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Stresses in suspension bridges

William M. Claypool

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W. M. CLAYPOOL.

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A suspension bridge is one in which the roadway over the stream or span to be crossed is suspended from chains or wire ropes.
A suspension bridge consists of the towers or piers over which the main chains or cables pass; the anchorages to which the ends of the cables are attached; the main chains or cables from which the roadway is suspended; the suspending rods or chains which connect the roadway with the cable and the roadway.
The sub-structure consists of the foundations, piers &c. The super-structure consists of the roadway and the chains or cables.

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Towers-
The towers, frequently termed
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used. The particular kind or form of
towers will depend to some extent upon
the locality and character of surroundings.
Their dimensions will depend upon
their height and the amount of strains
they will have to resist.
Where the cables pass over the towers are
saddles. One construction of saddles
in which the cable passes over friction
rollers rigidly attached to the top of the
pier, allows the cable to slip backwards
and forwards over it with comparatively
little friction, so that the stress on the
cable may be taken as equal on
both sides of the saddle.
In another construction the chain
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In the first form of saddles the
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leaves the pier at an equal inclination on each side, and even when the bridge is designed with an equal slope of chain on both sides of the pier, a change in the distribution of weight due to any passing load, will cause some departure from the equal slope of the chains, and therefore from the truly vertical pressure on the piers. This departure is easily allowed for in the design of the bridge piers.
The friction on the saddle renders the assumption of equal stresses on each side slightly incorrect, and with this type of saddle, care must be taken to provide against the wear produced by the motion of the chain.
In the second type, the use of rollers under the solid saddle leaves the motion of the saddle very free; its resultant pressure on the tower is always vertical, and the chains may

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leave the tower at any angle, equal or unequal.
The chain must in no case be rigidly attached to the pier, unless the pier, or rather support in this instance, is free to rock on its base, or for example, when the place of the pier is taken by iron struts working on a horizontal axis.

<u>Anchorage</u> —
If the shore or bank be of rock, a vertical passage should be excavated and a strong iron plate placed in the bottom and firmly imbedded in the side of the passage.
Through this plate the ends of the cables are passed and secured on the under side.
After the cables are put in place, the passage should be filled with concrete and masonry.
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anchorage, a heavy mass of masonry should be built of large blocks of cut-stone well bonded together for this purpose. In this case it is well to construct a passage way so that the chains and fastenings may at any time be examined. The mass of masonry or the natural rock to which the ends of the cables are fastened is frequently called the abutment.
Its stability must be greater than the tension of the cables. Its weight and thickness must be sufficient to prevent its being over turned, and its center of resistance must be in safe limits. The calculations in regard to the anchorage, when it is artificial, properly belong with the suspension bridge, but I have left them out, since they can be very appropriately included in masonry.

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Cables -

These may be made of iron bars connected by eye bar and pin joints, of iron links &c, but the custom now is to use wire ropes or cables. The smallest number of cables is two, one to support each side of the roadway. Generally more than two are used, since, for the same amount of material, they offer at least the same resistance, are more accurately manufactured, are liable to less danger of accident, and can be more easily put in place and replaced than a single cable of an equal amount of material.

Great care is taken to give each wire the same degree of tension. To ensure this, it used to be thought necessary to strain each wire separately over the actual piers, or piers similarly placed, and bind them together when hanging, strained by their own weight with the dip proposed for the bridge. It was also thought essential that each rope

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should be an aggregate of parallel wires, not spun, as in a rope. Experiment, however, has shown that wire ropes spun with a machine which does not put a twist in each wire, but lays it helically and untwisted, and with no straight central wire, are as strong as wire ropes of equal weight made with straight wires.

It is the custom now to make the cable of wire 1/6" to 1/5" in diameter, and bring them to a cylindrical shape by a spiral wrapping of wire. The wires are coated with varnish before being bound up, and the cable itself is suitably protected from atmospheric influences.

<u>Suspension</u> <u>Rods</u> –

When the cable is composed of links or bars, they are attached directly to them. If of rope, the suspension rod is attached to a collar of iron of suitable shape bent around the cable, or to a saddle piece resting on it. When there are two cables,

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care must be taken to distribute the load upon the cables according to their degree of strength.

Roadway –

The roadway bearers are supported by the suspension rods. On the bearers are laid longitudinal joists, and on them the planking, or the planking is laid directly on the roadway bearers. The latter are stiffened by diagonal ties of iron placed horizontally between each pair of roadway bearers.

General Principles –

The great merit of a suspension bridge is its cheapness, arising from the comparatively small quantity of material required to carry a given passing load across a given span. This cheapness may be seen more clearly by considering an example. A man might cross a chasm of 100' by hanging to a steel wire .21" in diameter, dipping 10';

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lighter than a beam, the saving in this respect will, for small spans, be more than compensated by the expense of the anchorages.

