Vetting Online Resources that Support Teachers' Promotion of Mathematical Habits of Mind

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Abstract: *In this report, four secondary school mathematics teachers and one university mathematics teacher educator present a tool designed to evaluate the usefulness and relevance of online resources at helping teachers develop practical conceptions of mathematical habits of mind as described by the Common Core's* Standards for Mathematical Practice. *Descriptions for how the tool was developed, its components parts, and an example of its use are provided in detail.*

Keywords: *habits of mind, common core, teaching materials, assessment*

1 Introduction

As U.S. districts, schools, and teachers focus on improved and sustained implementation of college and career ready mathematics standards for K-12 students, including the Common Core State Standards for Mathematics (CCSSM) (NGA Center & CCSSO, 2010), the number of online resources designed to guide and support enactment continues to expand in breadth and depth. Even with such increased support, many teachers realize that faithful implementation and assessment of CCSSM or aligned standards can be formidable endeavors. Along with changes in mathematics content standards and their progressions, college and career ready standards place increased emphasis on powerful mathematical ways of thinking, reasoning, modeling, and communicating (e.g., Common Core Standards for Mathematical Practice, Nebraska Mathematical Processes).

1.1 Mathematical Habits of Mind

As described in Common Core documents, the Standards for Mathematical Practice specify the mathematical habits of mind or processes students should develop while learning mathematics content (NGA Center & CCSSO, 2010; Parker & Novak, 2012). Such mathematical ways of operating "change through the grades [and courses] as students grow in mathematical maturity and in the sophistication with which they apply mathematics" (Zimba & Lovanio, 2012).

Table 1: Alphanumeric Identifier and Title of Standards for Mathematical Practice

Identifier	Title
MP1	Make sense of problems and preserve in solving them
MP2	Reason abstractly and quantitatively
MP3	Construct viable arguments and critique the reasoning of others
MP4	Model with mathematics
MP5	Use appropriate tools strategically
MP6	Attend to precision
MP7	Look for and make use of structure
MP8	Look for an express regularity in repeated reasoning

Table 1 displays the alphanumeric identifier (or code) and title for each of the eight mathematical practices. For a comprehensive description of each mathematical practice, see the Common Core State Standards Initiative (2018) website.

Although the Internet is replete with resources claiming to deconstruct or 'unpack' Common Core mathematical content standards (e.g., Kentucky Department of Education, Public Schools of North Carolina) or provide teachers with sample problems, tasks, lessons, and assessments aligned to these same content standards (e.g., *CPALMS, Illustrative Mathematics, PARCC Mathematics Practice Tests*), there are few online resources designed to help teachers develop practical conceptions of the Standards for Mathematical Practice. By 'practical conceptions' we mean conceptions that allow teachers to: (1) differentiate among the mathematical practices, (2) understand how the mathematical practices connect and interact with one another, (3) understand and anticipate where in students' attempts to solve a problem or task students might engage in a particular mathematical practice or practices, (4) understand what engagement in a particular mathematical practice or practices 'looks like' both verbally (during discussions) and in written work (when students show their work or explain their thinking and reasoning), and (5) manage instruction to both model and promote engagement in a particular mathematical practice or combination of practices. In this report, four secondary school (grades 9–12) mathematics teachers and one university mathematics teacher educator, the 'research team,' present a rubric designed to evaluate the relevance and usefulness of online resources in helping teachers develop practical conceptions of mathematical habits of mind as described by the Common Core's Standards for Mathematical Practice.

2 Methods

The first stage in developing the "Mathematical Practices Online Resources Rubric" involved examining a wide array of existing online resources related to CCSSM or aligned college and career ready mathematics standards. All resources examined were accessible through common web browsers and free to everyone. The team's initial list of potential resources was generated through four different methods:

1. Results from Google searches of the phrases 'Standards for Mathematical Practice,' 'mathematical practices,' 'mathematical habits of mind,' and similar phrases adding the terms: 'Resources for the,' 'Understanding the,' 'Implementing the,' and 'Assessing the.'

2. Exploring resources at widely known (in U.S.) mathematics standards-related websites, including: *Achieve the Core, Illustrative Mathematics, Inside Mathematics, Mathematics Assessment Project, Partnership for Assessment of Readiness for College and Careers* (PARCC), and *Smarter Balanced Assessment Consortium* (SBAC).

