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Decimal Dilemmas: Interpreting and Addressing Misconceptions

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Abstract: *In this article, a student's misconception of multiplication and division of decimals is analyzed and findings are presented from preservice teachers' interpretations of that misconception. The authors then highlight common decimal misconceptions, outline two strategies for addressing such misconceptions in the classroom, and include final remarks connecting the professional noticing framework with addressing misconceptions in mathematics.*

Keywords. *Teacher preparation, student misconceptions, decimal arithmetic*

1 Jeremy's Misconception

Imagine that you are teaching students about multiplication and division of decimals. One of your students, Jeremy, notices that when he enters 0.4×8 into a calculator, his answer is less than 8, and when he enters $8 \div 0.4$, his answer is greater than 8. He is confused by this and asks you for a new calculator. *What is Jeremy's misconception?*

1.1 Two responses

When faced with this scenario, teachers' interpretations range from generic and overgeneralized to mathematically precise and comprehensive. Compare the following two interpretations as examples:

1. It is likely that Jeremy does not know the concept of a decimal point and needs to be taught how to multiply and divide decimals.

2. He knows that when you multiply two whole numbers that the products are greater and when you divide them, the quotients are less. So he extends this thinking to decimals as well. But he doesn't understand decimals are a special form of fractions and when you multiply and divide with fractions or decimal fractions, the "rule" that had worked with whole numbers doesn't apply to fractions or decimal fractions.

The second interpretation is likely from a more skilled teacher as it fully captures the mathematical misconception of expecting multiplication to result in a product greater than either factor and division to result in a quotient that is less than the dividend. While the first interpretation notes that Jeremy was mistaken, it does not identify the misconception with the same accuracy, mathematical precision, or clarity as the second teacher.

Misconceptions of this type are consistent with the mathematical content embodied in Common Core State Standards for Mathematics (CCSSM) 7.EE.3.3, 6.NS.3, and 5.NBT.7.

1.2 Mathematical Noticing

The act of identifying, understanding, and responding to students' misconceptions relates directly to professional noticing. Jacobs, Lamb, and Philipp (2010) define professional noticing of children's mathematical thinking as "a set of interrelated skills including (a) attending to children's strategies, (b) interpreting children's understandings, and (c) deciding how to respond on the basis of children's understandings" (p. 172). Teachers attend to student thinking when they examine students' work or listen to their mathematical discussions. They interpret the students' thinking when they analyze what students know and don't know mathematically, based on their work or statements, just as the two teachers above were interpreting what Jeremy does and does not understand about multiplying and dividing decimals after he asked for a new calculator. Teachers use this insight to inform instructional decisions that build on students' knowledge and address misconceptions. In this article, we present findings from preservice teachers' interpretation of Jeremy's misconception, highlight common decimal misconceptions, outline two strategies for addressing such misconceptions in the classroom, and include final remarks connecting the professional noticing framework with addressing misconceptions in mathematics.

2 Analyzing Jeremy's Misconception

As part of a larger study, preservice elementary teachers (PSETs) in a mathematics methods course at two universities in the south-central United States were administered a mathematics assessment. The data were collected and recorded via an online survey, where eight items were selected from released items from the TEDS-M (Teacher Education and Development Study in Mathematics) assessment (Tatto et al., 2012). The eight-item online survey was given to 45 preservice teachers who were enrolled in the mathematics methods courses.

Specifically, responses to Jeremy's misconception problem were chosen for analysis for three reasons. First, this problem was applicable to real life teaching situations and is something these preservice teachers will encounter in their future classrooms. Second, this problem contains critical mathematics content for students to succeed at the upper elementary and middle school grades. Third, a wide variety of responses were recorded for this problem and the stronger responses were easily distinguishable from the lower scoring responses. Responses were then scored according to the TEDS-M scoring guide with scores varying from 0 to 2, 2 being the highest possible score. Among the 45 interpretations about Jeremy's misconception, 23 received a score of zero, 5 received a score of 1, and 17 received a score of 2.

Responses that received a score of zero for this problem were blank/off-topic or were not related to understanding of decimal numbers, decimal multiplication/division or use of a calculator. Responses that received a score of 1 were those that suggested Jeremy considers 0.2 as a whole number or addressed that multiplication always gives a larger answer or division always gives a smaller answer, but do not address both. Finally, the responses that received a score of 2 were those that suggested Jeremy's misconception is that multiplication always gives a larger answer and division always gives a smaller answer. They had to address both operational explanations to receive this high score. Table 1 provides examples of responses of each score.