The disadvantages of the suspension bridge are numerous. A change in the distribution of the load causes a very sensible deformation of the structure, for the cable of the suspension bridge must adapt its form to the new position of the load, whereas in the beam the deformation is hardly sensible, equilibrium being attained by a new distribution of the stresses through the material.

This flexibility of the suspension bridge renders it unsuitable for the passage of a railway train at any considerable speed. The platform rises up as a wave in front of any rapidly advancing load, and the masses in motion produce stresses much greater than those which would result from

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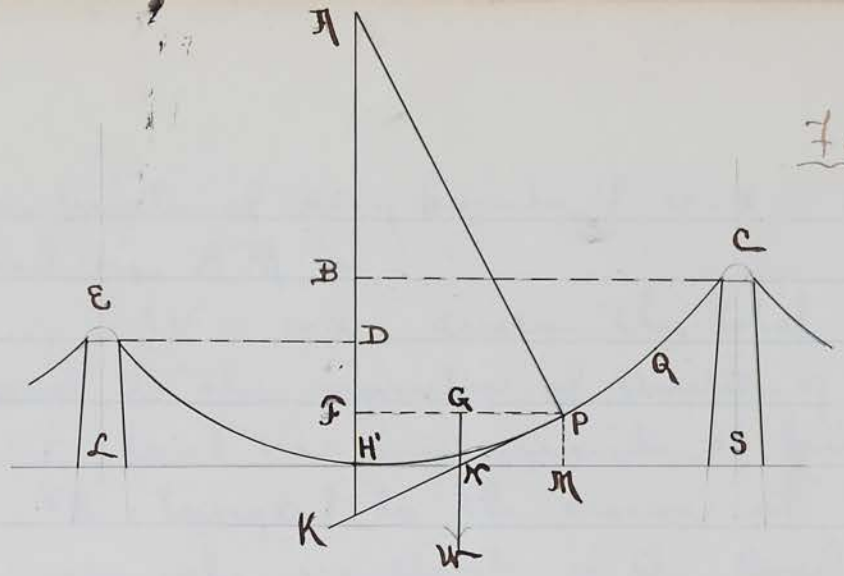

Fig. 1.

Let EH'C be cable of a suspension bridge carrying a load which extends over the whole span. In practice the load carried by a suspension bridge cable is uniform in intensity in reference to a horizontal line. Theoretically this assumption would not do, but the load is so nearly uniform per foot of span that it is taken to be exactly so.

Let $ED + BC = l$ = span

$BH' = h_1$ = hight of highst tower

$DH' = h_2$ = " " lower tower.

w = load for horizontal foot

x = distance measured horizontally from H', the lowest point in the cable.

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The ordinate of any point P is x, then the load on H'M is

$$W = wx,$$ since the total load is equal to the number of units of length into the load on one unit of length. Draw PK tangent to the curve at P, then, since the resultant of the load between P and H' acts through the point of intersection of the tangents at P and H', and the load and tensions on the chain at P and H' are respectively proportional to the sides of a triangle parallel to their directions, the cable tensions at P and H' and the direction of W must intersect in one point.
Since w is uniform along x, the resultant direction of W passes through N, half way between H' and M.
Therefore, FH' = H'K, or, since H'K is the sub-tangent, the abscissa, FH', of the curve is equal to the sub-tangent, hence, the curve is the

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Therefore FH' = H'K, or, since H'K is the sub-tangent, the abscissa, FH', of the curve is equal to the sub-tangent, hence, the curve is the

ordinary parabola.
Also –
It is known that the horizontal component of the tension of a cable wire be a constant quantity if– the loading, as is assumed in this case, be vertical; let that component be denoted by H.
Let the right triangle GNP be taken for the triangle of forces at P, in which NP represents the cable tension at P, GN the load W = wx, and GP the constant horizontal component H.
PH being normal to the curve at P, the Δ's HPF and GNP will be similar, and we have the proportion –

$$\frac{HF}{GP} = \frac{FP}{GN} = \frac{x}{wx} = \frac{1}{w} = \text{a constant.}$$

HF is the sub normal of the curve of the cable, and since it is constant, the curve must be the common parabola.
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direct function of its length, the curve assumed by the mean fibre of the cable will be a catenary. If it be a direct function of its span it will be a parabola. But the weight resting on the main chains is neither a direct function of the length of the cable nor of the span, but a function of both. The curve is, therefore, neither a catenary nor a parabola. But since the roadway, which forms the principal part of the load, is distributed very nearly uniformly over the span, the curve approaches nearer the parabola, and in practice, is usually regarded as such a curve.

