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26 May 2010, 4:45 pm - 6:45 pm

## A Parametric Study on Soil-Pile Kinematic Interaction in Layered Soils

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Fifth International Conference on

## Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics and Symposium in Honor of Professor I.M. Idriss

May 24-29, 2010 • San Diego, California

### A PARAMETRIC STUDY ON SOIL-PILE KINEMATIC INTERACTION IN LAYERED SOILS

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#### ABSTRACT

In pile-foundation seismic design, until a few years ago, only inertial loads applied by the overstructure at the pile cap were taken into account, neglecting the dynamic interaction. So, good approaches to evaluate inertial interaction effects have been developed, but not for kinematic interaction. Modern national and international codes require to take into account the dynamic soil-pile-overstructure interaction (inertial and kinematic), without giving any information about kinematic interaction evaluation criteria.

In this work a practical method based on the Winkler foundation model is applied to analyse the seismic response of single piles. The analysis is focused on a single pile embedded in a two-layered soil profile. The two layers are  $h_1$  and  $h_2$  thick, with two different shear moduli,  $G_1$  and  $G_2$ , respectively. The system is subjected to a conventional dynamic input motion. Different sharp stiffness contrast, expressed in terms of variation of the ratio  $G_1/G_2$  are investigated. A parametric study on the influence trained on the maximum bending moment, by the depth of the discontinuity between the two layers and by the pile slenderness, is brought about. The results are presented in terms of bending moments expressed in a dimensionless form.

#### INTRODUCTION

Pile foundation behaviour under seismic conditions is a main problem in engineering practise, especially for as regards the most urbanised areas, where skyscrapers, viaducts and other man made works are usually founded on piles.

Until a few years ago, in professional seismic analyses, piles were designed taking into account only inertial loads applied by the overstructure at the pile cap, neglecting the dynamic interaction effects. Recent national and international seismic regulations and guidelines, among which European Technical Code, EC8 (EN 1998-1, 2003; EN 1998-5, 2003); AASHTO, 1983; JSCE, 1988; AFGP, 1990; and the Italian D.M. 14/01/2008 (the latter in effect since 01/07/2009), recognize the importance of dynamic soil-structure interaction and require to take into account both inertial and kinematic interactions for particular situations related to the soil type, the seismicity of the area, and the importance of structure.

However, while good approaches to evaluate inertial interaction effects have been developed and applied worldwide, not the same can be said about kinematic interaction: different approaches have been proposed but they

led to different evaluations of the kinematic bending moments on pile foundations, and no suggestions on criteria for its evaluation can be found in seismic regulations.

In the last years many researches on kinematic interaction aroused to better understand those phenomena, but over all to offer to practitioners simple design procedures. The analysis methods proposed in technical literature can be divided into four groups: coupled methods with a continuous modelling, linearly equivalent approaches, decoupled methods (Winkler model) and simplified methods.

In this work a practical simplified method based on the “Beam on Dynamic Winkler Foundation” model, that can be included among decoupled methods, is applied to analyse the seismic response of single piles embedded in a two-layered soil. The two layers are  $h_1$  and  $h_2$  thick and are characterised by two different Young moduli,  $E_1$  and  $E_2$ , respectively. The system is subjected to a conventional dynamic input motion located at the bedrock.

A new dimensionless expression for bending moment, that allows to take into account the mechanical features of the

interface, in terms of Young moduli, is proposed and applied to present the results.

A parametric study on the influence trained on the maximum bending moment, by the depth of the discontinuity between the two layers and by the pile slenderness, is brought about investigating on different sharp stiffness contrast, expressed in terms of variation of the ratio  $E_1/E_2$ . Both the conditions of  $E_1/E_2 > 1$  and  $E_1/E_2 < 1$  are investigated.

## ABOUT SOIL-PILE DYNAMIC INTERACTION

Piles dynamic behaviour is a typical case of soil-foundation-overstructure dynamic interaction, affected by the non linear behaviour of the soil rounding piles and by kinematic effects linked to ground shaking.

For a fixed head pile, in homogeneous soil the maximum bending moment is generally reached at the pile cap, but many authors, like Mizuno [1987], Tokimatsu et al. [1996], Matsui and Oda [1996], show a series of case histories in which seismic pile damage are not only near the pile cap, but also along the pile shaft, at remarkable depths, without specific situations like liquefaction phenomena, so that they could not be attributed to inertial effects.

Mizuno [1987] identified four main causes of seismic damage to pile foundations. They are soil liquefaction and soil permanent displacements, but also “kinematic interaction”, due to soil deformations arising from seismic waves crossing, and “inertial interaction”, caused by the inertial forces due to the over-structure. Kinematic and inertial interaction, in reality, happen both together, influencing each other, and together represent soil-pile-overstructure dynamic interaction.

A seismic S-wave, propagating vertically in a soil layer without any foundation, causes only horizontal displacements in the soil, but free-field seismic ground motion,  $u_{ff}(t)$ , are influenced by the pile presence. In fact the pile reflects and refracts the waves, so that the pile head horizontal displacement,  $u_p(t)$ , will be different from the free-field surface motion, and a rotation  $\phi_p$  of the pile head will take place.

Gazetas [1984] and Fan et al. [1991], investigate kinematic interaction effects with an extensive parametric analysis on single piles and pile groups, under harmonic excitation in homogeneous and layered soils, and with linear elastic piles, and proposed two factors to synthetically quantify those effects:

$$I_u = |u_p| / u_{ff} \quad (1)$$

$$I_\phi = |\phi_p| \cdot d / (2 \cdot u_{ff}), \quad (2)$$

being  $d$  pile diameter.

When kinematic interaction is neglected  $|u_p| = u_{ff}$  and  $|\phi_p| = 0$ , so  $I_u = 1$  and  $I_\phi = 0$ . At low frequencies (high wave length) pile head follows free field motion; for intermediate values  $I_u$  decreases quickly with growing frequencies; for higher frequencies pile displacements are more and more different from free field motions, and  $I_u$  fluctuates around a constant value.

Kinematic interaction is negligible for a pile embedded in a very stiff soil site, but in case of soft soil layers, relative soil-pile displacements may happen along the pile shaft and displacement profiles can be very different from that of the free-field displacements. So, kinematic bending moments and shear stresses arise.

Gazetas and Mylonakis [1998] observe that kinematic bending moments are mainly influenced by seismic motion frequency content with respect to the soil deposit natural frequency. They observe also that bending moments are influenced by pile-soil relative stiffness and, when the pile is embedded in a multi-layered deposit, by the stiffness contrast between two adjacent layers, and by the ratio between the layer interface depth and the pile active length. This effect is more remarkable when piles are embedded in soil layers with strong mechanical discontinuities, and the highest stress levels are reached especially near interfaces between two layers with different stiffness features (Maiorano & Aversa [2006]; Cairo & Dente [2007]).

