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Probabilistic Response of Multi-support Structures on Non-uniform Soil Conditions

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Probabilistic Response of Multi-support Structures on Non-uniform Soil Conditions

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SYNOPSIS Conventional seismic design criteria take values of internal forced and other response variables as those provided by an evelope to the values of those variables produced by in-phase motion of all supports. In structures extended in plan, such as long bridges, or founded on heterogeneous formations or irregular topography, such as dams, differences in ground motion among different supports may give to differences as compared with those produced by conventional analysis. In this paper ground motion is represented as stochastic process with evolutionary intensity and frequency content. A criterion for determining design responses, based on the variance of the response of the structure is proposed. Proportionality criterion depends on cross-correlations between displacements and accelerations ocurring at supports. The proposed criterion is illustrated by applying it to a continuous bridge supported on piles embedded in a variable depth layer of soft clay.

INTRODUCTION

Response values used for the seismic design of civil structures are ordinarily obtained under the assumption that all supports move in phase. However, recent studies (Esteva et al, 1980; Ruiz and Esteva, 1980) show that those values may differ qualitatively and quantitatively from those predicted when phase differences among support motions are accounted for. In the above mentioned papers a probabilistic criterion has been developed, based on representing seismic motion by means of time-segments of Gaussian stationary processes and taking design values of responses proportional to the variances of the corresponding transient response variables at the instant the excitation ends. In the present paper attention is focused on the formulation of theoretical models to describe out-of-phase ground motion at sites characterized by diverse local conditions. Non-stationarity of motion is taken into account.

SEISMIC RESPONSE TO OUT-OF-PHASE GROUND MOTIONS

The response of a linear structural system subjected to out-of-phase support motions can be obtained as follows:

$$y(t) = \sum_s x_s(t)\, Y_s + \sum_s \int_0^t \eta_s(\zeta)\, \ddot{x}_s(t-\zeta)\, d\zeta \qquad (1)$$

In this equation,

$x_s(t)$ = displacement of support s at instant t

$$\eta_s(t) = \sum_j a_{js} Z_j h_j(t)$$

Y_s = static response produced by a unit displacement of supports

a_{js} = participation factor of mode j for the configuration produced by a unit displacement of supports

Z_j = response of interest for mode j at an arbitrary scale

$h_j(t)$ = unit impulse response fuction for mode j

From eq. 1 and the relation $x(t) = \int_0^t \ddot{x}(\zeta)(t-\zeta)H(t-\zeta)d\zeta$, where $H(\cdot)$ is the Heaviside step function, one obtains the covariance function of $y(t)$:

$$R_y(t_1,t_2) = \sum_s \sum_r \int_0^{t_1}\int_0^{t_2} R_{sr}(\zeta_1,\zeta_2)\, f_s(t_1-\zeta_1)\, f_r(t_2-\zeta_2)\, d\zeta_1\, d\zeta_2 \qquad (2)$$

In this equation, $f_s(t)=Y_s d(t)+\eta_s(t)$; $d(t)=tH(t)$ and $R_{sr}(\zeta_1,\zeta_2)$ is the cross-correlation function of $\ddot{x}_r(t)$ and $\ddot{x}_s(t)$. This function can be obtained from a stochastic process model of a train of waves arriving at the rock-soil interface, as shown in fig. 1.

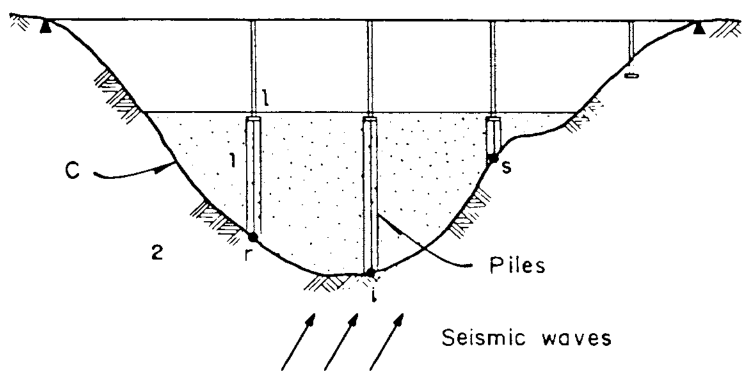

Fig 1. Structural system

RELATION BETWEEN SEISMIC WAVES AND SURFACE GROUND MOTION

Attention will be centered on the particular case when a train of vertically traveling SV waves arrive at the soil-rock interface, the slopes of which are so small that the conventional shear-beam model of wave propagation can be applied for predicting the surface ground motion at the location of each support. The stiffening effect of piles is ignored. Thus, if $\ddot{x}_\ell(t)$ and $\ddot{u}_\ell(t)$ denote respectively the accelerations a the soil surface and at the rock surface in the absence of soil, both at the vertical going through support ℓ, one obtains

