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PROBABILISTIC ANALYSIS OF WIND RESPONSE OF TALL STRUCTURES SUPPORTED BY FLEXIBLE FOUNDATIONS

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ABSTRACT: The effect of soil-structure interaction on the response of structures to dynamic loads has long been recognized and the deterministic approach is usually used for its evaluation. In most soil-structure interaction analyses, the soil shear wave velocity is used to characterize the stiffness of the soil and the foundation system. In practice, the shear modulus of the soil is difficult to evaluate and the natural spatial variability and the measurement technique affect its measured value. Probabilistic concepts are used to evaluate the significant design parameters of tall structures and to examine the sensitivity of their wind response to the variation of the soil shear wave velocity used in the analysis. In this study, the dynamic response of tall structures and the base bending moment of R/C TV towers, as an example of a tall shell structure, are evaluated accounting for soil-structure interaction. A probabilistic approach is used to account for the uncertainties in the shear modulus of the soil underneath the foundation and the design wind speed on the calculated response and base bending.

INTRODUCTION

Foundation flexibility has a significant effect on the behavior of tall structures such as R/C TV-towers. Therefore, dynamic soil-structure interaction is an essential part of the analysis and design of these structures. Because different tall shell structures such as chimneys, cooling towers and TV-towers behave differently under dynamic loading conditions, each type of structure should be considered separately and its design guidelines should be established accordingly. This issue is not adequately addressed in most of the national building codes used in practice (e.g NBCC 1995). This inadequacy is compounded by the fact that dynamic soil-structure interaction analyses rely on parameters evaluated from field measurements and/or empirical correlations that frequently involve large uncertainties.

Novak (1974, 1977) and Novak and El Hifnawy (1983) examined the response of tall reinforced concrete chimneys supported on flexible foundations to gusting wind. Galsworthy and El Naggar (2000) considered the across wind response of R/C chimneys while accounting for soil-structure interaction (*SSI*). Halabian and El Naggar (1999) investigated the seismic response of R/C TV-towers considering *SSI* and its effects on the natural frequencies and the base forces due to earthquakes. These studies highlight the significant effect of *SSI* on the dynamic response of tall structures.

Conceptually, the easiest way to analyze *SSI* for dynamic excitation is to model a significant part of the soil around the embedded structure and to apply the dynamic forces to this complex model. However, this approach (referred to as the direct approach) involves a large number of dynamic degrees of freedom that results in a large computer storage requirement and significant running time. Alternatively, if the principle of the superposition is assumed to be valid in a *SSI* analysis, it is computationally more efficient to use the substructuring approach. This approach subdivides the entire system into two parts: superstructure and substructure. The dynamic analysis for the superstructure is performed using the impedance functions of the substructure. Both approaches use soil dynamic parameters such as soil shear wave velocity. The impedance functions for a given foundation (substructure) may differ significantly depending on the value of the shear wave velocity used in the analysis. Therefore, the uncertainties in the soil shear wave velocity may have a remarkable effect on the calculated response of the superstructure.

Wind forces, one of the most significant lateral dynamic forces on tall structures, may be influenced by the *SSI* effect. To design tall structures for wind, both mean pressure and the gust part of wind fluctuations must be considered. For all slender buildings in which the wind response is predominantly governed by a single mode, the gust effect can be represented by the gust factor proposed by Davenport (1967). In this approach, the dynamic effects of wind are approximated by equivalent static loads (mean wind pressures) magnified by "the gust factor". The formulation of dynamic gust factors is built on some meteorological parameters such as mean wind speed, and some structural parameters such as natural frequencies of the structure whose evaluation involves some uncertainties (Halabian and El Naggar 2000). Natural frequencies in the principal modes of the structure depend on the structural characteristics (e.g. modulus of elasticity of the structure's material) and the foundation stiffness. Therefore, the behavior of tall structures under strong winds was investigated in the current study using a probabilistic formulation of the model and accounting for soil-structure interaction in the analysis. The variation of base bending moment due to uncertainties in the soil shear wave velocity and wind speed for towers with flexible foundations have been examined.