3. Exploring resources at noted mathematics education professional organization websites, such as *Association of Mathematics Teacher Educators* (AMTE), *National Council of Teachers of Mathematics* (NCTM), and the *National Council of Supervisors of Mathematics* (NCSM).

4. Exploring resources at various U.S. mathematics standards-related state websites, including the Arizona Department of Education, New York State Education Department, Ohio Department of Education, Public Schools of North Carolina, and Utah State Board of Education.

The resultant list of over 70 distinct online resources was partitioned among the research team. Each team member examined their respective list of resources, divided their list among 'Yes,' 'No,' and 'Maybe' categories, and characterized the strengths (reasons why resource is beneficial to teachers) and weaknesses (areas where resource was lacking) for each 'Yes' resource. Next, members exchanged resources from their respective 'Maybe' categories and identified those 'Maybe' resources that should be transferred to 'Yes'—addressing the strengths and weaknesses for each such resource. Once only those resources identified as a 'Yes' remained, the entire team reviewed each such resource in an attempt to determine: (a) whether there were common characterizations team members utilized to classify their 'Yes' resources, and (b) whether the remaining resources could be classified or partitioned in terms of their importance, usefulness, application or in some other propitious manner. Team discussions resulted in the decision to sort the remaining 'Yes' resources in terms of the situation in which a teacher might utilize the resource: (1) as part of a mathematical practice directory or general mathematical practice resource; (2) for course planning or curriculum development, typically employed prior to the start of the school year; (3) for unit or lesson planning and implementation; (4) for conferences with parents, guardians, or administrators.

The final steps in analysis involved: deciding (as a team) whether each resource would remain on the final 'Yes' list, modifying the resource's strengths and weaknesses if needed, and determining where the resource fit in the team's situational categorization-scheme. Throughout the sorting, confirmation, and categorization processes, the research team had many debates as to the usefulness and type of support each resource provided. Such discussions and their resolutions were important, because the team intended resources be useful to actual pre-tertiary teachers, teaching in mathematics classrooms, with pre-tertiary students and genuine school, district, and state directives and mandates.

2.1 Mathematical Practices Online Resources Rubric

Upon review of the strengths and weaknesses of resources on the initial 'Yes' lists, several characteristics were commonly indicated by team members—characteristics that separated a strong resource from a weak one. Such characteristics became the main components of the "Mathematical Practices Online Resources Rubric."

1. **Ease of access and navigation:** The resource must be found through web searches for resources that include the mathematical practices. Can it be found through common word strings (e.g., 'mathematical practices') in a variety of web browsers? Are there relevant keywords or operators that allow the resource to be found more quickly? In addition, the site must be easy to navigate. Are there suitable links within the resource (e.g., dropdown menus, sidebar) that direct teachers to mathematical practice-related activities, lessons, tasks, assessments, etc.?

2. **Regular updates:** Examines whether or not the resource is regularly updated with new, amended, or enhanced material. This includes ascertaining whether the resource link remains active over time (e.g., whether website has relocated, link becomes broken or expires). The component's focus also involves examining whether a resource stays up-to-date and relevant as state, district, and school directives and mandates (e.g., standards, curricula, and

assessments), and research-based recommendations for mathematics teaching and learning, evolve.

3. **Operationalization of Mathematical Practices:** Examines how well the resource helps teachers operationalize each mathematical practice. This could range from simply listing each mathematical practice (as in Table 1), to providing a detailed description of each practice standard, including but not limited to: a rationale for its usefulness, an expanded description that helps conceptualize the standard, or an explanation for which concepts and skills a mathematical practice may be engaged. In addition, the component examines whether the resource supports operationalization of all practices at multiple grade levels and courses.

4. **Student engagement:** Articulates how students might engage in and exhibit engagement in one or more mathematical practice. This component examines how well the resource assists teachers in imagining how students might engage in and exhibit engagement in one or more mathematical practice. Does the resource provide examples of students' thought processes? Does the resource illustrate how specific or combinations of mathematical practices might be exhibited by students for specific mathematics content?

5. **Teacher engagement:** Articulates how a teacher might model engagement in one or more mathematical practice during instruction. This component focuses on practical strategies that help teachers during instruction. Such strategies could be in the form of videos or transcripts of sample teachers productively implementing one or more mathematical practice. The component also involves general strategies for promoting student engagement in one or more mathematical practice (e.g., how to promote student engagement and exhibition of a mathematical practice in student written work).

