Analysis of Liquefiable Sand Deposits Using Gravel Drains

Zhi-ying Xu

Hohai University, China

Proceedings: Second International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, March 11-15, 1991, St. Louis, Missouri, Paper No. 3.5

Analysis of Liquefiable Sand Deposits Using Gravel Drains

Zhi-ying Xu
Professor of Geotechnical Engineering, Hohai University, Nanjing, Jiangsu

Synopsis: In this paper, based on the simplified assumption about seismic pore water pressure a partial differential equation of 3-D axi-symmetrical problem which takes into account of the generation, diffusion and dissipation of seismic pore water pressure is presented. This equation is then solved by the method of separation of variables and δ-function and a mathematical formula for residual pore water pressure is obtained. The formula can be used in the calculation and design of the gravel drains installed in the sand layer prone to liquefy. The computed results by this formula are presented in a series of charts which can be conveniently used in design.

INTRODUCTION

In recent years, the particular attention has been paid to the use of the columnar gravel drain method as a measure against liquefaction of the sand deposits. As shown in Fig.1, the residual water can flow into the gravel drain,

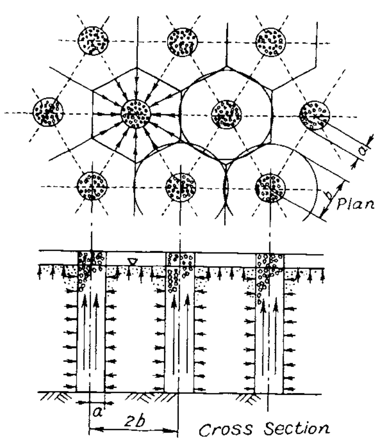

Fig.1 Arrangement of Gravel Drain System

which will obviously increase the average permeability of the ground, so that pore water pressures generated by cyclic loading may be dissipated very fast, thus considerably reducing the liquefaction potential. The gravel drains can be installed without vibrations and noises and, therefore, the technique has an advantage over the compaction methods when a ground near the existing structures has to be improved. When designing gravel drains for certain ground conditions, it is necessary to calculate the excess pore water pressure developing in the ground with the installed gravel drains during earthquake motion, and to determine the intervals of gravel drains having a certain diameter. This problem was firstly studied by Seed and Booker (1977) with FEM. Seveal studies followed them (Iai, et al., 1986; Matsubara, et al., 1988; Kawamura, et al., 1988). A mathematical formula of pore water pressure for condition of purely radial drainage has been given by auther (Xu, 1985). In this formula, however, the vertical drainage was not taken into account. Therefore, an analysis considering both the radial drainage and vertical drainage in the ground with installed gravel drains will be given in this paper.

DIFFERENTIAL EQUATION AND ITS SOLUTION

Fig.1 shows the arrangement of gravel drains installed in a sand layer. The diameter of the columnar gravel drains is 2a and their spacing centre to centre is 2b. Suppose that the layer is subjected to an earthquake consisting of N_{eq} uniform stress cycles of the same amplitude for a period of time t_d, the seismic induced pore water pressure is generated gradually in accompanying with its diffusion and dissipation. The residual water flows towards the columnar gravel drain and ground surface. If the pore water flow obeys Darcy's law and the coefficient of volume compressibility of sand is a constant, then the pore water pressure in sand layer is governed by the following equation (Seed, 1977):

$$\frac{\partial u}{\partial t}=c_v\frac{\partial^2 u}{\partial z^2}+c_h\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)+\frac{\partial u_g}{\partial N}\frac{\partial N}{\partial t} \tag{1}$$

Its initial and boundary conditions are as following:

$$u\big|_{t=0}=0 \tag{2}$$

$$\left.\begin{aligned} u\big|_{z=0}&=0, & \frac{\partial u}{\partial z}\Big|_{z=H}&=0 \\ u\big|_{r=a}&=0, & \frac{\partial u}{\partial r}\Big|_{r=b}&=0 \end{aligned}\right\} \tag{3}$$

where: u = excess pore water pressure
u_g = seismic induced pore water pressure
r = raidal distance from columnar gravel drain centre
z = vertical distance from ground surface
H = thickness of sand layer

t = time
c_v = $k_v/\gamma_w m_v$, called coefficient of consolidation in vertical direction
c_h = $k_h/\gamma_w m_v$, called coefficient of consolidation in horizontal direction
k_v = coefficient of permeability in vertical direction
k_h = coefficient of permeability in horizontal direction
γ_w = unit weight of water
m_v = volume compressibility of sand
N = cycles of alternating shear stress