MODELING SOIL-STRUCTURE INTERACTION

Shallow footings (Fig. 1a) or deep foundations (Fig. 1b) can support tall structures. The flexibility of the foundation influences the dynamic characteristics of these structures such as natural frequencies (see Halabian and El Naggar 1999) and therefore *SSI* should be accounted for. The equilibrium equations for free damped vibration analysis of the structure including *SSI* using the substructuring technique, can be written in a matrix form as

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{0\} \tag{1}$$

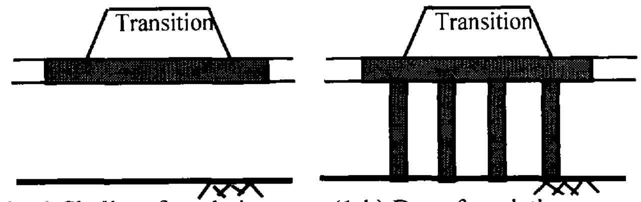
where [M], [C], [K] are mass, damping and stiffness matrices of the entire system, respectively, and {u} is the displacements' vector. The mass, stiffness and damping matrices include the corresponding matrices for the two subsystems shown in Fig. 2 (i.e. substructure and superstructure) and are defined as (Clough and Penzien (1993))

$$[M] = \begin{bmatrix} [M_{SS}] & [M_{SI}] \\ [M_{IS}] & [M_{II}] \end{bmatrix}$$

$$[C] = \begin{bmatrix} [c_{SS}] & [c_{SI}] \\ [c_{IS}] & [c_{II}]+[c_{ff}] \end{bmatrix}$$

$$[K] = \begin{bmatrix} [K_{SS}] & [K_{SI}] \\ [K_{IS}] & [K_{II}]+[K_{ff}] \end{bmatrix} \tag{2}$$

where the common nodes at the interface of the superstructure and substructure are defined with "I" and subscript "S" defines the other nodes within the superstructure medium. The subscripts "ff" represent the corresponding parameters for substructure system.



(1-a) Shallow foundation (1-b) Deep foundation

Figure 1 Type of foundation

Impedance Functions of the Foundation

The proper evaluation of the dynamic stiffness and damping (impedance function) of the substructure is important to accurately analyze the response of structures subjected to dynamic loads. The foundation impedance functions depend on the dynamic soil properties.

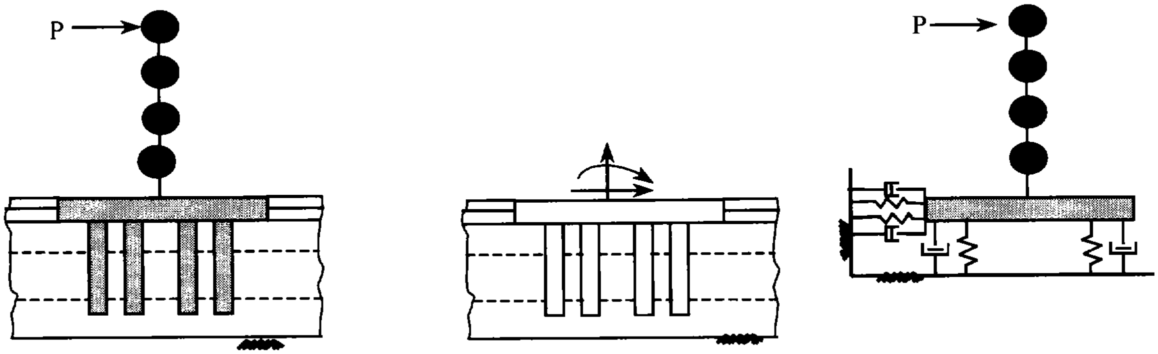
A number of approaches are available to calculate the impedance functions of both shallow and deep foundations. Most of these approaches are based on the assumption of elastic or viscoelastic soil continuum. The impedance function of a foundation system is a complex quantity that has a real part, K_1, representing the stiffness and imaginary (out of phase) component, K_2, representing the damping. The impedance function of the foundation in each vibration mode can be written as:

$$K = K_1 + iK_2 \tag{3}$$

The impedance function can also be expressed using the stiffness constant, k ($k=K_1$), and the constant of equivalent viscous damping, $c=\mathrm{Im}(K/\omega)=K_2/\omega$, where ω = frequency of loading in radians. For shallow foundations that are commonly used for tall structures such as TV-towers, the constants k and c can be evaluated using elastic half-space theory. The principal advantages of this model are that it accounts for energy dissipation through elastic waves, provides for systematic analysis and describes soil properties using basic constants such as shear modulus, material damping ratio and Poisson's ratio. The theoretical concepts and analytical approaches for surface foundation on viscoelastic half space can be found in Veletsos and Wei (1971) and Pais and Kausel (1985). The approach used in the current study assumed the soil shear modulus to be constant with depth.

Stick Model of the Superstructure

In practice, the response of tall structures is commonly analyzed using a lumped-mass (stick) model (Fig. 3). In this model, a series of beam elements and some lumped masses represent the superstructure. Thus, each element is represented by two degrees of freedom, and the displacements' vector is: $\{u\} = \{u_b, \psi_b, u_1, u_2, \ldots, u_n\}^T$, and the matrices [M], [K] and [C] are given as



a) System considered b) Substructure c) Sueperstructure

Figure 2 Substructuring approach

$$[M]=\begin{bmatrix} m_b+m_0 & 0 & 0 \\ 0 & I_b & 0 \\ 0 & 0 & [m] \end{bmatrix};$$

$$\text{in which} \quad [m]=\begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix}$$

$$[K]=\left[\begin{array}{c:c} \begin{bmatrix} k_{uu}+(k_{uu})_0 & k_{u\psi}+(k_{u\psi})_0 \\ k_{\psi u}+(k_{\psi u})_0 & k_{\psi\psi}+(k_{\psi\psi})_0 \end{bmatrix} & \begin{bmatrix} (k_{uu})_{01} & (k_{u\psi})_{01} & \vdots & \\ & & \vdots & [0] \\ (k_{\psi u})_{01} & (k_{\psi\psi})_{01} & \vdots & \end{bmatrix} \\ \hdashline \begin{bmatrix} (k_{uu})_{01} & (k_{u\psi})_{01} \\ (k_{\psi u})_{01} & (k_{\psi\psi})_{01} \\ \cdots & \cdots \\ [0] & \end{bmatrix} & [k_s] \end{array}\right]$$

$$[C]=\left[\begin{array}{c:c} \begin{bmatrix} c_{uu}+(c_{uu})_0 & c_{u\psi}+(c_{u\psi})_0 \\ c_{\psi u}+(c_{\psi u})_0 & c_{\psi\psi}+(c_{\psi\psi})_0 \end{bmatrix} & \begin{bmatrix} (c_{uu})_{01} & (c_{u\psi})_{01} & \vdots & \\ & & \vdots & [0] \\ (c_{\psi u})_{01} & (c_{\psi\psi})_{01} & \vdots & \end{bmatrix} \\ \hdashline \begin{bmatrix} (c_{uu})_{01} & (c_{u\psi})_{01} \\ (c_{\psi u})_{01} & (c_{\psi\psi})_{01} \\ \cdots & \cdots \\ [0] & \end{bmatrix} & [c_s] \end{array}\right] \quad (4)$$