The extent to which an online resource satisfies the research team's situational categorization-scheme and the number of situational categorization-schemes satisfied also play a role in scoring the "Mathematical Practices Online Resources Rubric."

6. **Can be used as a mathematical practice general resource:** This component focuses on resources that explicate the mathematical practices. Such resources support teachers' conceptions of what engagement and exhibition of engagement 'looks like' during verbal interactions and written work. Resources should articulate characteristics of one or more mathematical practice, and the development and connections among combinations of practices.

7. **Can be used for curriculum development:** This component focuses on resources that can be utilized by teachers when developing curriculum maps, pacing guides, scope and sequence, or specific mathematics learning targets across a mathematics department. Resources should provide information about how the mathematical practices can be incorporated into and developed throughout a map or guide, and within and across grade levels and courses.

8. **Can be used for lesson or unit planning and implementation:** This component focuses on resources that support teachers' seamless enactments of instruction that engages students in one or more mathematical practice. Does the resource have sample lessons or units that incorporate the mathematical practices? Does the resource provide grade level or course-appropriate classroom activities that productively connect the mathematical practices with mathematical content? Does the resource provide sample activities appropriate for multiple grade levels or courses?

9. **Can be used for conferences with parents, guardians or administrators:** This component identifies resources teachers can use to make the mathematical practices more accessible to parents or guardians, or to support teachers' formal or informal observations by department, school, or district administrators.

The rubric uses a four-point Likert scale for each of the nine components. The scale ranges from 1 ('not satisfying' or 'lacking' a component) to 4 ('satisfying' or 'fulfilling all aspects of' a component). Once all nine components are evaluated, the first two component scores are summed to arrive at the 'Technical Aspects of Resource' sub-total (out of eight points). Next, scores from components #3 through #5 are summed to arrive at an interim 'Support from Resource' score (out of 12 points). This score is then doubled, yielding the 'Support from Resource' sub-total (out of 24 points). The reason for making 'Support' components of greater weight than 'Technical' components is due to the rationale behind creating the rubric—to evaluate the relevance and usefulness of online resources in helping teachers develop practical conceptions of the Common Core's Standards for Mathematical Practice. The sum of the 'Technical Aspects of Resource' sub-total and 'Support from Resource' sub-total gives the level of effectiveness of the resource, or 'Resource Effectiveness' total (out of 32), where higher scores indicate higher levels of effectiveness. Finally, individual scores on components #6 through #9 provide information regarding 'Resource Application(s),' where any individual component receiving a score of '3' or '4' indicates a productive application or situational category for the resource. Figure 1 illustrates the rubric template.

Effectiveness and Applications of Online Resource	Rating (1, 2, 3, or 4) 1 - not satisfying / lacking a component 4 - satisfying / fulfilling all aspects of a component
Resource:	
Website (URL):	
Grade Level(s) Targeted:	
Technical Aspects of Resource (rate each component: 1, 2, 3, or 4)	Sub-total:
1. Ease of access and navigation	
2. Regular updates	
Support from Resource (rate each component: 1, 2, 3, or 4)	Sub-total (×2):
3. Operationalization of Mathematical Practices	
4. Student engagement	
5. Teacher engagement	
Resource Effectiveness (higher sums, or total, indicate higher levels of effectiveness)	Total:
Resource Application(s) (rate each component: 1, 2, 3, or 4)	
6. Can be used as a mathematical practice general resource	
7. Can be used for curriculum development	
8. Can be used for lesson or unit planning and implementation	
9. Can be used for conferences with parents, guardians, or administrators	
Situational Use(s):	

Fig. 1: *Mathematical Practices Online Resources Rubric Template.*

2.2 Sample Rubric Scoring

Figure 2 below and the subsequent discussion, provide the completed rubric for the website *Implementing the Mathematical Practice Standards* (Education Development Center, 2016) and a rationale for how each component was scored. The *Implementing the Mathematical Practice Standards* (Education Development Center, 2016) resource was easy to access from several web browsers and navigation was straightforward and well-organized; the resource received a 4 on component #1 ('Ease of access and navigation'). The website has not updated their mathematical practice illustrations—which include a mathematical content and practice-aligned task, student dialogue, teacher reflection questions, and student materials—since August 2016; the resource received a 1 on component #2. The sum of the first two components yield a score of five (4 + 1) out of eight on Technical Aspects of Resource.

