
Sparking Surprise: A Negative Number Game to Promote Productive Concepts of Addition and Subtraction

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Abstract: We describe a game that uses inexpensive and accessible materials to encourage students to make important discoveries about integer operations. Go-High/Go-Low gives students studying negative numbers a fun way to not only practice addition, subtraction, and ordering numbers, but also to discover the surprising idea that subtraction can actually lead to a greater number, depending on the number subtracted.

Keywords: Negative Numbers, Integers, Games, Classroom Discourse

1 Introduction

Are you looking for a way to help students recognize that subtraction does not always decrease the value of a number? At the 2018 OCTM conference, Dan Meyer’s keynote focused on creating opportunities to evoke surprise (Meyer, 2018). Games can offer a way to do this in the mathematics classroom. Julie created the game Go-High/Go-Low to encourage her secondary students to discover that addition doesn’t always produce a larger result and, likewise, that subtraction doesn’t always produce a smaller one. These are surprising ideas for many students because prior experiences with whole numbers reinforce faulty generalizations about addition and subtraction operations.

We need to help students learn that operations interact in complex ways with different types of numbers. Thus, the goal of Go-High/Go-Low is to help students develop more productive generalizations of arithmetic operations. To foster these productive generalizations, the game first asks players to think about how to make a number larger, then how to make a number smaller. In this article, we discuss how to play Go-High/Go-Low; we provide suggestions for leading class discussions to bring out the most important mathematical ideas during game play; and we share seventh-graders’ feedback regarding their experiences with the game. Go-High/Go-Low offers rich opportunities for students to think about integer operations. Specifically, the game helps students:

- Pay attention to the sign of a number and its magnitude,
- Order integers from least to greatest,
- Practice adding and subtracting positive and non-positive integers,
- Discover that subtracting a negative number increases the value and adding a negative number decreases the value (MP 7 *Look for and Make Use of Structure*; CCSM, 2010; ODE, 2017).

2 How to Play

Go-High/Go-Low requires only a few inexpensive, easy-to-find materials. Each *group* of two or three students needs:


- 1 direction sheet (see appendix),
- 1 positive/negative sign die (e.g., a six-sided die with “-” on three faces and “+” on three faces), and
- 1 numeral die (e.g., a six-sided die with numerals 1–6 written on each face).

Each student needs:

- 1 game board (see appendix) and
- 1 pencil.

To set up each part the game, students roll a sign die and then a numeral die to obtain individual starting numbers / scores. Students write their starting scores on their own game boards. On each turn they roll a sign die and numeral die again, record the outcome, and decide whether to add or subtract the starting score and the new roll to make the largest (or least) number possible. Each time they roll, students write their equations so their group can verify whether they agree with the calculations. Figure 1 shows a sample game board from the Greatest Number part of the game.

First Roll



Greatest Number Game

The player with the greatest (largest) number after 4 turns wins.

$\begin{array}{c} \text{1st roll} \\ \downarrow \\ -2 \end{array}$

=

$\begin{array}{c} \text{2nd roll} \\ \downarrow \\ \end{array}$

=

$\begin{array}{c} \text{3rd roll} \\ \downarrow \\ \end{array}$

=

$\begin{array}{c} \text{4th roll} \\ \downarrow \\ \end{array}$


=

Final score

Write the names and final scores of the students in your group **in order from least to greatest**. Circle the winner.

<input style="width: 40px; height: 20px;" type="text"/>	Name _____	<input style="width: 40px; height: 20px;" type="text"/>	Name _____
Score		Score	

Second Roll



Greatest Number Game

The player with the greatest (largest) number after 4 turns wins.

$\begin{array}{c} \text{1st roll} \\ \downarrow \\ -2 \end{array}$

+

$\begin{array}{c} \text{2nd roll} \\ \downarrow \\ 3 \end{array}$

=

$\begin{array}{c} \downarrow \\ 1 \end{array}$

=

$\begin{array}{c} \text{3rd roll} \\ \downarrow \\ \end{array}$

=

$\begin{array}{c} \text{4th roll} \\ \downarrow \\ \end{array}$


=

Final score

Write the names and final scores of the students in your group **in order from least to greatest**. Circle the winner.

