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SYNOPSIS Earthquake-induced vibration in earth dams in a direction parallel to the dam axis is studied using analytical elastic models. The nonhomogeneity of the dam materials is taken into account by assuming specific variations of the elastic moduli along the depth due to the confining pressure. Based on the models, a rational procedure is developed to estimate dynamic stresses and strains (both shear and normal) and corresponding elastic moduli and damping factors for earth dams from their hysteretic responses to real earthquakes, utilizing the hysteresis loops from the crest and base records. This leads to a study of the variation of stiffness and damping properties with the strain levels of different loops. Finally, an analysis of real earthquake performance of an earth dam, in the longitudinal direction, yields data on the shear moduli, damping factors, and non-linear constitutive relations for the dam materials; the Ramberg-Osgood nonlinear stress-strain curves are then fitted to these data.

INTRODUCTION

The problem of longitudinal vibrational behavior of earth dams during earthquakes is of practical interest in that such vibrations may lead to transverse cracks, which may later enlarge and produce catastrophic conditions under further earthquake impulses and other stresses. The San Fernando earthquake of February 9, 1971 (M_L= 6.3) caused a transverse crack on the Santa Felicia Earth Dam crest at the east abutment (Abdel-Ghaffar and Scott, 1979). This dam is comparatively large (236.5 ft high) and was constructed with modern design details and construction methods. The depth of the crack, approximately one-sixteenth of an inch in width, is not known. Investigation has implied that this narrow crack was caused by the dynamic strains induced by longitudinal vibration resulting from the earthquake and not by any settlement. Fortunately, the crack does not seem to be structurally significant. Both observation of this crack and previous engineering investigation have prompted a research effort dealing with the earthquake response characteristics of earth dams in the longitudinal direction; this paper is part of the effort.

THE ANALYTICAL MODELS

Two-dimensional analytical models are developed for evaluating natural frequencies and modes of vibrations, and for estimating earthquake induced strains and stresses (both shear and normal or axial) on earth dams in a direction parallel to the dam axis. Each model is a nonhomogeneous elastic wedge of finite length (with symmetric triangular section) in a rectangular canyon, resting on a rigid foundation and is subjected to uniform longitudinal ground motion (acceleration) $\ddot{w}_g(t)$ (Fig. 1). The equation

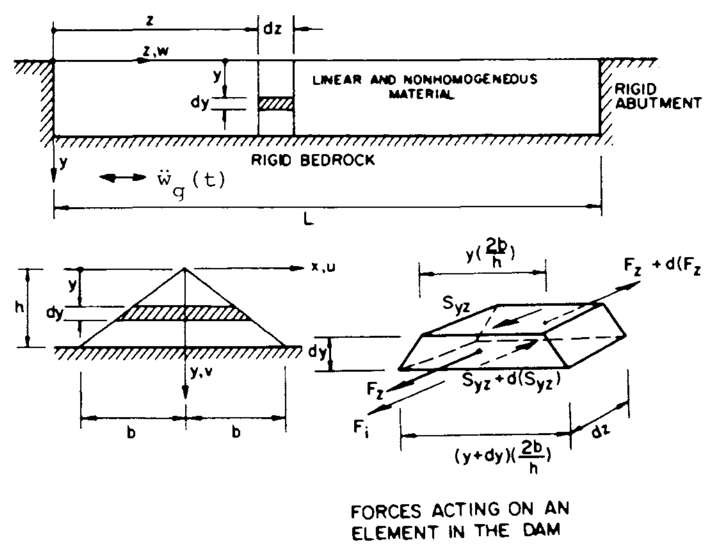


Figure 1. The model considered in the longitudinal vibration analysis

of motion of the dam can be written as

$$\rho \frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} - \frac{1}{y}\frac{\partial}{\partial y}\left[G(y)\frac{\partial w}{\partial y}\, y\right] - \frac{1}{y}\frac{\partial}{\partial z}\left[\eta G(y)\frac{\partial w}{\partial z}\, y\right] = -\rho\ddot{w}_g(t) \qquad , \qquad (1)$$

