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Natural Frequency of Vibrating Foundations on Layered Soil System-An Experimental Investigation

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NATURAL FREQUENCY OF VIBRATING FOUNDATIONS ON LAYERED SOIL SYSTEM-AN EXPERIMENTAL INVESTIGATION

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ABSTRACT

This paper presents model block vibration tests results on two and three layered soil system underlain by rigid layer. Using sand and sawdust in different positions (either at top or at middle or at bottom) and thickness (es) inside a tank, different layered soil bed are prepared and vibration tests are conducted on each prepared bed using Lazan Type oscillator. A large number of response curves (frequency vs. amplitude) are obtained on different layered system. Maximum amplitudes (resonant amplitude) and corresponding frequencies (resonant frequency) are tabulated to study the effect of layering on resonant frequencies. Effects of position and thickness (es) of the layer/layers on natural frequencies are found to be significant. Natural frequency increases significantly due to presence of thin stiff layer at top compared to natural frequency of bottom layer treating it to be half space whereas it decreases significantly due presence of thin soft layer at top. Several other significant observations are made and presented in the paper. Natural frequencies of the soil foundation on different layered soil systems used for the experimental investigations are predicted using static equivalent stiffness of the system and compared with experimental results. Predicted results are found to be in good agreement with the experimental results.

INTRODUCTION

The subject of vibratory response of foundations has attracted the attention of several researchers since the classical work by Lamb (1904) and Reissner (1936). Methods are now available not only for computing the response of machine foundations on the surface of the elastic half space but also for embedded foundations and foundations on piles. Amongst different methods which have been developed since then for the analysis of machine foundations, Lysmer analog [Lysmer (1965), Lysmer and Richart (1966)] is popular because of its simplicity. The essential parameters in this model for the dynamic analysis of foundations are: equivalent mass, equivalent stiffness and equivalent damping. Lysmer's method has been proved to be quite satisfactory for the analysis of machine foundations in low to medium frequency range [Richart et al (1970)].

Vibrating mass can approximately be estimated assuming total mass of foundation and machines as vibrating mass. Some amount of soil adjacent to the foundation vibrates in phase with the foundation. However, it has been proved that it gives satisfactory results even neglecting the vibrating soil mass. Whereas stiffness and damping parameters are critical and it depend on several factors, namely, shape and size of the foundation, contact stress distribution below the foundations, depth of embedment, soil properties, nonhomogeneities and nonlinearity of the soil etc. Most of the above factors have been studied by the past investigators [Gazetas (1991)]. However, nonhomogeneities in the soil systems have not been addressed adequately in the past. A few investigators namely, Warburton (1957), Novak and Beredugo (1972), Kausel (1974), Hadjian and Luco (1977), Kagawa and Kraft (1981), Gazetas (1983), Sridharan et el (1990), Baidya (1992), Baidya and Sridharan (1994) and Muralikrishna et al. (1997) have considered the nonhomogeneities (in the form of layering) of soil for the analysis of machine foundation soil system. However, there are still ample scopes for investigation to improve the understanding of dynamic response of foundations on nonhomogeneous soil system.

In this paper an attempt is made to study the effect of layering on the resonant frequency of the vibrating footing on different layered soil systems. Natural frequency of the foundations on layered soil system as used in the experimental investigation are also predicted using available methods and its applicability is verified using present experimental results.

EXPERIMENTAL PROGRAM

A tank of size 1.7mx1.7mx1.35m constructed on the concrete floor with brick wall all around is used for the investigation. A concrete footing of size 400mmx400mmx100mm is used as model footing and Lazan type oscillator is used to induce vibration to the footing. Using sand and saw dust in different positions (either at top or bottom or middle) and thickness (es) inside the tank different layered beds are prepared as shown in Table 1. Table 1(a) presents Program for two layered soil system. As per above schedule tests are conducted in two different series, i. e., a stiff layer (sand) over a soft layer (sawdust) and vice versa. Table 1(b) presents program for three layered soil system. In three layered systems also tests are conducted in two different series, i. e., a soft layer between two stiff layer and vice versa. In two layered system, thickness of 1st and 2nd layer and total thickness are variable whereas, in three layered system thickness of 1st, 2nd and 3rd layer are variable but total thickness is always constant (1200mm). Two different static weights i.e. 6.6kN and 8.0kN and three different eccentricities in the oscillator are used to simulate different vibrating mass and dynamic force respectively.