Now if any two points, P and Q, be considered fixed, and the portion PQ of the cable carries the same intensity of load as before, we have - a cable carrying a load whose intensity along a straight line and direction are uniform.

Hence - if - a perfectly flexible cable

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carry a load uniform in direction and intensity in reference to a straight line, the cable will assume the form of an ordinary parabola whose axis will be parallel to the loading.

<u>Parameter of Curve</u> –

From fig. 1 we have the equation of the curve –

$$x^2 = 2py, (1)$$ in which $2p$ is the parameter.

Let $BC = x_1$, $ED = x_2$, then

$$x_1^2 = 2ph_1\ (h_1 = H'B),\ \therefore x_1 = \sqrt{2ph_1}\ (2)$$
$$x_2^2 = 2ph_2,\ (h_2 = DH'),\ \therefore x_2 = \sqrt{2ph_2}\ (3)$$

Then, multiplying together equations (2) and (3),

$$x_1 x_2 = 2p\sqrt{h_1 h_2},\ \text{and}\ 2x_1 x_2 = 4p\sqrt{h_1 h_2}.\ (4)$$

Hence –

$$(x_1+x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2 = l^2 = 2p(\sqrt{h_1}+\sqrt{h_2})^2$$
$$= 2p(h_1 + 2\sqrt{h_1h_2} + h_2).\ (5).$$
$$l^2 = 2p(h_1 + 2\sqrt{h_1h_2} + h_2)\ (6)$$
$$\therefore p = \frac{l^2}{2(\sqrt{h_1}+\sqrt{h_2})^2} = \frac{l^2}{2(h_1 + 2\sqrt{h_1h_2} + h_2}\ (6)$$

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Let BC = x_1, ED = X_2, then
$x_1^2 = 2ph_1$ (h_1=H'B) , therefore $x_1 = \sqrt{2ph_1}$ (2)
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$$x_2\ x_1 = 2p\sqrt{h_1h_2} = \text{and } 2x_2\ x_1 = 4p(\sqrt{h_1} + \sqrt{h_2})^2\ (4)$$

Hence

$$(x_1+x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2 = l^2 = 2p(\sqrt{h_1} + \sqrt{h_2})^2 (5)$$
$$l^2 = 2p(h_1 + 2\sqrt{h_1h_2} + h_2\ (6)$$

$$\therefore \quad p = \frac{l^2}{2(\sqrt{h_1} + \sqrt{h_2})^2} = \frac{l^2}{2(h_1+2\sqrt{h_1h_2}+h_2}\ (6)$$

If the towers are of the same height, then $h_1 = h_2 = h$, and equation (6) becomes—

$$p = \frac{l^2}{8h} \quad --- (7)$$

Horizontal distances from lowest point of curve to points of support.

The horizontal distance from the lowest point of the cable to the highest tower is, Fig 1, $BC = x_1$, so also ED, the lower tower, $= x_2$

$$x_1 = \sqrt{2ph_1} \quad --- (2)$$

But $p = \frac{l^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}$ (6). Substituting this value of p in (2) above—

$$x_1 = \sqrt{2ph_1} = \left[2h_1 \frac{l^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}\right]^{1/2} = \left[\frac{l^2 h_1}{(\sqrt{h_1}+\sqrt{h_2})^2}\right]^{1/2}$$

$$= \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} \quad --- (8)$$

In a similar, we have from (3) —

$$x_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}} \quad --- (9)$$

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$$x_1 = \sqrt{2ph_1} = \sqrt{\frac{2h_1 l^2}{2(h_1+h_2)^2}} = \sqrt{\frac{l^2 h_1}{(h_1+h_2)^2}}$$

$$= \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\text{----(8)}$$

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If $h_1 = h_2$; $x_1 = x_2 = \frac{l}{2}$ ---(10)

<u>Inclination of cable at any point</u> –

Since $KH' = H'F = y$, if i is the inclination to a horizontal line of the curve at any point P, then we have from the Δ FPK, (FK = 2y),

$$x \tan i = 2y;$$

$$\tan i = \frac{2y}{x}, \therefore \sec i = \sqrt{1+\frac{4y^2}{x^2}} \text{ ---(11)}$$

At the tops of the towers

$$\tan i_1 = \frac{2h_1}{x_1} \text{ --(12)}. \quad \tan i_2 = \frac{2h_2}{x_2} \text{ ---(13)}$$

If $h_1 = h_2$, $\tan i_1 = \tan i_2 = \frac{4h}{l}$ ----(14)

<u>Resultant tension at any point of cable</u> –

It has been shown that if the loading on a cable is uniform in direction, the component of cable tension normal to that direction will be constant at all points of the cable. Let the resultant tension at the lowest point of the cable be this constant component, denoted by H.