<input style="width: 40px; height: 20px;" type="text"/>	Name _____	<input style="width: 40px; height: 20px;" type="text"/>	Name _____
Score		Score	

Third Roll



Greatest Number Game

The player with the greatest (largest) number after 4 turns wins.

$\begin{array}{c} \text{1st roll} \\ \downarrow \\ -2 \end{array}$

+

$\begin{array}{c} \text{2nd roll} \\ \downarrow \\ 3 \end{array}$

=

$\begin{array}{c} \downarrow \\ 1 \end{array}$

-

$\begin{array}{c} \text{3rd roll} \\ \downarrow \\ -4 \end{array}$

=

$\begin{array}{c} \downarrow \\ 5 \end{array}$

=

$\begin{array}{c} \text{4th roll} \\ \downarrow \\ \end{array}$

=

Final score

Write the names and final scores of the students in your group **in order from least to greatest**. Circle the winner.

<input style="width: 40px; height: 20px;" type="text"/>	Name _____	<input style="width: 40px; height: 20px;" type="text"/>	Name _____
Score		Score	

Fig. 1: Sample game board from the Greatest Number part of the game.

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Students do not have to help each other use a winning strategy, but they are responsible for each group member's learning by ensuring each person correctly calculates solutions. We recommend grouping students into heterogeneous trios by achievement level to help ensure that at least one member will accurately assess equation accuracy. The game board was designed to include extra practice with ordering integers by requiring each student to write the group's scores in order from least to greatest.

3 Suggestions to Bring Out the Mathematics of the Game

Go-High/Go-Low should be played after students have already begun to learn how to add and subtract negative numbers. Doing so helps ensure that the game will simultaneously provide students with opportunities to generalize ideas while practicing calculations to develop fluency. Although students can generate numbers using a random number generator on Excel or a computer app, they typically enjoy using real dice. We recommend that students use two separate die, one with only the sign and one with only numeral magnitudes such as 1 to 6 (not dot dice). This encourages students to focus on the meaning of the sign notation and to encourage student abstraction of integers as numbers.

After explaining how to play, it is helpful to model two or more turns for students. Let them know they should be ready to discuss the strategies used to win each game. After all groups have had a chance to finish the Greatest Number Game (with some groups likely also playing the Least Number Game), we recommend that you call your class back together for an intermediate discussion of strategies. As students play and discuss mathematical ideas, we suggest using the practices of Monitoring, Selecting, Sequencing, and Connecting (NCTM, 2014; Stein & Smith, 2011). We discuss ways to implement each of these practices in the following sections.

3.1 Monitor Student Thinking

As students play, we suggest first quickly monitoring groups to ensure that each student is taking a turn rolling the dice to play their own game board. It is a common misunderstanding for a group to think everyone in the group will record the same rolls and write down the same numbers, which eliminates the competitive nature of the game. As you circulate, also make sure each student checks the accuracy of their competitor's equations. They do not have to help their opponent learn a winning strategy, but each group is responsible for ensuring the accuracy of all written equations. Once you ensure everyone knows how to play the game, monitor student thinking (a) to recognize when all groups have finished the Greatest Number part of the game and (b) to select which students' work to share in the discussions described next.

3.2 Selecting Student Thinking

As you monitor student thinking to prepare for whole group discussions, we suggest noticing which students are:

- Calculating accurately and using the best strategy for the game on each turn;
- Calculating accurately, but may not yet have noticed the strategy that subtracting a negative number makes a greater number or adding a negative number makes a lesser number; and
- Making calculation mistakes.

3.3 Sequencing and Connecting Student Thinking

Next we suggest when to pause students' play to sequence three discussions of student thinking.

3.3.1 Phase 1: Calculation accuracy check-in after the Greatest Number Game

As you walk around monitoring students' thinking, we suggest copying an example of one or two student equations that were incorrectly calculated and an example of an equation that was correctly calculated (regardless of strategy). Post these examples on the board without student names. Pose the question to the class to discuss in their group: *Which of these equations are correct? Why?* Help students discuss the processes and solutions using any models students have previously used (e.g., Walk-It-Off Number Line Model (Nurnberger-Haag, 2007), traditional number line model, chip model), or other ways of thinking about integer arithmetic.