Table 1: *Sample Responses from "Jeremy's Misconception" Problem*

Score	**Sample Responses**	**Frequency of score among PSETs**
0	He does not understand decimals. When a number comes after a decimal point it is smaller than a whole number.	23
1	[He thinks of] 0.2 as a whole number and not a rational number.	5
2	He probably assumes that multiplication is always related to a bigger number and assumes that division is always related to a smaller number. He is probably just not used to multiplying and dividing with decimals - which are just parts and not always whole numbers.	17

To build upon preservice teachers' interpretations of Jeremy's misconception and their own understanding of decimals, we discussed more decimal misconceptions and presented the two teaching strategies outlined in this article. Informed by the work of Grossman, Compton, Igra, Ronfeldt, Shahan, & Williamson (2009), we developed personalized scenarios as "representations of practice" in this article.

3 Decimal Misconceptions

In addition to Jeremy's misconception where he generalized the multiplication and division patterns of whole numbers to decimals, there are other common decimal errors. One misconception students often have about decimals is, "The longer the number, the larger the number" (Karp, Bush, & Dougherty, 2014, p. 23). Students use whole number thinking when examining numbers to the right of a decimal. For example, a student might incorrectly reason that $3.175 > 3.4$ because 175 is greater than 4. A related misconception is, the longer the decimal, the smaller the number (Griffin, 2016). This error is due to students thinking that $2.725 < 2.7$ because thousandths is smaller than tenths.

Another misconception students have about adding or subtracting decimals is that numbers to the right of the decimal cannot be regrouped into whole numbers. $5.4 + 2.8 = 7.12$ is an example of this type of error. This occurs when students view the decimal part of a number as separate from the whole number. Another common error is for students to overgeneralize similarities between decimals and fractions (Griffin, 2016). This misconception leads to students incorrectly converting $\frac{4}{5}$ to $.45$ or $\frac{1}{2}$ to $.2$.

When these decimal misconceptions go unnoticed in grades 4-5, this presents problems later, in middle school, when the mathematics content builds upon foundational ideas. To avoid and address such misconceptions, we provide two teaching strategies, namely (a) posing purposeful tasks and questions and (b) varying representations.

4 Posing Purposeful Tasks and Questions

Effective teaching of mathematics involves posing meaningful tasks along with purposeful questions to ascertain and advance students' thinking (NCTM, 2014). Such questioning, focused on particular aspects of students' strategies, provides students with a reflective opportunity as they translate their thoughts to verbalizations and mathematizing. Number strings can build students' understanding from a familiar mathematical expression to one that is less familiar and more advanced, encouraging place value relationships that are crucial to overcoming Jeremy's misconception regarding decimal division and multiplication. Empson and Levi (2011) build children's understanding of decimal and fraction division from the context of equal sharing in whole number contexts. Following this line of thinking, we can present children with strings of equations to look for patterns.

Consider these two strings of expressions:

$4 \times 10 =$	$4 \div 10 =$
$4 \times 1 =$	$4 \div 1 =$
$4 \times 1.0 =$	$4 \div 1.0 =$
$4 \times 0.1 =$	$4 \div 0.1 =$
$4 \times 0.10 =$	$4 \div 0.10 =$
$4 \times 0.01 =$	$4 \div 0.01 =$

The learning goal of these strings is not so much about each individual product or quotient but more about relationships. Therefore, students might use a calculator or a spreadsheet to solve each expression. Looking first at the column on the left for patterns across the products in relation to the second factor, the conversation below might ensue.

SHANNA : Hmm . . . I don't understand. I thought multiplying always gave a bigger number, but the answers seem to be getting smaller.

MS. GARCIA : Do you think you could use the base ten materials to help you think about this?

SHANNA : But we don't have any pieces that are smaller than a unit. How can we do that?

MS. GARCIA : Could we imagine the flat as one unit? What would be the value of a long? Of a tiny cube?

SHANNA : Well, there are 10 longs in a flat. So I guess a long would be one tenth. And, since there are one hundred tiny cubes in a flat, that would be a hundred . . . no, I mean one-hundredth.

MS. GARCIA : Okay, so try some of those equations using the base ten pieces.

SHANNA : If a flat is one, then 4 times a flat would be four flats, or 4. And, if a long is one tenth, then four of those are one, two, three, four tenths. And that is smaller than a flat.