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We have seen that $H = wAF$. But AF is the sub-normal of the curve, and we know from General geometry, that the subnormal to the parabola is equal to one half the parameter $2p$, or equal to p. Hence, $H = wp$ --- (15)

Substituting in the above the value of p as found in equation (6), we have —

$$H = \frac{wl^2}{2(\sqrt{h_1}+\sqrt{h_2})^2} = \frac{wl^2}{2(h_1 + 2\sqrt{h_1 h_2} + h_2)} \text{ --- (16)}$$

Let R represent the resultant tension at any point, then from the triangle of forces, GNP,

$$PN = GP \sec i, \text{ or}$$

$$R = H \sec i = H\sqrt{1+\frac{4y^2}{x^2}} \text{ --- (17)}$$

by substituting the value of $\sec i$ in (11).

At the tops of the towers the tensions are

$$R_1 = H\sqrt{1+\frac{4h_1^2}{x_1^2}} \text{ ---- (18)}$$

$$R_2 = H\sqrt{1+\frac{4h_2^2}{x_2^2}} \text{ ---- (19)}$$

We have seen that H = wAF. But AF is the sub-normal of the curve, and we know from General geometry, that the subnormal to the parabola is equal to one half the parmeter 2p, or equalt to p. Hence, H=wp----(15)

Substituting in the above the value of p as found in equation (6), we have:

$$H = \frac{wl^2}{2(h_1+h_2)^2} = \frac{wl^2}{2(h_1+2\sqrt{h_1h_2}+h_2)}\text{----(16)}$$

Let R represent the resultant tension at any point, then from the triangle of forces, GNP,

PN=GPsec(i), or

$$R = Hsec(i) = H\sqrt{1+\frac{4y^2}{x^2}}\text{----(17) but}$$

substituting the value of sec(i) in (11).

At the tops of the towers the tensions are:

$$R_1 = H\sqrt{1+\frac{4h_1^2}{x_1^2}}\text{------(18)}$$

$$R_2 = H\sqrt{1+\frac{4h_2^2}{x_2^2}}\text{------(19)}$$

If $h_1 = h_2$, then $x_1 = x_2 = \frac{l}{2}$, and from Equation (16) –

$$H = \frac{wl^2}{8h} \text{ ---(20)}$$

Also $R_1 = R_2 = H\sqrt{1+\frac{16h^2}{l^2}}$ --- (21)

<u>Length of cables between a known point and the vertex, or between vertex and a point at which the inclination to a horizontal line is i.</u>

From the calculus we have the formula for the rectification of plane curves

$dz = \sqrt{dx^2 + dy^2}$, in which z represents the length of curve, and x and y the general coördinates.

From the equation of the curve

$$x^2 = 2py, \text{ we have}$$

$$dy^2 = \frac{x^2 dx^2}{p^2}, \text{ and}$$

$$dz = \frac{1}{p}(p^2+x^2)^{1/2} dx$$

To integrate this expression, apply formula C of reduction –

If $L_1 = L_2$, then $x_1 = x_2 = \frac{l}{2}$, and from
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$dy^2 = \frac{x^2 dx^2}{pz}$, and

$dz = \frac{1}{p}\sqrt{p^2 + x^2}dx$

To integrate this expression, apply formula C of reduction.

$$y = \int x^m(a+bx^n)^p\,dx = \frac{x^{m+1}(a+bx^n)^p + anp\int x^m(a+bx^n)^{p-1}\,dx}{np+m+1} \quad (\mathfrak{C})$$

and we have —

$$z = \frac{x\sqrt{x^2+p^2}}{2p} + \frac{p}{2}\int\frac{dx}{\sqrt{p^2+x^2}} \quad \text{--}(22)$$

To integrate $\frac{dx}{\sqrt{p^2+x^2}}$, put $z = x+\sqrt{p^2+x^2}$ --(23)

then $dz = dx + \frac{x\,dx}{\sqrt{p^2+x^2}} = dx\left(1+\frac{x}{\sqrt{p^2+x^2}}\right)$

then —

$$dz = \frac{x+\sqrt{p^2+x^2}}{\sqrt{p^2+x^2}}\,dx$$

Now we have —

$$\frac{dz}{z} = \frac{\frac{x+\sqrt{p^2+x^2}}{\sqrt{p^2+x^2}}\,dx}{x+\sqrt{p^2+x^2}} = \frac{dx}{\sqrt{p^2+x^2}} \quad \text{--}(24)$$

$\int\frac{dz}{z} = \int\frac{dx}{\sqrt{p^2+x^2}} = \log z$. Restoring the value of z —

$$\int\frac{dx}{\sqrt{p^2+x^2}} = \log\left(x+\sqrt{p^2+x^2}\right) \quad \text{---}(25)$$

