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# SHEAR WAVE VELOCITY RELATIONS FOR SILTY AND GRAVELLY SOILS

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## ABSTRACT

Shear wave velocity  $v_s$ , dynamic shear modulus  $G_{max}$ , and damping characteristics are important parameters required for both static and dynamic response analyses of earth structures. Traditional indirect methods for estimation of these parameters based on void ratio, relative density, and mean effective stress have been successful for rather narrowly graded soils, but not for the most commonly found silty and gravelly soils. Their direct application to determine the above characteristics for silty and gravelly soils are not satisfactory. A primary reason for this is that global void ratio is not a good measure of intergrain contact density for granular mixes. A simple array of two-sized particle system with large size disparity is presented to highlight the relative roles of intercoarser and interfiner grain contacts on mechanical response parameters of such granular mixes. New parameters, namely equivalent intergranular void ratio  $(e_c)_{eq}$ , and equivalent interfine void ratio  $(e_f)_{eq}$  are introduced as indices of active intergrain contacts. They are related to shear modulus and  $v_s$  of silty and gravelly soils.

## INTRODUCTION

Shear wave velocity ( $v_s$ ), dynamic shear modulus  $G_{max}$ , and damping characteristics of soils are important parameters required for dynamic site response analysis as well as for design and performance evaluation of earth structures and foundations. Proper choice of such input soil parameters is an essential part of ground motion studies and determination of design ground accelerations. Several indirect methods have been developed, based on theoretical studies on uniform spherical arrays, laboratory studies on (clean) sands and/or field observations, to determine  $v_s$  and  $G_{max}$  (Duffy and Mindlin 1957, Hardin and Richart 1963, Hardin and Drenevich 1972, Seed et al. 1986, Goddard 1990, Lo Presti and Jamiolkowski 1998). Direct field test methods such as cross-hole and SASW techniques (Stokoe and Woods 1972, Stokoe et al. 1988) have also been developed. The indirect methods relate the above material characteristics to void ratio, relative density or some insitu parameters such as SPT blow counts and/or CPT resistance which are considered to be related to relative density. For example, widely used relationship for  $v_s$  (m/s) is of the form:

$$v_s = v_{so} \left( \frac{\sigma'}{p_a} \right)^\alpha \quad \text{where} \quad v_{so} = A p_a^\alpha (e_m - e)$$

or  $v_{so} = B e^{-x}$  (1)

The dynamic shear modulus  $G_{max}$  relationships are of the form:

$$G_{max} = C \frac{(e_m - e)^2}{1 + e} \left( \frac{\sigma'}{p_a} \right)^\beta \text{ MPa}; \quad (2)$$

$$G_{max} = 219 K_{2max} (\sigma')^{0.5} \text{ kPa} \quad [\sigma' \text{ in kPa}] \quad (3)$$

where  $A=9.1$  for rounded grains and  $6.2$  for angular grains,  $C$ = constant (about 70 for rounded grains and 32 for angular grains),  $\alpha=0.25$ ,  $\beta$  = constant (about 0.5),  $e$  = void ratio,  $e_m=2.17$  for rounded grains and 2.97 for angular grains,  $\sigma'$ =mean normal stress ( $\text{kN/m}^2$ );  $v_{so}$ = normalized shear wave velocity [ $v_s$  at  $\sigma'=p_a$ =atmospheric pressure= $100 \text{ kN/m}^2$ ];  $x=1.3$  for sands, all approximately. The  $K_{2max}$  depends on relative density and it varies from about 30 for loose sands and about 75 for dense sands. For gravel it ranges from 80 to 180. Other factors such as aging, cementation (Saxena et al. 1988), packing, anisotropy, etc. also affect  $v_s$  and  $G_{max}$ .

The underlying tenet has been that global void ratio  $e$  (or relative density) is an index of active contacts among the soil grains, and hence it can be correlated with  $v_s$  and  $G_{max}$ .

## OBSERVATIONS

While such indirect correlations have proven successful for rather clean sands their wide spread applicability to natural

soils, which often contain silty and gravely sandy soils, have proven unsatisfactory.