where ρ is the uniform mass density of the dam material, $w(y,z,t)$ is the longitudinal vibrational displacement (relative to the base of the dam), c is the damping coefficient, $G(y)$ is the shear modulus (which varies along the depth), and $\eta[= E(y)/G(y) = 2(1 + \nu)]$ is an elastic constant (where $E(y)$ is the modulus of elasticity and ν is the Poisson's ratio of the dam material). Variations in the elastic moduli with depth (below the crest) are expressed in the form

$$G(y) = G_0\left(\frac{y}{h}\right)^{\ell/m} \quad , \quad \left(\frac{\ell}{m} = 0, 1, \frac{1}{2}, \frac{1}{3}, \frac{2}{5}\right) \quad , \qquad (2\text{-a})$$

$$G(y) = G_0\left[(1 - \varepsilon)\frac{y}{h} + \varepsilon\right] \quad , \quad \varepsilon = \frac{G_1}{G_0} \quad , \qquad (2\text{-b})$$

where G_0 and G_1 are the shear moduli of the dam material at the base and at the crest, respectively, h is the height of the dam, ℓ/m is an index which defines the type of stiffness variation with depth. Natural frequencies and associated mode shapes corresponding to the above specific values of ℓ/m as well as to the cases where $\varepsilon = 0.5$ and $\varepsilon = 0.6$ were studied by Abdel-Ghaffar and Koh (1981). In addition, results of full-scale dynamic tests, of both ambient and forced vibrations (Abdel-Ghaffar, Scott, and Craig, 1980) and real earthquake observations of some existing dams are utilized to confirm the suggested models. The agreement between the experimental and earthquake data and the theoretical results is reasonably good. It was found that the models which take into account the effect of variation with depth of both the shear modulus and the modulus of elasticity of the dam material are the most appropriate representations for predicting the dynamic characteristics in the longitudinal direction.

EARTHQUAKE INDUCED STRAINS AND STRESSES

Using the generalized coordinates and the principle of mode superposition, expressions of the magnitude and distribution of shear strains in the $(n,r)^{th}$ mode are given by

$$\gamma_{n,r}(y,z,t) = \frac{\partial w}{\partial y} =$$

$$\frac{4P_{n,r}}{r\pi h\omega_{n,r}\sqrt{1 - \zeta_{n,r}^2}}\,\Psi_{n,r}\left(\frac{y}{h}\right)\sin\left(\frac{r\pi z}{L}\right)V_{n,r}(t) \quad ,$$

$$n = 1,2,3,\ldots \quad , \quad r = 1,3,5,\ldots \quad , \qquad (3)$$

where $P_{n,r}$ is the participation factor resulting from the modal configuration along the depth (Abdel-Ghaffar and Koh, 1981), $\zeta_{n,r}$ is the modal damping factor, $\omega_{n,r}$ is the (n,r)th natural frequency, $\Psi_{n,r}(y/h)$ is defined as the modal participation and distribution of shear strain along the depth, $\sin(r\pi z/L)$ is the modal configuration along the crest, and $V_{n,r}$ is the earthquake response of a damped single-degree-of-freedom oscillator (the convolution or Duhamel integral).

The shear strain modal participation factors $\Psi_{n,r}$ or Ψ_n (with $n = 1,2,3,4$) in the y-direction for the case where $\varepsilon = 0.5$ and for a value of $\beta_r[=\eta(r\pi h/L)^2]$ equal to 5 is shown in Fig. 2. The coefficient β_r depends on the Poisson's ratio ν of the dam material, the dam's depth-to-length ratio (h/L), and the order, r, of the modal configuration along the longitudinal axis of the dam (low $\beta_r(= 0\text{-}10)$ implies low modal order (r = 1 to 3) or a very long dam, while higher values of $\beta_r(>10)$ indicate a short, high dam or higher modes along the crest). The basic characteristics of the shear strain modal participation functions, Ψ_n, (i.e., the maximum values and their locations and the relative modal contributions) can be easily extracted from Fig. 2. In general, Fig. 2 shows significant contributions from higher modes along the depth $(n \geq 2)$.