Therefore —

$$z = \frac{x\sqrt{p^2+x^2}}{2p} + \frac{p}{2}\log\left(x+\sqrt{p^2+x^2}\right) + C$$

Estimating the arc from the vertex, it being the origin, $C = -\frac{p}{2}\log p$

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Estimating the arc from the vertex, it being the origin $C = -\frac{P}{2}\log p$

Thus the corrected integral is -

$$z = \frac{x\sqrt{p^2+x^2}}{2p} + \frac{p}{2}\log\left[\frac{x+\sqrt{p^2+x^2}}{p}\right] \text{---}(26)$$

Now by substituting in the above for p its value $\frac{x^2}{2y}$, the equation can be put in the form -

$$z = \frac{x^2}{4y}\left[\frac{2y}{x}\sqrt{1+\frac{4y^2}{x^2}} + \log\left(\frac{2y}{x} + \sqrt{1+\frac{4y^2}{x^2}}\right)\right] \text{---}(27)$$

Now we have seen that $\frac{x^2}{4y} = \frac{p}{2}$; $\frac{2y}{x} = \tan i$; $\sqrt{1+\frac{4y^2}{x^2}} = \sec i$, and by substituting these values, eq. (27) becomes -

$$z = \frac{p}{2}\left[\tan i \sec i + \log(\tan i + \sec i)\right] \text{---}(28)$$

In the above formulas the Naperian logarith is used, since the modulus is 1. Since the above formulae were deduced for the distances from the vertex to any particular point, the total length of cable will be found by substituting for y, h_1; and for x, x_1 in equation (27); or i_1 for i in eq. (28); then x_2 and h_2 for x and y in (27), or i_2 for i in (28) and adding the results. Denoting these

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$\sqrt{1+\frac{4y^2}{x^2}} = sec(i)$, and by substituting these values, eq. (27) becomes

$$z = \frac{p}{2}[tan(i)sec(i) + log(tan(i) + sec(i))]\text{----}(28)$$

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$$l_1 + l_2 \text{ --- } (29)$$

A formula, which is close enough for practical purposes, and which is frequently used, is deduced as follows.

In fig 1. suppose H'P is an arc of a circle whose radius is R. The coordinates x and y are the same as before. The expression for a circular arc in the integral calculus is

$$\int \frac{dx}{\left(1 - \frac{x^2}{R^2}\right)^{1/2}} = \int \frac{dx}{1 - \frac{x^2}{2R^2}}, \text{ approximately,}$$

considering R very large as compared with x.

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Suspension rods —

It is usually assumed in the calculations relating to suspension rods, that the cable lies in a vertical plane, and that the suspension rods are vertical. Since in all cases the suspension rods are parallel to each other, the above assumption does not affect the generality of the results. If the rods are inclined, the true lengths can be found by multiplying the values obtained by the secant of the inclination of the rods to a vertical line.

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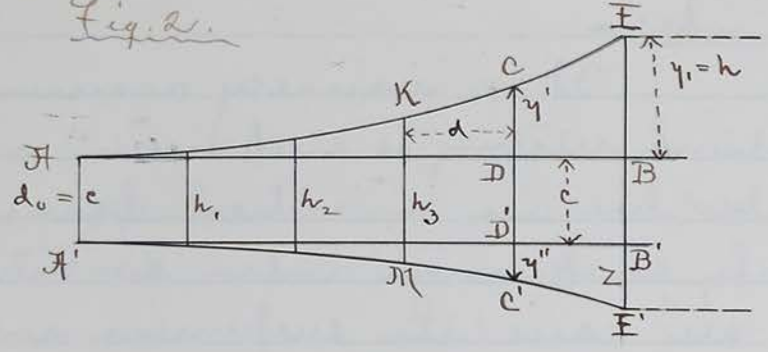
Fig. 2.



Let the ordinate $AD = x$ be measured from A toward B, and $DC = y$ perpendicular to it. Let $AB = x_1$ = the half span. The curve of the cable is the parabola as before.