MS. GARCIA : Do you think multiplying by one-hundredth will be smaller still?

A similar conversation might ensue around the right column.

SHANNA : Okay, I see how when I multiply by a decimal, I get an answer smaller than the whole number. Now I'm puzzled about division. Look at this. Four divided by 0.1 resulted in 40. How is that possible?

MS. GARCIA : Think back to the language we have used when we are dividing two whole numbers such as forty-five divided by five. What do we want to know?

SHANNA : How many fives are in forty-five?

MS. GARCIA : Yes, now let's see if that works for decimals. Which base ten blocks will you use?

SHANNA : Four flats?

MS. GARCIA : Now what do you want to find out?

SHANNA : How many one-tenths there are ... it's forty.

MS. GARCIA : Very interesting! How did you get that?

SHANNA : I multipied four times ten.

MS. GARCIA : So do you think four divided by one-tenth equals forty?

SHANNA : Yes, because there are ten tenths in each one.

MS. GARCIA : But why did you use multiplication to solve a division problem?

SHANNA : It was an easier way to solve it, because I could see it.

Posing probing questions offers teachers an avenue to guide students to more deeply inspect portions of an approach or conjecture. Using the number strings above, students might be further challenged to consider the tasks 4×0.1 and $4 \div 0.1$. These tasks provide the occasion for students to consider, more deeply, the relationships between decimal multiplication and division. At the onset, student-pairs might be tasked with conjecturing whether the product or the quotient will be larger. As the pairs of students share their reasoning, the teacher should focus on posing probing questions aimed at bringing the students' understanding of the mathematics at hand to the forefront.

In the previous conversation, Ms. Garcia's responses rely heavily on questioning (rather than explaining) in the context of physical materials to help Shanna construct a more robust understanding of arithmetic involving decimals and connections between division and multiplication. Additionally, the teacher amplifies this opportunity for sense-making by posing several questions that are particularly open-ended (e.g., "What do we want to know?"; "How did you get that?"). Such questioning strategies in the context of meaningful tasks set the stage for addressing misconceptions.

5 Varying the Representations

The use of multiple representations is widely considered to be a productive means to foster a deeper understanding of mathematics among students (Yeong, Dougherty, & Berkaliev, 2015). Moreover, CCSSM requires fifth grade students to use strategies steeped in "concrete models" and "drawings," as well as symbolic and verbal representations, to negotiate arithmetic tasks involving decimals (CCSSI, 2010, p. 35). Teachers' use of varied representations of mathematical ideas is foundational to facilitating the construction of mathematical meaning (NCTM, 2014). Constructing meaning develops from seeing the relationships among the quantitative (materials or drawings), symbolic (equations, expressions, numerals), and verbal (both written and oral). In order to address misconceptions, students should understand the reasoning behind the algorithm, rather than just execute a sequence of procedural steps whether these steps are executed with materials, symbols, or verbally. To understand such algorithms, a variety of concrete and visual representations may be used to represent multiplication of whole numbers, decimals, and fractions in concert with the symbolic and verbal representations, as well as across various concrete representations. For example, teachers may use concrete examples, such as base ten blocks, or visual representations such as array models to help make visible the quantities that exist behind the numerals.

One example of these multiple representations occurred in Mr. Jackson's sixth grade math class. Mr. Jackson was reviewing students' work and when he got to Kim's paper, he realized that she had multiplied 2.3×3.9 incorrectly (see Figure 1). In interpreting her work, he notes that she has incorrectly applied the standard algorithm for multiplication. He could see her error occurred in the second line where she recorded her answer, 69, to 3×2.3. Kim moved the decimal two places, as instructed, but she did not realize that her answer of 2.76 did not make sense. Due to this misconception, Mr. Jackson was able to apply this problem using grid paper and partial products during class the next day so that Kim could visually see why 2.76 was not a realistic answer to the problem (see Figure 2). Using a rectangular array and partial products can enable students' estimation strategies and visually show the incorrect reasoning behind a common error.