where n is the number of lumped masses of the superstructure as shown in Fig. 3. The matrices $[k_s]$ and $[c_s]$ list all the stiffness and damping constants of the frame (beam) elements representing the structure. The subscripts 0 and 01 represent the stiffness and damping coefficients of the structure's node at the superstructure-foundation interface and cross terms between this node and the first node of the structure above the foundation, respectively. The effect of foundation flexibility is accounted for through stiffness, damping and mass submarices as shown in Eqs. (4). In these matrices, m_b and I_b are mass and mass moment of inertia of the foundation system, $k_{uu}, k_{u\psi}, k_{\psi u}, k_{\psi\psi}$ are stiffness coefficients of the foundation and $c_{uu}, c_{u\psi}, c_{\psi u}, c_{\psi\psi}$ are its damping coefficients. Finally, u_b and Ψ represent the horizontal deflection and rotation of the foundation.

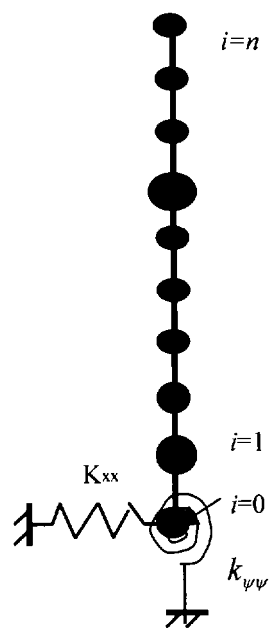


Figure 3 Stick model with foundation springs

VARIATIONS OF FIRST NATURAL FREQUENCY WITH Vs AND Ec

Natural frequencies of structures are usually influenced by the elastic modulus of their material. Galsworthy and El Naggar (1997) and Halabian and El Naggar (2000) showed that the first natural frequency of R/C tall structures is strongly influenced by the flexibility of the supporting soil. In the current study, the sensitivity of the first natural frequency of R/C tall structures to the variation of the soil shear wave velocity, V_s, and the concrete elastic modulus, E_c, is examined. For this purpose, the Milad TV-tower in Tehran is used as an example for R/C tall structures. The geometric data of the tower and the site geological data were made available to the authors. Milad TV-tower is 435m high with a thirteen-story heavy building and a 120 m tube antenna. It has a flexible shallow foundation that consists of a mat footing and a transition structure between the shaft and mat footing. For purposes of analysis, the soil is assumed to be a homogeneous visco-elastic halfspace. The shear wave velocity of the soil is assumed to be constant with depth and its value is varied from 100 m/sec to 500 m/sec to represent possible values of real soils.

The global system matrices were assembled using the approach outlined above and the first natural frequency, n_0, of the tower was obtained by solving the eigenvalue problem (Eq.1). Figure 4 shows the variation of the first natural period of the structure, $T_0=1/n_0$ with V_s for different values of the structure's modulus of elasticity, E_c. It can be noted from Fig. 5 that the effect of the foundation flexibility is to increase the first natural period. For example, the natural period of the tower with E_c = 4.0E07 KN/m^2 increased by approximately 50% as V_s varied from 100 m/sec to 300 m/sec. This effect is more pronounced for higher values of concrete modulus as can be seen in for Fig. 4.

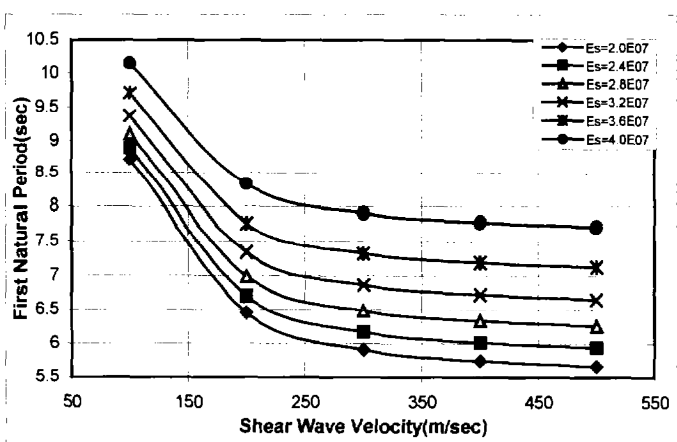


Figure 4 Variation of first natural frequency with soil shear wave velocity

Figure 4 shows general trends for the variation of the tower's natural period with V_s and E_c. However, to perform a probabilistic analysis on the effect of the variation of V_s *and* E_c on the behaviour of the tower an analytic formula is required. For complex real structures such a formula does not exist. Alternatively, one can use a formula that satisfies the physical aspects of the problem and best fits the results of the analysis. For a generalized SDOF system, the natural frequency is