The relative importance of kinematic or inertial interaction depends on the features of the structure, of its foundation, of the soil foundation and of seismic waves (Maiorano & Aversa [2006]). Inertial interaction effects, generally, concentrate in a very narrow frequencies interval around the fundamental natural frequency of the foundation-overstructure system interacting with the soil through the impedance functions (Mylonakis et al., 2006).

Ardita et al. [2009] observe that kinematic bending moments in case of weak soil deposits, for which, typically, a Gibson soil behaviour, with linearly increasing mechanical properties, can be assumed, could reach very high values near the ground level, where pile-soil relative stiffness is greater. However, the high kinematic bending moment values are often glossed over by inertial bending moments, that reach their highest values near the ground level. Moreover, Di Laora [2009] observes that kinematic and inertial bending moments are not in phase, and the phase angle depends on the relation between the fundamental natural frequency of the system and the frequency content of the seismic solicitation. So, it may happen that when inertial effects are maximum kinematic bending moments are negligible and vice-versa, and it become very difficult to distinguish each effect.

## ANALYSIS METHODS PROPOSED IN TECHNICAL LITERATURE

In technical literature many methods are proposed to evaluate kinematic interaction effects and, in particular, the arisen maximum bending moments. They can be grouped into four categories:

1. coupled methods with a continuous modelling,
2. linearly equivalent approaches,
3. decoupled methods (Winkler model)
4. simplified methods.

In coupled methods the soil, the pile and the overstructure are included in a unique model and analysed numerically, for example, by FEM, FDM or BEM (Wu and Finn [1997], Maiorano & Aversa [2006], Grassi and Massimino [2009]). Several aspects, like the non linearity of soil or of structural elements can be taken into account, but they request, as input, constitutive parameters not always available in current applications.

Linearly equivalent approaches are a simplification in which a elastic solution is applied updating the input parameters step by step with the strain level evolution.

In every case, such computational effort is nullified if seismic input is not well defined in terms of amplitude, duration, frequency content and main direction of seismic waves propagation.

*Decoupled methods* are based on “method of substructure” (Gazetas and Mylonakys, 1998) with the hypothesis that the pile follow the soil free-field motion (Margason and Holloway [1977]; NEHRP [1997]) As they need a superimposition of the effects, they are based on the hypothesis of soil and overstructure linear behaviour. Those approaches analyse, separately, first soil-foundation kinematic interaction, choosing the foundation input motion; then they define a impedance functions, that describe the system dynamic stiffness; at last the previously determined input motion is applied to the overstructure, through the interface, to determine inertial interaction. The impedance functions, defined for each degree of freedom of the soil-foundation system, represent the dynamic stiffness and the geometric and hysteretic damping. They are functions of the geotechnical features, of the stratigraphy, of the foundation geometry and of the excitation frequency. Mylonakis et al [2006] propose a simple closed form solution valid in the hypothesis of linearly elastic behaviour of the system.

For example, in a “Beam on Dynamic Winkler Foundation” approach, the pile is modelled as a beam embedded in a visco-elastic soil. Soil-pile interaction is represented by a system of springs and dampers distributed along the pile. Dobry & O'Rourke [1983]; Nikolaou et al. [1995]; Nikolaou et al. [2001]; Sica et al. [2007]; Castelli et al. [2008] hypotesize a linear-elastic soil behavior, while Conte & Dente, [1988], [1989], Castelli & Maugeri [2007]; Maiorano et al., [2007];

Cairo et al. [2008], introduce a non-linear and hysteretic behaviour, that better represents soil response to great deformations, but it should not be forgotten that superimposition effects is not rigorously valid in this case.

Simplified solutions have been proposed by Margason and Holloway [1977], Dobry and O'Rourke [1983], Nikolau e Gazetas [1997] and NEHRP [1997], to evaluate the maximum bending moment due to kinematic interaction, but their application is not widespread in engineering practise.

*Simplified methods* make use of simple empirical formulas to evaluate kinematic bending moments.

Margason and Holloway [1977] and NEHRP [1997] formulas base on the simplified assumption that pile moves as free-field motion (1-D seismic waves propagation analysis) of soil under S-waves propagating vertically. The maximum moment value  $M$  is:

$$M = E_p \cdot I_p \cdot \frac{1}{R} \quad (3)$$

being  $E_p \cdot I_p$  the pile bending stiffness and  $1/R$  the maximum bending, that can be expressed in one of the following manners:

$$1/R \cong 2 \cdot \Delta u_{ff} / \Delta z^2 \quad (\text{Margason and Holloway, 1977}) \quad (4)$$

$$1/R \cong a_{ff} / V_s^2 \quad (\text{NEHRP, 1997}) \quad (5),$$

Where

$\Delta u_{ff}$  is the maximum relative displacement between two soil points with a  $\Delta z$  dept difference,

$a_{ff}$  is the maximum free-field acceleration at the soil surface

$V_s$  is the soil S-wave velocity.

However these methods disregard kinematic interaction, and consequently do not take into account pile-soil relative stiffness,  $L/d$  ratio and radiative damping. Moreover they can not be applied at the interface between different stiffness soil layers, where in theory deformations are discontinuous, so bending is infinite.

Dobry and O'Rourke [1983] propose one of the most famous Winkler model to evaluate the maximum bending moment at the interface between two different stiffness layers. They assume a infinitely long pile ( $d$  = pile diameter) and two infinitely thick layers, elastic behaviour for both pile and soil, uniform shear stress  $\tau$  in both the soil layers ( $\gamma_1/\gamma_2 = G_1/G_2$ ). Pile is analysed as a beam on a spring bed with  $k_1 = 3 \cdot G_1/d$  and  $k_2 = 3 \cdot G_2/d$  respectively. They propose the following equation:

$$M = 1.86 \cdot (E_p \cdot I_p)^{3/4} \cdot (G_1)^{1/4} \cdot \gamma_1 \cdot F \quad (6)$$

Where:

$$F = \frac{(1 - C^{-4}) \cdot (1 + C^3)}{(1 + C) \cdot (C^{-1} + 1 + C + C^2)} \quad (7)$$

$$C = \left( \frac{G_2}{G_1} \right)^{1/4} \quad (8);$$

$$\gamma_1 = \frac{\tau}{G_1} \quad (\text{Dobry and O'Rourke, 1983}) \quad (9)$$

$$\text{or } \gamma_1 = \frac{\rho_1 \cdot H_1}{G_1} \cdot a_{\max,s} \quad (\text{Dente, 2005}), \quad (10)$$

being  $\tau$  is the maximum shear stress derived by a one-dimensional free-field analysis,  $a_{\max,s}$  the maximum acceleration at the soil surface and finally  $H_1$  the upper layer thickness.