In order to solve Eq.(1), it is necessary to determine $\partial u_g/\partial N$ and $\partial N/\partial t$. The value of $\partial u_g/\partial N$ can be found from undrained tests. The relationship between u_g and N can be expressed approximately for practical purpose in the following form (Prater, 1979)

$$\frac{u_g}{\sigma_0'} = \frac{N}{N_l} \tag{4}$$

where N_l = cycles required to cause initial liquefaction under the given stress condition
σ_0' = initial effective stress

Thus

$$\frac{\partial u_g}{\partial N} = \frac{\sigma_0'}{N_l} \tag{5}$$

$\partial N/\partial t$ can be determined from N_{eq} and t_d

$$\frac{\partial N}{\partial t} = \frac{N_{eq}}{t_d} \tag{6}$$

where: N_{eq} = equivalent number of uniform stress cycles
t_d = duration of time of earthquake shaking

Substituting Eq.(5) and (6) into Eq.(7), we get

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z} + c_h\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + \frac{\sigma_0'}{N_l}\frac{N_{eq}}{t_d} \tag{7}$$

Substituting $\sigma_0' = \gamma' z$ (in which γ' is submerged unit weight of sand) into Eq. (7) and putting $c_h = kc_v$, we obtain:

$$\frac{\partial u}{\partial t} - c_v\frac{\partial^2 u}{\partial z^2} - kc_v\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = \frac{\gamma' z\, N_{eq}}{N_l\, t_d} \tag{1$'$}$$

The physical meaning for the solved problem of Eqs.(1)', (2) and (3) is as follows: at start of earthquake the excess pore water pressure u is zero everywhere in ground, and then increase with time under earthquake motion, $\gamma' z\, N_{eq}/N_l\, t_d$ corresponds to force source of seismic induced pore water pressure, and varies continuously from time zero to time t, the continuous source is considered to be superposed of a series of successively instantaneous force source $N_{eq}/N_l \cdot \gamma' z/t_d \cdot d\tau\, \delta(t-\tau)$:

$$\frac{N_{eq}}{N_l}\frac{\gamma' z}{t_d} = \int_0^t \frac{N_{eq}}{N_l}\frac{\gamma' z}{t_d}\,\delta(t-\tau)\,d\tau \tag{8}$$

Then based upon the principle of superposition, the solution of equation (1)' also should be the superposition of effect of force source. Therefore

$$u(r,z,t) = \int_0^t v(r,z,t,\tau)\,d\tau \tag{9}$$

where $v(r, z, t, \tau)$ is pore water pressure induced by instantaneous force source which applies at time τ and generates instantaneous force $N_{eq}/N_l \cdot \gamma' z/t_d \cdot \delta(t-\tau)$.

The problem now reduces to the solution of following equation with following initial and boundary conditions:

$$\frac{\partial v}{\partial t} - c_v\left[\frac{\partial^2 v}{\partial z^2} + k\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right)\right] = \frac{\gamma' z\, N_{eq}}{N_l\, t_d}\,\delta(t-\tau) \tag{10}$$

$$v\Big|_{t=0} \tag{11}$$

$$v\Big|_{z=0} = 0\,, \qquad \frac{\partial v}{\partial z}\Big|_{z=H} = 0 \tag{12}$$

$$v\Big|_{r=a} = 0\,, \qquad \frac{\partial v}{\partial r}\Big|_{r=b} = 0 \tag{13}$$

The solution of Eqs.(10) - (13) is equivalent to the solution of following equations:

$$\frac{\partial v}{\partial t} - c_v\left[\frac{\partial^2 v}{\partial z^2} + k\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right)\right] = 0 \tag{14}$$

$$v\Big|_{t=\tau=0} = \frac{N_{eq}\,\gamma' z}{N_l\, t_d} \tag{15}$$

$$v\Big|_{z=0} = 0 \qquad \frac{\partial v}{\partial z}\Big|_{z=H} = 0 \tag{16}$$

$$v\Big|_{r=a} = 0 \qquad \frac{\partial v}{\partial r}\Big|_{r=b} = 0 \tag{17}$$