$$\ddot{x}_\ell (t) = \int_0^t g_\ell (\zeta) \, \ddot{u}_\ell (t-\zeta) \, d\zeta \qquad (3)$$

where $g_\ell(\zeta)$ is the unit impulse response function which transforms $\ddot{u}_\ell$ into $\ddot{x}_\ell$ in accordance with the model depicted in fig. 2 (Tsai, 1969; Ruiz and Esteva, 1980).

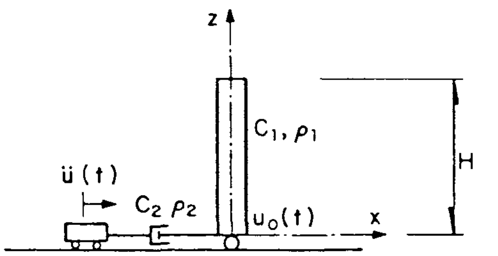

Fig 2 . One - dimensional model

According to our assumptions, $\ddot{u}_\ell$ and $\ddot{u}_K$ at two different supports differ only in their time origin: $\ddot{u}_\ell(t) = \ddot{u}(t-\zeta_\ell)$, $\ddot{u}_K(t) = \ddot{u}(t-\zeta_K)$. From this condition, one obtains the cross correlation function R_{sr}

$$R_{sr}(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} R_{\ddot{u}} (\zeta_1-\zeta_s,\zeta_2-\zeta_r) \, g_s (t_1-\zeta_1) \, g_r (t_2-\zeta_1) \, d\zeta_1 \, d\zeta_2 \qquad (4)$$

where $R_{\ddot{u}}(\zeta_1,\zeta_2)$ is the auto-correlation function of $\ddot{u}$, which is related to its evolutionary spectral density $G_{\ddot{u}}(w,t)$ as follows:

$$R_{\ddot{u}} (\zeta_1, \zeta_2) = \int_{-\infty}^{\infty} G_{\ddot{u}} (w, \zeta_1) \, e^{iw(\zeta_2-\zeta_1)} dw \qquad (5)$$

From the analysis of a number of acelerograms recorded on firm ground, Arias (1979) proposed for $G_{\ddot{u}}(w,t)$ expressions of the form given by eq. 6:

$$G_{\ddot{u}} (w,t) = K(t) \left(e^{-w^2K_1^2(t)} - e^{-w^2K_2^2(t)}\right) \qquad (6)$$

The parameters of this equation are estimated by fitting the observed values of the integrals of $\dot{u}$, $\ddot{u}^2$ and $\ddot{u}^2$ with their expected values predicted from eq. 6.

RESPONSE VARIANCES AND DESIGN VALUES

From practical considerations it appears reasonable to take design values of response variables proportional to the maximum values attained by the respective standard deviations while ground motion lasts. Thus, if β^2 is the maximum variance of ground acceleration during the earthquake and $A(p)$ is the design value of that acceleration (for a probability (p) of being exceeded), and if σ_y^2 and $y^*(p)$ are the corresponding values associated with a response variable y, the assumption proposed implies that if the design criterion adopted is based on equal exceedance probabilities for all design responses, then the ratio of the design value of y to the specified peak ground acceleration should equal σ_y/β.

The evolutionary spectral density given by eq. 6 provides a reasonable representation of an earthquake accelerogram for the purpose of estimating the response of short-and moderate-period systems; however, it does not lead to accurate estimates of the variances of quantities sensitive to ground displacements (such as the response of long-period structures or the stresses produced by phase differences among support displacements). This drawback can be overcome by adopting spectral density functions which agree with observations in the low frequency range or by introducing corrective factors to the individual terms in eq. 1. The latter approach is advocated here, as a simpler (and cruder) alternative to a previous proposal by Esteva et al (1980). The corrective factors can be obtained by calibration with respect to the ratios of peak values of u_r and $\ddot{u}_r$ and the corresponding maxima of their variance functions. This is accomplished if the design value of y is made equal to the square root of $\max_t R_y(t,t)$ obtained by means of eq. 2, with f_s and f_r determined as follows:

$$f_i(t) = \gamma_i \, d(t) \frac{D_i}{\alpha_i} + \eta_i(t) \frac{A_i}{\beta_i}, \quad i = s,r \qquad (7)$$