An evolution of this model is proposed by Mylonakis [1999] that take into account the two soil layers thickness and the dynamic nature of the seismic input. The kinematic moment is calculated by the pile bending strain,  $\varepsilon_p$  that is generated in the fibres of the external radius  $r$ ; in particular:

$$M = E_p I_p \left( \frac{\varepsilon_p}{\gamma_1} \right) \cdot \gamma_1 \cdot \Phi \cdot r \quad (4)$$

Where

$$\frac{\varepsilon_p}{\gamma_1} = 1.5 \left( \frac{k_1}{E_p} \right)^{1/4} F \quad (5)$$

$$F = \frac{(C^2 - C + 1)[(2\lambda_1 h_1 - 1)C(C - 1) - 1]}{2C^4 \lambda_1 h_1} \quad (6)$$

$$\lambda_1 = \left( \frac{k_1}{4E_p I_p} \right)^{1/4}; \quad k_1 = \frac{3G_1}{d} \quad (7)$$

being:  $\Phi$  a factor that takes into account the seismic input frequency and varies between 1 and 1.2; and  $C$  the coefficient of Dobry & O'Rourke [1983] (see expression (2)).

Nikolau and Gazetas [1997] propose the two following closed form solutions to evaluate the maximum stationary bending moment at the interface between two different stiffness layers, deriving from harmonic excitations  $a_r \cdot \exp(i \cdot \omega \cdot t)$  with various frequencies at the base of the layer:

$$\max M(\omega) = 0.042 \cdot \tau_{\text{interf}} \cdot d^3 \cdot \left( \frac{L}{d} \right)^{0.30} \cdot \left( \frac{E_p}{E_1} \right)^{0.65} \cdot \left( \frac{V_{s2}}{V_{s1}} \right)^{0.5} \quad (11)$$

$$\max M(\omega) = 2.7 \cdot 10^{-7} \cdot E_p \cdot d^3 \cdot \left( \frac{a_r}{g} \right) \cdot \left( \frac{L}{d} \right)^{1.30} \cdot \left( \frac{E_p}{E_1} \right)^{0.7} \cdot \left( \frac{V_{s2}}{V_{s1}} \right)^{0.3} \cdot \left( \frac{H_1}{L} \right)^{1.25} \quad (12)$$

where  $\tau_{\text{interf}} \cong a_{ff} \cdot \rho_1 \cdot H_1$  is an estimation of the shear stress developing at the interface between the two layers,

$\varepsilon_M = \frac{M}{E_p \cdot \left( \frac{\pi \cdot d^4}{64} \right)} \cdot \frac{d}{2}$  is the maximum strain due to the

bending moment acting on the pile and  $a_r$  is the maximum seismic acceleration at the bedrock.

Those equations are rigorously valid when  $H_1 > L_a$ , being  $L_a \cong 1.5 \cdot (E_p/E_s)^{1/4} \cdot d$  (Randolph, 1981) the pile active length.

Pile response to real earthquakes is less onerous than the one obtained by the described simplified approaches, because of the time variation of seismic solicitations. The authors suggest the following approximate expression to evaluate the real maximum bending moment  $\max M(t)$  for a real seismic excitation:

$$\max M(t) = \eta \cdot \max M(\omega) \quad (13)$$

The reduction factor  $\eta$ , varies between 0 and 1, depending on the earthquake time-length, in terms of number of equivalent cycles  $N_c$ , on the ratio between the earthquake main period  $T_p$  and the soil system natural period  $T_s$ , and on the effective soil-pile system damping  $\beta_{\text{eff}}$ . Nikolau et al. [1995] and Nikolau et al. [2001] propose the following expressions:

$$\eta = \begin{cases} 0.04N_c + 0.23 & \text{for } T_1 \approx T_p \\ 0.015N_c + 0.17 \approx 0.2 & \text{for } T_1 \neq T_p \end{cases} \quad (14)$$

All the simplified methods allow to evaluate the kinematic component of the pile maximum bending moment, but do not give any information about the seismic action variation caused by kinematic interaction.

Some application of those methods (Dente, 2005) highlight that NEHRP [1997] and Dobry and O'Rourke [1983] methods overestimate very much the bending moments obtained by the

more rigorous equations proposed by Nikolau and Gazetas [1997].

Aversa et al [2005] analyse the various parameters influence in the previous simplified formulas. They find that  $\max M(t)$  in a two-layered soil increases with the increasing of pile-soil relative stiffness, of pile slenderness ratio,  $L/d$  and of the interface depth and of the ratio  $V_2/V_1$ , that expresses the stiffness-contrast. Moreover the value of  $\max M(t)$  is deeply influenced by absolute values of soil stiffness parameters, increasing for lower  $G_1$  values.

To analyse pile dynamic behaviour, the application of rigorous analytical tools would be desirable, but in design practise this is too onerous, especially when a frequency domain seismic analysis is brought about, as pile response should be evaluated with so a high frequencies number (thousands) that it would be enough to cover the seismic signal frequency content. Moreover, to reinforce a pile we need to know the maximum bending moment reached along the pile shaft, and this problem could be easily solved by the application of simplified methods.

#### THE APPLIED MATHEMATICAL MODEL

Maugeri et al. [2009] and Ardita et al. [2009] present and apply a BDWF simplified model to evaluate displacements, rotations, shear and bending moments along the pile shaft. For this work that model has been better developed and now it can rely on a deeper discretization of the soil deposit, so that various mechanical properties distribution along the pile shaft can be simulated. The soil around the pile is hypotised in free-field conditions (seismic S-waves propagate vertically, not influenced by pile presence) and it interact with the embedded pile by means of springs and dampers distributed along the pile shaft (Fig. 1). The interface is defined by an impedance function.

First of all, a dynamic input is applied at the bedrock and free-field displacements of undisturbed soil  $u_{ff}(z, t)$  are evaluated by a one dimensional S-wave propagation theory, assuming a linear hysteretic soil behaviour. Then free-field displacements are applied to the pile through the visco-elastic interface. The input parameters are functions of both the pile and the involved soil layers parameters features. Moreover the impedance function, that depends on soil and pile geometrical and physical parameters, must be defined. This is the most critical aspect of the modellation.

The spring and dumpers mechanical properties (stiffness  $k_x$  and viscosity  $c_x$ ) are functions of the oscillation frequency,  $\omega$ .

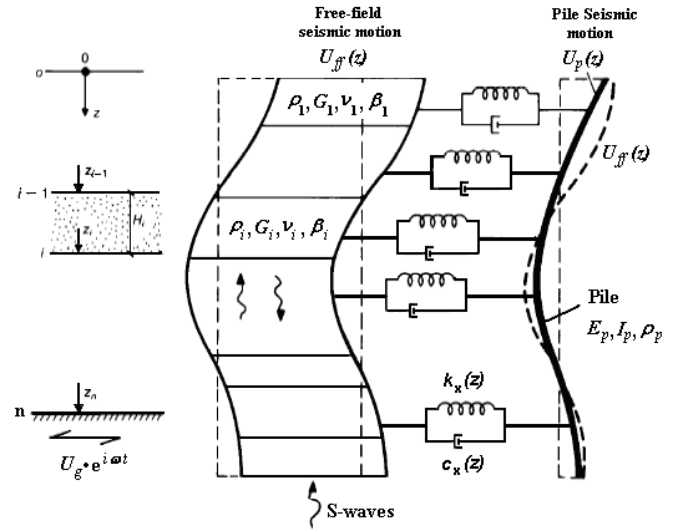


Fig. 1. BDFW model for a layered soil and a free head pile.