The Eqs. (14) - (17) can be solved by the method of separation variables. Letting $v(r, z, t) = Z(z)\cdot R(r)\cdot T(t)$ and substituting it into Eq.(14), we obtain three ordinary differential equations:

$$Z'' + \mu Z = 0 \tag{18}$$

$$r^2 R'' + rR' + \frac{\lambda - \mu}{k} r^2 R = 0 \tag{19}$$

$$T' + c_v \lambda T = 0 \tag{20}$$

where λ and μ are eigen values introduced in the separation process. The solutions satisfying boundary conditions (16) and (17) are presented as follows

$$Z_n(z) = C_n \sin \frac{(n+\frac{1}{2})\pi}{H} z \tag{21}$$

$$R_m(r) = C_m\left[J_0(\beta_m r) Y_0(\beta_m a) - J_0(\beta_m a) Y_0(\beta_m r)\right] \tag{22}$$

$$T_{nm}(t) = B_{nm} \exp(-c_v \lambda_{nm} t) \tag{23}$$

where J_0 and Y_0 is Bessel function of zero order of first and second kinds respectively; C_n, C_m, B_{nm} are arbitrary constants; $\beta_m \sqrt{(\lambda_{nm} - \mu_n)/k}$.

Substituting Eqs. (21), (22) and (23) into v= Z(z).R(r).T(t), we get following solution satisfying boundary conditions:

$$v(r,z,t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} A_{nm} \sin \frac{(n+\frac{1}{2})\pi z}{H} \cdot \\ \cdot \left[J_0(\beta_m r) Y_0(\beta_m a) - J_0(\beta_m a) Y_0(\beta_m r)\right] \exp(-c_v \lambda_{nm} t) \tag{24}$$

Substituting Eq.(15) into Eq.(16), the constant A_{nm} can be obtained, then substituting v(r,z,t) into Eq.(9) and integrating it between the limits of 0 and t, the solution of Eq.(1)′ with initial and boundary conditions is obtained finally :

$$u(r,z,t) = \frac{2\gamma' H a^2 N_{eq}}{\pi N_l t_d c_v} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{J_1^2(x_m K) \sin(n+\frac{1}{2})\pi \sin(n+\frac{1}{2})\pi \frac{z}{H}\left[J_0(x_m \frac{r}{a}) Y_0(x_m) - J_0(x_m) Y_0(x_m \frac{r}{a})\right]}{(n+\frac{1}{2})^2\left[k x_m^2 + (\frac{a}{H})^2 (n+\frac{1}{2})^2 \pi^2\right]\left[J_1^2(x_m K) - J_0^2(x_m)\right]} \cdot \\ \cdot \left\{1 - \exp\left[-c_v\left(\frac{k x_m^2}{a^2} + \frac{(n+\frac{1}{2})^2 \pi^2}{H^2}\right)t\right]\right\} \tag{25}$$

where J_1 is Bessel function of first order, first kind; K=b/a; x_m is the m-th root of the following equation:

$$J_1(Kx) Y_0(x) - J_0(x) Y_1(Kx) = 0 \tag{26}$$

in which Y_1 is Bessel function of first order, second kind.

ANALYSIS AND COMPUTATION

Eq.(25) is a theoretical formula to compute the excess pore water pressure in the ground with an installed system of gravel drains during earthquake. It can be seen that the pore water pressure for a given time and depth depends on the following dimensionless parameters: K=b/a and a/H —— ratios characterizing the geometric configuration of the gravel drains; N_{eq}/N_l —— a ratio characterizing the severity of earthquake shaking in relation to the liquefaction characteristics of sand; $kc_v t_d/a^2 = T_{ad}$, $kc_v t_d/b^2 = T_{bd}$ —— relative durations of earthquake to the consolidation properties of sand and $k = k_h/k_v$ —— a ratio characterizing the permeability anisotropy in sand. Generally speaking, the pore water pressure increases with the increase of b/a and N_{eq}/N_l , and decreases with the increase of T_{bd} T_{ad} , a/H and k.