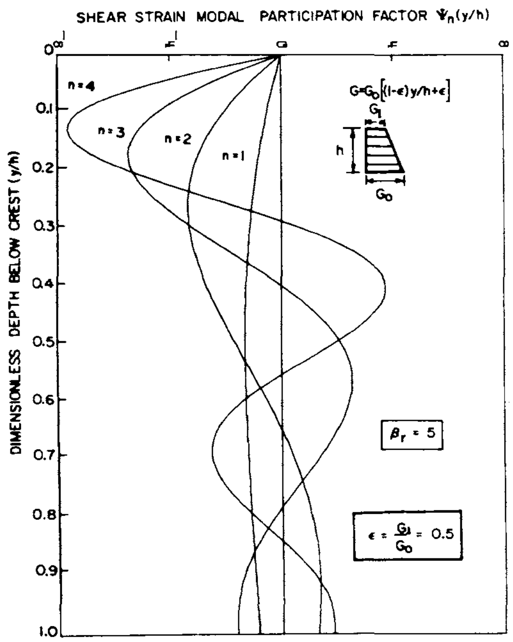


Figure 2

The modal shear stresses are given by

$$\tau_{n,r}(y,z,t) =$$

$$\frac{4G_0P_{n,r}}{r\pi h\omega_{n,r}\sqrt{1 - \zeta_{n,r}^2}}\,\Phi_{n,r}\left(\frac{y}{h}\right)\sin\left(\frac{r\pi z}{L}\right)V_{n,r}(t), \qquad (4)$$

where $\Phi_{n,r}(y/h)$ is the modal participation and distribution of shear stress. The function $\Phi_{n,r}$ or Φ_n is shown in Fig. 3 for the case $\varepsilon = 0.5$ and $\beta_r = 5$.

The magnitude and distribution of normal (axial) strains and stresses in the (n,r)th mode can be given by

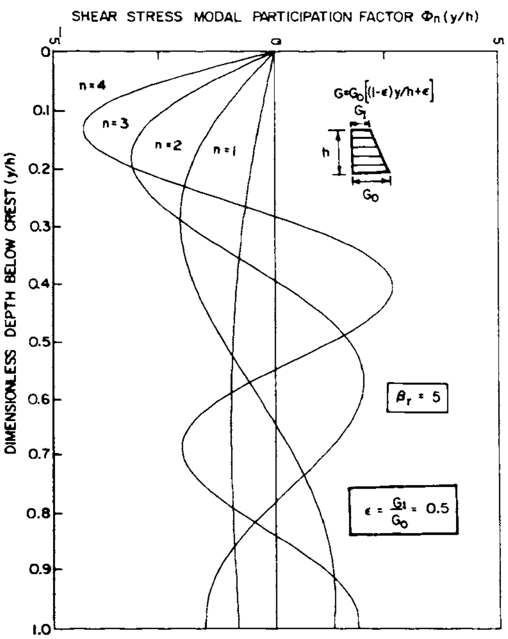


Figure 3

$$\epsilon_{n,r}(y,z,t) = \frac{\partial w}{\partial z} =$$

$$\frac{4P_{n,r}}{L\omega_{n,r}\sqrt{1 - \zeta^2_{n,r}}} Y_n\left(\frac{y}{h}\right)\cos\left(\frac{r\pi z}{L}\right)V_{n,r}(t) \quad , \qquad (5)$$

where $Y_n(y)$ is the modal configuration along the depth (Fig. 4).