$$x^2 = 2py,$$

$$y = \frac{x^2}{2p}$$

From Eq. (7) $p = \frac{l^2}{8h}$. Substituting —

$$y = \frac{4hx^2}{l^2}$$

$AB = x_1 = \frac{1}{2} l$, $\therefore l = 2x_1$, and we have

$$y = \frac{4hx^2}{l^2} = \frac{4hx^2}{4x_1^2} = h\frac{x^2}{x_1^2}$$

Now $h = y_1$ in Fig. II, or calling y, the ordinate and x the abscissa of any point in the curve above AB, we have the general equation —

$$y' = y_1 \frac{x^2}{x_1^2} \text{ ---(33)}$$

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From Eq. (1) $p = \frac{l^2}{8h}$. substituting:

$$y = \frac{4hx^2}{l^2}$$

$AB = x_1 = \frac{1}{2}l$, $\therefore$ l=2x, and we have

$$y = \frac{4hx^2}{l^2} = \frac{4hx^2}{4x_1^2} = L\frac{x^2}{x_1^2}$$

Now L=y, in Fig II, or calling y, the ordinate and x the abscissa of any point in the curve above AB, we have the general equation:

$$y' = y_1 \frac{x^2}{x_1^2}\text{-----(33)}$$

In the same manner for the lower curve or the camber

$$y'' = z\frac{x^2}{x_1^2} \text{ --(34), in which}$$

A'B' is taken as the axis of abscissae and z is the ordinate

Representing the length of any suspender as CC' by L, we have -

$$L = CC' = CD + DD' + D'C' \text{ --(35)}$$

Now DD' = h_0 = c, and taking the value of CD and D'C' as given in eqs. (33) and (34), $h = y' + y'' + c$ --(36)

From this we see that each suspender is composed of the constant length c and the two variable ones y' and y''.

Adding equations (33) and (34), and representing the sum of the variable lengths by y -

$$y = (y' + y'') = (y_1 + z)\frac{x^2}{x_1^2} \text{ --(37)}$$

Now let the suspenders all be the same distance apart, and represent this constant distance by d, then

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Adding equations (33) and (34), and representing the sum of the variable lengths by y --

$$y = (y' + y'') = (y_1 + 2)\frac{x^2}{x_1^2} \text{-----(37)}$$

Now let the suspenders all be the same distance apart, and represent this constant distance by d, then

$$h_2 = c + \frac{4d^2}{x_1^2}(y_1 + z)$$

$$h_3 = c + \frac{9d^2}{x_1^2}(y_1 + z)$$

$$h_{n-1} = c + \frac{(n-1)^2}{x_1^2}(y_1 + z)$$

$$h_n = c + \left(\frac{n^2d^2}{x_1^2} = 1\right)(y_1 + z) = c + y_1 + z \text{---}(38)$$

Since h_1 was assumed equal to h_2 in the above calculations, or the towers of the same height and equal to h, the lengths of the suspenders on each side of the lowest point in the cable will be equal, and having computed one side we use these values for the other. The vertical load which any rod carries multiplied by the secant of its inclination to a vertical line gives the stress on such rod.

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Deflection of cable for change in length, the span remaining the same.

In eq. (3)

$x.(1+\frac{2y^2}{3x^2})$, substitute x_1 for x and h_1 for y, and we have —

$$x_1(1+\frac{3h_1^2}{3x_1^2}) \text{----} (39)$$

Also substituting x_2 and h_2 for x and h in the same equation, it becomes

$$x_2(1+\frac{2h_2^2}{3x_2^2}) \text{---} (40)$$

Adding the above equations and denoting the two segments of the parabola by c_1 and c_2, we have the total length of cable —

$$c_1+c_2 = x_1+x_2+\frac{2}{3}\left(\frac{h_1^2}{x_1}+\frac{h_2^2}{x_2}\right) \text{---} (41)$$

Differentiating —

$$d(c_1+c_2) = \frac{4}{3}\left(\frac{h_1}{x_1}+\frac{h_2}{x_2}\right) dx \text{--} (42)$$

Now $h_1 - h_2$ being equal to a constant,

$$dh_1 = dh_2 = dh$$

$$\therefore dh = \frac{3d(c_1+c_2)}{4\left(\frac{h_1}{x_1}+\frac{h_2}{x_2}\right)} \text{--} (43)$$

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From whatever cause the cable may vary in length, this variation is to be put for $d(c_1+c_2)$ in equations (42) and (43), and then dh will be the corresponding deflection of the lowest point of the cable.

If the towers are of the same height -

$$c_1=c_2\,,\quad h_1=h_2\,,\quad x_1=x_2=\frac{l}{2}$$

Then we have from (42) -

$$2\,dc_1=\frac{16}{3}\frac{h_1}{l}\text{ --- }(44)$$

$$dh=\frac{3h_1}{16l}\,2\,dc_1\text{ --- }(45)$$

In equations (42) and (43) the assumption, though not strictly true, is that the lowest point of the cable remains at the same horizontal distance from the towers.

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In equations (42) and (43) the assumption, though not strictly true, is that the lowest point of the cable remains at the same horizontal distance from the towers.

To obtain the true length of the curve since the above relations were deduced from the approximate formula ( ), we must take the true equation for the length of the curve.