In order to illustrate the effect of system of gravel drains, for the case of N_{eq}/N_l =2, K=5, k=4, a/H=0.05 and various values of T_{bd} , the max. pore pressure ratio $(u/\sigma_0')_{max}$ (maximum value of u/σ_0' throughout the layer at time t) is plotted versus t/t_d in Fig.2. For a sand having a relatively low permeability coefficient or for the case of large drain spacing, say, e. g,

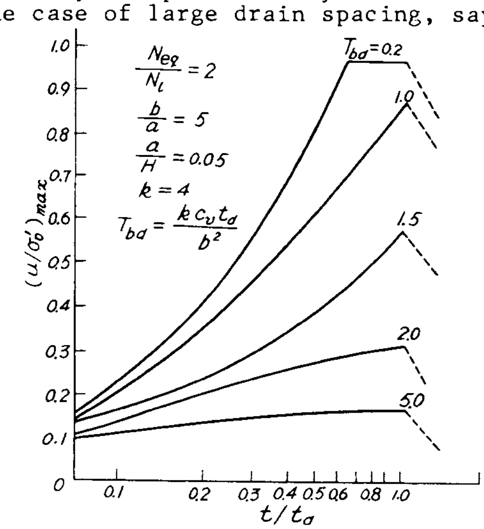

Fig.2 Effect of Drain Diameter and Spacing on Max. Pore Water Pressure Ratio

when a value corresponding to T_{bd} =0.2, is reached, the maximum pore water pressure ratio becomes so high that initial liquefaction will develop at some time between $t=(\frac{1}{2})t_d$ and $t = t_d$. The liquefied zone then continues to grow until the end of strong shaking. After this, no further excess pore water pressure is generated, the accumulated pore water pressure begin to dissipate, the maximum pore water pressure ratio drops steadily from the value one drown to zero. If the sand has a greater permeability or the drain spacing is smaller, corresponding to, say, T_{bd} =1.0, 1.5, the maximum pore water pressure ratio will be still less than one, and no liquefaction will occur in the sand. For a still higher premeability corresponding to T_{bd} =5, the pore water pressure only increase to a limit amount and stop to increase at the instant when the rate of dissipation of pore water pressure is almost equal to the rate of their generation.

In order to illustrate the variation of pore water pressure with time and space, computations have been made with Eq.(25) for the case of a= 0.32m, b=1.6m, e. g. K=5, H=8m, a/H=0.04, γ' = 10 kN/m^3, m_v =5 × 10^{-5}m^2/kN, k_v =0.8 × 10^{-5} m/s, k_h =3.2 × 10^{-5} m/s, N_{eq}/N_l =2, t_d =50s. The computed results are given in the Fig.3, Fig.4 and Fig.5. Fig.3 shows the excess pore water pressure ratio u/σ_0' in r-z plan at time t = 10s. It can be seen that the u/σ_0' near the drain is very small, thus the liquefaction-resistance of ground is considerably increased. Fig.4 represents pore water pressure at depth z = 4m and different distances from drain wall at various times. Fig.5 gives pore water pressure at r=0.8m and different depths at various times. In the design of columnar gravel drain system to pre-