After this mini-discussion to help the class identify calculation mistakes, encourage students to check their own work. Provide students with a minute or so in their groups to check that they agree with every calculation on each competitor's board. Students do not have to help peers use a winning strategy, but they are expected to help their classmates correctly implement arithmetic operations.

3.3.2 Phase 2: Sharing and Connecting Strategies

After completing the mini-discussion about accuracy and helping students check their work is a good time to elicit strategies for winning the Greatest Number Game. We suggest asking questions that focus on game strategy and foster productive generalization of addition and subtraction of integers, such as: *Now that everyone has had a chance to play the Greatest Number Game . . . Did anything surprise you about how to win this game? What strategies did you use to keep increasing your score?*

By monitoring strategies during play, you can call on a student who realized that if they rolled a negative number they should subtract it to make the result larger. If possible, select a student whom other students may not recognize as a strong mathematics student to share this important insight. Subtracting negatives is a surprising idea, so encourage others to ask questions of the student. Then elicit additional strategies by asking the class: *What other strategies or insights did you have about how to win the Greatest Game?* To encourage discussion, have several other students share and encourage students to ask each other questions or disagree with each other.

Next, it is helpful for students to apply what they learned through the discussion to play more rounds and then discuss their strategies and generalizations again. So send students back to finish or start playing the Least Number Game, depending on where each group paused the play for your discussion. We recommend handing out a second game board, because many groups will be able to play each game at least twice during a 45-minute lesson. Continuing to monitor student thinking during this round will make it possible to select students to share their strategies in the final discussion.

3.3.3 Phase 3: Connecting and Contrasting Student Strategies

We engage students in this phase of the lesson after most groups have completed two games of Go-High/Go-Low. It is important to leave about 10 to 15 minutes at the end of class for a closure discussion and exit ticket. In the closing discussion, we suggest asking similar questions for the Least Game and connecting the ideas and strategies students shared in order to generalize about adding and subtracting integers. Some questions to encourage generalizations about winning strategies include: *How would you explain the winning strategy for each game?* To encourage students to extend their learning beyond the game to productive generalizations about addition and subtraction, you might ask: *What did you learn about adding and subtracting integers by playing this game?* During the last three minutes of class, you might give students an exit ticket such as shown in Figure 2.

Is the following statement always, sometimes, or never true? Why?

Subtraction always makes a number less than the number you started with.

Fig. 2: Example of an exit ticket to assess individual understanding.

Doing so will help you understand which students might still hold onto thinking about operations in terms of whole numbers and which students were able to generalize about operations across number systems.

4 Suggestions to Differentiate

The Go-High/Go-Low game offers multiple ways to meet the needs of diverse learners. Although it is competitive, the game is not a race, which gives students time to think through their work. Also, a teacher could let groups choose which dice they use or intentionally provide different dice to different groups based on the challenge they need (e.g., six-sided numeral dice, 10-sided dice with numerals 0 to 9, 12-sided dice, 20-sided dice, etc.). Julie developed this game while teaching at a school for students with learning disabilities who often struggled with whole number calculations. To help students focus on the main purpose of the game—namely, to spark surprise as they determined the effects of adding or subtracting a negative number, it was most helpful for them to use six-sided numeral dice the first couple times they played, before using larger numbers to develop fluency. Whereas, if you know all of your students are adept at adding and subtracting whole numbers, you might choose 10, 12 or 20-sided numeral dice to offer additional challenges from the beginning. Moreover, teachers can modify the game to make it more collaborative by having pairs of students work together on one game board, competing against another pair.

5 Students Feedback about Go-High/Go-Low

Not surprisingly, most students enjoy playing games in math class. Experience and research clearly suggest that games offer an engaging way for students to learn mathematics (e.g., Cordova & Lepper, 1996). Recently, we observed the implementation of a seventh-grade unit on negative number operations as part of a larger research study. The observed students played Go-High/Go-Low and discussed strategies at two points in the unit: First after learning how to add and subtract using the Walk-It-Off number line model (Nurnberger-Haag, 2007), then again after learning a chip model and doing other activities throughout the unit as part of a unit review. During the unit, students played three different games. When the classroom teacher asked students in an online survey “What game did you like the best during the unit?” almost half the students chose Go-High/Go-Low. The following are responses of three students who preferred Go-High/Go-Low.