The 1-D propagation of S-wave theory is applied to determine free-field ground motion, under the hypothesis of a linearly hysteretic soil behaviour. The following equation expresses the above stating:

$$u_{ff}(z, t) = U_{ff}(z) \cdot \exp[i \cdot (\omega \cdot t + \alpha_{ff})] = \hat{U}_{ff} \exp(i \cdot \omega \cdot t) \quad (15)$$

Each layer is characterized by the following a complex S-wave velocity:

$$V_s^* = V_s \cdot \sqrt{1 + 2 \cdot i \cdot \beta} \quad (16)$$

where  $V_s = \sqrt{G/\rho}$  is the S-wave velocity and  $\beta$  is the hysteretic damping ratio.

The dynamic motion is a harmonic acceleration with the soil fundamental natural frequency, because this is the most severe case that could happen as it would bring about resonance phenomena. The iterative Rayleigh method is applied to determine the deposit natural frequency.

For free-field motion evaluation the one-dimensional S-wave propagation theory is applied, with the hypothesis of a linear-hysteretic soil behaviour.

$$u_{ff}(z, t) = U_{ff}(z) \cdot \exp[i \cdot (\omega \cdot t + \alpha_{ff})] = \hat{U}_{ff} \exp(i \cdot \omega \cdot t) \quad (17)$$

where:

$U_{ff}(z)$  is the free-field displacement modulus;

$\alpha_{ff} = \arctan[2 \cdot \xi \cdot \beta / (1 - \beta^2)]$  is the phase difference between the bedrock sollicitation and the soil response;

$\beta = \omega_f / \omega_s$  is the ratio between the solicitation frequency and the system fundamental natural frequency of a system made of the only soil without the embedded foundation;  $\xi$  is the hysteretic damping ratio.

Each layer is characterised by a complex shear wave velocity:

$$V_s^* = V_s \cdot \sqrt{1 + 2 \cdot i \cdot \xi} \quad (18)$$

Where  $V_s$  is the shear wave velocity.

So, a horizontal harmonic motion is applied at the bedrock and free-field ground motion are determined and then they are applied on pile foundations, without taking into account the stresses caused by the relative soil-pile displacements.

Pile response is governed by the following differential equation, that expresses the dynamic equilibrium of an infinitesimal element of the pile:

$$E_p \cdot I_p \cdot \frac{\partial^4 u_p}{\partial z^4} + m_p \cdot \frac{\partial^2 u_{pt}}{\partial t^2} = S_x \cdot (u_{ff} - u_p) \quad (19)$$

where  $t$  is the time,  $E_p$  and  $I_p$  are respectively the pile Young's modulus and moment of inertia, so that  $E_p I_p$  is the bending stiffness,  $m_p$  is the pile mass per length,  $u_{pt}$  is the total pile displacement and  $u_p$  is the relative pile displacement.

$$S_x = k_x + i \cdot \omega \cdot c_x \quad (20)$$

represents features of the interface by which free-field ground displacement transmits strains to pile.

As a first approximation, the spring stiffness  $k_x$  can be considered approximately frequency independent and expressed as multiple of the local soil Young's modulus  $E_s$ :

$$k_x \approx \delta \cdot E_s \quad (21)$$

where  $\delta$  a frequency independent coefficient assumed to be constant (i.e. the same for all layers and independent of depth), that will be called "pile-soil interaction coefficient".  $\delta$  has been determined through FEM by Kavaddas & Gazetas [1993]. The stiffness parameter  $c_x$  represents both radiative and material damping; the former arises from waves originating at the pile perimeter and spreading laterally outward and the latter from hysteretically-dissipated energy in the soil.

Solving eq. (19), pile deformations (displacements and rotations), bending moment and shear will be determined as functions of both depth  $z$  and time  $t$ .

Horizontal pile displacements can be determined also with the following equation:

$$u_p(z, t) = U_{pp}(z) \cdot \exp[i \cdot (\omega \cdot t + \alpha_p)] = \hat{U}_{pp}(z) \exp(i\omega t) \quad (22)$$

where:  $U_{pp}(z)$  is the pile displacement modulus;  $\omega$  is the excitation round frequency;  $\alpha_p$  is the phase difference between free-field displacement and pile response in terms of displacement.

Equation (19) can be alternatively written as follows:

$$\hat{U}_{pp}^{IV} - \lambda^4 \cdot \hat{U}_{pp} = \alpha \cdot \hat{U}_{ff} \quad (23)$$

$$\text{where } \lambda^4 = \frac{m_p \cdot \omega^2 - S_x}{E_p \cdot I_p}; \quad \alpha = \frac{S_x}{E_p \cdot I_p}$$

Equation (23) has the following general solution:

$$\hat{U}_{pp}(z) = \left[ e^{-\lambda z} \cdot e^{-\lambda z} \cdot e^{-\lambda z} \cdot e^{-\lambda z} \right] \cdot \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix} + s \cdot \hat{U}_{ff}(z) \quad (24)$$

where  $D_1, D_2, D_3, D_4$  are arbitrary constants to evaluate basing on the compatibility equations and the boundary conditions, while  $s = \alpha / (q^4 - \lambda^4)$   $q = \omega / V_s$

By eq. (24) the following equation can be obtained:

$$\begin{Bmatrix} \hat{U}_{pp}(z) \\ \hat{U}'_{pp}(z) \\ \hat{U}''_{pp}(z) \\ \hat{U}'''_{pp}(z) \end{Bmatrix} = \begin{bmatrix} e^{-\lambda z} & e^{\lambda z} & e^{-i\lambda z} & e^{i\lambda z} \\ -\lambda \cdot e^{-\lambda z} & \lambda \cdot e^{\lambda z} & -i \cdot \lambda \cdot e^{-i\lambda z} & i \cdot \lambda \cdot e^{i\lambda z} \\ \lambda^2 \cdot e^{-\lambda z} & \lambda^2 \cdot e^{\lambda z} & -\lambda^2 \cdot e^{-i\lambda z} & -\lambda^2 \cdot e^{i\lambda z} \\ -\lambda^3 \cdot e^{-\lambda z} & \lambda^3 \cdot e^{\lambda z} & i \cdot \lambda^3 \cdot e^{-i\lambda z} & -i \cdot \lambda^3 \cdot e^{i\lambda z} \end{bmatrix} \cdot \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix} + s \cdot \begin{Bmatrix} \hat{U}_{ff}(z) \\ \hat{U}'_{ff}(z) \\ \hat{U}''_{ff}(z) \\ \hat{U}'''_{ff}(z) \end{Bmatrix} \quad (25)$$

or concisely, for a pile element in the domain of the soil layer  $j$ :

$$\tilde{U}_{pj}(z) = \tilde{F}_j(z) \cdot \tilde{D}_j + s_j \cdot \tilde{U}_j(z) \quad (26)$$

The vector  $\tilde{U}_j(z)$  can be determined from the free-field displacement solution.