As before, let (c_1+c_2) be the known length of curve before variation takes place; let h_1 and h_2, x_1 and x_2 be the original heights of towers ~~also~~ segments of span also known.

Let y_1 and y_2 be the heights of the towers above the lowest point in the cable, after variation in its length has taken place.

x_1 and x_2 are still constants.

Let the variation in length of the cable be represented by v

Thus $v = -(c_1+c_2)+(c_1+c_2+v)$ ---(46)

$$v = \frac{x_1^2}{4y_1}\left[\frac{2y_1}{x_1}\sqrt{1+\frac{4y_1^2}{x_1^2}}+\log\left(\frac{2y_1}{x_1}+\sqrt{1+\frac{4y_1^2}{x_1^2}}\right)\right]$$

$$+\frac{x_2^2}{4y_2^2}\left[\frac{2y_2}{x_2}\sqrt{1+\frac{4y_2^2}{x_2^2}}+\log\left(\frac{2y_2}{x_2}+\sqrt{1+\frac{4y_2^2}{x_2^2}}\right)\right]-(c_1+c_2) \text{ ---(47)}$$

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But since $y_1 - y_2 = h_1 - h_2 =$ a constant, we can take the value of y_1 or y_2 and substitute in Equation (47), and we will then have only one unknown quantity in the right member, and this unknown is determined by trial.
(y_1 or y_2, h_1 or h_2 may be taken as the)
The first value of y_1 or y_2 taken may be h_1 or h_2 increased or decreased, as the case may be, by dh.
dh is taken from eq. (45)

$y_1 - h_1 = y_2 - h_2$ is the deflection sought,
The variation of length can be obtained at once from equation (47) when the new heights y_1 and y_2 are given.
When the towers are of the same height,

$$x_1 = x_2 = \frac{l}{2},\ c_1 = c_2,\ \text{and } y_1 = y_2 = h$$

Making substitutions of these values in equation (47), then results, after adding -

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$$v = \frac{2l^2}{16h}\left[\frac{4h}{l}\sqrt{1+\frac{16h^2}{l^2}} + \log\left(\frac{4h}{l} + \sqrt{1+\frac{16h^2}{l^2}}\right)\right] - 2c \quad (48)$$

If h is known, we can find v from the above equation. If v is known, h is found by trial, and v then results.

$h - h_1$ = deflection of the middle point of the truss.

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Pressure on Tower —

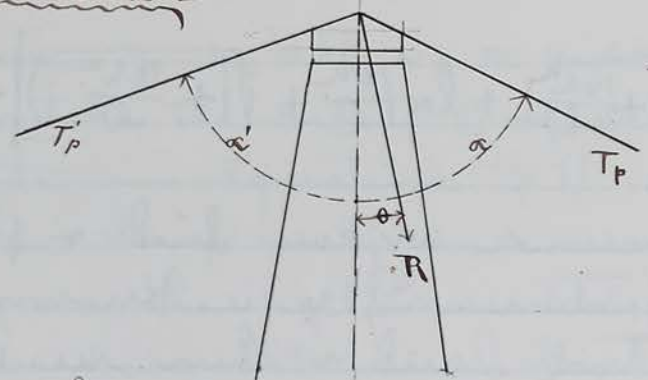

Fig. 3.

Let P_v = the vertical component of pressure on tower head
" P_h = " horizontal " " " " " "
" R = " resultant " " " "
" T_p and T'_p = Tensions of cable on different sides " "
" α, α' and θ represent inclinations to a vertical as shown

When we consider friction on the saddle —

$$P_v = T_p \cos\alpha + T'_p \cos\alpha' \; ; \quad (49)$$
$$P_h = T_p \sin\alpha - T'_p \sin\alpha' \; ; \quad (50)$$
$$R = \sqrt{P_v^2 + P_h^2} \quad (51) \; ; \qquad \cos\theta = \frac{P_v}{R} \quad (52)$$

When friction is not considered —

$$T_p = T'_p \, , \quad \therefore P_v = T_p(\cos\alpha + \cos\alpha') \; ;$$
$$P_h = T_p(\sin\alpha - \sin\alpha')$$
$$R = \sqrt{P_v^2 + P_h^2} \; ; \quad \cos\theta = \frac{P_v}{R}$$

If $\alpha = \alpha'$, $P_v = 2W$, $P_h = 0$, $R = 2W = P_v$
$\theta = 0$, W represents one half the weight of the load and structure.

Pressure on Tower

Fig. 3

Let P_L = the vertical component of pressure on tower head
" P_h = " horizontal " " " " " "
" R = " resultant " " " " " "
" T_p and T'_p = tension of cable on different* sides " "
" x, x' and θ represent inclinations to a vertical as shown.