**Shear Wave Velocity:** Experimental data for  $v_s$  for silty sands and gravely soils (e.g. Kokusho et al. 1995, Brignoli et al. 1997, Rollins et al. 1998) indicate that shear wave velocity for gravely soils and silty soils are often significantly smaller than the respective values for 'pure' sand, silt, or gravel at the same  $e$ . An example is shown in Figs.1-2. Fig.2 shows the  $v_{s0}$  (shear wave velocity at 100 kPa confining stress) data for Tone river sand (TRS) and three gravely soils (G25, G50, G75) prepared by mixing TRS with 25, 50 and 75% gravel by weight (Kokusho et al. 1995). The non-uniqueness of void ratio  $e$  to correlate with  $v_{s0}$  is clear. Low void ratio does not necessarily mean larger shear wave velocity. The measured  $v_{s0}$  is about 200m/s for the dense Gravely Soil G75 at  $e=0.3$  whereas  $v_{s0}$  is about the same for TRS at a very loose state at  $e=0.7$ .

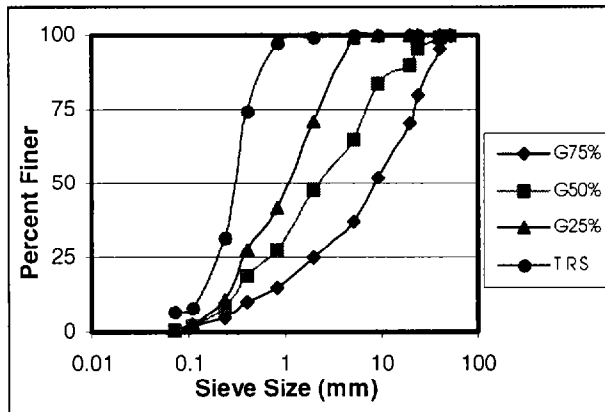


Fig.1 Grain size data – Sand-gravel mixes

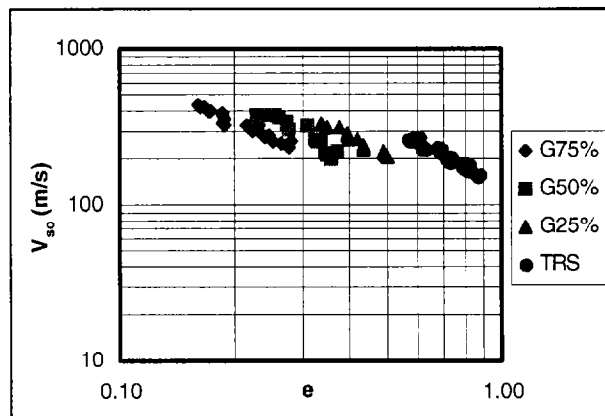


Fig.2 Influence of gravel content on  $V_{s0}$

**Shear Modulus:** Fig.3 shows a comparison of calculated  $G_{max}$  (Eq.2) and the measured  $G_{max}$  (Borden et al. 1996) for two non-plastic silty soils at 10 and 19% silt content by weight. Each soil was tested at two different confining stresses (50 and 100 kPa). The calculated values of  $G_{max}$  using Eq.2 deviate from the measured ones. The deviation increases with an increase in silt content.

Fig.4 shows four sets of secant shear modulus  $G_{0.05}$  (measured at 0.05% axial strain) data for a sand-silt mix at different silt

contents obtained from a series monotonic triaxial tests on large size specimens (typically 74 mm diameter and 150 – 160 mm height) prepared using a single host sand (OS55) (OS-F55, Foundry Sand from US Silica Company, Illinois,  $e_{max}=0.80$ ,  $e_{min}=0.60$ ,  $d_{50}=0.24mm$ ) mixed with different amounts of non-plastic crushed silica fines (Sil co sil #40, 99.9% passing sieve #200,  $d_{50}=0.007mm$ ) at (a) 0%, (b) 7%, (c) 15%, (d) 25% fines by dry weight (denoted as os00, os07, os15 and os25, respectively) (Thevanayagam et al. 1999). All specimens were consolidated to an initial effective isotropic confining stress of 100 kPa. Again, the silty sand specimens show lower  $G_{0.05}$  values when compared against clean sands at the same global void ratio  $e$ .

**Possible Reasons:** Recently, it has been brought to the attention that physical nature of silty sands and gravely sands are entirely different from clean sand, 'pure' silt or gravel (Thevanayagam 1998, 1999a-c). As the void ratio and proportion of the coarser and finer grains content of these soils change the nature of their microstructure also changes.