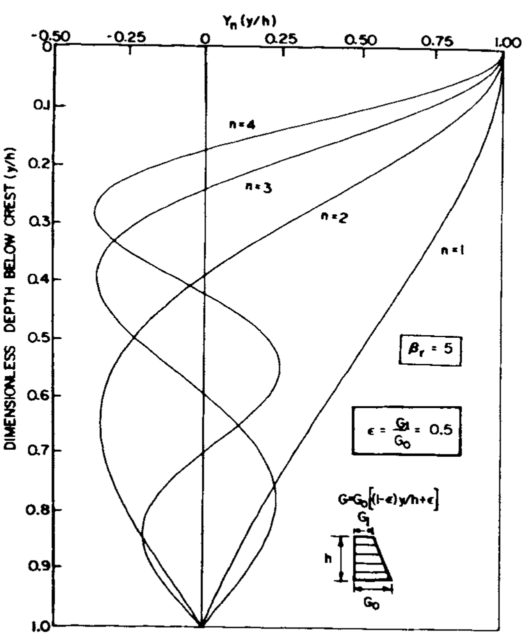


Figure 4 Normal strain modal participation factor.

The modal normal stresses are given

$$\sigma_{n,r}(y,z,t) =$$

$$\frac{4\eta G_0 P_{n,r}}{L\omega_{n,r}\sqrt{1 - \zeta^2_{n,r}}} \Gamma_{n,r}\left(\frac{y}{h}\right)\cos\left(\frac{r\pi z}{L}\right)V_{n,r}(t) \quad , \qquad (6)$$

where the normal stress modal participation function $\Gamma_{n,r}(y/h)$ is shown in Fig. 5 (as Γ_n) of the case $\epsilon = 0.5$ and $\beta_r = 10$.

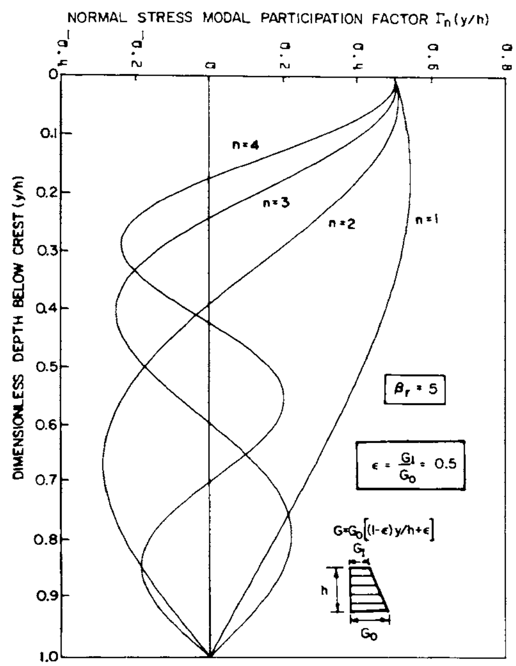


Figure 5

It can be seen from Figs. 4 and 5 and Eqs. 5 and 6 that the maximum dynamic normal strains and stresses occur near the top region of the dam at the end abutments (where $\cos(r\pi z/L) \simeq 1$); this may explain the Santa Felicia Dam crack mentioned previously.

UTILIZATION OF RESPONSE SPECTRA

The response spectra technique can be utilized for estimating maximum earthquake-induced longitudinal strains and stresses. The maximum value of the quantity $V_{n,r}(t)$ of Eqs. 3, 4, 5, and 6 is equal to the ordinate of the velocity spectrum S_v of the ground motion, corresponding to the natural frequency $\omega_{n,r}$ and the damping factor $\zeta_{n,r}$ of the (n,r)th mode, i.e.,

$$V_{n,r}(t)\big|_{max} = S_v(\omega_{n,r},\zeta_{n,r}) \quad . \qquad (7)$$

For earthquake-like excitations where little error is involved for $\zeta_{n,r} < 20\%$, it can be shown that

$$S_a = \omega_{n,r}^2 S_d = \omega_{n,r} S_v \quad , \tag{8}$$

where S_d and S_a are the ordinates of the calculated acceleration and displacement spectra, respectively.