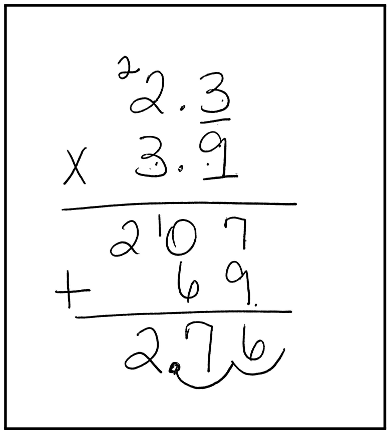

Fig. 1: *Kim's Practice Problem.*

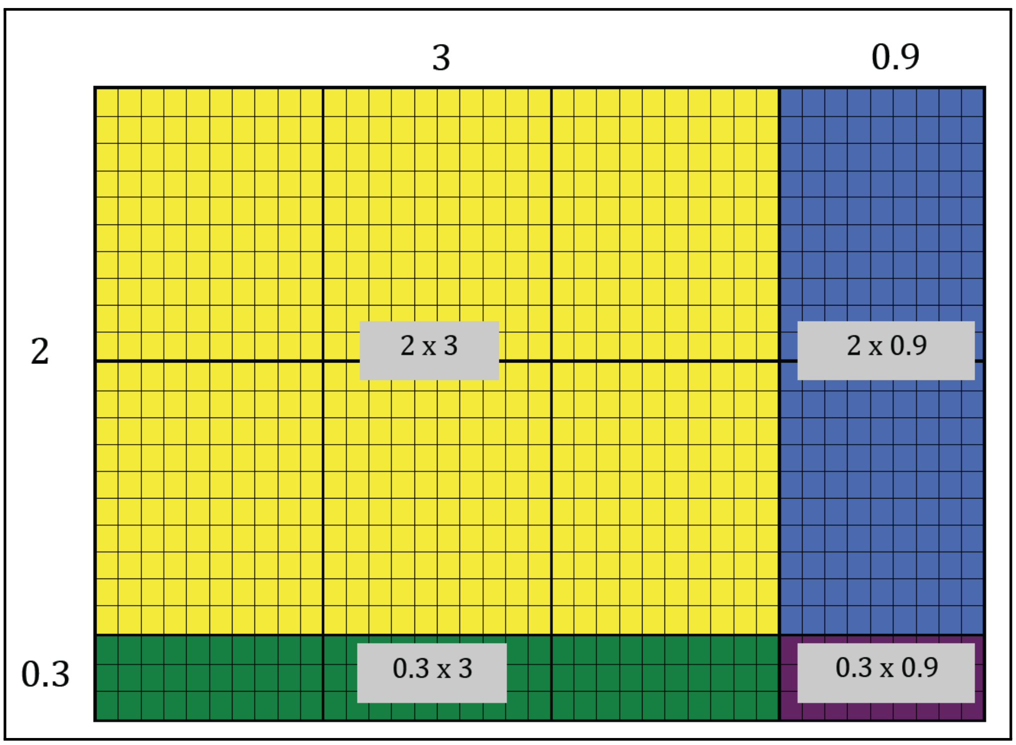

Fig. 2: *Decimal Multiplication Using Grid Paper.*

6 Final Remarks

Misconceptions can be difficult to detect. When teachers are unable to identify or understand a student's misconception, they might develop an incorrect assumption about that students mathematical knowledge. Misconceptions can be especially tricky when students are inconsistently getting the right answer by using a procedure that is not generalizable. For example, solving a multi-digit subtraction problem from left to right will produce a correct answer in cases where no regrouping is required or when an alternative algorithm is used (Jong, Dowty, Hume, & Miller, 2016). Attending to student thinking is essential when detecting the manifestation of a misconception that warrants an interpretation. To help students recognize and address misconceptions, teachers can introduce new concepts using multiple contexts and representations that support students' mathematical conceptual understanding, such as those outlined in this article. The use of multiple representations and explaining one's thinking will also make misconceptions highly visible for teachers to act on from their inception before they are fully formed.

The use of the three distinct components of professional noticing is an important strategy in identifying and addressing these misconceptions. Without attending to students' individual work or thinking while walking around the classroom, teachers might not realize that students are making mathematical errors that need to be addressed. Along similar lines, Mr. Jackson was able to easily interpret Kim's misconception of the standard algorithm on her bell ringer exercise. This alerted him to teach additional estimation strategies to assist students to check their answers for appropriateness. Finally, in the example with Ms. Garcia and Shanna, Ms. Garcia modeled effective deciding skills when she asked questions that assisted Shanna in thinking more deeply about the number strings instead of simply providing her with answers. It is critical that teachers examine students' work, both in-the-moment and written work, and unpack what they attend to and how they interpret the student work to develop productive and meaningful practices that will further students' mathematical thinking.

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