- “It was really fun. It gave me more practice on adding and subtracting without feeling pressured to do it fast.”
- “[I]t really made me think about what would give me the greatest or least number. Also it was the most fun to me.”
- “It was easy to get the lowest and the greatest. You do need to do more math, but it’s kinda easy.”

During post-interviews, we asked students if they recommended the use of each game and why. Most students said teachers should use Go-High/Go-Low because it was fun and helped students

learn. Many students even said they would play it outside of school if they had it! A number of students noted that they enjoyed rolling physical dice and appreciated the element of chance this created. They liked that the game was a combination of skill and luck. Although the game is competitive, it also allows students to think at their own pace, so those who need more time to reflect or have anxiety can enjoy the game without time pressure.

6 Final Thoughts

A benefit of Go-High/Go-Low is its flexible game play. Teachers can use the game in many different ways to meet the needs of their particular students. It can be used to launch whole-class discussions, as a review activity at a center, or as a free-choice activity available on a daily basis. Consider making Go-High/Go-Low available as an enrichment resource throughout the year to support students' mathematical fluency and understanding.

There are many ways to get the materials for the game. Different materials afford different learning opportunities, as described in Figures 3 and 4. Games are about playing. An important aspect of play is the opportunity to negotiate one's own rules and modes of play (Bogost, 2008). "Kids are particularly adept at inventing new games based on the constraints of their environment; if one listens closely to children at play, one of the most common things to overhear is the establishment of new rules" (Bogost, 2008, p. 120–121). Students can modify the rules of the game as they play during the year. For example, during the unit review, one seventh-grade group decided to challenge themselves by playing the Go-High/Go-Low Game using all four primary operations instead of just addition and subtraction.

Integer operations are often viewed as a skill-based topic without a lot of room for spontaneity, but with the introduction of learning activities such as games, they can be an arena for discovery and surprise! We hope you try Go-High/Go-Low with your students. We would love to hear from you about how the game provided opportunities to spark your students' surprise and what they learned as they played. We're also interested in ways that you and your students modified the game to support fun and mathematics learning.

Alternative Materials for Separate Signs and Magnitude

(e.g., – and then 7)

Rolling separate sign and magnitude dice emphasizes the role that each part of a signed number plays. In other words, we don't know what kind of number it is until we roll the sign die. The game was designed to encourage students to focus on the sign in this way, because students commonly overlook or "ignore" the sign of a number.

Options for Positive and Negative Signs (dice):

- Buy "addition/subtraction" dice, but use them as positive and negative signs
- Buy blank dice or spinner sets:
 - For equal odds, write 3 negative and 3 positive signs opposite each other.
 - To increase the likelihood that students have to think about negative numbers (sometimes with equal odds, students will get mostly positive rolls which provides less opportunity to think about the intended learning goals), write 4 negative signs and 2 positive signs.

Options for Magnitude of the Number:

- Use numeral dice (not dot dice) of any size you wish students to practice. Examples:
 - Six-sided cubes (1-6 numerals)
 - 10-sided (0 to 9 numerals)
 - 12-sided (1-12 numerals)
 - 20-sided (0-19 numerals or 1 to 20)
- Print unsigned numeral cards on card stock in any range.

Fig. 3: How to make or buy the tools for the game in ways that encourage students to pay attention to the sign of positive and negative numbers.

Alternative Materials to “Roll” Number with the Sign and Magnitude Together (e.g., -7)

If you don't have access to physical dice or as an extension after students have played the game with separate dice, you might use a random number generator such as random.org. It is crucial that students see the positive or negative sign as part of the structure of the number. That is, -7 is a single valid number rather than just the number 7 with a negative sign added on. To emphasize this idea, the random number generator validates not only that each number rolled is an integer because the website calls them integers, but the number appears automatically as a valid number. To differentiate, students could also choose which number range they use to play, or teachers can assign groups to use certain numbers.