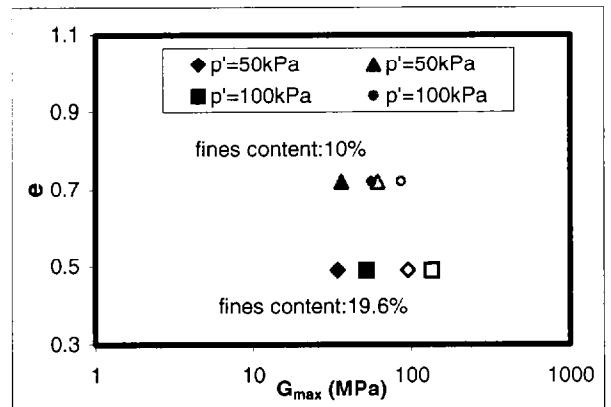


Fig.3  $G_{max}$ : measured vs. estimated (Eq.2) (symbols: solid = measured; open = calculated;  $p'$ =eff. confining stress)

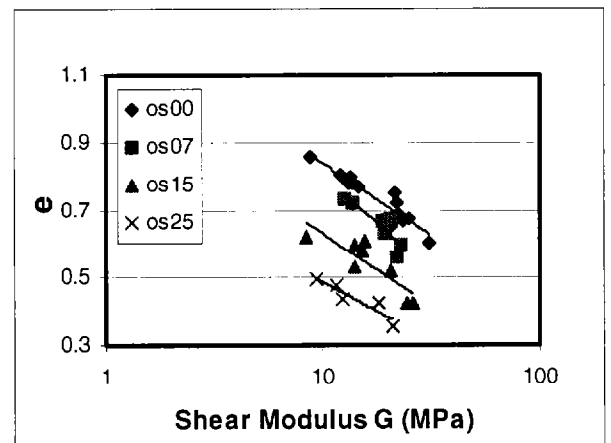


Fig.4  $e$  vs.  $G_{0.05}$  -- sand and silty sand (os00=0% fines content; os07=7% fines content)

The relative participation of the particles of very different sizes in the internal interparticle contact also changes. Due to particle size disparity and availability of pores larger than

some particles, at low finer grains content (FC) some of the finer particles may remain inactive or move between pores without significantly affecting or contributing to the force chain. Yet they contribute to the global void ratio. Alternately when there is a sufficient amount of finer grains the coarser grains become dispersed contributing much less to the force chain than to the global void ratio. Global void ratio  $e$  ceases to be an index to represent the nature of *contact density* of active particles. The traditional use of  $e$  to compare and correlate with  $G_{max}$  or  $v_{so}$  of granular mixes containing different amounts of finer grains ceases to be valid. The same holds for relative density.

In general the stress-strain behavior of granular mixes are affected by a critical combination of *intergranular and interfine contacts*. New *indices of active contacts* are needed to represent the nature of intergrain contacts in order to characterize the behavior of such soils.

Using a two-sized particle mix as a model, this paper highlights the nature of the microstructure of granular mixes. Based on this such granular mixes are classified into certain groups (Fig.5) depending on the relative frictional contributions at the intergranular and interfine grain contact level. Equivalent intergranular ( $e_c$ )<sub>eq</sub>, and equivalent interfine ( $e_f$ )<sub>eq</sub> void ratios (Fig.6) are introduced as primary indices of contact density for the various groups. Global void ratio is introduced as a secondary index. These new indices are related to  $v_s$  and  $G_{max}$  at different silt or gravel contents. The underlying theme is that contact is the mechanism by which a granular medium responds to external excitations. The density of contacts may be expressed using the above indices.

**CONCEPTUAL FRAMEWORK**

*Soil Microstructure:* Consider a two-size gap graded particle system shown in Fig.5. The microstructure of such a granular mix can be constituted by many different ways. Each one of them leads to a different internal contact arrangement and therefore to a different internal force chain structure and different stress-stain response. Among infinite variations, four extreme limiting categories of microstructure are as follows: The first category (Fig.5a) is obtained when the finer grains are fully confined within the void spaces between the coarser grains with no contribution whatsoever in supporting the coarser grain skeleton. The second category (Fig.5b) is applicable when the coarser grains are fully dispersed in the finer grain matrix. The third category (Figs.5c-d) is possible when the coarser and finer grains constitute a fully layered system where the coarser grain layers have no fines contained in them and vice versa. A fourth category (Fig.5e-f) is obtained when partial separation of coarser grains by the finer grains is present. The figures 5a, c, e and f are relevant at low finer grains content (FC). Figs. 5b and d are relevant at high FC. The case of layered soils (Fig.5c-d) is not discussed further.