$$n_0 = \frac{1}{2\pi}\sqrt{k^* / m^*} \qquad (5)$$

in which

$$k^* = \int_0^L E_c I(x)\,\psi''(x)^2 dx + \sum_i k_i \psi_i^2 ;$$

$$m^* = \int_0^L m(x)\psi(x)^2 dx + \sum_i m_i \psi_i^2 + \sum_i j_i \psi_i'^2 \qquad (6)$$

where $E_c I(x), m(x)$ are the flexural stiffness and mass of the structure per unit length, and $\psi(x)$ is the generalized displacement. The first parts of the k^* and m^* expressions in Eq. 6 represent the structure's contribution in the total stiffness and mass, respectively. The second parts represent the foundation's contributions. If one assumes that the natural period of the tower is a function of V_s and E_c, as Fig. 4 suggests, and considering Eq. 6, this function should obey the following rules:

i) if V_s tends to zero and infinity, the tower's natural period should also tend to infinity and period corresponding to the fixed base case, respectively.

ii) if E_c tends to zero and infinity, the tower's natural period should reach a value that corresponds to that of the first natural period of soil and zero, respectively.

A formula that may be fitted to some data and thus satisfy these rules may be written in the form:

$$n_0 = \left(\frac{1}{\dfrac{a}{\sqrt{bV_s^2 + cE_cV_s}} + d} \right) \sqrt{eE_c + f} \qquad (7)$$

where *a, b, c,d, e* and *f* are curve fitting constants. In this formula the constant *e* represents the contribution of the generalized displacement in the flexural stiffness that varies with the foundation stiffness as boundary conditions. The effect of soil stiffness is characterized by shear wave velocity $V_s = \sqrt{G_s / \rho}$ through a second order polynomial relationship in terms of V_s, where G_s = shear modulus and ρ = mass density of the soil. Due to existing interaction in the degrees of freedom in the interface between foundation and structure, the second term of this modification factor is in terms of the structure's modulus of elasticity. The foundation stiffness modeled as discrete springs in the second part of Eq. 6, is represented by the constant *f*. This term varies with soil shear wave velocity and is also affected by the first bracket in Eq. 7. Using 60 data points obtained from the same number of dynamic analyses, the following values were obtained for the curve fitting constants: a=3.25; b=0.003125; $c=1.25\times10^{-8}$; d=0.566; $e=2.4\times10^{-10}$; f=0.0035.

PROBABILISTIC ANALYSIS OF STRUCTURE RESPONSE TO WIND

The total along wind response of a tall structure is represented in terms of mean response and along wind response to turbulence. According to the approach proposed by Davenport (1967), the along wind response of slender structures to turbulence may be obtained by multiplying the mean response by a factor called the gust factor, G. This approach (pseudo static analysis) was used to analyze the response of the Milad TV-tower considering *SSI*. Figure 5 shows the variation of the gust factor with the soil shear wave velocity. It can be noted from Fig. 5 that the gust factor increased as V_s decreased for all E_c values that were considered in this study. This means that the effect of the foundation flexibility is to increase the dynamic component of the structure's response to wind loading.