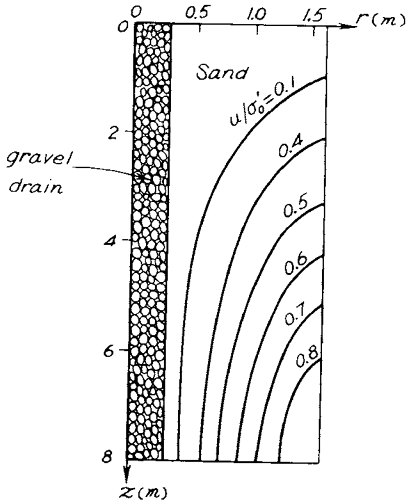

Fig.3 Variation of Pore Water Pressure Ratio with Space

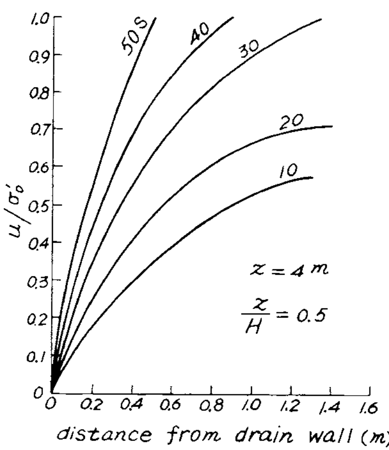

Fig.4 Variation of Pore Water Pressure Ratio with Time

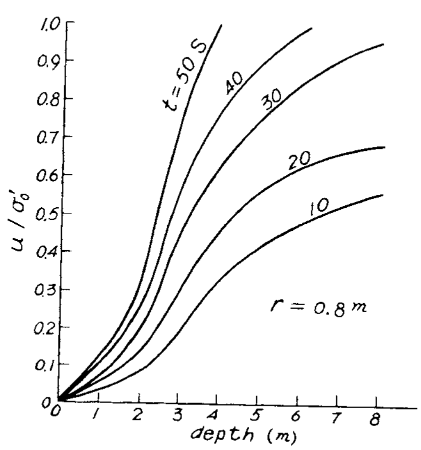

Fig.5 Variation of Pore Water Pressure Ratio with Time

vent liquefaction, it would be helpful to know what spacing of drain should be chosen for a given soil and a given diameter of drain in order to limit the maximum pore water pressure ratio within a given value. To facilitate this analysis, a series of curves for the condition of a/H=0.05 and k=4 are given in Fig.6. For the other values of a/H and k the corresponding curves have been also prepared by the author. For any particular soil and given diameter of sand drain, N_{eq}/N_l and T_{ad} are known, and thus, the value a/b corresponding to a given maximum allowable value $(u/\sigma_0')_{max}$ can be determined directly from these curves.

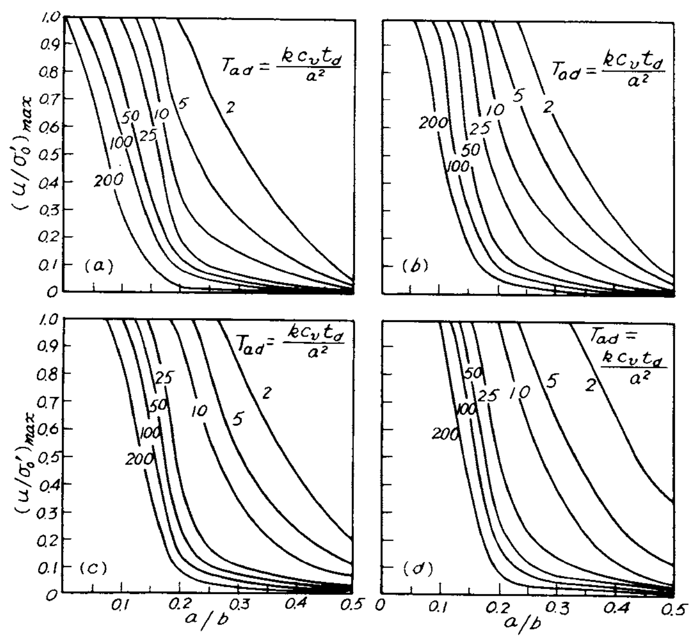

Fig.6 Relationship between Max. Pore Pressure Ratio and Drain System Parameters
for: (a) N_{eq}/N_l =1; (b) N_{eq}/N_l =2; (c) N_{eq}/N_l =3; (d) N_{eq}/N_l =4

CONCLUSION

In many cases, the columnar gravel drain system offers an effective and economical measure for the preventation of liquefaction of sand layer. An axial symmetrical mathematical solution presented in this paper provides a convenient basis for designing an effective gravel drain system in such cases. Generally, the analyses can readily be made by hand calculation even without computer, and for most practical cases, the curves as shown in Fig.6 will provide an adequate basis for design and selection of suitable drain system to effectively stabilize a potentially liquefiable sand layer.

REFERENCES

Seed, H. B. and Booker, J. R. (1977), "Stabilization of Potentially Liquefiable Sand Deposits Using Gravel Drains",Proc. ASCE, Vol.103, No. GT7, 757-768.

Iai, S.and Koizumi, K. (1986), "Estimation of Earthquake Induced Excess Pore Water Pressure for Gravel Drains", Proc. of the Seventh Japan Earthquake Engineering Symposium, 679-684.

Matsubara, K., Mihara. M. and Tsujita, M. (1988). "Analysis of Gravel Drain Against Liquefaction and Its Application to Design", Proc. of Ninth World Conference on Earthquake Engineering, Tokyo-kyoto. Vol.III, 249-254.

Xu Z. Y. (1985), "Calculation of Anti-liquefaction of Sand by Gravel Drains", Selected Works of Geotechnical Engineering, China Building Industry Press, Beijing, 274-277.

Prater, E. G. and Studer, J. (1979), "Some consideration on the Seismic Behaviour of Rockfill Dam.", Thirteenth International Congress on Large Dam, New Delhi. Vol. 2, 1097-1113.