Use a free random number generator such as random.org/integers to “roll” an integer within any range:

1. In Part 1, set it to “Generate random integers.” Enter the values such as to . Format in column.
2. In Part 2, click [Get Numbers](#). This is the first person's “roll.”

The screenshot shows the 'Part 1: The Integers' section of the website. It includes a form with the following elements:

- A label 'Part 1: The Integers' in blue.
- A text input field containing '1' followed by the text 'random integers (maximum 10,000)'.
- A text input field containing '-20' followed by the text 'and' and another text input field containing '20', followed by the text '(both inclusive; limits ±1,000,000,000)'.
- A text input field containing '1' followed by the text 'column(s)'.
- A label 'Part 2: Go!' in blue.
- A text message: 'Be patient! It may take a little while to generate your numbers...'.
- Three buttons: 'Get Numbers', 'Reset Form', and 'Switch to Advanced Mode'.

3. Each student just has to click [Again!](#) on the result screen for every turn.

Fig. 4: How to use a random number generator to encourage students to learn that each number rolled is an integer and see each numeral as a valid number.

References

- Bogost, I. (2008). The rhetoric of video games. In K. Salen (Ed.), *The Ecology of Games: Connecting Youth, Games, and Learning* (pp. 117-140). Cambridge, MA: The MIT Press. doi: 10.1162/dmal.9780262693646.117
- Cordova, D. I. & Lepper, M. R. (1996). Intrinsic motivation and the process of learning: Beneficial effects of contextualization, personalization, and choice. *Journal of Educational Psychology*, 88(4), 715–730.
- Lappan, G., Phillips, E. D., Fey, J. T., Friel, S. N., Grant, Y., & Stewart, J. (2014). *Connected Mathematics 3*. Boston, MA: Pearson Education, Inc.
- Meyer, D. (2018). Recipes for Mathematical Surprise. Keynote presented at the *Ohio Council of Teachers of Mathematics 68th Annual Conference*. Akron, OH.
- National Council of Teachers of Mathematics (NCTM). (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.
- National Governors Association Center for Best Practices and Council of Chief State School Officers (2010). *Common Core State Standards Mathematics (CCSM)*. Washington, DC: National Governors Association Center for Best Practices and Council of Chief State School Officers.
- Nurnberger-Haag, J. (2007). Integers made easy: Just walk it off! *Mathematics Teaching in the Middle School*, 13, 118–121.
- Ohio Department of Education (2017). Ohio Learning Standards-Mathematics. Retrieved from <https://education.ohio.gov/getattachment/Topics/Learning-in-Ohio/Mathematics/Ohio-s-Learning-Standards-in-Mathematics/MATH-Standards-2017.pdf>
- Stein, M. & Smith, M.S. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions* (2nd ed.). Reston, VA: National Council of Teachers of Mathematics.



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Name: _____

Go-High! Go-Low!

Directions:

- On your turn, roll a sign die and a magnitude die.
- Starting with your second turn, decide whether to **add** or **subtract** your new roll.
- Write your equation. Everyone has to agree that everyone's equations are mathematically correct, but whether you give strategy hints is up to you!

Greatest Number Game

The player with the greatest (largest) number after 4 turns wins.

1st roll
↓
_____ □ _____ = _____
2nd roll
↓
_____ □ _____ = _____
3rd roll
↓
_____ □ _____ = _____
4th roll
↓
_____ □ _____ = _____
Final score

Write the names and final scores of the students in your group **in order from least to greatest**. Circle the winner.

□	_____	□	_____	□	_____
Score	Name	Score	Name	Score	Name

Least Number Game

The player with the least (smallest) number after 4 turns wins.

1st roll
↓
_____ □ _____ = _____
2nd roll
↓
_____ □ _____ = _____
3rd roll
↓
_____ □ _____ = _____
4th roll
↓
_____ □ _____ = _____
Final score

Write the names and final scores of the students in your group **in order from least to greatest**. Circle the winner.

□	_____	□	_____	□	_____
Score	Name	Score	Name	Score	Name