When we consider friction on the saddle

$$P_L = T_P cos(\alpha) + T'_p cos(\alpha'); \; (49)$$
$$P_h = T_p sin(\alpha) - T'_p sin(\alpha'); \; (50)$$
$$R = \sqrt{P_L^2 + P_h^2}; \; (51) \qquad cos(\theta) = \frac{P_L}{R}; \; (52)$$

When friction is not considered

$$T_p = T'_p \therefore P_L = T_p(cos(\alpha) + cos(\alpha'))$$
$$P_h = T_p(sin(\alpha) - sin(\alpha'))$$
$$R = \sqrt{P_L^2 + P_h^2}$$
$$cos(\theta) = \frac{P_L}{R}$$

If $\alpha = \alpha', P_L = 2W, P_h = 0, R = 2W = P_L$,
$\theta = 0$, then W represents one half the weight of the load and structure.

<u>Bracing to resist heavy traveling load</u> —

Various methods have been proposed, and some of them tried, to enable a suspension bridge to resist the action of a heavily traveling load so as to undergo no more disfigurement than a girder. To ensure this in a bridge of several bays, the piers must be made very strong, and the chains securely fastened to them.

The best way of bracing is by means of auxilary girders, or a pair of straight girders of any convenient form hung from the cable by suspending rods and supporting the cross joists of the platform. These girders should be supported at each end, and also fastened down, as there are certain positions of the rolling load which would tend to raise one end of the truss. The ends should be free to move horizontally, however.

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By preventing a change in the form of the cable, which is accomplished by the stiffening truss, we not only prevent very injurious undulations, but also lessen the work of computing stresses, which would be very difficult if the cable did not retain the same form. The cable will assume the same parabolic curve only when there is a uniform pull on the suspension rods from end to end.
Let T be the uniform pull on any suspension rod, and t its intensity per unit of span. Now if p represents one panel length in the truss, T = pt
Let w be the fixed load per unit of span sustained by the cables, and w' the moving load sustained by them;
Let l be the span; R the reaction at B (fig.); R' the reaction A. Suppose the moving load to pass on from B.
Let x, be the distance from B to the front of the moving load. The load is supposed continuous.

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Let x_1 be the distance from B to the front of
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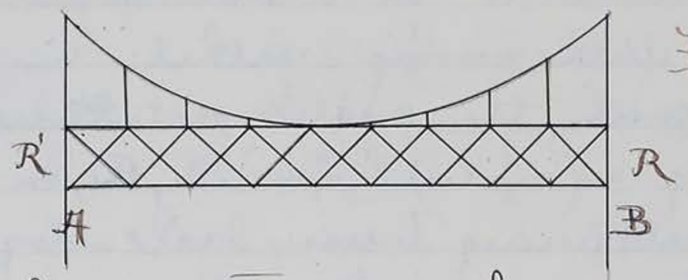

Fig. 4.

All the forces acting on the truss are vertical in their direction and we have two equations of equilibrium, indicating that the sum of the external vertical forces and, that the sum of these moments about any point must each equal zero

$$wl + w'x_1 - tl - R - R' = 0 \text{--} (53)$$

Thus by taking the moments about the head of the moving load –

$$(w+w')\frac{x_1^2}{2} - t\frac{x_1^2}{2} - Rx_1 + (t-w)\frac{(l-x_1)^2}{2} + R'(l-x_1) = 0 \text{--} (54)$$

In each of the above equations are three unknown quantities, t, R and R'. If any one of these is known, the others may found from the two conditional equations.

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Therefore —

$$R = -R'$$

Substituting this value in Equation (53),

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Substituting this value in Equation (53), and solving for z,

$$z = w + w'\frac{x_1}{l} \text{ --- } (55)$$

From substituting the same in (54),

$$R = -R' = \frac{w'x_1}{2}\left(1 - \frac{x_1}{l}\right) \text{ -- } (56)$$

In (56), if $x_1 = l$, or $w' = 0$, both reactions become zero, $R = -R' = 0$

It is also seen that R and R' are numerically equal, but have opposite directions.

R' is a downward reaction and will show the amount of anchorage required.

Differentiating (56) and finding value of x_1,

$$\frac{dR}{dx_1} = \frac{w'}{2} - \frac{w'x_1}{2l} - \frac{w'x_1}{2l}, \text{ and } x_1 = \frac{l}{2}.$$

Now substituting $x_1 = \frac{l}{2}$ in equation (56)

$$R = \frac{w'l}{8} \text{ --- } (57)$$

Equation shows the greatest shear to be provided for at either end of the truss, and also the maximum amount of anchorage to be provided for.

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The matter on stresses proper is very limited, on account of space.

Respectfully Submitted
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Rolla, Mo., June 7th, 1884.

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