In this equation, D_i, A_i, α_i^2 and β_i^2 are respectively the peak values of u_i, $\ddot{u}_i$, var u_i and var $\ddot{u}_i$. The first two values are obtained from the design response spectra on rock, and the last two are given by the following equations:

$$\alpha_i^2 = \max_t \int_0^t \int_0^t R_{ii} (\zeta_1,\zeta_2) d(t-\zeta_1) d(t-\zeta_2) d\zeta_1 d\zeta_2 \qquad (8a)$$

$$\beta_i^2 = \max_t R_{\ddot{i}\ddot{i}}(t,t) \qquad (8b)$$

APPLICATION

Suppose it is of interest to obtain a design value for the relative displacement between the adjacent ends of girders A and B of the bridge schematized in fig. 3.

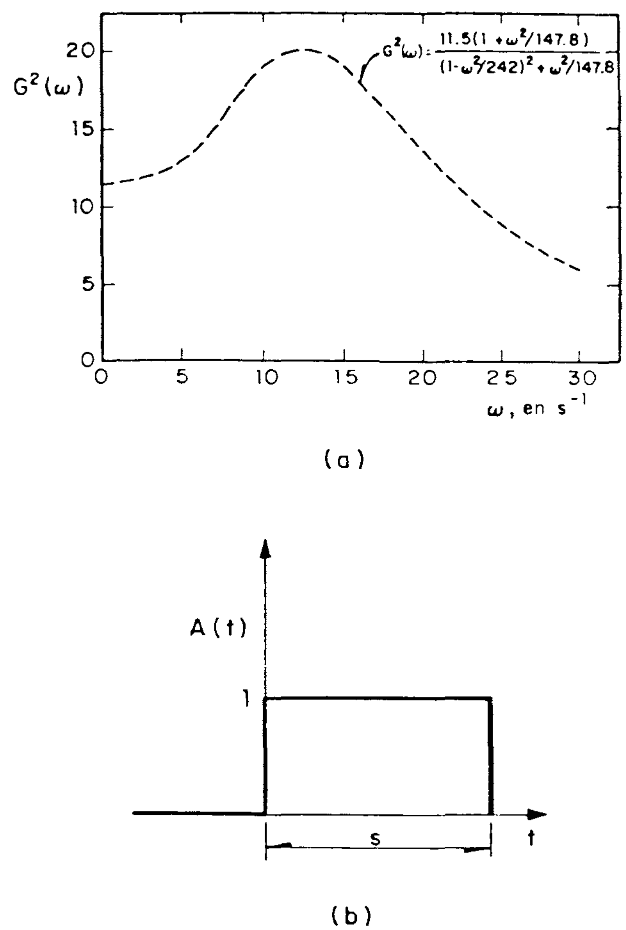

Fig 3. Characteristics of the movement

Assume the excitation to be a train of SV waves, traveling vertically along the underlying rock formation, such that the accelerogram at the rock surface in the absence of the soil above it would be $\ddot{u}(t-\zeta_s)$, where ζ_s is a time lag which depends on the vertical coordinate of the rock surface directly under support s. Suppose also that the spectral density of $\ddot{u}$ is given by $G(w, t) = A(t)G(w)$, with A and G as shown in fig. 4.

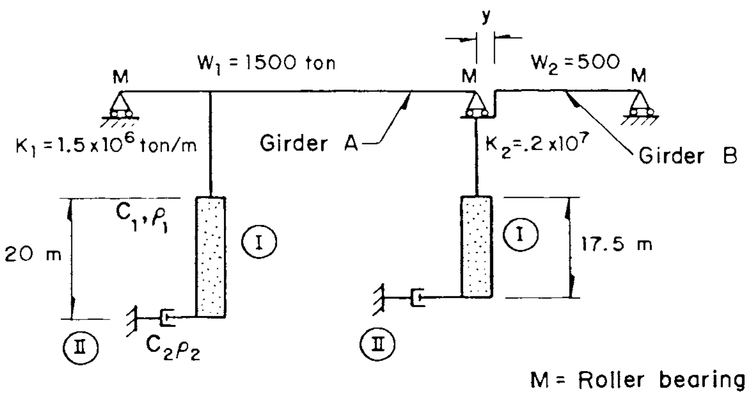

Fig 4 Structural model

The intermediate supports in fig. 3 include each a soil prism and a damper intended to represent the influence of local soil conditions, in accordance with fig. 2. Energy feedback from soil to rock is accounted for by means of the dampers tying the base of the soil prisms to the rock surface. The coefficient of viscous friction of each damper is equal to $\rho_2 c_2 A$ (Tsai, 1969), where ρ_2 is the mass density of the rock, c_2 the velocity of propagation of shear waves on it, and A the cross-section area of the soil prism. Soil-structure interaction was ignored; that is, the accelerogram at the soil surface was obtained from $\ddot{u}$, independently of the properties of the superstructure.