Table 1(a) Experimental program for two layered soil system

Thickness of bottom layer, h_2 (mm)	Thickness of top layer, h_1 (mm)
400	200, 400, 600, 800 and 950
600	200, 400, 600 and 750
800	200, 400 and 550
1000	200 and 350
1350	-------------

Table 1(b) Experimental program for three layered soil system

Thickness of bottom layer, h_3 (mm)	Thickness of intermediate layer, h_2 (mm)	Thickness of top layer, h_1 (mm)
400	600, 400 and 800	200, 400 and 600
600	400 and 200	200 and 400
800	200	200

MATERIALS AND METHODS

To simulate the field condition, naturally available river sand as moderately stiff material (medium fine sand, $\phi=36^0$ from direct shear test, $\gamma=17$ kN/m^3) and locally available sawdust as extreme soft material ($\gamma=2.6$ kN/m^3) are selected. Essential material properties for the analysis are elastic/shear modulus (E or G) and Poisson's ratios (μ) which have been presented elsewhere in this paper. A concrete tank of size 1.7 m x 1.7 m x 1.35 m with bottom concrete base, representing it as rigid boundary, is used to conduct the tests. To maintain the uniform condition throughout the test programme, the tank has been filled in steps of 200 mm and each layer is compacted using a compactor by constant compactive effort (to achieve density approximately 17 kN/m^3 for sand and 2.6 kN/m^3 for saw dust). After reaching required level, the surface of the stratum is levelled properly. The concrete footing is then placed in the middle of the tank. A rigid mild steel plate is tightly fixed on the concrete footing to facilitate load-fixing arrangement. Oscillator is placed over the plate and numbers of mild steel ingots are placed on the top of the oscillator to provide required static weight. Sufficient rubber packing is given to avoid noise during the test and for tight fixing. Whole set-up is rigidly tightened by nuts and bolts. Proper care is taken to maintain the c.g. of whole system and the footing to lie in the same vertical line. In this investigation, 6.6 and 8.0 kN static weights are used to simulate the two different foundation weights. Three different eccentric angles,θ are used and the eccentric angle, θ is related to eccentric moment in N-m is given by

$$W_e e = 0.9 \sin(\theta/2) \qquad (1)$$

and dynamic force in N corresponding to this θ at any frequency is given by

$$Dynamic\ force = \frac{W_e e}{g}\omega^2 = 0.9/g\ \omega^2 \qquad (2)$$

where, W_e is the eccentric weight, e is the eccentricity of the weight, g is the acceleration due to gravity and ω is the circular frequency of vibration. The oscillator is connected through a flexible shaft to a variable DC motor (3 H.P. range up to 3000 rpm). A B & K vibration pickup (Bruel & Kjaer made) is placed on top of the footing to measure the amplitude with the B&K vibration meter. Figure 1 shows the schematic diagram of experimental set-up.

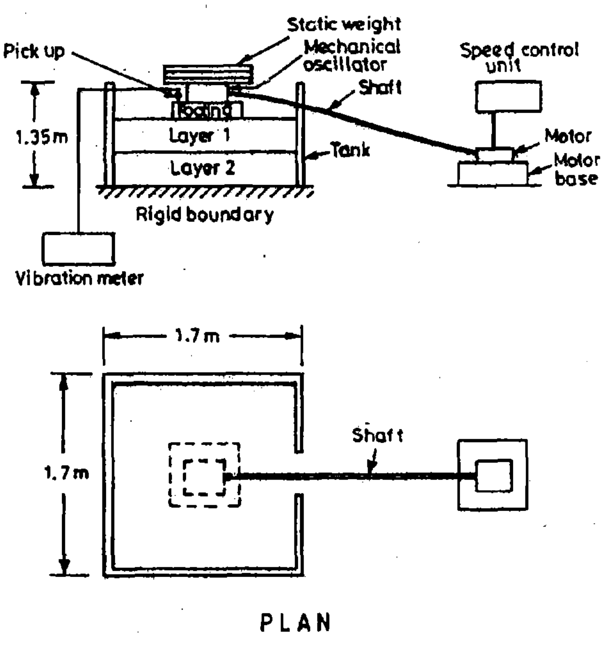

Fig.1 Schematic diagram of experimental set up.

