage required. Differentiating (56) and finding value of _ _,

$$\frac{dR}{dx_1} = \frac{w'}{2} - \frac{w'x_1}{2l} - \frac{w'x_1}{2l}, \text{ and } x_1 = \frac{l}{2}$$
Now substituting $x_1 = \frac{l}{2}$ in equation (56)
$$R = \frac{w'l}{8} - ---(57)^2$$

Equation shows the greatest shear to be provided for at either end of the truss, and also the maximum amount of anchorage to be provided for.

The matter on Atrine proper is very limited, on account of spore. Respectfully Inbuitted-W, M. Claypool. Rucea, mo., Jun 7", 1884.

The matter on stresses <u>proper</u> is very limited, on account of space.

Respectfully Submitted W. M. Claypool

Rolla, Mo., June 7th 1884.