In the case of a multy-layered soil profile with  $N$  layers ( $j = 1, 2, \dots, N$ ), eq. (26) will be a system of  $4 N$  equations

with  $4N$  arbitrary constants  $\tilde{D}_1, \tilde{D}_2, \tilde{D}_3, \tilde{D}_4$  that could be evaluated by compatibility equations between pile and soil and boundary conditions.

Compatibility equations express that at the  $(N - 1)$  soil layer and pile interfaces, the pile deflection  $u_p$ , rotation  $\theta$ , bending moment  $M$ , and shear force  $Q$  must be continuous: these compatibility requirement can be expressed by the following  $4(N - 1)$  equations (at a arbitrary interface  $j$ )

$$\tilde{U}_{pj}(z_j) = \tilde{U}_{p(j+1)}(z_j) \quad (27)$$

For as regards boundary conditions it can be observed that at the pile top, in the case of a free head pile,  $M(0,t) = 0$  and  $Q(0,t) = 0$  while for hinged head pile  $\Theta(0,t) = 0$  and  $Q(0,t) = 0$ . At the pile tip, in the case of a pile hinged at the bedrock,  $M(z_N,t) = 0$  and  $u_p(z_N,t) = u_g(t)$ , while in the case of a floating pile,  $M(z_N,t) = 0$  and  $Q(z_N,t) = 0$ , being  $M$  and  $Q$  pile bending moment and shear.

Thus a set of  $4N$  equations can be obtained and they can be solved for the constants  $\tilde{D}_1, \tilde{D}_2, \dots, \tilde{D}_N$ . Once these constants are evaluated, pile displacements, bending moments, shear forces, etc. can be obtained directly from eq. (26), since pile displacements, rotations, bending moments and shear can be expressed as follows:

$$u_{pp}(z,t) \quad (28)$$

$$\Theta(z,t) = u'_{pp}(z,t) \quad (29)$$

$$M(z,t) = -E_p \cdot I_p \cdot u''_{pp}(z,t) \quad (30)$$

$$Q(z,t) = -E_p \cdot I_p \cdot u'''_{pp}(z,t) \quad (31)$$

#### THE ANALYSED CASES

A computer code has been written following the mathematical formulation presented in the previous paragraph.

Firstly the natural frequency  $\omega_s$  of the soil deposit is determined by the iterative procedure suggested by Rayleigh. Then a harmonic input is applied at the bedrock with the natural frequency for each case analysed to induce a free field motion along the soil profile utilised in the analysis. Finally the free-field is introduced in eq. (19) to determine displacements, rotations, bending moments and shear forces along the pile shaft. For as regards soil-pile interface features, they have been calculated according to eq. (20).

A system made of a fixed head single floating pile, with length  $L = 20\text{m}$  and diameter  $d = 0,60\text{ m}$ , embedded in a two layer

soil deposit, thick  $h_1$  and  $h_2$  respectively, has been studied (Fig. 2). The soil deposit lies on a rigid bedrock and is subjected to vertically propagating S-waves, producing a horizontal harmonic motion. With the aim of investigating on the influence of the mechanical discontinuity position and of the materials mechanical features on the pile response, a parametrical analysis have been made on  $h_1/h_2$ ,  $E_1/E_p$ , and  $E_1/E_2$ .

The cases  $h_1/L = 0,1 ; 0,2; 0,4$  have been analysed.

For as regards boundary conditions  $M = 0$  and  $Q = 0$  have been fixed at the pile tip. The value of the fundamental natural frequency has been assumed as bedrock acceleration frequency.

For as regards mechanical parameters, the pile has been represented as a linearly elastic beam with a mass density  $\rho_p = 2400\text{ kg/m}^3$  and a Young modulus  $E_p = 25 \cdot 10^3\text{ Mpa}$ .

The soil has been hypothesised as a linearly hysteretic solid made of two layers, respectively with Young's modulus  $E_1$  and  $E_2$ , damping ratio  $\xi_1 = \xi_2 = 10\%$ , mass density  $\rho_1 = \rho_2 = 1900\text{ kg/m}^3$  and a Poisson's ratio  $\nu_1 = \nu_2 = 0.40$ .

The following cases have been analysed:

$$E_1 [\text{MPa}] = 25; 125; 250; 1250 \quad (32)$$

$$E_1/E_p = 0.001; 0.005; 0.01; 0.05 \quad (33)$$

$$E_2/E_1 = 0.1; 0.5; 1.0; 2.0; 5.0; 10.0 \quad (34)$$

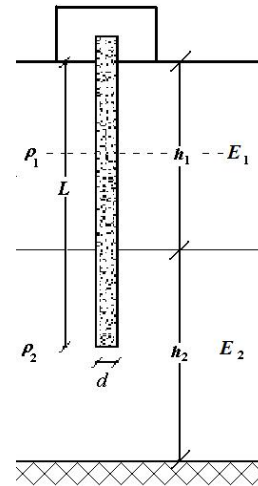


Fig. 2. The calculation model



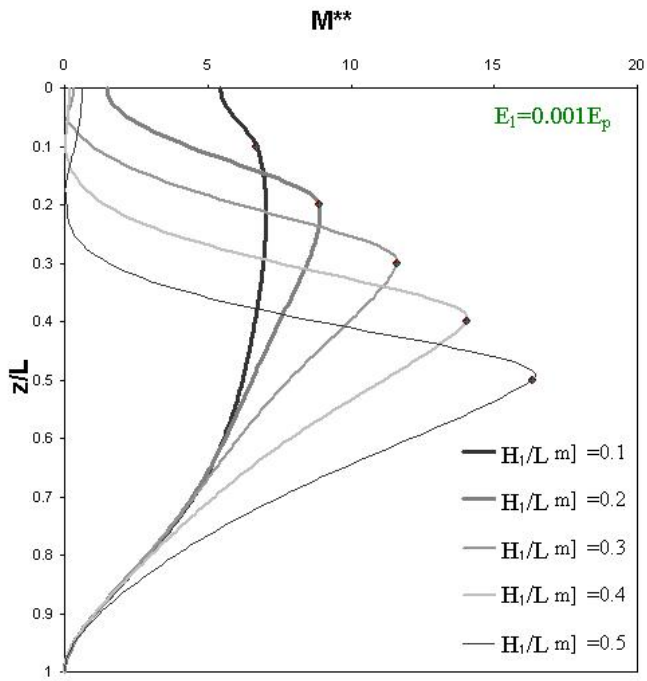


Fig. 3. Dimensionless maximum bending moments versus dimensionless layer depth for  $E_1/E_p = 0.001$  and  $E_1/E_2 = 10$ .