It is apparent from Fig.5 that global void ratio is not a suitable common index to characterize the mechanical response of the

entire spectrum of cases shown. A simple solution may be to consider the various cases as a composite mix of coarser and finer grain skeletons with respective intergranular and interfine void ratios  $e_c$  [ $= (e+fc)/(1-fc)$ ] and  $e_f$  [ $= e/fc$ ] as defined in Fig.6 (Thevanayagam 2000).

*Index of Active Contacts:* Transition from Fig.5a to Fig.5b occurs naturally with an increase in FC beyond a threshold value ( $FC_{th}$ ). The category shown in Fig.5a is possible only if: (1) the size of the finer grains is much smaller than the possible minimum pore opening size in the coarser grain skeleton. For spherical particles this implies that  $D/d > 6.5$  where  $d$  and  $D$ =sizes of finer and coarser grains, respectively, and (2) the intergranular voids are not completely filled with the fines ( $FC < FC_{th}$ ). From a conceptual standpoint  $FC_{th}$  is expected to occur when the interfine void ratio  $e_f$  decreases below  $e_{max,HF}$  (Thevanayagam 1999c, 2000):

$$FC_{th} \leq FC_{th,max} = \frac{100e_c}{1 + e_c + e_{max,HF}} \% = \frac{100e}{e_{max,HF}} \% \quad (4)$$

where  $e_{max,HF}$  = the maximum void ratio of the pure silt beyond which it has no appreciable strength, and  $e_c$  = intergranular void ratio (Fig.6). The rationale is that, as  $e_f$  reaches below  $e_{max,HF}$ , the finer grains are packed close enough so that direct finer-grain to finer-grain friction becomes active.

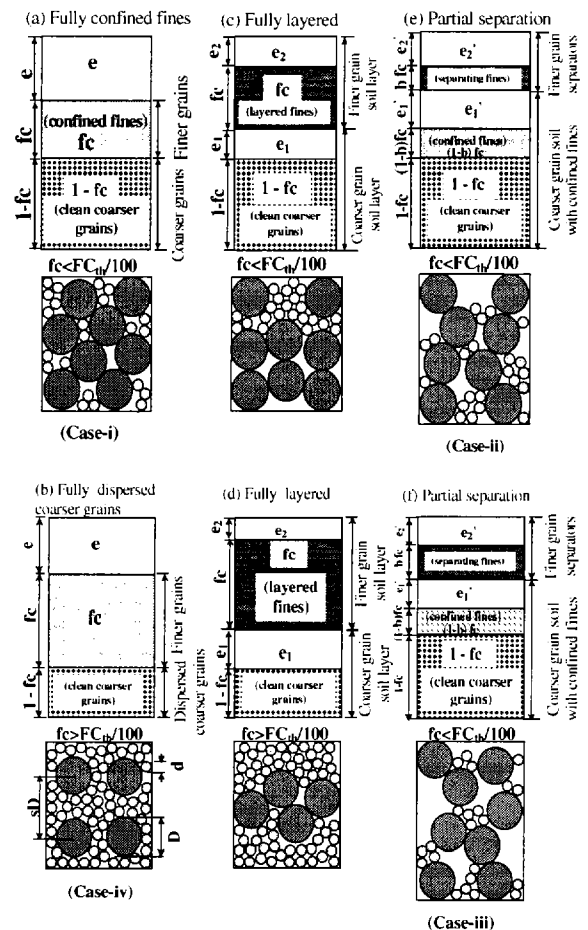


Fig.5 Microstructure and intergranular matrix phase diagram

At  $FC < FC_{th}$ , primarily the intergranular contacts between the coarser grains affect the mechanical response. Further, if considering the separating fines' contribution to active contact as "b" ( $0 < b < 1$ ) (Fig.5e-f), it is expected the mechanical behavior of such mixes would be different and generally stronger than that of the host coarser grain soil at the same  $e_c$ . Then relevant contact index would be equivalent intergranular void ratio ( $e_c$ )<sub>eq</sub>:

$$(e_c)_{eq} = \frac{e + (1-b)fc}{1 - (1-b)fc}; \quad 0 < b < 1 \quad (5)$$

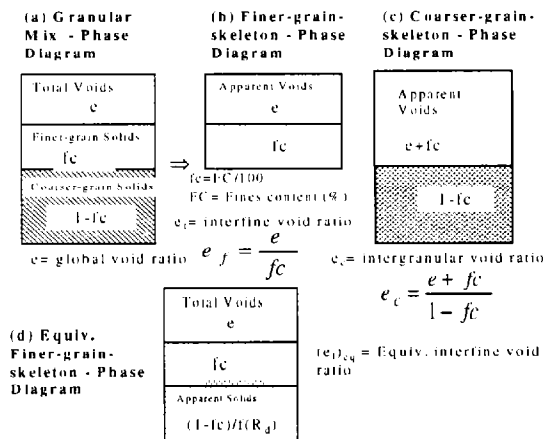
When  $FC > FC_{th}$  the finer grains begin to play a rather important role. The coarser grains begin to disperse (Fig.5b) and provide a sort of reinforcement effect until they are separated sufficiently apart when FC exceeds a limiting fines content  $FC_l$ . The  $FC_l$  is given by (Thevanayagam 1999c, 2000):

$$FC_l \geq 100 \left[ 1 - \frac{\pi(1+e)}{6s^3} \right] \% = 100 \left[ \frac{\frac{6s^3}{\pi} - 1}{\frac{6s^3}{\pi} + e_f} \right] \%; \quad e_f \leq e_{max,HF} \quad (6)$$

where  $s = 1 + a/R_d$ ,  $R_d = D/d =$  size disparity ratio, and  $a = 10$  (approximately). At  $FC_{th} < FC < FC_l$ , the reinforcement effect by the coarser grains must also be introduced to obtain an equivalent interfine void ratio ( $e_f$ )<sub>eq</sub> as the index of active contacts. For a two size particle system with large size disparity, an approximate expression for ( $e_f$ )<sub>eq</sub> can be derived (Thevanayagam 1999c, 2000):

$$(e_f)_{eq} = \frac{e}{fc + \frac{1-fc}{R_d^m}} < e_f \quad (7)$$

where  $fc = FC/100$ , and  $m =$  a coefficient. Beyond  $FC_l$  the behavior is entirely governed by the finer grains. The interfine void ratio  $e_f$  may be considered as an index of active contacts.



$$(e_f)_{eq} = \frac{e}{fc + (1-fc)/f(R_d, etc.)}$$

$$(e_c)_{eq} = \frac{e + (1-b)fc}{1 - (1-b)fc} \quad 0 < b < 1$$

Fig.6 Intergranular contact indices

## PROPOSED RELATIONS FOR $v_s$ AND $G_{max}$

*Shear wave velocity:* Based on the above considerations, with due consideration for the dependence of  $v_s$  on the nature of contacts (Duffy and Mindlin 1957, Goddard 1990, Thevanayagam 1999c),  $v_s$  of granular mixes may be given by the following, reflecting its dependence on contact index  $e_c$ :

$$v_s = v_{so} \left( \frac{\sigma'}{p_a} \right)^{\alpha}; \quad v_{so} = A p_a^{\alpha} (e_m - e_{eq})$$

$$\text{or } v_{so} = B e_{eq}^{-x} \quad (8)$$

where  $e_{eq} = (e_c)_{eq}$  for  $FC < FC_{th}$ ;  $e_{eq} = (e_f)_{eq}$  for  $FC > FC_{th}$ . Eq.8 may also be conveniently presented in terms of  $e$  by substituting the proposed expressions for  $e_{eq}$ :

$$v_{so} = A_{eq} p_a^{\alpha} ((e_m)_{eq} - e) \quad (9a)$$

$$\text{or } v_{so} = B_{eq} e^{-x} \quad \text{for } FC > FC_{th} \text{ and}$$

$$v_{so} = B(1 - (1-b)fc)^x (e + (1-b)fc)^{-x} \quad \text{for } FC < FC_{th} \quad (9b)$$

where  $A_{eq} = A/(1 - (1-b)fc)$  and  $(e_m)_{eq} = e_m(1 - (1-b)fc) - (1-b)fc$  for  $FC < FC_{th}$ ;  $A_{eq} = A/[fc + (1-fc)/(R_d)^m]$  and  $(e_m)_{eq} = e_m [fc + (1-fc)/(R_d)^m]$  for  $FC > FC_{th}$ ;  $B_{eq} = B[fc + (1-fc)/(R_d)^m]^x$  for  $FC > FC_{th}$ .