The maximum shear strains and stresses in the first mode (n = 1, r = 1), which occur in the central region of the dam (z ≃ L/2), along the y-axis (the depth axis) of Fig. 1, can be written as (with the aid of Eqs. 3, 4, and 8)

$$\gamma_{1,1}|_{max} = \left[\frac{4P_{1,1}}{\pi h} \Psi_{1,1}|_{max}\right] S_d \tag{9}$$

and

$$\tau_{1,1}|_{max} = \left[\frac{4G_0 P_{1,1}}{\pi h \omega_{1,1}^2} \Phi_{1,1}|_{max}\right] S_a \tag{10}$$

Similarly, the maximum tensile or compressive strains, and stresses in the first mode, occurring in the top region near the crest at its ends (where z ≃ L or cos(rπz/L) ≃ 1), are given (with the aid of Eqs. 5, 6, and 8) by

$$\varepsilon_{1,1}|_{max} = \left[\frac{4P_{1,1}}{L} \Upsilon_1|_{max}\right] S_d \quad , \tag{11}$$

and

$$\sigma_{1,1}|_{max} = \left[\frac{4\eta G_0 P_{1,1}}{L\omega_{1,1}^2} \Gamma_{1,1}|_{max}\right] S_a \tag{12}$$

It is important to note that the use of the response spectra technique may lead to inaccuracies in ascertaining the true influence of material nonlinearity on the dam response since the technique provides only single-valued estimates of stresses and strains induced by earthquakes. The manner in which the amplitudes of an earth dam's motion vary with time have a major role in the dam's earthquake response characteristics.

IDENTIFICATION OF CONSTITUTIVE RELATIONS, ELASTIC MODULI, AND DAMPING FACTORS OF EARTH DAMS FROM THEIR EARTHQUAKE RECORDS

Like all soils, materials of earth dams develop nonlinear inelastic stress-strain relationships when subjected to earthquake loading conditions. By using earthquake response records of the crest and the base (structural and input ground motions) together with the results of the above-mentioned analytical models, these stress-strain relationships can be estimated for the dam's materials. The strain-dependent elastic moduli and damping factors have already been determined in this manner for the upstream-downstream recorded motion of a modern earth dam by Abdel-Ghaffar and Scott (1979a,b), using existing analytical shear-beam models (Ambraseys, 1960).

The idea is illustrated in Fig. 6 and can be summarized in the following steps:

(1) By using the earthquake records, the experimental results and the analytical models, the fundamental frequency in the longitudinal direction can be identified.

(2) By using very narrow band-pass digital filtering around the fundamental frequency (usually the primary) of the crest and base records, the pure fundamental mode response can be obtained.

(3) By treating the filtered modal response as that of a single-degree-of-freedom (SDOF) hysteretic structure (with nonlinear restoring force $F(x,\dot{x})$ equal to $-M(\ddot{x} + \ddot{w}_g(t))$; x, $\dot{x}$, and $\ddot{x}$ are the relative displacement, velocity, and acceleration, respectively, and M is the mass) the hysteresis loops, which show the relationship between the relative displacement of the crest with respect to the base and the absolute acceleration of the dam, can be obtained.

(4) By using the elastic longitudinal analytical models, the shear stresses and shear strains (Eqs. 9, 10) can be determined as functions of the maximum absolute acceleration and maximum relative displacement, respectively, for each hysteresis loop, and consequently, equivalent (secant) shear moduli and damping factors can be determined from the slope and the area, respectively, of the loop. Since it was assumed that each hysteresis loop is a response of an SDOF oscillator, and in order to get a qualitative picture of the dynamic shear strain and stress from the hysteretic response, the value of S_d and S_a in Eqs. 9 and 10 are assumed to be the maximum relative displacement, $(w(t))_{max}$, and the maximum absolute acceleration, $(\ddot{w}(t) + \ddot{w}_g(t))_{max}$, respectively, for each hysteresis loop.

(5) Finally, the data so obtained permit development of typical stress-strain curves which are then approximated by the Ramberg-Osgood analytical models and/or the hyperbolic curves. The data can also be compared with those previously available from soil-dynamic laboratory investigations and can be combined with those obtained from the analysis of the upstream-downstream vibrations (Abdel-Ghaffar and Scott, 1979) to give informative materials to both earthquake and the geotechnical engineers.