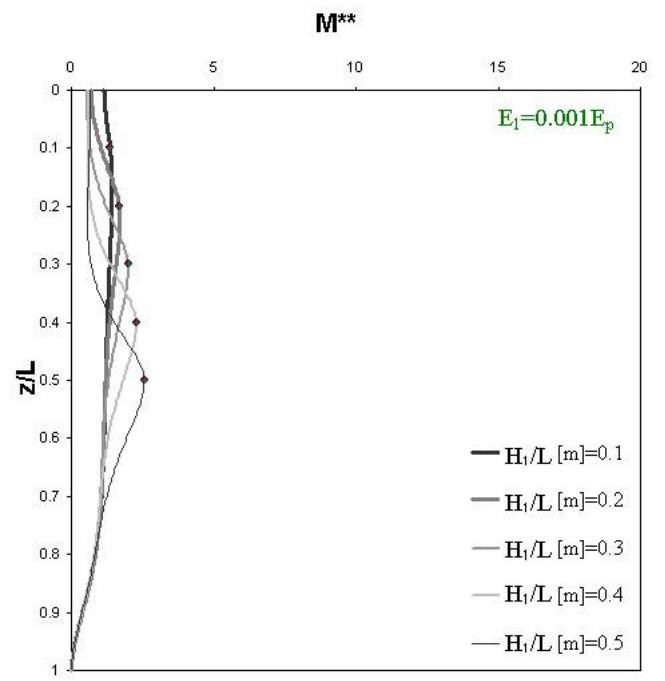


Fig. 5. Dimensionless maximum bending moments versus dimensionless layer depth for  $E_1/E_p = 0.001$  and  $E_1/E_2 = 2$ .

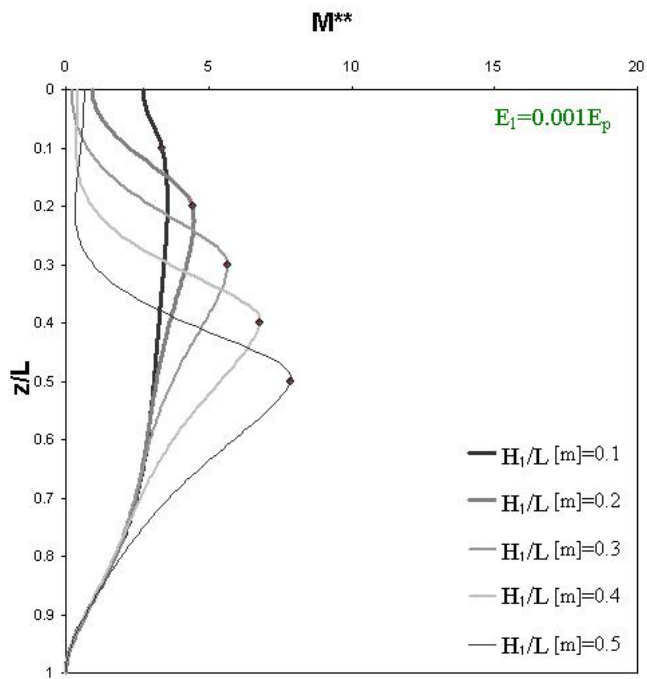


Fig. 4. Dimensionless maximum bending moments versus dimensionless layer depth for  $E_1/E_p = 0.001$  and  $E_1/E_2 = 5$ .

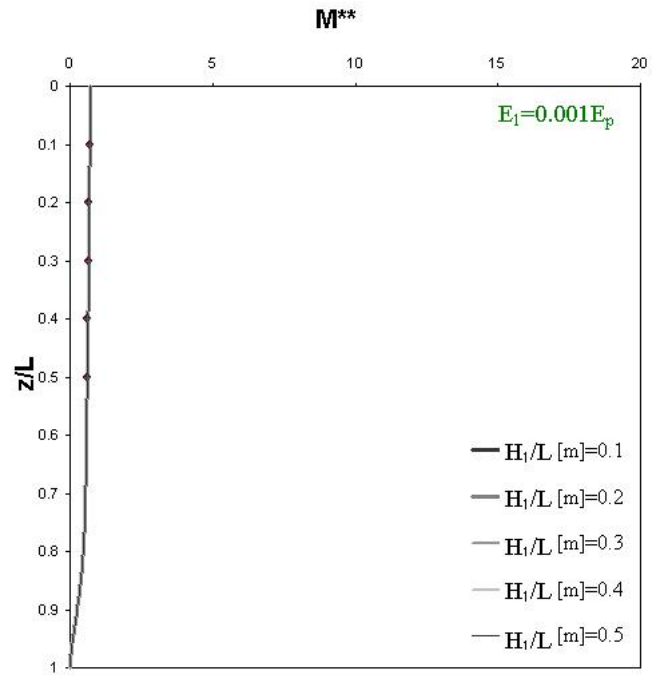


Fig. 6. Dimensionless maximum bending moments versus dimensionless layer depth for  $E_1/E_p = 0.001$  and  $E_1/E_2 = 1$ .

Kavvadas and Gazetas [1993] propose a dimensionless expression by which representing the calculation results.

$$M^*(z,t) = \frac{M(z,t)}{\rho_p \cdot d^4 \cdot A} \quad (35)$$

where  $A = \ddot{u}_g = \omega^2 \cdot u_g$  is the harmonic amplitude.

However, this expression does not allow to take into account the mechanical features of the interface, in terms of Young moduli. In this work a new dimensionless expression is proposed to represent bending moments along the pile shaft.

$$M^{**}(z,t) = \frac{M(z,t)}{\rho_p \cdot d^4 \cdot A} \cdot \frac{E_1}{E_p} \quad (36)$$

In Fig. 3, 4, 5, 6, 7 and 8, the dimensionless bending moments distribution along the pile shaft are plotted for the case  $E_1/E_p = 0.001$ , for different layer interface depth.

The  $E_1/E_2$  ratios are assumed constant in each plot. They have been determined fixing the Young modulus  $E_1$  in the upper layer, so that, to vary the  $E_1/E_2$  ratios, simply  $E_2$  was changed in each analysis.

The analysis shows that the bending moments at the interface are greater than those at the pile head. Furthermore the greater is the ratio  $E_1/E_2$ , the greater is the moment at the interface between the two layers.

In Fig. 10, the dimensionless maximum bending moments at the pile head are plotted versus the dimensionless layer interface depth for different  $E_1/E_2$  ratios.