The oscillator is then run slowly through a motor using speed control unit to avoid sudden application of high magnitude dynamic load. Thus the foundation is subjected to vibration in the vertical direction. Frequency and corresponding amplitude of vibration are recorded by photo tachometer and vibration meter respectively. To obtain a foundation response and locate the resonant peak correctly, the amplitudes are noted at a frequency interval of 25-50 rpm. Frequency corresponding to maximum displacement amplitude from the response curve is considered as resonant frequency.

RESULTS AND DISCUSSION

Frequencies and corresponding amplitudes of vibrating footings on each prepared layered bed and for each static weight and eccentricity are obtained. Finally, frequency versus amplitude curves are plotted to locate the resonant frequency and amplitude correctly. It is observed that with the increase in thickness of top sawdust layer (thickness of bottom sand layer being constant) resonant frequencies decrease and resonant amplitude increase for a particular static weight and eccentricity. Figure 2 shows the variation of response with the variation of thickness of top sawdust layer when thickness of sand layer is 400mm and W = 8 kN and $\theta = 4^0$. Reduction in resonant frequency and increase in resonant amplitude with the increase in thickness of top sawdust layer (bottom sand layer thickness being constant) are due to two reasons: (i) with the increase in thickness of soft top layer, stiffness of the whole system reduces which results in decrease in resonant frequency and increase in resonant amplitude (ii) with the increase in thickness of top layer total thickness increases i.e., depth of the position of rigid layer increases which results in further reduction in the stiffness. Similar results are obtained on two layered soil system but reversing the position of materials i.e., sawdust as bottom layer and sand as top layer. Figure 3 presents the variation of response with the variation of thickness of top sand layer (thickness of bottom sawdust layer being constant as 400mm) and for w = 8 kN and $\theta = 4^0$. It can be seen from the Figure 3 that with the increase of thickness of top sand layer resonant frequencies increase and amplitude decrease. Effect of position of rigid layer and effect of increase in thickness of top layer on the response are also present for this case but its effect on the response is in opposite direction. With the increase of depth of rigid layer stiffness reduces whereas with the increase in thickness of top sand layer stiffness increases. But the effect of increase of thickness of top layer is more compared to effect of increase in depth of rigid layer. Hence, due to combined effect, stiffness of the whole system increases resulting in increase in resonant frequency and decrease in resonant amplitude.

Results are also obtained on three layered soil system as per program given in Table 1(b). Figure 4 presents variation of response with the variation of thickness of top and intermediate layer, thickness of bottom sand layer and total thickness being constant as 400mm and 1200mm respectively for W = 6.6 kN and $\theta = 4^0$. It can be seen from the figure that with the increase in thickness of top sand layer and decrease in thickness of intermediate sawdust layer, resonant frequency increases and resonant amplitude decreases. With the increase in thickness of the top sand layer and decrease of thickness of intermediate sawdust layer (thickness of bottom layer and total thickness being constant), stiffness of the whole system increases which results in increase in resonant frequency. Figure 5 presents similar results on three layered soil system where sawdust is used as top and bottom layer and sand is used as intermediate layer. With the increase of thickness of top sand layer, reverse effects are observed compared to Figure 4 which is quite obvious. It is mentioned earlier that tests are conducted on each layered bed using two static weights and three eccentricities. However, due to want of more space all results could not be presented. Resonant frequencies obtained from the response curves on different layered system are presented in Tables 2 - 5 for both static weights and for $\theta = 8^0$ only.

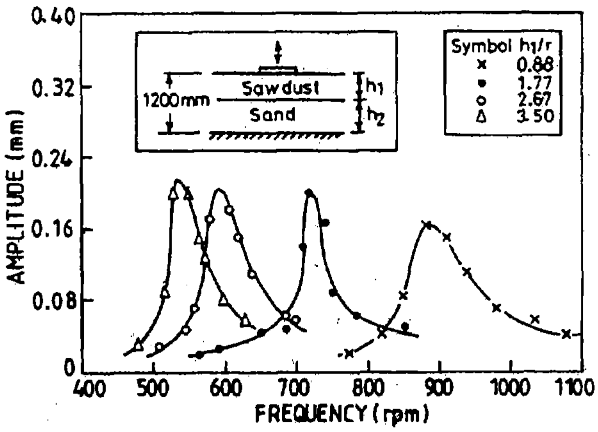

Fig.2 Variation of response on two-layered system due to variation of thickness of top sawdust layer.