APPLICATION OF THE ANALYSIS

Longitudinal Dynamic Shear Stress-Strain Relations for Santa Felicia Earth Dam

The Santa Felicia Dam is equipped with two accelerographs (one on the central region of the crest and the other at the base) that yielded data on how it responsed to two earthquakes.

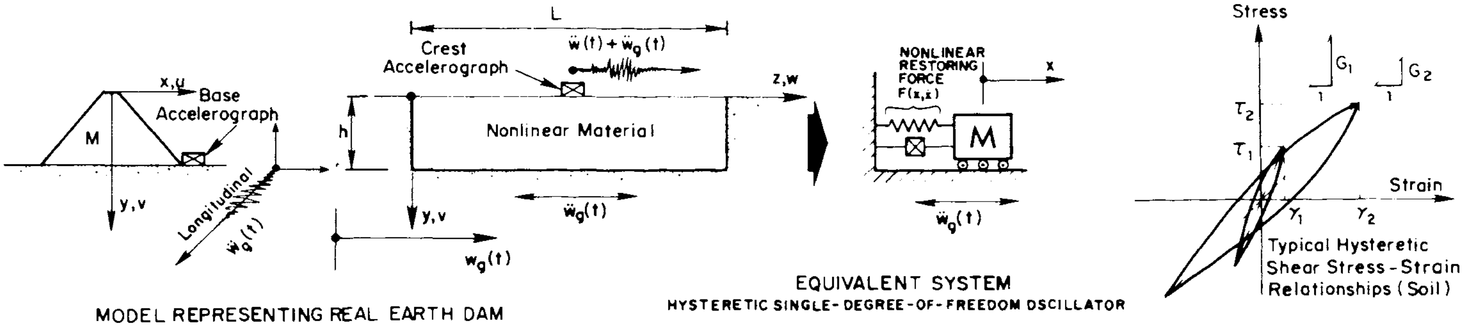
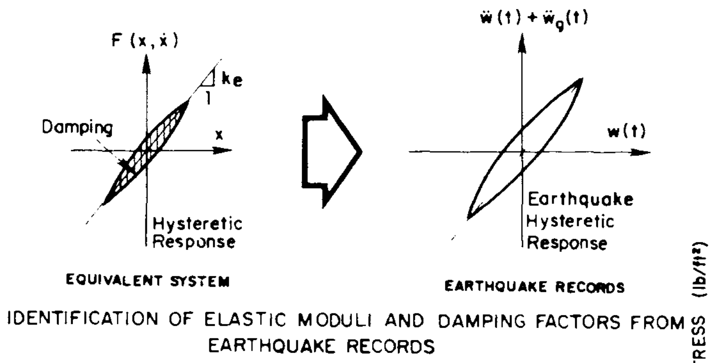