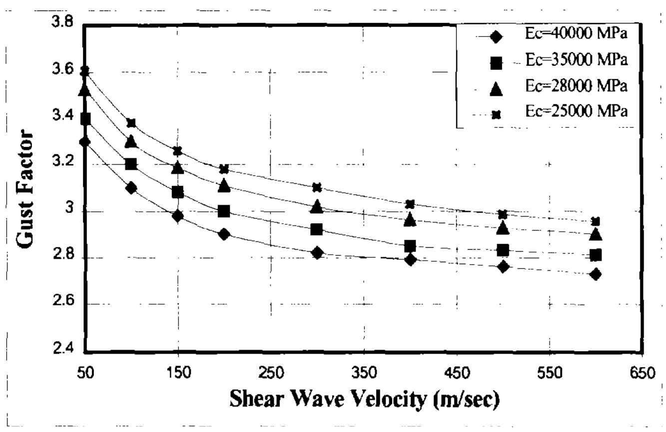


Figure 5 Variation of gust factor with soil shear wave velocity

Typical records show that the mean velocity remains approximately constant throughout the record and that the amplitude of the fluctuations about the mean values remains approximately the same at the same height of structure. Therefore, assuming that the behaviour of the structure is linear elastic, the base bending moment, as one of the most important parameters in the design of slender structures, would be a function of the wind pressure at each level, i.e.

$$M = f(\overline{p}_z(z), D(z), G) \tag{8}$$

in which $\overline{p}_z(z)$, $D(z)$ and G are the mean wind pressure, a horizontal dimension of the structure and gust factor at level z, respectively. A closer look at Eq. 8 reveals that the base bending moment of a tall structure depends on three main variables: the mean design wind velocity, $\overline{U}_{10}$; the modulus of elasticity of structure, E_c; and soil shear wave velocity, V_s, i.e. $M = M(V_s, E_c, \overline{U}_{10})$. The values of these variables are uncertain in all practical situations and the effect of their variation on base bending has to be evaluated in a probabilistic form. Since these variables are not correlated, the base bending mean value, E(M), and variance, Var(M), can be evaluated using the second moment approximation (Taylor series expansion method), i.e.

$$E(M) \cong M(m_{V_s}, m_{E_c}, m_{\overline{U}_{10}}) + [\frac{\partial^2 M}{\partial V_s^2}\Big|_{m_{V_s}, m_{E_c}, m_{\overline{U}_{10}}} \sigma_{V_s}^2 + \frac{\partial^2 M}{\partial E_c^2}\Big|_{m_{V_s}, m_{E_c}, m_{\overline{U}_{10}}} \sigma_{E_c}^2 + \frac{\partial^2 M}{\partial \overline{U}_{10}^2}\Big|_{m_{V_s}, m_{E_c}, m_{\overline{U}_{10}}} \sigma_{\overline{U}_{10}}^2] \tag{9}$$

$$Var(M) \cong [(\frac{\partial M}{\partial V_s}\Big|_{m_{V_s}, m_{E_c}, m_{\overline{U}_{10}}})^2 \sigma_{V_s}^2 + (\frac{\partial M}{\partial E_c}\Big|_{m_{V_s}, m_{E_c}, m_{\overline{U}_{10}}})^2 \sigma_{E_c}^2 + (\frac{\partial M}{\partial \overline{U}_{10}}\Big|_{m_{V_s}, m_{E_c}, m_{\overline{U}_{10}}})^2 \sigma_{\overline{U}_{10}}^2] \tag{10}$$

where $m_{V_s}, m_{E_c}, m_{\overline{U}_{10}}$ are the mean of shear wave velocity, structural modulus of elasticity and design wind speed, respectively; and $\sigma_{V_s}, \sigma_{E_c}, \sigma_{\overline{U}_{10}}$ are the standard deviation of shear wave velocity, concrete strength and design wind speed, respectively.

EXAMPLE CALCULATIONS

The probabilistic base bending moment of the Milad TV-tower was calculated using the proposed approach. Tehran environmental information inclusive of wind speed records, roughness factor, and wind pressure distribution along the height were made available to the authors to be used in this study. The key input parameters used in the probability analysis are listed in Table 1. The uncertainty associated with soil shear velocity is typical for Tehran terrain.