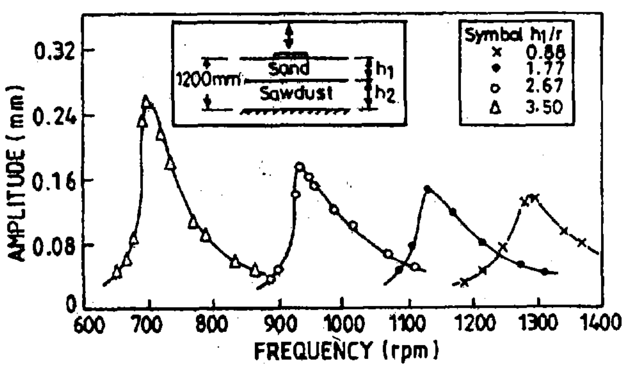

Fig.3 Variation of response on two-layered system due to variation of thickness of top sand layer.

ANALYSIS OF TEST RESULTS AND COMPARISONS

For the analysis of vibrating footing on layered soil system, a suitable model has to be chosen. Lysmer's analog found to be very effective for foundation on half space particularly in low

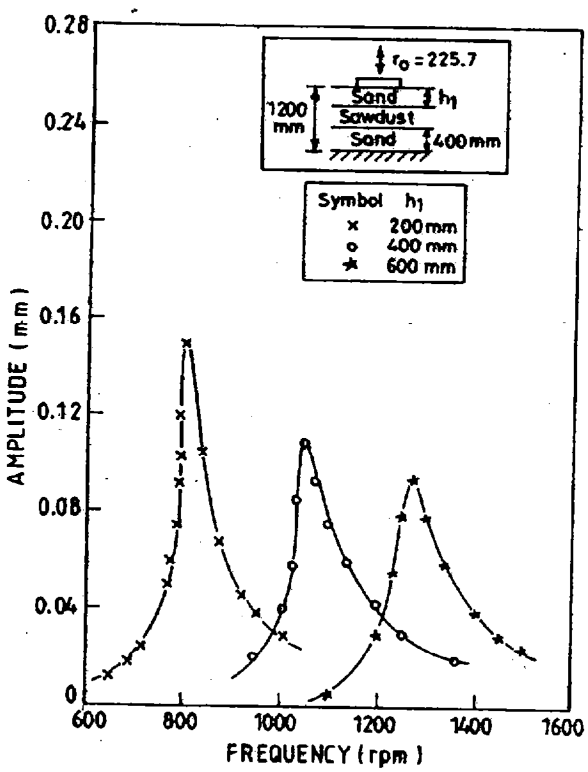

Fig.4 Variation of response on three-layered system for the variation of top layer (intermediate layer sawdust and top & bottom layer is sand).

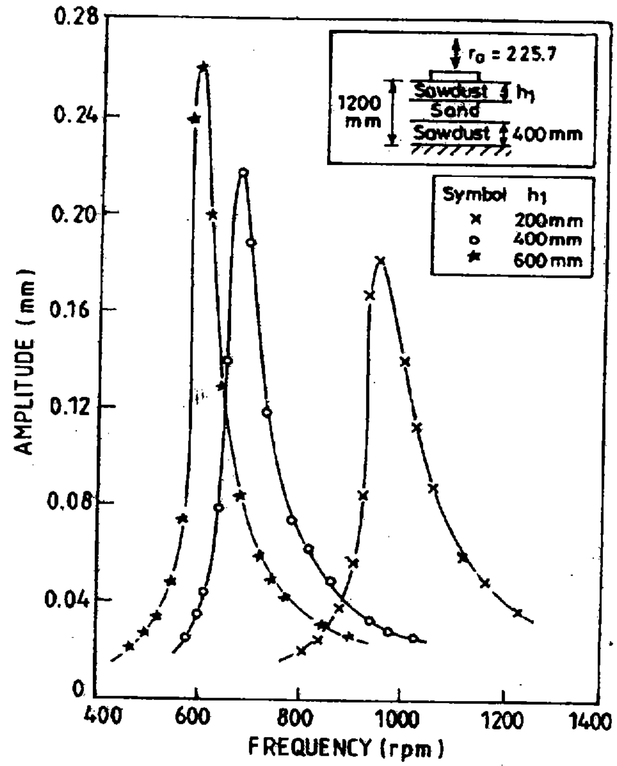

Fig.5 Variation of response on three-layered system for the variation of top layer (intermediate layer sand and top and bottom layer is sawdust.).