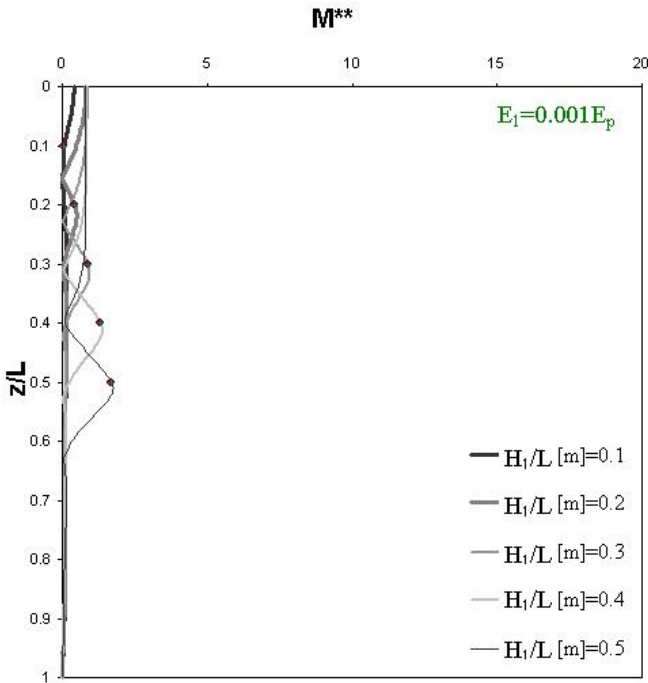


Fig. 7. Dimensionless maximum bending moments versus dimensionless layer depth for  $E_1/E_p = 0.001$  and  $E_1/E_2 = 0.2$ .

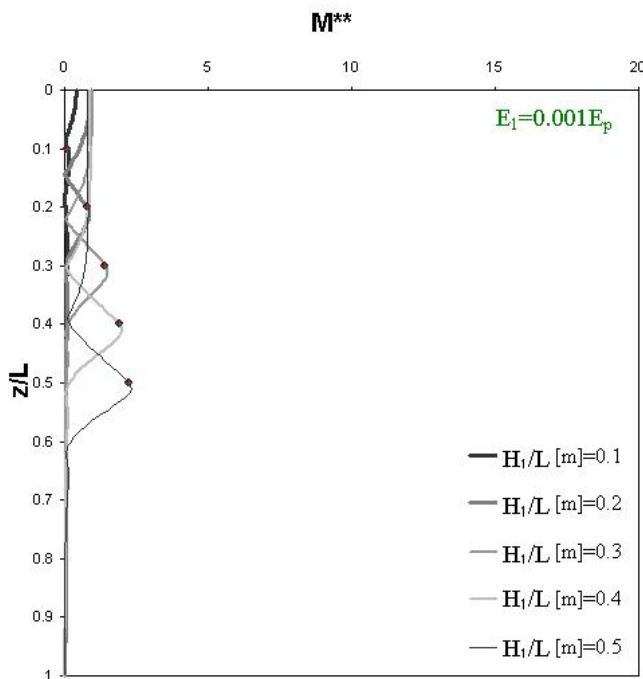


Fig. 8. Dimensionless maximum bending moments versus dimensionless layer depth for  $E_1/E_p = 0.001$  and  $E_1/E_2 = 0.1$ .

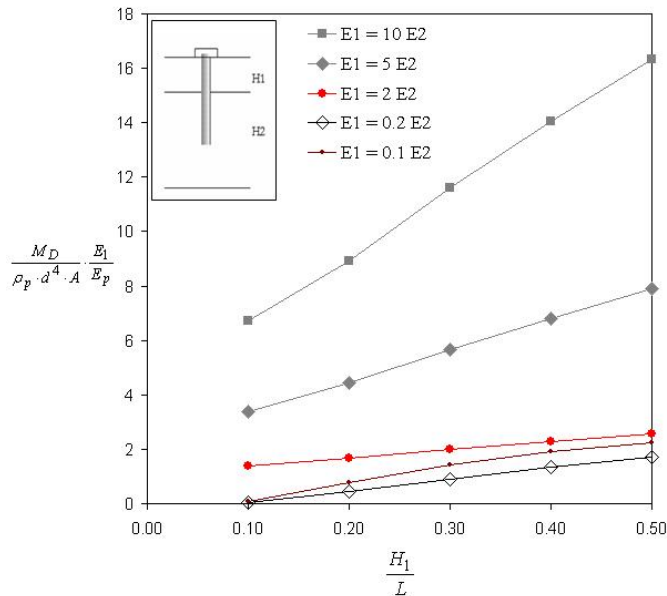


Fig. 9. Dimensionless maximum bending moments at the interface between the two layers versus dimensionless layer interface depth for different  $E_1/E_2$  ratios.

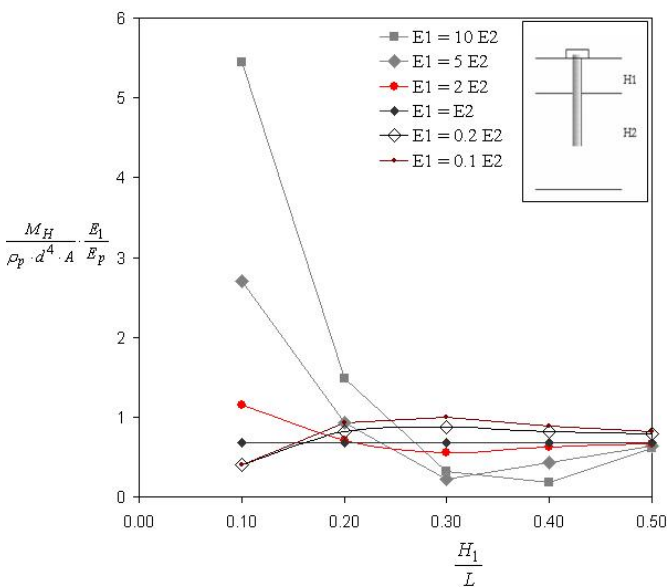


Fig. 10. Dimensionless maximum bending moments at the pile head versus dimensionless layer interface depth for different  $E_1/E_2$  ratios.

In this case it can be observed that the bending moments induced at the pile head could be severe when the interface between the two layers is closer to the ground surface. However, this behaviour is often covered by inertial interaction effects at the pile head.

The analyses performed in this study can be useful for a first assessment of the maximum bending moments at the pile head or close to a discontinuity interface, once the geometrical and mechanical properties of the soil-pile system are known. However it is known in literature that the results obtained using a single harmonic input method may overestimate the maximum bending moment along the pile.