Table 1 Key parameters

Parameter	Mean, μ	Standard Deviation, σ	Distribution Function
Soil Shear Wave Velocity, V_s(m/s)	50 – 500	(0.1-0.5) μ	Normal
Wind Mean Speed, $\overline{U}_{10}$ (m/sec)	18.1	2.61	Gumble

The analyses were performed for different values of mean and standard deviations of V_s and the results are shown in Figs.6 and 7. It can be noted from Fig. 6 that the mean base bending decreased as V_s increased. It can also be noted from the figure that as the standard deviation of V_s increased so did the mean base bending. Phoon and Kulhawy (1999) investigated the natural variations in soil material properties and how they are measured. They concluded that coefficients of variation of soil stiffness (which is a function of V_s) are very high (up to 70%). In this range, the effect of the variability in V_s is to increase the base bending moment by up to 20%. Figure 7 shows that the standard deviation of the base bending moment decreased as V_s increased but the effect of the standard deviation of V_s on it is negligible.

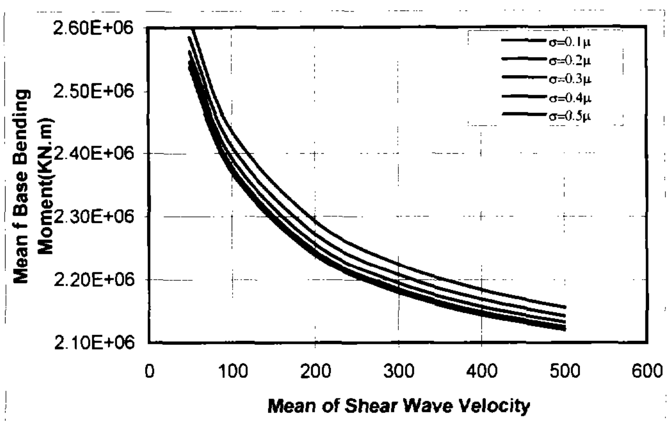


Figure 6 Variation of mean of base bending moment with mean of soil shear wave velocity

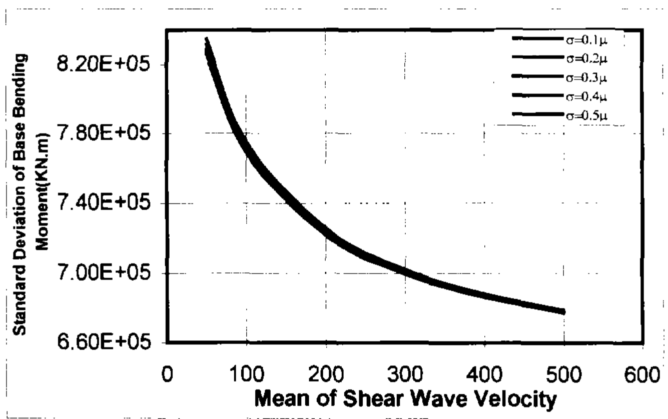


Figure 7 Variation of Standard deviation of base bending moment with mean of soil shear wave velocity

CONCLUSION

The foundation flexibility alters the dynamic characteristics of structures, and consequently influences their response to environmental (dynamic) loads. The response to wind loading and the resulting base bending moment represent important considerations in the design of tall structures. The effects of uncertainties of the value of soil shear wave velocity, represented in the form of mean value and standard deviation, on the response of R/C tall structures were evaluated. Based on the results, it was concluded that both the dynamic response of the tower (represented by the gust factor) and the base bending moment increase as the shear wave velocity decrease. For the practical range of soil shear wave velocity, both the tower response and the base bending moment may increase by up to 20% due to the foundation flexibility. Therefore, the foundation flexibility should be included in the analysis and design of tall structures subjected to severe wind loading.

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