Effectiveness and Applications of Online Resource	Rating (1, 2, 3, or 4) 1 - not satisfying / lacking a component 4 - satisfying / fulfilling all aspects of a component
Resource: *Implementing the Mathematical Practice Standards*	
Website (URL): http://mathpractices.edc.org/	
Grade Level(s) Targeted: Grade 4-12	
Technical Aspects of Resource (rate each component: 1, 2, 3, or 4)	Sub-total: 5
1. Ease of access and navigation	4
2. Regular updates	1
Support from Resource (rate each component: 1, 2, 3, or 4)	Sub-total (×2): 9 (×2) = 18
3. Operationalization of Mathematical Practices	3
4. Student engagement	4
5. Teacher engagement	2
Resource Effectiveness (higher sums, or total, indicate higher levels of effectiveness; total out of 32)	Total: 23
Resource Application(s) (rate each component: 1, 2, 3, or 4)	
6. Can be used as a mathematical practice general resource	4
7. Can be used for curriculum development	2
8. Can be used for lesson or unit planning and implementation	3
9. Can be used for conferences with parents, guardians, or administrators	1
Situational Use(s): Mathematical practice general resource; Lesson or unit planning and implementation	

Fig. 2: *Implementing the Mathematical Practice Standards Example.*

Each illustration provides commentary explicating how students engaged in and exhibited engagement in the associated mathematical practices (from the sample dialogue). There are currently 31 illustrations, none of which involve Kindergarten through Grade 3, only five of which involve MP4 and eight involve MP6 (see Table 1); the resource received a 3 on component #3. Each illustration articulates how individual or combinations of mathematical practices might be exhibited by students for specific mathematics content by focusing on students' thought processes; the resource received a 4 on component #4. Although each illustration prompts teachers to think about and identify evidence of students (in dialogue) engaging mathematical practices, they do not provide practical strategies to help teachers promote student engagement; the resource received a 2 on component #5. The sum of components #3 through #5 yields 9 points (3 + 4 + 2), which is doubled due to each of these components being of greater weight than the two technical components (#1 and #2), giving a Support from Resource score of 18 (9 × 2) out of 24. Finally, the Technical Aspects of Resource and Support from Resource scores are summed to yield a total of 23 (5 + 18) points out of 32.

The illustrations support teachers' conceptions of what engagement and exhibition of engagement 'looks like' during verbal interactions, through transcripts of student dialogue—also in written work, but to a lesser degree; the resource received a 4 on component #6. Although illustrations are useful for designing lessons focused on specific content, no information is provided about how the mathematical practices can be incorporated into and develop throughout a sequence of lessons, unit, grade level or course; the resource received a 2 on component #7. Mathematical tasks, sample student dialogue, and teacher reflection questions support the development of activities and lessons that focus on the mathematical content and practice standards. There are only a minimum number of illustrations for certain grade levels (e.g., four illustrations for Grade 4, 12 Grade 7 illustrations); the resource received a 3 on component #8. Mathematical tasks and sample student dialogue were not concise enough to be beneficial to administrators during observations or parents (guardians) during conferences; the resource received a 1 on component #9. The resource received at least a 3 only on Resource Application components #6 and #8 (Can be used as a mathematical practice

general resource; Can be used for lesson or unit planning and implementation). Therefore, the *Implementing the Mathematical Practice Standards* website (Education Development Center, 2016) received 23 effectiveness points (out of 32; or 71.9%), where higher scores indicate higher levels of effectiveness, and is productive for use as a mathematical practice general resource and for lesson or unit planning and implementation.

3 Conclusion

Although the research team initially envisioned creating a web space with descriptions of and links to highly-rated online resources, we realized the ever-evolving nature of school, district, and state directives and mandates produced an environment where resources move (change URL), become obsolete, or non-existent (expired link). Therefore, we modified our focus to developing a tool that allowed users to evaluate the relevance and effectiveness of any resource they may encounter, with regards to the Standards for Mathematical Practice (see Table 1). Although the rubric is focused on information and material beneficial to mathematics teachers, the rubric should also be of benefit to mathematics coaches, district mathematics coordinators, mathematics teacher educators, and other Kindergarten to Grade 12 and university mathematics education personnel.

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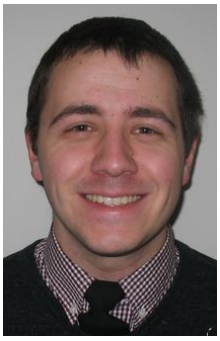

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