Table 1. The seven different soil types reported in EC8 (EN 1998-1, 2003)

Soil type	Description of the stratigraphic profile	PARAMETERS		
		$V_{S30}$ [m/s]	$N_{SPT}$	$C_u$ [kPa]
A	Rock – < 5m weak material	> 800	-	-
B	Very dense sand and gravel, very stiff clay; $h > 10m$	360 - 800	> 50	> 250
C	Deep deposits medium dense to dense sand; stiff clay; $h = 10 - 100 m$	180 - 360	15 - 50	70 - 250
D	Loose to medium dense sand, soft to firm cohesive soil	< 180	< 15	< 70
E	Alluvium C or D, $h = 5 - 20 m$ above rock (A)			
S1	Soft clay /silts $h > 10m$ with high PI	< 100 (indicative)	-	10 - 20
S2	Liquefiable soils, sensitive clay; any soil type not listed above			

Table 2. The soil types analysed by Maugeri et al. (2009).

	Soil type C (Tab. 1)				Soil type D (Tab. 1)			
	300	150	150	150	150	100	100	100
$V_{S1}$ [m/s]	300	150	150	150	150	100	100	100
$V_{S2}$ [m/s]	300	300	400	600	150	200	300	400
$V_{S2}/V_{S1}$	1	2	2.67	4	1	2	3	4
$V_{S30}$ [m/s]	300	200.00	218.18	240.00	150.00	133.33	150.00	160
$w_s$ [1/s]	17.99	11.413	12.686	14.173	8.994	7.608	8.783	9.448

Table 3. The Italian seismic records used to calibrate the model and the corresponding values of the operative harmonic amplitude “ *A* ” (in units of *g*) (Maugeri et al., 2009).

File name	Date	Seismic station	Earthquake	Main Dir.	$V_{s2}/V_{s1}$							
					Values of A for soil type C (Tab. 1)				Values of A for soil type D (Tab. 1)			
					1	2	2.67	4	1	2	3	4
<b>A-TMZ270</b>	06/05/1976	Tolmezzo-Diga Ambiesta	Friuli	WE	0.14	0.14	0.12	0.16	0.12	0.09	0.14	0.14
<b>A-TMZ000</b>	06/05/1976	Tolmezzo-Diga Ambiesta	Friuli	NS	0.07	0.05	0.06	0.08	0.05	0.04	0.05	0.05
<b>A-STU270</b>	23/11/1980	Sturno	Campano-Lucano	WE	0.08	0.07	0.07	0.10	0.07	0.08	0.07	0.06
<b>A-STU000</b>	23/11/1980	Sturno	Campano-Lucano	NS	0.09	0.13	0.10	0.03	0.11	0.09	0.11	0.12
<b>A-AAL018</b>	26/09/1997	Assisi-Stallone	Umbria-Marche	NS	0.08	0.06	0.07	0.10	0.04	0.03	0.03	0.05
<b>E-NCB090</b>	06/10/1997	Nocera Umbra-Biscontini	Umbria-Marche (aftershock)	WE	0.02	0.02	0.02	0.03	0.02	0.02	0.03	0.02
<b>E-NCB000</b>	06/10/1997	Nocera Umbra-Biscontini	Umbria-Marche (aftershock)	NS	0.03	0.03	0.03	0.03	0.02	0.01	0.02	0.03
<b>R-NCB090</b>	03/04/1998	Nocera Umbra-Biscontini	Umbria-Marche (aftershock)	WE	0.02	0.02	0.02	0.03	0.01	0.01	0.02	0.02
<b>J-BCT000</b>	14/10/1997	Borgo-Cerreto Torre	Umbria-Marche (aftershock)	NS	0.03	0.03	0.03	0.04	0.02	0.02	0.02	0.02
<b>J-BCT090</b>	14/10/1997	Borgo-Cerreto Torre	Umbria-Marche (aftershock)	WE	0.05	0.04	0.05	0.06	0.04	0.04	0.05	0.05
<b>E-AAL018</b>	06/10/1997	Assisi-Stallone	Umbria-Marche (aftershock)	WE	0.03	0.03	0.03	0.03	0.02	0.01	0.02	0.02
<b>B-BCT000</b>	26/09/1997	Borgo-Cerreto Torre	Umbria-Marche	NS	0.03	0.02	0.03	0.03	0.02	0.02	0.03	0.02
<b>B-BCT090</b>	26/09/1997	Borgo-Cerreto Torre	Umbria-Marche	WE	0.03	0.02	0.03	0.05	0.02	0.01	0.02	0.02
<b>TRT000</b>	11/09/1976	Tarcento	Friuli (aftershock)	NS	0.04	0.04	0.04	0.05	0.02	0.03	0.02	0.02
<b>C-NCB000</b>	03/10/1997	Nocera Umbra-Biscontini	Umbria-Marche (aftershock)	NS	0.02	0.02	0.02	0.03	0.01	0.01	0.01	0.01
<b>C-NCB090</b>	03/10/1997	Nocera Umbra-Biscontini	Umbria-Marche (aftershock)	WE	0.02	0.02	0.02	0.03	0.01	0.01	0.01	0.01
<b>R-NC2090</b>	03/04/1998	Nocera Umbra 2	Umbria-Marche (aftershock)	WE	0.03	0.03	0.03	0.03	0.02	0.01	0.02	0.02
<b>R-NC2000</b>	03/04/1998	Nocera Umbra 2	Umbria-Marche (aftershock)	NS	0.02	0.02	0.02	0.02	0.01	0.01	0.02	0.02

For example, Maugeri et al. [2009] bring about a back analysis to obtain the values of the operative harmonic amplitude “*A*” for soil type C and soil type D for the soil types reported in Tab. 2 and for a series of chosen scenario earthquakes.. Operative “*A*” values obtained by the authors for eighteen different scenario earthquakes are listed in Tab. 3.

## CONCLUSIONS

In this work, a method has been proposed to analyse the seismic response of a single pile embedded in a layered soil. The mathematical model is based on a dynamic-Winkler type approach.

The case of a single fixed head floating pile embedded in a two-layered soil profile has been analysed for various  $E_1/E_2$  ratios, being  $E_1$  and  $E_2$ , respectively the Young moduli in the upper and in the lower layer. The analysis has been carried out for different soil layer interface depth. For this aim a new dimensionless expression for the bending moments that takes into account soil-pile stiffness ratio, has been proposed.

Results show that, for the analysed cases, the layer discontinuity depth seems to play an important role on the entity of the bending moments, whose maximum values are always reached very close to the layer interface.

The analyses show also that the bending moments obtained when the ratio  $E_1/E_2$  is greater than 1, seem to be much more severe than those obtained with  $E_1/E_2$  lower than 1.

Of course, it must be underlined that the excitation frequency can sensitively condition results, depending on being near or far from the fundamental natural frequency.

Moreover, it should be remembered that a great role on bending moments induced by kinematic interaction is played by the amplitude of the harmonic excitation utilised in this kind of analyses. In fact, previous studies shown that to investigate on the effects, in terms of kinematic bending moments, induced on a pile by an assigned accelerogram, the operative harmonic amplitude that should be used could be much less than the peak acceleration recorded for that accelerogram (see Tab. 3).

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