It is readily apparent that Eq.9 is similar to Eq.1 but with different coefficients depending on FC. The differences are manifestations of differences in intergrain contact density in granular mixes containing different proportions of coarser and finer grains. Direct use of Eq.1, without due consideration for the dependence of the coefficients in Eq.1 on FC can be misleading. Details are presented elsewhere (Thevanayagam 1999e).

*Shear Modulus:* Since  $G_{max}$  is a mechanical response parameter that is dependent on active contact density of the particles,  $G_{max}$  relationship for silty sands (or sandy gravels) and sandy silts (or gravelly sands) could be related to ( $e_c$ )<sub>eq</sub> and ( $e_f$ )<sub>eq</sub>, respectively:

$$G_{max} = C \frac{(e_m - (e_c)_{eq})^2}{1 + (e_c)_{eq}} \left( \frac{\sigma'}{p_a} \right)^b \text{ MPa}; \quad \text{for } FC < FC_{th} \quad (10a)$$

$$G_{max} = C \frac{(e_m - (e_f)_{eq})^2}{1 + (e_f)_{eq}} \left( \frac{\sigma'}{p_a} \right)^b \text{ MPa}; \quad \text{for } FC > FC_{th} \quad (10b)$$

## EVALUATION

*Shear Wave Velocity:* Consider the sand/gravel mixes shown before in Figs.1-2. In this case sand is the finer grain and gravel is the coarser grain. The gradation data (Fig.1) was decomposed into two gradation curves, one for gravel and the other for sand. Then the respective  $D_{50}$  and  $d_{50}$  were determined (Table 1).  $FC_{th}$  was estimated for each specimen assuming  $e_{max,HF} = 0.966$  ( $= e_{max}$  for TRS). FC (=sand content) exceeded  $FC_{th}$  for most specimens. The relevant contact index void ratio ( $e_f$ )<sub>eq</sub> was calculated assuming  $m = 0.45$ .

Table 1: Sand-gravel mix data

Soil	D <sub>50</sub> (mm)	d <sub>50</sub> (mm)	R <sub>d</sub>	Range e	FC (%)	FC <sub>th</sub> (%)	Contact Index
TRS		0.34		0.55-0.90	100		e
G25	3	0.7	4.3	0.33-0.50	75	35-52	(e <sub>f</sub> ) <sub>eq</sub>
G50	7	0.7	10	0.22-0.37	50	24-38	(e <sub>f</sub> ) <sub>eq</sub>
G75	15	0.6	25	0.16-0.28	25	17-29	(e <sub>i</sub> ) <sub>eq</sub>

FC=sand content

Fig.7 shows the same  $v_{so}$  data shown in Fig.2, but plotted against  $(e_f)_{eq}$ . The  $v_{so}$  data for all sand/gravel mixes collapse into a single narrow band along with that for TRS sand. It correlates well with  $(e_f)_{eq}$  better than with  $e$ . It shows clearly that the direct use of  $e$  to estimate  $v_{so}$  (Eq.1) for gravely soils can be very unconservative.

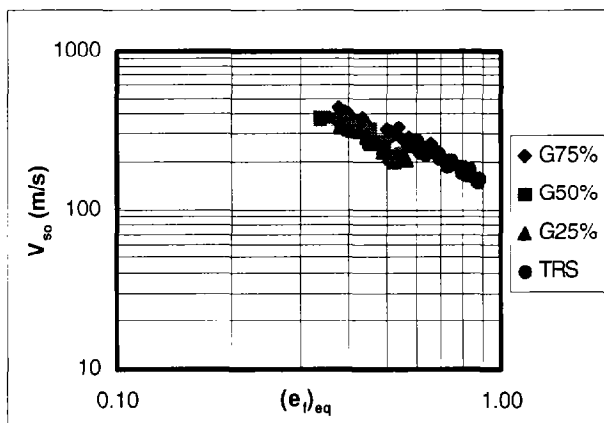


Fig.7  $v_{so}$  versus  $(e_f)_{eq}$  relations

**Shear Modulus:** Fig.8 presents a comparison of the re-calculated  $G_{max}$  values for two silty soils using Eq.10 in terms of  $(e_c)_{eq}$  against the same measured values shown in Fig.3. The  $(e_c)_{eq}$  values were calculated assuming  $b=0.25$ . The calculated