Figure 6



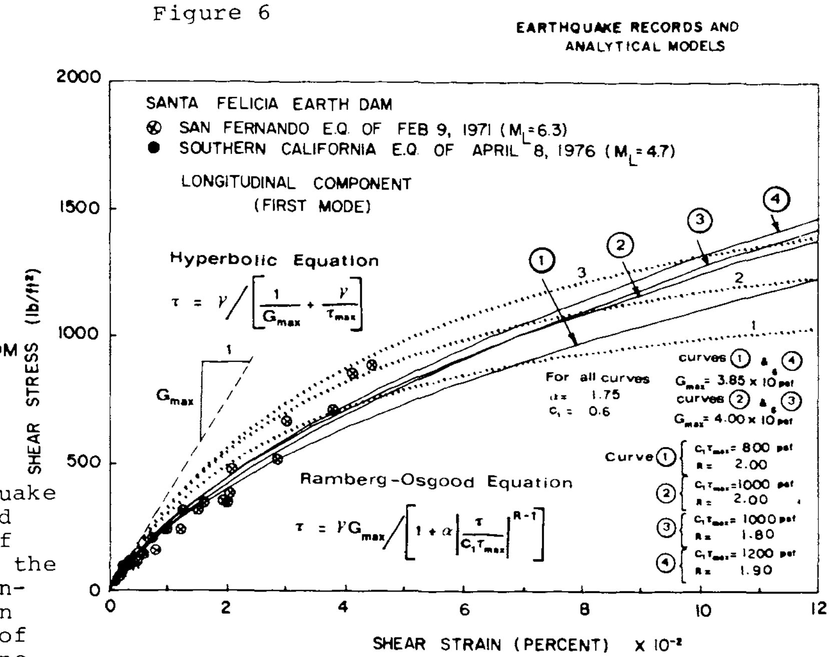


Figure 7

Amplification spectra of the dam's two earthquake records which were computed (Abdel-Ghaffar and Scott, 1979) by dividing Fourier amplitudes of acceleration of the crest records by those of the base records (to indicate the resonant frequencies and to estimate the relative contribution of different modes) revealed that the values of the resonant frequencies vary slightly from one earthquake to the other. In addition, amplification spectra of the upstream-downstream direction showed that the dam responded primarily in its fundamental mode in that direction, but the spectra of the longitudinal component are lacking pronounced single peaks.

The first longitudinal frequency determined from the amplification spectra of the 1971 earthquake is 1.35 Hz (1.27 Hz for the 1976 earthquake), ρ = 4.02 lb-sec^2/ft^4, ν = 0.45, and h/L = 236.5/912.5 = 0.26; this gives $\beta_r = 1.92\ r^2$, r = 1,2,3,... (or β_1 = 1.92 for the first mode). The first-mode maximum shear strains for the analytical models of various stiffness variations occur at about 0.4 - 0.7 of the dam height (except for the linear case, where $\ell/m = 1$, in which the maximum occurs at the crest). The maximum shear stress occurs at about 0.7 - 0.8 of the dam height.

The maximum strains and stresses (shear and normal) for each hysteresis loop, of the two earthquakes were estimated with the aid of the low-strain analytical models (where an average value of cases where ℓ/m = 1/3, 2/5, 1/2, 1 was taken). The time dependence of the hysteretic behavior was determined for only the first 25 secs. The estimated shear strain and stress for each hysteresis loop are shown (as circles) in Fig. 7; the data show an initial slope, G_{max}, at the origin ranging from 3.5 to 4.10 (x 10^6 psf). The non-linear stress strain curves (Ramberg-Osgood (R-O) curves) are adopted here to fit the data (shown as solid curves in Fig. 7). Numerical values of the parameters which adjust the position and shape of the R-O curves can be chosen, from Fig. 7, to represent the relations for earth dam materials (similar to Santa Felicia Dam).

Taking G_{max} and τ_{max} from curves 1, 2, and 3 of Fig. 7, hyperbolic shearing stress-shearing strain curves are determined and shown on Fig. 7 as dotted curves. The hyperbolic fit deviates considerably after the low-strain range, indicating the R-O curves are a better match for the overall behavior.

Shear Moduli and Damping Factors for Santa Felicia Earth Dam

The relationship between the estimated secant shear modulus and the dynamic shear strain is shown by the semilog plots of Fig. 8 for the two earthquakes. Figure 8 shows also the decrease of secant modulus, G, with strain, γ, for the same special values of G_{max}, α, $C_1\tau_{max}$, and R of curves 1, 2, 3, and 4 of Fig. 7. The R-O