The soil formation is assumed homogeneous, with properties ρ_1, c_1. The structure is defined by its masses m_1 and m_2, as well as by the linear stiffnesses k_1 and k_2 of the columns. The girders are taken as infinitely stiff. Three different cases were analized, determined by the ratio $\rho_2 c_2/\rho_1 c_1$, taken as 500, 13.33 and 5.33 for cases 1 to 3, respectively. As shown in fig. 3, the thicknesses of the upper layer are 20m and 17.5m for the left and right supports, respectively.

A simplified version of the criterion of proportionality between variances and design responses was adopted, as follows:

$$\left(\frac{D_d}{D_s}\right)^2 = \frac{\sigma_d^2}{\sigma_s^2} \qquad (9)$$

In this equation, D_d is the design value of the relative displacement of interest, and D_s is the peak ground displacement of the design earthquake; σ_d^2 and σ_s^2 are respectively the variances of each of those displacements at the end of the excitation interval. The quantities included in this proportionality include terms sensitive to both structural response and ground displacement, and therefore it would have been more adequate to adopt the criterion of correcting the functions $f_i(t)$ in accordance with eq. 7. The approach adopted can be justified if most of the contribution to D_d stems from the differences between the ground displacements at both intermediate supports, (which is the case for very stiff structures), or if the structural system response is specially sensitive to low frequency waves (which is the case for very flexible structures). A systematic study of the range of validity of eq. 9 and of the relative values of the contributions of structural deformations and ground displacements to D_d is still to be done.

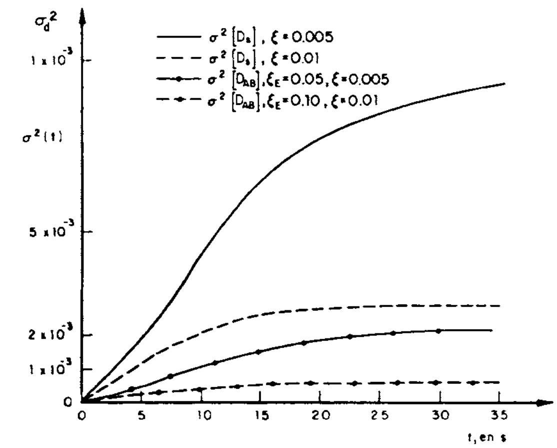

Fig 5. Response variance

Fig. 5 shows the variance $\sigma_d^2(t)$ for various combinations of the damping ratios of structure (ξ_E = 0.01, 0.05) and soil (ξ = 0.005, 0.01). In all combinations the ratio of the maximum values of σ_d and σ_s approximately equals 0.5, and it is not very sensitive to the duration of the excitation as shown by the following table

VALUES OF σ_d/σ_s

ξ_E	ξ	s = duration, sec				
		5	10	15	20	35
.05	0.005	0.005	0.49	0.49	0.49	0.48
0.1	0.01	0.48	0.44	0.46	0.46	0.47

CONCLUDING REMARKS

A probabilistic model has been proposed for estimating the seismic response of multi-support structures subjected to out-of-phase ground motion. In the lack of simultaneous records of earthquake ground motions at near-by points, the excitation is described by means of probabilistic models of the arriving seismic waves, and linear analysis criteria for the prediction of the influence of local conditions.

This paper gives an introductory formulation of the problem as well as a criterion for analysis, which is illustrated by its application to a simple case, representative of a typical practical problem. It is concluded that irregular local conditions may give place to significant discrepancies in the simultaneous ground motions at the different supports, and that those differences may seriously affect structural response, both qualitatively and quantitatively.

The criterion proposed is sufficiently simple as to permit its application to practical design problems, in spite of its obvious limitation of dealing only with linear systems.

Its accurancy must be calibrated by comparing its results with those arising from step-by-step response analysis. Some variants of the general criterion must be studied, for instance alternative ways of selecting the instantaneous or averaged values of the response variances which are best related with the values corresponding to given probabilities of being exceeded.

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