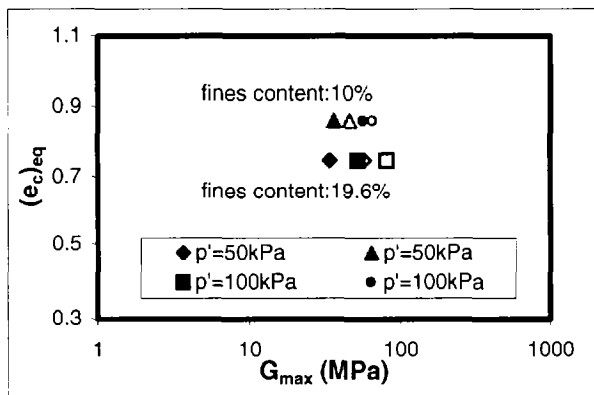


Fig.8  $G_{max}$ : measured vs. estimated (Eq.10a)  
(symbols: solid = measured; open = calculated;  
 $p'$ =eff. confining stress)

$G_{max}$  values in Fig.8 are in closer agreement with the measured values than those in Fig.3. The equivalent intergranular void ratio  $(e_c)_{eq}$  represents more closely the active contacts than

global void ratio  $e$ . Therefore  $(e_c)_{eq}$  appears to correlate with  $G_{max}$  better than  $e$  versus  $G_{max}$ .

The newly proposed active contact indices were also evaluated using the  $G_{0.05}$  data shown before in Fig.4. Fig.9 shows the same  $G_{0.05}$  data plotted versus  $(e_c)_{eq}$  ( $b=0.25$ ). The data for all soils collapse into a narrow band with that of the host clean sand in Fig. 9, as opposed to the different trend lines for each soil observed in Fig.4. Again,  $(e_c)_{eq}$  correlates better with shear modulus than does the global void ratio  $e$ .

Although not shown in this paper, the above contact indices [ $(e_c)_{eq}$  and  $(e_f)_{eq}$ ] have also been found to correlate with many other mechanical properties of granular mixes (Thevanayagam et al. 2000a, 2000b, 2001).

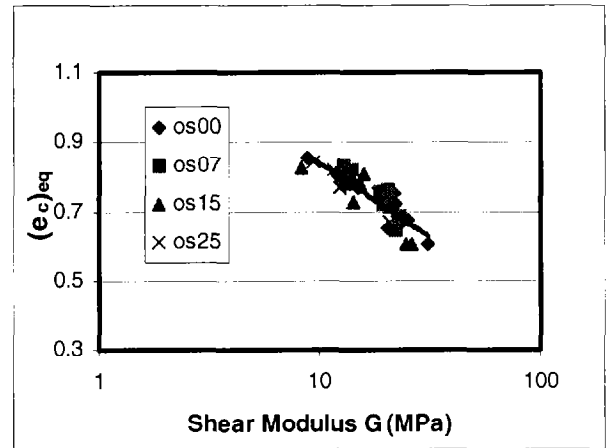


Fig.9  $(e_c)_{eq}$  vs.  $G_{0.05}$  -- sand and silty sands  
(os00=0% fines content; os07=7% fines content)

## CONCLUSION

The limitation of indirect methods for estimating shear wave velocity and  $G_{max}$  based on global void ratio  $e$  is examined. These indirect methods of estimating material parameters, even for preliminary design purposes, based on density or void ratio of a soil can yield unconservative values for  $v_s$  and  $G_{max}$ . The reason for this is that void ratio or density of a soil is indeed an index of volume or mass density. Mechanical soil parameters are primarily affected by contact density (per grain). A simple framework is presented to take into account the relative contribution of coarser and finer grains in a soil to its shear wave velocity and shear modulus. Based on the above simple framework, two new intergrain contact indices [ $(e_c)_{eq}$  and  $(e_f)_{eq}$ ] have been proposed for correlation with  $v_s$  and  $G_{max}$ . These contact indices correlate well with the measured  $v_{so}$  and  $G_{max}$  data for silty and gravely soils. For soils containing low finer grains content (FC) less than a threshold value ( $FC_{th}$ ),  $v_{so}$  and  $G_{max}$  correlate well with  $(e_c)_{eq}$ . For soils containing large finer grains content,  $v_{so}$  and  $G_{max}$  correlate well with  $(e_f)_{eq}$ . There is no unique correlation with global void ratio  $e$  in either case.

Further work is needed to examine the applicability of these contact indices for different soils.

## ACKNOWLEDGMENTS

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