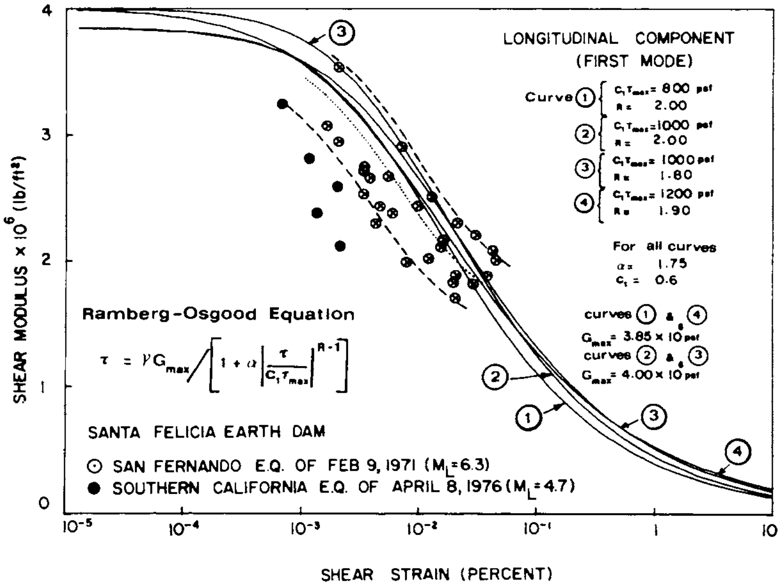


Figure 8

curves match the estimated earthquake results and also the results of low-strain full-scale tests (Abdel-Ghaffar, Scott, and Criag, 1980) very well indicating both the reliability of the developed longitudinal analytical models in predicting earthquake induced stresses and strains and the applicability of the R-O curves to represent the stiffness relations for earth dams.

The relationship between the estimated equivalent viscous damping factor determined from the area of each hysteresis loop and the corresponding shear-strain amplitude are shown in Fig. 9; shown also are the damping factors evaluated, by integration, from the R-O curves (where the area of the hysteresis loop formed is a measure of the hysteretic damping occurring in the dam

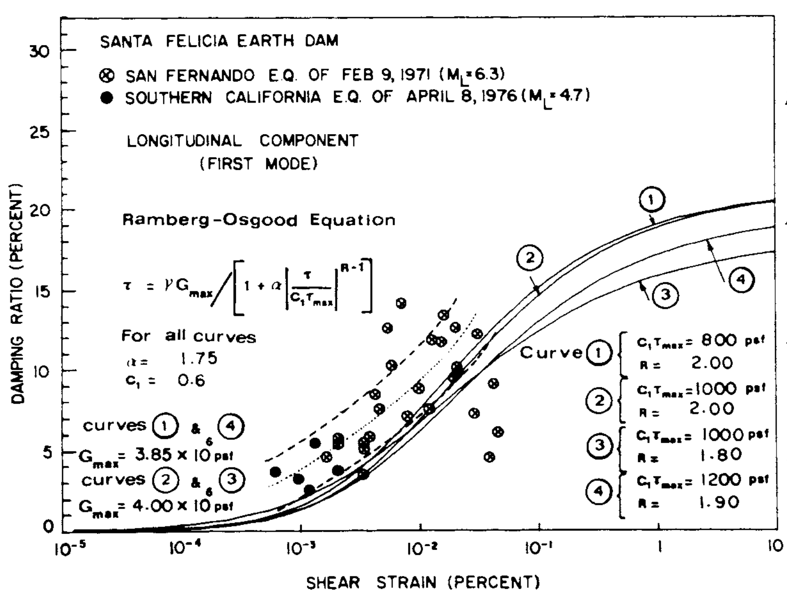


Figure 9

materials) for the same special conditions of curves 1, 2, 3, and 4 of Fig. 7. The R-O curves, which represent the best fit of the three sets of data of Figs. 7 and 8, represent a lower bound of the earthquake damping data.

CONCLUSIONS

Dynamic strains and stresses (both shear and normal) induced in a wide class of earth dams by longitudinal component of earthquake ground motions are investigated. Reasonable estimates of the dynamic stress-strain curves (nonlinear strain-softening (or yielding) type) and the strain-dependent elastic moduli and damping for earth dam materials can be obtained by using the proposed dynamic analysis procedure, earthquake records (on and in the vicinity of dams), and the adoption of the Ramberg-Osgood-type curves. These estimates would be useful for any study of the dam's earthquake-response characteristics; in addition, the variation of material properties with depth should be taken into account for any realistic dynamics study. Figures 7 to 9 should provide a good guide to the material properties in the dynamic analysis of any earth dam composed predominantly of rolled-fill, essentially cohesionless material, with or without a relatively thin core. Despite the value of this study, however, further research to accurately assess and mitigate potentially adverse effects of seismic shaking on earth dams is needed.

ACKNOWLEDGMENTS

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