Table 2 Comparisons between observed and predicted natural frequency on two layered soil system (bottom layer being sand and top layer sawdust)

Bottom layer	Top layer	W = 6.6 kN		W = 8.0 kN	
		f_o♦	f_p♠	f_o	f_p
400*	200	17.0	14.0	16.2	13.1
	400	12.5	10.9	12.0	10.2
	600	10.5	9.8	10.0	9.2
	800	9.3	9.4	8.8	8.8
	950	9.2	9.1	8.7	8.5
600	200	16.2	13.9	15.8	13.0
	400	12.3	10.8	12.0	10.1
	600	10.3	9.8	9.8	9.2
	750	9.7	9.4	9.2	8.2
800	200	15.8	13.8	15.0	12.9
	400	12.5	10.8	12.0	10.1
	550	11.5	10.0	10.3	9.3
1000	200	14.8	13.7	14.3	12.9
	350	12.7	12.2	12.2	10.5
1350	-----	31.3	31.3	30.2	30.1

* Thickness of the layer in mm, ♦observed natural frequency and ♠ predicted natural frequency

Table 3 Comparisons between observed and predicted natural frequency on two layered soil system (bottom layer being sawdust and top layer sand)

Bottom layer	Top layer	W = 6.6 kN		W = 8.0 kN	
		f_o♦	f_p♠	f_o	f_p
400*	200	14.3	13.1	13.8	12.3
	400	17.0	16.4	16.8	15.6
	600	20.1	19.3	19.3	18.2
	800	21.8	21.6	20.9	20.4
	950	23.2	22.9	22.1	21.7
600	200	12.8	12.0	12.5	11.2
	400	15.8	14.9	15.6	14.0
	600	18.9	17.8	18.4	16.7
	750	20.4	19.5	19.6	18.4
800	200	12.3	11.4	11.8	10.6
	400	15.8	14.1	15.2	13.3
	550	17.4	16.2	16.8	15.2
1000	200	11.8	11.0	11.4	10.3
	350	13.8	13.0	13.3	12.2
1350	-----	8.8	8.8	8.3	8.2

* Thickness of the layer in mm, ♦observed natural frequency and ♠ predicted natural frequency

Table 4 Comparison between observed and predicted natural frequency on three layered soil system (top and bottom layer being sand and intermediate layer being sawdust)

Bottom layer	Middle layer	Top layer	W = 6.6 kN		W = 8.0 kN	
			f_o	f_p	f_o	f_p
400	600	200	13.2	11.9	12.7	11.1
400	400	400	17.2	16.1	16.5	15.1
400	200	600	20.7	22.2	20.0	21.0
600	400	200	14.3	12.9	13.8	12.0
600	200	400	19.7	18.9	19.0	17.8
800	200	200	18.7	15.4	18.3	14.4

Table 5 Comparison between observed and predicted natural frequency on three layered soil system (top and bottom layer being sawdust and intermediate layer being sand)

Bottom layer	Middle layer	Top layer	W = 6.6 kN		W = 8.0 kN	
			f_o	f_p	f_o	f_p
400	600	200	15.5	12.4	15.0	11.7
400	400	400	11.0	10.2	10.5	9.5
400	200	600	10.0	9.3	9.7	8.5
600	400	200	13.2	11.6	12.7	10.8
600	200	400	10.5	9.7	9.7	9.0
800	200	200	11.2	10.4	10.7	9.7

to medium frequency range. An attempt is made to anlyse the foundation on layered soil system using the same with a simple modification. Layered system can be lumped into an equivalent system and its equivalent stiffness, K_e can be obtained. If the layered system is idealised as equivalent mass-spring-dash pot system with equivalent parameters, governing equation of motion takes the form as,

$$m\ddot{x} + C_e\dot{x} + K_e x = m_e e\omega^2 \sin(\omega t) \quad (3)$$

Where, C_e is equivalent damping coefficient, K_e is equivalent stiffness, m is total vibrating mass, m_e is eccentric rotating mass, e is the eccentricity, x is displacement amplitude t is time variable and ω is operating frequency in rad/sec. Solutions for maximum displacement amplitude, x_m and natural frequency, ω_n can be obtained as,

$$\frac{x_m m}{m_e e} = \frac{1}{2D_e\sqrt{\left(1-D_e^{\,2}\right)}} \quad (4)$$

$$\omega_n = \omega_r\sqrt{\left(1-2D_e^{\,2}\right)} \quad (5)$$

Using measured amplitude from different layered soil system and other input parameters in Equation 4, equivalent damping, D_e is obtained and found it to consistently low (less than 10%) for almost all cases. Hence, observed resonant frequency can be approximated as natural frequency (from Fig 5 $\omega_n \cong \omega_r$ when damping is low). Observed resonant frequency presented as f_o in Tables 2-5 are also observed natural frequency as well. Natural frequency of different layered soil systems considered in the present investigation can be obtained obtaining equivalent stiffness of the system. Baidya and Sridharan (1994) given a solution for obtaining equivalent stiffness of the multi-layered system as,

Table 6 Shear modulus of sand and sawdust

Static weight (kN)	θ in degrees	Shear modulus, G (kn/m²)	
		sand	Sawdust
6.6	4	17400	1970
	8	16600	1930
	12	15900	1860
8.0	4	20100	2130
	8	18600	2040
	12	17400	1960

γ_{sand} =17 kN/m^3, $\gamma_{sawdust}$ = 2.6 kN/m^3, μ_{sand} = 0.3 and $\mu_{sawdust}$ =0.0

$$K_e = \frac{1}{\sum_{i=1}^{n} 1/K_i} \quad (6)$$

$$K_i = \frac{\pi G_i r}{\left[F\right]_{h_i/r}^{h_{i-1}/r}} \quad (7)$$

$$F = \frac{1-\mu}{2}\tan^{-1}(h/r) - \frac{1}{4}\frac{(h/r)}{1+(h/r)^2} \quad (7a)$$

It can be seen from the above equations that shear modulus and Poisson's ratio of the soils are needed to obtain equivalent stiffness of the different layered soil systems. Hence, shear modulus values of the sand and sawdust are obtained from a separate test conducted for the similar conditions and presented in Table 6. Poisson's ratio is assumed as 0.3 for sand and 0.0 for sawdust. Finally, obtaining equivalent stiffness using Equations 6 & 7 and shear modulus vales from table 6, natural frequencies of different layered soil systems are obtained as,

$$f = \frac{1}{2\pi}\sqrt{K_e/m} \quad (8)$$

Obtained natural frequencies by this way for different conditions are presented as f_p in Tables 2-5 for different layered soil systems. It can be seen from the above tables that except for a few cases the predicted and observed frequencies are in good agreement for all type of layered systems considered in the study. Maximum differences between observed and predicted frequencies are observed when a a thin soft layer at top or at depth and between two stiff layers. When a thin soft layer at top, it get strained more which results in decrease in shear modulus. But amount of decrease in shear modulus due to this effect could not be considered and hence the differences. On the other hand, when the soft layer at depth or between two stiff layers, shear modulus of the material increases due to extra confinement. But a constant shear modulus for a material is used irrespective of the positions and thickness in the analysis which caused differences. The differences between observed and predicted results are, however, within the acceptable limits. Hence, it can be concluded that natural frequency of the layered soil system can be predicted satisfactorily from the static equivalent stiffness of the system.

CONCLUSIONS

From the model block vibration test results in vertical mode from two and three layered soil systems underlain by rigid layer the following conclusions are drawn: (i) effect of layering on natural frequency is very significant and presence of even a very thin soft layer at top reduces the natural frequency significantly (ii) positions and thickness (es) of individual layer influences the natural frequency to a great extent, for example, natural frequency is higher when stiff layer at top than when soft layer at top, individual and total thickness being constant (iii) position of rigid layer influences the natural frequency significantly especially when it is at shallow depth. Presence of rigid layer at great depths (more than 3B) does not have much influence on natural frequency but may have influence on damping.

Natural frequencies of different layered soil systems considered in the study is also predicted based on static equivalent stiffness of the system and compared with the experimental results. Encouraging agreement between observed and predicted results are found. The method proposed in this paper for the prediction of natural frequency of the vibrating footing may prove useful particularly for preliminary calculations in the conceptual stage of the design process.

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